# MEDICAL INFERENCE SYSTEM ON ARTHRITIS USING FUZZY RELATIONAL THEORY

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# Table of Contents

# List of Tables

# List of Figures

# ABSTRACT

Fuzzy relational theory is a branch in the study of fuzzy sets theory. According to this theory, mapping among two sets can be carried out in four ways, which are represented by circle product, sub-triangular product, super-triangular product and square product. Each of these products can be abstracted into inference structures, which is useful in designing the inference engine of an expert system.

Based on the revised version [DeBaets and Kerre 1993, Hallam and Yew 1998a] of fuzzy relational theory, a series of 18 inference structures, namely sub-K inference structures (denoted as K1 to K18) are constructed. Based on these inference structures, a medical expert system is developed. This Arthritic disease diagnosis system is a two-level diagnosis system. The first level of diagnosis diagnose a patient according to the distribution of abnormalities in hands and wrists, whereas the second level diagnose a patient according to the signs and symptoms of a patient. It is designed in such a way so that level 1 diagnosis may short list possible diseases for level 2 and reduces the system work loads.

The system has been tested with a number of patients and found that most inference structures shows good results in both level. Among all, inference structures K2 and K16 show the best result in level 1 and level 2 diagnosis respectively.

# Chapter 1

# INTRODUCTION

## 1.0 INTRODUCTION

Aristotelian two-valued logic has met with a great challenge when logicians in the 1920s and 1930s worked out multivalued logic that dealt with true, false and value in between it. The study of vagueness on logic and sets theory started since then. Eventually, fuzzy set theory - a set theory that successfully dealt with uncertainty and vagueness was proposed by Lofti Zadeh [1965].

Compared to traditional sets theory (crisp sets), this revolutionary fuzzy sets theory is a formal mathematical theory whose objects -- fuzzy sets -- are sets with boundaries that are not precise [George 1995]. Every member belongs to a fuzzy set to some degree of membership, compared to two-state membership situations (member and non-member) in crisp sets. This special characteristic of fuzzy sets theory makes it suitable for the representation of uncertainty and vagueness, which is always present in the reasoning process of human everyday life as well as professional tasks. Therefore, fuzzy sets theory is useful in modeling human decision making process and it is applied widely in designing computer-based expert systems.

Since 1960s, several expert systems based on fuzzy sets or fuzzy logic have been developed to assist people in multiple domains, such as robotics designing and controlling [Kaoru, Yoshinori & Witold 1985], analysis and forecasting related to business activities [Tay 1994] and even in some household electrical appliances like washing machine. It is very popular because of the ability to handle information which is not precise or full of vagueness.

## 1.1 PROBLEM STATEMENT

Developing a medical expert system is always not an easy work to physicians and computer scientists. One of the toughest parts is to understand and simulate the decision making process of a medical expert. The result of this research -- the reasoning methods of the medical expert will become the core of the expert system, i.e. the inference engine.

Computer scientists have proposed different methods to design the inference engine, and all of them show different performance. Among all, Bandler and Kohout [1977] has proposed a theory called fuzzy relational theory which is believed to be able to work well as an inference engine in multiple domains, including medical diagnosis. However, this needs to be proven and validated, especially after the original theory has been revised twice.

## 1.2 OBJECTIVE

The main objective of this dissertation is to evaluate the performance of fuzzy triangular products as an inference engine of an expert system.

To achieve this objective, a medical diagnosis system based on fuzzy relational theory is needed. These fuzzy triangular products will work as the inference engine of the expert system. The objective of this dissertation is reached if this medical expert system works well.

## 1.3 THESIS ORGANIZATION

In the second chapter of this dissertation, an introduction to fuzzy sets theory is given. The traditional set theory -- crisp sets theory is also discussed here as a comparison to fuzzy sets theory. The concept of relation between sets, as well as fuzzy relational theory are also discussed in this chapter.

To make the theory applicable, an inference structure must be built. In chapter 3, sets of inference structures, namely B inference structures and K inference structures are abstracted. Some concepts such as fuzzy implication operators and checklist paradigm which are useful in abstracting inference structures are also discussed here.

Hallam and Yew [1998b] have found another weak point of the inference structures, and the cause of this weakness comes from the fuzzy implication operators which are not

pseudo-strictly monotonic. Chapter 4 discusses this finding and the solution proposed by Hallam [1999].

Chapter 5 presents the design of the proposed system. A few constraints and difficulties in designing a medical diagnosis system has been considered. To increase the efficiency of the system, a hierarchical diagnosis system has been developed.

An evaluation of the diagnosis system is performed and the result and analysis are presented in chapter 6. The ranking of each inference structure is also discussed before the conclusion in chapter 7. A few recommendations for further research are also proposed in the last chapter.

## 1.4 LITERATURE SURVEY

Fuzzy sets theory was used widely in medical field. Generally, five main types of fuzzy based applications are used in medical environments. These include :

a) **Pattern recognition** : Electronic imaging system such as X-ray, computed tomography or Magnetic Resonance Imaging (MRI) are used to image internal structures of patient body. Klein et. al [1996] have implemented a fuzzy classifier to classify focal liver lesions. 143 hepatic lesions were tested and the classifier classified them into three groups : hemangioma, other benign processes and malignant lesions. The result has been

compared with the humans observation and found that the system gave a high accuracy (90.2%) achievement.

b) **Controller** : Fuzzy sets theory has also been used to develop patient monitoring and controlling systems. Bellazzi et al. [1995] have developed a system to assist outpatients affected by Insulin Dependent Diabetes Mellitus. In this system, the status of patients will be monitored by a module of the system, and the Level Control Module, which consists of a fuzzy sets controller, will determine the next insulin dosage, depending on the actual blood glucose measurement and a certain predefined insulin delivery protocol.

c) **Diagnosis** : In this case, fuzzy sets theory is used to model medical experts diagnosis process, which always deals with uncertainty and vagueness. Several systems have been developed using this approach: these include diagnosis of cardiovascular diseases [Yew 1995], coronary artery stenosis [Sztandera et al. 1996] and more [Bellamy 1997], [Colombet, Jaulent, Diebold and Degoulet 1998], [Youngdo and Neo 1998], [Belacel, Vincke, Scheiff and Boulassel 2001]. Most of these systems show good performance.

d) **Analysis** : It is important to understand the characteristics of diseases in order to cure patients. Observation and analysis are important and hence characteristics of diseases can be discovered. Sedbrook et. al. [1993] have implemented a visual fuzzy cluster system in discovering relationship among patients with acute upper respiratory infections. As like implementation of most fuzzy systems, this system works well and encouraging result was obtained.

e) **Administration** : Management of a medical organization is important so that the effectiveness of the organization is high. However, as in other types of organizations, management process is always dealt with uncertainty and vagueness. To solve such a problem, again fuzzy sets theory or fuzzy logic can be applied. Kachukhashvili et al. [1995] have developed a system based on fuzzy sets theory to solve the problem of resource allocation among consulting rooms in the outpatient division of a hospital, and hope that the system can minimize patients' queues as well as physicians' idle time. The result gained by the system is quite acceptable.

Fuzzy sets theory is widely applied in designing medical expert systems, particularly as an inference engine in the diagnosis of consulting software. Anyway, different implementation techniques may apply to suit the need of the system. One of the techniques that relies on mathematical theory of relations is proposed by Bandler and Kohout [1977].

**Fuzzy relational theory** is one of the branches in fuzzy systems studies. The study of fuzzy relations involves the relationship between elements, which can be represented with degree of relationship. This theory works with two classes of objects that sharing another class of features, can be applied as a tool for medical diagnosis, as well as information retrieval.

Consider a patient with unknown disease showing certain number of signs and symptoms, say S1, S2, S3 and S4. Each sign/symptom is related to the patient with a relation, i.e. how "strong" is the sign/symptom found on the patient. If S1 is "pain", then we can ask "How painful is the patient?", "very painful", "slightly painful" or others? A number (degree) can be given to represent the relation between the patient and "pain" (S1); as well as other

signs and symptoms. On the other hand, each sign and symptom has a relation with each disease. This relation, which also represents a number (degree of relationship) shows the possibility of a disease developing such sign/symptom. From these two relations (relation of patient→sign/symptom and relation of sign/symptom→disease), we can obtain the composition of relations by four ways, which correspond to four relational products [Bandler and Kohout 1977]. These relational products are : circle product, sub-triangle product, super-triangle product and square product. A fuzzy relational based medical inference engine can be built using these relational products.

DeBeats and Kerre [1993] have revised the definitions of the relational products and found that the last three products (sub-triangle product, super-triangle product and square product) are uncompleted. Non-emptiness condition was not mentioned in the definition of these products and this may yield suprising result if they are applied in inference.

On the other hand, a lot of empirical works have been done in investigating the properties of fuzzy implication operators of these relational products. Most of these operators were listed and evaluated by Hallam [1999] and he found that improper choice of fuzzy implication operator will derive fuzzy logical connectives which is not pseudo-strictly monotonic, thus will cause unreliable inference result.

A new fuzzy implication operator has been suggested [Hallam 1999] and corresponding logical connectives have been generated. With the new set of logical connectives, medical diagnosis with fuzzy relational products could produce more reliable results.

## 1.5 CHAPTER SUMMARY

Fuzzy relational theory is well designed and has the potential to work as a good inference engine in a medical expert system. In the later chapters of this dissertation, the concept of this theory will be discussed, as well as revisions and contributions to this theory. However, the theory is useless if it cannot be proven or it is not working well as it is believed. A system must be developed and tested to show the performance of the theory working in the background. This is also done in the later part of this dissertation.

# Chapter 2

# FUZZY SETS AND FUZZY RELATION

## 2.0 INTRODUCTION

In this chapter, a theoretical background of fuzzy sets theory as well as fuzzy relational inference structures will be given. Some common symbols that will be applied throughout this dissertation are also introduced here.

This chapter starts with a brief explanation on crisp sets theory and then a comparison with fuzzy sets theory. Fuzzy relational product is defined in the later section of this chapter, where application of some logical connective and their problems is discussed together.

## 2.1 CRISP SETS

A set is a group of objects showing similar characteristic [Vaught 1995]. Ordinary set theory, or **crisp sets** are sets that deal with two types of membership status : member or nonmember of the set. Examples of crisp sets are set of universities, set of natural numbers and set of working days :

$A$ = { UM, USM, UPM, ... } (local universities)

$B$ = { 1, 2, 3, 4, ... } (natural numbers)

$C$ = { Monday, Tuesday, Wednesday, Thursday, Friday } (working days)

An empty set is a set without any member, denoted by $\varnothing$. A **universal set** is denoted by letter $V$, which represents all the possible elements of concern in each particular context or application. To define a set in a universe, a **characteristic function** is used to map all elements in the universe to element of set {0,1}. If $\mu_C$ is the characteristic function of set $C$, we can write :

*Definition 2.1*

$$\mu_C : c \rightarrow \{ 0,1 \}$$

To map an element d to set $C$: If $c$ is a member of set $C$ ($c \in C$), we will get $\mu_c(c) = 1$. Otherwise, if $c$ is not a member of set $C$ ($c \notin C$), we will get $\mu_C(c) = 0$.

Number of members in a set is represented as $n$. Using the example above, we can write $n(C) = 5$ and $n(B) = \infty$.

If all elements in set $A$ are also members of set $B$, we say that set $B$ is a **superset** of set $A$, or set $A$ is a **subset** of set B :

***Definition 2.2***

$$A \subseteq B$$

***Definition 2.3***

If set $A$ and set $B$ share the same members, i.e. every member in set $A$ is also a member in set $B$ and vice-versa ( $A \subseteq B$ and $B \subseteq A$ ) , we say that set $A$ and set $B$ are **equivalent**, $A = B$. Otherwise, $A \neq B$.

***Definition 2.4***

The word **proper subset** refers to the situation that A is a subset of set $B$ and $A$ is not equivalent with set B, i.e. $A \subseteq B$ and $A \neq B$ hold true. We write that :

$$A \subset B$$

***Definition 2.5***

The **power set** of a set $A$, $P(A)$ is defined as the set of all subsets of set $A$. Number of members of a power set grows with increasing number of members in the set :

$$n(\ P(A)\ ) = 2^{n(A)}$$

***Definition 2.6***

**Complement** of a set $A$, $\overline{A}$ is a set which contains all elements in the universe except members of set $A$. Whereas, **relative complement** (also called **difference**) of a set $A$ with

respect to set $B$ is a set containing all members of set $B$ that do not exist in set $A$. We denote the set as $B - A$ :

$$B - A = \{ x \mid x \in B \text{ and } x \notin A \}$$

***Definition 2.7***

A set containing all members of set $A$ and set $B$ is a **union** of set $A$ and $B$ :

$$A \cup B = \{ x \mid x \in B \text{ or } x \in A \}$$

The concept of union is also applicable to a family of sets, $A_i$ for example, where $i \in I$ :

$$\bigcup_{i \in I} A_i = \{ x \mid x \in A_i \text{ for some } i \in I \}$$

***Definition 2.8***

Another common operator for crisp sets is **intersection**. An intersection of two sets $A$ and $B$ is a set containing elements that are members of both sets :

$$A \cap B = \{ x \mid x \in B \text{ and } x \in A \}$$

This concept also can be applied to a family of sets, $A_i$ for example, where $i \in I$ :

$$\bigcap_{i \in I} A_i = \{ x \mid x \in A_i \text{ for all } i \in I \}$$

With the above operators, we can express some important properties (Table 2.1) that are held by all crisp sets.

*Table 2.1 : Properties of Crisp Sets with Union And Intersection Operators*

| | |
|---|---|
| Involution | $\bar{\bar{A}} = A$ |
| Commutative | $A \cup B = B \cup A$ |
| | $A \cap B = B \cap A$ |

| | |
|---|---|
| Associative | $(A \cup B) \cup C = A \cup (B \cup C)$ |
| | $(A \cap B) \cap C = A \cap (B \cap C)$ |
| Distributivity | $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$ |
| | $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ |
| Idempotence | $A \cap A = A$ |
| | $A \cup A = A$ |
| Absorption | $A \cup (A \cap B) = A$ |
| | $A \cap (A \cup B) = A$ |
| Law of Contradiction | $A \cap \bar{A} = \varnothing$ |
| Law of Excluded Middle | $A \cup \bar{A} = X$ ($X$ is a universal set) |
| De Morgan's Laws | $\overline{A \cap B} = \bar{A} \cup \bar{B}$ |
| | $\overline{A \cup B} = \bar{A} \cap \bar{B}$ |

***Definition 2.9***

For a set of real numbers $A$, if there exists a number $x$ such that $x \geq a$ for all $a \in A$, we say that $x$ is the **upper bound** of $A$. Furthermore, if no number less than $x$ is an upper bound of $A$, we call $x$ as **supremum** of $A$. We denote this supremum as sup($A$).

***Definition 2.10***

Similarly, we can define **lower bound** and **infimum** as follow : If $x$ is a number such that $x \leq a$ for all $a \in A$, $x$ is the lower bound of $A$. If no number greater than $x$ is a lower bound of $A$, then $x$ is the infimum of $A$ and denoted as inf($A$).

*Definition 2.11*

**Cartesian product** of two crisp sets $A$ and $B$ is denoted by $A \times B$. The result of Cartesian product among two sets is a set that consists of pairs of members of set $A$ and set $B$ in sequence. i.e. :

$$A \times B = \{ (a,b) \mid a \in A \text{ and } b \in B \}$$

As like union and intersection, Cartesian product can also be generalized to perform on a family sets $\{A_i \mid i \in I\}$ :

$$\underset{1 \le i \le n}{\mathrm{X}} A_i = \{ (a_1, a_2, a_3, ..., a_n) \mid a_i \in A_i \text{ for every } i = 1, 2, 3, ..., n \}$$

## 2.2 FUZZY SETS

### 2.2.1 BASIC CONCEPTS

Obviously, crisp sets theory that strictly divide objects into two groups (member or nonmember) is insufficient to describe the real physical world, which vagueness, uncertainty or imprecise definition present more often than not. For example, we can easily classify numbers such as 1, 2, 3 and so on as natural number, but how to strictly classify a distance as a "far" distance? Is 100 meter a far distance? How about 100 kilometer? If 100 kilometer is considered as a member in the set "far", then how about a distance of 100 light years (approximately $9.5 \times 10^{14}$ kilometer)? So, "far" is not a concept we can describe with simple definition and class it as member or nonmember. Some other real world examples where crisp

sets theory fails to describe well include fuzzy quantifier, such as "many", "very much", "few", and linguistic variables like "pain", "red colour" and "beautiful".

There are many other fuzzy concepts in our daily life, three main sources of this fuzziness are due to [Beliakov 1996] :

1) Inexact conditions of observation.

2) Classification in an under-dimensioned or over-dimensioned universe.

3) The inter-subject differences with respect to the membership functions.

To handle these fuzziness, fuzzy sets theory was developed. Every fuzzy set in universe $X$ is a collection of ordered pairs which consist of an element and membership degree of the element :

$$A = \{ ( Ax/x ) \}$$

where $Ax \rightarrow [0, 1]$ is the membership function of A for $x$

*and* $x \in A$

Membership function of set $A$ maps every element in the universe to a real number in an interval of [0,1] to show their **grade/degree of memberships**. The grade of membership shows how "exact" is an element belonging to a set. If $Ax$=1 then $x$ is a member of $A$. If $Ax$=0 then $x$ does not belong to $A$. If $Ax$ takes a value between 0 and 1, then $x$ partially belongs to fuzzy set $A$. A larger value denotes that the element is more likely a member in the set. With the definition of membership functions in fuzzy sets, fuzzy data can be handled easily.

The development of fuzzy sets theory does not mean that crisp sets theory is going to be obsolete. Crisp sets theory with characteristic function mapping all elements in the universe to set {0, 1} rather than interval [0, 1] by membership function of fuzzy sets theory

is considered as a special case in fuzzy sets theory. Or we can say, crisp sets theory is a subset of fuzzy sets theory. Furthermore, some concepts and operators of crisp sets are borrowed and applied in fuzzy sets theory after alteration.

### 2.2.2 TERMINOLOGY AND OPERATORS

Basic terminology and operators of fuzzy sets have been discussed by [DeBaets and Kerre 1994, George and Yuan 1995, Novak 1986] and summarized as below :

*Definition 2.12*

The **α-cut** of a fuzzy set $A$ produced a crisp set where the members of this crisp set are all the elements in $A$ with degree of membership equals or greater then α, where $\alpha \rightarrow [0, 1]$ :

$$^{\alpha}A = \{ x \mid Ax \geq \alpha \}$$

*Definition 2.13*

The **support** of a fuzzy set $A$ is a crisp set such that all elements in fuzzy sets $A$ with degree of membership greater then zero will become a member of the crisp set. In other words, it is equal to 0-cut :

$$\text{Supp}(A) = \{ x \mid Ax \neq 0 \}$$

*Definition 2.14*

The **kernel** of a fuzzy set $A$ is another crisp set such that all elements in $A$ with degree of membership equal to one is a member of the crisp set. In other words, it is equivalence with 1-cut (α-cut with $\alpha=1$) :

$$\text{Ker}(A) = \{ x \mid Ax = 1 \} = {}^{1}A$$

*Definition 2.15*

**Height** of a fuzzy set $A$, denoted as hgt($A$) is the value of largest membership degree obtained by members in the set, whereas the **plinth**, plt($A$) is the lowest membership degree obtained by members in the set :

$$\text{hgt}(A) = \sup_{x \in X} A(x)$$

$$\text{plt}(A) = \inf_{x \in X} A(x)$$

We say that a set is **normal** if height of the set is 1, otherwise, it is **subnormal**.

The three basic operators in crisp sets, i.e. **standard complement, standard union** and **standard intersection** are also applicable in fuzzy sets theory after generalization. The standard complement of a fuzzy set with respect to universal set $V$ is defined as :

*Definition 2.16*

$$\bar{A}x = 1 - Ax \qquad \text{for all } x \in X$$

Zadeh [1971] has proposed that standard intersection and standard union for two set $A$ and $B$ can be defined for all elements $x$ in the universe using MIN and MAX operators as follow :

*Definition 2.17*

$C = A \cap B$ iff $Cx = \min(Ax, Bx)$ standard intersection

$C = A \cup B$ iff $Cx = \max(Ax, Bx)$ standard union

And both can be generalized to work with a family of sets, $A_i$ where $i \in I$ :

$$C = \bigcap_{i \in I} A_i \quad \text{iff } Cx = \inf_{i \in I} A_i(x)$$

$$C = \bigcup_{i \in I} A_i \quad \text{iff } Cx = \sup_{i \in I} A_i(x)$$

Although standard complement, standard intersection and standard union were defined for fuzzy sets, some properties of crisp sets described in Table 3.1 does not hold in fuzzy sets. Law of contradiction and law of excluded middle as defined for crisp sets theory is violated in fuzzy sets theory, this can be proved easily :

for law of contradiction, $A \cap \bar{A} = \varnothing$

But $\overline{Ax} = 1 - Ax$

So, $Ax \cap \overline{Ax} = \min(Ax, 1\text{-}Ax) \neq 0$ unless $Ax \in \{0, 1\}$

Obviously, the law of contradiction only works on crisp sets.

For law of excluded middle, $A \cup \bar{A} = X$

Similarly, with $\overline{Ax} = 1 - Ax$

We can write $Ax \cup \bar{A}x = \max(Ax, 1\text{-}Ax) \neq 1$ unless $Ax \in \{0, 1\}$

So, it is clear that both laws are only true when working with crisp sets.

### 2.2.3 TRIANGULAR NORMS AND CONORMS

Instead of standard intersection and standard union of fuzzy sets, Alsina et. al. [1983] have introduced the theory of triangular norms and triangular conorms into the world of fuzzy sets, as an alternative to the above operators. The terms "triangular norms" and "triangular

conorms", now have accepted widely as equivalent to the class of "fuzzy intersection" and "fuzzy union" respectively [George and Yuan, 1995].

***Definition 2.18***

A function $\tau : [0, 1]^2 \rightarrow [0, 1]$ is called a triangular norms if these properties holds:

i) $T(a,1) = a$ (boundary condition)

ii) $T(a,b) \leq T(a,c)$ if $b \leq c$ (monotonicity)

iii) $T(a,b) = T(b,a)$ (commutativity)

iv) $T(a, T(b,c)) = T(T(a,b),c)$ (associativity)

where a, b, c $\in$ [0, 1]

It is clear that from the first 3 properties of the triangular norms,

$\tau(0, 1) = \tau(1, 0) = 0$ ( from boundary condition and commutativiy )

$\tau(1, 1) = 1$ ( from boundary condition )

$\tau(0, 0) = 0$ ( from monotonicity )

And this has shown that intersection of crisp sets is fully embedded into fuzzy intersection, where intersection of crisp sets is a very special case that degree of membership of both sets are 1 or 0.

Some additional properties which may hold by some triangular norms include :

i) T is a continuous function (continuity)

ii) $T(a,a) < a$ (subidempotency)

iii) $T(a,b) < T(c,d)$ if $a < c$ and $b < d$ (strict monotonicity)

***Definition 2.19***

A function $\perp : [0, 1]^2 \rightarrow [0, 1]$ is called a triangle conorms if these properties hold:

i) $\perp(a,0) = a$ (boundary condition)

ii) $\perp(a,b) \leq \perp(a,c)$ if $b \leq c$ (monotonicity)

iii) $\perp(a,b) = \perp(b,a)$ (commutativity)

iv) $\perp(a, \perp(b,c)) = \perp(\perp(a,b),c)$ (associativity)

where a, b, c $\in$ [0, 1]

It is clear that from the first 3 properties of the triangular conorms,

$\perp(0, 1) = \perp(1, 0) = 1$ ( from boundary condition and commutativiy )

$\perp(1, 1) = 1$ ( from monotonicity and boundary condition)

$\perp(0, 0) = 0$ ( from boundary condition)

As in the case of triangular norms, union of crisp sets is also fully embedded into fuzzy union, where crisp sets are special cases that the degree of membership of both sets are 1 or 0.

Some additional properties which may hold by some triangular conorms include :

i) $\perp$ is a continuous function (continuity)

ii) $\perp(a,a) > a$ (superidempotency)

iii) $\perp(a,b) < \perp(c,d)$ if $a < c$ and $b < d$ (strict monotonicity)

$$K16 = \min\left( \frac{1}{n}\sum_{j=1}^{n}(R_{ij} \rightarrow S_{jk}) , \text{OrBot}(AndBot(R_{ij}, S_{jk})) \right)$$

$$K17 = \min\left( \frac{1}{n}\sum_{j=1}^{n}(R_{ij} \rightarrow S_{jk}) , \frac{1}{n}\sum_{j=1}^{n}(AndTop(R_{ij}, S_{jk})) \right)$$

$$K18 = \min\left( \frac{1}{n}\sum_{j=1}^{n}(R_{ij} \rightarrow S_{jk}) , \frac{1}{n}\sum_{j=1}^{n}(AndBot(R_{ij}, S_{jk})) \right)$$

$$\forall \alpha = 19 \ldots 36, K\alpha = \max\left( \Phi_2(R_{ij} \rightarrow S_{jk}) , \Theta_3(\Phi_4(R_{ij}, S_{jk})) \right)$$

Where

| | | | |
|---|---|---|---|
| PlyTop, $\rightarrow$ | = | Min(1,1-$a$+$b$) | = $I_L$ |
| PlyBot, $\rightarrow$ | = | Max($b$,1-$a$) | = $I_{KD}$ |
| AndTop | = | Min($a$,$b$) | |
| AndBot | = | Max(0, $a$+$b$-1) | |
| OrTop | = | Min(1,$a$+$b$) | |
| OrBot | = | Max($a$,$b$) | |

These structures should work well as the core of inference engine of an expert system. However, defects have been found. Below, we are going to discuss some cases where sub K inference structures show their weakness and make the result of inference not reasonable and unreliable.

We will illustrate an example from medical diagnosis, where A is a set of patients, B is a set of signs and symptoms and C is a set of diseases. We have R as the relation from patients to signs and symptoms, which varies among patients. We also have S as the relation from signs and symptoms to diseases, which are stored in the knowledge base :

$$R \subseteq A \times B$$

and $$S \subseteq B \times C$$

### 4.1.1 Case $R_{ij} \leq S_{jk}$

Of course this could be a common case, but it brings a big challenge to those inference structures, especially K19 to K36. In such case, the upper bound of $R_{ij} \rightarrow S_{jk}$ is `min`(1, 1 - $R_{ij}$ + $S_{jk}$ ) = 1. Thus, for K19 to K36, which take `max` as the outer connective, the result will always be 1. Surely, this is not reasonable and we should not put K19 to K36 into consideration while designing inference engine.

Below are some examples which will result in the case :

| | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 0.8 | 0.5 | 0.6 |
| $A_2$ | 0.4 | 0.7 | 0.7 |
| $A_3$ | 0.5 | 0.1 | 0.6 |

| | $C_1$ |
|---|---|
| $B_1$ | 0.9 |
| $B_2$ | 0.8 |
| $B_3$ | 0.7 |

The bounds of the result :

| | $A_1C_1$ | $A_2C_1$ | $A_3C_1$ |
|---|---|---|---|
| Upper Bound | 1 | 1 | 1 |

On the following, we will only consider inference structures that take `min` as $\Phi_1$.

### 4.1.2 Case $R_{ij} + S_{jk} \leq 1$

This is also a common case but affect all inference structures that take AndBot as $\Phi_4$.

$$\texttt{AndBot}(R_{ij}, S_{jk}) = \max(0, R_{ij} + S_{jk} - 1) = 0$$

All inference structures that take AndBot as 4th logical connective (i.e. K2, K4, K6, K8, K10, K12, K14, K16 and K18) will have 0 as inference result, since these inference structures will take the lowest value ($\Phi_1$ = min) from both argument. So, no matter what are $\Phi_2$ and $\Theta_3$ representing, and also the actual value of $R_{ij}$ *and* $S_{jk}$ , as long as this condition is true ($R_{ij} + S_{jk} \leq 1$), both the upper and lower bound of the inference will be 0.

These are examples that have unreasonable inference result because of this weakness:

| | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 0.1 | 0.4 | 0.6 |
| $A_2$ | 0.3 | 0.5 | 0.7 |
| $A_3$ | 0.0 | 0.3 | 0.2 |

| | $C_1$ |
|---|---|
| $B_1$ | 0.7 |
| $B_2$ | 0.5 |
| $B_3$ | 0.3 |

The bounds of the result :

| | $A_1C_1$ | $A_2C_1$ | $A_3C_1$ |
|---|---|---|---|
| Upper Bound | 0 | 0 | 0 |
| Lower Bound | 0 | 0 | 0 |

### 4.1.3 Case $R_{ij} \leq S_{jk}$, $R_{ij} + R_{ip} \geq 1$ and $R_{ij} + S_{jk} \geq 1$ ( $\forall j \neq p$ )

Inference structures K 1, K7 and K13 which $\Phi_1$, $\Theta_3$ and $\Phi_4$ are `min`, `OrTop` and `AndTop` respectively will get the impact of this weakness :

$\texttt{AndTop}(R_{ij}, S_{jk}) = \min(R_{ij}, S_{jk}) = R_{ij}$, so, $\texttt{OrTop}(\texttt{AndTop}(R_{ij}, S_{jk})) = 1$

For the upper bound of these inference structures, we take $I_L$ as implication operator :

K1 $= \min(\text{AndTop}(R_{ij} \rightarrow S_{jk}), \text{OrTop}(AndTop(R_{ij}, S_{jk})))$

$= \min(\min(1), 1) = 1$

K7 $= \min(\text{AndBot}(R_{ij} \rightarrow S_{jk}), \text{OrTop}(AndTop(R_{ij}, S_{jk})))$

$= \min(1, 1) = 1$

K13 $= \min\left(\frac{1}{n}\sum_{j=1}^{n}(R_{ij} \rightarrow S_{jk}), \text{OrTop}(AndTop(R_{ij}, S_{jk}))\right)$

$= \min\left(\frac{1}{n}\sum_{j=1}^{n}(1), 1\right) = 1$

So, upper bound of all theses inference will be 1, regardless the actual value of $R_{ij}$ *and* $S_{jk}$ .

For the lower bound, use $I_{KD}$ as implication operator :

K1 $= \min(\text{AndTop}(R_{ij} \rightarrow S_{jk}), \text{OrTop}(AndTop(R_{ij}, S_{jk})))$

$= \min(\min(S_{jk}), 1) = \min(S_{jk})$

K7 $= \min(\text{AndBot}(R_{ij} \rightarrow S_{jk}), \text{OrTop}(AndTop(R_{ij}, S_{jk})))$

$= \min(\text{AndBot}(S_{jk}), 1) = \text{AndBot}(S_{jk})$

K13 $= \min\left(\frac{1}{n}\sum_{j=1}^{n}(R_{ij} \rightarrow S_{jk}), \text{OrTop}(AndTop(R_{ij}, S_{jk}))\right)$

$= \min\left(\frac{1}{n}\sum_{j=1}^{n}(S_{jk}), 1\right) = \frac{1}{n}\sum_{j=1}^{n}(S_{jk})$

Obviously, the result of the lower bound inferences will totally depend on $S_{jk}$ , which are taken from the knowledge base.

In this case, the data taken from the real world ($R_{ij}$), i.e. how strong was a sign/symptom found on a patient have no influence on the result of inferences.

These are examples that have unreasonable inference result because of this weakness:

| | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 0.7 | 0.5 | 0.8 |
| $A_2$ | 0.5 | 0.4 | 0.5 |
| $A_3$ | 0.6 | 0.6 | 0.7 |

| | $C_1$ |
|---|---|
| $B_1$ | 0.8 |
| $B_2$ | 0.6 |
| $B_3$ | 0.9 |

The bounds of the result (K1) :

| | $A_1C_1$ | $A_2C_1$ | $A_3C_1$ |
|---|---|---|---|
| Upper Bound | 1 | 1 | 1 |
| Lower Bound | 0.6 | 0.6 | 0.6 |

The bounds of the result (K7) :

| | $A_1C_1$ | $A_2C_1$ | $A_3C_1$ |
|---|---|---|---|
| Upper Bound | 1 | 1 | 1 |
| Lower Bound | 0.8 | 0.8 | 0.8 |

The bounds of the result (K13) :

| | $A_1C_1$ | $A_2C_1$ | $A_3C_1$ |
|---|---|---|---|
| Upper Bound | 1 | 1 | 1 |
| Lower Bound | 0.77 | 0.77 | 0.77 |

### 4.1.4 Case $R_{ij} \leq \min_j(S_{jk})$ and $R_{ij} + S_{jk} \geq 1$

This are other defects of K3, K9 and K15, which uses $\Theta_3$ and $\Phi_4$ as OrBot and AndTop respectively.

$$\text{AndTop}(R_{ij}, S_{jk}) = \min(R_{ij}, S_{jk}) = R_{ij}$$

$$\text{OrBot}(\text{AndTop}(R_{ij}, S_{jk})) = \max_j(R_{ij})$$

For upper bounds,

$$\text{K3} = \min(\text{AndTop}(R_{ij} \rightarrow S_{jk}), \text{OrBot}(AndTop(R_{ij}, S_{jk})))$$

$$= \min(\min(1), \max_j(R_{ij})) = \max_j(R_{ij})$$

$$\text{K9} = \min(\text{AndBot}(R_{ij} \rightarrow S_{jk}), \text{OrBot}(AndTop(R_{ij}, S_{jk})))$$

$$= \min(\text{AndBot}(1), \max_j(R_{ij})) = \max_j(R_{ij})$$

$$\text{K15} = \min\left(\frac{1}{n}\sum_{j=1}^{n}(R_{ij} \rightarrow S_{jk}), \text{OrBot}(AndTop(R_{ij}, S_{jk}))\right)$$

$$= \min\left(\frac{1}{n}\sum_{j=1}^{n}(1), \max_j(R_{ij})\right) = \max_j(R_{ij})$$

Whereas for lower bounds :

$$\text{K3} = \min(\text{AndTop}(R_{ij} \rightarrow S_{jk}), \text{OrBot}(AndTop(R_{ij}, S_{jk})))$$

$$= \min(\min_j(S_{jk}), \max_j(R_{ij})) = \max_j(R_{ij})$$

$$\text{K9} = \min(\text{AndBot}(R_{ij} \rightarrow S_{jk}), \text{OrBot}(AndTop(R_{ij}, S_{jk})))$$

$$= \min\left(\max_j\left(0, \sum_{i=1}^{n}(S_{jk}) - (n-1)\right), \max_j(R_{ij})\right) = \max_j(R_{ij})$$

$$\text{K15} \quad = \min\left( \frac{1}{n} \sum_{j=1}^{n} (R_{ij} \rightarrow S_{jk}) \, , \, \text{OrBot}(AndTop(R_{ij}, S_{jk})) \right)$$

$$= \min\left( \frac{1}{n} \sum_{j=1}^{n} (S_{jk}) \, , \, \max_{j} (R_{ij}) \right) = \max_{j} (R_{ij})$$

In this case, the result of both upper and lower bound will be the biggest value taken from real world, i.e. $\max_{j} (R_{ij})$. Data from the knowledge base will have no influence on the result of inferences. Furthermore, only the biggest value of the experimental data will affect the final result, this is not reasonable because the result of inference will be based on a single data.

These are examples that have unreasonable inference result because of this weakness:

| | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 0.3 | 0.5 | 0.4 |
| $A_2$ | 0.7 | 0.2 | 0.1 |
| $A_3$ | 0.7 | 0.7 | 0.7 |

| | $C_1$ |
|---|---|
| $B_1$ | 0.7 |
| $B_2$ | 0.8 |
| $B_3$ | 0.9 |

The bounds of the result (K3):

| | $A_1C_1$ | $A_2C_1$ | $A_3C_1$ |
|---|---|---|---|
| Upper Bound | 0.5 | 0.7 | 0.7 |
| Lower Bound | 0.5 | 0.7 | 0.7 |

The bounds of the result (K9):

| | $A_1C_1$ | $A_2C_1$ | $A_3C_1$ |
|---|---|---|---|
| Upper Bound | 0.5 | 0.7 | 0.7 |
| Lower Bound | 0.5 | 0.7 | 0.7 |

The bounds of the result (K15):

| | $A_1C_1$ | $A_2C_1$ | $A_3C_1$ |
|---|---|---|---|
| Upper Bound | 0.5 | 0.7 | 0.7 |
| Lower Bound | 0.5 | 0.7 | 0.7 |

### 4.1.5 Case $S_{jk} \leq R_{ij}$

This case affects K5 and K17 in different ways, but both of them share the same cause, i.e. AndTop( $R_{ij}$ , $S_{jk}$ ) = min($R_{ij}$ , $S_{jk}$) = $S_{jk}$.. We will consider both one by one.

K5 : For the case $R_{ij} \geq \max_h(S_{hk})$ ; 1 - $R_{ij} \geq R_{ij}$ - $S_{jk}$ and 1 - $R_{ij} \leq S_{jk}$ , we have :

Upper bound of K5,

$$\text{K5} = \min\left( \text{AndTop}(R_{ij} \rightarrow S_{jk}) , \frac{1}{n} \sum_{j=1}^{n} (AndTop(R_{ij}, S_{jk})) \right)$$

$$= \min\left( \min(1, 1 - R_{ij} + S_{jk}) , \frac{1}{n} \sum_{j=1}^{n} (S_{jk}) \right)$$

Since $1 - R_{ij} \geq R_{ij} - S_{jk} \Rightarrow 1 - R_{ij} + S_{jk} \geq R_{ij}$

On the other hand, $R_{ij} \geq \max_h(S_{hk})$

$\Rightarrow 1 - R_{ij} + S_{jk} \geq \max_h (S_{hk})$

$\Rightarrow 1 - R_{ij} + S_{jk} \geq \frac{1}{n} \sum_{j=1}^{n} (S_{jk})$

So,

$$\text{K5} = \frac{1}{n} \sum_{j=1}^{n} (S_{jk})$$

For the lower bound, PlyBot = $I_{KD}$

$$\text{K5} = \min\left( \text{AndTop}(R_{ij} \rightarrow S_{jk}) , \frac{1}{n} \sum_{j=1}^{n} (AndTop(R_{ij}, S_{jk})) \right)$$

Since $1 - R_{ij} \leq S_{jk}$ ,

$$\text{K5} = \min\left( \min_j(S_{jk}) , \frac{1}{n} \sum_{j=1}^{n} (S_{jk}) \right)$$

$$= \min_j (S_{jk})$$

K17 : The inequality hold true for all $R_{ij}$ and $S_{jk}$ :

$$I_L ( R_{ij} , S_{jk} ) \geq I_{KD} ( R_{ij} , S_{jk} ) \geq \min ( R_{ij} , S_{jk} ) = S_{jk}$$

And it still holds true for the mean of the terms :

$$\frac{1}{n} \sum_{j=1}^{n} ( I_L( R_{ij} , S_{jk} )) \geq \frac{1}{n} \sum_{j=1}^{n} ( I_{KD} ( R_{ij} , S_{jk} )) \geq \frac{1}{n} \sum_{j=1}^{n} ( \min ( R_{ij} , S_{jk} )) = \frac{1}{n} \sum_{j=1}^{n} ( S_{jk} )$$

From K17,

$$\text{K17} = \min ( \frac{1}{n} \sum_{j=1}^{n} (R_{ij} \rightarrow S_{jk}) , \frac{1}{n} \sum_{j=1}^{n} (AndTop(R_{ij},S_{jk})) )$$

$$= \min ( \frac{1}{n} \sum_{j=1}^{n} (R_{ij} \rightarrow S_{jk}) , \frac{1}{n} \sum_{j=1}^{n} (\min(R_{ij},S_{jk})) )$$

$$= \frac{1}{n} \sum_{j=1}^{n} (S_{jk})$$

for both upper and lower bound of the inferences.

In this case, both upper and lower bounds of K5 and K17 will depend on data from knowledge base only. Upper bound of K5, both upper and lower bound of K17 will be the mean of value in knowledge base, whereas the lower bound of K5 will be the smallest value in knowledge base.

These are examples that have unreasonable inference result because of this weakness:

| | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 0.6 | 0.7 | 0.6 |
| $A_2$ | 0.7 | 0.6 | 0.8 |
| $A_3$ | 0.6 | 0.6 | 0.7 |

| | $C_1$ |
|---|---|
| $B_1$ | 0.4 |
| $B_2$ | 0.5 |
| $B_3$ | 0.6 |

The bounds of the result (K5):

| | $A_1C_1$ | $A_2C_1$ | $A_3C_1$ |
|---|---|---|---|
| Upper Bound | 0.5 | 0.5 | 0.5 |
| Lower Bound | 0.4 | 0.4 | 0.4 |

The bounds of the result (K17):

| | $A_1C_1$ | $A_2C_1$ | $A_3C_1$ |
|---|---|---|---|
| Upper Bound | 0.5 | 0.5 | 0.5 |
| Lower Bound | 0.5 | 0.5 | 0.5 |

### 4.1.6 Case : AndBot ( $R_{ij} \rightarrow S_{jk}$ ) As First Term

This case affects K7, K8, K9, K10, K11 and K12 which AndBot($R_{ij} \rightarrow S_{jk}$) becomes the first term in min function and take min as the outer connective.

According to Proposition 3.1, when number of ($R_{ij} \rightarrow S_{jk}$) increased, AndBot($R_{ij} \rightarrow S_{jk}$) can be generalized into :

$$\underset{i}{\text{AndBot}}(a_i) = \max_i \left(0, \sum_{i=1}^{n} (a_i) - (n-1)\right)$$

When $n$ is increasing, ($n$-1) will increase faster than $\sum_{i=1}^{n} (a_i)$, for $\forall a_i < 1.0$. In another words, $\sum_{i=1}^{n} (a_i) - (n-1)$ will become smaller and smaller and eventually become 0 or negative number.

Thus, $\underset{i}{\text{AndBot}}(a_i) = \max_i \left(0, \sum_{i=1}^{n} (a_i) - (n-1)\right) = 0$ while $n$ is increased up to a certain level. For example, for all the $a_i$=0.9, $n$=10 is enough to make the term become 0, as well as the final result of inferences.

Clearly, the habit of AndBot($R_{ij} \rightarrow S_{jk}$) will make inference structures K7, K8, K9, K10, K11 and K12 become not reliable especially when the number of data increase.

## 4.2 REASON BEHIND THE WEAKNESS OF INFERENCE STRUCTURES

Every logical connectives may or may not present the pseudo-strict monotonic property, which has discussed briefly in last chapter (Table 3.4). A logical connective, CON is pseudo-strict monotonic if :

$$a_1 \neq a_2 \Rightarrow \text{CON}(a_1, b) \neq \text{CON}(a_2, b)$$

and

$$b_1 \neq b_2 \Rightarrow \text{CON}(a, b_1) \neq \text{CON}(a, b_2)$$

where,

$\forall(a_1, a_2, a) \neq (0, 0, 0)$ and $\forall(b_1, b_2, b) \neq (0, 0, 0)$ for CON = AND

$\forall(a_1, a_2, a) \neq (0, 0, 0)$ and $\forall(b_1, b_2, b) \neq (1, 1, 1)$ for CON = PLY

$\forall(a_1, a_2, a) \neq (1, 1, 1)$ and $\forall(b_1, b_2, b) \neq (1, 1, 1)$ for CON = OR

Clearly, all PLY, AND and OR operators appear in inference structures above are non pseudo-strict monotonic. Hallam and Yew [1998b] have also shown proofs with empirical method that the main reason of the defects of inference structures are caused by the non pseudo-strictly monotonic property of logical connectives. The lack of this property, causing an operator to generate results depend only on one of the two arguments, despite the other argument may bring meaningful information.

It is important to have pseudo-strict monotonic property in logical connectives, however, most existing logical connectives does not fulfill the requirement.

## 4.3 GENERATING PSEUDO-STRICTLY MONOTONIC FUZZY IMPLICATION

It is rather safe to improve the original inference structures compared to reconstruct a new theory, and this is what most scientists do since long time ago.

The basic concept of interval value inference proposed by Bandler and Kohout [1986a, 1986b] is good, but the well known implication operators $I_{KD}$ and $I_L$ do not perform well as expected. As an improvement, Hallam [1999] has generated a family of fuzzy implication operators as a substitution of $I_{KD}$ and $I_L$.

From the checklist paradigm, it is clear that $I_{KD}$ and $I_L$ are the lower and upper bound of an inference, respectively. Bandler and Kohout also proposed that $I_{KDL}$ is the expected value of such inference. So :

$$I_{KD} \leq I_{KDL} \leq I_L$$

From here, 2 implication operators $I_X$ and $I_Y$ can be generated, where $I_{KD} \leq I_X \leq I_{KDL}$ and $I_{KDL} \leq I_Y \leq I_L$.

Using dichotomous division,

$$I_X = \max\left[\frac{(b)+(1-a+ab)}{2}, \frac{(1-a)+(1-a+ab)}{2}\right]$$

$$= \max\left[\frac{1-a(b+1)+b}{2}, 1-a+\frac{ab}{2}\right]$$

Also,

$$I_Y = \min\left[\frac{(1)+(1-a+ab)}{2}, \frac{(1-a+b)+(1-a+ab)}{2}\right]$$

$$= \min\left[1-\frac{a(1-b)}{2}, 1-a+\frac{b(1+a)}{2}\right]$$

Of course, using the same method, we can have infinite number of fuzzy implication operators generated in the interval [ $I_{KD}$ , $I_X$ ], [ $I_X$ , $I_{KD}$ ] , [ $I_{KD}$ , $I_Y$ ] and [ $I_Y$ , $I_L$ ].

**a) Between $I_{KD}$ and $I_X$**

We can have infinite number of fuzzy implication operators which stand between $I_X$ and $I_{KD}$. We can generalize it as :

$$I_{KD_p} = \max\left[\frac{(2^{p-1}-1)[1+a(b-1)]+(2^{p-1}+1)b}{2^p}, (1-a)+\frac{(2^{p-1}-1)(ab)}{2^p}\right]$$

and

$$I'_{KD_p} = \max\left[\frac{1-a(b+1)+(2^p-1)b}{2^p}, 1-a+\frac{ab}{2^p}\right]$$

Where $I'_{KD_p}$ stands between $I'_{KD_{p-1}}$ and $I_X$, whereas $I_{KD_p}$ stands between $I_{KD_{p-1}}$ and $I_{KD}$. In another words, The increasing of $p$ will make $I'_{KD_p}$ move toward $I_{KD}$, and $I_{KD_p}$ move toward $I_X$.

If $p = 1$,

$$I_{KD_1} = \max[b, 1-a] = I_{KD}$$

and

$$I'_{KD_1} = \max\left[\frac{1-a(b+1)+b}{2},\ 1-a+\frac{ab}{2}\right] = I_X$$

If $p = \infty$

$$I_{KD_\infty} = \max\left[\frac{1-a(b+1)+b}{2},\ 1-a+\frac{ab}{2}\right] = I_X$$

and

$$I'_{KD_p} = \max[b, 1-a] = I_{KD}$$

**b) Between $I_X$ and $I_{KDL}$**

Similar to fuzzy implication operators between $I_{KD}$ and $I_X$, we can have infinite number of fuzzy implication operators between $I_X$ and $I_{KDL}$ :

$$I_{X_p} = \max\left[\frac{(2^p-1)[1+a(b-1)]+b}{2^p}, (1-a)+\frac{(2^p-1)(ab)}{2^p}\right]$$

and

$$I'_{KDL_p} = \max\left[\frac{(2^{p-1}+1)[1+a(b-1)]+(2^{p-1}-1)b}{2^p}, (1-a)+\frac{(2^{p-1}+1)(ab)}{2^p}\right]$$

Where $I'_{KDL_p}$ stands between $I'_{KDL_{p-1}}$ and $I_X$, whereas $I_{X_p}$ stands between $I_{X_{p-1}}$ and $I_{KDL}$. In another words, the increasing of $p$ will make $I'_{KDL_p}$ move toward $I_{KDL}$, and $I_{X_p}$ move toward $I_X$.

If $p = 1$,

$$I_{X_1} = \max\left[\frac{[1+a(b-1)]+b}{2}, (1-a)+\frac{(ab)}{2}\right] = I_X$$

and

$$I'_{KDL_1} = \max[(1+a(b-1))+b, (1-a)+ab] = 1-a+ab = I_{KDL}$$

If $p = \infty$

$$I_{X_\infty} = \max[(1 + a(b-1)) + b, (1-a) + ab] = 1 - a + ab = I_{KDL}$$

and

$$I'_{KDL_p} = \max\left[\frac{[1 + a(b-1)] + b}{2}, (1-a) + \frac{(ab)}{2}\right] = I_X$$

**c) Between $I_{KDL}$ and $I_Y$**

Another family of fuzzy implication operators can be generated between $I_Y$ and $I_{KDL}$ :

$$I_{KDL_p} = \min\left[1 - \frac{(2^{p-1}+1)(a)(1-b)}{2^p}, (1-a) + \frac{b[(2^{p-1}-1) + (2^{p-1}+1)(a)]}{2^p}\right]$$

and

$$I_{Y_p} = \min\left[1 - \frac{(2^p - 1)(a)(1-b)}{2^p}, (1-a) + \frac{b[1 + (2^p - 1)a]}{2^p}\right]$$

Where $I_{KDL_p}$ stands between $I_{KDL_{p-1}}$ and $I_Y$, whereas $I_{Y_p}$ stands between $I_{Y_{p-1}}$ and $I_{KDL}$. In another words, the increasing of $p$ will make $I_{KDL_p}$ move toward $I_Y$, and $I_{Y_p}$ move toward $I_{KDL}$.

If $p = 1$

$$I_{KDL_1} = \min[1 - a(1-b),\ 1 - a + ab] = I_{KDL}$$

and

$$I_{Y_1} = \min\left[1 - \frac{(a)(1-b)}{2},\ (1-a) + \frac{b[1+a]}{2}\right] = I_Y$$

If $p = \infty$

$$I_{KDL_\infty} = \min\left[1 - \frac{a(1-b)}{2}, (1-a) + \frac{b(1+a)}{2}\right] = I_Y$$

and

$$I_{Y_\infty} = \min[1 - a(1-b),\ 1 - a + ab] = I_{KDL}$$

**d) Between $I_Y$ and $I_L$**

Lastly, another family of fuzzy implication operators can be generated between $I_Y$ and $I_L$ :

$$I_{L_p} = \min\left[1 - \frac{a(1-b)}{2^p}, (1-a) + \frac{b[(2^p - 1) + a]}{2^p}\right]$$

and

$$I'_{L_p} = \min\left[1 - \frac{(2^{p-1} - 1)(a)(1-b)}{2^p}, (1-a) + \frac{b[(2^{p-1} + 1) + (2^{p-1} - 1)(a)]}{2^p}\right]$$

Where $I_{L_p}$ stands between $I_{L_{p-1}}$ and $I_L$, whereas $I'_{L_p}$ stands between $I'_{L_{p-1}}$ and $I_Y$. In another words, the increasing of $p$ will make $I'_{L_p}$ move toward $I_Y$, and $I_{L_p}$ move toward $I_L$.

If $p = 1$

$$I_{L_1} = \min\left[1 - \frac{a(1-b)}{2}, (1-a) + \frac{b[1+a]}{2}\right] = I_Y$$

and

$$I'_{L_1} = \min[1, 1 - a + b] = I_L$$

If $p = \infty$

$$I_{L_\infty} = \min[1, (1-a) + b] = I_L$$

and

$$I'_{L_\infty} = \min\left[1 - \frac{a[1-b]}{2}, (1-a) + \frac{b[1+a]}{2}\right] = I_Y$$

Now, by choosing different value of $p$, we can have infinite pairs of fuzzy implication operators. Properties held by these operators are listed in Table 4.1 :

*Table 4.1 : Properties of Fuzzy Implication Operators*

| Axioms | Name | Description |
|---|---|---|
| 1 | Boundary Condition | $I(0,0) = I(0,1) = I(1,1) = 1$; $I(1,0) = 0$ |
| 2 | Dominance of falsity | $\forall a,b \in [0, 1]$, $I(0,b) = I(a,1) = 1$ |
| 3 | Neutrality of truth | $\forall b \in [0, 1]$, $I(1,b) = b$ |
| 4 | Antagonism of falsity | $\forall a \in [0, 1]$, $I(a,0) = 1-a$ |
| 5 | Contrapositivity | $\forall a,b \in [0, 1]$, $I(a,b) = I(1-b,1-a)$ |
| 6 | Monotonicity in first argument | $\forall a_1,a_2,b \in [0, 1]$, for $a_1 \le a_2$ $I(a_1,b) \ge I(a_2,b)$ |
| 6' | Pseudo-Strict Monotonicity in first argument | For $b \neq 1$ and $\forall a_1,a_2 \in [0, 1]$, $a_1 < a_2 \Rightarrow I(a_1,b) > I(a_2,b)$ |
| 7 | Monotonicity in second argument | $\forall b_1,b_2,a \in [0, 1]$, for $b_1 \le b_2$ $I(a,b_1) \le I(a,b_2)$ |
| 7' | Pseudo-Strict Monotonicity in second argument | For $a \neq 0$ and $\forall b_1,b_2 \in [0, 1]$, $b_1 < b_2 \Rightarrow I(a,b_1) < I(a,b_2)$ |
| 8 | Exchange Property | $\forall a,b,x \in [0, 1]$, $I(a, I(b,x)) = I(b, I(a,x))$ |

| | | |
|---|---|---|
| 9 | Continuity | I is a continuous function from the unit interval to the unit interval. |

Among all properties shared by these operators, property 6' and 7' are most remarkable. These pseudo-strict monotonicity properties are not exhibited on traditional fuzzy implication operators $I_{KD}$, $I_L$ as well as $I_{KDL}$.

## 4.4 CONSTRUCTING PSEUDO-STRICTLY MONOTONIC FUZZY LOGICAL CONNECTIVES

With the fuzzy implication operators generated in last section, we can generate corresponding connectives using the relation :

$$\text{NOT } a = \bar{a} = 1 - a$$

$$a \text{ AND } b = \overline{a \rightarrow \bar{b}} = 1 - [\, a \rightarrow (1 - b)\,]$$

$$a \text{ OR } b = \bar{a} \rightarrow b = (1 - a) \rightarrow b$$

a) Defining $\rightarrow$ as $I_X$

$$\text{AND}_x = \max\left[\frac{ab}{2}, a + \frac{(1+a)(b-1)}{2}\right]$$

$$\text{OR}_x = \max\left[b + \frac{a(1-b)}{2}, a + \frac{b(1-a)}{2}\right]$$

b) Defining $\rightarrow$ as $I_Y$

$$\text{AND}_Y = \min\left[a + \frac{b(1-a)}{2}, \frac{a(b+1)}{2}\right]$$

$$\text{OR}_Y = \min\left[1 - \frac{(1-a)(1-b)}{2}, a + b - \frac{ab}{2}\right]$$

c) Defining $\rightarrow$ as $I_{KD_p}$

$$\text{AND}_{KD_p} = \min\left[\frac{b(2^p-1) + a(2-b)}{2^p}, a + \frac{a(b-1)}{2^p}\right]$$

$$\text{OR}_{KD_p} = \max\left[b + \frac{(2^{p-1}-1)a(b-1)}{2^p}, a + \frac{(2^{p-1}-1)b(1-a)}{2^p}\right]$$

d) Defining $\rightarrow$ as $I'_{KD_p}$

$$\text{AND}'_{KD_p} = \min\left[\frac{(2^{p-1}-1)ab + (2^{p-1}+1)b}{2^p}, a + \frac{a(2^{p-1}-1)(b-1)}{2^p}\right]$$

$$\text{OR}'_{KD_p} = \max\left[\frac{a(b+1) + b(2^p-2)}{2^p}, a + \frac{b(1-a)}{2^p}\right]$$

e) Defining $\rightarrow$ as $I_{X_p}$

$$\text{AND}_{X_p} = \min\left[\frac{(2^{p-1}+1)ab + (2^{p-1}-1)b}{2^p}, \frac{a[2^{p-1}(b+1) + b - 1]}{2^p}\right]$$

$$\text{OR}_{X_p} = \max\left[b + \frac{(2^p-1)a(1-b)}{2^p}, a + \frac{(2^p-1)b(1-a)}{2^p}\right]$$

f) Defining $\rightarrow$ as $I'_{KDL_p}$

$$\text{AND}'_{KDL_p} = \min\left[ab - \frac{b(a+1)-1}{2^p}, ab + \frac{a(1-b)}{2^p}\right]$$

$$\text{OR}'_{KDL_p} = \max\left[b + \frac{(2^{p-1}+1)a(1-b)}{2^p}, a + \frac{(2^{p-1}+1)b(1-a)}{2^p}\right]$$

g) Defining $\rightarrow$ as $I_{KDL_p}$

$$AND_{KDL_p} = \min\left[\frac{(2^p-1)ab}{2^p}, ab + \frac{(1-a)(b-1)}{2^p}\right]$$

$$OR_{KDL_p} = \min\left[\frac{(2^{p-1}+1)(a+b-ab)+(2^{p-1}-1)}{2^p}, b - \frac{(2^{p-1}+1)a}{2^p}\right]$$

h) Defining $\rightarrow$ as $I_{Y_p}$

$$AND_{Y_p} = \min\left[\frac{(2^{p-1}+1)ab}{2^p}, \frac{(2^{p-1}-1)(a+b-1)+(2^{p-1}+1)ab}{2^p}\right]$$

$$OR_{Y_p} = \min\left[\frac{(2^p-1)(a+b-ab)+1}{2^p}, (a+b) - \frac{(2^p-1)ab}{2^p}\right]$$

i) Defining $\rightarrow$ as $I_{L_p}$

$$AND_{L_p} = \min\left[\frac{(2^p-1)ab}{2^p}, \frac{(2^{p-1}+1)(a+b-1)+(2^{p-1}-1)ab}{2^p}\right]$$

$$OR_{L_p} = \min\left[\frac{2^p-1+a+b-ab}{2^p}, a+b-\frac{(2^p-1)ab}{2^p}\right]$$

j) Defining $\rightarrow$ as $I'_{L_p}$

$$AND'_{L_p} = \min\left[\frac{ab}{2^p}, (a+b-1) + \frac{(1-a)(1-b)}{2^p}\right]$$

$$OR'_{L_p} = \min\left[\frac{(2^{p-1}-1)(a+b-ab)+(2^{p-1}+1)}{2^p}, (a+b) - \frac{(2^{p-1}-1)ab}{2^p}\right]$$

Clearly, we have infinite sets of inference structures, depending on how we choose fuzzy implication operators, fuzzy logical connective as well as the value of $p$ for some implication operators.

## 4.5 CHAPTER SUMMARY

Pseudo-strict monotonic property is an important property for an inference structure. Inference structures without this property may produce wrong inference results in some special cases. In this chapter, new families of fuzzy implication operators are generated using dichotomous division. With these fuzzy implication operators, new fuzzy logical connectives as well as inference structures are generated.

# Chapter 5

# IMPLEMENTATION : HIERARCHICAL FUZZY INFERENCE SYSTEM

## 5.0 INTRODUCTION

The usefulness of a theory is greatly dependent on empirical performance. Fuzzy inference structures based on sub-K triangle inference templates and fuzzy logical connectives such as $I_L$ and $I_{KD}$ have been deployed as inference engine of medical diagnosis systems [Yew, 1995]. As the theory of sub-K inference templates have some theoretical enhancement, it is worth while to have another experiment on the performance of these inference structures. Hence, an arthritic diseases diagnosis system has been setup for the purpose.

## 5.1 PROBLEMS IN DESIGNING KNOWLEDGE BASE FOR ARTHRITIC DISEASES

[Bullough and Bansal 1988], [Mongey and Hess 1988], [Renner and Weinstein 1988], [Resnick 1995a]

Arthritis is a general name for a group of diseases that afflict joints. Table 5.1 lists all the arthritic diseases that will be considered in this system. Most of these diseases show both systemic and local manifestations.

*Table 5.1 : Arthritic diseases that are included in this system.*

| | |
|---|---|
| 1 | Rheumatoid Arthritis |
| 2 | Degenerative Joint Disease (Osteoarthritis) |
| 3 | Gouty Arthritis |
| 4 | Psoriatic Arthritis |
| 5 | Reiter's Syndrome |
| 6 | Calcium Pyrophosphate Dihydrate Crystal Deposition Disease (CPPD) |
| 7 | Scleroderma |
| 8 | Polymyositis |
| 9 | Diffuse Idiopathic Skeletal Hyperostosis |
| 10 | Systemic Lupus Erythematosus |
| 11 | Calcium Hydroxyapatite Crystal Deposition Disease |
| 12 | Multicentric Reticulohistiocytosis |
| 13 | Renal Osteodystrophy |
| 14 | Rheumatic Fever |

| | |
|---|---|
| 15 | Wilson's Disease |
| 16 | Amyloidosis |
| 17 | Acromegaly |
| 18 | Relapsing Polychondritis |
| 19 | Behcet's Syndrome |

There are 206 joints in our body. However, not all joints are involved in arthritic diseases, with some joints showing an increased predilection while other joints do not. In this study, only joints/regions in hand and wrist (Figure 5.1 and Figure 5.2) that are often afflicted by arthritis will be considered (see Table 5.2).

*Table 5.2 : Target area / joints that are commonly afflicted by arthritis.*

| Sites | Joints |
|---|---|
| Hand[1] | Distal interphalangeal |
| | Proximal interphalangeal |
| | Metacarpophalangeal |
| Wrist | Radiocarpal compartment |
| | Inferior Radioulnar compartment |
| | Midcarpal compartment |
| | Common carpometacarpal compartment |
| | First carpometacarpal compartment |

[1] These 3 joints only exist for each second to fifth digits for both hands, but the thumbs only have 2 joints (interphalangeal and metacarpophalangeal).

*Figure 5.1 : Radiographic findings of Rheumatoid Arthritis of the hand [Resnick 1995a]*

*Figure 5.2 : Radiographic of wrist - this patient has Rheumatoid Arthritis and CPPD Crystal Deposition [Resnick 1995a]*

The radiological diagnosis of arthritic diseases has to deal with some difficulties [Bellamy, 1997, Jain 1998] :

**i. Clinical appearance of a disease varies with the stage of the disease**

Generally diseases show different clinical manifestations at different stages of the evolution of the diseases and this applies to the arthritic diseases as well. For example, local pain and tenderness over the sacroiliac joints can be a prominent manifestation in the early stage of Ankylosing Spondylitis. However, this symptom may become mild or even disappear completely with further development of the disease. [Resnick and Niwayama, 1995a]

**ii. Diseases usually manifest themselves differently in different patients.**

This is a common difficulty in diagnosis of many different kinds of diseases including arthritis. For example, Psoriatic Arthritis may appear as monoarticular, pauciarticular or even polyarticular disease in different patients. [Resnick and Niwayama, 1995b]

**iii. Patients were afflicted by two or more diseases at the same time.**

Two or more arthritic diseases may develop simultaneously in one individual and complicate the pattern of each disease. For example, coexistence of Rheumatoid Arthritis and Calcium Pyrophosphate Dihydrate Crystal Deposition Disease (CPPD), Rheumatoid Arthritis and Ankylosing Spondylitis is not rare [Resnick and Niwayama,

1995c]. Furthermore, arthritis and non-arthritic diseases may afflict a patient at the same time.

Besides these difficulties, we have to bear in mind that arthritic diseases do not only show systemic signs and symptoms. Systemic abnormalities are also not sufficient to identify one's arthritic illness. Although diagnosis that is only based on systemic criteria has been worked out for some arthritic related diseases [Leitich, Adlassing and Kolarz, 1996], more descriptive criteria are still needed for a more rigorous diagnosis [Resnick and Niwayama, 1995a]. A more specific diagnosis of arthritic diseases will be based on radiological findings at related site. This has given rise to another difficulty in the diagnosis of arthritic diseases :

**iv. A disease shows different manifestation in different joint of the same patient.**

This is a common situation in arthritic diseases, e.g. in Rheumatoid Arthritis, bony ankylosis is a common manifestation in wrist, especially in midcarpal compartment [Resnick and Niwayama, 1995c], but it is rare for involvement of the knee.

These difficulties make the knowledge base of the system more complicated. Therefore, more information about the patients as well as diseases is needed from the users to make the system perform properly. However, this raises another problem :

v. **A large amount of data input reduces system efficiency.**

Assume $D$ is the number of diseases to be considered and $d$ indicates index of the diseases, each disease may show up with $M_d$ signs/symptoms, then the number of questions (corresponding to signs/symptoms of the diseases) to be answered for every diagnosis is equal to $\sum_{d=1}^{D} M_d$. This could be a big number when the number of diseases increased. Diagnosticians have to spend a lot of time to input all medical findings into the systems. All questions regarding the patient's condition asked by the system have to be answered one by one although some may be irrelevant to the disease(s) being considered for a particular patients. Further, with the increasing number of input, the number of mappings required to associate input and output will grow in exponential fashion. This will certainly increase computational processing time in addition to more machine cycles needed for the mapping procedure. Therefore, it is important to decrease the number of input of the system as well as the size of the knowledge base so that the system is not only an effective system, but also a system with high efficiency.

All these points must be noted while developing a medical diagnosis system, including a system for diagnosing arthritic diseases. In subsequent sections, the design of the system will be discussed and so the methods of overcoming these problems clarified.

*Table 5.3 : List of signs and symptoms of arthritis*

| | |
|---|---|
| 1. | synovial hypertrophy |
| 2. | accumulation of intra-articular fluid |
| 3. | soft tissue edema |
| 4. | osteochondral destruction in inflammatory pannus |
| 5. | indistinctness of asseous outline |

6. fusiform soft tissue swelling
7. periarticular osteoporosis
8. marginal erosion
9. erosion and swelling
10. ulnar styloid process extend around styloid proces.
11. synovial infalmmation
12. proliferative synovitis
13. tendinitis of extensor carpi ulnaris tendon
14. loss of inter osseous space
15. subchondral eburnation
16. apron-like marginal osteophytes
17. joint space narrowing
18. marginal osteophytes
19. uniform narrowing of interosseous space
20. radial subluxation of the metacarpal base
21. synovial calcification
22. sclerosis
23. cysts formation
24. bony collapse
25. arthropathy
26. progressive destruction of joints
27. marginal erosion at distal interphalangeal joints & proceed centrally
28. dorsal subluxation at interphalangeal of the thumbs
29. sesamoid destruction at first metacarpophalangeal joints
30. tuftal resorption in one or more terminal phalanges
31. extensive osteolysis in proximal segments of hands
32. osteoporosis
33. osseous excrescences
34. eccentric local soft tissue prominence
35. regional or periarticular osteoporosis
36. subchondral and marginal erosions
37. the pannus of granulation tissue causes joints narrowing
38. large erosions of inter carpal and carpometacarpal joints
39. new bone formation
40. osseous destruction of portions of the carpal bones and ulna
41. soft tissue resorption of fingertips
42. subcutaneous calcification in hands
43. osseous destruction in hand
44. bony erosion of the phalanges
45. soft tissue swelling in metacarpophalangeal and interphalangeal joints
46. bony erosions in inferior radioulnar
47. bony erosions in inferior metacarpophalangeal
48. bony erosions in inferior distal interphalangeal
49. bony erosions in inferior ulnar styloid
50. flexion deformities of metacarpophalangeal joints
51. radial subluxation or dislocation at intrephalangeal of the thumb
52. broadening and arrow heading of distal interphalangeal tufts
53. increased cortical width of tubular bones
54. enlarged sesamoid bones hyperostosis along the distal end of radius

55. soft tissue swelling
56. articular space widened/narrowed
57. resorption of the phalanges
58. swelling
59. erythema
60. brown tumors
61. osteosclerosis
62. chondrocalcinosis
63. periostitis
64. fractures
65. soft tissue and vascular calcification
66. deposits in or near the tendons of the flexor carpiunaris
67. deposits in or near the tendons of the flexor carpiradialis
68. deposits in or near the tendons of the common flexors
69. calcification with the flexor carpiulnaris tendon
70. subchondral bone fragmantation
71. cortical irregularities
72. less of bone density in hand
73. osteopenia in hand
74. cartilage calcification
75. lytic lesions
76. phatologic fractures
77. osteonecrosis
78. soft tissue nodules and swelling
79. subchondral cysts and erosions
80. joint subluxation and contractures
81. neuropathic osteoarthropathy
82. soft tissue thickening of fingers
83. thickening and squaring of the phalanges and metacarpals
84. tubulation of the shafts of the phalanges
85. prominence of the ungual tufts
86. calcification of articular cartilage
87. joint space narrowing at metacarphopahalangeal
88. osseous erosions of metacarphopahalangeal
89. "arthritis mulitans"
90. osseous erosion
91. swelling of metacarpophalangeal joints
92. stiffness of metacarpophalangeal joints
93. swelling of proximal interphalangeal joints
94. stiffness of interphalangeal joints
95. inflammation of tendon sheats of metacarpophalangeal
96. inflammation of tendon sheats of proximal interphalaneal
97. osteoporosis about proximal intrphalangeal
98. osteoporosis about metacarpophalangeal
99. joint space not nerrowed
100. hyperextension at proximal interphalngeal
101. flexion at distal interphalngeal
102. hyperextension at interphalngeal of the thumbs
103. osteoporosis and cysts formation at metacarpal heads

## 5.2 DESIGN OF THE KNOWLEDGE BASE

[Lim, Yew, Ng and Abdullah]

A large and complicated fuzzy knowledge base is required to overcome the above difficulties, especially (i), (ii), (iii) and (iv). This knowledge base, including information of diseases as in Table 5.1, will record all possible diagnostic clues of these diseases. These will include signs and symptoms (Table 5.3) of diseases on all listed joints (Table 5.2). With this information, difficulty (iv) could be overcome.

With a fuzzy inference engine, imprecise information brought by difficulties (i), (ii) and (iii) can be handled [Bellamy, 1997] if the variation of signs and symptoms is not large. For any disease that may show large variation in signs and symptoms in different situations, another set of disease profile has to be stored. This profile, even though it represents the same disease, will be considered as another type of disease by the inference engine. For instance, rheumatic fever may develop as an acute disease or insidious disease. Acute onset of rheumatic fever may be characterized by fever, night sweats, headaches and joint pain, however insidious onset may be characterized by pallor, fatigue, anorexia, weight loss and muscular pain [Resnick, 1995a]. These two sets of profiles will be stored separately in the knowledge base.

Development of such a large fuzzy knowledge base will be a good solution to difficulty (i), (ii) and (iii). However, as discussed in difficulty (v), this will also reduce the efficiency of the system.

A hierarchical system could provide a good solution to overcome this problem. With a hierarchical system, diagnosis will be broken down into several levels.

On the first level of the hierarchy, several questions will be asked with the purpose of narrowing down the scope of the diagnosis. The mapping process as illustrated in Figure 5.3 will be applied. As a response to this input, a list of diseases will be generated by the inference engine using the knowledge base. This output - diseases in the generated list would be the suspected diseases that afflict the patient.

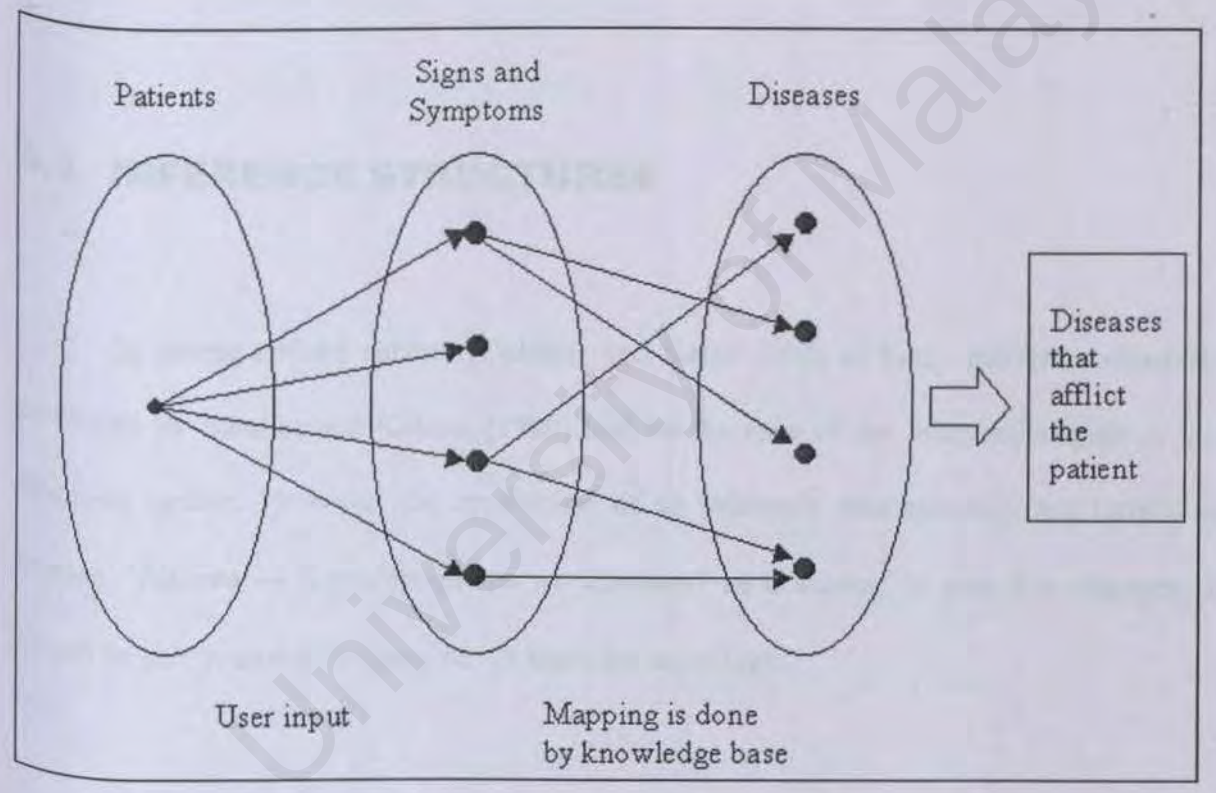


*Figure 5.3 : The process of matching signs and symptoms of a patient with the knowledge base.*

In the second level of the hierarchy, only questions that are related to the diseases in the previously generated list will be asked. Data gathered from these questions will be

mapped to the knowledge base again to generate another list, with fewer numbers of diseases compared to the previous one.

This process is repeated until the last level - the end of the hierarchy. In each level, the inference engine acts as a fuzzy filter for screening those "unqualified" diseases. Disease(s) in the last generated list will be the actual disease(s) affecting the patient. If no disease is listed then the medical findings are not consistent with any of the arthritic diseases in the knowledge base.

## 5.3 INFERENCE STRUCTURES

Of course revised version [DeBaets and Kerre 1993] of fuzzy inference structures developed by Bandler and Kohout [1980] will be the core of the inference engine in this diagnosis system. However, the application of an inference structure may not limited to relation "Patients → Signs/Symptoms → Diseases" as discussed in past few chapters. It should be able to extend to cases which share the same logic.

The first 18 inference structures generated from sub-K inference templates, which use min as the outer connective (K1 - K18) will be tested in the system. For the sake of simplicity and to speed up the process of calculation, the fuzzy implication operators which is proposed by Hallam [1999] will be applied :

$$I_X = \max\left[\frac{1-a(b+1)+b}{2},\ 1-a+\frac{ab}{2}\right]$$

$$I_Y = \min\left[1-\frac{a(1-b)}{2},\ 1-a+\frac{b(1+a)}{2}\right]$$

Therefore, $AND_x$, $OR_x$, $AND_Y$ and $OR_Y$ will be :

$$AND_x = \max\left[\frac{ab}{2}, a+\frac{(1+a)(b-1)}{2}\right]$$

$$OR_x = \max\left[b+\frac{a(1-b)}{2}, a+\frac{b(1-a)}{2}\right]$$

$$AND_Y = \min\left[a+\frac{b(1-a)}{2}, \frac{a(b+1)}{2}\right]$$

$$OR_Y = \min\left[1-\frac{(1-a)(1-b)}{2}, a+b-\frac{ab}{2}\right]$$

## 5.4 IMPLIMENTATION

Morphology of the articular lesions and their distribution in the body are amongst the two main clues for a radiologist to arrive at an accurate radiological diagnosis on arthritic diseases [Resnick, 1995a]. With this "target area" approach, these two main parameters of diagnosis can form a two-level fuzzy hierarchical inference system.

In the first level of the hierarchy, the distribution of articular lesions in hand and wrist will be considered. This is reasonable because most arthritic diseases will show prominent menifestations in hands and wrists. Arthritic diseases have a remarkable tendency to afflict specific joints. For example, rheumatoid arthritis always afflicts joints in hands except distal

interphalangeal joints. In contrast, for scleroderma and polymyositis, joints in hands are seldom afflicted except distal interphalangeal joints and interphalangeal joints of the thumbs.

The knowledge base can be built based on this remarkable predilection. For each disease $D_k$, a set of membership functions ($\mu_{jk} : X \rightarrow [0,1]$) is given, corresponding to each joint $J_j$, where:

$\mu_{jk}$ - degree to which joint $J_j$ will be involved with disease $D_k$ .

Consider $L$ as the relation between set of patients $P$ and their joints that are involved with arthritis; and $N$ as relation between each joint $J$ and arthritic diseases $D$ (Figure 5.4). The composition of these relations, based on sub-K product which is discussed in last section, is given by :

$$L \lhd_k N = \min[\inf(L \rightarrow N), \sup \top(L, N)]$$

will help to define the illness associated with the patient. As the output of this process, a list of possible diseases will be generated. A 'fuzzy filter' will then be employed to eliminate some diseases that show low possibility after this mapping. The list of diseases with high possibility, $D'$ will become the scope of diagnosis in the next level. Diseases that are not in this list will not be considered further.

The diagnosis will come to the second level of the hierarchy after $D'$ has been determined. In this level, a knowledge base that describes the relation between manifestation of all diseases in all joints is needed. With the knowledge base, only manifestations related to diseases $D'$ and occurring in joints $J'$ will be selected from the universal of manifestation $M$. The selected manifestation, denoted as $M'$ will be used for further diagnosis.

The diagnostician will be requested to answer questions according to his/her observations. Questions regarding manifestations $M'$ will be proposed and the answer given by the diagnostician will be defined as relation between patient and manifestations, $R$.

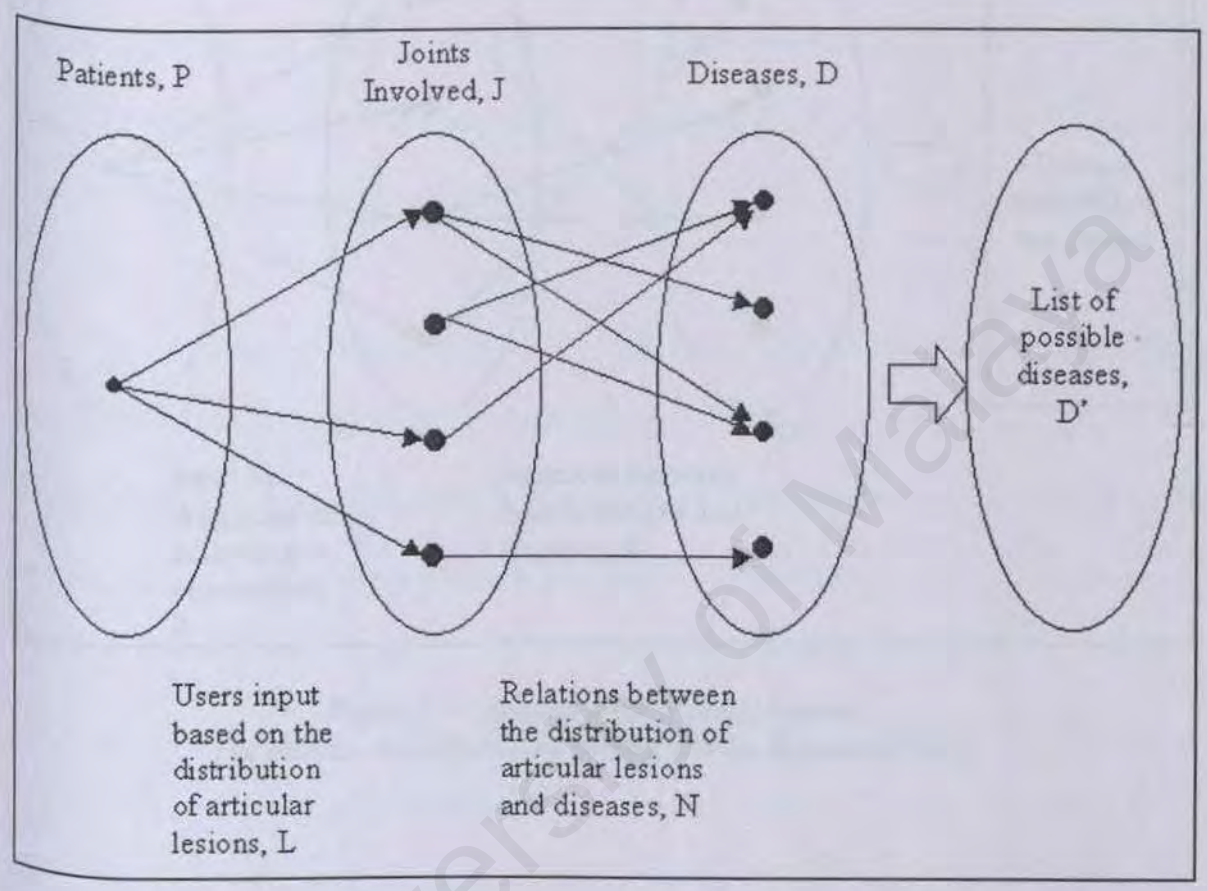


*Figure 5.4 : First level of the hierarchical inference system. Diagnostician is required to select the problematic articulars from a list and D', list of possible diseases will be generated*

$S$, the relation between selected manifestation $M'$ and disease $D'$ is predefined in the knowledge base. Having established the relationship $R$ and $S$, the composition of relations, which is similar in the first level of hierarchy, will be used to determine the disease/diseases which afflict the patient (Figure 5.5).

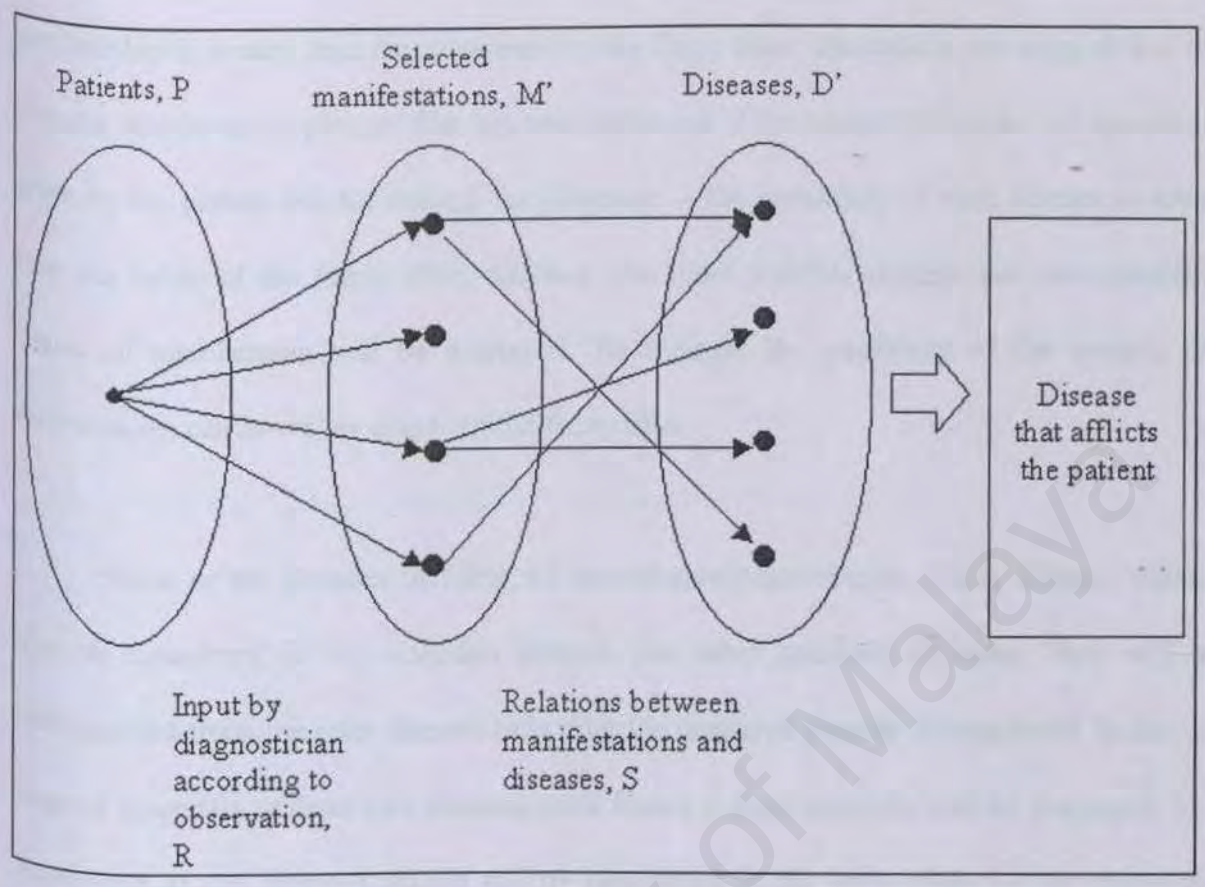


*Figure 5.5 : Second level of the hierarchy.*
*The disease that afflicts the patient will be determined here.*

The actual output of the mapping should be in the form of a disease-possibility pair. Each disease in *D'* will get a score (degree of membership) showing the possibility that the patient is suffering from a disease. A fuzzy filter should be employed again to filter diseases with low possibility.

There may be only a single disease, more then one disease or even no disease found after the filtering. If a single disease is found, then that should be the suspected disease that afflicts the patient and the possibility given. If more then one disease is found, then the patient may be suffering from more than one disease. In this case, the system will display the

disease with highest possibility as the suspected disease. For other diseases with the degree of membership is greater than the value used by the fuzzy filter, the system will suggest that the diseases coexist in the patient. The last case will occur if the meaningful signs and symptoms given by the patient are not enough for judgment -- the possibility of each disease is lower than the value of the fuzzy filter. Anyway, the most possible disease and corresponding degree of membership will be displayed. To increase the sensitivity of the system, the diagnostician can lower the degree of the fuzzy filter.

Some of the diseases in Table 5.1 are relatively uncommon. These diseases should also be considered in the diagnosis process like other common diseases. They will be distinguished from common diseases only after the diagnosis process is completed. In the last stage of diagnosis, if these rare diseases were found, a short reminder will be displayed. The occurrence of the diseases should not be considered at the early stage of the diagnosis, otherwise all diseases with low occurrence will be filtered very early, no matter what signs and symptoms are associated with the patient.

**Example :**

For patient $P_1$, diagnosis can be performed based on the distribution of articular lesions on joints ($J_1$, $J_2$) and signs/symptoms ($S_1$, $S_2$, $S_3$) for a list of diseases ($D_1$ and $D_2$).

Assume that we have these value to perform a diagnosis based on the distribution of articular lesions on joints $J_1$ and $J_2$:

| | $J_1$ | $J_2$ |
|---|---|---|
| $P_1$ | 0.7 | 0.5 |

| | $D_1$ | $D_2$ |
|---|---|---|
| $J_1$ | 0.9 | 0.1 |
| $J_2$ | 0.4 | 0.6 |

Using $I_Y$ as implication operator to calculate the upper bound :

$K3(D_1) = \min[\text{AndTop}(I_Y(.7,.9), I_Y(.5,.4)), \text{OrBot}(\text{AndTop}(.7,.9), \text{AndTop}(.5,.4))]$

$= \min(0.87, 0.72) \quad = 0.72$

$K3(D_2) = \min[\text{AndTop}(I_Y(.7,.1), I_Y(.5,.6)), \text{OrBot}(\text{AndTop}(.7,.1), \text{AndTop}(.5,.6))]$

$= \min(0.37, 0.52) \quad = 0.37$

Using $I_X$ as implication operator to calculate the upper bound :

$K3(D_1) = \min[\text{AndTop}(I_X(.7,.9), I_X(.5,.4)), \text{OrBot}(\text{AndTop}(.7,.9), \text{AndTop}(.5,.4))]$

$= \min(0.49, 0.72) \quad = 0.49$

$K3(D_2) = \min[\text{AndTop}(I_X(.7,.1), I_X(.5,.6)), \text{OrBot}(\text{AndTop}(.7,.1), \text{AndTop}(.5,.6))]$

$= \min(0.28, 0.52) \quad = 0.28$

The possibility for this patient to suffer from diseases D1 and D2 are within the range [0.72, 0.49] and [0.37, 0.28] respectively. This is clear that the chance for this patient to afflict by $D_2$ is low. Thus, we can proceed to another level in the hierarchy by running a diagnosis on $D_1$ based on the value below :

| | $S_1$ | $S_2$ | $S_3$ |
|---|---|---|---|
| $P_1$ | 0.4 | 1 | 0.9 |

| | $D_1$ |
|---|---|
| $S_1$ | 0.1 |
| $S_2$ | 0.3 |
| $S_3$ | 0.6 |

Using $I_Y$ as implication operator to calculate the upper bound :

$K3 = \min[\text{AndTop}(I_Y(.4,.1), \text{AndTop}[I_Y(1,.3), I_Y(.9,.6)]),$

$\text{OrBot}(\text{AndTop}(.4,.1), \text{OrBot}[\text{AndTop}(1,.3), \text{AndTop}(.9,.6)])]$

$= \min(0.42, 0.83) \quad = 0.42$

Using $I_X$ as implication operator to calculate the lower bound :

$$K3 = \min[\text{AndTop}(I_X(.4,.1), \text{AndTop}[I_X(1,.3), I_X(.9,.6)]), \text{OrBot}(\text{AndTop}(.4,.1), \text{OrBot}[\text{AndTop}(1,.3), \text{AndTop}(.9,.6)])]$$

$$= \min(0.31, 0.83) = 0.31$$

So, the possibility of this patient to suffer from disease $D_1$ is between the range [0.42, 0.31]. The schematic diagram of this system is shown in Figure 5.6.

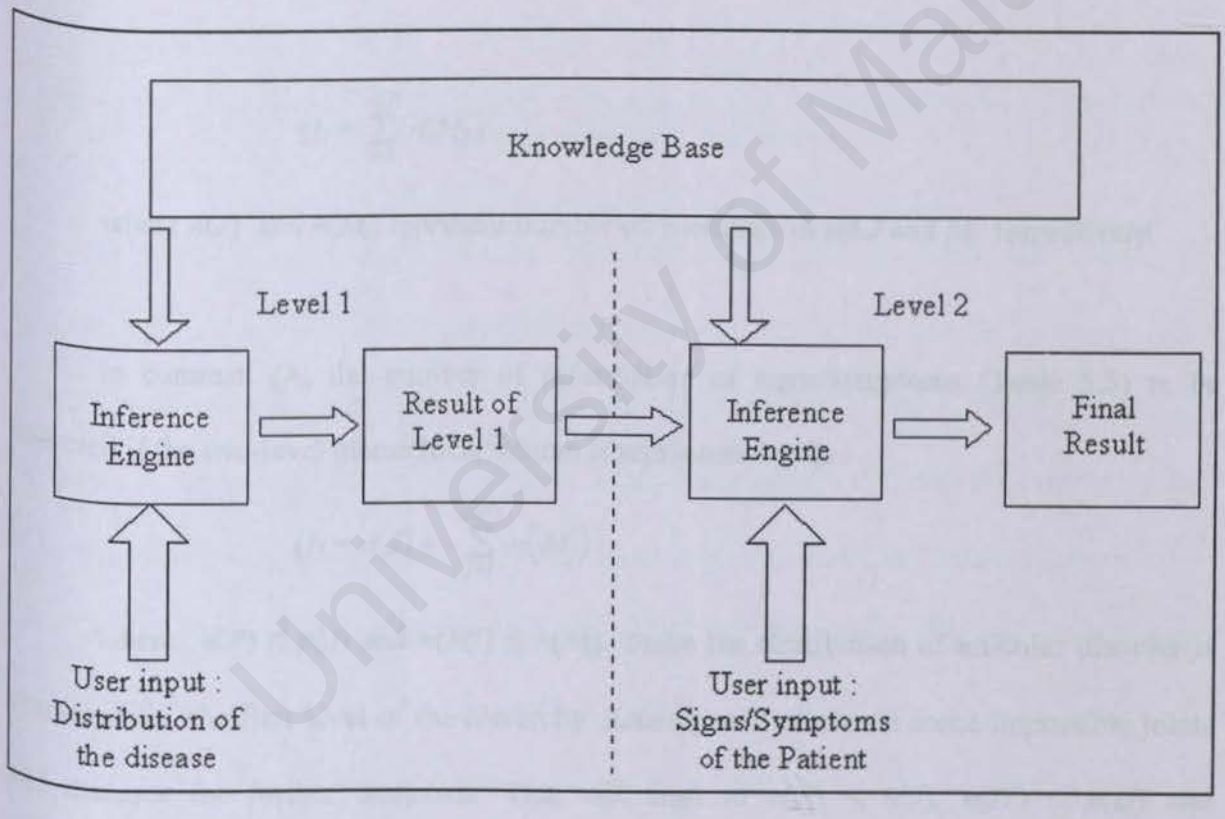


*Figure 5.6 : Work flow of the hierarchical fuzzy inference system*

## 5.5 CHAPTER SUMMARY

The hierarchical system will increase the efficiency of diagnosis in two ways,

a. fewer inputs are needed for a single diagnosis,

b. decrease number of mappings between manifestations and diseases.

Systems that do not implement such a hierarchy will need to collect information of all signs and symptoms for each joint. In other words, Information that will be collected for every diagnosis, $Q_1$ can be represented with :

$$Q_1 = \sum_{j=1}^{n(J)} n(M_j)$$

where $n(J)$ and $n(M_j)$ represent number of members in set $J$ and $M_j$ respectively.

In contrast, $Q_2$, the number of information of signs/symptoms (Table 5.3) to be collected if the two-level hierarchical system is represented with :

$$Q_2 = n(J) + \sum_{j=1}^{n(J')} n\left(M_j'\right)$$

where $n(J') \leq n(J)$ and $n(M_j') \leq n(M_j)$. Since the distribution of articular disorder is characteristic, the first level of the hierarchy certainly will eliminate some impossible joints and diseases for further diagnosis. This will lead to $n(J') < n(J)$, $n(D') < n(D)$ and $n(M_j') < n(M_j)$.

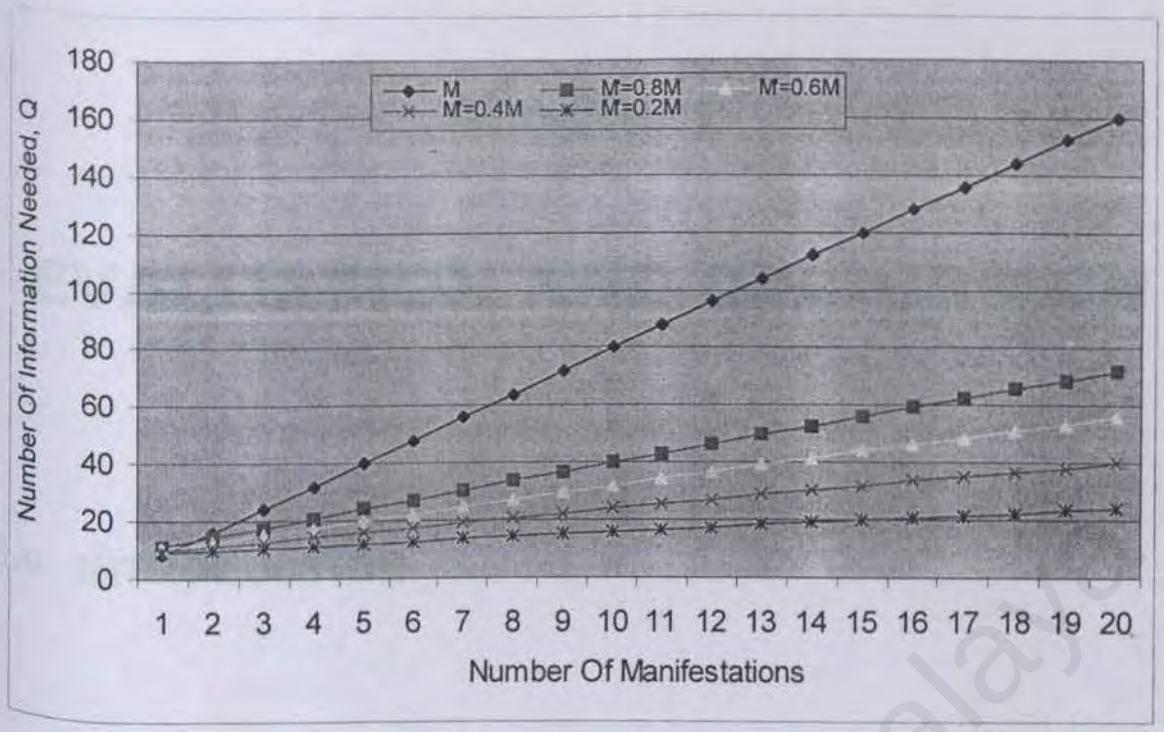


*Figure 5.7 : Increment of Q ($Q_1$ and $Q_2$) when the number of manifestations (M and M') increased. Number of joints, J = 8 and J' = 4.*

In Figure 5.7, we show that the increment of $Q_1$ and $Q_2$ when the number of manifestations increased. To make the explanation simple, we assume that the all joints show up same number of manifestations, i.e. $n(M_x) = n(M_y)$ for all $x$ and $y$ which are members of $D$. Furthermore, the number of joint to consider is fixed at a value of 25 and the number of $M_j'$ and $J'$ are considered as a fraction of $M_j$ and $J$, respectively. From the figures, it is obvious that once $n(J')$ and $n(M_j')$is smaller, $Q_2$ will be small compared to $Q_1$, thus fewer inputs are needed and fewer mappings.

The efficiency of the system can also be increased by assigning a higher degree of the fuzzy filter in the first level of hierarchy, but this will lead to decreased sensitivity of the system.

# Chapter 6

# EVALUATION AND DISCUSSION

## 6.0 INTRODUCTION

The proposed hierarchical fuzzy inference system has been built and has been evaluated. In this chapter, the results of the test are presented as well as the matrix of the evaluation is defined. The performance of the inference structures involved in this test is discussed at the end of this chapter.

# 6.1 MATRIX OF EVALUATION

## 6.1.1 TRADITIONAL EVALUATION METHODS

Traditionally, the "two by two table method" is used in analyzing dichotomized subjects in medical field. In this method, a two by two table like the one below was created (Figure 6.1):

| | Disease present | Disease absent | |
|---|---|---|---|
| Test result positive | True Positive (TP) | False Positive (FP) | All positives |
| Test result negative | False Negative (FN) | True Negative (TN) | All negatives |
| Total | All diseased | All normal | All patients |

*Figure 6.1 : A Traditional 2 X 2 table*

The result of a test might be positive (the test claim that the subject tested is suffered with the disease) or negative (the test claim that the subject tested is free from the disease). But the test result might not be always correct. For a test with positive result, the test can be correct (true positive) or wrong (false positive). Similarly, for a test with negative result, the test also can be correct (true negative) or wrong (false negative).

In a two by two table, TP, FP, TN and FN represent the numbers that reflect the clinical scenario. The sum of TP, FP, TN and FN is 1 after normalization. With these values, we can define some terms regarding the performance of the system :

$$\text{Sensitivity} = \frac{TP}{TP + FN} = \frac{TP}{\text{All diseased}}$$

$$\text{Specificity} = \frac{TN}{TN + FP} = \frac{TN}{\text{All normal}}$$

These values reflect the accuracy of an inference system. However, since the method is based on dichotomous theorem, it is facing the same problem with traditional crisp set theory. Therefore, the method is not suitable for evaluating an inference system which is dealing with fuzzy sets theory.

There are also other methods such as Bayes Theorem Methods (Table 6.1) and Likelihood Ratios (Table 6.2) which are also popular in evaluating a medical inference system, but these methods are also useless in this case because they are also based on the same TP, FP, TN and FN.

*Table 6.1 : Bayes Theorem Method*

$$\text{Positive predictive value} = \frac{TP}{TP + FP} = \frac{TP}{\text{All tested positive}}$$

$$\text{Negative predictive value} = \frac{TN}{TN + FN} = \frac{TN}{\text{All tested negative}}$$

*Table 6.2 : Likelihood Ratio*

Likelihood ratio for a positive test result

$$= \frac{\text{Probability of a positive test in patient with disease}}{\text{Probability of a positive test in patient without disease}}$$

$$= \frac{\text{sensitivity}}{1 - \text{specificity}}$$

Likelihood ratio for a negative test result

$$= \frac{\text{Probability of a negative test in patient with disease}}{\text{Probability of a negative test in patient without disease}}$$

$$= \frac{1 - \text{sensitivity}}{\text{specificity}}$$

### 6.1.2 EVALUATION METHOD FOR A FUZZY SYSTEM

Instead of simply mark a diagnosis as correct (true) or wrong (false), the result of an inference is presented with a number within the range [0, 1]. To determine whether an inference is accepted, the range is divided into three bands, i.e. "accepted band", "rejected band" and "undecided band". Result of a diagnosis may fall into one of these three bands. Accepted band means that the result of an inference is accepted to be positive, whereas rejected band means that the inference shows negative result. There are chances that an inference shows some evidence of abnormality but it is still not strong enough to be accepted, so it will fall into the undecided band.

The threshold of an inference structure can be defined depending on the design of the inference structure and the field of application. In this system, the acceptance band is defined

as the range of [0.7, 1], whereas the rejection band is defined as the range [0, 0.3]. Any value within these two bands are considered as undecided band.

To evaluate the performance of a fuzzy inference system, Yew [1995] has provided the main concept of Fuzzy True Acceptance, Fuzzy False Acceptance, Fuzzy True Rejection and Fuzzy False Rejection.

Let's assume that for a single diagnosis on one patient,

$S$ = Cardinality of the set with all diseases which are going to test. This depends on the number of diseases selected from knowledge base (at current hierarchy level) for a diagnosis.

$A$ = Cardinality of the accepted set (diagnosis result fall into accepted band). This depends on experimental results.

$R$ = Cardinality of the rejected set (diagnosis result fall into rejection band). This depends on experimental results.

$C$ = Number of correct diagnoses. This number describes the real situation of a patient.

So, we can define :

***Definition 6.1:***

a) **Fuzzy True Acceptance (FTA)** : The degree of correctness of accepted diagnosis. The value depends on A, the cardinality of the accepted set and the existence of the correct diagnosis in this set.

IF accepted set does not contains actual diseases OR $A$=0 THEN

FTA = 0

ELSE

FTA = 1/$A$

ENDIF

FTA is not defined if $C = 0$.

*Definition 6.2:*

b) **Fuzzy False Acceptance (FFA)** : The proportion of wrongly accepted diagnosis and number of incorrect diagnosis in the knowledgebase at the particular hierarchy level.

FFA = (number of wrongly accepted diagnosis) / ($S$ - $C$)

FFA is not defined if the test only involve all correct diagnosis (i.e. $S = C$).

*Definition 6.3:*

c) **Fuzzy True Rejection (FTR)** : The proportion of correctly rejected diagnosis and the number of incorrect diagnosis in the knowledgebase at the particular hierarchy level.

FTR = (number of incorrect diagnoses rejected) / ($S$ - $C$)

FFA is not defined if the test only involve all correct diagnosis (i.e. $S = C$).

*Definition 6.4:*

d) **Fuzzy False Rejection (FFR)** : The degree of rejected correct diagnosis.

FFR = Number of rejected correct diagnoses / $C$

FFR should be 0 or 1 unless the patient has more than 1 diseases. FFR also not defined for the case $C = 0$

Also, we can define Mean True Acceptance, Mean False Acceptance, Mean True Rejection and Mean False Rejection as references on the performance of an inference structures over a few test.

Assume that $T$ is the total number of cases tested using a particular inference structure, we can define :

***Definition 6.5:***

a. Mean True Acceptance (MTA) = $\frac{1}{T}\sum_j \text{FTA}_j$

b. Mean False Acceptance (MFA) = $\frac{1}{T}\sum_j \text{FFA}_j$

c. Mean True Rejection (MTR) = $\frac{1}{T}\sum_j \text{FTR}_j$

d. Mean False Rejection (MFR) = $\frac{1}{T}\sum_j \text{FFR}_j$

For a good performing fuzzy inference system, it should produce results with high MTA, high MTR, low MFA and low MFR.

It is easier to illustrate the concept of FTA, FFA, FTR and FFR through examples. Here, a few examples are given. In these example, 5 diseases are considered, they are Rheumatoid Arthritis (RA), Reiter's Syndrome (RS), Calcium Pyrophosphate Dehydrate Crystal Deposition Disease (CPPD), Systemic Lupus Erythematosus (SLE) and Behcet's Syndrome (BS). Among these diseases, RA is correct diagnose.

**Example 6.1**

Accepted diagnoses : RA

Undecided diagnoses : BS

Rejected diagnoses :CPPD, RS, SLE, BS

FTA = 1/1 = 1 FFA = 0/(5-1) = 0

FTR = 4/(5-1) = 1 FFR = 0/(5-1) = 0

**Example 6.2**

Accepted diagnoses : RA, SLE

Undecided diagnoses : BS

Rejected diagnoses : CPPD, RS

FTA = 1/1 = 1 FFA = 1/(5-1) = 0.25

FTR = 3/(5-1) = 0.75 FFR = 0/(5-1) = 0

**Example 6.3**

Accepted diagnoses : -

Undecided diagnoses : RA, SLE, BS

Rejected diagnoses : CPPD, RS

FTA = 0 FFA = 0/(5-1) = 0

FTR = 2/(5-1) = 0.5 FFR = 0/(5-1) = 0

**Example 6.4**

Accepted diagnoses : BS, RS

Undecided diagnoses : CPPD

Rejected diagnoses : SLE, RA

FTA = 0 FFA = 2/(5-1) = 0.5

FTR = 1/(5-1) = 0.25 FFR = 1/(5-1) = 0.25

## 6.2 RESULTS AND ANALYSIS

### 6.2.1 RESULTS OF EACH INFERENCE STRUCTURE

10 cases were tested using this system, all of them are patients of different kinds of joint diseases, and another one (patient 3) is free from any joint diseases. These patients are :

*Table 6.3 : Patients and their actual disease*

| Patient | Actual Disease |
|---|---|
| 1 | Gouty Arthritis |
| 2 | Gouty Arthritis |
| 3 | Health (no disease) |
| 4 | Rheumatoid Arthritis |
| 5 | Scleroderma |
| 6 | Osteoarthritis |
| 7 | Calcium Pyrophosphate Dehydrate Crystal Deposition Disease |
| 8 | Osteoarthritis |
| 9 | Rheumatoid Arthritis |
| 10 | Rheumatoid Arthritis |

All 18 inference structures generated from sub-K inference templates, and with `min` as outer connectives are tested. The results of these inference structures are presented as below:

i) K1

Table 6.4 presents the number of diseases tested, accepted, rejected and undecided in level 1 (based on distribution of abnormalities) and level 2 (based on signs and symptoms) diagnosis. The situation of actual diagnoses in each level is also shown in the last two columns. A, U and R are representing the actual diagnoses are fallen into accepted band, undecided band or rejected band respectively.

*Table 6.4 : Diagnosis result using inference structure K1*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 3 | 13 | 3 | 4 | 1 | 2 | 1 | U | A |
| 2 | 19 | 1 | 13 | 5 | 2 | 0 | 1 | 1 | U | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 2 | 17 | 2 | 0 | 1 | 1 | U | U |
| 5 | 19 | 1 | 12 | 6 | 2 | 1 | 0 | 1 | U | A |
| 6 | 19 | 1 | 11 | 7 | 4 | 1 | 0 | 3 | A | A |
| 7 | 19 | 2 | 9 | 8 | 2 | 0 | 2 | 0 | A | U |
| 8 | 19 | 1 | 11 | 7 | 4 | 0 | 3 | 1 | A | U |
| 9 | 19 | 0 | 5 | 14 | 5 | 0 | 2 | 3 | U | U |
| 10 | 19 | 0 | 2 | 17 | 2 | 0 | 1 | 1 | U | U |

Diseases which are accepted in level 1 are qualified to enter level 2. For the purpose of evaluating the performance of inference structures in both level, if the actual diagnoses are not accepted in the fist level, it is also picked to enter the second level. If none of the diseases are accepted in the first level, the actual diagnoses and all diseases that are not rejected will be tested in level 2. To avoid the case $S = C$, all tests are run with more than one disease. For patient 3, all diseases are tested in level 2.

Form table 6.4, we will get:

$MTA_1(K1) = 0.28$

$MTR_1(K1) = 0.57$

$MFA_1(K1) = 0.03$

$MFR_1(K1) = 0.00$

$MTA_2(K1) = 0.33$

$MTR_2(K1) = 0.74$

$MFA_2(K1) = 0.00$

$MFR_2(K1) = 0.00$

Subscript 1 and 2 denotes result of diagnosis for level 1 or level 2.

ii) K2

Table 6.5 shows the result of diagnosis using inference structure K2.

*Table 6.5 : Diagnosis result using inference structure K2*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 0 | 15 | 4 | 15 | 1 | 6 | 8 | U | A |
| 2 | 19 | 1 | 12 | 6 | 2 | 0 | 1 | 1 | U | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 5 | 8 | 6 | 5 | 1 | 0 | 4 | A | A |
| 5 | 19 | 1 | 12 | 6 | 2 | 1 | 0 | 1 | U | A |
| 6 | 19 | 1 | 11 | 7 | 5 | 1 | 0 | 4 | A | A |
| 7 | 19 | 2 | 9 | 8 | 2 | 0 | 2 | 0 | A | U |
| 8 | 19 | 1 | 11 | 7 | 3 | 0 | 2 | 1 | A | U |
| 9 | 19 | 0 | 5 | 14 | 5 | 0 | 2 | 3 | U | U |
| 10 | 19 | 0 | 2 | 17 | 2 | 0 | 1 | 1 | U | U |

With these value, we can calculate:

$MTA_1(K2) = 0.30$

$MTR_1(K2) = 0.52$

$MFA_1(K2) = 0.04$

$MFR_1(K2) = 0.00$

$MTA_2(K2) = 0.33$

$MTR_2(K2) = 0.78$

$MFA_2(K2) = 0.00$

$MFR_2(K2) = 0.00$

**iii) K3**

The result of diagnosis using K3 inference template is shown in Table 6.6:

*Table 6.6 : Diagnosis result using inference structure K3*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 3 | 13 | 3 | 4 | 1 | 2 | 1 | U | A |
| 2 | 19 | 1 | 14 | 4 | 2 | 0 | 1 | 1 | U | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 2 | 17 | 2 | 0 | 1 | 1 | U | U |
| 5 | 19 | 1 | 12 | 6 | 2 | 1 | 0 | 1 | U | A |
| 6 | 19 | 1 | 11 | 7 | 4 | 1 | 0 | 3 | A | A |
| 7 | 19 | 2 | 9 | 8 | 2 | 0 | 2 | 0 | A | U |
| 8 | 19 | 1 | 11 | 7 | 3 | 0 | 2 | 1 | A | U |
| 9 | 19 | 0 | 5 | 14 | 5 | 0 | 2 | 3 | U | U |
| 10 | 19 | 0 | 2 | 17 | 2 | 0 | 1 | 1 | U | U |

So, we will have:

$MTA_1(K3) = 0.28$

$MTR_1(K3) = 0.56$

$MFA_1(K3) = 0.03$

$MFR_1(K3) = 0.00$

$MTA_2(K3) = 0.33$

$MTR_2(K3) = 0.76$

$MFA_2(K3) = 0.00$

$MFR_2(K3) = 0.00$

iv) K4

Table 6.7 presents the result of diagnosis using inference structures K4

*Table 6.7 : Diagnosis result using inference structure K4*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 3 | 12 | 4 | 4 | 1 | 2 | 1 | U | A |
| 2 | 19 | 1 | 12 | 6 | 2 | 0 | 1 | 1 | U | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 2 | 17 | 2 | 0 | 1 | 1 | U | U |
| 5 | 19 | 1 | 13 | 5 | 2 | 1 | 0 | 1 | U | A |
| 6 | 19 | 1 | 11 | 7 | 4 | 1 | 0 | 3 | A | A |
| 7 | 19 | 2 | 9 | 8 | 2 | 0 | 2 | 0 | A | U |
| 8 | 19 | 1 | 11 | 7 | 3 | 0 | 2 | 1 | A | U |
| 9 | 19 | 0 | 5 | 14 | 5 | 0 | 2 | 3 | U | U |
| 10 | 19 | 0 | 2 | 17 | 2 | 0 | 1 | 1 | U | U |

$MTA_1(K4) = 0.28$

$MTR_1(K4) = 0.57$

$MFA_1(K4) = 0.03$

$MFR_1(K4) = 0.00$

$MTA_2(K4) = 0.33$

$MTR_2(K4) = 0.76$

$MFA_2(K4) = 0.00$

$MFR_2(K4) = 0.00$

v) K5

Table 6.8 presents the result of diagnosis using inference structures K5

*Table 6.8 : Diagnosis result using inference structure K5*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 0 | 0 | 19 | 7 | 0 | 0 | 7 | R | R |
| 2 | 19 | 0 | 0 | 19 | 6 | 0 | 0 | 6 | R | R |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 0 | 19 | 3 | 0 | 1 | 2 | R | U |
| 5 | 19 | 0 | 0 | 19 | 7 | 0 | 0 | 7 | R | R |
| 6 | 19 | 0 | 0 | 19 | 7 | 0 | 1 | 6 | R | U |
| 7 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | R |
| 8 | 19 | 0 | 0 | 19 | 4 | 0 | 1 | 3 | R | U |
| 9 | 19 | 0 | 0 | 19 | 5 | 0 | 0 | 5 | R | R |
| 10 | 19 | 0 | 2 | 17 | 2 | 0 | 1 | 1 | U | U |

$MTA_1(K5) = 0.00$

$MTR_1(K5) = 0.99$

$MFA_1(K5) = 0.00$

$MFR_1(K5) = 0.89$

$MTA_2(K5) = 0.00$

$MTR_2(K5) = 1.00$

$MFA_2(K5) = 0.00$

$MFR_2(K5) = 0.56$

**vi) K6**

Table 6.9 presents the result of diagnosis using inference structures K6

*Table 6.9 : Diagnosis result using inference structure K6*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 0 | 0 | 19 | 7 | 0 | 0 | 7 | R | R |
| 2 | 19 | 0 | 0 | 19 | 6 | 0 | 0 | 6 | R | R |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 0 | 19 | 3 | 0 | 1 | 2 | R | U |
| 5 | 19 | 0 | 0 | 19 | 7 | 0 | 0 | 7 | R | R |
| 6 | 19 | 0 | 0 | 19 | 7 | 0 | 1 | 6 | R | U |
| 7 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | R |
| 8 | 19 | 0 | 0 | 19 | 4 | 0 | 1 | 3 | R | U |
| 9 | 19 | 0 | 0 | 19 | 5 | 0 | 0 | 5 | R | R |
| 10 | 19 | 0 | 2 | 17 | 2 | 0 | 1 | 1 | U | U |

$MTA_1(K6) = 0.00$

$MTR_1(K6) = 0.99$

$MFA_1(K6) = 0.00$

$MFR_1(K6) = 0.89$

$MTA_2(K6) = 0.00$

$MTR_2(K6) = 1.00$

$MFA_2(K6) = 0.00$

$MFR_2(K6) = 0.56$

vi) K7

Table 6.10 presents the result of diagnosis using inference structures K7

*Table 6.10 : Diagnosis result using inference structure K7*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 1 | 11 | 7 | 2 | 0 | 2 | 0 | U | U |
| 2 | 19 | 0 | 10 | 9 | 10 | 1 | 1 | 8 | U | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 0 | 19 | 3 | 0 | 1 | 2 | R | R |
| 5 | 19 | 1 | 8 | 10 | 2 | 1 | 0 | 1 | R | A |
| 6 | 19 | 1 | 8 | 10 | 4 | 1 | 0 | 3 | A | A |
| 7 | 19 | 2 | 9 | 8 | 2 | 0 | 2 | 0 | A | U |
| 8 | 19 | 1 | 3 | 15 | 3 | 0 | 1 | 2 | A | R |
| 9 | 19 | 0 | 1 | 18 | 2 | 0 | 2 | 0 | U | U |
| 10 | 19 | 0 | 0 | 19 | 2 | 0 | 0 | 2 | R | R |

$MTA_1(K7) = 0.28$

$MTR_1(K7) = 0.72$

$MFA_1(K7) = 0.00$

$MFR_1(K7) = 0.33$

$MTA_2(K7) = 0.22$

$MTR_2(K7) = 0.59$

$MFA_2(K7) = 0.01$

$MFR_2(K7) = 0.33$

**vi) K8**

Table 6.11 presents the result of diagnosis using inference structures K8

*Table 6.11 : Diagnosis result using inference structure K8*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 0 | 13 | 6 | 13 | 0 | 2 | 11 | U | U |
| 2 | 19 | 0 | 10 | 9 | 10 | 0 | 2 | 8 | U | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 0 | 19 | 2 | 0 | 0 | 2 | R | R |
| 5 | 19 | 1 | 8 | 10 | 2 | 1 | 0 | 1 | R | A |
| 6 | 19 | 1 | 8 | 10 | 4 | 1 | 0 | 3 | A | A |
| 7 | 19 | 2 | 9 | 8 | 2 | 0 | 2 | 0 | A | U |
| 8 | 19 | 1 | 3 | 15 | 4 | 0 | 2 | 2 | A | R |
| 9 | 19 | 0 | 1 | 18 | 2 | 0 | 2 | 0 | U | U |
| 10 | 19 | 0 | 0 | 19 | 2 | 0 | 0 | 2 | R | R |

$MTA_1(K8) = 0.28$

$MTR_1(K8) = 0.72$

$MFA_1(K8) = 0.01$

$MFR_1(K8) = 0.33$

$MTA_2(K8) = 0.22$

$MTR_2(K8) = 0.71$

$MFA_2(K8) = 0.00$

$MFR_2(K8) = 0.33$

vi) K9

Table 6.12 presents the result of diagnosis using inference structures K9

*Table 6.12 : Diagnosis result using inference structure K9*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 1 | 12 | 6 | 2 | 0 | 2 | 0 | U | U |
| 2 | 19 | 0 | 10 | 9 | 10 | 1 | 1 | 8 | U | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | R |
| 5 | 19 | 1 | 8 | 10 | 2 | 1 | 0 | 1 | R | A |
| 6 | 19 | 1 | 8 | 10 | 4 | 1 | 0 | 3 | A | A |
| 7 | 19 | 2 | 10 | 7 | 2 | 0 | 2 | 0 | A | U |
| 8 | 19 | 1 | 3 | 15 | 3 | 0 | 1 | 2 | A | R |
| 9 | 19 | 0 | 1 | 18 | 2 | 0 | 2 | 0 | U | U |
| 10 | 19 | 0 | 0 | 19 | 2 | 0 | 0 | 2 | R | R |

$MTA_1(K9) = 0.28$

$MTR_1(K9) = 0.71$

$MFA_1(K9) = 0.02$

$MFR_1(K9) = 0.33$

$MTA_2(K9) = 0.22$

$MTR_2(K9) = 0.64$

$MFA_2(K9) = 0.01$

$MFR_2(K9) = 0.33$

**vi) K10**

Table 6.13 presents the result of diagnosis using inference structures K10

*Table 6.13 : Diagnosis result using inference structure K10*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 1 | 12 | 6 | 2 | 0 | 2 | 0 | U | U |
| 2 | 19 | 0 | 10 | 9 | 10 | 1 | 1 | 8 | U | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | R |
| 5 | 19 | 1 | 8 | 10 | 2 | 1 | 0 | 1 | R | A |
| 6 | 19 | 1 | 8 | 10 | 4 | 1 | 0 | 3 | A | A |
| 7 | 19 | 2 | 9 | 8 | 2 | 0 | 2 | 0 | A | U |
| 8 | 19 | 1 | 3 | 15 | 3 | 0 | 1 | 2 | A | R |
| 9 | 19 | 0 | 1 | 18 | 2 | 0 | 2 | 0 | U | U |
| 10 | 19 | 0 | 0 | 19 | 2 | 0 | 0 | 2 | R | R |

$MTA_1(K10) = 0.28$

$MTR_1(K10) = 0.72$

$MFA_1(K10) = 0.02$

$MFR_1(K10) = 0.33$

$MTA_2(K10) = 0.22$

$MTR_2(K10) = 0.64$

$MFA_2(K10) = 0.01$

$MFR_2(K10) = 0.33$

vi) K11

Table 6.14 presents the result of diagnosis using inference structures K11

*Table 6.14 : Diagnosis result using inference structure K11*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 0 | 0 | 19 | 7 | 0 | 0 | 7 | R | R |
| 2 | 19 | 0 | 0 | 19 | 6 | 0 | 0 | 6 | R | R |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 0 | 19 | 2 | 0 | 0 | 2 | R | R |
| 5 | 19 | 0 | 0 | 19 | 5 | 0 | 0 | 5 | R | R |
| 6 | 19 | 0 | 0 | 19 | 4 | 0 | 1 | 3 | R | U |
| 7 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | R |
| 8 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | R |
| 9 | 19 | 0 | 0 | 19 | 2 | 0 | 0 | 2 | R | R |
| 10 | 19 | 0 | 0 | 19 | 2 | 0 | 0 | 2 | R | R |

$MTA_1(K11) = 0.00$

$MTR_1(K11) = 1.00$

$MFA_1(K11) = 0.00$

$MFR_1(K11) = 1.00$

$MTA_2(K11) = 0.00$

$MTR_2(K11) = 1.00$

$MFA_2(K11) = 0.00$

$MFR_2(K11) = 0.89$

**vi) K12**

Table 6.15 presents the result of diagnosis using inference structures K12

*Table 6.15 : Diagnosis result using inference structure K12*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 0 | 0 | 19 | 7 | 0 | 0 | 7 | R | R |
| 2 | 19 | 0 | 0 | 19 | 6 | 0 | 0 | 6 | R | R |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 0 | 19 | 2 | 0 | 0 | 2 | R | R |
| 5 | 19 | 0 | 0 | 19 | 5 | 0 | 0 | 5 | R | R |
| 6 | 19 | 0 | 0 | 19 | 4 | 0 | 1 | 3 | R | U |
| 7 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | R |
| 8 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | R |
| 9 | 19 | 0 | 0 | 19 | 2 | 0 | 0 | 2 | R | R |
| 10 | 19 | 0 | 0 | 19 | 2 | 0 | 0 | 2 | R | R |

$MTA_1(K12) = 0.00$

$MTR_1(K12) = 1.00$

$MFA_1(K12) = 0.00$

$MFR_1(K12) = 1.00$

$MTA_2(K12) = 0.00$

$MTR_2(K12) = 1.00$

$MFA_2(K12) = 0.00$

$MFR_2(K12) = 0.89$

**vi) K13**

Table 6.16 presents the result of diagnosis using inference structures K13

*Table 6.16 : Diagnosis result using inference structure K13*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 14 | 4 | 1 | 14 | 2 | 5 | 7 | A | A |
| 2 | 19 | 6 | 10 | 3 | 6 | 0 | 1 | 5 | A | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 2 | 17 | 0 | 2 | 2 | 0 | 0 | A | A |
| 5 | 19 | 10 | 7 | 2 | 10 | 1 | 0 | 9 | A | A |
| 6 | 19 | 11 | 6 | 2 | 11 | 1 | 2 | 8 | A | A |
| 7 | 19 | 2 | 9 | 8 | 2 | 0 | 2 | 0 | A | U |
| 8 | 19 | 12 | 5 | 2 | 12 | 1 | 3 | 8 | A | A |
| 9 | 19 | 7 | 11 | 1 | 7 | 0 | 2 | 5 | A | U |
| 10 | 19 | 2 | 17 | 0 | 2 | 1 | 0 | 1 | A | A |

$MTA_1(K13) = 0.24$

$MTR_1(K13) = 0.21$

$MFA_1(K13) = 0.32$

$MFR_1(K13) = 0.00$

$MTA_2(K13) = 0.56$

$MTR_2(K13) = 0.70$

$MFA_2(K13) = 0.11$

$MFR_2(K13) = 0.00$

**vi) K14**

Table 6.17 presents the result of diagnosis using inference structures K14

*Table 6.17 : Diagnosis result using inference structure K14*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 0 | 15 | 4 | 15 | 2 | 5 | 8 | U | A |
| 2 | 19 | 2 | 13 | 4 | 3 | 0 | 1 | 2 | U | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 2 | 16 | 1 | 2 | 1 | 1 | 0 | A | A |
| 5 | 19 | 10 | 7 | 2 | 10 | 1 | 0 | 9 | A | A |
| 6 | 19 | 10 | 7 | 2 | 10 | 1 | 2 | 7 | A | A |
| 7 | 19 | 2 | 9 | 8 | 2 | 0 | 2 | 0 | A | U |
| 8 | 19 | 12 | 5 | 2 | 12 | 1 | 3 | 8 | A | A |
| 9 | 19 | 4 | 10 | 5 | 4 | 0 | 2 | 2 | A | U |
| 10 | 19 | 2 | 16 | 1 | 2 | 1 | 0 | 1 | A | A |

$MTA_1(K14) = 0.23$

$MTR_1(K14) = 0.26$

$MFA_1(K14) = 0.21$

$MFR_1(K14) = 0.00$

$MTA_2(K14) = 0.61$

$MTR_2(K14) = 0.67$

$MFA_2(K14) = 0.01$

$MFR_2(K14) = 0.00$

**vi) K15**

Table 6.18 presents the result of diagnosis using inference structures K15

*Table 6.18 : Diagnosis result using inference structure K15*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 18 | 1 | 0 | 18 | 4 | 5 | 10 | A | A |
| 2 | 19 | 12 | 6 | 1 | 12 | 1 | 1 | 10 | A | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 2 | 17 | 0 | 2 | 2 | 0 | 0 | A | A |
| 5 | 19 | 14 | 4 | 1 | 14 | 1 | 0 | 13 | A | A |
| 6 | 19 | 15 | 3 | 1 | 15 | 1 | 1 | 13 | A | A |
| 7 | 19 | 2 | 9 | 8 | 2 | 1 | 1 | 0 | A | A |
| 8 | 19 | 15 | 3 | 1 | 15 | 2 | 2 | 11 | A | A |
| 9 | 19 | 11 | 8 | 0 | 11 | 0 | 2 | 9 | A | U |
| 10 | 19 | 2 | 17 | 0 | 2 | 1 | 0 | 1 | A | A |

$MTA_1(K15) = 0.21$

$MTR_1(K15) = 0.17$

$MFA_1(K15) = 0.46$

$MFR_1(K15) = 0.00$

$MTA_2(K15) = 0.58$

$MTR_2(K15) = 0.71$

$MFA_2(K15) = 0.13$

$MFR_2(K15) = 0.00$

**vi) K16**

Table 6.19 presents the result of diagnosis using inference structures K16

*Table 6.19 : Diagnosis result using inference structure K16*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 13 | 2 | 4 | 13 | 1 | 5 | 7 | A | A |
| 2 | 19 | 7 | 9 | 3 | 7 | 0 | 1 | 6 | A | U |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 2 | 16 | 1 | 2 | 2 | 0 | 0 | A | A |
| 5 | 19 | 13 | 5 | 1 | 13 | 1 | 0 | 12 | A | A |
| 6 | 19 | 14 | 4 | 1 | 14 | 1 | 1 | 12 | A | A |
| 7 | 19 | 2 | 9 | 8 | 2 | 1 | 1 | 0 | A | A |
| 8 | 19 | 15 | 3 | 1 | 15 | 1 | 3 | 11 | A | A |
| 9 | 19 | 11 | 6 | 2 | 11 | 0 | 2 | 9 | A | U |
| 10 | 19 | 2 | 17 | 0 | 2 | 1 | 0 | 1 | A | A |

$MTA_1(K16) = 0.23$

$MTR_1(K16) = 0.22$

$MFA_1(K16) = 0.39$

$MFR_1(K16) = 0.00$

$MTA_2(K16) = 0.72$

$MTR_2(K16) = 0.72$

$MFA_2(K16) = 0.01$

$MFR_2(K16) = 0.00$

**vi) K17**

Table 6.20 presents the result of diagnosis using inference structures K17

*Table 6.20 : Diagnosis result using inference structure K17*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 0 | 0 | 19 | 7 | 0 | 0 | 7 | R | R |
| 2 | 19 | 0 | 0 | 19 | 6 | 0 | 0 | 6 | R | R |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 10 | 9 | 10 | 0 | 1 | 9 | U | U |
| 5 | 19 | 0 | 0 | 19 | 11 | 0 | 0 | 11 | R | R |
| 6 | 19 | 0 | 0 | 19 | 4 | 0 | 1 | 3 | R | U |
| 7 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | R |
| 8 | 19 | 0 | 0 | 19 | 4 | 0 | 1 | 3 | R | U |
| 9 | 19 | 0 | 0 | 19 | 5 | 0 | 0 | 5 | R | R |
| 10 | 19 | 0 | 8 | 11 | 8 | 0 | 1 | 7 | U | U |

$MTA_1(K17) = 0.00$

$MTR_1(K17) = 0.91$

$MFA_1(K17) = 0.00$

$MFR_1(K17) = 0.78$

$MTA_2(K17) = 0.00$

$MTR_2(K17) = 1.00$

$MFA_2(K17) = 0.00$

$MFR_2(K17) = 0.56$

**vi) K18**

Table 6.21 presents the result of diagnosis using inference structures K18

*Table 6.21 : Diagnosis result using inference structure K18*

| P | Level 1 Diagnosis | | | | Level 2 Diagnosis | | | | Result Of Diagnosis For Level | |
|---|---|---|---|---|---|---|---|---|---|---|
| | No. Of Diseases Involved | Accepted | Undecided | Rejected | No. Of Diseases Involved | Accepted | Undecided | Rejected | 1 | 2 |
| 1 | 19 | 0 | 0 | 19 | 7 | 0 | 0 | 7 | R | R |
| 2 | 19 | 0 | 0 | 19 | 6 | 0 | 0 | 6 | R | R |
| 3 | 19 | 0 | 0 | 19 | 19 | 0 | 0 | 19 | - | - |
| 4 | 19 | 0 | 10 | 9 | 5 | 0 | 1 | 4 | U | U |
| 5 | 19 | 0 | 0 | 19 | 11 | 0 | 0 | 11 | R | R |
| 6 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | U |
| 7 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | R |
| 8 | 19 | 0 | 0 | 19 | 4 | 0 | 1 | 3 | R | U |
| 9 | 19 | 0 | 0 | 19 | 4 | 0 | 0 | 4 | R | R |
| 10 | 19 | 0 | 8 | 11 | 3 | 0 | 1 | 2 | U | U |

$MTA_1(K18) = 0.00$

$MTR_1(K18) = 0.97$

$MFA_1(K18) = 0.00$

$MFR_1(K18) = 0.78$

$MTA_2(K18) = 0.00$

$MTR_2(K18) = 0.11$

$MFA_2(K18) = 0.00$

$MFR_2(K18) = 0.67$

### 6.2.2 COMPARING AND ANALYZING PERFORMANCE OF INFERENCE STRUCTURES

An inference structure always performs two tasks at the same time, i.e. :

1) accepting only all correct diagnoses
2) rejecting only all incorrect diagnoses

For the first task, we have two measurements to describe how well an inference structure performs : MTA indicates how well an inference structure accepts correct diagnoses and MFA indicates whether incorrect diagnoses can be accepted simultaneously.

On the other hand, MTR and MFR describes how well an inference structure performs the second task. MTR indicates how well is the inference structure rejects incorrect diagnoses, whereas MFR indicates whether the correct diagnoses is also rejected. For a good inference structure. It should have high MTA, low MFA, high MTR and low MFR.

Table 6.22 lists MTA, MTR, MFA and MFR of all the sub-K inference structures which have been tested. The performance of the inference structures in level 1 and level 2 are evaluated separately. As usual, we will use 0.7 and 0.3 as acceptance or rejection boundaries of the performance of inference structures.

*Table 6.22 : MTA, MTR, MFA and MFR of all the sub-K inference structures*

| Inference Structures | LEVEL 1 | | | | LEVEL 2 | | | |
|---|---|---|---|---|---|---|---|---|
| | MTA | MTR | MFA | MFR | MTA | MTR | MFA | MFR |
| K1 | 0.28 | 0.57 | 0.03 | 0.00 | 0.33 | 0.74 | 0.00 | 0.00 |
| K2 | 0.30 | 0.52 | 0.04 | 0.00 | 0.33 | 0.78 | 0.00 | 0.00 |
| K3 | 0.28 | 0.56 | 0.03 | 0.00 | 0.33 | 0.76 | 0.00 | 0.00 |
| K4 | 0.28 | 0.57 | 0.03 | 0.00 | 0.33 | 0.76 | 0.00 | 0.00 |
| K5 | 0.00 | 0.99 | 0.00 | 0.89 | 0.00 | 1.00 | 0.00 | 0.56 |
| K6 | 0.00 | 0.99 | 0.00 | 0.89 | 0.00 | 1.00 | 0.00 | 0.56 |
| K7 | 0.28 | 0.72 | 0.02 | 0.33 | 0.22 | 0.59 | 0.01 | 0.33 |
| K8 | 0.28 | 0.72 | 0.01 | 0.33 | 0.22 | 0.71 | 0.00 | 0.33 |
| K9 | 0.28 | 0.71 | 0.02 | 0.33 | 0.22 | 0.64 | 0.01 | 0.33 |
| K10 | 0.28 | 0.72 | 0.02 | 0.33 | 0.22 | 0.64 | 0.01 | 0.33 |
| K11 | 0.00 | 1.00 | 0.00 | 1.00 | 0.00 | 1.00 | 0.00 | 0.89 |
| K12 | 0.00 | 1.00 | 0.00 | 1.00 | 0.00 | 1.00 | 0.00 | 0.89 |
| K13 | 0.24 | 0.21 | 0.32 | 0.00 | 0.56 | 0.70 | 0.11 | 0.00 |
| K14 | 0.23 | 0.26 | 0.21 | 0.00 | 0.61 | 0.67 | 0.01 | 0.00 |
| K15 | 0.21 | 0.17 | 0.46 | 0.00 | 0.58 | 0.71 | 0.13 | 0.00 |
| K16 | 0.23 | 0.22 | 0.39 | 0.00 | 0.72 | 0.72 | 0.01 | 0.00 |
| K17 | 0.00 | 0.91 | 0.00 | 0.78 | 0.00 | 1.00 | 0.00 | 0.56 |
| K18 | 0.00 | 0.97 | 0.00 | 0.78 | 0.00 | 1.00 | 0.00 | 0.67 |

Graphs of these measurements can be plotted to get a clearer picture on the performance of these inference structures. Below, figure 6.2 and figure 6.3 shows the performance of sub-K inference structures in level 1, whereas figure 6.4 and 6.5 show the performance in level 2.

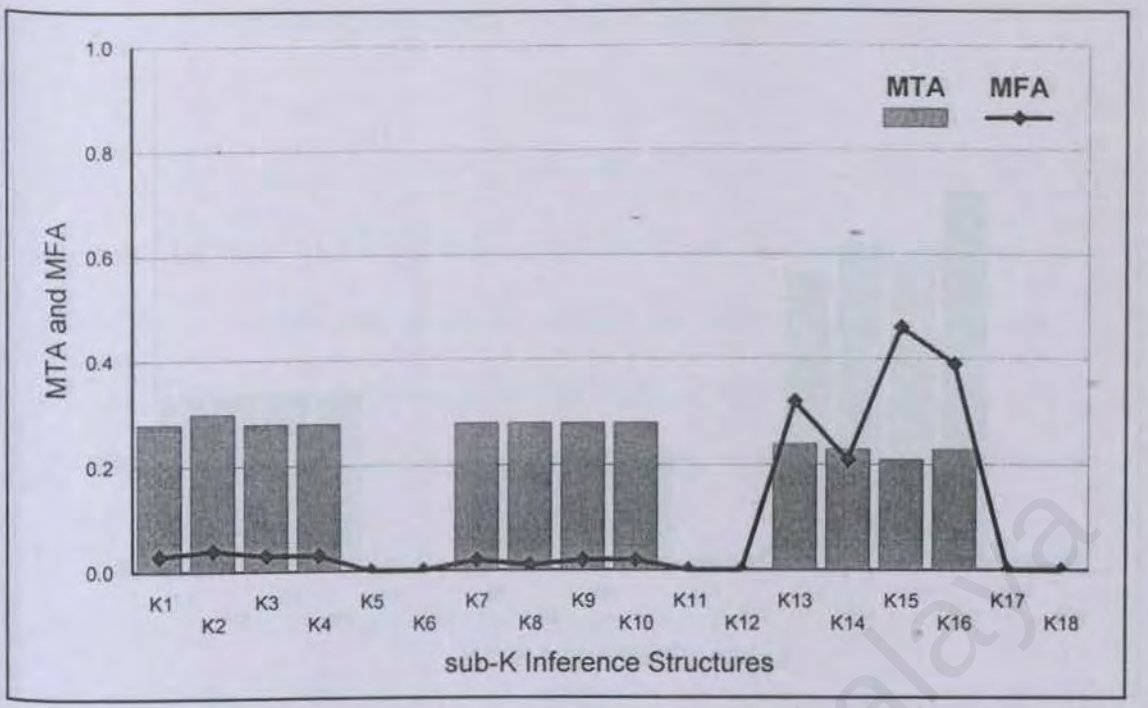


*Figure 6.2 : MTA and MFA in level 1*

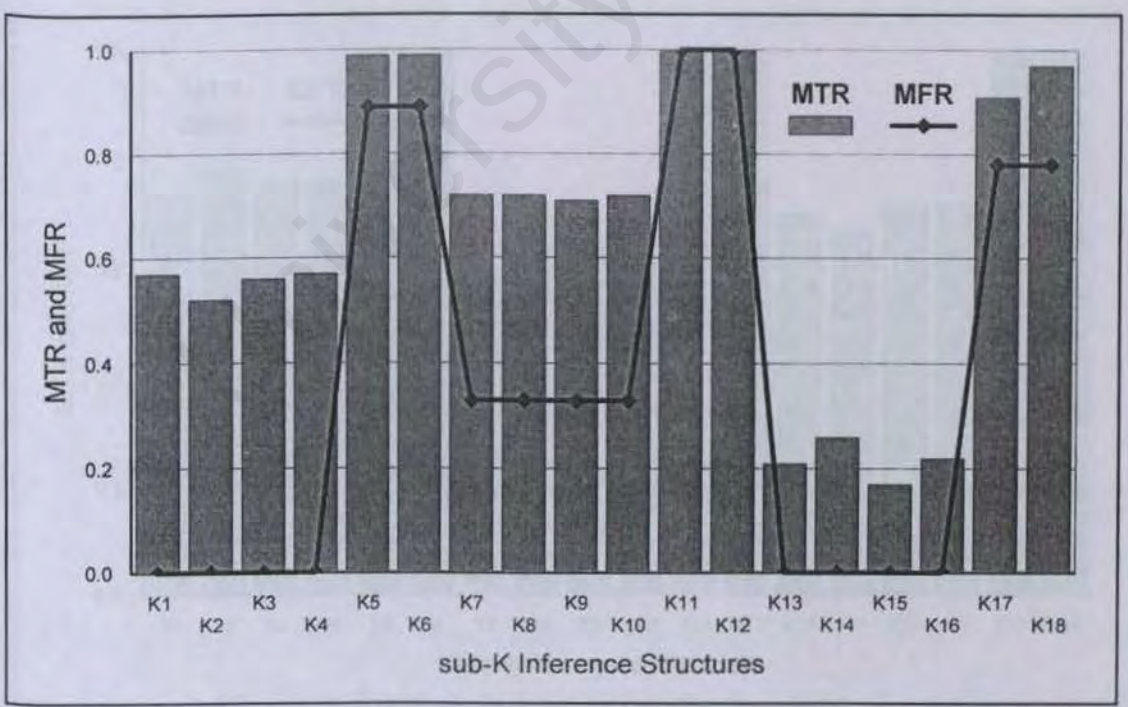


*Figure 6.3 : MTR and MFR in level 1*

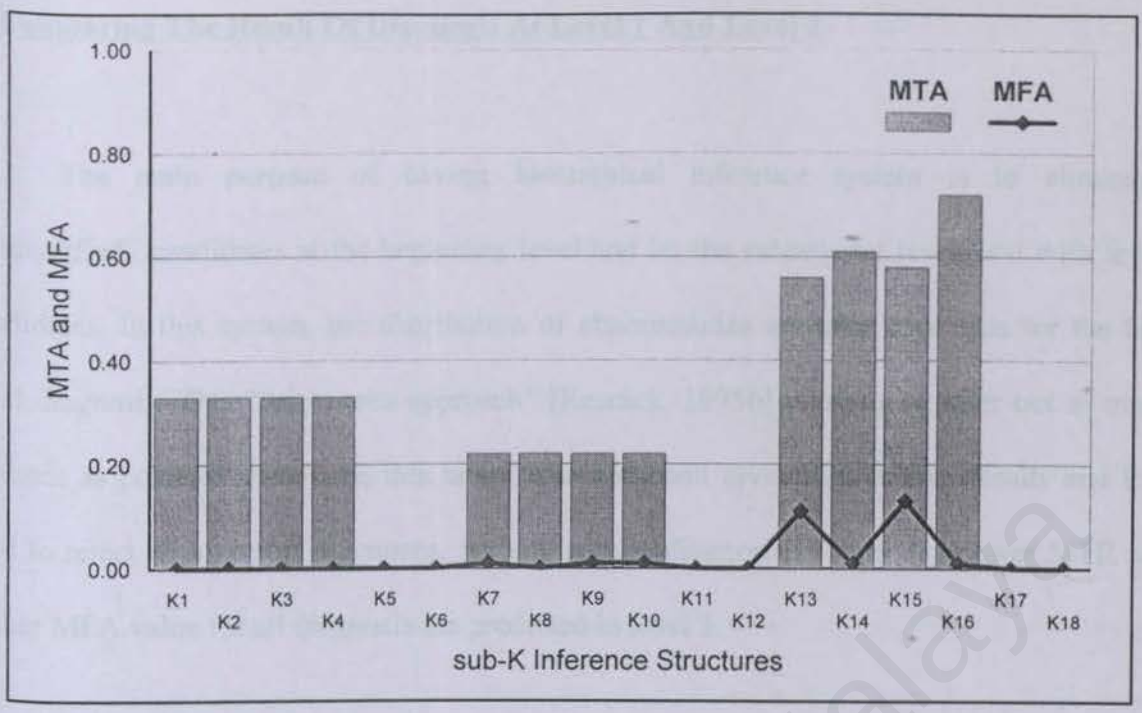


*Figure 6.4 : MTA and MFA in level 2*

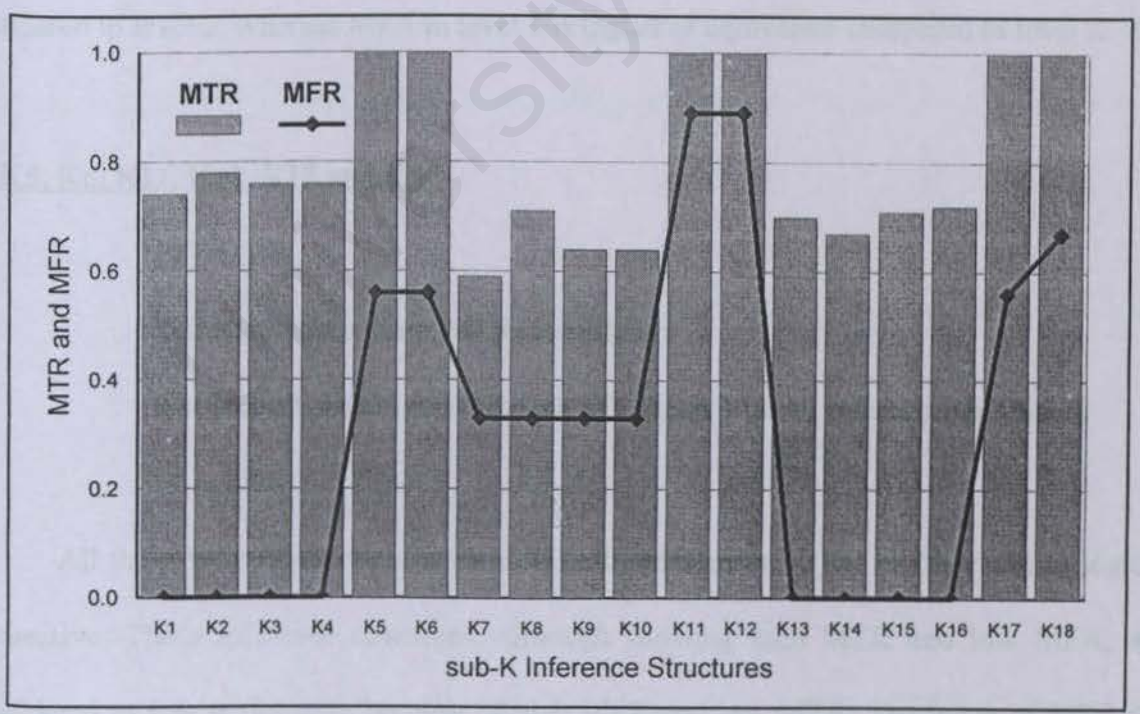


*Figure 6.5 : MTR and MFR in level 2*

**a) Comparing The Result Of Diagnosis At Level 1 And Level 2**

The main purpose of having hierarchical inference system is to eliminates "unqualified" candidates at the beginning level and let the subsequent level deal with fewer candidates. In this system, the distribution of abnormalities are used as a clue for the first level diagnosis. This "target area approach" [Resnick, 1995b] is trying to filter out as many diseases as possible. However, this target area approach gives less precise results and it is hard to reject all incorrect diagnoses. So, compare to diagnosis in level 2, a lower MTR and higher MFA value for all diagnosis are predicted in level 1.

The experimental results confirm that the prediction is true for all inference structures, except K5, K6, K11, K12, K17 and K18, which do not perform well and will be discussed later. For others inference structures, the results shows lower or equivalent MTR in level 1 compared to level 2, whereas MFA in level 1 is higher or equivalent compared to level 2.

**b) K5, K6, K11, K12, K17 and K18**

The advantages : high MTR; low MFA

The limitations : high MFR; low MTA (get 0 for all inference structures)

All these inference structures employs arithmetic mean ($\Sigma$) as the third fuzzy logical connective. These inference structures, although showing high MTR and low MFA, are considered as low performing because of high MFR and low MTA. The value of MFR for these inference structures are in the range of 0.56 to 1, whereas have 0 for MTA of all. In

other words, they tend not to accept all diagnoses, either rejecting them or making them as undecided.

**c) K1, K2, K3 and K4**

The advantages : high MTR; low MFA and MFR

The limitation : medium MTA

These inference structures, which using AndTop as their second fuzzy logical connective, work as "conservatives" inference structures. All these structures have 0 for MFR, low MFA (range from 0 to 0.04) and high MTR (0.5X for level 1 and 0.7X for level 2). However, they do not perform well when come to the value of MTA -- the highest value of MTA in level 2 is only 0.33. These inference structures are described as "conservative" because they are hard to wrongly reject correct diagnoses and almost never accepts a wrong diagnoses. However, the acceptance of correct diagnoses seems a bit low here.

**d) K7, K8, K9 and K10**

The advantages : high MTR; low MFA

The limitations: medium MTA and MFR

These inference structures which employing AndBot as second fuzzy logical connective do not perform well in this test. With having MFR value of 0.33 for all inference structures, and MTA in the range of 0.22 to 0.28, the number of correctly accepted diseases is lower than the number of wrongly rejected diseases.

**d) K13, K14, K15 and K16**

The advantages : MTA - high in level 2 and medium in level 1;
MTR - high in level 2 and medium in level 1;
MFA - low in level 2 and medium in level 1;
Low MFR.

The limitation : MTA and MTR still need improvements.

Overall, these inference structures, especially K16 have the best performance among all. The common entity among these inference structures is that it employs arithmetic mean ($\Sigma$) as the second fuzzy logical connective. Although these inference structures have medium value of MTA, MTR and MFA in level 1, but this is acceptable because they are just trying to reduce the number of candidates for level 2 and they do not wrongly reject any correct diagnoses in this level. Anyway, it will be better if the value of MTA and MTR can be increased, especially in level 2.

## 6.3 DISCUSSION

### 6.3.1 RANKING OF INFERENCE STRUCTURES

The ranking of inference structures can be done in two ways : rank according to each measurement (MTA, MFA, MTR, MFR) separately, or assume all the four measurements as one measurement system to all inference structures. In either way, we must rank the performance of inference structures at level 1 and level 2 separately.

Table 6.23 to 6.30 shows the ranking of inference structures in level 1 and level 2 according to each measurement separately. Inference structures with higher MTA, higher MTR, lower MFA or lower MFR is ranked higher.

*Table 6.23 : Ranking of inference structures in level 1 based on MTA*

| Ranking | Inference Structures | MTA |
|---|---|---|
| 1 | K2 | 0.30 |
| 2 | K1, K3, K4, K7, K8, K9, K10 | 0.28 |
| 3 | K13 | 0.24 |
| 4 | K16, K14 | 0.23 |
| 5 | K15 | 0.21 |
| 6 | K5, K6, K11, K12, K17, K18 | 0.00 |

*Table 6.24 : Ranking of inference structures in level 1 based on MFA*

| Ranking | Inference Structures | MFA |
|---|---|---|
| 1 | K5, K6, K11, K12, K17, K18 | 0.00 |
| 2 | K8 | 0.01 |
| 3 | K7, K9, K10 | 0.02 |
| 4 | K1, K3, K4 | 0.03 |
| 5 | K2 | 0.04 |
| 6 | K14 | 0.21 |
| 7 | K13 | 0.32 |
| 8 | K16 | 0.39 |
| 9 | K15 | 0.46 |

*Table 6.25 : Ranking of inference structures in level 1 based on MTR*

| Ranking | Inference Structures | MTR |
|---|---|---|
| 1 | K11, K12 | 1.00 |
| 2 | K5, K6 | 0.99 |
| 3 | K18 | 0.97 |
| 4 | K17 | 0.91 |
| 5 | K7, K8, K10 | 0.72 |
| 6 | K9 | 0.71 |
| 7 | K1, K4 | 0.57 |
| 8 | K3 | 0.56 |
| 9 | K2 | 0.52 |
| 10 | K14 | 0.26 |
| 11 | K16 | 0.22 |
| 12 | K13 | 0.21 |
| 13 | K15 | 0.17 |

*Table 6.26 : Ranking of inference structures in level 1 based on MFR*

| Ranking | Inference Structures | MFR |
|---|---|---|
| 1 | K1, K2, K3, K4, K13, K14, K15, K16 | 0 |
| 2 | K7, K8, K9, K10 | 0.33 |
| 3 | K17, K18 | 0.78 |
| 4 | K5, K6 | 0.89 |
| 5 | K11, K12 | 1 |

*Table 6.27 : Ranking of inference structures in level 2 based on MTA*

| Ranking | Inference Structures | MTA |
|---|---|---|
| 1 | K16 | 0.72 |
| 2 | K14 | 0.61 |
| 3 | K15 | 0.58 |
| 4 | K13 | 0.56 |
| 5 | K1, K2, K3, K4 | 0.33 |
| 6 | K7, K8, K9, K10 | 0.22 |
| 7 | K5, K6, K11, K12, K17, K18 | 0 |

*Table 6.28 : Ranking of inference structures in level 2 based on MFA*

| Ranking | Inference Structures | MFA |
|---|---|---|
| 1 | K1, K2, K3, K4, K5, K6, K8, K11, K12, K17, K18 | 0 |
| 2 | K7, K9, K10, K14, K16 | 0.01 |
| 3 | K13 | 0.11 |
| 4 | K15 | 0.13 |

*Table 6.29 : Ranking of inference structures in level 2 based on MTR*

| Ranking | Inference Structures | MTR |
|---|---|---|
| 1 | K5, K6, K11, K12, K17, K18 | 1 |
| 2 | K2 | 0.78 |
| 3 | K3, K4 | 0.76 |
| 4 | K1 | 0.74 |
| 5 | K16 | 0.72 |
| 6 | K8, K15 | 0.71 |
| 7 | K13 | 0.7 |
| 8 | K14 | 0.67 |
| 9 | K9, K10 | 0.64 |
| 10 | K7 | 0.59 |

*Table 6.30 : Ranking of inference structures in level 2 based on MFR*

| Ranking | Inference Structures | MFR |
|---|---|---|
| 1 | K1, K2, K3, K4, K13, K14, K15, K16 | 0 |
| 2 | K7, K8, K9, K10 | 0.33 |
| 3 | K5, K6, K17 | 0.56 |
| 4 | K18 | 0.67 |
| 5 | K11, K12 | 0.89 |

To determine an inference structure with the best performance, we must rank all inference structures using all measurements with the following order of priority : MTA,

MTR, MFA and MFR. Table 6.31 and 6.32 shows the ranking at level 1 and level 2 respectively.

*Table 6.31 : Ranking of inference structures according to results in Level 1*

| Ranking | Inference Structures | Measurement | | | |
|---|---|---|---|---|---|
| | | MTA | MTR | MFA | MFR |
| 1 | K2 | 0.30 | 0.52 | 0.04 | 0.00 |
| 2 | K8 | 0.28 | 0.72 | 0.01 | 0.33 |
| 3 | K7, K10 | 0.28 | 0.72 | 0.02 | 0.33 |
| 4 | K9 | 0.28 | 0.71 | 0.02 | 0.33 |
| 5 | K1, K4 | 0.28 | 0.57 | 0.03 | 0.00 |
| 6 | K3 | 0.28 | 0.56 | 0.03 | 0.00 |
| 7 | K13 | 0.24 | 0.21 | 0.32 | 0.00 |
| 8 | K14 | 0.23 | 0.26 | 0.21 | 0.00 |
| 9 | K16 | 0.23 | 0.22 | 0.39 | 0.00 |
| 10 | K15 | 0.21 | 0.17 | 0.46 | 0.00 |
| 11 | K11, K12 | 0.00 | 1.00 | 0.00 | 1.00 |
| 12 | K5, K6 | 0.00 | 0.99 | 0.00 | 0.89 |
| 13 | K18 | 0.00 | 0.97 | 0.00 | 0.78 |
| 14 | K17 | 0.00 | 0.91 | 0.00 | 0.78 |

*Table 6.32 : Ranking of inference structures according to results in Level 2*

| Ranking | Inference Structures | Measurement | | | |
|---|---|---|---|---|---|
| | | MTA | MTR | MFA | MFR |
| 1 | K16 | 0.72 | 0.72 | 0.01 | 0.00 |
| 2 | K14 | 0.61 | 0.67 | 0.01 | 0.00 |
| 3 | K15 | 0.58 | 0.71 | 0.13 | 0.00 |
| 4 | K13 | 0.56 | 0.70 | 0.11 | 0.00 |
| 5 | K2 | 0.33 | 0.78 | 0.00 | 0.00 |
| 6 | K3, K4 | 0.33 | 0.76 | 0.00 | 0.00 |
| 7 | K1 | 0.33 | 0.74 | 0.00 | 0.00 |
| 8 | K8 | 0.22 | 0.71 | 0.00 | 0.33 |
| 9 | K9, K10 | 0.22 | 0.64 | 0.01 | 0.33 |
| 10 | K7 | 0.22 | 0.59 | 0.01 | 0.33 |
| 11 | K5, K6, K17 | 0.00 | 1.00 | 0.00 | 0.56 |
| 12 | K18 | 0.00 | 1.00 | 0.00 | 0.67 |
| 13 | K11, K12 | 0.00 | 1.00 | 0.00 | 0.89 |

In level 1, the highest score in MTA is only 0.30 by K2. It seems that the result is not so good but actually it is still acceptable because the performance is restricted by the ability of target area approach.

The performance of K7, K10, K8 and K9 are comparable to K2 in level 1. MTA of these inference structures are only a bit lower than K2 (0.28 vs. 0.30), but having much higher MTR (0.71 or 0.72 vs. 0.52). So, the result of these inference structures should not be ignore during application.

K16 has the best performance in level 2. Compare to other inference structures in this level, the gap of performance is obvious: 0.11 higher score in MTA than the second (K14) in this level. Even without comparison, the score of 0.72 for MTA is also considered high and K16 should become the only choice if inference structures in level 2.

### 6.3.2 PERFORMANCE COMPARISON

Yew [1995] had developed a medical diagnosis system based on Bandler and Kohout's fuzzy relational theory before the theory was improved [Hallam 1999]. Since this dissertation is based on the improved version of fuzzy inference structures, it is worth while to compare the results of both the systems.

In Yew's system, 19 inference structures were evaluated by using 4 methods. Among these 19 inference structures, some used max as the outermost logical connective, which has been discarded by Hallam [1998]. In addition, this system uses different set of notation for its inference structures as compared to Yew's one. Table 6.33 shows a comparison of notation between these two systems.

*Table 6.33 : Comparison of notation between this system and Yew's system*

| No | This System | Yew's System |
|---|---|---|
| 1 | K1 | $K13_{YEW}$ |
| 2 | K2 | $K14_{YEW}$ |
| 3 | K3 | $K18_{YEW}$ |
| 4 | K4 | $K19_{YEW}$ |
| 5 | K5 | Not tested |
| 6 | K6 | Not tested |
| 7 | K7 | Not tested |
| 8 | K8 | Not tested |
| 9 | K9 | $K16_{YEW}$ |
| 10 | K10 | $K15_{YEW}$ |
| 11 | K11 | Not tested |
| 12 | K12 | Not tested |
| 13 | K13 | $K12_{YEW}$ |
| 14 | K14 | $K10_{YEW}$ |
| 15 | K15 | $K9_{YEW}$ |
| 16 | K16 | $K7_{YEW}$ |
| 17 | K17 | $K1_{YEW}$ |
| 18 | K18 | $K2_{YEW}$ |

In Yew's system, the test was ran using both fuzzy and crisp data. Also, for a sign/symptom found on a patient but not existing in the knowledge base of the particular disease, Yew has tested two methods, i.e. ignore this sign/symptom (without material paradox) or treat it as the sign/symptom has 0 value for the disease (with material paradox).

In Table 6.34 to Table 6.41, a comparison between the result from level 2 of this system and result from disease diagnosis (deterministic) of Yew's system. Since Yew has tested his system using 4 difference methods (fuzzy data without paradox, crisp data without paradox, fuzzy data with paradox and crisp data with paradox), results of all these methods are compared.

On the other hand, Yew has tested his system with a range of acceptance threshold (0.6 to 0.9), so the MTA with 0.7 as the threshold (denoted as $MTA_{YEW}$) and the biggest MTA in this range (denoted as $MTA_{MAX}$) were listed in these tables. Also, MTR (denote $MTR_{YEW}$) at threshold 0.7 and $MTR_{ATR}$ (denote MTR at the threshold of maximum $MTA_{MAX}$). In both systems, rejection threshold is set at 0.3.

We look for higher MTA and MTR, lower MFA and MFR in the comparison.

*Table 6.34 : Comparing MTA and MTR of this system with Yew's system (fuzzy data, no paradox)*

| No | Inference Structures | This System | | Yew's System | | |
|---|---|---|---|---|---|---|
| | | MTA | MTR | $MTA_{YEW}$ | $MTA_{MAX}$ | $MTR_{YEW}$ |
| 1 | K1 / $K13_{YEW}$ | 0.33 | 0.74 | 0.19 | 0.19 | 0.56 |
| 2 | K2 / $K14_{YEW}$ | 0.33 | 0.78 | 0.03 | 0.24 | 0.67 |
| 3 | K3 / $K18_{YEW}$ | 0.33 | 0.76 | 0.19 | 0.19 | 0.56 |
| 4 | K4 / $K19_{YEW}$ | 0.33 | 0.76 | 0.16 | 0.25 | 0.66 |
| 5 | K9 / $K16_{YEW}$ | 0.22 | 0.64 | 0.02 | 0.03 | 0.56 |
| 6 | K10 / $K15_{YEW}$ | 0.22 | 0.64 | 0.10 | 0.10 | 0.66 |
| 7 | K13 / $K12_{YEW}$ | 0.56 | 0.70 | 0.23 | 0.23 | 0.56 |
| 8 | K14 / $K10_{YEW}$ | 0.61 | 0.67 | 0.03 | 0.23 | 0.67 |
| 9 | K15 / $K9_{YEW}$ | 0.58 | 0.71 | 0.23 | 0.23 | 0.56 |
| 10 | K16 / $K7_{YEW}$ | 0.72 | 0.72 | 0.38 | 0.38 | 0.66 |
| 11 | K17 / $K1_{YEW}$ | 0.00 | 1.00 | 0.21 | 0.21 | 0.56 |
| 12 | K18 / $K2_{YEW}$ | 0.00 | 1.00 | 0.10 | 0.10 | 0.67 |

*Table 6.35 : Comparing MFA and MFR of this system with Yew's system (fuzzy data, no paradox)*

| No | Inference Structures | This System | | Yew's System | | |
|---|---|---|---|---|---|---|
| | | MFA | MFR | $MFA_{YEW}$ | $MFA_{ATR}$ | $MFR_{YEW}$ |
| 1 | K1 / $K13_{YEW}$ | 0.00 | 0.00 | 0.13 | 0.13 | 0.00 |
| 2 | K2 / $K14_{YEW}$ | 0.00 | 0.00 | 0.04 | 0.02 | 0.00 |
| 3 | K3 / $K18_{YEW}$ | 0.00 | 0.00 | 0.13 | 0.13 | 0.00 |
| 4 | K4 / $K19_{YEW}$ | 0.00 | 0.00 | 0.05 | 0.01 | 0.00 |
| 5 | K9 / $K16_{YEW}$ | 0.01 | 0.33 | 0.06 | 0.01 | 0.00 |
| 6 | K10 / $K15_{YEW}$ | 0.01 | 0.33 | 0.01 | 0.01 | 0.00 |
| 7 | K13 / $K12_{YEW}$ | 0.11 | 0.00 | 0.14 | 0.14 | 0.00 |
| 8 | K14 / $K10_{YEW}$ | 0.01 | 0.00 | 0.07 | 0.02 | 0.00 |
| 9 | K15 / $K9_{YEW}$ | 0.13 | 0.00 | 0.14 | 0.14 | 0.00 |
| 10 | K16 / $K7_{YEW}$ | 0.01 | 0.00 | 0.05 | 0.05 | 0.00 |
| 11 | K17 / $K1_{YEW}$ | 0.00 | 0.56 | 0.12 | 0.12 | 0.00 |
| 12 | K18 / $K2_{YEW}$ | 0.00 | 0.67 | 0.05 | 0.05 | 0.00 |

*Table 6.36 : Comparing MTA and MTR of this system with Yew's system (crisp data, no paradox)*

| No | Inference Structures | This System | | Yew's System | | |
|---|---|---|---|---|---|---|
| | | MTA | MTR | $MTA_{YEW}$ | $MTA_{MAX}$ | $MTR_{YEW}$ |
| 1 | K1 / $K13_{YEW}$ | 0.33 | 0.74 | 0.12 | 0.13 | 0.58 |
| 2 | K2 / $K14_{YEW}$ | 0.33 | 0.78 | 0.11 | 0.13 | 0.58 |
| 3 | K3 / $K18_{YEW}$ | 0.33 | 0.76 | 0.11 | 0.13 | 0.58 |
| 4 | K4 / $K19_{YEW}$ | 0.33 | 0.76 | 0.11 | 0.13 | 0.58 |
| 5 | K9 / $K16_{YEW}$ | 0.22 | 0.64 | 0.04 | 0.04 | 0.63 |
| 6 | K10 / $K15_{YEW}$ | 0.22 | 0.64 | 0.04 | 0.04 | 0.63 |
| 7 | K13 / $K12_{YEW}$ | 0.56 | 0.70 | 0.13 | 0.15 | 0.58 |
| 8 | K14 / $K10_{YEW}$ | 0.61 | 0.67 | 0.13 | 0.15 | 0.58 |
| 9 | K15 / $K9_{YEW}$ | 0.58 | 0.71 | 0.13 | 0.15 | 0.58 |
| 10 | K16 / $K7_{YEW}$ | 0.72 | 0.72 | 0.13 | 0.15 | 0.58 |
| 11 | K17 / $K1_{YEW}$ | 0.00 | 1.00 | 0.13 | 0.15 | 0.58 |
| 12 | K18 / $K2_{YEW}$ | 0.00 | 1.00 | 0.13 | 0.15 | 0.58 |

*Table 6.37 : Comparing MFA and MFR of this system with Yew's system (crisp data, no paradox)*

| No | Inference Structures | This System | | Yew's System | | |
|---|---|---|---|---|---|---|
| | | MFA | MFR | $MFA_{YEW}$ | $MFA_{ATR}$ | $MFR_{YEW}$ |
| 1 | K1 / $K13_{YEW}$ | 0.00 | 0.00 | 0.13 | 0.03 | 0.00 |
| 2 | K2 / $K14_{YEW}$ | 0.00 | 0.00 | 0.13 | 0.03 | 0.00 |
| 3 | K3 / $K18_{YEW}$ | 0.00 | 0.00 | 0.13 | 0.03 | 0.00 |
| 4 | K4 / $K19_{YEW}$ | 0.00 | 0.00 | 0.13 | 0.03 | 0.00 |
| 5 | K9 / $K16_{YEW}$ | 0.01 | 0.33 | 0.07 | 0.08 | 0.64 |
| 6 | K10 / $K15_{YEW}$ | 0.01 | 0.33 | 0.07 | 0.08 | 0.55 |
| 7 | K13 / $K12_{YEW}$ | 0.11 | 0.00 | 0.15 | 0.03 | 0.00 |
| 8 | K14 / $K10_{YEW}$ | 0.01 | 0.00 | 0.15 | 0.03 | 0.00 |
| 9 | K15 / $K9_{YEW}$ | 0.13 | 0.00 | 0.15 | 0.03 | 0.00 |
| 10 | K16 / $K7_{YEW}$ | 0.01 | 0.00 | 0.15 | 0.03 | 0.00 |
| 11 | K17 / $K1_{YEW}$ | 0.00 | 0.56 | 0.14 | 0.03 | 0.00 |
| 12 | K18 / $K2_{YEW}$ | 0.00 | 0.67 | 0.14 | 0.03 | 0.00 |

*Table 6.38 : Comparing MTA and MTR of this system with Yew's system (fuzzy data, with paradox)*

| No | Inference Structures | This System | | Yew's System | | |
|---|---|---|---|---|---|---|
| | | MTA | MTR | $MTA_{YEW}$ | $MTA_{MAX}$ | $MTR_{YEW}$ |
| 1 | K1 / $K13_{YEW}$ | 0.33 | 0.74 | 0.05 | 0.05 | 0.56 |
| 2 | K2 / $K14_{YEW}$ | 0.33 | 0.78 | 0.05 | 0.13 | 0.67 |
| 3 | K3 / $K18_{YEW}$ | 0.33 | 0.76 | 0.05 | 0.08 | 0.56 |
| 4 | K4 / $K19_{YEW}$ | 0.33 | 0.76 | 0.10 | 0.17 | 0.66 |
| 5 | K9 / $K16_{YEW}$ | 0.22 | 0.64 | 0.00 | 0.05 | 0.56 |
| 6 | K10 / $K15_{YEW}$ | 0.22 | 0.64 | 0.05 | 0.12 | 0.66 |
| 7 | K13 / $K12_{YEW}$ | 0.56 | 0.70 | 0.09 | 0.09 | 0.56 |
| 8 | K14 / $K10_{YEW}$ | 0.61 | 0.67 | 0.08 | 0.13 | 0.67 |
| 9 | K15 / $K9_{YEW}$ | 0.58 | 0.71 | 0.14 | 0.18 | 0.56 |
| 10 | K16 / $K7_{YEW}$ | 0.72 | 0.72 | 0.18 | 0.18 | 0.66 |
| 11 | K17 / $K1_{YEW}$ | 0.00 | 1.00 | 0.00 | 0.00 | 0.98 |
| 12 | K18 / $K2_{YEW}$ | 0.00 | 1.00 | 0.00 | 0.00 | 1.00 |

*Table 6.39 : Comparing MFA and MFR of this system with Yew's system (fuzzy data, with paradox)*

| No | Inference Structures | This System | | Yew's System | | |
|---|---|---|---|---|---|---|
| | | MFA | MFR | $MFA_{YEW}$ | $MFA_{ATR}$ | $MFR_{YEW}$ |
| 1 | K1 / $K13_{YEW}$ | 0.00 | 0.00 | 0.06 | 0.18 | 0.00 |
| 2 | K2 / $K14_{YEW}$ | 0.00 | 0.00 | 0.03 | 0.12 | 0.00 |
| 3 | K3 / $K18_{YEW}$ | 0.00 | 0.00 | 0.06 | 0.07 | 0.00 |
| 4 | K4 / $K19_{YEW}$ | 0.00 | 0.00 | 0.01 | 0.04 | 0.00 |
| 5 | K9 / $K16_{YEW}$ | 0.01 | 0.33 | 0.02 | 0.10 | 0.00 |
| 6 | K10 / $K15_{YEW}$ | 0.01 | 0.33 | 0.01 | 0.05 | 0.00 |
| 7 | K13 / $K12_{YEW}$ | 0.11 | 0.00 | 0.07 | 0.07 | 0.00 |
| 8 | K14 / $K10_{YEW}$ | 0.01 | 0.00 | 0.05 | 0.02 | 0.00 |
| 9 | K15 / $K9_{YEW}$ | 0.13 | 0.00 | 0.08 | 0.03 | 0.00 |
| 10 | K16 / $K7_{YEW}$ | 0.01 | 0.00 | 0.03 | 0.03 | 0.00 |
| 11 | K17 / $K1_{YEW}$ | 0.00 | 0.56 | 0.04 | 0.00 | 0.10 |
| 12 | K18 / $K2_{YEW}$ | 0.00 | 0.67 | 0.04 | 0.00 | 0.10 |

*Table 6.40 : Comparing MTA and MTR of this system with Yew's system (crisp data, with paradox)*

| No | Inference Structures | This System | | Yew's System | | |
|---|---|---|---|---|---|---|
| | | MTA | MTR | $MTA_{YEW}$ | $MTA_{MAX}$ | $MTR_{YEW}$ |
| 1 | K1 / $K13_{YEW}$ | 0.33 | 0.74 | 0.07 | 0.07 | 0.58 |
| 2 | K2 / $K14_{YEW}$ | 0.33 | 0.78 | 0.07 | 0.07 | 0.58 |
| 3 | K3 / $K18_{YEW}$ | 0.33 | 0.76 | 0.07 | 0.07 | 0.58 |
| 4 | K4 / $K19_{YEW}$ | 0.33 | 0.76 | 0.07 | 0.07 | 0.58 |
| 5 | K9 / $K16_{YEW}$ | 0.22 | 0.64 | 0.00 | 0.00 | 0.63 |
| 6 | K10 / $K15_{YEW}$ | 0.22 | 0.64 | 0.00 | 0.00 | 0.63 |
| 7 | K13 / $K12_{YEW}$ | 0.56 | 0.70 | 0.07 | 0.07 | 0.58 |
| 8 | K14 / $K10_{YEW}$ | 0.61 | 0.67 | 0.07 | 0.11 | 0.58 |
| 9 | K15 / $K9_{YEW}$ | 0.58 | 0.71 | 0.06 | 0.11 | 0.58 |
| 10 | K16 / $K7_{YEW}$ | 0.72 | 0.72 | 0.06 | 0.11 | 0.58 |
| 11 | K17 / $K1_{YEW}$ | 0.00 | 1.00 | 0.00 | 0.00 | 0.98 |
| 12 | K18 / $K2_{YEW}$ | 0.00 | 1.00 | 0.00 | 0.00 | 0.98 |

*Table 6.41 : Comparing MFA and MFR of this system with Yew's system (crisp data, with paradox)*

| No | Inference Structures | This System | | Yew's System | | |
|---|---|---|---|---|---|---|
| | | MFA | MFR | $MFA_{YEW}$ | $MFA_{ATR}$ | $MFR_{YEW}$ |
| 1 | K1 / $K13_{YEW}$ | 0.00 | 0.00 | 0.13 | 0.13 | 0.00 |
| 2 | K2 / $K14_{YEW}$ | 0.00 | 0.00 | 0.13 | 0.13 | 0.00 |
| 3 | K3 / $K18_{YEW}$ | 0.00 | 0.00 | 0.13 | 0.13 | 0.00 |
| 4 | K4 / $K19_{YEW}$ | 0.00 | 0.00 | 0.13 | 0.13 | 0.00 |
| 5 | K9 / $K16_{YEW}$ | 0.01 | 0.33 | 0.07 | 0.00 | 0.64 |
| 6 | K10 / $K15_{YEW}$ | 0.01 | 0.33 | 0.07 | 0.00 | 0.55 |
| 7 | K13 / $K12_{YEW}$ | 0.11 | 0.00 | 0.23 | 0.23 | 0.00 |
| 8 | K14 / $K10_{YEW}$ | 0.01 | 0.00 | 0.23 | 0.05 | 0.00 |
| 9 | K15 / $K9_{YEW}$ | 0.13 | 0.00 | 0.24 | 0.08 | 0.00 |
| 10 | K16 / $K7_{YEW}$ | 0.01 | 0.00 | 0.24 | 0.31 | 0.00 |
| 11 | K17 / $K1_{YEW}$ | 0.00 | 0.56 | 0.03 | 0.00 | 0.00 |
| 12 | K18 / $K2_{YEW}$ | 0.00 | 0.67 | 0.03 | 0.00 | 0.00 |

The tables above (from 6.34 to 6.41) show that this system performs much better than its predecessor. Generally, MTA of this system is much higher than $MTA_{YEW}$ or even $MTA_{MAX}$. The only inference structures that show lower MTA are K17 and K18, which do not perform well in this system. Besides, MTA are higher than $MTA_{YEW}$ with a range of 0.12 to 0.58, and even $MTA_{MAX}$ with a range of 0.08 to 0.38 in the case of using fuzzy data and no material paradox.

Overall, MTR are also higher than $MTR_{YEW}$ besides MTR for K10, which shows 0.02 lower while fuzzy data is used. In both systems, MFA and MFR show low value in most situations. This indicates that the false acceptance and false rejection rate are low in both systems.

It is clear that there is a great improvement in performance compared to its predecessor, and shows a much better acceptance rate for correct diagnoses. An improvement is also shown in other measurements such as MTR, MFA and MFR.

### 6.3.3 APPLICATION OF INFERENCE STRUCTURES

As stated in last section, inference structures can be categorized into 4 groups. All inference structures with arithmetic mean as the third logical connective (K5, K6, K11, K12, K17 and K18) are categorized into a low performance group. Three other groups can be easily found since all inference structures in the particular group show similar performance. The groups are :

Group A : K1, K2, K3, K4

Group B : K7, K8, K9, K10

Group C : K13, K14, K1, K16

One may also notice that all inference structures in group A are using AndTop as second fuzzy logical connective, whereas group B uses AndBot and group C uses arithmetic mean.

Inference structures from group C have the best performance among all in level 2, especially K16. However, their performance in level 1 is not as good as other inference structures, for example inference structures from group A. To improve the efficiency of the system, it would be a good idea if a hybrid inference is used in the hierarchical system : use any inference structures from group A for level 1 diagnosis and K16 in the second level of the diagnosis.

## 6.4 CHAPTER SUMMARY

The test result of the system developed here is presented in this chapter. From the result, it is clear that the performance of the system is good, especially using inference structure K16. With this inference structure, the mean true acceptance (MTA) of diseases can reach a high point of 0.72. The result of the test is also contrasted with its predecessor -- the system developed by Yew [1995] and it is proved that the improved set of inference structures show better performance than the old one.

# Chapter 7

# CONCLUSION AND FURTHER RESEARCH

## 7.0 INTRODUCTION

After all the theory, methodology and results are presented in last few chapters, it comes to the last part of the dissertation. A conclusion of the previous works is presented in this chapter. Last but not least, suggestions of future research are also presented here.

## 7.1 CONCLUSION

The world of fuzzy sets is a world of understanding and abstracting human being reasoning process. Fuzzy relational theory, a branch of fuzzy sets theory which was developed by Bandler and Kohout [1980] and revised by DeBaets and Kerre [1993] is believed to be able to simulate the human's reasoning process well.

In this dissertation, the basic concept of fuzzy sets as well as fuzzy relational theory and fuzzy relational products are discussed. To put the fuzzy relational theory into application, one must abstract the fuzzy relational products into inference templates. Among all, the following K inference templates were developed :

Sub-K inference template : $(R \triangleleft_k S)_{ik} = \min\left[\inf_j (R_{ij} \rightarrow S_{jk}),\ \sup_j \mathrm{T}(R_{ij}, S_{jk})\right]$

Super-K inference template : $(R \triangleright_k S)_{ik} = \min\left[\inf_j (S_{jk} \rightarrow R_{ij}),\ \sup_j \mathrm{T}(R_{ij}, S_{jk})\right]$

Square-K inference template : $(R \diamond_k S)_{ik} = \min[(R \triangleleft_k S)_{ik},\ (R \triangleright_k S)_{ik}]$

This inference templates are defined in terms of t-norms and t-conorms. To make these templates usable, all this logical connectives must be defined to construct applicable inference structures. Hallam [1998] has found that the original and common used logical connectives are not capable to become a good connectives here because lack of an important property, i.e. pseudo-strict monotonic property.

A set of fuzzy logical connectives with pseudo-strict monotonic property have been developed [Hallam 1998a]. With this connectives, one can build inference structures using any inference templates easily.

To prove that these fuzzy logical connectives as well as fuzzy relational theory are working well, an arthritic diseases diagnosis system based on sub-K inference templates has been developed. In this system, a number of 18 inference structures developed from sub-K inference templates are tested in two levels.

In the first level of the system, inference structures are implemented to find out suspected diseases based on the distribution of abnormalities in hand and wrist. These suspected diseases will be brought to the second level for a more accurate diagnosis. From the test results, one can see that some inference structures are showing good performance but some are not.

Good performance : K2, K7, K10, K8 and K9.

Average performance : K1, K4, K3, K13, K14, K16 and K15

Poor performance : K11, K12, K5, K6, K18 and K17

The list of suspected diseases is brought to the second level of these two-level diagnosis system. In this level, inference structures infer is based on signs and symptoms shown on the patient. In this level, inference structures are required to reject all incorrect diseases and only accept the correct diseases. Generally, inference structures shows good results in this level :

Excellence performance : K16

Good performance : K14, K15, K13

Average performance : K2, K3, K4, K1, K8, K9, K10, K7

Poor performance : K11, K12, K5, K6, K18 and K17

The result of this system is also compared with its predecessor, another medical diagnosis system based on fuzzy relational theory and fuzzy logical connectives before revision. The comparison shows that the performance of current system is much better than its predecessor. The reason of improvement should be dedicated to the application of fuzzy logical connectives which are pseudo strictly monotonic.

## 7.2 FURTHER RESEARCH

The following are some of the suggestions for further research or improvement on the inference theory and the diagnosis system. These suggestions could not be carried out in this thesis due to the time constraint.

i) In the current system, a knowledge base editor has been constructed to let users key in or edit the knowledge base easily. However, it is better if an error feedback mechanism can be set up. This feedback mechanism will start a process of unsupervised learning when an error inference occurs. Once a user tells the system that the inference is wrong, the feedback mechanism reads input data (distribution of abnormalities of the patient in level 1, or signs and symptoms shown on the patient in level 2) and calibrate data in the

knowledge base according to these input data. The performance of the system could be increased after a few training sessions.

ii) For a sign or symptom found on a patient, how do we know we should assign 0.7 or 0.8, or even 0.9 for this finding? The assignment of degree of signs and symptoms is a subjective process. Different users may give different value for the same finding, and this will cause the system to show different performance with different users. To avoid this problem, an image recognition front-end is proposed. Instead of relying on different users to read X-ray film and key in different sets of data, this image recognition front-end system will read film based on a predefined method. With this, the problem of subjectivity of human reader can be avoided.

iii) Definition of implication operators $I_X$ and $I_Y$ as

$$I_X = \max\left[\frac{1-a(b+1)+b}{2},\ 1-a+\frac{ab}{2}\right]$$

$$I_Y = \min\left[1-\frac{a(1-b)}{2},\ 1-a+\frac{b(1+a)}{2}\right]$$

is working well, but it is too complicated and might result in complicated AndTop, AndBot, OrTop and OrBot operators. This is fine if an inference does not involve many data, but if the inference is going to deal with a high amount of data, the inference process will be quit slow. Since AndTop, AndBot, OrTop and OrBot are generated based on implication operators $I_X$ and $I_Y$, it is better if simpler but good performance implication operators can be found.

iv) It is good to test the inference system on different diseases to confirm the performance of these inference structures. Of course different knowledge base have to be designed for different type of disease.

v) The combination of fuzzy relational theory and such as neural network is recomended. There are two types of Artificial Neural Network (ANN), anmely supervised and unsupervised neural network. Supervised neural network can be trained with suitable input data, which could help to direct the search optimally. The fuzzy information, hance giving better output. Where suitable training data is not available, the unsupervised neural network can be used.

The layer of neural network built above the fuzzy layer, provides a refined, intelligent and a more directed search and it is hoped that the performance can be enhanced.

vi) Inference engine of this system is classified as case-based reasoning inference engine. It is interesting if this system can incorporated the advantages of rule-based reasoning [Golding 1996].

# Appendix A

# MEDICAL INFERENCE SYSTEM FOR ARTHRITIS

## A.0 INTRODUCTION

Medical Inference System For Arthritis is a package developed for this research. This package, which was developed using pure Java programming language, enables users to manage patient information, run a diagnosis based on predefined inference structures and knowledge base, and edit the contents of the knowledge base.

As described in chapter 5, the package run a diagnosis in two levels. The first level is based on the distribution of abnormalities in hands and wrists of the patient, and the second level is based on the signs and symptoms shown by the patient. Level 1 diagnosis short lists possible diseases, so that level 2 diagnosis can deal with lesser signs and symptoms.

## A.1 FEATURES

I. Patient Management

- Add new patient records
- Edit patient records
- Patient records includes : patient name, addresses, race, gender, birth date, occupation, allergic, blood group as well as a field for physicians to store patients history or comment.
- Browse through patients information

II. Diagnosis

- Run a two-level diagnosis : level 1 based on the distribution of abnormalities, and the level 2 based on the strength of each sign/symptom.
- Diseases with the upper bound score higher or equal to 70% in the level 1 diagnosis will automatically be selected to enter level 2 diagnosis.
- Diseases with the upper bound score lower than 70% in the level 1 diagnosis can be manually selected to level 2 diagnosis.
- In the level 2 diagnosis, the accepted diagnoses will be highlighted with red colour.
- Capable to run diagnosis using a wide range of inference structures. Although only inference structures derives from sub-K inference template is demonstrated in chapter 6, but other inference templates as well as inference structures are prepared in the system. Below are all the options for an inference structure:

  Relational products = { Sub-B, Super-B, Sub-K, Super-K }

  Connective 1 = { min, max }

Connective 2 = { AndTop, AndBot, Arithmetic means }

Connective 3 = {OrTop, OrBot, Arithmetic means }

Connective 4 = {AndTop, AndBot }

So, there are totally 3 X 2 X 3 X 3 X 2 = 108 inference structures in the system.

III. Knowledge Base Editor

- Retrieve information in the knowledge base
- Edit information about existing diseases, this includes the distribution of abnormalities and signs/symptoms.
- Add new diseases.

## A.3 SYSTEM REQUIREMENTS

- Since the package is developed with Java, a platform independence programming language, theoretically it runs on any machine with JRE 1.1.7B installed.
- VGA or higher resolution graphic device with at least 256 colours, 640 X 480 resolution.
- Hard disk space : 250KB (minimum), extra space is needed for patient data and new diseases in knowledge base.

# Appendix B

# "MEDICAL INFERENCE SYSTEM FOR ARTHRITIS" USER GUIDES

## B.0 INTRODUCTION

To start the system, simply run the Java class file Arthritis.class on a computer with JRE 1.1.7B. The program contains the code of inference and patient management.

## B.1 PATIENT MANAGEMENT

Patient management works can be performed using "Patient" menu in Arthritis class.

- **Add new users** : Using "Patient" pull down menu, new patients can be added. (Figure B.1 and B.2)

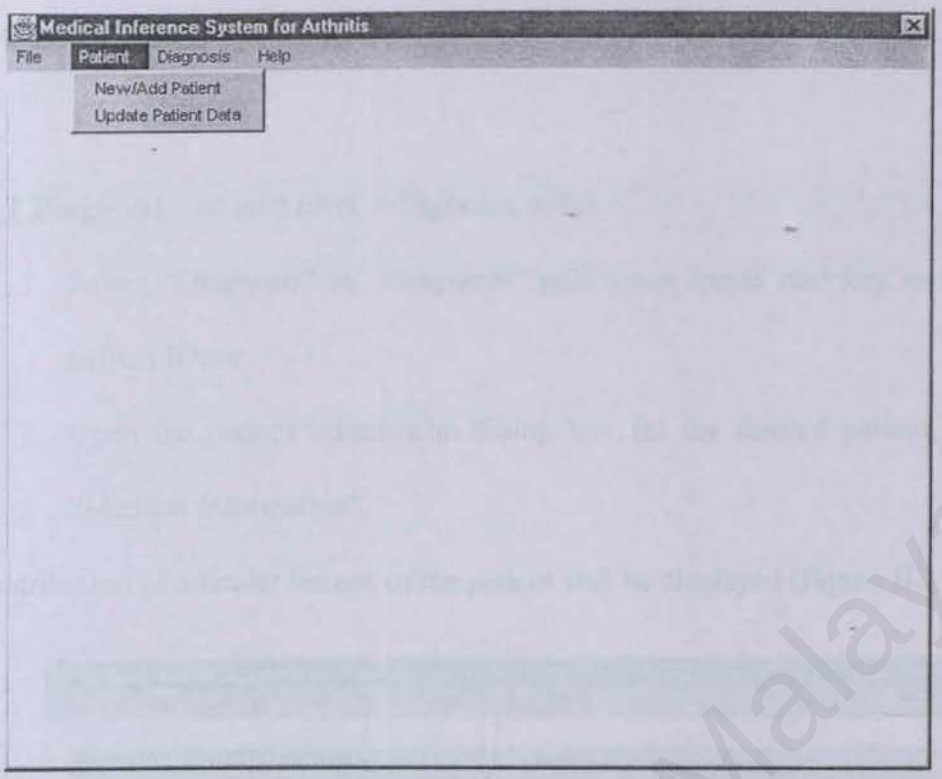


*Figure B.1 : Add new patient or edit patient records using Patient menu in Arthritis.class*

- **Edit patient's information** : By selecting "Update Patient Data" in Patient pull down menu (Figure B.1), the patient information dialog box (Figure A.2) will appear and particular of a patient can be edited.

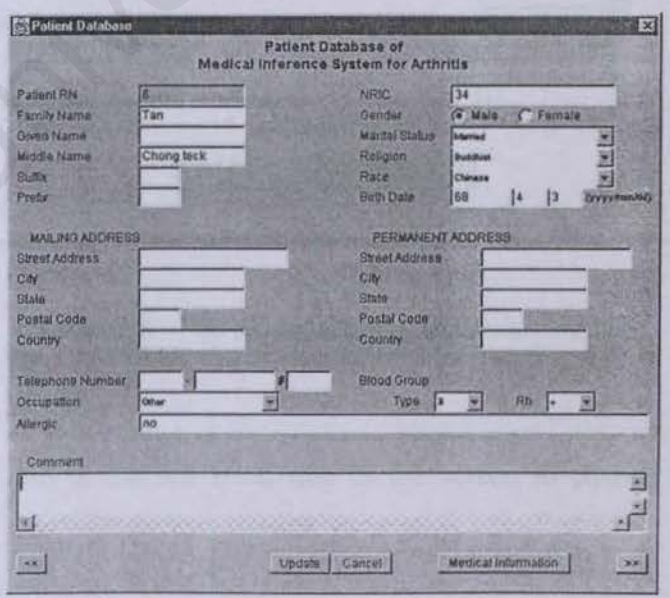


*Figure B.2 : Patient information dialog box*

## B.2 DIAGNOSIS

- **Level 1 Diagnosis** : To start level 1 diagnosis, either :

    1. Select "Diagnosis" in "Diagnosis" pull down menu, and key in the desired patient ID, or
    2. Open the patient information dialog box for the desired patient, and select "Medical Information"

The distribution of articular lesions of the patient will be displayed (Figure B.3).

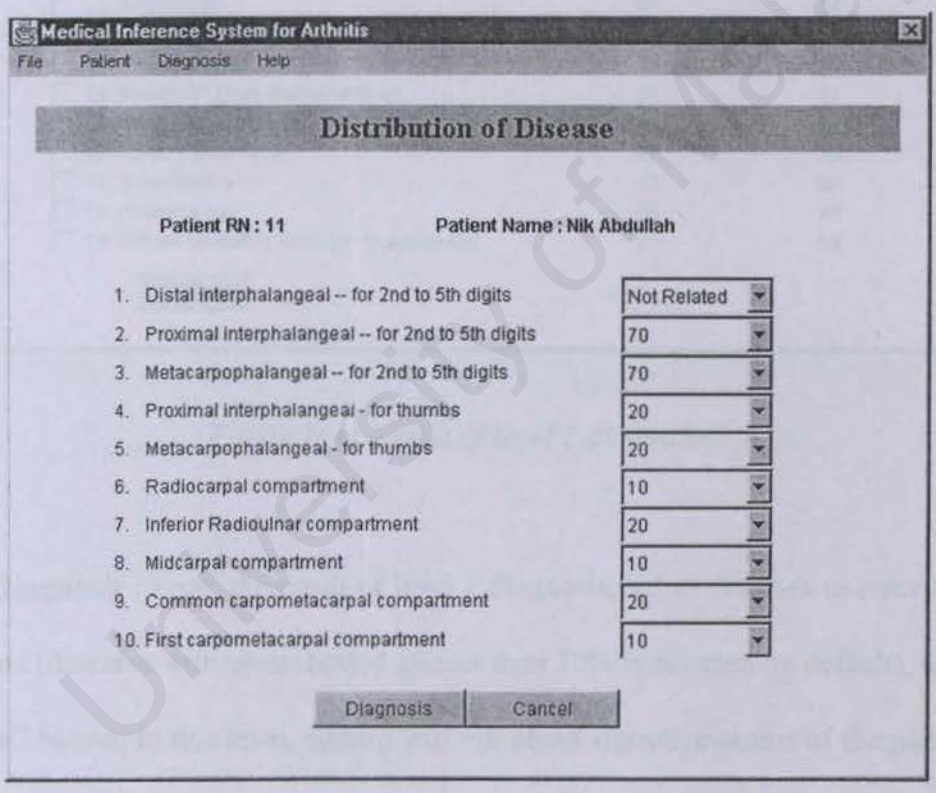


*Figure B.3 : The distribution of articular lesions*

Use the pull down lists at the left hand side of the screen to change the score of each joints.

To run the diagnosis, just press "Diagnosis" button and the result will be displayed (Figure B.4).

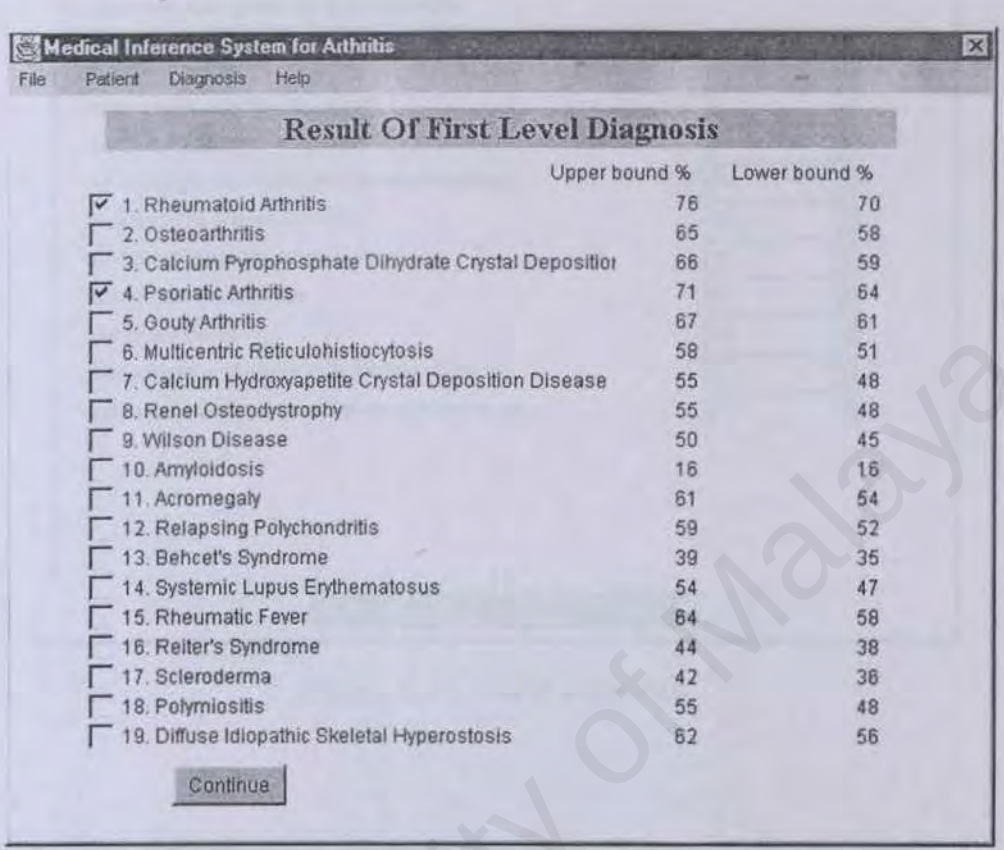


*Figure B.4 : Result of level 1 diagnosis.*

- **Level 2 Diagnosis** : From the result of level 1 diagnosis, select diseases to enter the level 2 diagnosis (diseases with upper bound greater than 70% is selected by default), and press "Continue" button. In this level, system will ask about signs/symptoms of the patient, and the user answers with selecting appropriate degree in the pull down list (Figure B.5). To get the result of diagnosis, just press "Diagnosis" button and the result will displayed (Figure B.6). In this level, the accepted diagnoses is highlighted with red colour.

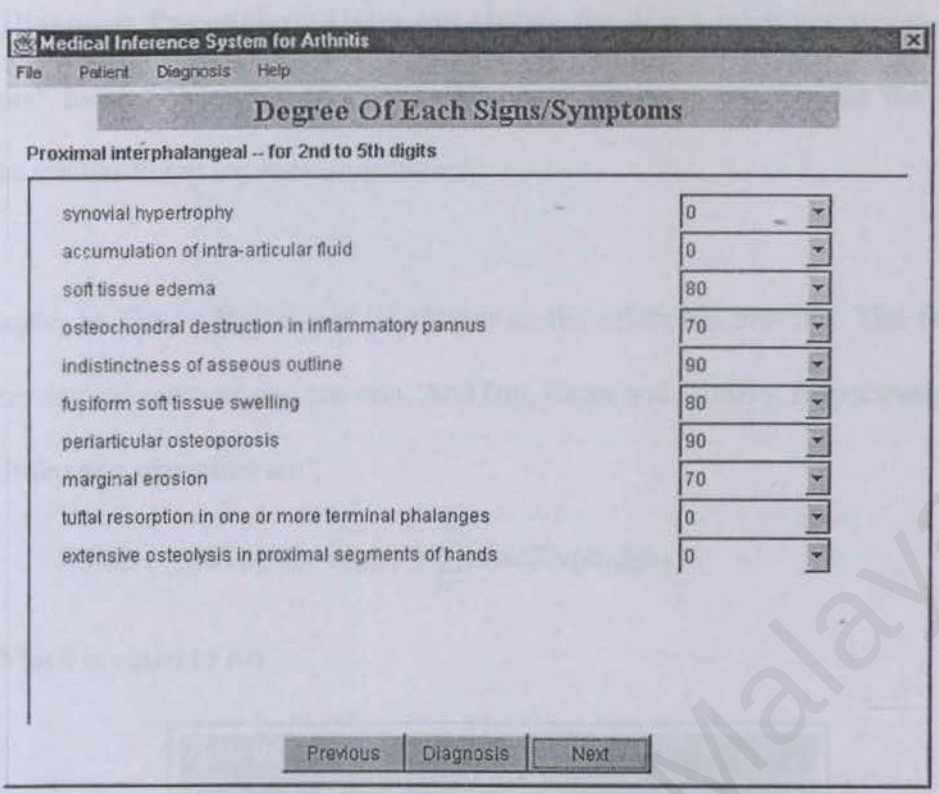


*Figure B.5 : level 2 diagnosis.*

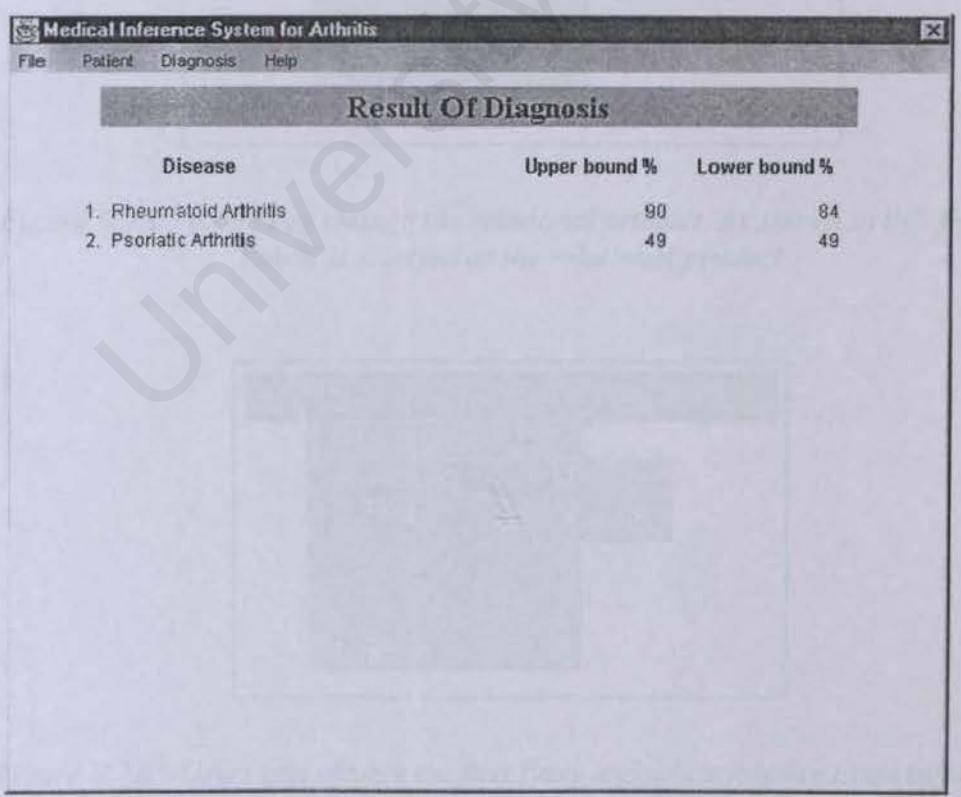


*Figure B.6 : Final result of a diagnosis*

- **Setting Diagnosis Parameter** : Users can change the desire inference structures using "Diagnosis" menu. As shown in Figure B.7 (A to E), users can change the relational product as well as fuzzy logical connectives.

For example, in Figure B.7, Sub-K is chosen as the relational product. The first to the forth fuzzy logical connectives are min, AndTop, mean and AndBot respectively. So, the selected inference structures are :

$$\min\left( \text{AndTop}(R_{ij} \rightarrow S_{jk}) , \frac{1}{n}\sum_{j=1}^{n}(AndBot(R_{ij}, S_{jk})) \right)$$

Which is equal to K6.

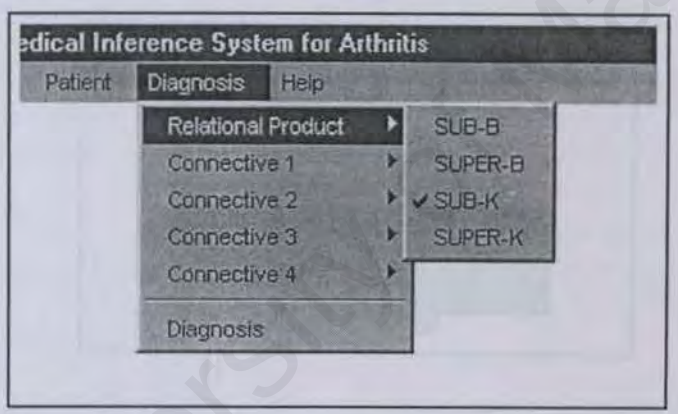


*Figure B.7A : Users can change the relational product. As shown in this figure, Sub-K is selected as the relational product*

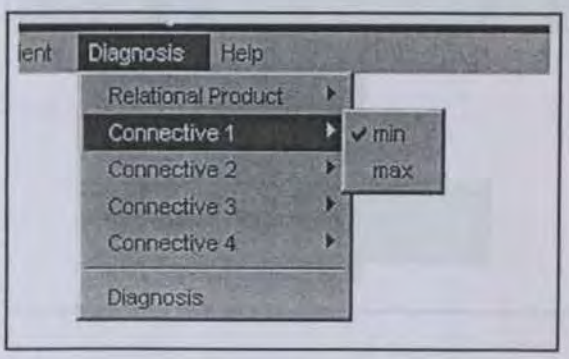


*Figure B.7B : Users can change the first fuzzy logical connective in an inference template. As shown in this figure, min is selected as the connective*

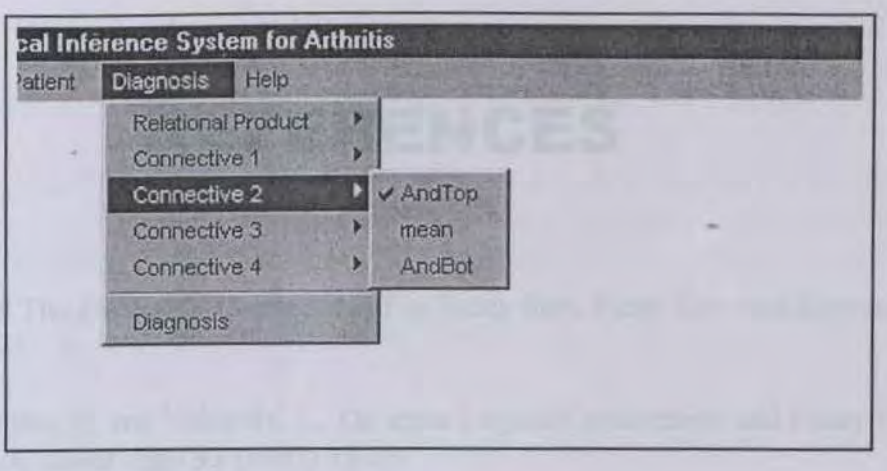


*Figure B.7C : Users can change the second fuzzy logical connective in an inference template. As shown in this figure, AndTop is selected as the connective*

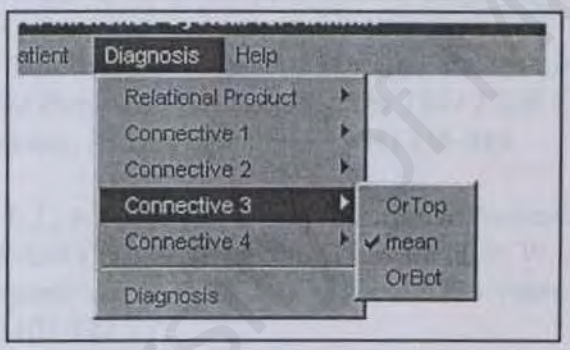


*Figure B.7D : Users can change the third fuzzy logical connective in an inference template. As shown in this figure, Arithmetic mean is selected as the connective*

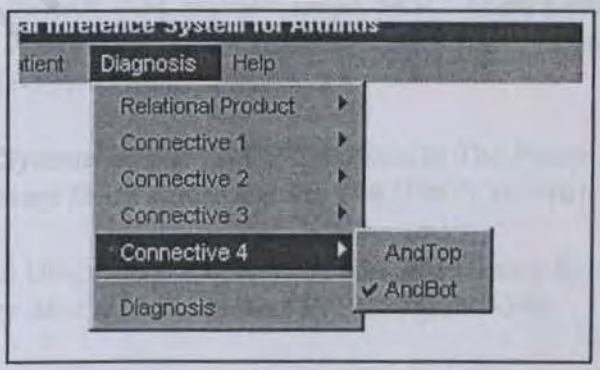


*Figure B.7E : Users can change the forth fuzzy logical connective in an inference template. As shown in this figure, AndBot is selected as the connective*

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