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THE INCIDENCE QUOTIENTS OF
3-HOMOGENEOUS SIMPLICIAL
COMPLEXES

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Abstract

The objective of this research is to investigate the incidence quotients of 3-homogeneous simplicial complexes (3HSCs). A geometrical interpretation is given during the investigation.

The existence of a 3HSC whose incidence quotient is a cyclic group of order n , in which n is any given nonzero multiple of 3, is established. In the special case, namely when $n = 2^m - (-1)^m$ for some integer $m \geq 3$, some complexes relate to the multiplicative order of -2 modulo p , where p is an odd prime dividing n .

Finally, we observe that some *virtual* triangles could have been added to a given 3HSC, yet the structure of its incidence quotient being essentially invariant. We define a closed complex as a 3HSC which has been *saturated* with these virtual triangles. Some further properties of closed complexes are also given.

Key words: complex, connected, hyper connected, incidence quotient, equivalence classes of vertexes, full equivalence classes, subdivision, cyclic complex, binary expansion, Bachmann-Farey's expansion, binary tree, forest, cap, closed complex, closure

Abstrak

Penumpuan penyelidikan berkisar pada hasil bahagi insidens bagi kompleks bersimpleks 3-homogen (KS3H). Suatu interpretasi geometri disertakan dalam proses pengajian.

Kewujudan KS3H yang mempunyai hasil bahagi insidens berstruktur kumpulan kitaran bertertib n , di mana n merupakan suatu gandaan 3 tak sifar yang diberi, dipamerkan. Dalam kes khusus, iaitu apabila $n = 2^m - (-1)^m$ bagi suatu integer $m \geq 3$, kompleks tersebut dikaitkan dengan peringkat -2 modulo p , di mana p suatu nombor perdana yang ganjil yang membahagi n .

Akhirnya, kita dapati mungkin wujudnya segitiga *maya* yang boleh ditambahkan kepada suatu KS3H yang diberi, manakala struktur hasil bahagi insidens KS3H tersebut pada asasnya tak berubah. Kita mendefinisikan suatu kompleks sebagai suatu KS3H tertutup sekiranya tidak lagi boleh ditambahkan sebarang segitiga *maya* yang mempunyai ciri yang dinyatakan. Sifat-sifat tertentu bagi kompleks tertutup juga disertakan.

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