THE INCIDENCE QUOTIENTS OF 3-HOMOGENEOUS SIMPLICIAL COMPLEXES

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June 15, 1999

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DISSERTATION PRESENTED FOR THE DEGREE OF MASTER OF SCIENCE UNIVERSITY OF MALAYA KUALA LUMPUR (1999)
Acknowledgments

I gratefully acknowledge my indebtedness to Dr. Thomas Bier for his patience and meticulous care with which he examined the first draft of this material, corrected a host of errors, and made numerous suggestions which I believe improved substantially the contents of this dissertation.

I am also greatly indebted to colleagues at Institute of Mathematical Sciences, especially to Dr. Mohamed Rizal Wahiddin who recommended me for the tutorship, friends, and my family, from whom I have always received invaluable support.

Kwa Kiam Heong, 1999.
Abstract

The objective of this research is to investigate the incidence quotients of 3-homogeneous simplicial complexes (3HSCs). A geometrical interpretation is given during the investigation.

The existence of a 3HSC whose incidence quotient is a cyclic group of order \( n \), in which \( n \) is any given nonzero multiple of 3, is established. In the special case, namely when \( n = 2^m - (-1)^m \) for some integer \( m \geq 3 \), some complexes relate to the multiplicative order of \(-2\) modulo \( p \), where \( p \) is an odd prime dividing \( n \).

Finally, we observe that some virtual triangles could have been added to a given 3HSC, yet the structure of its incidence quotient being essentially invariant. We define a closed complex as a 3HSC which has been saturated with these virtual triangles. Some further properties of closed complexes are also given.

**Key words:** complex, connected, hyper connected, incidence quotient, equivalence classes of vertexes, full equivalence classes, subdivision, cyclic complex, binary expansion, Bachmann-Farey's expansion, binary tree, forest, cap, closed complex, closure
Abstrak

Penumpuan penyelidikan berlakar pada hasil bahagi insidens bagi kompleks bersimpleks 3-homogen (KS3H). Suatu interpretasi geometri disertakan dalam proses pengajian.

Kewujudan KS3H yang mempunyai hasil bahagi insidens berstruktur kumpulan kitaran bertubuh \( n \), di mana \( n \) merupakan suatu gandaan 3 tak sifar yang diberi, dipamerkan. Dalam kes khusus, iaitu apabila \( n = 2^m - (-1)^m \) bagi suatu integer \( m \geq 3 \), kompleks tersebut dikaitkan dengan peringkat \(-2\) modulo \( p \), di mana \( p \) suatu nombor perdana yang ganjil yang membahagi \( n \).

Akhirnya, kita dapat mungkin wujudnya segitiga \( maya \) yang boleh ditambahkan kepada suatu KS3H yang diberi, manakala struktur hasil bahagi insidens KS3H tersebut pada asasnya tak berubah. Kita mendefinisikan suatu kompleks sebagai suatu KS3H tertutup sekitaranya tidak lagi boleh ditambahkan sebarang segitiga \( maya \) yang mempunyai ciri yang dinyatakan. Sifat-sifat tertentu bagi kompleks tertutup juga disertakan.
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