Proof. Let $x_1x_2x_3, y_1y_2y_3 \in max(K)$ be of the same type such that $x_i \equiv y_i$ for each i. It is then clear that the sequence $x_1x_2x_3, x_1x_2y_3, x_1y_2y_3, y_1y_2y_3$ is in $max(K)$, showing $x_1x_2x_3$ and $y_1y_2y_3$ belong to the same hyper component of K.

Let $\delta, \delta' \in max(K)$ be in the same hyper component of K, say H. Since H is hyper connected, there is a sequence $\delta = \delta_1, \delta_2, \cdots, \delta_n = \delta'$ in $max(H)$ such that $|\delta_i \frown \delta_{i+1}| = 2$ for $i = 1, 2, \cdots, n-1$. Since $|\delta_i \frown \delta_{i+1}| = 2$, it is readily derived that δ_i and δ_{i+1} are of the same type for each i. It follows at once that $\delta = \delta_1$ and $\delta' = \delta_n$ are of the same type.

Since any hyper component of K consists of exactly one type of triangles, and since any triangle of this type must belong to the hyper component, we get that $h(K) = t(K)$.

q.e.d.

It is easily seen in general that $t(K) \leq h(K)$ for any complex K.

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