CHAPTER 2

REVIEW OF LITERATURE

2.0 Introduction

This study attempts to find out the computational errors in division of whole number of a sample of Form One students. The literature review in this study focuses on the following aspects:

- The meaning of the division process
- The Division algorithms
- Error patterns in division computation
- Individual interviews in diagnosis of errors

2.1 The Meaning of the Division Process

The Curriculum Standards for School Mathematics (NCTM,1989) encompasses "Concept of Whole-Number Operations" and "Whole-Number Computation" as two of its thirteen standards for K-4 mathematics. Students are expected to develop meaning for the operations by modelling and discussing a rich variety of problem situations. Besides, they should be able to relate mathematical language and symbolism of operations to problem situations and informal language. Understanding the meaning of division process is essential before the students are introduced to the division algorithm. Very often, errors in division are due to student's misconceptions or lack of understanding of the division concept.

To understand division process, two different types of situations are suggested: *measurement* situation and *partition* situation (Bates & Rousseau, 1986; Burton, 1992; Fischbein, Nello, & Marino, 1985; Grossnickle & Brueckner, 1959; Reys et al.,1995).

(a) In *measurement* situation, the number in each group is known and one needs to determine the number of groups. For example, 12 ÷ 3 means: How many groups of 3s are in 12? This can be related to a problem situation such as: When 12 marbles are packed into boxes with 3 in each box, how many boxes can one get? The measurement situation finds out how many equal groups are there in a set.

(b) In *partition*, or sharing situation, one looks for the number within each of several equal groups. For example, 12 ÷ 3 means: If 12 is divided into 3 equal groups, how many are there in each group? This concept can be related to a problem situation such as: 12 candies are shared among 3 brothers, how many candies will each boy get? In short, the partition situation refers to equal sharing in a set.

These two division situations can be modelled by using manipulatives (Beattie, 1986; Burton 1992; Vest, 1985). In the measurement situation, the answer can be obtained by repeatedly removing a group of 3s from a set of 12 counters. While in the partition situation, the procedure is to deal a set of 12 counters to 3 children. If the divisor is small, many teachers are found to prefer the partition situation to the measurement situation in illustrating the meaning of division. Some teachers felt that the concept of equal sharing relates better to the children's daily encounters in sharing. However, when the divisor is big, such as in 51 ÷ 17, the measurement situation might provide a better picture of the division process. For example, partitioning a set of 51 marbles into 17 piles is not as practical as the measurement situation of making piles of 17 marbles from a box of 51 marbles. Nevertheless, both situations are necessary in order to understand the meaning of division process. The students should have the ability to relate division process to both problem situations.

Vest (1986) found that children in the third and fourth grades were able to distinguish between the two division situations. Burton (1992) interviewed 117 grade two pupils using a set of 12 measurement and partition situation problems. His study found that partition problems were not much more difficult than measurement problems for these children. Nevertheless, earlier findings by Anghileri and Johnson (1988) indicated that primary children found partition problems more difficult than measurement problems, although the differential difficulties disappeared after grade five. In another study, Bell, Fischbein, and Greer (1984), found that 27 out of the 28 correct responses for writing stories for the division expression 18 ÷ 3, comprised stories of the partition type. The findings showed that the partition model was the preferred structure in the 12 and 13-year-olds. Hence, these findings indicate that there is no general agreement as to which division situation is easier for the children.

In Malaysian schools, the division concept is introduced in primary two. The Teacher's Guide for Primary Two (Malaysia, 1983) introduced division process through partition situation. Measurement situation, which was only considered at the later stage, was often ignored by most teachers. However, the revised Primary School Integrated Curriculum (Malaysia, 1994a) placed the measurement situation before the partition situation. No particular reason was given for the switch in the order. To the knowledge of the researcher, no study had been done in Malaysia on the comparison between the two situations in division operation. Nevertheless, informal observations by the researcher during the in-service and the pre-service courses in a teacher training college revealed that many teachers preferred to use the partition situation when introducing the meaning of division operation in their classroom instructions. One plausible explanation is the term "divide", which is translated as "*bahagi*" in the Malay language, and that implies "equal sharing". Many of the in-service and pre-service teachers were of the opinion that "equal sharing" should be more appropriate in introducing the meaning of division operation. Informal interviews with some teachers further revealed that many of them used only the partition situation in their instructions. They believed that introducing the measurement situation might confuse the students.

This observation was supported by the findings of Tirosh and Graeber (1990). Their study, which was based on individual interviews on 21 pre-service teachers in America, found that many of them were relatively unfamiliar with the measurement interpretation of the division. They relied heavily on the partition model. This is in line with their earlier findings (Tirosh and Graeber, 1989), which indicated that American preservice elementary teachers tend to think predominantly in the partition model.

2.2 The Division Algorithms

Besides understanding the meaning of the division process, the Curriculum Standards for Grade K-4, on "Whole number computation", require the students to be able to model, explain, and develop reasonable proficiency with basic facts and algorithms (NCTM, 1989). The students need to acquire considerable proficiency in using algorithm for basic division computation. Although calculators are easily available for computation, the importance of algorithm in basic computation should not be de-emphasized. The algorithm is useful as it provides a concise and efficient procedure in computation.

In division computation, two algorithms are generally used: the *distributive* algorithm (also referred to as the standard algorithm), and the *subtractive* algorithm.

(a) The *distributive* algorithm is the more commonly used algorithm (Reys et al., 1995). An example of the distributive algorithm is shown below:

$$\begin{array}{r l l}
94 & & \\
4\overline{)376} & & \\
\underline{36} & \rightarrow & \text{How many 4s are in 37?} \quad 4\text{x}9=36 \\
16 & & \\
\underline{16} & \rightarrow & \text{How many 4s are in 16?} \quad 4\text{x}4=16
\end{array}$$

When 376 is divided by 4, the first step is to take 37 as the partial dividend, and then working out, "How many 4s are there in 37?" The answer 9 is then written down as the quotient figure in the tens place. The remainder 1 (37 –36) is written down, and 6 from the ones digit in the dividend is then brought down, resulting in 16 as the next partial dividend. Similarly, the procedure to obtain the next quotient is to find out "How many 4s are there in 16?" The answer 4 is written down as the next quotient figure in the ones place. As there is no remainder (16 –16), the quotient for 376 divided by 4 is 94.

(b) The *subtractive* algorithm involves repeated subtraction of the partial products. An example of the algorithm is illustrated below:

$$\begin{array}{rcl} 4\overline{)376} & & \\ -\underline{200} & \rightarrow & 50 \times 4 = 200 \\ 176 & & \\ -\underline{160} & \rightarrow & 40 \times 4 = 160 \\ 16 & & \\ -\underline{16} & \rightarrow & \underline{4} \times 4 = \ 16 \\ & & 94 \end{array}$$

When 376 is divided by 4, the first step is to find out "How many 4s are there in 376?" An estimate is made of the number of 4s in the dividend. Assuming there are fifty 4s, 50 multiplied by 4 is 200, subtracting 200 from 376 gives a remainder of 176. However, 176 still contains 4s. Hence, the same procedure is repeated to find out "How many 4s are there in 176?" Assuming there are forty 4s, 40 multiplied by 4 is 160, subtracting 160 d from 176 gives a remainder of 16. The same procedure is repeated by asking, "How many 4s are there in 16?" This time, 4 multiplied by 4 is 16, and gives no remainder. Therefore, the total number of 4s in 376 is 94 (50 + 40 + 4 = 94) (Grossnickle & Brueckner, 1959; Reys et al., 1995).

Van Engen and Gibb (1956) compared the effect of using each of the two algorithms among the low and high achieving pupils. The findings showed that the subtractive algorithm had some beneficial effects on performance in division, especially for low achievers. The subtraction algorithm might be easier for them because there was no necessity to make correction on the quotient figure if the estimation made is too small. However, they also found that high achievers showed little differences in performance between the two methods. According to Gallahan and Glennon (1975), the findings of Van Engen and Gibb resulted in many elementary school teachers using the subtractive algorithm in initial introduction of the division process, and then gradually switching to the distributive algorithm at a later stage. Nevertheless, another study by Kratzer and Willoughby in 1973, seemed to favour the distributive approach over the subtractive approach (cited by Gallahan & Glennon, 1975)

Both distributive and subtractive algorithms have their merits and demerits. The distributive algorithm is neat and concise. But when the distributive algorithm is used, it is noted that some students tend to consider each partial dividend independently without referring to its place value. On the other hand, the subtractive algorithm has the merit of *being closely related to the measurement model of division as repeated* subtraction. Moreover, it does not require bringing down new dividend digit for each new partial dividend. However, the subtractive algorithm is not as concise as the distributive algorithm. This method is also new to many teachers (Reys et al., 1995).

In Malaysian schools, distributive algorithm is used exclusively in the textbooks and in the teacher's guides. The researcher's informal interactions with the in-service teachers revealed that majority of the teachers were not exposed to the subtractive algorithm. Consequently, the teachers seldom used subtractive algorithm.

Although algorithm is important in computation, teachers should not focus their instructions only on the mastery of the mechanics of algorithm. Students also need to understand the rationale behind each step of the algorithm. The reason for teaching algorithm should change from merely obtaining the correct answers through the rote manipulation of symbols to understanding the meaning of the operations (Beattie, 1986). Very often, students rely too heavily on memorized algorithm and use very little reasoning when performing their computation (Monroe & Clark, 1998). Reasoning is fundamental in knowing and learning mathematics (NCTM, 1989).

In building conceptual understanding of the meaning of division process, current studies suggested the use of manipulative materials in the instructional process (Beattie, 1986; Burton, 1992; Steffe and Cobb, 1998; Vest, 1985). Manipulative materials also help in building the procedural understanding of the algorithm by associating the steps of an algorithm with the actions on the manipulative materials. Mathematics teachers should make use of this facility to make connections between the meanings of an operation, the associated manipulation of materials, the verbal explanations accompanying the manipulations, and the steps of algorithm (Beattie, 1986).

It is noted that inappropriate instructional process of the teachers could have attributed to the difficulties encountered by some students in division computation. Some teachers spend insufficient time in activities that help to develop conceptual understanding in division operation before moving on to drills and practice on algorithm in division computation (Downes & Paling, 1968). Steffe and Cobb (1998) suggested that division algorithm must be based on the child's concept and existing scheme. They found that children's elementary division scheme was built on their counting scheme. The division algorithm should therefore be viewed as resulting from a series of reorganizations, which started with the child's elementary counting algorithm. The teacher should therefore provide opportunities for children to reflect and abstract the concepts from their problem solving activities that help them to construct non-counting methods. Learning would be effective only if instruction is in harmony with the student's schemes.

Reys et al. (1995) on the other hand attributed the difficulty in division computation to the inherent nature of the division algorithm. While the computations for addition, subtraction, and multiplication begin from right to left, division computation starts from left to right. Moreover, the algorithm in division computation necessitates the use of not only the division facts, but also the multiplication and subtraction facts. Added to this difficulty, the algorithm also requires making the correct estimation for the quotient. Their observations were supported by other studies on computational errors in division computations, which indicated that basic fact error (division, subtraction, and multiplication facts) and making incorrect estimation were among the common errors in division (Buswell & John, 1926; Grossnickle, 1936,1939; Lim, 1980; Stefanich & Rokusek, 1992; Williams & Whitaker, 1937).

2.3 Error Types in Division Computation

Children's errors in computation often follow a consistent pattern (Ashlock, 1976; Cox, 1975; Grossnickle, 1936, 1939; West, 1971). These error patterns indicate certain rules in their computations (Rudnistsky, Drickamer & Handy, 1981). Thus recognizing the error pattern is important in diagnostic and remedial teaching (Ashlock, 1976; Fowler, 1980; Reisman, 1978).

Burrow (1976) made a review of literature on errors in computation with whole number from the period of 1917 to 1976, he listed a total of 35 addition errors, 31 subtraction errors, 56 multiplication errors, and 72 division errors. For the errors in division, they are mainly errors identified by Buswell and John (1926), and Grossnickle (1936, 1939).

Buswell and John identified 41 division errors from a sample 352 third to sixth graders using the diagnostic interview technique. Among the errors listed, the most frequent ones were:

(a) Errors in division combinations
(b) Errors in subtraction
(c) Errors in multiplication
(d) Used remainder larger than divisor
(e) Neglected to use remainder within example
(f) Omitted zero resulting from another digit
(g) Used wrong operation
(h) Omitted digit in dividend
(i) Omitted final remainder

(j) Omitted zero resulting from zero in dividend

(k) Used too large a product

Grossnickle analyzed errors made by 453 students in grades five to eight, in long division with a one-digit divisor, and he compiled a list of 57 errors. He classified these errors into six types, in decreasing frequency of occurrence:

(a) Errors in combination (basic fact)

(b) Errors resulting from the use of remainders

(c) Errors resulting from zero

(d) Errors caused by faulty procedure

(e) Errors resulting from lapses of attention

(f) Errors resulting from bringing down

Except for errors resulting from lapses of attention, the other five classifications are systematic errors. Grossnickle's classification of errors was also employed in the recent study by Stefanich and Rokusek (1992) on computational errors in division.

The findings by Buswell and John, and Grossnickle indicated that errors in basic facts were the most frequent errors, followed by errors in the use of remainder, and errors involving the use of zero. Particularly, in Grossnickle's study, errors in combination, omission of final zero in quotient only, using a remainder greater than the divisor, and dropping the remainder when zero is final in the quotient only; made up about 60% of the total errors in division computation.

William and Whitaker's (1937) study on 516 children from grades four to eight found that for division computation, 63% of pupils showed errors in combination, 29% used remainder larger than divisor, 23% omitted final remainder, 28% omitted zero resulting from another digit. The study indicated that the more frequent errors in the division computation were: errors in combination, remainder errors, and zero errors. These observations supported the earlier findings of Grossnickle.

In Malaysia, Lim (1980) made a similar study on 237 primary four pupils using Cox's instrument for identifying systematic errors. The findings showed that in the division computation: 28% used remainder larger than the divisor, 20% omitted zero not final in quotient, and 14% used wrong operation. Nevertheless, Lim's findings did not indicate errors in combination as a frequent error. Instead, he found using wrong operation as a frequent error.

In another study, Schonell & Schonell (1965) identified six types of errors in division. Among these errors, only "carried wrong number" was not an additional error (cited by Burrow,1976). Grossnickle (1939) followed up with another study on computational errors in division by two-digit divisors. Although he recorded 24 types of errors made by the sample of 221 fourth-graders, no additional error pattern was identified. Nevertheless, his study indicated that the fourth-graders showed a high constancy of errors in division.

Lankford (1974) conducted individual interviews on 176 seventh-grade pupils to find out their computational strategies. He also made comparisons between good computers and poor computers. He found that children who were poor in computation also had difficulty in remembering the conventional operational algorithms. Moreover, they had difficulty matching the correct algorithms with the computational questions. Consequently, they devised simple “shortcut” procedures that would give them quick answers. This observation was supported by Fowler (1980), who found that many children soon forgot the mathematical rationalization for a procedure, but retained its mechanical operation. Robert (1968) classified such defective algorithm as one of the five failure strategies of the pupils.

Based on the review of literature on the error patterns in division of whole numbers, the researcher has grouped the error patterns under: basic fact errors, wrong operations, remainder difficulties, zero errors, and inversion of orders for discussions.

2.3.1 Basic Fact Errors

Division algorithm requires knowledge of basic number facts in division, multiplication, and subtraction. Buswell and John’s (1926) findings indicated that basic fact error was the most frequent error in division computation. Their findings were supported by Grossnickle (1936) who found that basic fact error alone accounted for about 40% of the errors in division computation. Grossnickle also found that about 80% of the multiplication fact errors in the division computation were the correct products to other combinations. For example, for a response such as 8 x 4= 24, the incorrect product 24 is actually the correct multiplication product for 8 x 3= 24. The incorrect response is only one factor from the correct product. His study found that about 53% of students exhibited this pattern of error. The findings indicated that students often derived their multiplication products from known multiplication facts.

Division facts are closely related to the multiplication facts. As division is the inverse of multiplication, division facts can be derived from multiplication facts. Kalin (1983) used division fact test and interviews to explore the nature of fourth-graders' thinking on how they arrived at their division facts. He identified two strategies that were used by the children: the *multiplication* and the *solution* strategies.

In the *multiplication* strategy, the pupil obtained 6 ÷ 3 = 2 because 3 multiplied by 2 was 6. In *solution* strategy, the pupil multiplied the divisor by a number as close to the dividend as possible, then using adding on method to get the final answer. For example, to get 27 ÷ 3 = 9, he started with 7 x 3 = 21, then adding on 3s to 21, he obtained 24, 27. He used his fingers to keep count of the number of 3s added on and found that he needed to add 2 more 3s to obtain 27. The student used fingers to help him in counting. This indicates that children build their division scheme from their elementary counting scheme (Steffe and Cobb, 1998).

However Lankford's (1974) study on seventh-graders using individual interviews found that more frequently, in division exercise such as $27\overline{)81}$, there was little thinking in the pattern of "27 x ? =81". The thinking was usually expressed as: "81 divided by 27", "27 goes into 81" and "How many 27s in 81?"

These findings indicated that fourth-graders and seventh-graders use different thinking pattern in their strategy to arrive at the basic division fact. The findings also suggests that seventh-graders have developed their non-counting division scheme, whereas the fourth-graders still need to rely on their elementary counting scheme to obtain their basic division fact.

It is not easy to differentiate between division fact error and multiplication fact error, since the division facts are often derived from multiplication facts. Nevertheless, in division algorithm, the division facts and multiplication facts may be differentiated as shown in the following examples:

(a) Division fact error:

$$\begin{array}{r} 8 \\ 3\overline{)27} \end{array}$$

The error is due to incorrect division fact: 27 $\div$ 3 = 8.

(b) Multiplication error:

$$\begin{array}{r} 8 \\ 3\overline{)27} \\ \underline{26} \\ 1 \end{array}$$

The error in this example is due to incorrect multiplication fact: 3 x 8 = 26 (Reisman, 1978).

For subtraction facts, Grossnickle (1936) found that error in subtraction combination decreased significantly from grade five through grade eight. As they progressed to grade eight, most of the pupils had acquired considerable proficiency in subtraction facts.

Roberts (1968) classified basic fact errors as obvious computational errors. He suggested that pupils' obvious computational errors arose from being unable to recall basic number facts. His study showed that 18% of the pupils made errors in computation due to errors in basic number facts.

In another study on 198 third and fourth-graders, using an 84-item arithmetic test, Engelhardt (1977) found that basic fact errors accounted for 38% of the computational errors in the four basic operations. In contrast, Lim's (1980) study did not indicate basic fact errors as among the more common errors in division computation. In fact, basic fact error was not observed in Stefanich and Rokusek's (1992) study on 25 students in grade four.

Nevertheless, the above studies indicated that basic fact errors in multiplication, division, and subtraction were the major causes of division computational errors. However, errors in subtraction facts decline as pupils progress to higher grade. Most pupils derive their division facts from the multiplication facts.

2.3.2 Wrong Operation

Roberts (1968) defines wrong operation as the pupil's attempt to respond by performing an operation other than the one that is required to solve the problem. He found that using wrong operation accounted for 18% of the total errors in computation in his sample of 766 third-graders.

Earlier study by Buswell and John (1926) indicated that using wrong operation occurred in 22% of the pupils in the first quartile, and

11% of the pupils in the fourth quartile. This observation suggests that higher achieving pupils are less inclined to use the wrong operation in their computation as compared to the low achieving pupils.

Lim (1980) found that using the wrong operation was the third most common error. Examples of error due to wrong operation observed in his study are:

(a) Multiplication:

Multiplying each digit of the dividend by the divisor.

$$\begin{array}{r} 182412 \\ 6\overline{)342} \end{array}$$

(b) Subtraction

Subtracting the divisor from each of the dividend digits.

$$\begin{array}{r} 56 \\ 2\overline{)78} \end{array}$$

(c) Addition

Adding the divisor to each digit of the dividend.

$$\begin{array}{r} 101110 \\ 8\overline{)232} \end{array}$$

Other recent studies also indicated the use of wrong operation by the pupils in division computation. The report on the National Assessment of Education Progress (NAEP) of 1980 found that 6% of the 9-year-olds did an operation other than division for the question $6\overline{)18}$, and 3% made a similar error for $3\overline{)42}$ (cited by McKillip,1981). Engelhardt (1977) found that using the wrong operation accounted for 4% of the errors in computation in the four operations, for the third to sixth grade pupils in his study. Nevertheless, the report on the NAEP of 1980 indicated that using wrong operations did not occur in the 13-year-olds (cited by McKillip, 1981).

2.3.3 Remainder Difficulties

Unlike other operations, which require only recalling basic number combinations, division algorithm requires the estimation of the quotient figure. In division algorithm, the quotient digit should give the partial product that is either equal to the partial dividend (in the case where there is no remainder), or less than the partial dividend (when there is remainder). Moreover, if there is a remainder, the remainder resulting from subtracting partial product from the partial dividend should be less than the divisor. Remainder errors in division computation are often caused by difficulty in making the correct estimation of the quotient digit. This error may also occur during regrouping process to obtain the remainder. Buswell and John (1926) listed nine errors in remainder difficulties as:

(a) Used remainder larger than divisor
(b) Neglected to use remainder within example
(c) Wrote remainders within example
(d) Omitted final remainder
(e) Used too large a product
(f) Used remainder without new dividend figure
(g) Wrote all remainders at end of example
(h) Added remainder to quotient
(i) Added remainder to next digit of dividend

Grossnickle (1936) identified another two remainder errors not listed before:

(a) Final reminder reduced in fraction form and numerator of fraction written as remainder

(b) Last partial dividend written as a remainder

Grossnickle noted that remainder error was the second most frequent error, next only to basic fact errors. This error constituted about 24% of the total errors made. In particular, he found that "leaving a remainder bigger that the divisor" was the most frequent error in remainder difficulties. This error was sometimes followed by error in "using remainder as a new partial dividend without the new dividend figure".

Lim (1980) also found that "using remainder greater than the divisor" was the most frequent error in division in his study. The NAEP of 1980, showed that 5% of the 13 -year-olds did not include the remainder as part of the answer for the question $6\ \overline{)608}$ (cited by McKillip, 1981). Stefanich and Rokusek (1992) found that approximately 11% of the fourth-graders in his study committed remainder errors.

Regrouping is sometimes necessary in computing the remainders within the computation and final remainder. Some regrouping errors that occur in subtraction of partial product from partial dividend are: "omitting carried number" and "carried the wrong number". Regrouping errors that occur in bringing down dividend digit include: "failure to bring down new dividend digit" and "bringing down two digits at a time". Engelhardt (1977) found that 22% of the fourth-graders made errors in regrouping in the four operations. Stefanich and Rokusek's (1992) study indicated that errors in regrouping accounted for 25 % of the errors.

Remainders were often wrongly interpreted by children (Lankford, 1974). Studies on problem solving in realistic problem situation indicated that the presence of remainders increased problem difficulty (Burton, 1992).

2.3.4 Zero Errors

Pupils are often confused in the use of zero in computation. Buswell and John (1926) identified five zero errors as:

(a) Omitted zero resulting from another digit
(b) Omitted zero resulting from zero in dividend
(c) Added zero to dividend when quotient was not a whole number
(d) Wrote rows of zeros
(e) Dropped zero from divisor and not from dividend

To further differentiate between the different zero errors, Grossnickle (1936) constructed his test in such a way that zero occurred in five different positions in dividends and quotients. Zeros were found: (i) final in the quotient only, (ii) final in dividend only, (iii) final in both dividend and quotient, (iv) not final in quotient only, and (v) not final in dividend only. He listed fifteen errors resulting from zero (in decreasing frequency of occurrence) as:

(a) Omitted final zero in quotient

(b) Zero final in quotient only, final dividend figure was written for quotient figure

(c) Omitted final zero in both dividend and quotient only

(d) Omitted zero not final in quotient only

(e) Extra zeros written in quotient because each remainder treated as a new partial dividend

(f) After zero not final in quotient only, dividend disregarded and zero written in quotient

(g) After zero not final in quotient only, dividend written as remainder

(h) Omitted zero in the quotient when final in both dividend and quotient, but added zero as a remainder

(i) Zero not final in quotient only, dividend figure written in quotient

(j) Zero final in quotient only, extra zero annexed to dividend

(k) Example completed only to zero, not final in quotient only

(l) Remainder not carried to next dividend figure when zero was not final in quotient only

(m)Zero written for remainder when zero was not final in quotient only

(n) Last two quotient figures interchanged when zero was final in both dividend and quotient

(o) Zero and next quotient figures interchanged when zero was not final in quotient only

His findings indicated that the greatest number of zero errors occurred when "zero is final in the quotient only", as in error (a), (b), (j), and (n). Among these errors, "omitting final zero in quotient only" occurred most frequently and accounted for about 50% of the total zero errors.

Another common zero error is "omitted zero not final in quotient". The report on numeracy by City and Guilds of London Institute mentioned that when a zero appeared between two digits (embedded zero) in the answer of a division computation, 49% of the candidates omitted it (cited by Barr, 1983). Barr also noted the study by the Mathematics Education Group at Brunei University on "the Embedded Zero task" in the division operation. The findings indicated that for the question 4669 ÷ 23, about 38% of the subjects aged between 13+ to 27+ gave the incorrect answer of 23 when they omitted the embedded zero in the quotient. The NAEP report of 1980 also found that approximately 4% of the students omitted the zero in the quotient in the question $28\overline{)3052}$, and 3% did the same for $6\overline{)608}$ (cited by McKillip, 1981). McKillip further suggested that the error may be due to the inaccurate placement of quotient digits or simply forgetting to write the zero.

Engelhardt (1982) suggested that errors in omitting zeros might be caused by difficulty with concept of zero as placeholder. He cited the following examples of zero errors involving zero as placeholders:

(a) Omitting place-holding zero in dividend

```
       1 1 R 6
12 )1308
     12
      18
      12
       6
```

In this example, the embedded zero in the dividend was omitted.

(b) Inserting zero as placeholder unnecessarily

```
    30
 2 )6
    6
    0
    0
    0
```

An extra zero was added to the quotient as placeholder.

Some "zero errors" are caused by confusion over the identity element involving zero (the identity element for addition) and one (the identity element for multiplication) in the four operations (Engelhardt,1977; Lankford, 1974). Stefanich and Rokusek(1992), in particular, found that zero errors due to confusion of zero and one as identity element contributed to approximately 18% of the systematic error made by the fourth-graders.

2.3.5 Inversion of Order

The algorithm in division computation involves many steps. Moreover, the students need to adhere to the order in the sequence of numbers in carrying out the various computational steps. Roberts (1968) found that many pupils had not yet internalised the concept of order of sequence to develop proficiency in multi-step processes. He cited the following examples that showed inappropriate inversion of order:

(a) Reversed subtrahend and minuend:

Reversing subtrahend and minuend was a common error found in subtraction computation, as shown in the following example,

$$\begin{array}{r} 332 \\ \underline{-175} \\ 246 \end{array}$$

The pupil subtracted in each column in the direction, which offered the least difficulty. In this case, he subtracted the smaller digit from the bigger digit regardless of whether it was minuend or subtrahend. He had avoided the regrouping step (Ashlock, 1976; Reisman, 1978; Roberts, 1968).

(b) Reversed divisor and dividend:

Roberts also found that in division computations, the single-digit divisor was often used as a dividend or alternated as a divisor or dividend as in the following example:

$$\begin{array}{r} 421 \\ 8\overline{)248} \end{array}$$

By inversion of the divisor and dividend, that is 2 ÷ 8 to 8 ÷ 2, the pupils obtained a quick response and avoided regrouping.

Other studies also supported these findings (Engelhardt,1977; Grossnickles, 1936; Lankford, 1974). Engelhardt, in particular, found that in computations involving the four operations, inappropriate inversion occurred in 21% of the fourth-graders.

2.4 Individual Interviews in Diagnosing Errors

Although written paper and pencil test is an important tool in diagnosing a pupil's error in computation, individual interview has the added advantage of enabling a teacher to explore the thinking process associated with a pupil's computation. Error patterns displayed by students are sometimes caused by using incorrect rules or procedures in computations (Rudnisky et al., 1981). These incorrect rules used may not be discernible in their written computations. Individual oral interviews are sometimes necessary to uncover the sources of errors (Cox, 1975; Liedtke, 1988; Shaw and Pelosi, 1983).

Lankford (1974) interviewed 176 seventh-grade pupils to explore the computational strategies used by the pupils. He suggested that knowledge of a pupil's thinking as he compute might be successfully determined by employing carefully conducted interviews. From his study, he produced a list of guidelines for teachers in conducting individual interviews with the pupils.

Cox (1975) suggested that oral interviews should be conducted after preliminary analysis of computational errors based on a written test. The teacher should encourage the pupil to tell what he thinks as he works the problem.

According to Liedtke (1988), an interview setting can furnish specific and detailed information about a student's cognitive strengths and weaknesses besides his affective needs. It is important to find out what the student does not know, or what has been learned incorrectly. Moreover, interview has the added advantage over written test because various adjustments can be made during the interview. He also suggested that interview protocol should use questions that begin with "Show me how you would..." or "Try to...", rather than "Can you..." which tend to get the response "No!" Furthermore, an interviewer should also avoid comments such as "Look again." and "Are you sure?" that hinted of incorrect response.

Rudnistky et al. (1981) recommended using dialogues or "talking mathematics" with children. They cited four ways of locating a proper level of difficulty and adjustment of content in an interview:

(a) *Illustration*, which involves asking students to represent or define a concept in terms that are more concrete than those used in an earlier response.

(b) *Redirection*, which entails changing the content of a line of questioning or adjusting the difficulty of the content.

(c) *Particularization*, which implies the use of an example to help the student to explain a procedure.

(d) *Generalization*, which comprises an attempt to elicit a rule for a specific type of computation and apply the rule to a different example.

These findings indicated that the search for computation errors must go beyond the standard paper and pencil test to determine the thinking processes involved in the computations.