# OPTIMAL CONTROL OF A RUMOR PROPAGATION MODEL WITH DIFFERENT PROPAGATION DEGREES IN SOCIAL NETWORK

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RESEARCH REPORT SUBMITTED TO THE
FACULTY OF ENGINEERING
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**OPTIMAL CONTROL OF A RUMOR PROPAGATION MODEL WITH DIFFERENT PROPAGATION DEGREES IN SOCIAL NETWORK**

## ABSTRACT

Rumor is a social interaction of information, and its development is of great significance to human beings. In this paper, by studying the D-K model and a rumor model spreading with rumor latent period, deduces the rumor model with different propagation degrees of the spreaders. The two equilibrium points in the system are found through derivation. In real life, enterprises often ignore the reasonable planning of the cost of rumor control. By means of public education and media technology using by the authorities to debunk rumors, an optimal control problem is established. The Pontryagin's maximum principle is combined with the Hamiltonian function. Bang-bang control enables the linear control set U to get the optimal solution in the nonlinear cost problem. MATLAB was used for simulation, and the visualization results of the rumor model were obtained. Finally, the research on rumor is summarized to optimize the cost of rumor control.

Keywords: rumor model; equilibrium point; maximum principle; Bang-Bang control; simulation.

**KAWALAN OPTIMUM MODEL PEMBIAKAN KHABAR ANGIN DENGAN PEMBIAKAN S DALAM RANGKAIAN SOSIAL**


**ABSTRAK**

Khabar angin interaksi sosial maklumat, dan pembangunannya adalah besar kepada manusia. Dalam kertas ini, dengan mengkaji model D-K dan model khabar angin khabar angin yang tersebar dengan tempoh spacent, deduces model khabar angin dengan darjah pembiakan sppembaca. Kedua-dua titik snum dalam sistem dijumpai rhoi rheo. Dalam kehidupan sebenar, Syarikat sering mengabaikan perancangan yang munasabah kos kawalan khabar angin. Dengan cara pendidikan awam dan teknologi media dengan menggunakan pihak berkuasa untuk membuat khabar angin, masalah kawalan yang optimum ialah seicrin. Semua prinsip Pontyagin digabungkan dengan fungsi Hamiltonian. Letupan-letupan kawalan ke atas kawalan linear menetapkan anda untuk mendapatkan penyelesaian yang optimum di strus. MATLAB telah digunakan untuk simulasi, dan Sai res s model khabar angin oleh waswed. Akhirnya, kajian ke atas khabar angin adalah periuk untuk mengoptimumkan kos kawalan khabar angin.



Kata kunci: model khabar angin; titik keseimbangan; prinsip maksimum; Letupan-letupan kawalan; Simulasi.

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## LIST OF SYMBOLS AND ABBREVIATIONS

| | | |
|---|---|---|
| RTO | : | Real-time optimization |
| RFE | : | rumor-free equilibrium |
| REE | : | rumor-endemic equilibrium |

## LIST OF APPENDICES

# CHAPTER 1: INTRODUCTION

## 1.1 Background of study

Rumors are thought to be 'brain sensations'. As a typical social phenomenon, rumors exist in every aspect of life. Especially in emergencies and various crises, the role of rumors should not be underestimated. Traditional rumors are spread from person to person. In the past 20 years, electronic information technology and Internet technology have made rapid development, and the existence form, propagation way and propagation means of rumors have undergone fundamental changes. The rapid development of the Internet does bring a lot of convenience to our life, but it also makes information spread quickly through the Internet. This includes a wealth of misinformation. From Facebook to Twitter, people share all kinds of false images.

During the COVID-19 pandemic in 2020, rampant rumors forced countries to establish mechanisms to check and combat fake news, and even the World Health Organization had a regularly updated, fact-busting web page. Interestingly, in the face of false information, there is an unexpected "equality of all", even scholars and intellectuals with a solid academic foundation seem not to be immune to disinformation. During the epidemic, the news broke out in China that shuanghuanglian oral liquid could resist disease and inhibit bacteria, and had a preventive effect on COVID-19. After that, shuanghuanglian was sold out by all the major pharmacies in China during the period. We are well aware of the damage that disinformation can do to our society, but we still believe in it inescapably, even the so-called "smart people".

What makes rumors such force majeure in the Internet Age. Part of the problem stems from the nature of the rumors themselves. We are bombarded with information every day, so we often rely on our intuition to determine whether information is true or not. Disseminators of fake news often use some simple techniques to make information feel "real." This prevents us from using critical thinking to verify the authenticity of sources. Eryn Newman of The Australian National University has shown that attaching a picture to an article increases confidence in its accuracy, even if it is unrelated to the article's content. For example, a normal picture of a virus appears alongside the text of a new treatment. The picture does not prove the article itself, but it helps people visualize the general situation. So we see this "processing fluency" as a sign of being right. Even the simple repetition of a sentence, whether it's the same paragraph of text or multiple pieces of information, can increase "authenticity" by increasing familiarity. And people mistake that familiarity for authenticity. So the more we see in our news feeds, the more likely we are to think it's real, even if initially skeptical.

As early as 1947, American psychologist Gordon Willard Allport believed that there are two basic conditions for rumor: first, the content of rumor must have some importance to the listener and the rumormonger. Secondly, the real thing must be covered up with some vagueness. Importance means that if a rumor has no significant impact on people's lives, people will be less inclined to spread it . Ambiguity means that the less easily a rumor can be seen, the harder it is to tell the truth, the easier it will spread, similar to the incubation period in epidemiology. The more difficult it is to catch the symptoms of an epidemic before it develops, the more harmful it is. After that, Allport and Bosman give a rumor determination formula: rumor = importance × ambiguity; Then, in 1953, Crosfield revised it, arguing that rumor = importance ×

ambiguity × public critical power. As importance and ambiguity approach zero, rumors are no longer threatening. This surface, if people think a lot, can tell whether more information shared is true or false. As the saying goes, "Rumors stop with wise men."

Therefore, rumors can be controlled. When rumors rapidly spread on social networks, we usually adopt two methods to control the spread of rumors: control the channels through which rumors are spread and the release of authoritative information by public authorities. However, these two methods both require high cost in daily life. Control the propagation channels means stop the rumor from reaching out to influential people. On the one hand, the official release of authoritative information means the invocation of a large number of social resources, such as network resources, human resources and media resources. On the other hand, this requires us to spend a lot of energy in cooperation and coordination with media companies.

Large-scale studies of the rumor problem began in World War II, although their spread showed great similarities to infectious diseases. However, unlike a large number of studies on infectious disease models, researches on the dynamic mechanism of rumor propagation are very limited. Therefore, the study of complex network rumor propagation will have far-reaching significance in the coming decades.

## 1.2 Problem Statement

From the above, it can be seen that the spread of rumors is very complicated with various ways. Some rumors are made by people who are in trouble or who have made unsubstantiated statements without malice. This is largely due to their misunderstanding of something. However, there are also some rumors spread by people with ulterior motives. Some are seeking their own selfish interests, some are grandstanding, and some are spreading malicious remarks for their own political purposes. What's more, it is for hurtful words. Seemingly reliable information seems to come from a real message. Moreover, because of the cognitive, psychological and stance bias of each person, the spread of rumors has different influences on each person. No matter for what reason, who is spreading rumors, whether intentionally or unintentionally, the spread of rumors is of great harm to social media.

The harm of rumor is as follows:

1. It will cause social shock, endanger the public security and harm the public interest.
2. Disturb people's mind, psychological and behavior.
3. Destroy the credibility of the government, damage the government image.

Due to the destructive power of rumors that people try to control rumors in various ways. However, in the process of controlling rumors, there is little research on the cost of controlling rumors. Often an execution strategy requires a high execution cost, if the cost of disseminating the truth is reduced is a very necessary research problem.

This paper will aim at the cost of rumor control, using a special control method, so as to achieve the purpose of minimizing the cost of rumor propagation control and Increase strategic returns. Meanwhile, based on the previous research, the rumor model is further complicated. Someone who spread the rumor will first in a latent period, thinking whether spreading the rumor or not. People who spread the rumor have different propagation degrees.

### 1.3 Objective

The main objectives of this work:

1. To analyze the D-K rumor model and ABCD-type model, and improve the models by adding in different degrees of rumor spreaders.
2. To analyze the equilibrium points of the improved ABCDE-type model.
3. To add proportional control variables, and analyze the property of the ABCDE-type model under the control.
4. To optimize the benefit function of rumor control in the expected time by Bang-Bang control in order to minimize the cost of rumor control.

### 1.4 Scope of study

The scope of this work is to establish a rumor propagation model with propagation latency and different propagation degrees through the original model. Moreover, the control factors of intervention are added to the new model to obtain the cost and benefit of rumor propagation through investigation. The optimal control method can minimize the cost of rumor propagation and reduce the number of rumor propagation.



# CHAPTER 2: LITERATURE REVIEW

### 2.1 D-K Model

As early as in the 1960s, Daley and Kendall put forward the mathematical model of rumor propagation in the form of published paper. The Later researchers named it D-K model. The model analyzes rumor propagation by means of stochastic process. The D-K model assumes a closed population of N(t), the population is assumed to be uniformly distributed. The closely related populations were divided into three kinds of people, they are ignorants, spreaders, and stiflers. A(t) stands for the population of ignorants at some time, which means people who haven't heard or been exposed to a rumor. B(t) stands for the number of spreaders at one time; it stands for those who have heard the rumor and are continually reading it to others. C(t) represent for the population of stiflers at some time. It means someone who has heard a rumor and confirmed that the rumor is not true and no longer spreads it. The A(t) + B(t) + C(t) = N(t). The model also assumes that propagation of the rumor occurs between ignorants and spreaders. When a 'spreader' contacts an 'ignorant ', the 'ignorant' person becomes infected and becomes a propagator. When two spreaders contact each other, they lose interest and become a 'stifler'. Stiflers are defined as no longer spread.

Next, Cintron-Arias and Castillo-Chavez give the following deterministic version of the D-K model:

$$\begin{cases} \dfrac{dA}{dt} = -a_0 \dfrac{AB}{N} \\ \dfrac{dB}{dt} = a_0 \dfrac{AB}{N} - b_0 \dfrac{B(B+C)}{N} \\ \dfrac{dC}{dt} = b_0 \dfrac{B(B+C)}{N} \end{cases} \quad \ldots\ldots ①$$

This model is good for explaining the D-K model, because it's a deterministic model.

But this model also has many disadvantages, such as no inflow or outflow of classes. In addition, the model ignores the influence of individual personality on rumor propagation. We assume that one ignorant will immediately become a propagator after being exposed to the rumor, ignoring the time to judge whether the rumor is true or false. The model also assumes that when you hear a rumor, you will spread it. It ignores the possibility of not wanting to spread. When the spreader becomes a stifler, even if the rumor is heard again, it will not be spread again. Although this model still has many shortcomings, it still has far-reaching significance for the advancement of rumor research.

## 2.2 D-K Model with latent period

An paper published by Liangan Huo, Tingting Lin, Chongjun Fan, ChenLiu and JunZhao in 2015 named 'Optimal control of a rumor propagation model with latent period in emergency event' propose a new model for us. This model is derived from the D-K model which has four different classes. Their model is more general than the D-K model. The model suggests that it takes time for people to consider , from hearing rumors to spreading them. Not everyone spreads rumors. Because different people have different personalities, they behave differently after hearing rumors. Some people believe that rumors are 'truth' and these people actively spread rumors. Others think that rumors are not credible and they choose not to spread rumors after thinking about it. The model also takes into account the effect of class inflow and outflow on the system, including the emigration population and the emigration population.

The following is the detailed explain of the model：

The model is a result of ABCD type, which represents four different groups: the ignorant group, the latent group, the spreader group and the stifler group. Each population at time t is defined as A(t), B(t), C(t), D(t). Class A represents for those who have not been exposed to the rumor, which is called ‘ignorants’ ;Class C represents for those who is spreading the rumor, they are called ‘spreaders’; Class D is the population for someone who has been heard the rumor and no longer spreading it. They are called ’ stiflers‘. On this basis, a latent class B is added, which means that a person will enter an incubation period after hearing a rumor, consider the truth of the rumor, and decide whether to spread it. At time t the total population is defined as N (t).Then N(t)=A(t)+B(t)+C(t)+D(t).

The transformation of the relationships between the classes is shown in the figure below:

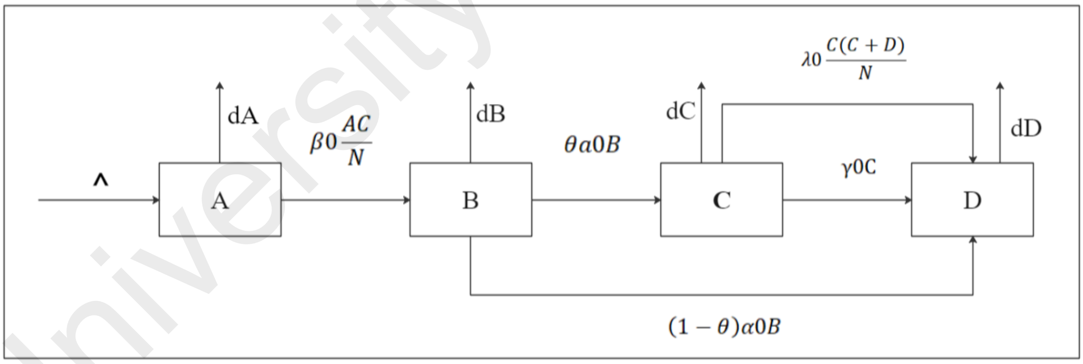



**Figure 2.1: State diagram of ABCD Model**

As can be seen from the figure, the model is no longer a model of closed population. We assume that there are some people flowing into the model all the time, and there are some people flowing out from all classes, Λ(t) is a positive number indicating the number of people flowing into the system at time t, that is the population of immigration; The positive constant $\mu \in (0,1]$ is for emigration rate, then we can easily get, the system tends to equilibrium when $\Lambda(t)-\mu N(t)=0$. In this model, the rumor-hierarchy changes only when someone is contacted with a spreader. When the ignorant and a spreader contact, we assume that the transmitter of rumors at a constant rate, an ignorant will enter an incubation period after hearing the rumor to judge the correctness of the rumor. Within a very small time interval (t, t+ Δt), $\beta 0 \frac{AC}{N}$ ignorants will change their class to latent-group, where $\beta_0 \in (0,1]$ is an positive value which represent the changing rate from ignorant-group. People in the latent-group need time to think about whether the rumor is true or not. Some people think the rumor is true, and they become transmitters. Some people think the rumor is false, they stop spreading the rumor and become the stifler. $\theta \in (0,1]$ indicating the rumor belief rate. $\alpha_0 \in (0,1]$ is an positive value which represent the changing rate from latent-group. When two spreaders contact each other, we assume that they will spread rumors to each other until both of them get tired, and eventually they both lose interest and become stiflers. When a spreader is contacted by a stifler, the stifler knows that the rumor is false, does not continue to spread the rumor and informs the spreader, thus the spreader eventually becomes the stifler. The conversion equation for the spreader group should be $\lambda 0 \frac{C(C+D)}{N}$, $\lambda_0$ represent the changing rate from spreader-group. With the advent of the Internet era, media has become an efficient and effective way to refute rumors, and we can get relevant information through social media in many cases. Set $\gamma_0$ as the media rate, some people will lose interest in rumors through media, the population is $\gamma_0 C$.

Their ABCD model is an optimization of the D-K model. During the period, it is assumed that the propagation content of rumors is unchanged, that is, rumors will spread at any time and any place, then we can get the equation is the rumor system:

$$\begin{cases} \dfrac{dA}{dt} = \Lambda - \beta_0 \dfrac{AC}{N} - \mu A \\ \dfrac{dB}{dt} = \beta_0 \dfrac{AC}{N} - \alpha_0 B - \mu B \\ \dfrac{dC}{dt} = \theta \alpha_0 B - \lambda_0 \dfrac{C(C+D)}{N} - \gamma_0 C - \mu C \\ \dfrac{dD}{dt} = (1-\theta)\alpha_0 B + \lambda_0 \dfrac{C(C+D)}{N} + \gamma_0 C - \mu D \end{cases} \quad \text{......②}$$

However, their model also has shortcomings, that is, it does not consider the impact of the intensity of rumor propagation on the system.

In our daily life, different people treat rumors in different ways. Some people will only say a few words after hearing a rumor, while others will spread the rumor to each other, depending on the character of the rumor monger. In addition, the importance of rumors to the parties involved is also different, which greatly affects the propagation intensity of the spreaders. In order to overcome it, when I build my own model, I further subdivide the process of spreaders into light disseminators and intense disseminators. Light disseminators are less harmful to the society, while intense rumor disseminators may cause panic and do more harm to the society. Therefore, these people must be strictly controlled. In chapter 3, I will specifically analyze and explain my model.

## 2.3 Potragyin's Maximum Principle

The maximum principle of Pontryagin is also called the minimization principle of Pontryagin. The theory of optimal control is to find the optimal control signal from one state to another in the condition of limited input control. The theory was proposed in 1956 by Lev Pontryagin, a Russian mathematician, and his students. This is a special case of The Euler-Lagrange equation in the variation method.

### 2.3.1 Euler-lagrange equation

Variation method is a method of researching functional extremum. For example, the speed drop problem: how to design a slide to slide from the top to the bottom in the shortest time. These are the two points that are not on the same vertical plane, and they can form an infinite number of curves, one of these curves is the optimal solution. Due to the velocity is different from time to time. To analyze this problem, we can solve it through Euler-Lagrange equation:

$$\frac{\partial L}{\partial g} = \frac{d}{dx}\frac{\partial L}{\partial g'} = 0 \ldots\ldots ③$$

It is the core of variation method, both fixed boundary and movable boundary cannot be separated from it.

### 2.3.2 Pontragyin's Maximum Principle

In simple terms, this theorem means that in all possible controls, it is necessary to make the "control Hamiltonian" an extreme value and find an appropriate particular solution U* in all possible control sets U. This theorem states that the optimal control U* must satisfy the following conditions:

$$H(x^*(t), u^*(t), \lambda^*(t)) \equiv constant \ldots\ldots ④$$

If the final time is unlimited, then:

$$H(x^*(t), u^*(t), \lambda^*(t)) \equiv 0 \ldots\ldots ⑤$$

If the Maximum value principle of Pontriagin is satisfied on a certain trajectory, the principle must determine an optimal solution (a necessary condition). The Hamilton-Jacobi-Behrman equation provides sufficient and necessary conditions for the optimal solution, but this condition must be true in the whole state space.

### 2.3.3 Transversal conditions

The establishment of transversal conditions is often a problem encountered in the optimal control problem. People study the optimal control problem because of the actual background of variation principle in many fields of society and nature.

The society pursues the benefit, when the input is certain, hopes to produce the maximum or if you have a certain amount of output, you want the minimum input. When we use Pontryagin's maximum principle to define a Hamiltonian function, the transversal condition must be considered if the boundary is not fixed.

However, the constraint of transversal conditions is often related to the declaration of the actual problem. In some cases, the function of the state variable must be equal to zero or a fixed value at time T.

By Pontryagin's maximum principle , it is easy to derive the Hamiltonian formula:

The objective function is $V \equiv \int_0^T F(t, y, u)dt \ldots\ldots ⑥$

subject to $\frac{dy}{dt} = f(y(t), u(t)), 0 < t < T \ldots\ldots ⑦$

Use the Lagrange multiplier, the constrained optimization problem can be transformed into unconstrained optimization problem by using Lagrange multiplier method, then:

$$L = \int_0^T [F(t, y, u) + \lambda_t (f(t, y, u) - y_t)]dt, \lambda_t (f(t, y, u) - \dot{y}_t) = 0 \ldots\ldots ⑧$$

**V=L,**

$$L = \int_0^T [F(t, y, u) + \lambda_t f(t, y, u) - \lambda_t \dot{y}_t]dt \ldots\ldots ⑨$$

$$H(t, y, u, \lambda) = F(t, y, u) + \lambda_t f(t, y, u) \ldots\ldots ⑩$$

Use partial integral:

$$V = \int_0^T [H(t, y, u, \lambda) + \dot{\lambda}_t y_t] dt + \lambda_0 y_0 - \lambda_T y_T \ldots\ldots ⑪$$

We can use it to discuss about the transversal condition:

1. Vertical line problem (Based on Chiang pp. 181-184)

A horizontal ending point transversal condition can be consider as the T is fixed but the $y_T$ cannot be determined. For example, you deposit an asset in a bank and you want to withdraw it after three months, but you're not bound by any other conditions on the time you withdraw. When $y_t$ is free, in order to maximum the return on asset V, the shadow of $y_T$ must equal to 0.

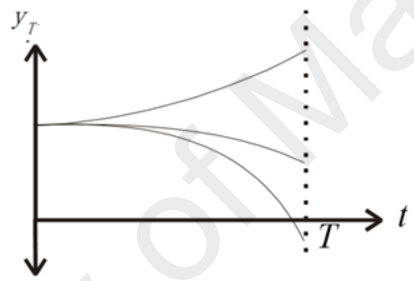



**Figure 2.2: Vertical line condition**

So at the final moment T, if $y_T$ can take any value, then the edge value of a change in $y_T$ must be zero, so $\partial V/\partial y_T = 0$, combine the Previous Hamiltonian formula we can get $\partial V/\partial y_T = -\lambda_T = 0$.

2. Horizontal terminal line

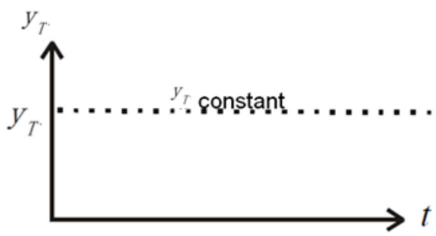



**Figure 3.1: Horizontal terminal condition**

Let's say that in another situation, you put an asset in the bank, you don't limit how long it can be withdrawn, but when you want to withdraw the asset, the asset has to be a certain state. This is a typical horizontal terminal problem where $y_T$ is fixed but T can choose any value. To solve this problem, let's apply the Hamiltonian formula.

$$V = \int_0^T [H(t, y, u, \lambda) + \dot{\lambda}_t y_t]dt + \lambda_0 y_0 - \lambda_T y_T \ldots\ldots ⑪$$

Since T varies, we take the partial derivative with respect to T.

$$\frac{\partial V}{\partial T} = [H(T, y_T, u_T, \lambda_T) + \dot{\lambda}_T y_T] - (\dot{\lambda}_T y_T + \lambda_T \dot{y}_T) \ldots\ldots ⑫$$

As you can see from the graph, the derivative of $y_T$ is zero. We use it to solve the optimal control problem and check the result. Hamiltonian equation should also be equal to 0.

## 2.4 Bang-Bang control

Rod control was first proposed by Pontryagin and belongs to the optimal control theory. It is a common integrated control in engineering field. As a hysteresis control, Bang-Bang control is often used to solve the problem of follow-up system control.

Compared with PID control, PIID control is simple and can achieve effective control even if the object model is not clear, but PID control has poor adaptability to model parameter change and interference. Bang-Bang control plays an important role in the follow-up system with large system deviation, which can increase the control force. Especially for the follow-up system with wide speed range, small static error and fast dynamic response, Bang-Bang control is a good choice. Bang-Bang control can be used in the research of soccer robot motion control, servo system and many other fields. Since their controllers are implemented by providing hysteresis, they are often used to control devices with binary inputs, such as a thermostat that can only be fully on or completely off.

Bang-Bang control can only be used to solve the control variable is a linear function, because Bang-Bang control is always the space state is divided into two areas, a region take the maximum control variables, a regional control variables take the minimum value, in the two areas of interface, control variable can take any value, we call it the 'switch face "of the system. The main thing that determines Bang-Bang control is the selection of the switch surface. Bang-Bang control is often used for maximum speed control problems and minimum fuel economy systems. With the participation of Hamiltonian, the control variable jumps between the minimum and the maximum within the range of a restricted control variable, and the number of times the control variable jumps from the maximum to the minimum is finite.

For example, when the follow-up system needs to carry out the turning motion, the regression with the maximum possible acceleration εm is required at a certain point, in which case the error |εm|<emax. When it reaches a certain point, it needs to decelerate with -εm, and when the speed is 0, the error is also 0, which needs to be done by Bang-Bang control.

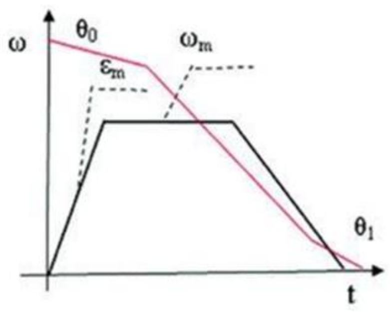



**Figure 2.4: Bang-Bang Control Example**

## 2.5 ODE 45

Ode is a function of Matlab specifically designed to solve differential equations.The solver has two types: variable step and fixed step. Different types have different solvers, among which ODE45 solver is one of variable step size and runge-Kutta algorithm is adopted. Other variable step solvers using the same algorithm are ODE23.

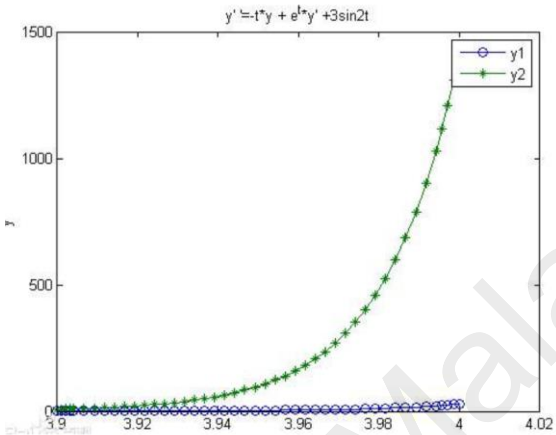



**Figure 2.5: ODE simulation diagram in MATLAB**

# CHAPTER 3: METHODOLOGY

## 3.1 Introduction

Rumors in social network are born in the virtual world, but due to the dissemination of mainstream media, rumors on the Internet reach more audiences. Although rumors abound on the Internet, people's control technology of rumors is not perfect. Like other technologies, the network technology itself is neutral, which will bring some negative effects while providing convenience and services for the society. As for the cost of rumor control technology, many enterprises ignore it. How to maximize the benefits brought by rumor control on the basis of effectively reducing rumor propagation is an issue that every enterprise should consider. Real-time optimization control can play an important role in rumor control systems that need timely response, high reliability and security requirements. In this chapter, we deduced a rumor model with disseminators of different propagation degrees through the accumulation of predecessors, and considered it in the propagation with latent period. The equilibrium points under this model are determined, and the rumor propagation control system within 10 days is analyzed by using the optimal control. At the same time, minimize the cost of rumor control to maximize the benefits of controlling rumor.

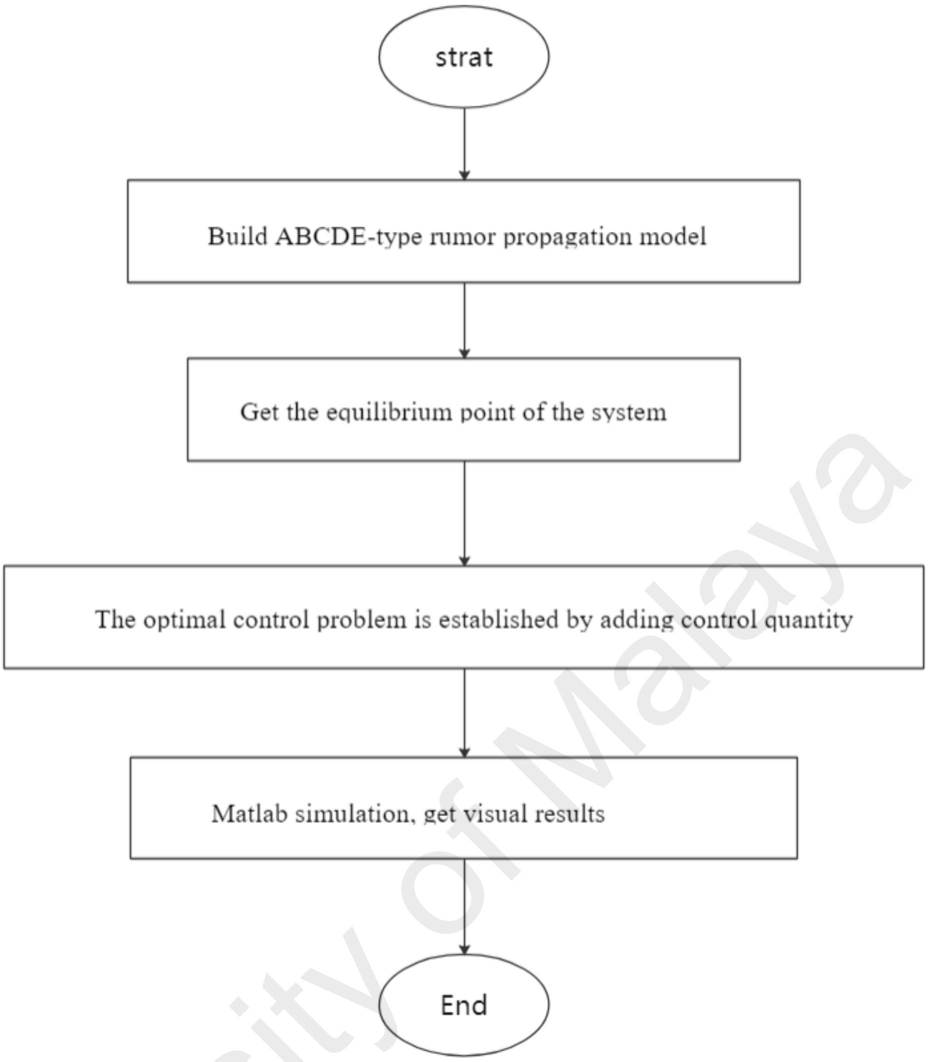



**Figure 3.1: Project flow for rumor model research**



## 3.2 Real-time optimization (RTO)

Real-time optimal strategy is used to study the system with strict requirements on response time. The performance of RTO varies according to the mathematical model established. RTO can ensure that a specific optimization is completed for a specific operating system within a certain time limit.

In this case, the optimizations were targeted at conversion rates between spreaders with different spreading degrees and stiflers, and the conversion rates between people in rumor incubation period and stiflers. In order to spread the truth, it is necessary to optimize the educational and media resources to control rumors at all times. However, the supply of resources is limited, and we cannot guarantee that sufficient resources can be provided at all times.

### 3.3 ABCDE-type Model

In the literature in Chapter 2, we have analyzed the existing models. The rumor propagation model with incubation period, published in 2015, takes into account the fact that people need to think about rumors based on the D-K model, that is, a new group B is considered to represent the population entering the incubation period of rumors. Adding both inflow and outflow classes increases the "liquidity" of the model, which is closer to the truth and greatly develops the D-K model. However, this model also has a shortcoming, that is, it ignores the influence of personality on individuals in the process of rumor propagation. After everyone hears the rumor and thinks about it, some people will choose to spread the rumor, while others decide not to spreading it. In that part of the spreaders, due to individual differences, the mode and intensity of spreaders will be different, so it is very necessary to subdivide the spreaders with different cases. In my model, disseminators are divided into two groups. One group is not affected by rumors, but only spreads rumors slightly. Others are strongly encouraged by rumors and firmly believe and spread them. Or the instigators and initiators of those rumors, they hope to get some benefits from the rumors, this part of the spreaders is more extreme, so the harm to the society is greater. These people must be strictly controlled in order to maintain the healthy development of public opinion. Further discussions will be made in the following process.

In this model, all the people in the rumor model are divided into five groups, namely A (t), B(t), C(t), D(t) and E(t). Each group represents a crowd. I call this model ABCDE-type model. A is for someone who has not been touched by the rumor, which is called Ignorants. B represents the population that heard the rumor and went into the incubation period. C is for the population who is affected by a rumor and spreads it slightly,it is called mild spreaders. D is the population who believes in or fancies rumors, call it severe spreaders. Finally, those who had heard the rumor and stopped spreading it were represented by E. We call this group the stiflers' group. The following diagram can be obtained:

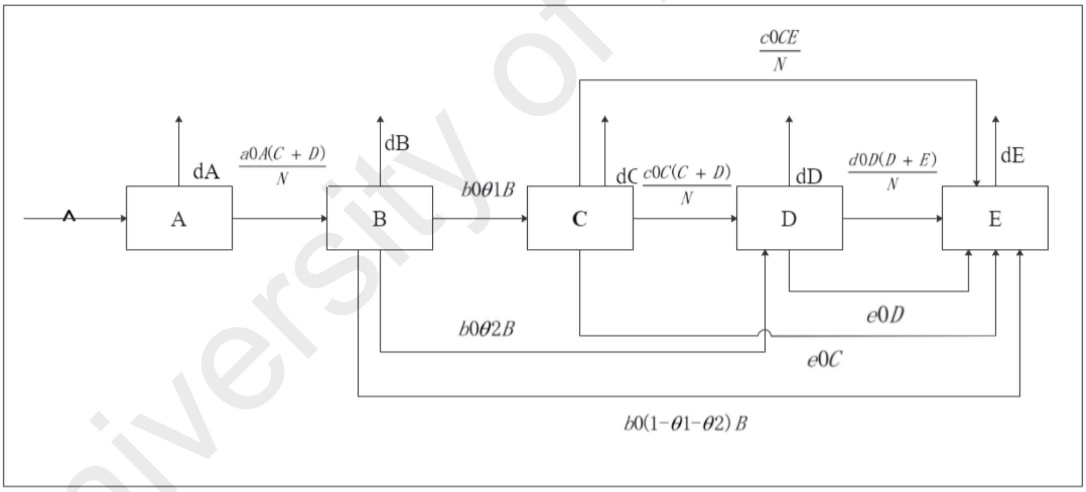



**Figure 3.2: State diagram of ABCDE-type Model**

We assume that the total population of the system is N, N=A+B+C+D+E. The population of immigration at any time is $\Lambda(t)$. We set the emigration rate of the population of the system at any given moment as $\mu(t)$, $\mu(t)\in(0,1]$. We assume that rumors must be spread by contact with people who are in mild or severe case spreading group. A person in the ignorant-group will enter a rumor spreading latent period when they are in contact with a mild spreader or severe spreader, thinking whether the rumor is the truth or not. In a very small time interval(t,t+ Δ t) , there is the population $\frac{a0A(t+\Delta t)[C(t+\Delta t)+D(t+\Delta t)]}{N(t+\Delta t)} - \frac{a0A(t)[C(t)+D(t)]}{N(t)}$ moving from the ignorant group to the latent-group. Where $a_0 \in (0,1]$ is a positive value indicating the conversion rate of ignorants. When an ignorant first entered into the latent-group B, he would go through a period of reflection. Some would consider the rumor to be false, so they would switch to the Stifler-group E and never spreading about it. Some people in the incubation period believe that the rumor is true, so they become spreaders. Under the influence of personality, some people become mild spreaders, while others become severe spreader, represented by $b_0\theta_1B$ and $b_0\theta_2B$ respectively. $\theta_1$ and $\theta_2$ are the conversion rates for becoming mild spreaders and severe spreaders, $0<\theta_1+\theta_2<1$. The population converted between the latency group and the Stifler group are represented by $b_0(1-\theta_1-\theta_2)B$, $b_0$ is a positive value represent the conversion rate from latent-group. When two mild spreaders come into contact, we assume that both parties spread the rumor at a constant rate. The repetition over and over will make both mild spreaders more convinced of the "truth" of the rumor, and both of them will turn into severe spreaders. When a mild and a severe spreader contact, the severe spreader believe the rumors is trust without any doubt. Some severe spreaders are instigators, they hope the others can also believe the rumor without any doubt. Rumors will spread among them at a certain rate, makes the mild spreader over and over again to deepen the impression. The mild spreader believes of the rumor originally, which will make them more convinced that the rumor is "true", and eventually become a severe spreader. Thus the population transition relationship between the mild-group and severe-group can be expressed as $\frac{c0C(C+D)}{N}$, $c_0 \in (0,1]$ represent the changing rate from mild-group. A stifler is a person who has heard a rumor, knows it's not true, and stops spreading it.

We assume that the system will not propagate twice, that is, a stifler will not believe the rumor even if he hears it again after confirming the rumor is not truth. Assume that rumor propagation will occur at any time and under any circumstances in this system, and the content of rumor propagation is constant in the whole process, so that stifler's relationship with others can be easily obtained. When a mild spreader is exposed to a stifler, the mild spreader spreads rumor to the stifler at a constant rate, but the stifler has determined whether the rumor is true or false and not spreading rumors any more.

People who are mild spreader spread the rumor again and again to the stifler, and eventually become tired and stop spreading it, thus becoming a stifler. $\frac{c0CE}{N}$ is the relationship between mild-spreader-group and stifler-group. Similarly, when a severe spreader contacts with a stifler, he will be informed of the truth of the rumor and converted to a stifler. The conversion relationship between them can be expressed as $\frac{d0DE}{N}$ and $d_0$ represents the conversion rate from severe-spreader group.

When two severe spreaders come into contact, we assume that both spreaders transmit rumor at a constant rate. The repeated propagation makes both feel tired, and eventually they stop spreading the rumor and both become stiflers. The development of the Internet has brought about a huge change in the way of media. The emergence of digitalization has given birth to new media technology, especially its influence on rumors in social network cannot be ignored. We introduce a media rate $e_0$. The spreaders will be influenced by the media when rumors spread in the social network.

Those mild and severe spreaders who read the news will be converted to stiflers, denoting the media-affected population in terms of $e_0C$ and $e_0D$ respectively.

Then We can derive that the differential equation for the system is:

$$\begin{cases} \frac{dA}{dt} = \Lambda - \frac{a_0A(C+D)}{N} - \mu A \\ \frac{dB}{dt} = \frac{a_0A(C+D)}{N} - b_0B - \mu B \\ \frac{dC}{dt} = b_0\theta_1B - \frac{c_0C(C+D)}{N} - \frac{c_0CE}{N} - e_0C - \mu C \\ \frac{dD}{dt} = b_0\theta_2B + \frac{c_0C(C+D)}{N} - \frac{d_0D(D+E)}{N} - e_0D - \mu D \\ \frac{dE}{dt} = b_0(1-\theta_1-\theta_2)B + \frac{c_0CE}{N} + \frac{d_0D(D+E)}{N} + e_0(C+D) - \mu E \end{cases} \quad \cdots\cdots ⑬$$

The range of all the parameters is listed as below:

**Table 3.1: Main parameters of ABCDE-type Model**

| | |
|---|---|
| $\theta_1 \in (0,1]$ | Probability of transition from latent period to a mild spreader |
| $\theta_2 \in (0,1]$ | Probability of transition from latent period to a severe spreader |
| $1-(\theta_1+\theta_2) \in (0,1]$ | Probability of transition from latent period to a stifler. |
| $a_0 \in (0,1]$ | The conversion rate from ignorant-group |
| $b_0 \in (0,1]$ | The conversion rate from latent-group |
| $c_0 \in (0,1]$ | The conversion rate from mild-spreader-group |
| $d_0 \in (0,1]$ | The conversion rate from severe-spreader-group |
| $e_0 \in (0,1]$ | The conversion rate from stifler-group |
| $\Lambda \in (0,+\infty]$ | The population of immigration in the system |
| $\mu \in (0,1]$ | The population rate of emigration in the system |

### 3.4 Equilibrium point

In an epidemic model, the population dynamics can be represented by $\frac{dN}{dt} = \Lambda - \mu N$, When t→+∞,the total population N→$\frac{\Lambda}{\mu}$. When N→ $\frac{\Lambda}{N}$ , in other words, the system has reached its maximum population carrying capacity. Let's make a=$\frac{A}{N}$, b=$\frac{B}{N}$, c=$\frac{C}{N}$, d=$\frac{D}{N}$, e=$\frac{E}{N}$, $a_1$=$\frac{a0}{\mu}$, $b_1$=$\frac{b0}{\mu}$, $c_1$=$\frac{c0}{\mu}$, $d_1$=$\frac{d0}{\mu}$, $e_1$=$\frac{e0}{\mu}$, τ=μt, then we can simplify the above differential equation:

$$\begin{cases} \frac{da}{d\tau} = 1 - a_1 a(c+d) - a \\ \frac{db}{d\tau} = a_1 a(c+d) - b_1 b - b \\ \frac{dc}{d\tau} = b_1 \theta_1 b - c_1 c(c+d) - c_1 ce - e_1 c - c \\ \frac{dd}{d\tau} = b_1 \theta_2 b + c_1 c(c+d) - d_1 d(d+e) - e_1 d - d \\ \frac{de}{d\tau} = b_1(1-\theta_1-\theta_2)b + c_1 ce + d_1 d(d+e) + e_1(c+d) - e \end{cases} \quad \cdots\cdots ⑭$$

From the simplified system of differential equations, it can be seen that Λ, μ and N are removed. Where a, b, c, d, e are respectively represent the proportion rate in the rumor system. We can easily get a+b+c+d+e=1. Rewrite τ as t and substitute e in with the formula e=1-a-b-c-d, then can get the rumor subsystem:

$$\begin{cases} \frac{da}{dt} = 1 - a_1 a(c+d) - a \\ \frac{db}{dt} = a_1 a(c+d) - b_1 b - b \\ \frac{dc}{dt} = b_1 \theta_1 b - c_1 c(1-a-b-c-d) - e_1 c - c \\ \frac{dd}{dt} = b_1 \theta_2 b + c_1 c(c+d) - d_1 d(1-a-b-c) - e_1 d - d \end{cases} \quad \cdots\cdots ⑮$$

The subsystem forms a four - dimensional space：

A={(a,b,c,d)∈$R^{4+}$|0< a+b+c+d <1|}

In order to get to the equilibrium point, all differential equations have to be equal to zero:

$$\begin{cases} 1-a_1a(c+d)-a=0 & (1) \\ a_1a(c+d)-b_1b-b=0 & (2) \\ b_1\theta_1b-c_1c(c+d)-c_1c(1-a-b-c-d)-e_1c-c=0 & (3) \\ b_1\theta_2b+c_1c(c+d)-d_1d(1-a-b-c)-e_1d-d=0 & (4) \end{cases} \quad \ldots\ldots ⑯$$

If I take (1)+(2), get $1-a-b_1b-b=0$, then convert the formula: $a=1-(b_1+1)b$……(5),

lug in the formula (5) into formulas (3) and (4), then:

$$\begin{cases} b_1\theta_1b-c_1c(c+d)-c_1c[1-1+(b_1+1)b-b-c-d]-e_1c-c=0 \\ b_1\theta_2b+c_1c(c+d)-d_1d[1-1+(b_1+1)b-b-c]-e_1d-d=0 \end{cases} \quad \ldots\ldots ⑰$$

Simplify the formula above then we can get:

$$\begin{cases} b_1\theta_1b-c_1c(c+d)-c_1c(b_1b-c-d)-e_1c-c=0 & (6) \\ b_1\theta_2b+c_1c(c+d)-d_1d(b_1b-c)-e_1d-d=0 & (7) \end{cases} \quad \ldots\ldots ⑱$$

Substitute formula (6) into (7) to solve c:

$$c=\frac{b_1\theta_1b}{1+e_1+c_1b_1b} \quad (8)\ldots\ldots ⑲$$

By the rumor system definition, $1+e_1+c_1b_1b>0$ is always true. Let's put the equation (8) back into the equation (7):

$$\left(d_1b_1b-\frac{b_1\theta_1b(d_1+c_1)}{1+e_1+c_1b_1b}+e_1+1\right)d=b_1\theta_2b+c_1\frac{(b_1\theta_1b)^2}{(1+e_1+c_1b_1b)^2} \quad \ldots\ldots ⑳$$

$$d=\frac{b_1\theta_2b(1+e_1+c_1b_1b)^2+c_1(b_1\theta_1b)^2}{(1+e_1+d_1b_1b)(1+e_1+c_1b_1b)^2-b_1\theta_1b(c_1+d_1)(1+e_1+c_1b_1b)} \quad \ldots\ldots ㉑$$

The formula above is true when the denominator is not 0. Therefore, we can express each state variable of the rumor subsystem with an algebraic expression of b

$$\begin{cases} a=1-(b_1+1)b \\ c=\dfrac{b_1\theta_1b}{1+e_1+c_1b_1b} \\ d=\dfrac{b_1\theta_2b(1+e_1+c_1b_1b)^2+c1(b_1\theta_1b)^2}{(1+e_1+d_1b_1b)(1+e_1+c_1b_1b)^2-b_1\theta_1b(c_1+d_1)(1+e_1+c_1b_1b)} \end{cases} \quad \ldots\ldots ㉒$$

It is easy to see that the system always passes a fixed point (1, 0, 0, 0). This point is called the rumor-free equilibrium (RFE) of the rumor system. In addition to a common equilibrium point RFE, there is a special equilibrium point which we call it rumor-endemic equilibrium (REE).

Since the calculation of REE is very complicated, we use MATLAB to solve it:

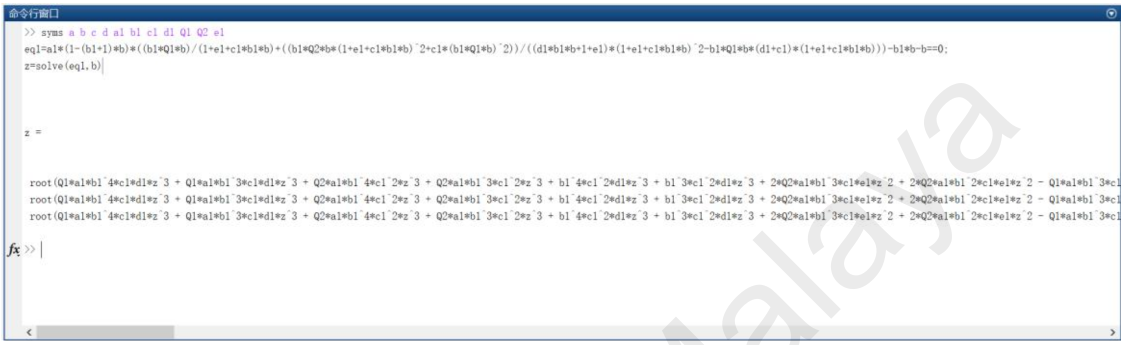



**Figure 3.3: MATLAB calculation**

As can be seen from the figure, REE calculation results are very complicated, each solution has dozens of components, because the highest power of equation b is to the third power, so there are three solutions. But these three solutions have the same magnitude. We use b* to represent the particular solution to b, so b* can be written as $b^* = X^3$, this also proves the uniqueness of REE, REE=(a*, b*, c*, d*).

Since the rumor system is a very complex nonlinear time-varying system, it is difficult to prove the stability of the equilibrium point REE and RFE by lyapunov second method and Lasalle's invariability principle. The stability of the system can be verified by MATLAB simulation and visualization.

First, we need to establish an optimal control problem.

### 3.5 Bang-Bang Control

When rumors break out in social network, they often need some intervention. We call it control. Rumors cannot be created out of thin air. People believe rumors, which are related to their own quality and education level. American communication scholar Teachino once put forward the theory of "knowledge gap", that is, people with high socioeconomic status usually get information faster than those with low socioeconomic status. Thus, the more information mass media transmit, the greater the knowledge gap between the two. Rational analysis of rumors and improvement of population quality play a role in suppressing rumors.$u_1$ is introduced as education rate to expand the model. Suppose that a person entering the incubation period, and gets educated, changes their conversion rate of becoming a stifler.

At the same time, the development of new media is derived in the digital era, new media technology is used to officially refute rumors, resist the spread of rumors, and control the conversion rate between spreaders and stiflers. Set $m=1-u_2$ as the media rate. $U_1$ and $U_2$ are used as the control variables to obtain the governing equation of the system:

$$\begin{cases} \frac{da}{dt} = 1 - a_1a(c+d) - a \\ \frac{db}{dt} = a_1a(c+d) - b_1\theta_1 b - b_1\theta_2 b - u_1b_1(1-\theta_1-\theta_2)b - b \\ \frac{dc}{dt} = b_1\theta_1 b - c_1c(c+d) - c_1ce - (1-u_2)c - c \\ \frac{dd}{dt} = b_1\theta_2 b + c_1c(c+d) - d_1d(d+e) - (1-u_2)d - d \\ \frac{de}{dt} = u_1b_1(1-\theta_1-\theta_2)b + c_1ce + d_1d(d+e) + (1-u_2)(c+d) - e \end{cases} \quad \text{......㉓}$$

Both $u_1$ and $u_2$ are proportional control.

Different intervention amounts will change the equilibrium point of the system. A successful scheme is considered to control rumor propagation with the minimum cost.

Suppose that a control scheme is optimal and its cost-benefit function should meet:

$$W(u_1(t),u_2(t)) = \int_{t0}^{tf}[B_0(a(t)+e(t)) - B_1(c(t)+d(t)) - B_2(u_1(t)+u_2(t))]dt \text{ ......㉔}$$

$B_0$ represents the weight coefficient of the population of the ignorant-group and the stifler-group. $B_1$ represents the weight coefficient of the population of the mild-spreader-group and the severe-spreader-group. $B_2$ represents the weight coefficient of control.

To minimize the cost of rumor control, the revenue function must be maximized:

$$W_{max}(u_1(t),u_2(t)) = \max[W(u_1(t),u_2(t)) \mid u_1(t),u_2(t) \in U] \text{ ......㉕}$$

In this way, we turn the extremum problem of the revenue function into the problem of finding the optimal solution of the control set U with respect to time T, and thus an optimal control problem is established.

The adjoint function is introduced by using the Pontryagin maximum principle.

The problem of maximizing the objective function of the system function is transformed into finding the specific solution of the Hamiltonian or Lagrangian function on the control set. Bang-Bang control optimizes control by pushing the value of the control variable to either the upper or lower boundary. The Hamiltonian should be:

$$\begin{aligned}
H = {} & B_0(a+e) - B_1(c+d) - B_2(u_1+u_2) + \lambda_1(1 - a_1ac - a_1ad - a) \\
& + \lambda_2\left[a_1ac + a_1ad - b_1\theta_1 b - b_1\theta_2 b - u_1b_1(1-\theta_1-\theta_2)b - b\right] \\
& + \lambda_3\left[b_1\theta_1 b - c_1c(c+d) - c_1ce - (2-u_2)c\right] \\
& + \lambda_4\left[b_1\theta_2 b + c_1c(c+d) - d_1d(d+e) - (2-u_2)d\right] \\
& + \lambda_5\left[c_1ce + d_1d(d+e) + (1-u_2)(c+d) + u_1b_1(1-\theta_1-\theta_2)b - e\right] \text{ ......㉖}
\end{aligned}$$

In the Bang-Bang control, $\{u_1(t),u_2(t) \mid u_1(t),u_2(t) \in U\}$ is linear with the slope.

A system of differential equations for the boundary solutions of

$\{u_1(t), u_2(t) \mid u_1(t), u_2(t) \in U\}$ can be listed:

$$\begin{cases} \dfrac{\partial H}{\partial u_1} = -B_2 + (\lambda_5 - \lambda_2) b_1 (1 - \theta_1 - \theta_2) b = 0 \\ \dfrac{\partial H}{\partial u_2} = -B_2 + (\lambda_3 - \lambda_5) c + (\lambda_4 - \lambda_5) d = 0 \end{cases} \quad \ldots\ldots ㉗$$

For the optimal control solution $u_1^*(t)$, $u_2^*(t)$, the relationship of its adjoint variables to the Hamiltonian should be satisfied:

$$\begin{cases} \dot{\lambda}_1 = -\dfrac{\partial H}{\partial a} = -B_0 + (\lambda_1 - \lambda_2) a_1 c + (\lambda_1 - \lambda_2) a_1 d + \lambda_1 \\ \dot{\lambda}_2 = -\dfrac{\partial H}{\partial b} = (\lambda_2 - \lambda_3) b_1 \theta_1 + (\lambda_2 - \lambda_4) b_1 \theta_2 + (\lambda_2 - \lambda_5) u_1 b_1 (1 - \theta_1 - \theta_2) + \lambda_2 \\ \dot{\lambda}_3 = -\dfrac{\partial H}{\partial c} = B_1 + (\lambda_1 - \lambda_2) a_1 a + (\lambda_3 - \lambda_4) c_1 (2c + d) + (\lambda_3 - \lambda_5) [c_1 e + (1 - u_2)] + \lambda_3 \\ \dot{\lambda}_4 = -\dfrac{\partial H}{\partial d} = B_1 + (\lambda_1 - \lambda_2) a_1 a + (\lambda_3 - \lambda_4) c_1 c + (\lambda_4 - \lambda_5) [d_1 (2d + e) + (1 - u_2)] + \lambda_4 \\ \dot{\lambda}_5 = -\dfrac{\partial H}{\partial e} = -B_0 + (\lambda_3 - \lambda_5) c_1 c + (\lambda_4 - \lambda_5) d_1 d + \lambda_5 \end{cases} \quad \ldots\ldots ㉘$$

Next, we will analyze the transversal conditions of the system, and it can be easily analyzed that the system's transversal condition satisfies the vertical line problem.

The system will reach a new equilibrium after going through the control process, and the numerical value of each group of the rumor model will remain unchanged when reaching the new stability. In other words, the value of each state variable does not change as it reaches the new steady state, but the time to reach system stability is free and varies. So $\partial W/\partial a_T=-\lambda_T=0$, $\partial W/\partial b_T=-\lambda_T=0$, $\partial W/\partial c_T=-\lambda_T=0$, $\partial W/\partial d_T=-\lambda_T=0$, $\partial W/\partial e_T=-\lambda_T=0$.

Finally, to explain how to use Bang-Bang control to find the solution that controls U1, U2, we need to use optimization techniques.

$\{u_1(t), u_2(t) \mid u_1(t), u_2(t) \in U\}$

Since $u_1$ and $u_2$ are linearly related to the system function, we can determine the value of the control quantity by taking the positive and negative of the partial derivative of the Hamiltonian with respect to the control quantity. $u_1$ and $u_2$ are proportional controls, and range from 0 to 1. then get the following two conditions:

1. $\frac{\partial H}{\partial u} > 0$, u=umax

$u_1^*(t) = u_1max$; u2*(t) =u2max.

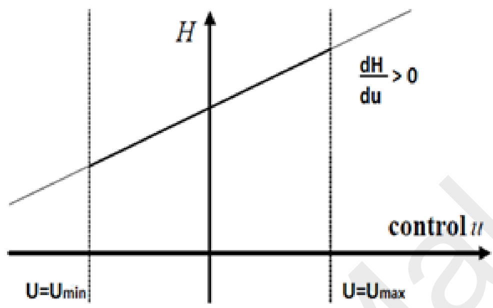



**Figure 3.4: Bang-Bang control principle diagram (1)**

2. $\frac{\partial H}{\partial u} < 0$, u=umin

$u_1^*(t) = u_1min$; $u_2^*(t) = u_2min$

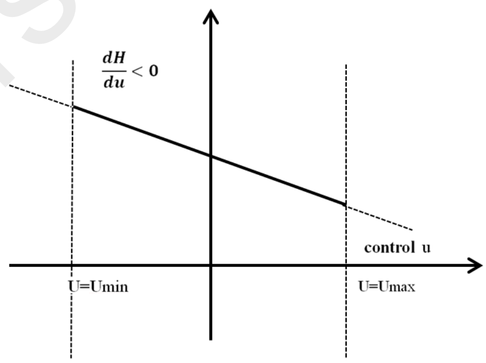



**Figure 3.5: Bang-Bang control principle diagram (2)**

# CHAPTER 4: RESULT AND DISCUSSION

## 4.1 Matlab simulation without control involved

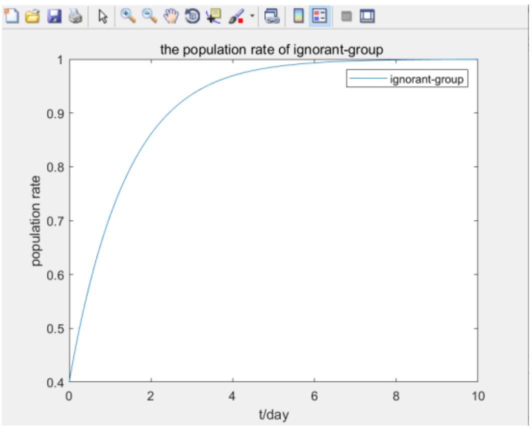
### 4.1.1 Parameters use in simulation

**Table 4.1: Parameters of ABCDE-type Model without control**

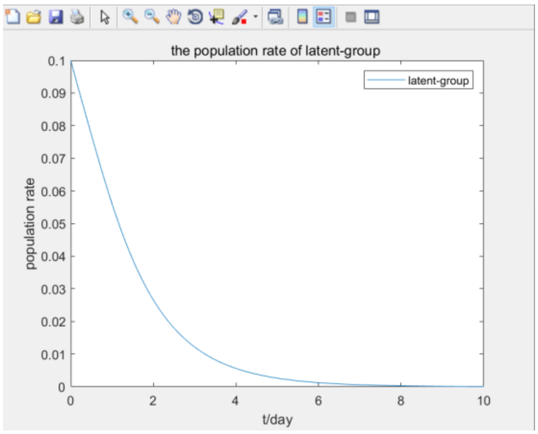
| | |
|---|---|
| $a0 \in (0,1]$ | The conversion rate from ignorant-group |
| $b0 \in (0,1]$ | The conversion rate from latent-group |
| $c0 \in (0,1]$ | The conversion rate from mild-spreader-group |
| $d0 \in (0,1]$ | The conversion rate from severe-spreader-group |
| $e0 \in (0,1]$ | The conversion rate from stifler-group |
| $\mu \in (0,1]$ | The population rate of emigration in the system |
| $\theta 1 \in (0,1]$ | Probability of transition from latent period to a mild spreader |
| $\theta 2 \in (0,1]$ | Probability of transition from latent period to a severe spreader |

### 4.1.2 Result and analysis

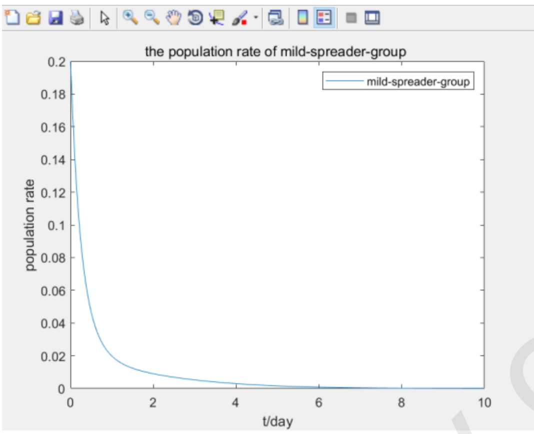
The ABCDE-type rumor propagation model is researched without Bang-Bang control in 10 days. Under the setting of different parameters, the natural changes of each group over time are shown as follows:



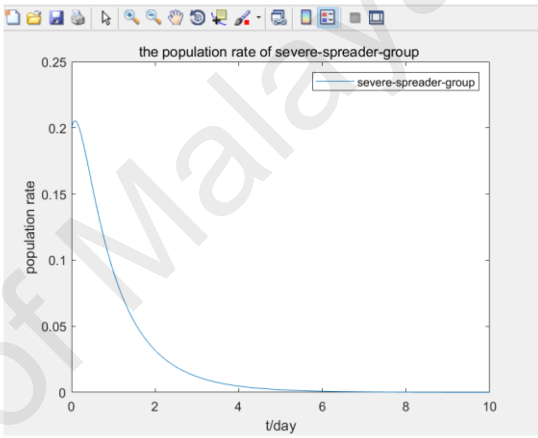

(a)



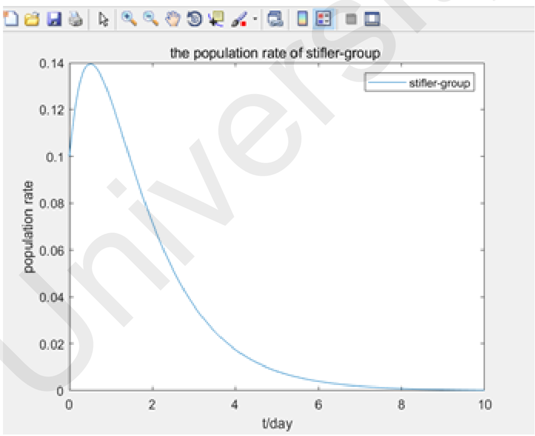

(b)




(c)




(d)

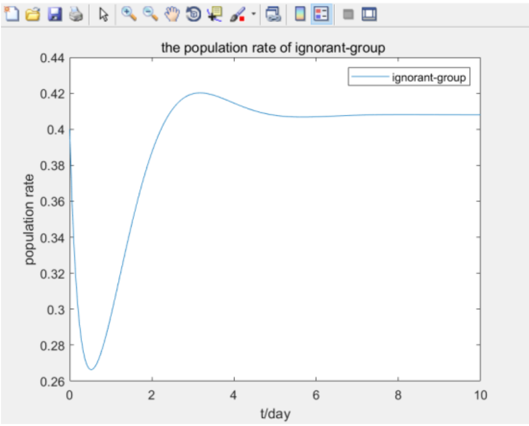



(e)

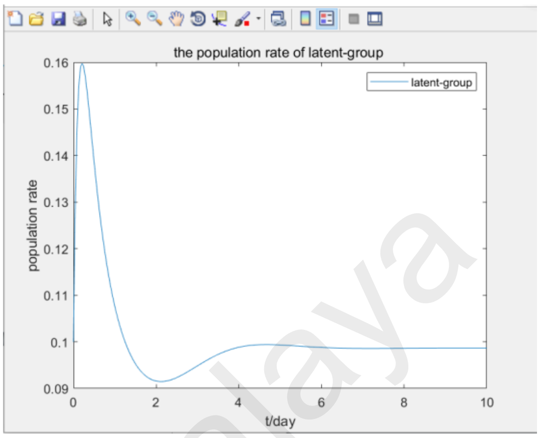
**Figure 4.1: Simulation result of ABCDE Model without control (1)**

$a_0$=0.1,$b_0$=0.2,$c_0$=0.5,$d_0$=0.05,$e_0$=0.03,μ=0.1,$\theta_1$=0.4, $\theta_2$=0.3.

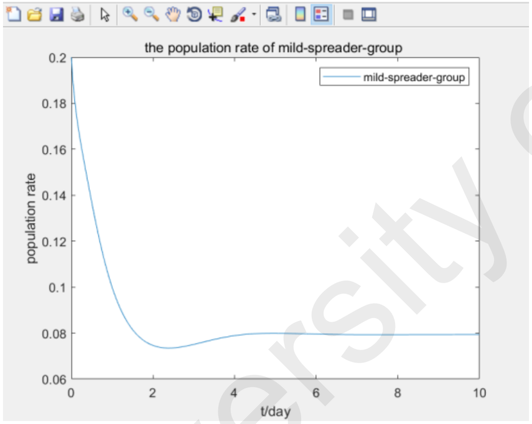
The basic regeneration $R_0$<1, even no control involved, rumors will eventually disappear. Finally, touch the point of RFE (1, 0, 0, 0, 0).



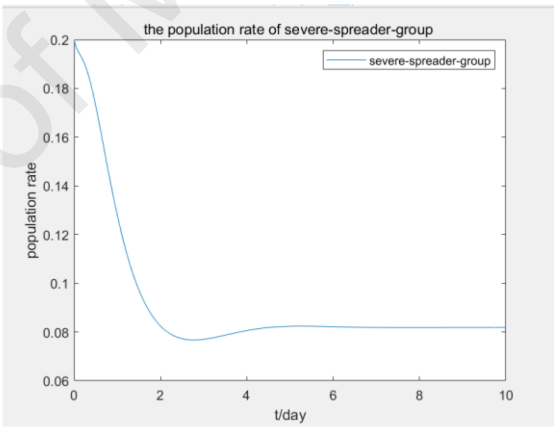

(a)



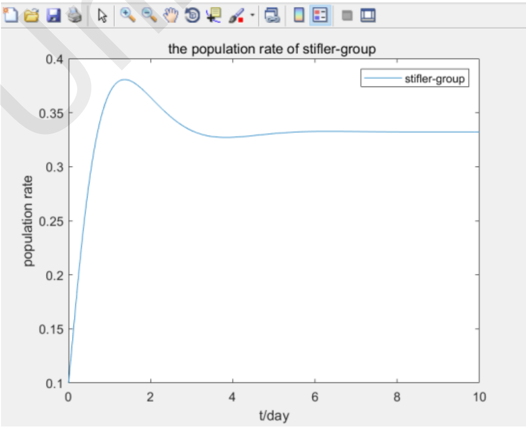

(b)




(c)




(d)




(e)

**Figure 4.2: Simulation result of ABCDE Model without control (2)**

$a_0$=0.9, $b_0$=0.5, $c_0$=0.2,$d_0$=0.15,$e_0$=0.05,μ=0.1,$\theta_1$=0.4,$\theta_2$=0.3.

The basic regeneration $R_0$>1, the spread of rumors will eventually stabilize at a fixed value, finally touch the point of REE. The spread of rumors will remain at a constant level. In this case, rumors do not automatically disappear over time.

## 4.2 Matlab simulation with Bang-Bang control

### 4.2.1 Parameters use in Bang-Bang control.

**Table 4.2: Parameters of ABCDE-type Model in control**

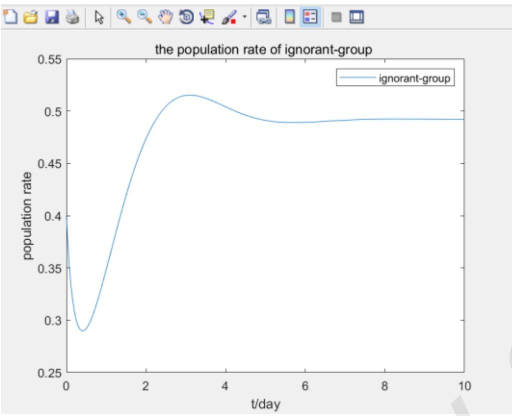
| | |
|---|---|
| d0∈(0,1] | The conversion rate from ignorant-group |
| a0∈(0,1] | The conversion rate from latent-group |
| c0∈(0,1] | The conversion rate from mild-spreader-group |
| d0∈(0,1] | The conversion rate from severe-spreader-group |
| e0∈(0,1] | The conversion rate from stifler-group |
| μ∈(0,1] | The population rate of emigration in the system |
| θ1∈(0,1] | Probability of transition from latent period to a mild spreader |
| θ2∈(0,1] | Probability of transition from latent period to a severe spreader |
| B0>0 | the weight coefficient of the population of the ignorant-group and the stifler-group |
| B1>0 | the weight coefficient of the population of the mild-spreader-group and the severe-spreader-group |

| | |
|---|---|
| B2>0 | the weight coefficient of control |
| u1∈(0,1] | Educational control coefficient |

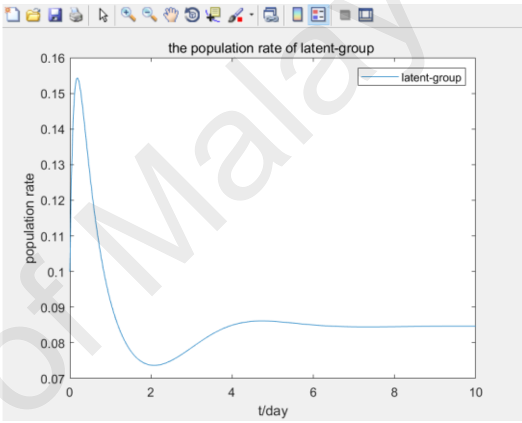
**Table 4.2 continued: Parameters of ABCDE-type Model in control**

| | |
|---|---|
| m=(1-u2) ∈(0,1] | Media technology control coefficient |

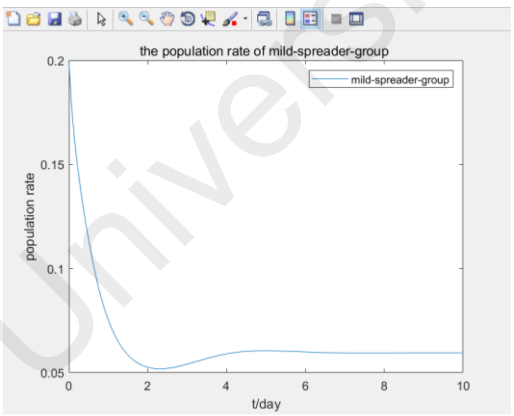
### 4.2.2 Result and analysis



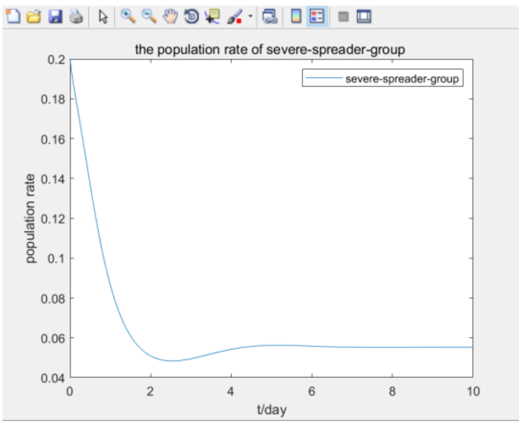

(a)




(b)

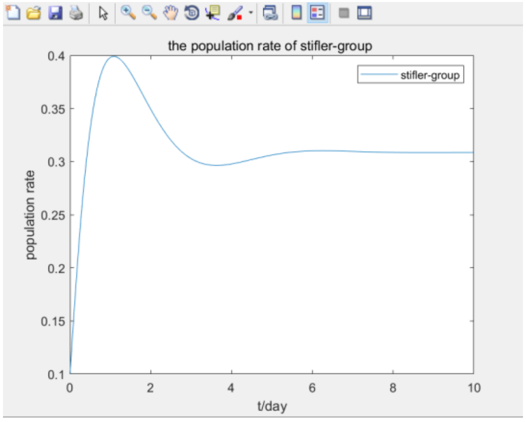



(c)




(d)

**Figure 4.3: Simulation result of ABCDE Model in Bang-Bang control**




(e)

**Figure 4.3 continued: Simulation result of ABCDE Model in Bang-Bang control**



$B_0=1, B_1=0.3, B_2=0.1, a_0=0.9, b_0=0.5, c_0=0.2, d_0=0.15, e_0=0.05, \mu=0.1, \theta_1=0.4, \theta_2=0.3$.

On the previous basis, public education and media technology are used to control the rumor, education control the conversion population from latent-group while the media technology change the process of spreaders into stiflers. Compared with Figure 4.1, the same system parameters were selected here and any equilibrium position can be selected for comparison. At the end of the graph line, in the 10th day, the states of each group of the system were analyzed in the following table:

**Table 4.3: Comparison of simulation result**

| | ingorants | latent period | mild | severe | stifler |
|---|---|---|---|---|---|
| without control | 0.408 | 0.09867 | 0.07936 | 0.08184 | 0.3321 |
| under control | 0.492 | 0.0847 | 0.0595 | 0.05526 | 0.3086 |
| comparison | 0.84↑ | 0.01383↓ | 0.01974↓ | 0.02658↓ | 0.0235↓ |

It can be seen from the table that, with the addition of control variables, on the tenth day, the population of ignorants increase and the population of spreaders and Stiflers decrease.

Steady state error of the system：

$$[(0.492+0.0847+0.0595+0.05526+0.3086)-1]\times 100\% = 0.06\%$$

Control of the system:

$$W=\int_0^{10}[(a+e)-0.3(c+d)-0.1(u_1+u_2)]dt$$

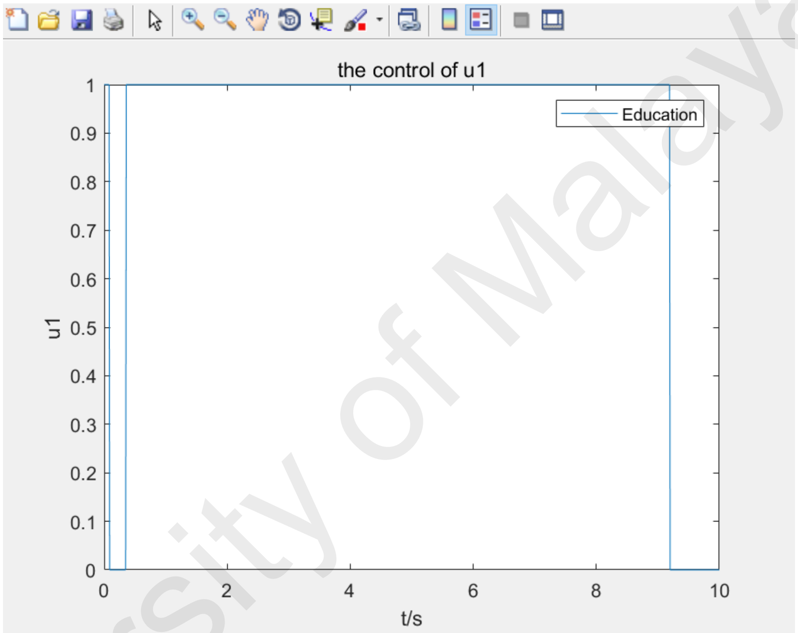



**Figure 4.4: Bang-Bang control of public education case (1)**

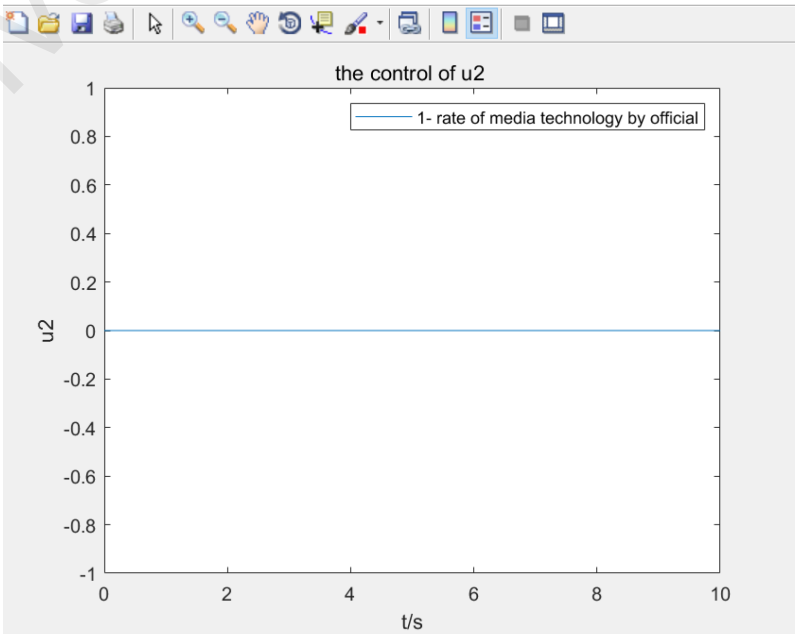


**Figure 4.5: Bang-Bang control of media technology by official case (1)**

Bang-Bang control is adopted to minimize the optimal solution of rumor control cost, as shown in the figure above. The conversion times ( 10 days ) $u_1$ jumps between the upper and lower bounds in finite times while u2 is always zero (Media rate is always 1).

$$W = \int_0^{10} [0.2(a + e) - 0.1(c + d) + 0.03(u_1 + u_2)]dt$$

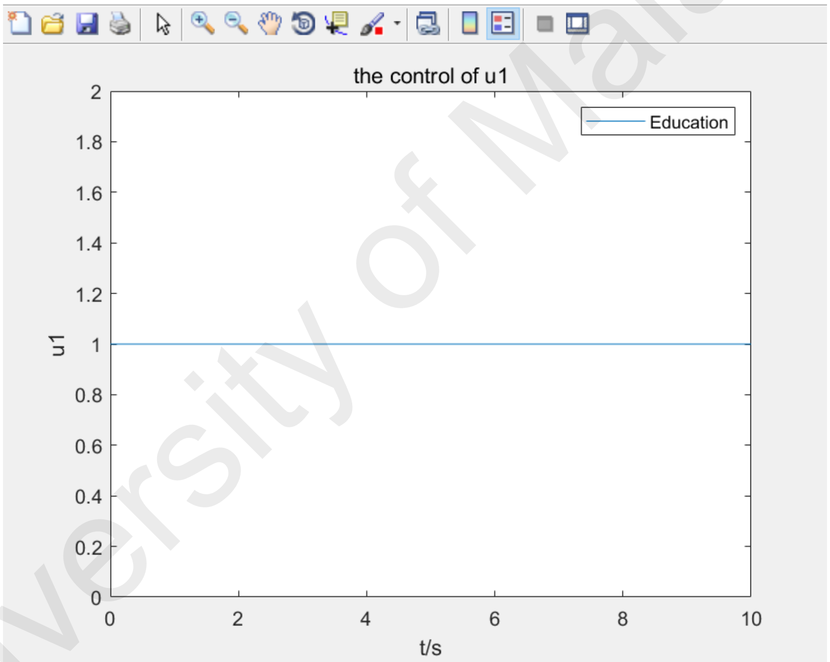



**Figure 4.6: Bang-Bang control of public education case (2)**

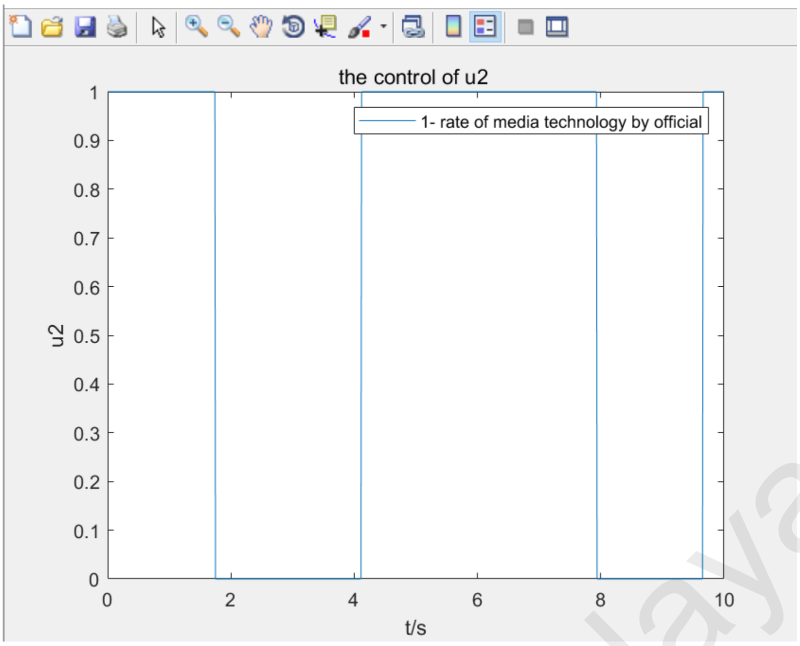



**Figure 4.7: Bang-Bang control of media technology by official case (2)**

B0=0.2,B1=0.1,B2=-0.03,a0=0.9,b0=0.5,c0=0.2,d0=0.15,e0=0.05,μ=0.1,θ1=0.4,θ2=0.

The weight coefficient of cost-benefit function is different, even if the same system and the same control, the bang-bang optimal solution will be different.

# CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

## 5.1 Conclusion

Rumors are the result of human interaction, and they can have a greater impact on social networks. The rational utilization of social resources can effectively reduce the harm brought by the spread of online rumors. By optimizing the rumor benefit function, we achieve the goal of minimizing the expenditure.

Through the D-K model with latent period, we improved the ABCD-type model by considering the personality of different rumor spreaders, a rumor model that contains the spreading latent period and spreaders with different propagation degrees was established. Due to the different personalities of different spreaders, the rumor was spreading in mild or in severe. We assumed that the content of the rumor is constant in the process of spreading. A person who has already experienced the rumor latent period will enter into different spreading groups, due to his own personality to choose whether to become a mild spreader or a severe spreader.

On this basis, the equilibrium point of the system is found through formula derivation, it is obviously illustrated that the system contains two different equilibrium points, REE and RFE. They are a certain point and a unique point respectively.

It is found that public education and media technology by official are two very effective ways to suppress rumor propagation. Therefore, we intervened the rumor system by adding proportional control variable. In the process of controlling the rumor spreading, the rationalization use of rumor resources is always ignored. By establishing the rumor control benefit function, an optimization problem is formed. In order to minimize the cost of rumor control, we must maximize the benefits of rumor control with limited resources. The maximum principle of Pontriagin is applied to obtain the optimal solution of the linear control set with Bang-Bang control in the Hamiltonian function. This kind of optimization method can be applied not only to rumor control, but also to similar problems in other fields. At the same time, in the process of Matlab simulation, the thought of iterative solving of optimal problems is also worth of reference for other optimization problems.

## 5.2 Research prospect

1. In my model, it is assumed that a person stops spreading rumors after becoming a stifler, but in fact, sometimes there will be secondary rumor spreading, which should be considered in future studies.

2. When using media technology to refute rumors, the difference in media technology control between mild and severe cases is not taken into account.

3. No scientific theory is used to judge the stability of the system equilibrium point.

4. The ABCDE-type rumor model is also universal, but sometimes rumors are generated in specific groups of people. In the future, special analysis will be carried out on special models.

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