INVENTORY MODELS FOR PERISHABLE ITEMS UNDER MARKDOWN POLICY

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INVENTORY MODELS FOR PERISHABLE ITEMS UNDER MARKDOWN POLICY

ABSTRACT

As expected, the demand for a fresh product depends on how fresh it is, therefore, it is important to take expiration date into consideration. Based on marketing and economic theory, several factors such as price, inventory level and advertisement play a crucial role in influencing the demand. Hence, we study the effect of these factors in influencing the demand in the inventory model. Since the demand for perishable product declines over time, markdown policy is offered to increase the demand and profit while reducing the inventory. Salvage value is incorporated to the deteriorating units. In this research, we extend previous works and develop three inventory models for perishable items under markdown policy. We show that the markdown policy is an appropriate strategy to sell and get rid of slow-moving or remaining inventory when the season ends and increase the annual profit. Moreover, we find the best time to offer markdown strategy. We obtain the optimum solution with the help of numerical examples. Sensitivity analysis is presented to illustrate the effectiveness of each model.

Keywords: inventory model, perishable items, expiration date, markdown policy, pricing strategy, advertisement frequency.

MODEL-MODEL INVENTORI BAGI BAHAN MUDAH ROSAK DI BAWAH POLISI PENURUNAN HARGA

ABSTRAK

Seperti yang dijangka, permintaan terhadap produk segar begantung kepada kesegaran produk tersebut, maka, tarikh luput perlu dititikberatkan. Berdasarkan teori pemasaran dan ekonomi, beberapa faktor seperti harga, tahap inventori dan pengiklanan memainkan peranan penting dalam mempengaruhi permintaan. Jadi, kami mengkaji kesan faktor-faktor tersebut dalam mempengaruhi permintaan model inventori. Oleh sebab permintaan bagi produk segar berkurang dengan masa, polisi penurunan harga diperkenalkan bagi meningkatkan kadar permintaan dan keuntungan serta mengurangkan inventori. Nilai sisaan bagi bahan yang merosot kualitinya dipertimbangkan. Dalam kajian ini, kami melanjutkan kerja-kerja sebelum ini dan membangunkan tiga model inventori bagi bahan mudah rosak di bawah polisi penurunan harga. Kami menunjukkan bahawa polisi penurunan harga merupakan strategi yang sesuai bagi menjual inventori yang bergerak perlahan dan lebihan inventori di akhir musim selain meningkatkan jumlah keuntungan tahunan. Tambahan pula, kami memperoleh masa terbaik untuk strategi penurunan harga dilaksanakan. Kami memperoleh penyelesaian yang optimum melalui contoh berangka. Analisis sensitf dibengtangkan bagi menunjukkan keberkesanan model.

Kata kunci: model inventori, bahan mudah rosak, tarikh luput, polisi penurunan harga, strategi harga, kekerapan iklan.

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CHAPTER 1: INTRODUCTION

This chapter discusses the research background, sets the aim and objectives of the research, states the thesis overview and summarizes the contribution of research.

Three inventory models have been developed in this research. The first model is an Economic Production Quantity (EPQ) model for delayed deteriorating items with price and inventory level dependent demand under markdown policy. The second model is an inventory model for a fresh product when demand depends on freshness, price, inventory level and expiration date under markdown policy. Lastly, the third model is an inventory model for delayed deteriorating items when demand depends on advertisement, price and inventory level under markdown policy.

1.1. Inventory problem

In the present, retailers face the challenge where it is a necessity to balance the need to maximize the profit with the need to clear end-of-life inventory. Following from there, a markdown policy is acquired. Markdown is necessary to help sell old and slow-moving inventory as well as reduce wastage. According to The National Retail Merchants Association (Carter, 2013), markdown is defined as a reduction in the originally marked retail price of the merchandise, primarily taken for clearance of poor selection, broken assortments, prior stock, for special events and to meet competition. In addition, many businesses in a diverse range of industries apply markdown policy in order to reduce wastage and losses when a fresh product approaches the end of its selling period. The selling period can be short, sometimes just a few hours or days.

According to (Teng & Chang, 2005), large piles of items displayed in a supermarket are often associated with sale items to enhance more profits. In practice, supermarket such as Tesco or Giant applies markdown on fresh goods after some time as the products approach their expiration date in order to increase the demand, clear end-of-life inventory, reduce unnecessary wastage as well as maximizing profit. Usually, these supermarkets will have their first stage of markdown earlier and later, sell the leftover with a salvage value which is the condition that we apply in this model. Salvage value is known as the estimated resale value of an asset at the end of a fresh product's life. Apart from that, markdown time can be tricky. There is no specific rule whether it is better to apply the markdown early or late in time. The amount of markdown given also has a significant effect on both total profit and demand pattern.

The consumption of fresh produce from low-income to high-income consumers is increasing significantly over time. Consumers are becoming more health-conscious and are eating more fresh goods such as vegetables, fruits, milk, bread and seafood than they did in the past. This scenario is also believed to occur due to the better living standard. According to James Russo, SVP, Global Consumer Insights at Nielsen (2015), "while economic concerns remain in the forefront for consumers, health and wellness concerns continue to increase in importance". The idea of consuming healthy food to maintain and improve our health helps in explaining the growing consumer's interest in fresh produce, for example, high consumption of fruits and vegetables helps to prevent chronic diseases.

Moreover, millennials nowadays see eating healthy habit as a trend. Whenever they eat in healthy cafes or cooking healthy foods at home, they feel like they are keeping up with the trend and millennials as we all know, are known for being the ones who are out to "change the world". This trend has also contributed to the increasing demand for fresh products. According to the Organic Trade Association, 52% of organic consumers are millennials. Besides, according to the market research provider Euromonitor International, the demand for healthier and fresh products has led to organic sales in developed markets such as US and UK as well as developing markets, from the Middle East and

Africa and Asia-Pacific regions. Following from here, the demand for fresh products has completely increased in recent times and requires researchers and retailers to give more attention to this particular area.

However, the main characteristic of these perishable items is they undergo deterioration process. The possible causes of deterioration are autoxidation process, poor storage conditions either as a result of cold or heat, transpiration and respiration normally by fruit and vegetable, the appearance of fungi, bacteria and viruses and many more. The loss due to deterioration is a critical aspect and should not be neglected. Fresh product normally has a very short lifetime and it is not safe to be consumed beyond its maximum lifetime. This specific characteristic is related to the expiration date which is a key to indicate the freshness condition of an item.

Majority of past researchers considered the effect of perishability into their models only from the retailer's point of view due to the deterioration process. Therefore, little consideration has been directed into the effect of perishability from the consumer's perspective. In reality, freshness can be considered as one of the main factors that affect the consumer's purchasing decision. As the age of the item increases, it brings a negative effect to the demand due to the misfortune on the item's quality since consumers always prefer to purchasing fresher items for a given price. Realistically, newly produced items attract more buyers. Hence, it is safe to say that the freshness affects consumers' demand pattern and the expiration date is needed in order to evaluate the freshness of a product.

It is undeniably true that advertisement plays an important role, especially in business and economy. We see hundreds of advertisements daily. There are various advertising mediums such as newspaper, television, radio and social media platform. The rise of social media networks gives more platform for the retailer to promote product or service. From the retailer's perspective, the advertisement is seen as a promotional tool in which it uses strategy and messaging about the product or service benefits to attract buyers.

Moreover, the advertisement from the angle of a retailer is seen to guarantee quick sale by bringing products to the attention of potential buyers which increases the sale momentum. Since advertisement normally displays the product's retail price, therefore, it makes it possible for a retailer to maintain the price for at least three to six month.

On the other hand, from the point of view of a consumer, advertisement provides new information and knowledge about a particular product or service. Each advertisement is a piece of information as it can convey a message, tell a story, provide a statistical profile and so on. It can also act as an informer and educator to the consumer. Moreover, advertisement helps in decision-making since there are a lot of similar products in the market nowadays. This scenario has attracted many researchers to conduct comprehensive studies on the effect of advertising in an inventory model.

A common Economic Order Quantity (EOQ) or an Economic Production Quantity (EPQ) model is used to determine the quantity a retailer or manufacturer should order or produce in order to minimize the total annual cost of setups and holding inventory per unit per unit time. Historically, inventory models assumed the demand rate to be constant. In reality, demand for physical goods may be time dependent, stock dependent and price dependent. Many researchers have extended these models by considering more realistic assumptions. In the past few years, significant literature on how inventory level and pricing strategy affect the demand pattern has been published in the field of operational research.

1.2. Research objectives

In this research, we extend the previous studies by developing three inventory models. In the first model, we apply markdown policy on an EPQ model when demand is dependent on price and inventory level. Moreover, we consider a delayed deteriorating item to make it more realistic. We extend on the management of fresh produce in the second model where we apply markdown policy in order to increase demand and reduce wastage and losses as fresh product approaches the end of their selling periods. We consider the deterioration process from consumer's perspective and took into consideration freshness and expiration date in the demand function. Lastly, in the third model, we consider demand to be dependent on advertisement together with price and inventory level under markdown policy to make the model more realistic since advertisement plays a huge role, especially in business and economy.

1. To develop inventory models for perishable items which consider:
   (a) Demand depending on inventory level and price under markdown policy.
   (b) Demand depending on freshness, price, inventory level and expiration date under markdown policy.
   (c) Demand depending on advertisement, inventory level and price under markdown policy.
2. To analyze the relationship between markdown offering time, markdown rate and annual profit and the effects of markdown policy on annual profit.
3. To determine the optimum quantities, optimum markdown time and optimum cycle time that maximize the total profit of the inventory model.

4. To demonstrate how the terms "deterioration" and "perishability" may affect an item.

1.3. Motivation for the research

This section discusses some of the factors that motivate the research undertaken. Previously, many published works that used different approaches and strategies to manage slow-moving and low demand perishable items. However, there is a dearth of research on the common strategy that has been used widely by retailers in different areas which is a markdown strategy.

Moreover, prior researches only emphasized and focused on finding the best markdown rate. However, in this research, we want to show that finding the markdown time is equally important as finding the markdown rate. From this finding, benefits will accrue to retailers and policymakers in order to find a better strategy in inventory management.

One way of managing inventory better is to comprehend the behavior of factors that affect the demand function. Many researchers have discussed the inventory management under markdown policy with a simpler demand function such as when demand depending on price and time. In this study, we feel the need to extend the demand function and make it as close as a real-life problem by considering advertisement frequency, freshness condition, expiration date as well as inventory level. There is an ambivalence when it comes to the relationship between annual profit and markdown rate which we discuss in this research.

When it comes to freshness, there is a need to enhance the importance of incorporating freshness condition and expiration date into the demand function which we address in this work since many previous researches failed to consider this condition from consumers' point of view.

1.4. Scope and limitations

This study focuses on perishable items with low demand and discusses the best markdown strategy in order to maximize profit and reduce inventory at the end of a season. Also, this study provides retailers with knowledge on how factors affecting demand such as price, advertisement frequency and inventory level play a huge role in inventory management.

However, there were certain limitations in undertaking this research work. This study only concerned on one markdown per cycle instead of multiple markdowns. Moreover, shortage problem is not considered in this study as it is assumed that retailers are able to cater all customers' demands and because every inventory models aim to clear out the remaining stock since all items are mandatory to be sold. Other than that, only constant deterioration function is used in all three models in this thesis.

1.5. Research contributions and its significance

To the author's knowledge, none of the previous models has considered several aspects as in this research and we are taking a new perspective on a previously studied topic. This study is an extension and combination of existing models by ((Teng & Chang, 2005); (Widyadana & Wee, 2007); (Srivastava & Gupta, 2013); (Feng et al., 2017)).

In particular, our research focuses on finding the best markdown strategy for inventory models since a little study has been carried out that links markdown policy with the management of perishable items. We extend the previous models and able to find the best markdown time together with the markdown rate. This research is able to demonstrate a concept in which it proves that markdown strategy is important and relevant in reducing inventory level as well as maximizing the profit.

Also, previous works on markdown policy focused on using simpler demand function.

Hence, in this study, we are able to extend the demand function to be depending on several factors such as inventory level, freshness condition, expiration date and advertisement.

A better understanding of the use of deterioration and perishability is demonstrated in this model. Deterioration process can be defined as the degradation of efficacy or physical features (color, consistency, odor, etc.) of a substance due to faulty packaging or abnormal storage conditions and we incorporated deterioration process in all models. Meanwhile, an item with expiration date and fixed lifetime perish once it passes its maximum lifespan and can no longer be used thus, must be disposed of properly and this is specifically discussed in the second model.

This research is able to contribute to a wide range of industries, from the small industry to the established industry as well as can be applied in practice by retailers and policymakers. It can contribute some ideas and theoretical knowledge on how to manage inventory model when it comes to fresh products as these items have their specific expiration date and a shorter lifetime. It is very important to wisely manage fresh products in order to maximize profit. This research developed inventory models that are appropriate and applicable for those perishable items with low demand in the market compared to other items.

Moreover, this research helps in finding the best strategy for retailers that sells a seasonal item over a finite schedule period while keeping in mind that these items undergo deterioration process and demand rate depends on several factors such as price, inventory level, freshness condition, expiration date and advertisement. Following from there, markdown is proven to be one of the best strategies not only to clear the inventory but also to reduce wastage. Other than finding the best markdown rate in order to maximize the profit, this research also helps to find the best markdown time and gives some ideas on the relationship between markdown time and annual profit which is helpful for retailers and policymakers in decision making.

Apart from that, markdown price also plays a huge factor in order to increase annual profit hence, this research also shows the impact of markdown price on the profit. Furthermore, in the age of social media marketing, the advertisement plays a huge role in an inventory model. Therefore, in this research, we discuss the appropriate advertisement strategies for a retailer.

Some possible extensions are identified and discussed in this research, hopefully, will encourage other researchers to extend the research.

1.6. Thesis overview

This paper is organized as follows and consists of six chapters:

- Chapter 1: This chapter contains a brief introduction of this research which includes problems and issues in inventory model, definitions, objectives and contributions of the research.

- Chapter 2: This chapter reflects both reviews of the previous researches and the fundamental relationship with this research.

- Chapter 3: This chapter discusses the mathematical formulation and development for the first inventory model which is an EPQ model of delayed deteriorating items with price and inventory level dependent demand under markdown policy.

- Chapter 4: This chapter presents the mathematical formulation and development for the second inventory model when demand depending on freshness condition of a fresh produce, inventory level, price, and expiration date under markdown policy.

- Chapter 5: This chapter develops the mathematical formulation and development for the third deteriorating inventory model when the demand rate is dependent on frequency of advertisement, price and inventory level under markdown policy.

- Chapter 6: This chapter concludes the thesis and presents suggestions for future research.

CHAPTER 2: LITERATURE REVIEW

The academic literature related to inventory model for perishable items is thematically organized around five main streams of research:

2.1. Inventory model for deteriorating item

It is a natural phenomenon that fresh product experiences deterioration which can be defined as damage, decay, spoilage, evaporation and loss of quality from its original condition. (Namhias, 1982), (Raafat, 1991) as well as (Goyal & Giri, 2001) provided a comprehensive study of published literature on deteriorating inventory models. Recently, (Chaudhary et al., 2018) presented the most updated literature review on inventory models for perishable products.

(Ghare & Schrader, 1963) initiated the study of deteriorating inventory where they proposed the classic inventory model with no shortages provided that the rate of deterioration is constant. (Covert & Philip, 1973)'s model later extended the EOQ model under the assumption of Weibull distribution for deterioration. (Cohen, 1977) developed an inventory model by considering exponentially decaying inventory.

(Dave & Patel, 1981) established an inventory model for deteriorating items in which demand rate is changing linearly with time. (Hollier & Mak, 1983) developed two mathematical models for an inventory system. They considered a constant deterioration rate and exponentially negative decreasing demand rate. The model of (Dave & Patel, 1981) is extended by (Sachan, 1984) by allowing shortages. Furthermore, (Wee, 1995) presented a model for joint pricing and replenishment policy for deteriorating inventory by taking into account the price elastic demand level which reduces over time. In this model, demand decreases linearly with time and cost of items.

(Benkherouf & Mahmoud, 1996) studied an inventory model for deteriorating items with increasing time-vary demand and shortages. (Liao et al., 2000) established an inventory model with deteriorating items under inflation when a delay in payment is permitted. They discussed the effects of the inflation rate, deterioration rate, initial-stock-dependent consumption rate as well as delay in payment. (Samanta & Roy, 2004) developed a continuous production control inventory model for deteriorating items with shortages. The authors assumed that the demand rate and production rate are constants while the distribution of the time to deterioration of an item follows the exponential distribution.

Realistically, most of the products have a time span for retaining their original state before the deterioration process of begins. According to (Wu et al., 2006), this phenomenon can be defined as non-instantaneous deterioration. Next, (Tao et al., 2010) developed an inventory model for deteriorating items with partial backlogging and permissible delay in payments. Recently published works include the research published by (Yang & Tseng, 2015) where they established a deteriorating inventory model for chilled food. This model used real deterioration rate data and temperature-dependent deterioration rate.

Meanwhile, (Lu et al., 2016) proposed an inventory system for perishable items with limited replenishment capacity when demand rate depends on the price as well as the stock quantity displayed. (Pervin et al., 2016) presented a more realistic optimal retailer's replenishment decisions for deteriorating items under a trade credit policy by using an economic product quantity model. A multi-item joint replenishment model for non-instantaneous deteriorating items is presented by (Ai et al., 2017), considering a constant demand rate. This model allowed full backlogging and considered piecewise function with exponential parts for its cost function.

2.2. Inventory model when demand depends on freshness condition and expiration date

(Fujiwara & Perera, 1993) were among the firsts to take into consideration the effect of product freshness on consumers demand. They presented EOQ models for deteriorating items which consider continuous deterioration of the usefulness of a product with an exponential penalty cost acts as a measure of the deterioration in utility. Also, (Sarker et al., 1997) studied the inventory model for perishable items when age negatively affected the demand function. It is an ordinary scenario that, for perishable goods, the inventory age has an adverse effect on demand. This is because of the loss of consumer confidence and attraction on such product.

(Hsu et al., 2006) studied on optimal lot sizing for deteriorating items with expiration date. Subsequently, (Bai & Kendall, 2008) established an EOQ inventory model for perishable products by letting the demand to be dependent on freshness and shelf-space. Demand is said to decline gradually to zero as the expiration date approaches. Next, (Qin et al., 2014) studied joint pricing and inventory control of fresh products and foods with deteriorating quality and physical quantity. (Teng et al., 2016) presented an inventory model for deteriorating items with expiration dates and advance payments. This paper considered the deterioration rate of a product gradually increases with the expiration date.

(Chen et al., 2016) then developed an economic order quantity model when the demand for fresh products depends on the freshness-expiration date and the volume displayed. Later, (Feng et al., 2017) established the pricing and lot-sizing policies for perishable goods where demand is a multivariate form of selling price, displayed stock and expiration date. This model is an extension of (Chen et al., 2016) model.

Following from that, (Herbon, 2017) investigated the effect of the common practice of the coexistence of multiple competing perishables, each with different age and price, all of which are associated with the same product. In this model, by assuming that consumers are analogous in their fondness, the author analyzed a deterministic version of the issue and the optimal pricing policy given any cycle length and any duration for which both types coexist on the shelf is obtained.

(Li et al., 2017) developed an inventory model by using a discounted cash-flow analysis by taking into consideration time value of money. Recently, (Fang et al., 2018) tried a different approach in which they applied the bundle pricing strategy for homogeneous fresh products with quality deterioration. The total annual profit is strictly concave in the unit price and strictly pseudo-concave in the period of replenishment.

2.3. Inventory model when demand depends on inventory level and price

The increasing number of displayed goods attracts more buyers, hence, increases the consumption rate. (Levin et al., 1972) observed that large piles of consumer goods displayed in a supermarket will result in customers buying more. (Silver & Peterson, 1985) discovered that sales made by retailers tend to be proportional to the amount of inventory displayed. In this line, (Baker & Urban, 1988) developed an EOQ model with a power form inventory level dependent demand.

Later, (Mandal & Phaujdar, 1989) established an EPQ model with linearly stock dependent. In addition, (Datta & Pal, 1990) developed an EOQ model with demand rate depending on the instantaneous stock amount displayed until the inventory level, L is achieved. After that demand rate becomes constant.

(Padmanabhan & Vrat, 1995) studied an inventory model for perishable items with stock-dependent selling rate. They investigated the effect of stock-dependent selling rate, perishability and partial backlog parameters on order quantity, profit and maximum inventory level. Moreover, (Datta & Paul, 2001) provided a multi-period EOQ model with stock-dependent and price-sensitive demand rate.

(Alfares, 2007) considered the inventory policy for a stock-dependent demand rate and storage-time dependent holding cost. It is assumed that the the cost of holding the item per unit time is an increasing function of the time spent in storage. Next, (Tao et al., 2010) established an optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. In this paper, they first modified (Wu et al., 2006) model by changing the objective function to maximizing the total profit. They also fixed a maximum inventory level as well as relaxed the zero end inventory restriction when shortages are not beneficial.

Furthermore, (Lee & Dye, 2012) presented a deteriorating inventory model with stock-dependent demand and controllable deterioration rate where preservation technology cost as a decision variable together with the replacement policy. (Omar & Zulkipli, 2014) considered demand to be deterministic and positively dependent on the level of items displayed in a just-in-time (JIT) system in which the manufacturer must supply the products in small quantities to minimize the buyers holding cost at the warehouse as well as accepting the supply of small quantities of raw materials to minimize its own holding cost.

Apart from that, pricing strategy is important especially in a competitive market nowadays. Some of the past researchers have proved that demand increases as the price decreases. Therefore, some researchers considered demand to be depending not only on inventory level but also price. To study the inventory management for non-instantaneous deteriorating items when the demand is both price and stock sensitive, see ((Teng & Chang, 2005); (You & Hsieh, 2007); (Wu et al., 2006); (Chowdhury, Ghosh, & Chaudhuri, 2014); (Chang, 2016); (Khurana & Chaudhary, 2016); (Mashud et al., 2018)).

2.4. Inventory model when demand depends on frequency of advertisement

To make the inventory model more realistic, we also study the effect of advertisement in the inventory model. (Bhunia & Maiti, 1997) established an inventory model for decaying items with selling price, advertising frequency and linear time-dependent demand, taking into account shortages.

(Pal et al., 2005) studied a deterministic inventory system for a single product when demand depending on selling price, advertising frequency and stock level (DSL) presented in a showroom or shop by allowing shortages and are backlogged partially. Moreover, the storage capacity is assumed to be limited.

(Shah & Pandey, 2009) developed a mathematical model for time dependent deteriorating item to determine optimal ordering policy when demand depends on displayed stock level and frequency of advertisement through media. (Wang, 2011) studied the changes in optimal decisions about the system parameters for the newsvendor model under general additive-multiplicative advertising-dependent demand distribution through comparative statics method.

(Aggarwal & Kumar, 2013) presented an inventory model where the demand of the product is dynamic over time and influenced by the level of advertising expenditure in an innovation diffusion environment. The diffusion of new products is made primarily by the external and internal influences. (Chowdhury et al., 2014) considered an inventory model in which the demand rate is a function stock level and the effort expended on advertising the product in which advertising spending is presumed to be a quadratic function of the salesmen's effort.

(Geetha & Udayakumar, 2016) dealt with an economic order quantity for non instantaneous items with advertisement and price dependent demand pattern considering shortages, partial backlogging and salvage value. The backlogging rate is depending on the waiting period for the next replenishment. (Manna et al., 2017) presented an economic production quantity (EPQ) model with imperfect production system in which demand is dependent on advertisement. They assumed the advertisement rate to be a function of time which has been increased with regard to time at a declining rate.

2.5. Inventory model considering markdown policy

(Urban & Baker, 1997) studied the single item economic order quantity model where demand depends on inventory level but eventually becomes constant after a certain time. The authors investigated how much discount can be provided on the selling price during the deterioration period to maximize the profit per unit time and how a pre-deterioration discount may affect the unit profit. In this line, (Burwell et al., 1997) studied an economic lot size model when demand depends on price by taking into consideration quality and freight discounts.

Next, (Smith & Achabal, 1998) established clearance pricing and inventory policies for retail chains. They took into account the effect of reduced assortment and seasonal changes on sales rates. Meanwhile, (Arcelus & Srinivasan, 1998) presented ordering policies under one time discount when demand is sensitive to price. Even on a one-time-only basis, the price reduction is believed to help increase sales.

(Papachristos & Skouri, 2003) developed an inventory model with deteriorating items, quantity discount, pricing and time-dependent partial backlogging. A quantity discount is an incentive offered to a buyer where the cost per unit of an item decreases when purchased in greater numbers. It is often offered by sellers to encourage buyers to purchase in larger quantities. Furthermore, (Widyadana & Wee, 2007) presented a deteriorating inventory model with price dependent demand and applies markdown policy in order to increase the profit. According to the authors, markdown policy is useful to increase the total profit, but the best markdown time and price depend are case dependent.

(Panda et al., 2009) established an EOQ model for perishable products with discounted selling price and stock dependent demand. In this model, they studied how much discount on selling price may be given during deterioration to maximize the profit per unit time and how a pre-deterioration discount affects the unit profit.

(Sana, 2010) presented an EOQ model for a perishable item with stock dependent demand and price discount rate. The author considered a stock-dependent inventory model for perishable items in a supermarket. (Srivastava & Gupta, 2013) developed an inventory model for deteriorating items with time and price-dependent demand which adopted the markdown policy to lower the inventory and to increase the profit. Markdown is also applicable on completely deteriorated items in this model.

Recently, (Sekar & Uthayakumar, 2019) proposed a continuous production inventory model for deteriorating items with preservation technology, rework and price-dependent demand under markdown policy. These authors applied the markdown strategy to minimize inventory cost and to get rid of end-of-life or end of the season inventory or to sell old or slow-moving inventory at the end of its life cycle.

2.6. Summary

The positive impact that markdown brings to an inventory model is known to many and has received the attention of researchers in this field. However, despite the fact that markdown is a common strategy and widely used by retailers, little research has been conducted that links markdown policy with perishable items management. Hence, there is a need to explore more on markdown policy which we discuss in this thesis as previous researchers mostly just focused on finding the best markdown rate without taking into consideration the importance of markdown time. Previous studies on markdown policy such as (Widyadana & Wee, 2007) and (Srivastava & Gupta, 2013) only focused on finding the markdown rate in which we extend and able to find the best markdown time too. Moreover, some recently published works discussed on other strategies to manage perishable items such as bundle pricing and advance payments but did not emphasize on markdown policy (Teng et al., 2016), (Li et al., 2017) and (Fang et al., 2018).

The terms “deterioration” and “perishability” are widely used and seem to be interchangeable in the literature. These terms are related to the physical state of items over time. Thus, this thesis clarify the definitions and differentiate the use of these two terms. Deterioration process can be defined as the reduce in the effectiveness or physical characteristics (color, consistency, odor, etc.) of a substance due to faulty packaging or abnormal storage conditions. Meanwhile, an item with expiration date and fixed lifetime perish once it passes its maximum lifespan and can no longer be used thus, must be disposed of properly.

Moreover, prior researches on markdown policy focused on using simpler demand function such as price and time. Hence, there is a need to extend the demand function and make it as close to a real-life problem because realistically, demand can be depending on several factors such as inventory level, freshness condition, expiration date and advertisement.

Apart from that, the effect of the freshness of an item is often studied only from retailers' point of view thus, we analyze the effect of it from consumers' perspective too as discussed by (Bai & Kendall, 2008), (Teng et al., 2016), (Chen et al., 2016) and (Feng et al., 2017). It is important to take into consideration the expiration date as it is the key to determine the freshness level of an item. On top of that, we extend the previous researches by incorporating markdown policy and study the effect of markdown strategy in perishable items management since their lifetime is normally shorter than other items.

The identified research gaps are addressed in which markdown policy is incorporated into all three inventory models. The first model deals with delayed deteriorating items when demand is depending on price and inventory level. Whereas, the second model focuses more on fresh goods such as milk, vegetables and fruits and takes into consideration freshness condition as well as expiration date into the demand function. Lastly, the third model incorporated not just price and inventory level into demand function, but also advertisement frequency.

In summary, this research aims to advance the present state of knowledge in the field of inventory mathematical modeling and management by extending the previous models and by considering more realistic assumptions with the objective to help the retailers maximize the total annual profit and policymakers in planning the best strategy.

CHAPTER 3: AN EPQ MODEL FOR A DELAYED DETERIORATING ITEM WITH PRICE AND INVENTORY LEVEL DEPENDENT DEMAND UNDER MARKDOWN POLICY.

3.1. Introduction

In this chapter, we will develop an Economic Production Quantity (EPQ) model for a delayed deteriorating item with price and inventory level dependent demand under markdown policy. This model is not only relevant to help sell slow-moving and end-of-season inventory but also maximizes the annual profit. The objective of this model is to find the optimum markdown time, optimum quantities as well as optimum total cycle length that maximize the total profit. We will show how markdown can help to achieve the aim of the inventory model. We use differential equations to develop a mathematical formulation for the model. In order to demonstrate the effectiveness of the model, we will also use numerical examples and sensitivity analysis.

This model is an extension of (Teng & Chang, 2005) and (Srivastava & Gupta, 2013). We extend the demand function to be depending on price and inventory model as large items are usually affiliated with sale items to enhance better profits (Teng & Chang, 2005) and price is often considered as an important factor in marketing strategy. In addition, we incorporate markdown policy into the inventory model by applying the same approach as (Widyadana & Wee, 2007) and (Srivastava & Gupta, 2013).

3.2. Model's background

The following notations are used throughout this chapter:

1. α = Markdown rate.

2. δ = Production percentage.

3. ϵ = Index of price elasticity.

4. γ = Markdown percentage.

5. λc_p = The salvage value associated to the deteriorated units during the cycle time where $0 \leq \lambda < 1$.

6. θ = Constant deterioration rate.

7. c_0 = Ordering cost per order.

8. c_h = Inventory holding cost per unit per unit of time.

9. c_p = Production cost per unit.

10. $I(t)$ = Inventory level at time t.

11. K = Constant production rate.

12. p = Initial price.

13. Q_1 = Inventory level at t_1.

14. Q_2 = Inventory level at t_2.

15. T = Replenishment cycle time in units of time.

To develop the model, the following assumptions are used:

1. Demand is a function of price and inventory level. The demand rate at time t is assumed to be $D(I(t), p) = b(\alpha p)^{-\epsilon} + \beta I(t)$, where α, b and β are positive constants with α and β are between 0 and 1.

2. A single item is considered over a prescribed period of T units of time.

3. Shortage is not allowed.

4. Rate of deterioration, θ is constant at any time, where $0 \leq \theta < 1$.

5. The rate of production is constant and production stops after t_1.

6. There is no rework process on the deteriorated units.

7. All items are mandatory to be sold.

8. Only one-time markdown price at one planning period is applied.

9. Markdown price is known.

10. Item starts to deteriorate after production stops and markdown is offered after some time of product deterioration.

11. Production time is proportional to the cycle time which is equivalent to $t_1 = \delta T$.

12. Markdown is offered after some time of product deterioration. Markdown time is equivalent to $t_2 = \gamma(1 - \delta)T$.

3.3. Mathematical formulation

The behaviour of the inventory system at time t is illustrated in Figure 3.1. The production and supply start simultaneously and the production ends at time t_1 with the inventory level, Q_1 is reached. To make this model more practical, we assumed that there is no deterioration during the production up-time. In the interval $(t_1, t_1 + t_2)$, inventory level decreases due to demand and deterioration processes. However, the demand rate decreases with time. Thus, at $(t_1 + t_2)$, markdown is offered in order to increase the demand rate due to a reduction in selling price. The inventory level at time t over period $(0, T)$ is governed by these differential equations:

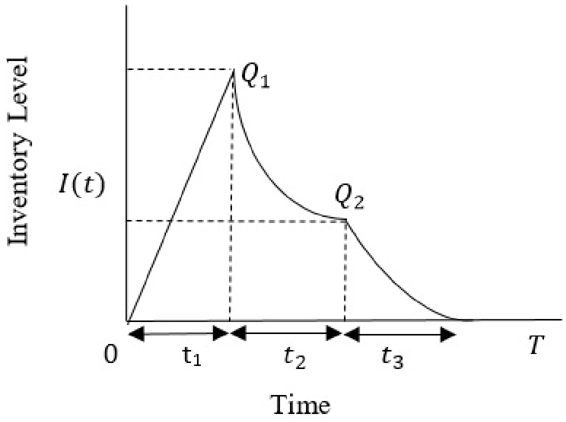

Figure 3.1: Graphical representation of the inventory system

The equations below are the combination and modification of the equations used by (Teng & Chang, 2005) and (Srivastava & Gupta, 2013).

At $0 \leq t \leq t_1$, production and supply processes start simultaneously and production stops at time t_1 once it reaches the quantity Q_1. Deterioration does not happen during this interval and markdown is not offered yet thus, there is no breakdown price. Hence, $\alpha = 1$

$$\frac{dI(t)}{dt} = K - \big(bp^{-\epsilon} + \beta I(t)\big). \tag{3.1}$$

At $0 \leq t \leq t_2$, inventory level decreases due to demand and deterioration process. Markdown is still not offered during this interval therefore, we have

$$\frac{dI(t)}{dt} + \theta I(t) = -(bp^{-\epsilon} + \beta I(t)). \tag{3.2}$$

Similarly, at $0 \leq t \leq t_3$, inventory level decreases due to demand and deterioration process. Realistically, demand rate decreases with time. Hence, in order to increase the demand rate markdown is offered during this interval. Thus, we have

$$\frac{dI(t)}{dt} + \theta(I(t)) = -(b(\alpha p)^{-\epsilon} + \beta I(t)), \quad 0 < \alpha < 1. \tag{3.3}$$

We solve the equations above by using the technique of an integrating factor as this method has been used by many established papers such as (Teng & Chang, 2005), (Widyadana & Wee, 2007) and (Srivastava & Gupta, 2013) to solve these types of equations as well as it is less tedious. The purpose of this derivation is to show the steps taken in order to obtain the equation of the inventory level, $I(t)$ for all intervals.

Solving the first differential equation,

$$\frac{dI(t)}{dt} = K - (bp^{-\epsilon} + \beta I(t)).$$

Rearrange,

$$\frac{dI(t)}{dt} + \beta I(t) = K - bp^{-\epsilon}. \tag{3.4}$$

Note that the integrating factor, $M(t) = e^{\beta t}$. Multiply equation (3.4) with the integrating factor and we obtain

$$\frac{dI}{dt}(e^{\beta t}) + \beta I(t)(e^{\beta t}) = (K - bp^{-\epsilon})(e^{\beta t}), \tag{3.5}$$

which is equivalent to

$$\frac{dI(t)}{dt}(e^{\beta t}) = (K - bp^{-\epsilon})(e^{\beta t}). \tag{3.6}$$

Integrate both sides of equation (3.6) with respect to t,

$$\begin{aligned}
\int \frac{de^{\beta t}I(t)}{dt} &= \int (K - bp^{-\epsilon})e^{\beta t}dt \\
e^{\beta t}I(t) &= K - bp^{-\epsilon}\int e^{\beta t}dt \\
e^{\beta t}I(t) &= (K - bp^{-\epsilon})\frac{e^{\beta t}}{\beta} + C \\
I(t) &= \frac{K - bp^{-\epsilon}}{\beta} + Ce^{-\beta t}. \qquad (3.7)
\end{aligned}$$

With boundary conditions $t = 0$ and $I(t) = 0$, solve equation (3.7) we obtain

$$\begin{aligned} I(0) &= \frac{K - bp^{-\epsilon}}{\beta} + Ce^{-\beta(0)} = 0 \\ \frac{K - bp^{-\epsilon}}{\beta} + C &= 0 \\ C &= -\frac{K - bp^{-\epsilon}}{\beta}. \end{aligned} \tag{3.8}$$

Substitute C into equation (3.7) we obtain,

$$\begin{aligned} I(t) &= \frac{K - bp^{-\epsilon}}{\beta} + (-\frac{K - bp^{-\epsilon}}{\beta})e^{-\beta t} \\ I(t) &= \frac{K - bp^{-\epsilon}}{\beta}(1 - e^{-\beta t}). \end{aligned} \tag{3.9}$$

Similarly, solving the second interval differential equation by using the same technique

$$\frac{dI(t)}{dt} + \theta I(t) = -(bp^{-\epsilon} + \beta I(t)).$$

Rearrange,

$$\frac{dI(t)}{dt} + (\theta + \beta)I(t) = -bp^{-\epsilon}. \tag{3.10}$$

Note that the integrating factor, $M(t) = e^{(\theta+\beta)t}$. Multiply equation (3.10) with the integrating factor we obtain,

$$\frac{dI(t)}{dt}(e^{(\theta+\beta)t}) + (\theta + \beta)I(t)(e^{(\theta+\beta)t}) = -bp^{-\epsilon}(e^{(\theta+\beta)t}) \tag{3.11}$$

which is equivalent to

$$\frac{dI(t)}{dt}(e^{(\theta+\beta)t}) = -bp^{-\epsilon}(e^{(\theta+\beta)t}). \tag{3.12}$$

Integrate both sides of equation (3.12) with respect to t,

$$\begin{aligned}\int \frac{d(e^{(\theta+\beta)t})I(t)}{dt} &= \int -bp^{-\epsilon}e^{(\theta+\beta)t}dt \\ e^{(\theta+\beta)t}I(t) &= -bp^{-\epsilon}\int e^{(\theta+\beta)t}dt \\ e^{(\theta+\beta)t}I(t) &= -bp^{-\epsilon}\frac{e^{(\theta+\beta)t}}{(\theta+\beta)}+C \\ I(t) &= \frac{-bp^{-\epsilon}}{(\theta+\beta)}+Ce^{-(\theta+\beta)t} \end{aligned} \tag{3.13}$$

with boundary condition $t=0, I(t)=Q_1$, equation (3.13) becomes

$$\begin{aligned} I(0) &= \frac{-bp^{-\epsilon}}{(\theta+\beta)}+Ce^{-(\theta+\beta)0}=Q_1 \\ \frac{-bp^{-\epsilon}}{(\theta+\beta)}+C &= Q_1 \\ C &= Q_1+\frac{bp^{-\epsilon}}{(\theta+\beta)}. \end{aligned} \tag{3.14}$$

Substitute C into equation (3.13) we obtain

$$\begin{aligned} I(t) &= \frac{-bp^{-\epsilon}}{(\theta+\beta)}+(Q_1+\frac{bp^{-\epsilon}}{(\theta+\beta)})e^{-(\theta+\beta)t} \\ &= Q_1e^{-(\theta+\beta)t}+\frac{bp^{-\epsilon}}{(\theta+\beta)}(e^{-(\theta+\beta)t}-1). \end{aligned} \tag{3.15}$$

Lastly, solving the third interval differential equation,

$$\frac{dI(t)}{dt}+\theta I(t)=-(b(\alpha p)^{-\epsilon}+\beta I(t)).$$

Rearrange,

$$\frac{dI(t)}{dt}+(\theta+\beta)I(t) = -b(\alpha p)^{-\epsilon}. \tag{3.16}$$

Note that the integrating factor, $M(t) = e^{(\theta+\beta)t}$. Multiply equation (3.16) with the integrating factor,

$$\frac{dI(t)}{dt}(e^{(\theta+\beta)t}) + (\theta+\beta)I(t)(e^{(\theta+\beta)t}) = -b(\alpha p)^{-\epsilon}(e^{(\theta+\beta)t}), \tag{3.17}$$

which is equivalent to

$$\frac{dI(t)}{dt}(e^{(\theta+\beta)t}) = -b(\alpha p)^{-\epsilon}(e^{(\theta+\beta)t}). \tag{3.18}$$

Integrate both sides of equation (3.18) with respect to t

$$\begin{aligned}
\int \frac{d(e^{(\theta+\beta)t})I(t)}{dt} &= \int -b(\alpha p)^{-\epsilon}e^{(\theta+\beta)t}dt \\
e^{(\theta+\beta)t}I(t) &= -b(\alpha p)^{-\epsilon}\int e^{(\theta+\beta)t}dt \\
e^{(\theta+\beta)t}I(t) &= -b(\alpha p)^{-\epsilon}\frac{e^{(\theta+\beta)t}}{(\theta+\beta)} + C \\
I(t) &= \frac{-b(\alpha p)^{-\epsilon}}{(\theta+\beta)} + Ce^{-(\theta+\beta)t}. \qquad (3.19)
\end{aligned}$$

With boundary condition $t = 0, I(t) = Q_2$, equation (3.19) becomes

$$\begin{aligned}
I(0) &= \frac{-b(\alpha p)^{-\epsilon}}{(\theta+\beta)} + Ce^{-(\theta+\beta)0} = Q_2 \\
\frac{-b(\alpha p)^{-\epsilon}}{(\theta+\beta)} + C &= Q_2 \\
C &= Q_2 + \frac{b(\alpha p)^{-\epsilon}}{(\theta+\beta)}. \qquad (3.20)
\end{aligned}$$

Substitute C into equation (3.19) we obtain,

$$\begin{aligned}
I(t) &= \frac{-b(\alpha p)^{-\epsilon}}{(\theta+\beta)} + (Q_2 + \frac{b(\alpha p)^{-\epsilon}}{(\theta+\beta)})e^{-(\theta+\beta)t} \\
&= Q_2 e^{-(\theta+\beta)t} + \frac{b(\alpha p)^{-\epsilon}}{(\theta+\beta)}(e^{-(\theta+\beta)t} - 1). \qquad (3.21)
\end{aligned}$$

It follows,

$$I(t)=\begin{cases}\frac{K-bp^{-\epsilon}}{\beta}(1-e^{-\beta t}), & 0\leq t\leq t_1.\\ \\ Q_1e^{-(\theta+\beta)t}+\frac{bp^{-\epsilon}}{(\theta+\beta)}(e^{-(\theta+\beta)t}-1), & 0\leq t\leq t_2\\ \\ Q_2e^{-(\theta+\beta)t}+\frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}(e^{-(\theta+\beta)t}-1), & 0\leq t\leq t_3.\end{cases}$$

Therefore, from Figure 1 the inventory level at time t_1 is

$$Q_1=\frac{K-bp^{-\epsilon}}{\beta}(1-e^{-\beta t_1}). \tag{3.22}$$

Similarly, at time t_2 and t_3 we have

$$Q_2=Q_1e^{-(\theta+\beta)t_2}+\frac{bp^{-\epsilon}}{(\theta+\beta)}(e^{-(\theta+\beta)t_2}-1) \tag{3.23}$$

and

$$Q_2=\frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}(e^{(\theta+\beta)t_3}-1). \tag{3.24}$$

3.4. Model development

In this model, the objective function consists of :

- Setup cost.
- Holding cost.
- Production cost.
- Deterioration cost.
- Salvage value.
- Total revenue.

3.4.1. Setup cost

Setup cost can be defined as a cost acquired to configure a machine for a production run and it is considered as a fixed cost.

The setup cost for this inventory model is

$$SC = c_0. \tag{3.25}$$

3.4.2. Holding cost

Holding cost can be defined as the additional costs associated with the storage and maintenance of an inventory per unit time. Holding cost is related with storing inventory that remains unsold.

The holding cost of inventory is

$$\begin{aligned}
HC &= c_h \times \Big[\int_0^{t_1} \frac{K - bp^{-\epsilon}}{\beta}(1 - e^{-\beta t})dt \\
&+ \int_0^{t_2} \big(Q_1 e^{-(\theta+\beta)t} + \frac{bp^{-\epsilon}}{\theta+\beta}(e^{-(\theta+\beta)t} - 1)\big)dt \\
&+ \int_0^{t_3} \big(Q_2 e^{-(\theta+\beta)t} + \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}(e^{-(\theta+\beta)t} - 1)\big)dt\Big] \\
&= c_h \times \Big[\frac{K - bp^{-\epsilon}}{\beta}(t_1 - \frac{1 - e^{-\beta t_1}}{\beta}) + \frac{Q_1}{\theta+\beta}(1 - e^{-(\theta+\beta)t_2}) \\
&+ \frac{bp^{-\epsilon}}{\theta+\beta}\frac{(1 - e^{(\theta+\beta)t_2})}{\theta+\beta} - \frac{bp^{-\epsilon}}{\theta+\beta}t_2 + \frac{Q_2}{\theta+\beta}(1 - e^{-(\theta+\beta)t_3}) \\
&+ \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}\frac{(1 - e^{-(\theta+\beta)t_3})}{\theta+\beta} - \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}t_3\Big].
\end{aligned} \tag{3.26}$$

Substitute Q_1 and Q_2 into equation (3.26),

$$\begin{aligned}
HC &= c_h \times \Big[\frac{K-bp^{-\epsilon}}{\beta}(t_1 - \frac{1-e^{-\beta t_1}}{\beta}) \\
&+ \frac{1}{\theta+\beta}\Big\{\frac{K-bp^{-\epsilon}}{\beta}(1-e^{-\beta t_1})\Big\}(1-e^{-(\theta+\beta)t_2}) \\
&+ \frac{bp^{-\epsilon}}{(\theta+\beta)^2}(1-e^{-(\theta+\beta)t_2}) - \frac{bp^{-\epsilon}}{\theta+\beta}t_2 \\
&+ \frac{1}{\theta+\beta}\Big\{\frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}(e^{(\theta+\beta)t_3}-1)\Big\}(1-e^{-(\theta+\beta)t_3}) \\
&+ \frac{b(\alpha p)^{-\epsilon}}{(\theta+\beta)^2}(1-e^{-(\theta+\beta)t_3}) - \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}t_3\Big]. \qquad (3.27)
\end{aligned}$$

We apply the power series expansion of the exponential function. In our model, the power of exponential function is small which is less than 1. Hence, the exponential terms are retained up to power of 2. It follows,

$$\begin{aligned}
HC &= c_h \times \Big[\frac{K-bp^{-\epsilon}}{2}t_1^2 + (K-bp^{-\epsilon})t_1t_2 - \frac{(K-bp^{-\epsilon})(\theta+\beta)t_1t_2^2}{2} \\
&- \frac{(K-bp^{-\epsilon})\beta t_1^2 t_2}{2} + \frac{(K-bp^{-\epsilon})\beta(\theta+\beta)t_1^2t_2^2}{4} - \frac{bp^{-\epsilon}t_2^2}{2} \\
&- \frac{b(\alpha p)^{-\epsilon}(\theta+\beta)^2t_3^4}{4} + \frac{b(\alpha p)^{-\epsilon}}{2}t_3^2\Big]. \qquad (3.28)
\end{aligned}$$

3.4.3. Production cost

Production cost refers to the cost of producing a product or supplying a service incurred by a company. Cost of production may include a range of expenditures, such as labor, raw materials, consumable supplies of manufacturing and overhead.

The production cost for this model is

$$PC = Kc_pt_1. \qquad (3.29)$$

3.4.4. Deterioration cost

The deterioration cost is often related to deteriorating units during the cycle time. The deterioration cost equation can be defined as

$$\begin{aligned}
DC &= c_p\theta \times \Big[\int_0^{t_2} \big(Q_1 e^{-(\theta+\beta)t} + \frac{bp^{-\epsilon}}{\theta+\beta}(e^{-(\theta+\beta)t} - 1)\big)dt \\
&+ \int_0^{t_3} \big(Q_2 e^{-(\theta+\beta)t} + \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}(e^{-(\theta+\beta)t} - 1)\big)dt\Big]. \\
&= c_p\theta \times \Big[\frac{Q_1}{\theta+\beta}(1 - e^{-(\theta+\beta)t_2}) + \frac{bp^{-\epsilon}}{\theta+\beta}\frac{(1 - e^{(\theta+\beta)t_2})}{\theta+\beta} - \frac{bp^{-\epsilon}}{\theta+\beta}t_2 \\
&+ \frac{Q_2}{\theta+\beta}(1 - e^{-(\theta+\beta)t_3}) + \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}\frac{(1 - e^{-(\theta+\beta)t_3})}{\theta+\beta} \\
&- \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}t_3\Big]. \qquad (3.30)
\end{aligned}$$

Replace Q_1 and Q_2 into equation (3.30),

$$\begin{aligned}
DC &= c_p\theta \times \Big[\frac{1}{\theta+\beta}\Big\{\frac{K - bp^{-\epsilon}}{\beta}(1 - e^{-\beta t_1})\Big\}(1 - e^{-(\theta+\beta)t_2}) \\
&+ \frac{bp^{-\epsilon}}{(\theta+\beta)^2}(1 - e^{-(\theta+\beta)t_2}) - \frac{bp^{-\epsilon}}{\theta+\beta}t_2 \\
&+ \frac{1}{\theta+\beta}\Big\{\frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}(e^{(\theta+\beta)t_3} - 1)\Big\}(1 - e^{-(\theta+\beta)t_3}) \\
&+ \frac{b(\alpha p)^{-\epsilon}}{(\theta+\beta)^2}(1 - e^{-(\theta+\beta)t_3}) - \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}t_3\Big]. \qquad (3.31)
\end{aligned}$$

Similarly as holding cost, by considering the exponential terms up to power of 2, then we have

$$\begin{aligned}
DC &= c_p\theta \times \Big[(K - bp^{-\epsilon})t_1t_2 - \frac{(K - bp^{-\epsilon})(\theta+\beta)t_1t_2^2}{2} - \frac{(K - bp^{-\epsilon})\beta t_1^2t_2}{2} \\
&+ \frac{(K - bp^{-\epsilon})\beta(\theta+\beta)t_1^2t_2^2}{4} - \frac{bp^{-\epsilon}t_2^2}{2} - \frac{b(\alpha p)^{-\epsilon}(\theta+\beta)^2t_3^4}{4} \\
&+ \frac{b(\alpha p)^{-\epsilon}t_3^2}{2}\Big]. \qquad (3.32)
\end{aligned}$$

3.4.5. Salvage Value

Salvage value is an asset's estimated resale value at the end of its helpful life. The salvage value is

$$
\begin{aligned}
SV &= \lambda c_p \theta \Big[\int_0^{t_2} \big(Q_1 e^{-(\theta+\beta)t} + \frac{bp^{-\epsilon}}{\theta+\beta}(e^{-(\theta+\beta)t} - 1) \big) dt \\
&+ \int_0^{t_3} \big(Q_2 e^{-(\theta+\beta)t} + \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}(e^{-(\theta+\beta)t} - 1) \big) dt \Big]. \\
&= \lambda c_p \theta \times \Big[\frac{Q_1}{\theta+\beta}(1 - e^{-(\theta+\beta)t_2}) + \frac{bp^{-\epsilon}}{\theta+\beta} \frac{(1 - e^{(\theta+\beta)t_2})}{\theta+\beta} - \frac{bp^{-\epsilon}}{\theta+\beta} t_2 \\
&+ \frac{Q_2}{\theta+\beta}(1 - e^{-(\theta+\beta)t_3}) + \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta} \frac{(1 - e^{-(\theta+\beta)t_3})}{\theta+\beta} - \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta} t_3 \Big]. \qquad (3.33)
\end{aligned}
$$

Similarly as before, substitute Q_1 and Q_2 into equation (3.33),

$$
\begin{aligned}
SV &= \lambda c_p \theta \times \Big[\frac{1}{\theta+\beta} \big\{ \frac{K - bp^{-\epsilon}}{\beta}(1 - e^{-\beta t_1}) \big\} (1 - e^{-(\theta+\beta)t_2}) \\
&+ \frac{bp^{-\epsilon}}{(\theta+\beta)^2}(1 - e^{-(\theta+\beta)t_2}) - \frac{bp^{-\epsilon}}{\theta+\beta} t_2 \\
&+ \frac{1}{\theta+\beta} \big\{ \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta}(e^{(\theta+\beta)t_3} - 1) \big\} (1 - e^{-(\theta+\beta)t_3}) \\
&+ \frac{b(\alpha p)^{-\epsilon}}{(\theta+\beta)^2}(1 - e^{-(\theta+\beta)t_3}) - \frac{b(\alpha p)^{-\epsilon}}{\theta+\beta} t_3 \Big]. \qquad (3.34)
\end{aligned}
$$

Likewise, as before we have

$$
\begin{aligned}
SV &= \lambda c_p \theta \times \Big[(K - bp^{-\epsilon}) t_1 t_2 - \frac{(K - bp^{-\epsilon})(\theta+\beta) t_1 t_2^2}{2} \\
&- \frac{(K - bp^{-\epsilon}) \beta t_1^2 t_2}{2} + \frac{(K - bp^{-\epsilon}) \beta (\theta+\beta) t_1^2 t_2^2}{4} - \frac{bp^{-\epsilon} t_2^2}{2} \\
&- \frac{b(\alpha p)^{-\epsilon} (\theta+\beta)^2 t_3^4}{4} + \frac{b(\alpha p)^{-\epsilon} t_3^2}{2} \Big]. \qquad (3.35)
\end{aligned}
$$

3.4.6. Total revenue

Total revenue consists of the revenue before markdown and the revenue when the markdown is offered. Thus, the revenue equation can be obtained as

$$\begin{aligned}
TR &= p \times \Big[\int_0^{t_1} \big(bp^{-\epsilon} + \beta I(t)\big)dt + \int_0^{t_2} \big(bp^{-\epsilon} + \beta I(t)\big)dt \\
&+ \int_0^{t_3} \big(b(\alpha p)^{-\epsilon} + \beta I(t)\big)dt\Big] \\
&= p \times \Big[bp^{-\epsilon}(t_1 + t_2) + b(\alpha p)^{-\epsilon}t_3 + \beta\big(\frac{K - bp^{-\epsilon}}{2}t_1^2 + (K - bp^{-\epsilon})t_1t_2 \\
&- \frac{(K - bp^{-\epsilon})(\theta + \beta)t_1t_2^2}{2} - \frac{(K - bp^{-\epsilon})\beta t_1^2t_2}{2} + \frac{(K - bp^{-\epsilon})\beta(\theta + \beta)t_1^2t_2^2}{4} \\
&- \frac{bp^{-\epsilon}t_2^2}{2} + \frac{b(\alpha p)^{-\epsilon}t_3^2}{2} - \frac{b(\alpha p)^{-\epsilon}(\theta + \beta)^2t_3^4}{4}\big)\Big]. \qquad (3.36)
\end{aligned}$$

Finally annual profit, AP, is equal to the total revenue plus salvage value minus holding cost, deterioration cost, production cost and set up cost. Hence,

$$\begin{aligned}
AP &= \frac{p}{T} \times \Big[bp^{-\epsilon}(t_1 + t_2) + b(\alpha p)^{-\epsilon}t_3 + \beta\big(\frac{K - bp^{-\epsilon}}{2}t_1^2 + (K - bp^{-\epsilon})t_1t_2 \\
&- \frac{(K - bp^{-\epsilon})(\theta + \beta)t_1t_2^2}{2} - \frac{(K - bp^{-\epsilon})\beta t_1^2t_2}{2} + \frac{(K - bp^{-\epsilon})\beta(\theta + \beta)t_1^2t_2^2}{4} \\
&- \frac{bp^{-\epsilon}t_2^2}{2} - \frac{b(\alpha p)^{-\epsilon}(\theta + \beta)^2t_3^4}{4} + \frac{b(\alpha p)^{-\epsilon}t_3^2}{2}\big)\Big] \\
&- \frac{c_h}{T} \times \Big[\frac{K - bp^{-\epsilon}}{2}t_1^2 + (K - bp^{-\epsilon})t_1t_2 - \frac{(K - bp^{-\epsilon})(\theta + \beta)t_1t_2^2}{2} \\
&- \frac{(K - bp^{-\epsilon})\beta t_1^2t_2}{2} + \frac{(K - bp^{-\epsilon})\beta(\theta + \beta)t_1^2t_2^2}{4} - \frac{bp^{-\epsilon}t_2^2}{2} \\
&- \frac{b(\alpha p)^{-\epsilon}(\theta + \beta)^2t_3^4}{4} + \frac{b(\alpha p)^{-\epsilon}}{2}t_3^2\Big] \\
&- \frac{c_p(1 - \lambda)\theta}{T} \times \Big[(K - bp^{-\epsilon})t_1t_2 - \frac{(K - bp^{-\epsilon})(\theta + \beta)t_1t_2^2}{2} \\
&- \frac{(K - bp^{-\epsilon})\beta t_1^2t_2}{2} + \frac{(K - bp^{-\epsilon})\beta(\theta + \beta)t_1^2t_2^2}{4} - \frac{bp^{-\epsilon}t_2^2}{2} \\
&- \frac{b(\alpha p)^{-\epsilon}(\theta + \beta)^2t_3^4}{4} + \frac{b(\alpha p)^{-\epsilon}t_3^2}{2}\Big] \\
&- \frac{Kc_pt_1}{T} - \frac{c_0}{T}. \qquad (3.37)
\end{aligned}$$

We can observe that AP is a function of t_1, t_2 and t_3. To reduce the complexity and to simplify the solution procedure, we optimize the AP function by following (Srivastava & Gupta, 2013) procedure where we rewrite,

$$\begin{aligned} t_1 &= \delta T, \\ t_2 &= \gamma(T - t_1) = \gamma(1-\delta)T, \\ t_3 &= T - (t_1 + t_2) = (1-\gamma)(1-\delta)T. \end{aligned}$$

In (Srivastava & Gupta, 2013), they only tried to vary T and fix the markdown time γ in order to find their optimal solution. However, in our case, we are able to find a better solution by varying both T and γ.

Substitute t_1, t_2 and t_3 into equation (3.37) and simplify, we obtain

$$\begin{aligned} AP &= p \times \Big[bp^{-\epsilon}\big(\delta + \gamma(1-\delta)\big) + b(\alpha p)^{-\epsilon}(1-\gamma)(1-\delta) \Big] \\ &+ (p\beta - c_h) \times \Big[\frac{(K - bp^{-\epsilon})\delta^2 T}{2} \Big] \\ &+ \big(p\beta - c_h - c_p(1-\lambda)\theta\big) \times \Big[(K - bp^{-\epsilon})\delta\gamma(1-\delta)T \\ &- \frac{(K - bp^{-\epsilon})(\theta+\beta)\delta\gamma^2(1-\delta)^2T^2}{2} - \frac{(K - bp^{-\epsilon})\beta\delta^2\gamma(1-\delta)T^2}{2} \\ &- \frac{bp^{-\epsilon}\gamma^2(1-\delta)^2 T}{2} + \frac{(K - bp^{-\epsilon})\beta(\theta+\beta)\delta^2\gamma^2(1-\delta)^2T^3}{4} \\ &- \frac{b(\alpha p)^{-\epsilon}(\theta+\beta)^2\big((1-\delta)(1-\gamma)\big)^4 T^3}{4} + \frac{b(\alpha p)^{-\epsilon}\big((1-\delta)(1-\gamma)\big)^2 T}{2} \Big] \\ &- Kc_p\delta - \frac{c_0}{T}. \end{aligned} \tag{3.38}$$

It follows that the necessary conditions for AP, a function of two variables to be maximized are $\frac{\delta AP}{\delta T} = 0$ and $\frac{\delta AP}{\delta \gamma} = 0$, where

$$
\begin{aligned}
\frac{\delta AP}{\delta T} &= (p\beta - c_h) \times \Big[\frac{(K-bp^{-\epsilon})\delta^2}{2}\Big] \\
&+ \big(p\beta - c_h - c_p(1-\lambda)\theta\big) \times \Big[(K-bp^{-\epsilon})\delta\gamma(1-\delta) \\
&- (K-bp^{\epsilon})(\theta+\beta)\delta\gamma^2(1-\delta)^2T - (K-bp^{-\epsilon})\beta\delta^2\gamma(1-\delta)T \\
&+ \frac{3(K-bp^{-\epsilon})\beta(\theta+\beta)\delta^2\gamma^2(1-\delta)^2T^2}{4} - \frac{bp^{-\epsilon}\big(\gamma^2(1-\delta)^2\big)}{2} \\
&- \frac{3b(\alpha p)^{-\epsilon}(\theta+\beta)^2\big((1-\gamma)(1-\delta)\big)^4T^2}{4} + \frac{b(\alpha p)^{-\epsilon}\big((1-\gamma)(1-\delta)\big)^2}{2}\Big] \\
&+ \frac{c_0}{T^2}.
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\delta AP}{\delta\gamma} &= p \times \Big[bp^{-\epsilon}(1-\delta) + b(\alpha p)^{-\epsilon}(-1+\delta)\Big] \\
&+ (p\beta - c_h) \times \Big[(K-bp^{-\epsilon})\delta(1-\delta)T\Big] \\
&+ \big(p\beta - c_h - c_p(1-\lambda)\theta\big) \times \Big[-(K-bp^{-\epsilon})(\theta+\beta)\delta\gamma(1-\delta)^2T^2 \\
&- \frac{(K-bp^{-\epsilon})\beta\delta^2(1-\delta)T^2}{2} + \frac{(K-bp^{-\epsilon})\beta(\theta+\beta)\delta^2(1-\delta)^2\gamma T^3}{2} \\
&- bp^{-\epsilon}\gamma(1-\delta)^2T - b(\alpha p)^{-\epsilon}(\theta+\beta)^2\big((1-\gamma)(1-\delta)\big)^3(-1+\delta)T^3 \\
&+ b(\alpha p)^{-\epsilon}\big((1-\gamma)(1-\delta)\big)(-1+\delta)T\Big].
\end{aligned}
$$

We test the optimal condition by using the function,

$$
\frac{\delta^2 AP}{\delta T^2}\frac{\delta^2 AP}{\delta\gamma^2} - \left(\frac{\delta^2 AP}{\delta T\delta\gamma}\right)^2 \quad > \quad 0
$$

where,

$$
\begin{aligned}
\frac{\delta^2 AP}{\delta T^2} &= \big(p\beta - c_h - c_p(1-\lambda)\theta\big) \times \Big[-(K - bp^{-\epsilon})(\theta+\beta)\delta\gamma^2(1-\delta)^2 \\
&- (K - bp^{-\epsilon})\beta\delta^2\gamma(1-\delta) - \frac{3}{2} b(\alpha p)^{-\epsilon}(\theta+\beta)^2\big((1-\gamma)(1-\delta)\big)^4 T \\
&+ \frac{3}{2}(K - bp^{-\epsilon})\beta(\theta+\beta)\delta^2\gamma^2(1-\delta)^2 T\Big] - \frac{2c_0}{T^3}.
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\delta^2 AP}{\delta \gamma^2} &= \big(p\beta - c_h - c_p(1-\lambda)\theta\big) \times \Big[-(K - bp^{-\epsilon})(\theta+\beta)\delta(1-\delta)^2 T^2 \\
&- bp^{-\epsilon}(1-\delta)^2 T + \frac{(K - bp^{-\epsilon})\beta(\theta+\beta)\delta^2(1-\delta)^2 T^3}{2} \\
&- b(\alpha p)^{-\epsilon}(\theta+\beta)^2 3\big((1-\gamma)(1-\delta)\big)^2(-1+\delta)(-1+\delta)T^3 \\
&+ b(\alpha p)^{-\epsilon}(-1+\delta)(-1+\delta)T\Big].
\end{aligned}
$$

Both functions must be less than zero in order to obtain the maximum annual profit.

3.5. Numerical examples

We illustrate the model by using numerical example with the following parameter values:

Example 1: Let c_0 = 100, p = 50, c_h = 0.05, c_p = 10, K = 200, β = 0.9, θ = 0.07, ϵ = 1.8, b = 10000, λ = 0.6 and δ = 0.2.

It is assumed that the value of markdown rate, α is varying from 0.5 to 0.9. By using equation (3.38) we are able to find the best markdown time γ and the best total cycle length T provided that all the optimality conditions are satisfied by using Excel Solver.

Table 3.1 shows the optimum solution for the problem. For example, when $\alpha = 0.5$ the maximum annual profit is 1268.1, the markdown percentage is 0.4195 which means

Table 3.1: Experimental result for Example 1

α	T^*	γ^*	Q_1^*	Q_2^*	AP^*
0.5	3.0	0.4195	85.8	83.2	1268.1
0.7	3.0	0.4282	88.0	46.8	1178.7
0.9	3.0	0.4284	89.5	31.0	1126.9

that by using the value of γ obtained the markdown time $t_2 = \gamma(1-\delta)T$ is 1.2585 and the cycle time is 3.0 with $Q_1 = 85.8$ and $Q_2 = 83.2$.

Observe that when α = 0.7, the maximum annual profit is 1178.7, the markdown percentage is 0.4282. Similarly, the markdown time $t_2 = \gamma(1-\delta)T$ is 1.2846 and the cycle time is 3.0 with Q_1 = 88.0 and Q_2 = 46.8.

Finally, when α = 0.9, the maximum total profit is 1126.9, the markdown percentage is 0.4284 which gives 1.2852 for markdown time and the cycle time is 3.0 with Q_1 = 89.5 and Q_2 = 31.0.

As per the above numerical result, note that when the value of α increases (lower reduction in price), the higher the markdown percentage which means it is better to apply markdown at a later time in order to maximize the annual profit since the impact is lesser compare to when the value of α is lower which indicates higher reduction in price. Moreover, the quantity Q_1 at t_1 is higher since more quantity is needed in order to maximize the annual profit due to the smaller impact of markdown for this case. The quantity for markdown Q_2 is lower because again, the small reduction in price gives a small impact in maximizing the annual profit hence, it is more profitable to have less quantity during this interval.

It can also be seen that if the markdown price value at an unknown markdown time is different, the profits are different but there is no change in the replenishment time. In all cases, the optimality conditions are satisfied.

From Table 3.1 observe that the effect of markdown price with an unknown time to optimize annual profit are case dependent. In this model, when the price is reduced to 50

$\%$ of its initial price, it dominates other markdown prices at any markdown rate in order to maximize the profit. A key element in markdown optimization is determining how items respond to price changes.

Table 3.1 also shows that if the markdown rate α is increased by 40% to 80%, there is a slight decrease in the annual profit by around 4% to 7%. Also, we can see that there is a slight increase in the markdown percentage γ by around 0.04% to 2% with the increase of markdown rate α. Hence, it is profitable to apply markdown later in time when the amount of markdown given is lower i.e., higher markdown rate.

3.5.1. Sensitivity analysis

The change in the values of parameters may happen due to uncertainties in any decision-making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in decision-making. The effect of changes in the parameter of the inventory model to the decision variables is shown in this subsection.

Table 3.2 to 3.11 give the sensitive analysis to examine the mathematical model's efficacy with regard to the parameters $b, \theta, K, \epsilon, p, \lambda, \beta, c_p, c_h c_0$ and δ on the optimum cycle time (T^*), optimum markdown percentage (γ^*), optimum quantities (Q_1^* and Q_2^*) as well as optimum annual profit. Also, the effect of changing the parameters are shown graphically in Figs. 3.2 to 3.12.

The sensitivity analysis for every parameter is tested by using the optimum value as well as the plus and minus 50 % from its optimum value to observe the effect of changes clearer.

Table 3.2: Effect of changes in b

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
b = 5000	3.0	0.4057	99.7	20.8	964.7
b = 10000	3.0	0.4284	89.5	31.0	1126.9
b = 15000	3.0	0.4304	77.9	50.8	1489.2

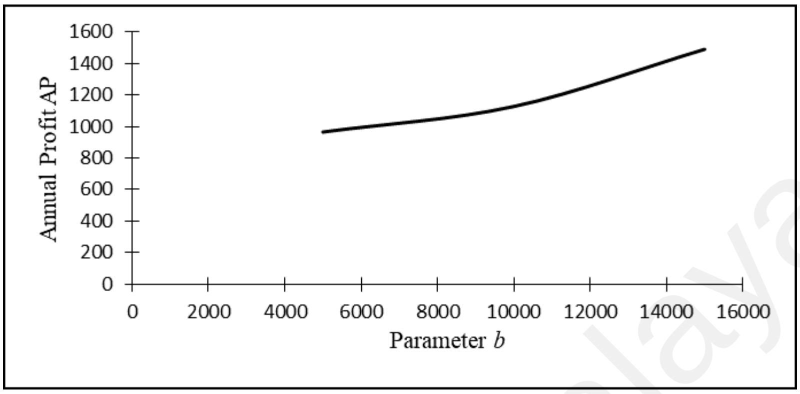

Figure 3.2: Effect of changes in b on the optimal annual profit

Table 3.3: Effect of changes in θ

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
θ = 0.04	3.0	0.4292	91.2	31.6	1156.9
θ = 0.07	3.0	0.4284	89.5	31.0	1126.9
θ = 0.10	3.0	0.4277	87.8	30.5	1098.3

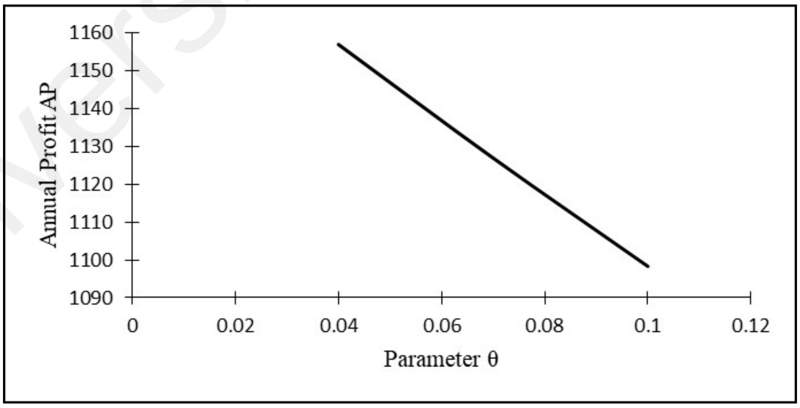

Figure 3.3: Effect of changes in θ on the optimal annual profit

Table 3.4: Effect of changes in K

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
K = 100	3.0	0.4300	39.4	26.1	726.2
K = 200	3.0	0.4284	89.5	31.0	1126.9
K = 300	3.0	0.4184	142.5	35.8	1539.1

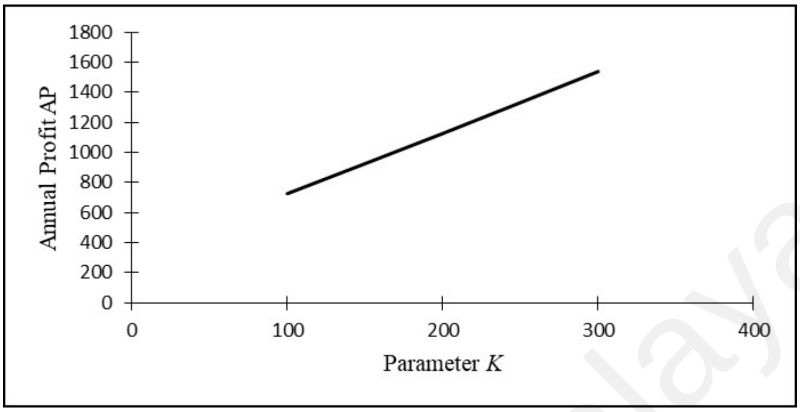

Figure 3.4: Effect of changes in K on the optimal annual profit

Table 3.5: Effect of changes in ϵ

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
ϵ = 0.9	2.7	0.4222	76.3	53.3	1548.2
ϵ = 1.8	3.0	0.4284	89.5	31.0	1126.9
ϵ = 2.7	7.0	0.2258	162.4	28.3	1034.6

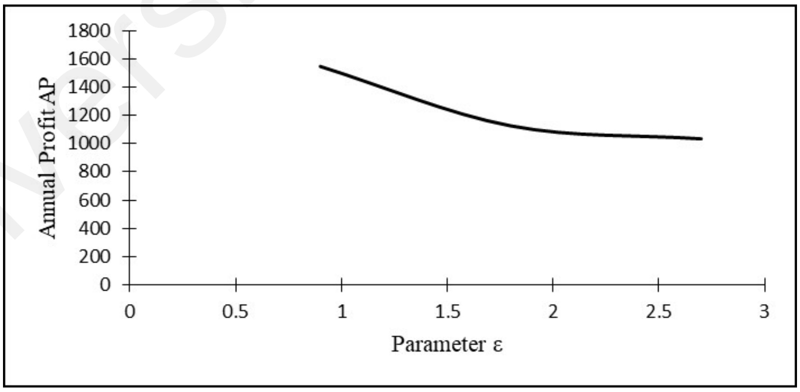

Figure 3.5: Effect of changes in ϵ on the optimal annual profit

Table 3.6: Effect of changes in p

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
p = 25	2.5	0.4073	71.0	67.8	819.4
p = 50	3.0	0.4284	89.5	31.0	1126.9
p = 75	3.4	0.4050	99.7	20.1	1656.4

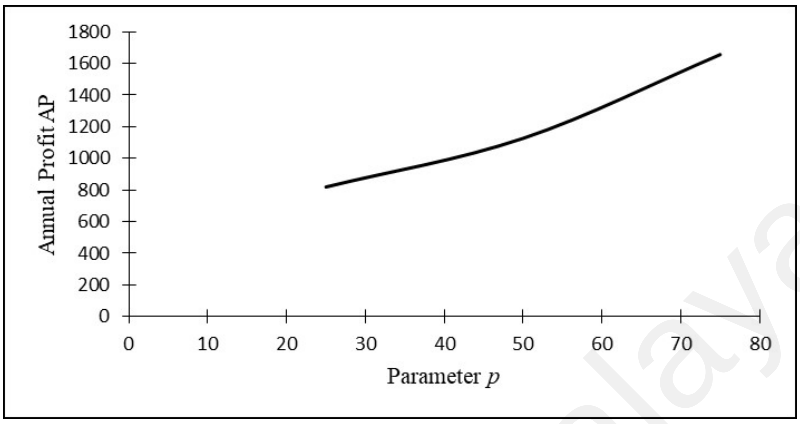

Figure 3.6: Effect of changes in p on the optimal annual profit

Table 3.7: Effect of changes in λ

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
λ = 0.3	3.0	0.4283	89.6	31.1	1124.2
λ = 0.6	3.0	0.4284	89.5	31.0	1126.9
λ = 0.9	3.0	0.4286	89.4	30.9	1129.7

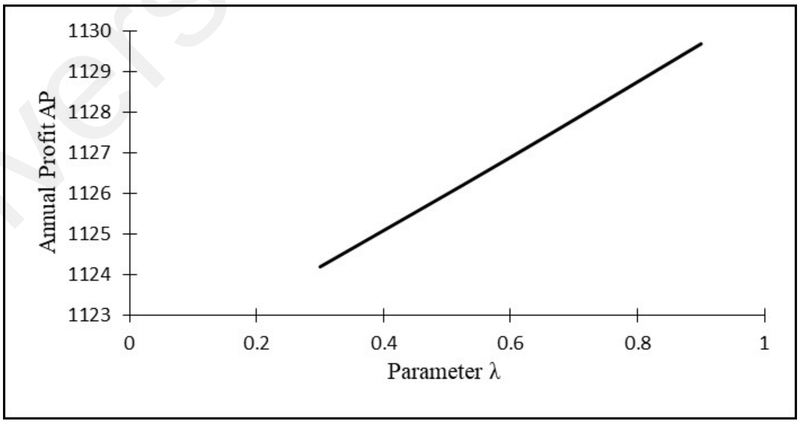

Figure 3.7: Effect of changes in λ on the optimal annual profit

Table 3.8: Effect of changes in β

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
$\beta = 0.3$	8.0	0.4264	244.0	83.4	1025.6
$\beta = 0.6$	4.0	0.4281	130.7	45.0	1104.3
$\beta = 0.9$	3.0	0.4284	89.5	31.0	1126.9

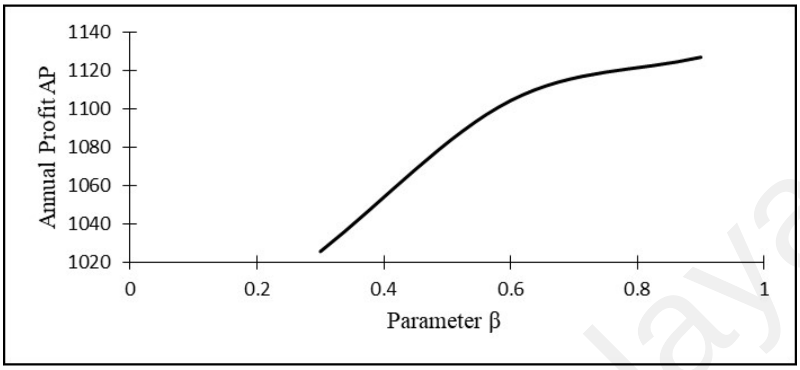

Figure 3.8: Effect of changes in β on the optimal annual profit

Table 3.9: Effect of changes in δ

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
$\delta = 0.10$	2.3	0.4264	39.7	23.6	676.9
$\delta = 0.20$	3.0	0.4284	89.5	31.0	1126.9
$\delta = 0.30$	4.5	0.4180	148.9	52.9	1882.9

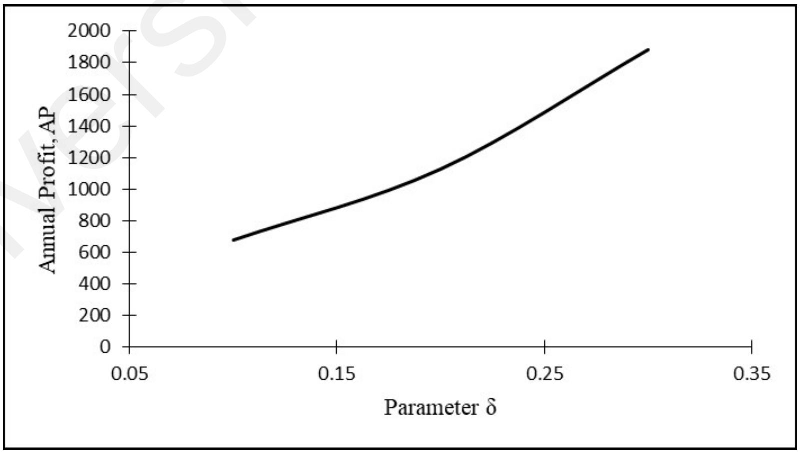

Figure 3.9: Effect of changes in δ on the optimal annual profit

Table 3.10: Effect of changes in c_0

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
c_0 = 50	3.0	0.4292	88.9	30.5	1143.5
c_0 = 100	3.0	0.4284	89.5	31.0	1126.9
c_0 = 150	3.0	0.4277	90.0	31.6	1110.6

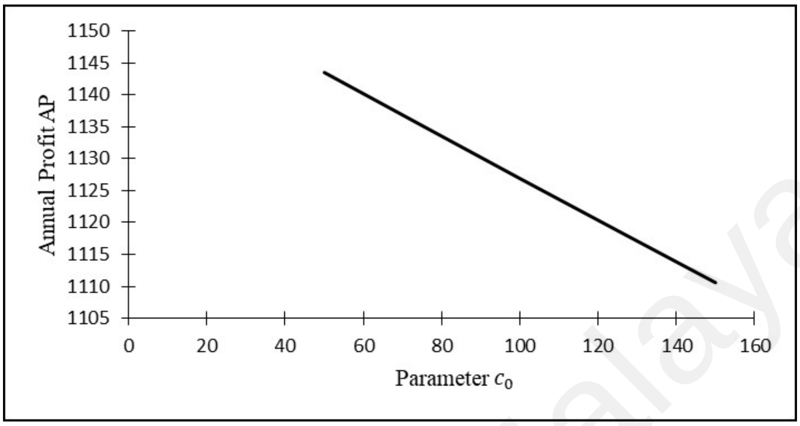

Figure 3.10: Effect of changes in c_0 on the optimal annual profit

Table 3.11: Effect of changes in c_h

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
c_h = 0.025	3.0	0.4284	89.5	31.0	1127.6
c_h = 0.050	3.0	0.4284	89.5	31.0	1126.9
c_h = 0.075	3.0	0.4284	89.5	31.0	1126.3

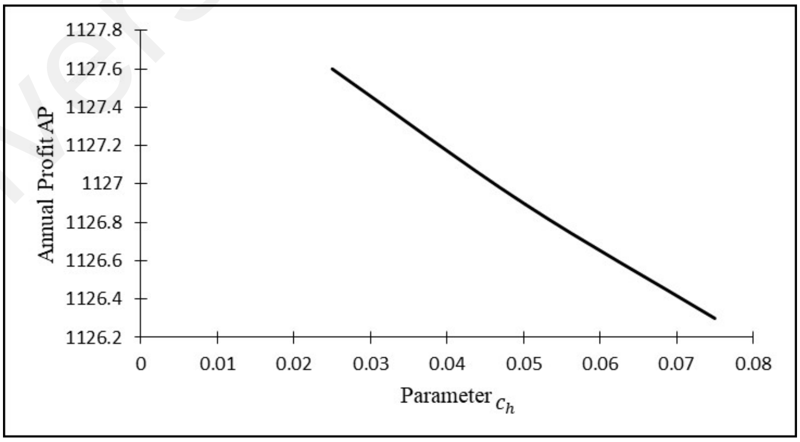

Figure 3.11: Effect of changes in c_h on the optimal annual profit

Table 3.12: Effect of changes in c_p

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
c_p = 5	3.0	0.4285	89.4	31.0	1328.8
c_p = 10	3.0	0.4284	89.5	31.0	1126.9
c_p = 15	3.0	0.4283	89.6	31.0	925.1

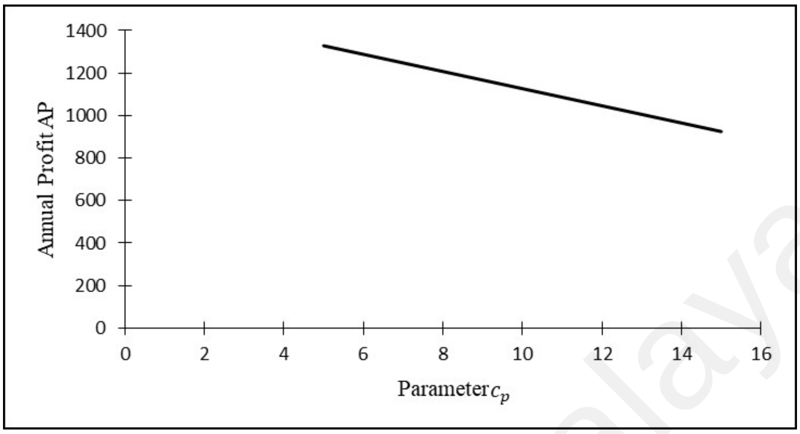

Figure 3.12: Effect of changes in c_p on the optimal annual profit

The sensitivity analysis reveals that:

1. A higher value of b, β or λ causes higher values of AP^* and γ^*.

2. Coversely, a higher value of θ, ϵ, c_p, c_h or c_0 causes lower values of AP^* and γ^*.

3. A higher value of p, K or δ causes a higher value of AP^* while a lower value of γ^*.

Simple interpretations of the above results are as follows:

1. The demand is high when the value of b (as well as β) is high. Therefore, higher demand gives higher annual profit and markdown is better to be applied at a later time since the demand is high. As for λ, the total inventory cost can be minimized if the salvage value is incorporated into the deteriorating items, which eventually maximized the annual profit.

2. Higher θ (as well as ϵ or c_p, c_h or c_0) leads to lower demand rate due to the higher rate of deterioration (as well as price increase or increase in cost) and so the annual profit eventually becomes lower. Therefore, markdown needs to be implemented

earlier since item deteriorates at a faster rate and in order to maximize the annual profit.

3. The value of annual profit is higher when the value of p (as well as δ and K) is higher. At a certain point, demand will eventually drop because of selling a deteriorated item with an expensive price. Therefore, markdown needs to be applied earlier if the value of p is high, in this case, in order to help clear the end of life inventory. The higher the δ (as well as K), it is assumed that the volume is also higher. Hence, the higher the volume, the higher the annual profit. Since the volume is high, markdown is best to be applied earlier in time in order to finish the end of life inventory and maximize annual profit.

3.6. Conclusion

In this study, we extend the previous published works by (Teng & Chang, 2005) and (Srivastava & Gupta, 2013) where a delayed deteriorating model when demand depends on both price and inventory level under markdown policy has been established. This model is relevant to help sell end-of-season and slow-moving inventory. In order to maximize the profit and reduce the stock specifically for slow-moving items, markdown policy is introduced.

The optimal replenishment time, optimal quantities, optimal markdown time as well as optimal annual profit have been derived. Unlike previous studies such as (Widyadana & Wee, 2007) and (Srivastava & Gupta, 2013), in this model, instead of fixing the markdown time, we considered the markdown time as a variable together with the cycle length. It also can be concluded that by finding the optimum markdown time that maximizes the Annual Profit, it gives a better profit than fixing the value of it.

Sensitivity analysis revealed that different types of the parameter have dissimilar effects on optimal values and this analysis can be a great help for retailers especially in decision making. Notice that based on this analysis, the annual profit is less sensitive to several parameters such as λ, c_h and c_0.

We can conclude that in this model, the higher the markdown rate (lesser reduction in price), the lower the annual profit. However, it contradicts previous studies such as (Widyadana & Wee, 2007) and (Srivastava & Gupta, 2013) that we refer to since they only considered demand rate to be dependent solely on price or price and time. Thus, it shows that markdown rate contributes significantly in optimizing the total profit and it is important for a policy maker to be very cautious in setting markdown rate because the optimum policy is distinct for different cases as it is said to be case dependent.

CHAPTER 4: INVENTORY MODEL FOR FRESH PRODUCT WHEN DEMAND DEPENDS ON FRESHNESS, PRICE, INVENTORY LEVEL AND EXPIRATION DATE UNDER MARKDOWN POLICY.

4.1. Introduction

In this chapter, we present an inventory model with demand for a perishable item depending on the freshness condition, price, stock level and expiration date. Previously, researchers implemented zero inventory strategy where retailers aspire to hold no on-hand inventory stock at the end of the cycle. The traditional zero ending is relaxed to enhance the potential profit. The objective is to maximize the annual profit of the inventory system by finding the optimum total cycle time and optimum quantities. We explore the best solution for the annual profit by numerical examples. We also demonstrate the relationship between markdown time and the annual profit. The sensitivity analysis will be a great help to see the effect of changes in the annual profit by varying some parameter values.

This model is developed based on the published works by (Teng & Chang, 2005), (Srivastava & Gupta, 2013) and (Feng et al., 2017). We aim to focus on fresh goods with low demand and shorter lifetime therefore, we extend the demand function to be depending on price, inventory model, freshness consideration as well as the expiration date. Price is an important factor in the demand based on marketing and economic theory (Banton, 2019). Moreover, since we are focusing on fresh items where the demand also depends on its freshness and expiration date. In addition, increasing inventory levels may encourage customers to buy more hence, enhance the profit.

We aim to study the effect of the freshness of an item from consumers' perspective and it is important to take into consideration the expiration date as it is the key to determine the freshness level of an item. In addition, we incorporate markdown policy into the inventory model by applying the same approach used by (Widyadana & Wee, 2007) and

(Srivastava & Gupta, 2013).

4.2. Model's background

The following notations are used throughout this chapter:

1. α = Markdown rate.
2. ϵ = Increase price rate.
3. γ = Markdown percentage.
4. c = Purchasing cost per unit.
5. c_0 = Ordering cost per order.
6. c_h = Holding cost per unit per unit of time.
7. $I(t)$ = Inventory level at time t.
8. m = Maximum lifetime (i.e., the time to its expiration date) in units of time.
9. p = Initial price.
10. Q_1 = Inventory level at 0.
11. Q_2 = Inventory level at t_1.
12. s = Salvage price per unit.
13. t_1 = Time at which the inventory level reaches Q_2.
14. T = Replenishment cycle time in units of time.
15. E = Ending-stock level in units, where $E \geq 0$.

To develop the model, the following assumptions are used:

1. Fresh product deteriorates continuously with time and has an expiration date. The freshness index is 1 at time 0 and gradually decreases over time. It becomes close to 0 when it is approaching the expiration date m. The freshness index at time t is as assumed to be :

$$f(t) = \frac{m-t}{m}, \quad 0 \leq t \leq m.$$

2. Demand at time t is assumed to be

$$D(I(t), p, t) = \big(b(\alpha p)^{-\epsilon} + \beta I(t)\big)\big(\frac{m-t}{m}\big), \quad 0 \leq t \leq t_1 \leq T,$$

   where α, b and β are positive constants with α and β are between 0 and 1. In addition, the total cycle time T must be less than or equal to the expiration date, m (i.e., $T \leq m$), otherwise $D(I(t), p, t) \leq 0$.

3. A single item is considered over a prescribed period of T units of time.

4. Shortage is not allowed.

5. Only one time markdown price at one planning period is applied.

6. Markdown price is known.

7. Markdown time is proportional to the cycle time where $t_1 = \gamma T$.

8. It may be profitable to maintain higher stocks (i.e., non-zero ending inventory) if the demand depends on the stock level and freshness condition, $E \geq 0$.

9. At the end of cycle time T (i.e., the inventory level reaches E units), the retailer sells all E units at a salvage price per unit.

10. We may assume that the quantity Q_1 is greater than or equal to quantity at t_1, Q_2. While Q_2 is greater or equal to ending inventory level, E such as $0 \leq E \leq Q_2 \leq$

Q_1.

4.3. Mathematical formulation

At time $t = 0$, the maximum inventory level, Q_1 is reached. The inventory level starts to decrease due to consumption. However, as time goes by, the freshness of fresh product continue to decrease and due to this, the demand decreases. In order to increase again the demand and help to sell fresh goods before they reach their expiration date, markdown is applied at time $t = t_1$. The ending inventory is relaxed to non-zero. Based on the assumptions, the behaviour of inventory system $I(t)$ at time t is illustrated in Figure 4.1 and the inventory level $I(t)$ at time t over period $(0, T)$ is governed by these differential equations:

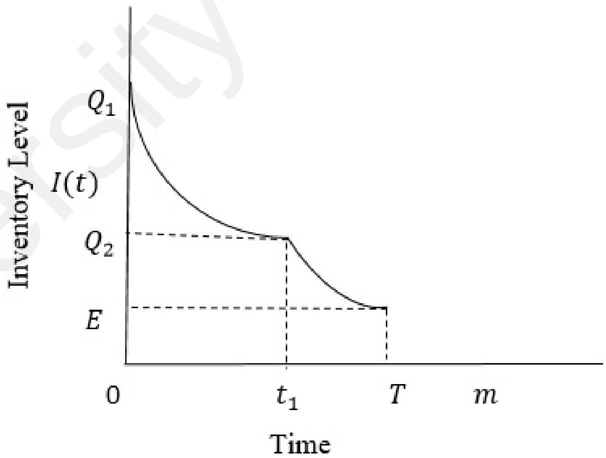

Figure 4.1: Graphical representation of the inventory system

The equations below are the combination and modification of the equations used by (Teng & Chang, 2005), (Srivastava & Gupta, 2013) and (Feng et al., 2017).

For $0 \leq t \leq t_1$, there is no breakdown price since markdown is not offered yet so

$\alpha = 1$.

$$\frac{dI(t)}{dt} = -\big(bp^{-\epsilon} + \beta I(t)\big)\big(\frac{m-t}{m}\big), \qquad 0 \le t \le t_1, \tag{4.1}$$

and

for $t_1 \le t \le T$, markdown is offered in order to increase the demand rate hence

$$\frac{dI(t)}{dt} = -\big(b(\alpha p)^{-\epsilon} + \beta I(t)\big)\big(\frac{m-t}{m}\big), \qquad 0 < \alpha < 1. \tag{4.2}$$

We solve the equations above by using the technique of an integrating factor as this method has been used by many established papers to solve these types of equations as well as it is less tedious. The purpose of this derivation is to show the steps taken in order to obtain the equation for the inventory level, $I(t)$ for each interval.

We first solve equation (4.1)

$$\frac{dI(t)}{dt} = -bp^{-\epsilon}\big(\frac{m-t}{m}\big) - \beta I(t)\big(\frac{m-t}{m}\big).$$

Rearrange,

$$\frac{dI(t)}{dt} + \beta I(t)\big(\frac{m-t}{m}\big) = -bp^{-\epsilon}\big(\frac{m-t}{m}\big). \tag{4.3}$$

To find the integrating factor

$$\mu(t) = e^{\int \beta\left(\frac{m-t}{m}\right)dt} = e^{\beta\left(t - \frac{t^2}{2m}\right)}. \tag{4.4}$$

Multiply equation (4.3) by the integrating factor obtained. This will give

$$\frac{dI(t)}{dt}e^{\beta\left(t-\frac{t^2}{2m}\right)} + \beta I(t)\big(\frac{m-t}{m}\big)e^{\beta\left(t-\frac{t^2}{2m}\right)} = -bp^{-\epsilon}\big(\frac{m-t}{m}\big)e^{\beta\left(t-\frac{t^2}{2m}\right)} \tag{4.5}$$

which is equivalent to

$$\frac{dI(t)}{dt}e^{\beta\left(t-\frac{t^2}{2m}\right)} = -bp^{-\epsilon}(\frac{m-t}{m})e^{\beta\left(t-\frac{t^2}{2m}\right)}. \tag{4.6}$$

Integrate both sides of the equation

$$\begin{aligned}\int \frac{d}{dt}e^{\beta\left(t-\frac{t^2}{2m}\right)}I(t) &= -bp^{-\epsilon}\int (\frac{m-t}{m})e^{\beta\left(t-\frac{t^2}{2m}\right)}dt \\ e^{\beta\left(t-\frac{t^2}{2m}\right)}I(t) &= -bp^{-\epsilon}\frac{e^{\beta\left(t-\frac{t^2}{2m}\right)}}{\beta}+C. \end{aligned} \tag{4.7}$$

Therefore, we obtain

$$I(t) = \frac{-bp^{-\epsilon}}{\beta}+Ce^{-\beta\left(t-\frac{t^2}{2m}\right)}. \tag{4.8}$$

Solving the differential equation with $I(0) = Q_1$,

$$\begin{aligned} I(0) &= \frac{-bp^{-\epsilon}}{\beta}+Ce^{-\beta\left(0-\frac{0^2}{2m}\right)} = Q_1 \\ \frac{-bp^{-\epsilon}}{\beta}+C &= Q_1 \\ C &= Q_1+\frac{bp^{-\epsilon}}{\beta}. \end{aligned} \tag{4.9}$$

Substitute C into equation (4.8). Hence, we get

$$I(t) = \frac{bp^{-\epsilon}}{\beta}(e^{-\beta(t-\frac{t^2}{2m})}-1)+Q_1e^{-\beta(t-\frac{t^2}{2m})}. \tag{4.10}$$

Substitute t_1 using the fact $I(t_1) = Q_2$,

$$\frac{bp^{-\epsilon}}{\beta}(e^{-\beta(t_1-\frac{t_1^2}{2m})}-1) \quad + \quad Q_1 e^{-\beta(t_1-\frac{t_1^2}{2m})} = Q_2$$
$$\frac{bp^{-\epsilon}}{\beta}(e^{-\beta(t_1-\frac{t_1^2}{2m})}-1) \quad + \quad Q_1 e^{-\beta(t_1-\frac{t_1^2}{2m})} - Q_2 = 0. \tag{4.11}$$

Hence, we obtain the order quantity, Q_1

$$Q_1 = Q_2 e^{\beta(t_1-\frac{t_1^2}{2m})} - \frac{bp^{-\epsilon}}{\beta}(1-e^{\beta(t_1-\frac{t_1^2}{2m})}) > Q_2. \tag{4.12}$$

The inventory level during the second interval is governed by this equation

$$\frac{dI(t)}{dt} = -\big(b(\alpha p)^{-\epsilon} + \beta I(t)\big)\big(\frac{m-t}{m}\big), \qquad t_1 \leq t \leq T \quad \text{and} \quad 0 < \alpha < 1.$$

Rearrange,

$$\frac{dI(t)}{dt} + \beta I(t)\big(\frac{m-t}{m}\big) = -b(\alpha p)^{-\epsilon}\big(\frac{m-t}{m}\big). \tag{4.13}$$

Similarly, we solve this equation by using the same technique. To find the integrating factor

$$\mu(t) = e^{\int \beta\left(\frac{m-t}{m}\right)dt} = e^{\beta\left(t-\frac{t^2}{2m}\right)}. \tag{4.14}$$

Multiply equation (4.13) by the integrating factor obtained. This will give

$$\frac{dI(t)}{dt} e^{\beta\left(t-\frac{t^2}{2m}\right)} = -b(\alpha p)^{-\epsilon}\big(\frac{m-t}{m}\big)e^{\beta\left(t-\frac{t^2}{2m}\right)}. \tag{4.15}$$

Integrate both sides of the equation

$$\begin{aligned}\int \frac{d}{dt} e^{\beta\left(t-\frac{t^2}{2m}\right)} I(t) &= -b(\alpha p)^{-\epsilon} \int \left(\frac{m-t}{m}\right) e^{\beta\left(t-\frac{t^2}{2m}\right)} dt \\ e^{\beta\left(t-\frac{t^2}{2m}\right)} I(t) &= -b(\alpha p)^{-\epsilon} \frac{e^{\beta\left(t-\frac{t^2}{2m}\right)}}{\beta} + C. \end{aligned} \tag{4.16}$$

Therefore, we obtain

$$I(t) = \frac{-b(\alpha p)^{-\epsilon}}{\beta} + Ce^{-\beta\left(t-\frac{t^2}{2m}\right)}. \tag{4.17}$$

Solving the differential equation with $I(T) = E$ we obtain

$$\begin{aligned} I(T) &= \frac{-b(\alpha p)^{-\epsilon}}{\beta} + Ce^{-\beta\left(T-\frac{T^2}{2m}\right)} = E \\ Ce^{-\beta\left(T-\frac{T^2}{2m}\right)} &= E + \frac{b(\alpha p)^{-\epsilon}}{\beta} \\ C &= \left(E + \frac{b(\alpha p)^{-\epsilon}}{\beta}\right) e^{\beta\left(T-\frac{T^2}{2m}\right)}. \end{aligned} \tag{4.18}$$

Substitute C into equation (4.17) then we get

$$I(t) = \frac{b(\alpha p)^{-\epsilon}}{\beta}\left(e^{\beta((T-t)-\frac{(T^2-t^2)}{2m})} - 1\right) + Ee^{\beta((T-t)-\frac{(T^2-t^2)}{2m})}. \tag{4.19}$$

Substitute t_1 using $I(t_1) = Q_2$,

$$Q_2 = \frac{b(\alpha p)^{-\epsilon}}{\beta}\left(e^{\beta((T-t_1)-\frac{(T^2-t_1^2)}{2m})} - 1\right) + Ee^{\beta((T-t_1)-\frac{(T^2-t_1^2)}{2m})}. \tag{4.20}$$

4.4. Model development

The objective function consists of:

- Holding cost.

- Ordering cost.
- Purchasing cost.
- Salvage value.
- Total revenue.

4.4.1. Holding cost

Holding cost can be defined as the additional costs associated with the storage and maintenance of an inventory per unit time. Holding cost is related with storing inventory that remains unsold.

To find the holding cost

$$\begin{aligned} HC &= c_h \times \Big[\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\Big] \\ &= c_h \times \int_0^{t_1} \Big(\frac{bp^{-\epsilon}}{\beta}(e^{-\beta(t-\frac{t^2}{2m})} - 1) + Q_1 e^{-\beta(t-\frac{t^2}{2m})}\Big)dt \\ &+ c_h \times \int_{t_1}^{T} \Big(\frac{b(\alpha p)^{-\epsilon}}{\beta}(e^{\beta((T-t)-\frac{(T-t)^2}{2m})} - 1) \\ &+ Ee^{\beta((T-t)-\frac{(T-t)^2}{2m})}\Big)dt. \end{aligned} \tag{4.21}$$

The integration for equation (4.21) seems to be too complicated and tedious to solve. For simplicity, we follow a simple approximation as in (Bai & Kendall, 2008), (Chen et al., 2016) and (Feng et al., 2017). Note that the average inventory level for the time interval [0,t_1] is approximated to be equal to $(Q_1 + Q_2)/2$ and the average inventory level for the time interval [t_1, T] approximately equals $(Q_2 + E)/2$. Therefore the holding cost approximately becomes

$$c_h \times \Big(\frac{1}{2}(Q_1 + Q_2)t_1 + \frac{1}{2}(Q_2 + E)(T - t_1)\Big). \tag{4.22}$$

4.4.2. Ordering cost

Ordering cost includes the expenses incurred to create and process an order to a supplier.

$$OC = c_0. \tag{4.23}$$

4.4.3. Purchasing cost

The purchasing cost is the most basic type of inventory cost. Retailers purchase inventory of finished goods prepared for resale as quickly as they receive it.

$$PC = cQ_1. \tag{4.24}$$

4.4.4. Salvage value

Salvage value is an asset's estimated resale value at the end of its helpful life. The salvage value is

$$SV = sE. \tag{4.25}$$

4.4.5. Total revenue

Total revenue consists of pre-markdown revenue and revenue when the markdown is offered. Thus, the revenue equation can be obtained as

$$TR = p(Q_1 - E) \tag{4.26}$$

Since the demand function is depending on price, stock level, freshness condition and expiration date hence, the objective must be to maximize the annual total profit because if the aim is to minimize the total cost, both inventory level and demand will decrease which eventually resulting in lower profit.

The annual profit, AP, for this problem can be defined as:

AP = revenue received + salvage value - purchasing cost - ordering cost - holding cost.

$$\begin{aligned} AP &= \frac{1}{T}\Bigg[p(Q_1 - E) + sE - cQ_1 - c_0 \\ &- c_h\Bigg(\frac{1}{2}(Q_1 + Q_2)t_1 + \frac{1}{2}(Q_2 + E)(T - t_1)\Bigg)\Bigg] \end{aligned} \tag{4.27}$$

where

$$Q_1 = Q_2 e^{\beta(t_1 - \frac{t_1^2}{2m})} - \frac{bp^{-\epsilon}}{\beta}(1 - e^{\beta(t_1 - \frac{t_1^2}{2m})}) > Q_2, \tag{4.28}$$

$$Q_2 = \frac{b(\alpha p)^{-\epsilon}}{\beta}(e^{\beta((T-t_1) - \frac{(T^2 - t_1^2)}{2m})} - 1) + Ee^{\beta((T-t_1) - \frac{(T^2 - t_1^2)}{2m})} \tag{4.29}$$

and

$$0 \leq t_1 \leq T \leq m. \tag{4.30}$$

Following from (Srivastava & Gupta, 2013), to reduce the complexity and to simplify the solution procedure we optimize the annual profit where we rewrite

$$t_1 = \gamma T.$$

Substitute t_1 into equation (4.27), then we obtain

$$\begin{aligned} AP &= \frac{1}{T}\Bigg[p(Q_1 - E) + sE - cQ_1 - c_0 \\ &- c_h\Bigg(\frac{1}{2}(Q_1 + Q_2)\gamma T + \frac{1}{2}(Q_2 + E)(T - \gamma T)\Bigg)\Bigg]. \end{aligned} \tag{4.31}$$

Since the main objective is to maximize the total profit, the necessary condition for AP to be maximum is obtained by taking the first derivative of AP with respect to T and equating it to zero where,

$$
\begin{aligned}
\frac{\delta TP}{\delta T} &= (p-c)\Bigg[\bigg(\frac{b(\alpha p)^{-\epsilon}}{\beta}+E\bigg)\bigg(\frac{e^{\beta(T-\frac{T^2}{2m})}\Big(\beta\big(1-\frac{T}{m}\big)\Big)}{T}-\frac{e^{\beta(T-\frac{T^2}{2m})}}{T^2}\bigg)\\
&+ \bigg(\frac{bp^{-\epsilon}}{\beta}-\frac{b(\alpha p)^{-\epsilon}}{\beta}\bigg)\bigg(\frac{e^{\beta(\gamma T-\frac{(\gamma T)^2}{2M})}\Big(\beta\big(\gamma-\frac{\gamma^2 T}{m}\big)\Big)}{T}-\frac{e^{\beta(\gamma T-\frac{(\gamma T)^2}{2M})}}{T^2}\bigg)\\
&+ \frac{bp^{-\epsilon}}{\beta T^2}\Bigg]+p\bigg[\frac{E}{T^2}\bigg]-c_h\bigg[\frac{\gamma}{2}\bigg(\frac{b(\alpha p)^{-\epsilon}}{\beta}+E\bigg)\bigg(e^{\beta(T-\frac{T^2}{2m})}\Big(\beta\big(1-\frac{T}{m}\big)\Big)\\
&+ e^{\beta((T-\gamma T)-\frac{(T^2-(\gamma T)^2)}{2m})}\Big(\beta\big((1-\gamma)-\frac{(T-\gamma^2 T)}{m}\big)\Big)\bigg)\\
&+ \bigg(\frac{bp^{-\epsilon}}{\beta}-\frac{b(\alpha p)^{-\epsilon}}{\beta}\bigg)\bigg(e^{\beta(\gamma T-\frac{(\gamma T)^2}{2M})}\Big(\beta\big(\gamma-\frac{\gamma^2 T}{m}\big)\Big)\bigg)\\
&+ \frac{(1-\gamma)}{2}\bigg(\frac{b(\alpha p)^{-\epsilon}}{\beta}+E\bigg)\bigg(e^{\beta((T-\gamma T)-\frac{(T^2-(\gamma T)^2)}{2m})}\Big(\beta\big((1-\gamma)\\
&- \frac{(T-\gamma^2 T)}{m}\big)\Big)\bigg)\bigg]-\frac{SE}{T^2}+\frac{c_0}{T^2}.
\end{aligned}
$$

We test the optimal condition by using the function

$$
\begin{aligned}
\frac{\delta^2 TP}{\delta T^2} &= (p-c)\Bigg[\bigg(\frac{b(\alpha p)^{-\epsilon}}{\beta}+E\bigg)\bigg(e^{\beta(T-\frac{T^2}{2m})}\Big(\beta\big(1-\frac{T}{m}\big)\Big)\Big(\beta\big(\frac{m-T}{mT}\big)\Big) \\
&- \frac{\beta e^{\beta(T-\frac{T^2}{2m})}}{T^2}-\frac{e^{\beta(T-\frac{T^2}{2m})}\beta\big(1-\frac{T}{m}\big)}{T^2}+\frac{2e^{\beta(T-\frac{T^2}{2m})}}{T^3}\bigg)-\frac{2bp^{-\epsilon}}{\beta T^3} \\
&+ \bigg(\frac{bp^{-\epsilon}}{\beta}-\frac{b(\alpha p)^{-\epsilon}}{\beta}\bigg)\bigg(e^{\beta(\gamma T-\frac{(\gamma T)^2}{2M})}\Big(\beta\big(\gamma-\frac{\gamma^2 T}{m}\big)\Big)\Big(\beta\big(\frac{m\gamma-\gamma^2 T}{mT}\big)\Big) \\
&- \frac{\beta\gamma e^{\beta(\gamma T-\frac{(\gamma T)^2}{2M})}}{T^2}-\frac{e^{\beta(\gamma T-\frac{(\gamma T)^2}{2M})}\Big(\beta\big(\gamma-\frac{\gamma^2 T}{m}\big)\Big)}{T^2}+\frac{2e^{\beta(\gamma T-\frac{(\gamma T)^2}{2M})}}{T^3}\bigg)\Bigg] \\
&- p\left[\frac{2E}{T^3}\right]-c_h\Bigg[\frac{\gamma}{2}\bigg(\frac{b(\alpha p)^{-\epsilon}}{\beta}+E\bigg)\bigg(e^{\beta(T-\frac{T^2}{2m})}\Big(\beta\big(1-\frac{T}{m}\big)\Big)^2 \\
&- \frac{\beta}{m}e^{\beta(T-\frac{T^2}{2m})}+e^{\beta((T-\gamma T)-(T^2-\frac{(\gamma T)^2}{2m}))}\Big(\beta\big((1-\gamma)-\frac{(T-\gamma^2 T)}{m}\big)\Big)^2 \\
&- \frac{\beta(1-\gamma^2)}{m}e^{\beta((T-\gamma T)-(T^2-\frac{(\gamma T)^2}{2m}))}\bigg) \\
&+ \bigg(\frac{bp^{-\epsilon}}{\beta}-\frac{b(\alpha p)^{-\epsilon}}{\beta}\bigg)\bigg(e^{\beta(\gamma T-\frac{(\gamma T)^2}{2M})}\Big(\beta\big(\gamma-\frac{\gamma^2 T}{m}\big)\Big)^2 \\
&- \frac{\beta\gamma^2}{m}e^{\beta(\gamma T-\frac{(\gamma T)^2}{2M})}\bigg) \\
&+ \frac{(1-\gamma)}{2}\bigg(\frac{b(\alpha p)^{-\epsilon}}{\beta}+E\bigg)\bigg(e^{\beta((T-\gamma T)-(T^2-\frac{(\gamma T)^2}{2m}))}\Big(\beta\big((1-\gamma) \\
&- \frac{(T-\gamma^2 T)}{m}\big)\Big)^2-\frac{\beta(1-\gamma^2)}{m}e^{\beta((T-\gamma T)-(T^2-\frac{(\gamma T)^2}{2m}))}\bigg)\Bigg] \\
&+ \frac{2SE}{T^3}-\frac{2c_0}{T^3}.
\end{aligned}
$$

4.5. Numerical examples

We demonstrated this model by using numerical example with the following parameter values.

Example 1: Let c_0 = 10, p = 70, c_h = 0.5, c = 10, β = 0.5, ϵ = 1.8, b = 10000, s = 3, m = 2 and E = 20. It is assumed that the value of α is varying from 0.7 to 0.9 and the value of γ is varying from 0.3 to 0.5.

By using equation (4.31) we are able to find the optimum total cycle length T which gives the optimum annual profit provided that all the optimality conditions are satisfied by using Excel Solver.

Table 4.1: Experimental result for Example 1

	$\gamma = 0.30$				$\gamma = 0.35$				$\gamma = 0.40$				$\gamma = 0.45$				$\gamma = 0.50$			
α	T^*	AP^*	Q_1^*	Q_2^*	T^*	AP^*	Q_1^*	Q_2^*	T^*	AP^*	Q_1^*	Q_2^*	T^*	AP^*	Q_1^*	Q_2^*	T^*	AP^*	Q_1^*	Q_2^*
0.7	0.893	958.4	37.0	31.5	0.894	959.2	37.0	30.8	0.896	956.6	37.0	27.0	0.898	950.4	36.9	29.2	0.900	940.7	36.8	28.4
0.8	0.925	876.4	36.2	30.7	0.926	882.1	36.3	29.3	0.927	884.4	36.4	29.3	0.928	883.2	36.4	28.5	0.929	878.7	36.3	27.8
0.9	0.950	819.0	35.7	30.1	0.951	828.0	35.8	29.4	0.951	833.7	35.9	28.7	0.952	836.0	36.0	28.1	0.953	835.0	36.0	27.4

The effect of markdown percentage γ on optimum solutions can easily be observed from Table 4.1 as well as Figure 4.2 to 4.4. In order to get the optimum annual profit, markdown time must not be either too early or too late.

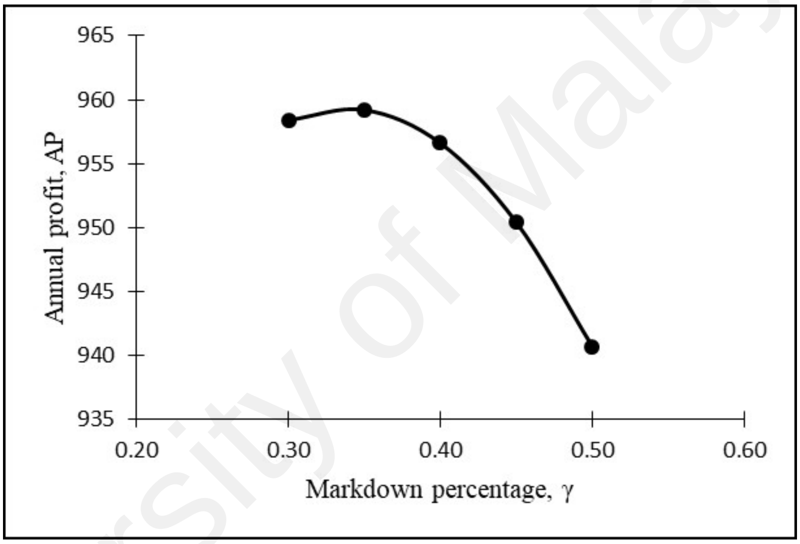

Figure 4.2: Effect of changes in γ on the optimum annual profit when α = 0.7

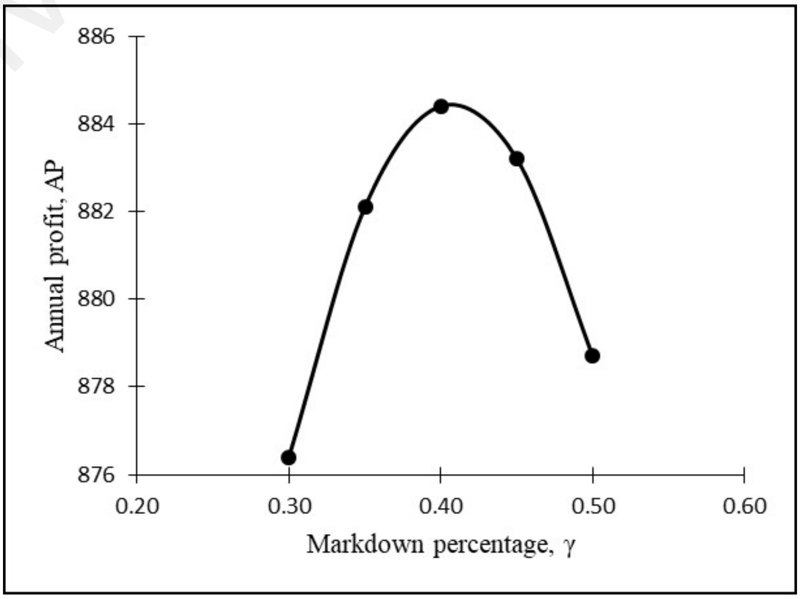

Figure 4.3: Effect of changes in γ on the optimum annual profit when α = 0.8

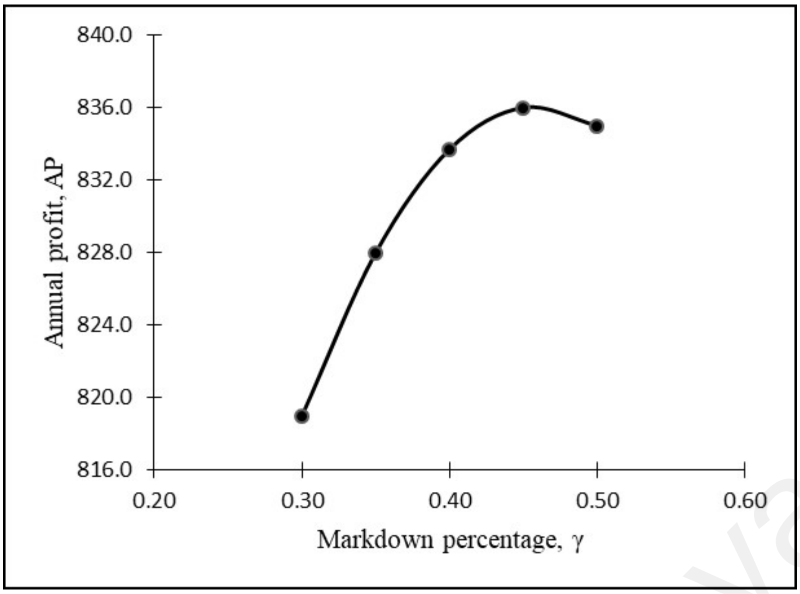

Figure 4.4: Effect of changes in γ on the optimum annual profit when α = 0.9

Table 4.1 demonstrates the optimal solution for the problem. For every markdown rate with the given parameter values, there is an optimum markdown time. For example, for α = 0.7, the maximum annual profit is 959.2 when γ = 0.35. The total cycle time is 0.894 hence, the markdown time $t_2 = \gamma(1-\delta)T = 0.3129$. For the case with α = 0.8, the maximum annual profit is 884.4 when γ = 0.40. The total cycle time is 0.927 hence, the markdown time is $t_2 = \gamma(1-\delta)T = 0.3706$. Whereas for α = 0.9, the optimum annual profit is 836.0 when γ = 0.45. The total cycle time is 0.952 and the markdown is applied at $t_2 = \gamma(1-\delta)T = 0.4283$.

Also, as per the above numerical result, notice that for the same value of γ = 0.3, when α = 0.7 the annual profit is 958.4 with Q_1 = 37.0 and Q_2 = 31.5. When α = 0.8 the annual profit is 876.4 with Q_1 = 36.2 and Q_2 = 30.7. Whereas, when α = 0.9 the annual profit 819.0 with Q_1 = 35.7 and Q_2 = 30.1.

Generally, note that when the value of α increases (lower reduction in price), the longer the total cycle length since more time needed to clear out the inventory which leads to a late markdown time. In addition, it means it is better to apply markdown later in time in order to maximize the annual profit since the impact of markdown is smaller compare to when the value of α is lower which indicates a higher reduction in price.

Moreover, the quantity Q_1 is lower as the value of α increases due to the shorter lifetime characteristic of a fresh item thus, it is not profitable to have a larger volume of Q_1 when the reduction of price is lower. The quantity for markdown Q_2 is also lower with the increase of α because again, the small reduction in price gives small impact in maximizing the annual profit hence, it is more profitable to have less quantity during this markdown offering time interval.

From Table 4.1 observe that the effect of markdown price with an unknown time to optimize annual profit are case dependent. In this model, when the price is reduced to 30% of its initial price, it dominates other markdown prices at any markdown rate in order to maximize the profit. A key element in markdown optimization is determining how items respond to price changes.

From Table 4.1, when we increase the value of markdown rate α by 14% to 28%, there is a slight decrease in the annual profit by around 5% to 8% irrespective of the value of markdown percentage. Also, notice that when the price is reduced to 30% of its initial price, it dominants other markdown prices in order to maximize the profit. This research result, however contradicts the previous researches such as (Widyadana & Wee, 2007) and (Srivastava & Gupta, 2013) that considered demand to be dependent solely on price or demand to be dependent on price and time without taking into consideration the effect of other factors such as inventory level and frequency of advertisement. Hence, we can say that the markdown policy impact in order to optimize annual profit is case dependent.

Also, our findings contradict (Srivastava & Gupta, 2013) when it comes to optimum markdown time. In their case, they claimed that the later they apply the markdown policy, the higher the profit. However, that is not the case for our research. We can observe that for every markdown rate, there is an optimum markdown time which is the best time to apply markdown policy as it must not be either too early or too late otherwise it won't help in maximizing the annual profit.

4.5.1. Sensitivity analysis

The shift in parameter values may occur owing to uncertainties in any scenario of decision-making. The sensitivity analysis will be of excellent assistance in decision-making to examine the consequences of these modifications. Using the same data as those in Example 1 and by fixing α = 0.8 and γ = 0.4, the effect of changes in the parameter of the inventory model to the decision variables can be seen in this section.

Table 4.2 to 4.11 give the sensitive analysis to examine the mathematical model's efficacy with regard to the parameters $b, \beta, \epsilon, p, s, c, c_0, c_h, m$ and E on the optimum cycle time (T^*), optimum quantities (Q_1^* and Q_2^*) as well as optimum annual profit. Also, the effect of changing the parameters are shown graphically in Figs. 4.5 to 4.14.

The sensitivity analysis for every parameter is tested by using the optimum value as well as the plus and minus 50 % from its optimum value to observe the effect of changes clearer.

Table 4.2: Effect of changes in b

Parameter	T^*	Q_1^*	Q_2^*	AP^*
$b = 5000$	1.0013	34.33	27.89	696.11
$b = 10000$	0.9266	36.38	29.27	884.37
$b = 15000$	0.8703	38.28	30.55	1073.30

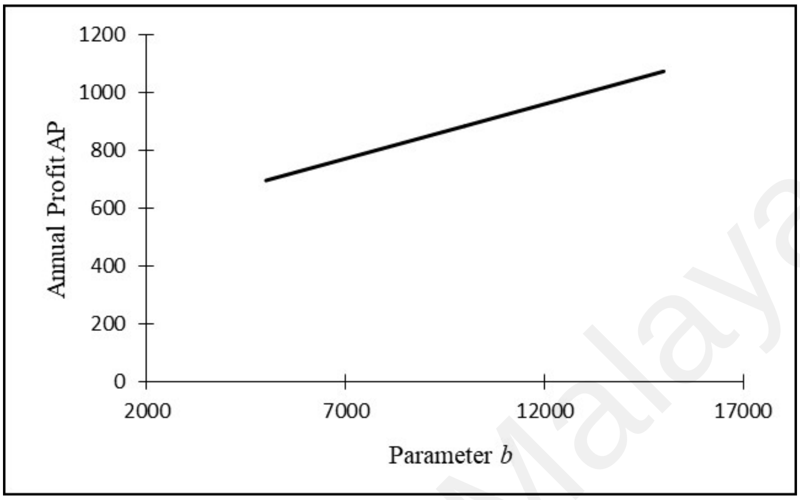

Figure 4.5: Effect of changes in b on the optimal annual profit

Table 4.3: Effect of changes in ϵ

Parameter	T^*	Q_1^*	Q_2^*	AP^*
$\epsilon = 0.9$	0.3660	118.41	80.60	1568.11
$\epsilon = 1.8$	0.9266	36.38	29.27	884.37
$\epsilon = 2.7$	1.1013	32.25	26.47	518.37

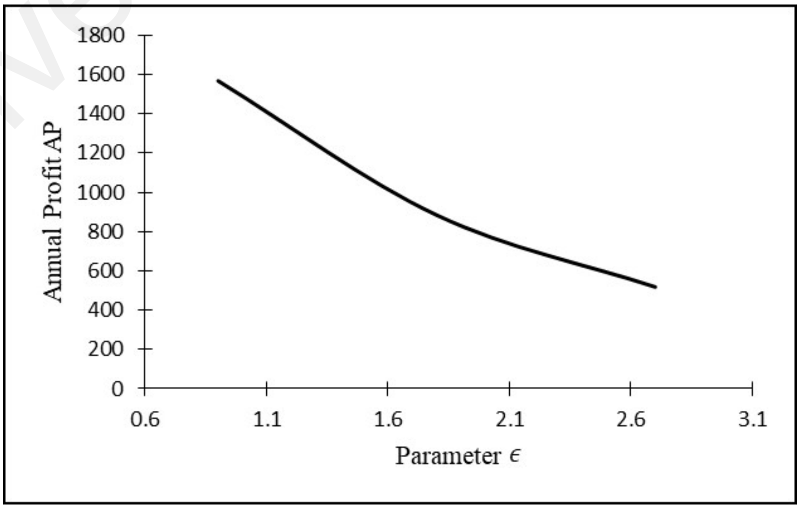

Figure 4.6: Effect of changes in ϵ on the optimal annual profit

Table 4.4: Effect of changes in p

Parameter	T^*	Q_1^*	Q_2^*	AP^*
p = 35	0.9767	53.87	39.77	695.17
p = 70	0.9266	36.38	29.27	884.37
p = 105	0.8623	32.15	26.79	1151.39

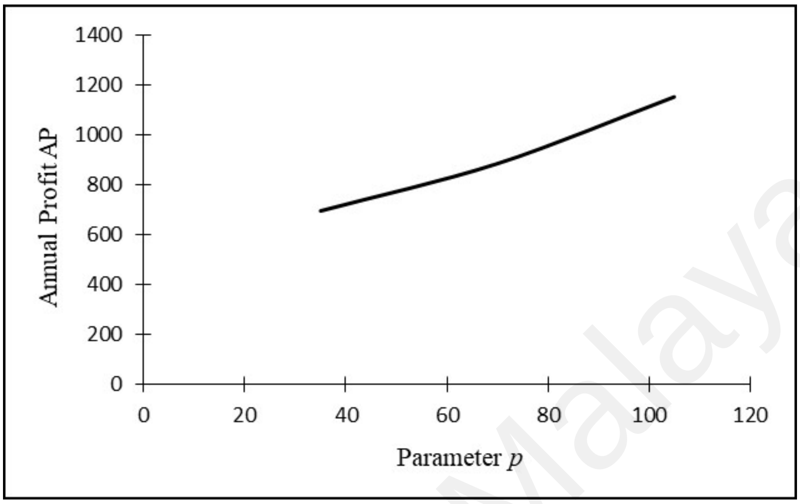

Figure 4.7: Effect of changes in p on the optimal annual profit

Table 4.5: Effect of changes in s

Parameter	T^*	Q_1^*	Q_2^*	AP^*
s = 1.5	0.9853	37.47	29.82	866.84
s = 3.0	0.9266	36.38	29.27	884.37
s = 4.5	0.8598	35.13	28.63	902.59

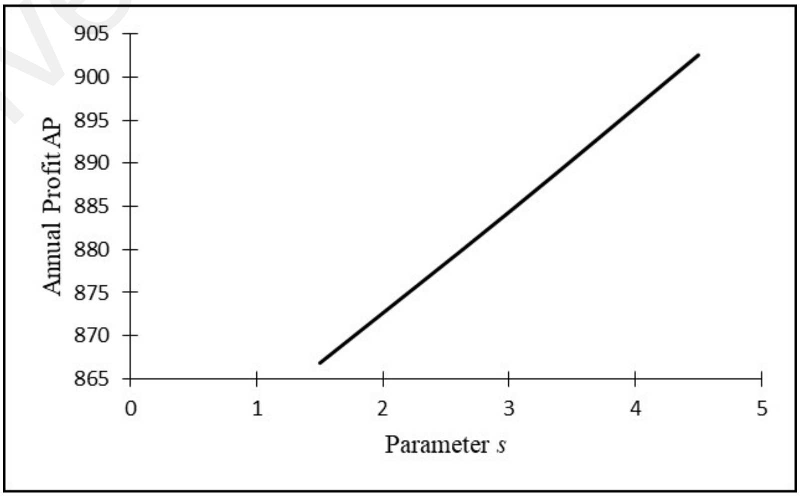

Figure 4.8: Effect of changes in s on the optimal annual profit

Table 4.6: Effect of changes in β

Parameter	T^*	Q_1^*	Q_2^*	AP^*
$\beta = 0.25$	0.9756	30.65	26.46	488.57
$\beta = 0.50$	0.9266	36.38	29.27	884.37
$\beta = 0.75$	0.9032	45.07	33.25	1389.99

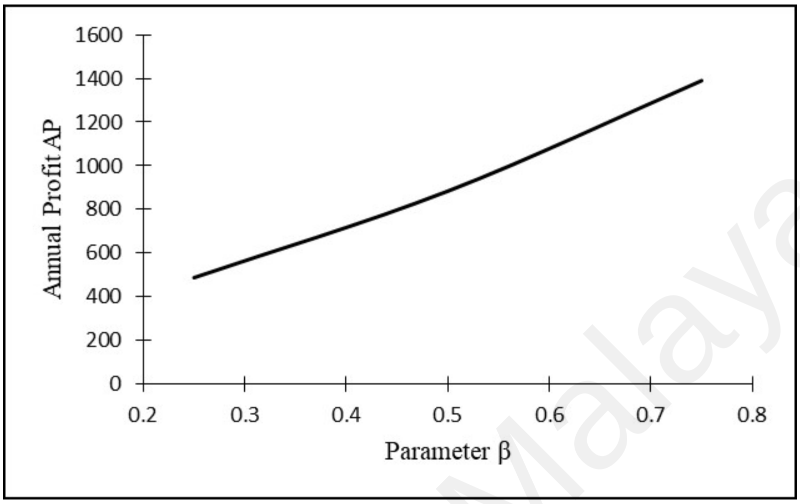

Figure 4.9: Effect of changes in β on the optimal annual profit

Table 4.7: Effect of changes in c

Parameter	T^*	Q_1^*	Q_2^*	AP^*
$c = 5$	0.6255	30.82	26.35	1031.57
$c = 10$	0.9266	36.38	29.27	884.37
$c = 15$	1.1341	40.25	31.18	746.81

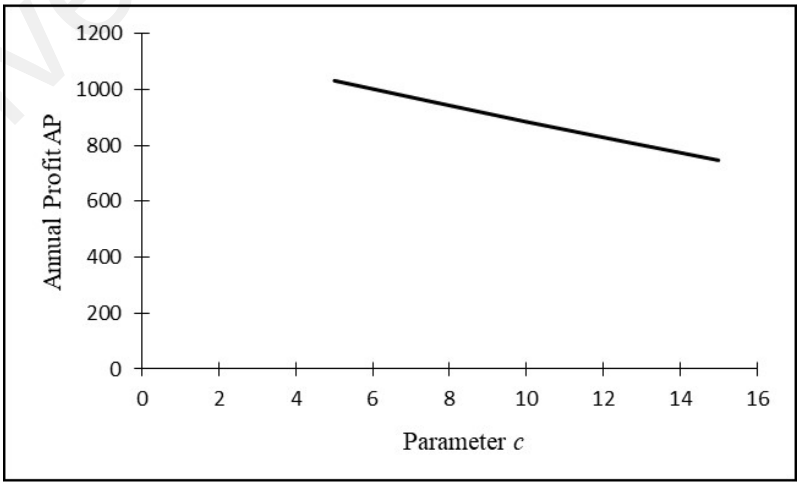

Figure 4.10: Effect of changes in c on the optimal annual profit

Table 4.8: Effect of changes in c_0

Parameter	T^*	Q_1^*	Q_2^*	AP^*
$c_0 = 5$	0.9161	36.18	29.17	887.36
$c_0 = 10$	0.9266	36.38	29.27	884.37
$c_0 = 15$	0.9369	36.57	29.36	881.41

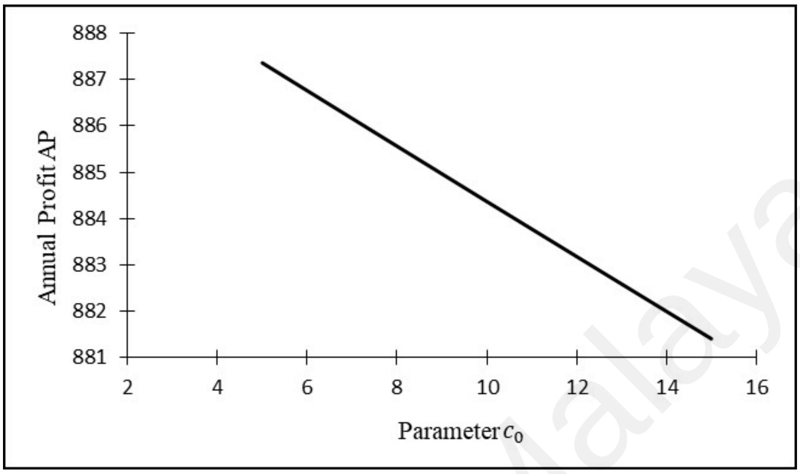

Figure 4.11: Effect of changes in c_0 on the optimal annual profit

Table 4.9: Effect of changes in c_h

Parameter	T^*	Q_1^*	Q_2^*	AP^*
$c_h = 0.25$	0.9287	36.41	29.29	891.83
$c_h = 0.50$	0.9266	36.38	29.27	884.37
$c_h = 0.75$	0.9246	36.34	29.25	876.92

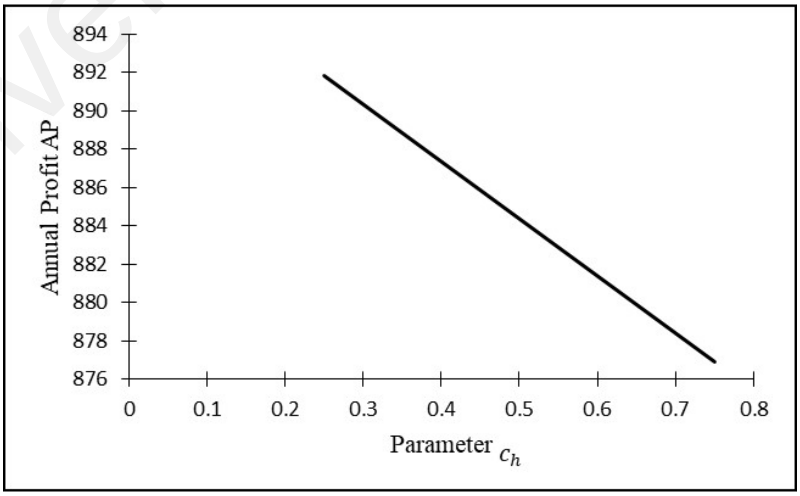

Figure 4.12: Effect of changes in c_h on the optimal annual profit

Table 4.10: Effect of changes in m

Parameter	T^*	Q_1^*	Q_2^*	AP^*
m = 1.0	0.5642	28.83	25.17	660.87
m = 2.0	0.9266	36.38	29.27	884.37
m = 3.0	1.3914	47.56	34.82	1064.03

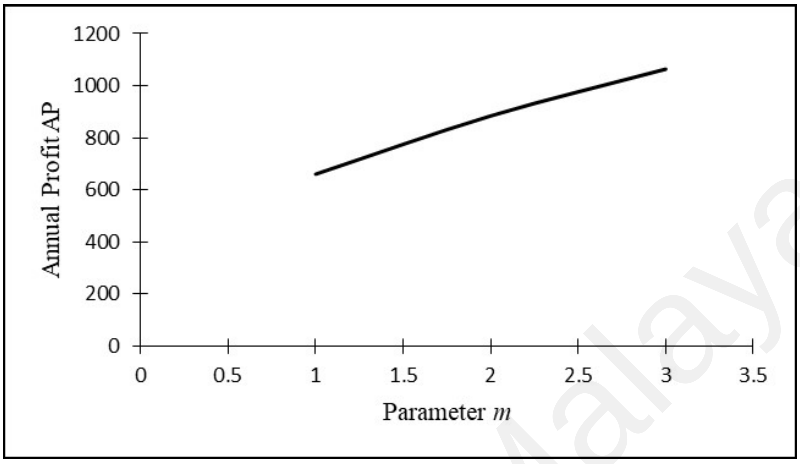

Figure 4.13: Effect of changes in m on the optimal annual profit

Table 4.11: Effect of changes in E

Parameter	T^*	Q_1^*	Q_2^*	AP^*
E = 10	0.8436	20.27	16.00	628.19
E = 20	0.9266	36.38	29.27	884.37
E = 30	0.9664	52.37	42.45	1141.05

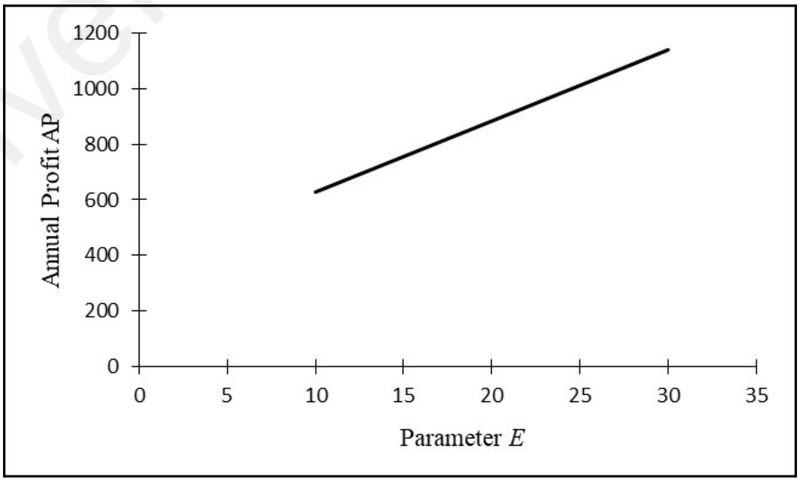

Figure 4.14: Effect of changes in E on the optimal annual profit

Based on Table 4.2 to Table 4.11, we observe that :

- When the rate of positive constant b increases, there is a decrease in the total cycle time but an increase in the annual profit.
- Increasing the index of price elasticity ϵ, increases the total cycle time and decreases the annual profit.
- Increasing the value of price p, decreases the total cycle time and increases the annual profit.
- Increasing the value of salvage price per unit s, decreases the total cycle time and increases the annual profit.
- Increasing the value of β, the total cycle time and the annual profit increase.
- When the value of purchasing cost per unit c increases, total cycle time increases while annual profit decreases.
- When the ordering cost per order c_0 increases, there is an increase in total cycle time and decreases in annual profit.
- When the holding cost per unit per unit of time c_h increases, there is a decrease in total cycle time and annual profit.
- Increase in the value of maximum lifetime in units of time m results in a positive change in the total cycle time and annual profit.
- Lastly, when the value of ending-stock level in units E increases, the total cycle time and annual profit increase.

The effect of changes in the first parameter shows that the value of b is allowed to vary while the other parameters remain constant. The annual profit gradually increases as the value of b increases. This implies that by increasing the value of b, demand rate increases, hence, slightly shorter cycle time is taken.

For the second parameter, the annual profit decreases if the price rate parameter ϵ increases. A higher value of ϵ means, an increase in price rate which lowers the demand. Thus, the retailer will lengthen the cycle time.

The impact of the modifications in the third parameter shows that the increase in price p causes the annual profit to increase but decreases the total cycle time. This is due to less time is required to complete the cycle.

Similarly, for the fourth parameter, the annual profit increases with the increase of salvage value s, however, the total cycle length decreases. This is because when the salvage value is incorporated into the deteriorating items, the total inventory cost can be minimized which eventually maximized the annual profit and less time is needed.

Meanwhile for the fifth parameter, the annual profit increases as the value of β increases which leads to a higher demand with a longer total cycle length.

The patterns for sixth and seventh parameters are similar to each other. The increase in c or c_0 causes the annual profit to decrease but increase the total cycle time. This is because it is more profitable to lengthen the cycle time since the ordering and purchasing cost are more expensive.

Whereas for the eighth parameter, the annual profit and total cycle length decrease as the value of holding cost increases. The reason is because it is more profitable to shorten the cycle length if the holding cost is high.

For the ninth parameter, the annual profit increases as the maximum lifetime m increases which implies a longer time is needed before deteriorating items meet their maximum lifetime, therefore a longer total cycle time.

Last but not least, the annual profit increases as the ending stock level E increases.

This is due to gaining more profit in having fresh items at the end of the inventory level which implies slightly longer total cycle time.

4.6. Conclusion

This model is an extension of (Teng & Chang, 2005), (Srivastava & Gupta, 2013) and (Feng et al., 2017). In this study, we established an inventory model where the demand depends on the price, freshness, expiration date as stock level under markdown policy. Freshness can be considered as one of the main factors that affects consumer's purchasing decision. As the age of the item increases, it brings a negative effect to the demand due to the loss in the product's quality since consumers always prefer to purchase fresher items for a given price. Demand decreases as the expiry date of a fresh product approaches. Due to this, a markdown policy is introduced. The markdown time should not be too early or too late otherwise it will not be able to help in maximizing the annual profit which is demonstrated in Table 4.1. Markdown strategy is an important tool which helps to clear out inventory before it approaches its maximum lifetime as well as proven to help in maximizing the annual profit of the inventory model. Moreover, to enhance the potential profit, we relaxed the traditional assumption that the inventory ends at zero at the end of the replenishment cycle by considering a non-zero ending inventory.

Moreover, for every markdown rate with the given parameter values, there is an optimum markdown time. In this case as shown in Table 4.1, the lower the markdown rate, the shorter the total cycle length which leads to an earlier optimum markdown time. This indicates that when the markdown rate is lower, it is better to offer markdown earlier and shorter cycle time is needed. In addition, we can also conclude that in this model, the higher the markdown rate (lesser reduction in price), the lower the annual profit. However, it contradicts previous studies such as (Widyadana & Wee, 2007) and (Srivastava

& Gupta, 2013) that we refer to since they only considered demand rate to be dependent solely on price or price and time.

The sensitive analysis section has shown that the total profit is more sensitive to b, β, ϵ, c, p m and E. Therefore, retailers must focus and pay more attention on these parameters in order to obtain the best policy. We have derived the optimum cycle time, optimum quantities and optimum annual profit. The results demonstrated that markdown rate gives significant contribution to optimize the annual profit and it is important for a policy maker to be very cautious in setting the markdown rate. This is because the optimum policy for distinct cases is distinct as it is case dependent.

CHAPTER 5: INVENTORY MODEL FOR DELAYED DETERIORATING ITEMS WHEN DEMAND DEPENDS ON ADVERTISEMENT, PRICE AND INVENTORY LEVEL UNDER MARKDOWN POLICY.

5.1. Introduction

In this chapter, we develop a delayed deteriorating inventory model when the demand rate is dependent on the frequency of advertisement, price and inventory level under markdown policy. We establish an inventory model which uses markdown policy as an appropriate strategy in selling slow-moving or remaining inventory at the end of a season especially in a rapid-evolving industry. The salvage value for deteriorating items is incorporated in this study. This model aims to maximize the annual profit by finding the optimum cycle time, optimum markdown time and optimal quantities. The optimal solution is obtained with the help of numerical examples. Sensitivity analysis of the annual profit function together with the managerial insight are provided to demonstrate the model.

This model is an extension and modification of the published works by (Teng & Chang, 2005), (Srivastava & Gupta, 2013), (Shah & Pandey, 2009) as well as our first model. Apart from price and inventory level, the other marketing parameter which has an impact on the demand is advertisement. It is commonly used by retailers in order to promote and provide information on their products and hence, to raise the demand. We are familiar with the positive impact that advertisement brings but there are also some limitations to it which we are able to observe in this model. In addition, we incorporate markdown policy into the inventory model by applying the same approach used by (Widyadana & Wee, 2007) and (Srivastava & Gupta, 2013).

5.2. Model's background

The following notations are used throughout this paper:

1. α = Markdown rate.
2. δ = Production percentage.
3. ϵ = Index of price elasticity.
4. γ = Markdown percentage.
5. λ = Rate of change of frequency of advertisement.
6. θ = Constant deterioration rate.
7. A = Frequency of advertisement in the cycle.
8. c_0 = Ordering cost per order.
9. c_h = Inventory holding cost per unit per unit of time.
10. c_p = Production cost per unit.
11. G = Cost of advertisement.
12. $I(t)$ = Inventory level at time t.
13. K = Constant production rate.
14. p = Initial price.
15. Q_1 = Inventory level at t_1.
16. Q_2 = Inventory level at t_2.
17. s = Salvage value for deteriorated units.

18. T = Replenishment cycle time in units of time.

To develop the model, the following assumptions are used :

1. Demand is a function of frequency of advertisement, price and inventory level. Demand at time t is assumed to be

$$D(I(t), p, A) = A^{\lambda}\big(b(\alpha p)^{-\epsilon} + \beta I(t)\big)$$

   where α, b and β are positive constants with α and β are between 0 and 1.

2. A single item is considered over a prescribed period of T units of time.

3. Shortage is not allowed.

4. Rate of deterioration, θ is constant at any time, where $0 \le \theta < 1$.

5. There is no rework process on the deteriorated units.

6. All items are mandatory to be sold.

7. Only one time markdown price at one planning period is applied.

8. Markdown price is known.

9. Production time is proportional to the cycle time which is equivalent to $t_1 = \delta T$.

10. After some period of product deterioration, markdown is offered. Markdown time is equivalent to $t_2 = \gamma(1-\delta)T$.

11. We may assume that the order quantity Q_1 is greater than or equal to quantity at t_1, Q_2. While Q_2 is greater or equal to ending inventory level, 0 such as $0 \le Q_2 \le Q_1$.

5.3. Mathematical formulation

Figure 5.1 shows the graphical representation of the inventory system. The production and supply begins at the same time and the production stops at time t_1 with the inventory level, Q_1 is reached. To make this model more realistic, there is no deterioration yet during the period $[0, t_1]$. Hence, the inventory level decreases owing to demand alone during the first interval. The deterioration only begins after t_1 therefore, the inventory level during interval $[t_1, t_1 + t_2]$ reduces due to deterioration process and demand processes. Realistically, the demand of an item will continue to decrease with time. Hence, at the time $(t_1 + t_2)$, markdown is offered in order to increase the demand rate.

Based on the assumptions, the inventory level $I(t)$ at time t over period $(0, T)$ is governed by these differential equations :

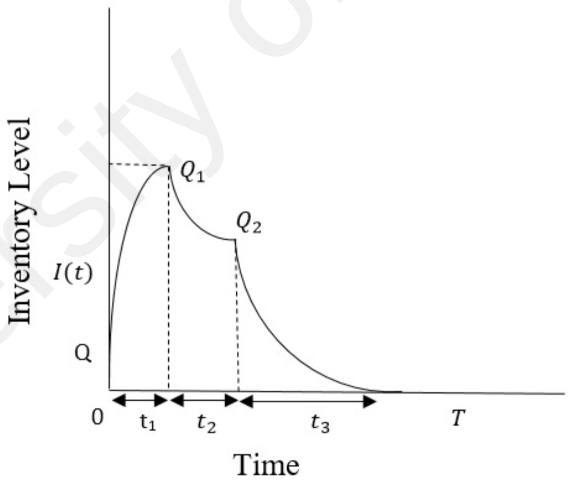

Figure 5.1: Graphical representation of the inventory system

The equations below are the combination and modification of the equations used by (Teng & Chang, 2005), (Srivastava & Gupta, 2013) and (Shah & Pandey, 2009).

$$\frac{dI(t)}{dt} = K - A^{\lambda}\big(bp^{-\epsilon} + \beta I(t)\big), \qquad 0 \le t \le t_1, \tag{5.1}$$

with $\alpha = 1$ (no markdown) and $I(0) = 0$. Deterioration does not begin at the instant of the arrival during this interval and markdown is not offered yet thus, there is no breakdown price.

Then inventory level depletes continuously due to demand and constant rate of deterioration of units.

$$\frac{dI(t)}{dt} + \theta I(t) = -A^{\lambda}\big(bp^{-\epsilon} + \beta I(t)\big), \qquad 0 \le t \le t_2, \tag{5.2}$$

with $\alpha = 1$ (no markdown) and $I(0) = Q_1$.

Similarly, at $0 \le t \le t_3$, inventory level decreases due to demand and deterioration process. Realistically, demand rate decreases with time. Hence, in order to increase the demand markdown is offered during this interval.

$$\frac{dI(t)}{dt} + \theta I(t) = -A^{\lambda}\big(b(\alpha p)^{-\epsilon} + \beta I(t)\big), \qquad 0 \le t \le t_3 \quad \text{and} \quad 0 < \alpha < 1, \tag{5.3}$$

with $I(0) = Q_2$.

We solve the equations above by using the technique of an integrating factor as this method has been used by many established papers to solve these types of equations as well as it is less tedious. The purpose of this derivation is to show the steps taken in order to obtain the equation for the inventory level, $I(t)$ for all intervals.

To solve the first interval differential equation,

$$\frac{dI(t)}{dt} = K - A^{\lambda}(bp^{-\epsilon} + \beta I(t)).$$

First we rearrange,

$$\frac{dI(t)}{dt} + A^{\lambda}\beta I(t) = K - A^{\lambda}bp^{-\epsilon}. \tag{5.4}$$

Note that the integrating factor, $M(t) = e^{A^\lambda \beta t}$. Multiply equation (5.4) with the integrating factor, we get

$$\frac{dI}{dt}(e^{A^\lambda \beta t}) + A^\lambda \beta I(t)(e^{A^\lambda \beta t}) = (K - A^\lambda b p^{-\epsilon})(e^{A^\lambda \beta t}), \tag{5.5}$$

which is equivalent to

$$\frac{dI(t)}{dt}(e^{A^\lambda \beta t}) = (K - A^\lambda b p^{-\epsilon})(e^{A^\lambda \beta t}). \tag{5.6}$$

Integrate both sides of the equation with respect to t,

$$\begin{aligned}
\int \frac{de^{A^\lambda \beta t} I(t)}{dt} &= \int (K - A^\lambda b p^{-\epsilon}) e^{A^\lambda \beta t} dt \\
e^{A^\lambda \beta t} I(t) &= (K - A^\lambda b p^{-\epsilon}) \int e^{A^\lambda \beta t} dt \\
e^{A^\lambda \beta t} I(t) &= (K - A^\lambda b p^{-\epsilon}) \frac{e^{A^\lambda \beta t}}{A^\lambda \beta} + C \\
I(t) &= \frac{K - A^\lambda b p^{-\epsilon}}{A^\lambda \beta} + C e^{-A^\lambda \beta t} \qquad (5.7)
\end{aligned}$$

with boundary conditions $t = 0$, $I(t) = 0$, solve equation (5.7) we obtain

$$\begin{aligned}
I(0) &= \frac{K - A^\lambda b p^{-\epsilon}}{A^\lambda \beta} + C e^{-A^\lambda \beta (0)} = 0 \\
\frac{K - A^\lambda b p^{-\epsilon}}{A^\lambda \beta} + C &= 0 \\
C &= -\frac{K - A^\lambda b p^{-\epsilon}}{A^\lambda \beta} \qquad (5.8)
\end{aligned}$$

Substitute C into equation (5.7),

$$\begin{aligned} I(t) &= \frac{K - A^\lambda bp^{-\epsilon}}{A^\lambda \beta} - (\frac{K - A^\lambda bp^{-\epsilon}}{A^\lambda \beta})e^{-A^\lambda \beta t} \\ I(t) &= \frac{K - A^\lambda bp^{-\epsilon}}{A^\lambda \beta}(1 - e^{-A^\lambda \beta t}). \end{aligned} \tag{5.9}$$

Next, by using the same method we solve the differential equation for the second interval

$$\frac{dI(t)}{dt} + \theta I(t) = -(A^\lambda bp^{-\epsilon} + A^\lambda \beta I(t)).$$

Solving the differential equation

$$\begin{aligned} \frac{dI(t)}{dt} &= -\theta I(t) - (A^\lambda bp^{-\epsilon} + A^\lambda \beta I(t)) \\ \frac{dI(t)}{dt} + (\theta + A^\lambda \beta)I(t) &= -A^\lambda bp^{-\epsilon} \end{aligned} \tag{5.10}$$

Note that the integrating factor, $M(t) = e^{(\theta + A^\lambda \beta)t}$. Multiply equation (5.10) with the integrating factor we obtain

$$\frac{dI(t)}{dt}(e^{(\theta + A^\lambda \beta)t}) + (\theta + A^\lambda \beta)I(t)(e^{(\theta + A^\lambda \beta)t}) = -A^\lambda bp^{-\epsilon}(e^{(\theta + A^\lambda \beta)t}) \tag{5.11}$$

which is equivalent to

$$\frac{dI(t)}{dt}(e^{(\theta + A^\lambda \beta)t}) = -A^\lambda bp^{-\epsilon}(e^{(\theta + A^\lambda \beta)t}). \tag{5.12}$$

Integrate both sides of the equation with respect to t,

$$\begin{aligned}
\int \frac{d(e^{(\theta+A^{\lambda}\beta)t})I(t)}{dt} &= \int -A^{\lambda}bp^{-\epsilon}e^{(\theta+A^{\lambda}\beta)t}dt \\
e^{(\theta+A^{\lambda}\beta)t}I(t) &= -A^{\lambda}bp^{-\epsilon}\int e^{(\theta+A^{\lambda}\beta)t}dt \\
e^{(\theta+A^{\lambda}\beta)t}I(t) &= -A^{\lambda}bp^{-\epsilon}\frac{e^{(\theta+A^{\lambda}\beta)t}}{(\theta+A^{\lambda}\beta)}+C \\
I(t) &= \frac{-A^{\lambda}bp^{-\epsilon}}{(\theta+A^{\lambda}\beta)}+Ce^{-(\theta+A^{\lambda}\beta)t} \qquad (5.13)
\end{aligned}$$

with boundary condition $t = 0, I(t) = Q_1$, equation (5.13) becomes

$$\begin{aligned}
I(0) &= \frac{-A^{\lambda}bp^{-\epsilon}}{(\theta+A^{\lambda}\beta)}+Ce^{-(\theta+A^{\lambda}\beta)0} = Q_1 \\
\frac{-A^{\lambda}bp^{-\epsilon}}{(\theta+A^{\lambda}\beta)}+C &= Q_1 \\
C &= Q_1+\frac{A^{\lambda}bp^{-\epsilon}}{(\theta+A^{\lambda}\beta)}. \qquad (5.14)
\end{aligned}$$

Substitute C into equation (5.13) we get

$$\begin{aligned}
I(t) &= \frac{-A^{\lambda}bp^{-\epsilon}}{(\theta+A^{\lambda}\beta)}+(Q_1+\frac{A^{\lambda}bp^{-\epsilon}}{(\theta+A^{\lambda}\beta)})e^{-(\theta+A^{\lambda}\beta)t} \\
&= Q_1e^{-(\theta+A^{\lambda}\beta)t}+\frac{A^{\lambda}bp^{-\epsilon}}{(\theta+A^{\lambda}\beta)}(e^{-(\theta+A^{\lambda}\beta)t}-1). \qquad (5.15)
\end{aligned}$$

Lastly, solving the third interval differential equation

$$\frac{dI(t)}{dt}+\theta I(t) = -(A^{\lambda}b(\alpha p)^{-\epsilon}+A^{\lambda}\beta I(t)).$$

Similarly,

$$\begin{aligned}
\frac{dI(t)}{dt} &= -\theta I(t)-(A^{\lambda}b(\alpha p)^{-\epsilon}+A^{\lambda}\beta I(t)) \\
\frac{dI(t)}{dt}+(\theta+A^{\lambda}\beta)I(t) &= -A^{\lambda}b(\alpha p)^{-\epsilon}. \qquad (5.16)
\end{aligned}$$

Note that the integrating factor, $M(t) = e^{(\theta+A^\lambda\beta)t}$. Multiply equation (5.16) by the integrating factor obtained,

$$\frac{dI(t)}{dt}(e^{(\theta+A^\lambda\beta)t}) + (\theta + A^\lambda\beta)I(t)(e^{(\theta+A^\lambda\beta)t}) = -A^\lambda b(\alpha p)^{-\epsilon}(e^{(\theta+A^\lambda\beta)t}) \tag{5.17}$$

which is equivalent to

$$\frac{dI(t)}{dt}(e^{(\theta+A^\lambda\beta)t}) = -A^\lambda b(\alpha p)^{-\epsilon}(e^{(\theta+A^\lambda\beta)t}) \tag{5.18}$$

Integrate both sides of equation (5.18) with respect to t

$$\begin{aligned}
\int \frac{d(e^{(\theta+A^\lambda\beta)t})I(t)}{dt} &= \int -A^\lambda b(\alpha p)^{-\epsilon}e^{(\theta+A^\lambda\beta)t}dt \\
e^{(\theta+A^\lambda\beta)t}I(t) &= -A^\lambda b(\alpha p)^{-\epsilon}\int e^{(\theta+A^\lambda\beta)t}dt \\
e^{(\theta+A^\lambda\beta)t}I(t) &= -A^\lambda b(\alpha p)^{-\epsilon}\frac{e^{(\theta+A^\lambda\beta)t}}{(\theta + A^\lambda\beta)} + C \\
I(t) &= \frac{-A^\lambda b(\alpha p)^{-\epsilon}}{(\theta + A^\lambda\beta)} + Ce^{-(\theta+A^\lambda\beta)t}. \qquad (5.19)
\end{aligned}$$

With boundary condition $t = 0, I(t) = Q_2$, equation (5.19) becomes

$$\begin{aligned}
I(0) &= \frac{-A^\lambda b(\alpha p)^{-\epsilon}}{(\theta + A^\lambda\beta)} + Ce^{-(\theta+A^\lambda\beta)0} = Q_2 \\
\frac{-A^\lambda b(\alpha p)^{-\epsilon}}{(\theta + A^\lambda\beta)} + C &= Q_2 \\
C &= Q_2 + \frac{A^\lambda b(\alpha p)^{-\epsilon}}{(\theta + A^\lambda\beta)}. \qquad (5.20)
\end{aligned}$$

Substitute C into equation (5.19) we get,

$$\begin{aligned}
I(t) &= \frac{-A^\lambda b(\alpha p)^{-\epsilon}}{(\theta + A^\lambda\beta)} + (Q_2 + \frac{A^\lambda b(\alpha p)^{-\epsilon}}{(\theta + A^\lambda\beta)})e^{-(\theta+A^\lambda\beta)t} \\
&= Q_2e^{-(\theta+A^\lambda\beta)t} + \frac{A^\lambda b(\alpha p)^{-\epsilon}}{(\theta + A^\lambda\beta)}(e^{-(\theta+A^\lambda\beta)t} - 1). \qquad (5.21)
\end{aligned}$$

It follows,

$$I(t) = \begin{cases} \frac{K - A^{\lambda} b p^{-\epsilon}}{A^{\lambda}\beta}(1 - e^{-A^{\lambda}\beta t}), & 0 \leq t \leq t_1. \\ \\ Q_1 e^{-(\theta + A^{\lambda}\beta)t} + \frac{A^{\lambda} b p^{-\epsilon}}{(\theta + A^{\lambda}\beta)}(e^{-(\theta + A^{\lambda}\beta)t} - 1), & 0 \leq t \leq t_2 \\ \\ Q_2 e^{-(\theta + A^{\lambda}\beta)t} + \frac{A^{\lambda} b (\alpha p)^{-\epsilon}}{\theta + A^{\lambda}\beta}(e^{-(\theta + A^{\lambda}\beta)t} - 1), & 0 \leq t \leq t_3. \end{cases}$$

Therefore, at time t_1

$$Q_1 = \frac{K - A^{\lambda} b p^{-\epsilon}}{A^{\lambda}\beta}(1 - e^{-A^{\lambda}\beta t_1}). \tag{5.22}$$

Similarly, at time t_2 and t_3

$$Q_2 = Q_1 e^{-(\theta + A^{\lambda}\beta)t_2} + \frac{A^{\lambda} b p^{-\epsilon}}{(\theta + A^{\lambda}\beta)}(e^{-(\theta + A^{\lambda}\beta)t_2} - 1) \tag{5.23}$$

and

$$Q_2 = \frac{A^{\lambda} b (\alpha p)^{-\epsilon}}{\theta + A^{\lambda}\beta}(e^{(\theta + A^{\lambda}\beta)t_3} - 1). \tag{5.24}$$

5.4. Model development

Now the objective function consists of:

- Setup cost.
- Advertisement cost.
- Holding cost.
- Production cost.
- Deterioration cost
- Salvage value.

- Total revenue.

5.4.1. Setup cost

Setup cost can be defined as a cost acquired to configure a machine for a production run and it is considered as a fixed cost.

To find setup cost, $OC = c_0$.

5.4.2. Advertisement cost

Advertisement cost is a type of financial accounting that covers expenses related with promoting an industry, entity, brand, product, or service.

Advertisement cost, $AC = A \times G$.

5.4.3. Holding cost

Holding cost can be defined as the additional cost involved in storing and maintaining a piece of inventory per unit time. Holding cost is related with storing inventory that remains unsold.

To find holding cost

$$\begin{aligned} HC &= c_h \times \Big[\int_0^{t_1} \frac{K - A^\lambda b p^{-\epsilon}}{A^\lambda \beta}(1 - e^{-A^\lambda \beta t})dt + \int_0^{t_2} \big(Q_1 e^{-(\theta + A^\lambda \beta)t} \\ &+ \frac{A^\lambda b p^{-\epsilon}}{\theta + A^\lambda \beta}(e^{-(\theta + A^\lambda \beta)t} - 1)\big)dt \\ &+ \int_0^{t_3} \big(Q_2 e^{-(\theta + A^\lambda \beta)t} + \frac{A^\lambda b (\alpha p)^{-\epsilon}}{\theta + A^\lambda \beta}(e^{-(\theta + A^\lambda \beta)t} - 1)\big)dt \Big]. \end{aligned} \tag{5.25}$$

Substitute Q_1 and Q_2 into equation (5.25) and integrate we get

$$\begin{aligned} HC &= c_h \times \Big[\frac{K - A^\lambda bp^{-\epsilon}}{A^\lambda\beta}(t_1 - \frac{1 - e^{-A^\lambda\beta t_1}}{A^\lambda\beta}) \\ &+ \frac{A^\lambda bp^{-\epsilon}}{(\theta + A^\lambda\beta)^2}(1 - e^{-(\theta + A^\lambda\beta)t_2}) \\ &+ \frac{1}{\theta + A^\lambda\beta}\big\{\frac{K - A^\lambda bp^{-\epsilon}}{\beta}(1 - e^{-A^\lambda\beta t_1})\big\}(1 - e^{-(\theta + A^\lambda\beta)t_2}) \\ &+ \frac{1}{\theta + A^\lambda\beta}\big\{\frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta + A^\lambda\beta}(e^{(\theta + A^\lambda\beta)t_3} - 1)\big\}(1 - e^{-(\theta + A^\lambda\beta)t_3}) \\ &- \frac{A^\lambda bp^{-\epsilon}}{\theta + A^\lambda\beta}t_2 + \frac{A^\lambda b(\alpha p)^{-\epsilon}}{(\theta + A^\lambda\beta)^2}(1 - e^{-(\theta + A^\lambda\beta)t_3}) - \frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta + A^\lambda\beta}t_3\Big]. \end{aligned} \tag{5.26}$$

We apply the power series expansion of the exponential function. In our model, the power of exponential function is small which is less than 1. Hence, the exponential terms are retained up to power of 2. It follows,

$$\begin{aligned} HC &= c_h \times \Big[\frac{K - A^\lambda bp^{-\epsilon}}{2}t_1^2 + (K - A^\lambda bp^{-\epsilon})t_1 t_2 \\ &- \frac{(K - A^\lambda bp^{-\epsilon})(\theta + A^\lambda\beta)t_1 t_2^2}{2} - \frac{(K - A^\lambda bp^{-\epsilon})A^\lambda\beta t_1^2 t_2}{2} \\ &+ \frac{(K - A^\lambda bp^{-\epsilon})A^\lambda\beta(\theta + A^\lambda\beta)t_1^2 t_2^2}{4} - \frac{A^\lambda bp^{-\epsilon}t_2^2}{2} \\ &- \frac{A^\lambda b(\alpha p)^{-\epsilon}(\theta + A^\lambda\beta)^2 t_3^4}{4} + \frac{A^\lambda b(\alpha p)^{-\epsilon}}{2}t_3^2\Big]. \end{aligned} \tag{5.27}$$

5.4.4. Production cost

Production cost refers to the cost of producing a product or supplying a service incurred by a company. Cost of production may include a range of expenditures, such as labor, raw materials, consumable supplies of manufacturing and overhead.

Production cost, $PC = Kc_p t_1$.

5.4.5. Deterioration cost

The deterioration cost is often related to deteriorating units during the cycle time.

The deterioration cost equation can be defined as

$$\begin{aligned} DC &= c_p\theta \times \Big[\int_0^{t_2} \big(Q_1 e^{-(\theta+A^\lambda\beta)t} + \frac{A^\lambda bp^{-\epsilon}}{\theta+A^\lambda\beta}(e^{-(\theta+A^\lambda\beta)t}-1)\big)dt \\ &+ \int_0^{t_3} \big(Q_2 e^{-(\theta+A^\lambda\beta)t} + \frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta+A^\lambda\beta}(e^{-(\theta+A^\lambda\beta)t}-1)\big)dt\Big]. \\ &= c_p\theta \times \Big[\frac{Q_1}{\theta+A^\lambda\beta}(1-e^{-(\theta+A^\lambda\beta)t_2}) - \frac{A^\lambda bp^{-\epsilon}}{\theta+A^\lambda\beta}t_2 \\ &+ \frac{A^\lambda bp^{-\epsilon}}{\theta+A^\lambda\beta}\frac{(1-e^{(\theta+A^\lambda\beta)t_2})}{\theta+A^\lambda\beta} + \frac{Q_2}{\theta+A^\lambda\beta}(1-e^{-(\theta+A^\lambda\beta)t_3}) \\ &+ \frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta+A^\lambda\beta}\frac{(1-e^{-(\theta+A^\lambda\beta)t_3})}{\theta+A^\lambda\beta} - \frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta+A^\lambda\beta}t_3\Big]. \end{aligned} \tag{5.28}$$

Replace Q_1 and Q_2 into equation (5.28)

$$\begin{aligned} DC &= c_p\theta \times \Big[\frac{1}{\theta+A^\lambda\beta}\big\{\frac{K-A^\lambda bp^{-\epsilon}}{A^\lambda\beta}(1-e^{-A^\lambda\beta t_1})\big\}(1-e^{-(\theta+A^\lambda\beta)t_2}) \\ &+ \frac{A^\lambda bp^{-\epsilon}}{(\theta+A^\lambda\beta)^2}(1-e^{-(\theta+A^\lambda\beta)t_2}) - \frac{A^\lambda bp^{-\epsilon}}{\theta+A^\lambda\beta}t_2 \\ &+ \frac{A^\lambda b(\alpha p)^{-\epsilon}}{(\theta+A^\lambda\beta)^2}(1-e^{-(\theta+A^\lambda\beta)t_3}) - \frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta+A^\lambda\beta}t_3 \\ &+ \frac{1}{\theta+A^\lambda\beta}\big\{\frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta+A^\lambda\beta}(e^{(\theta+A^\lambda\beta)t_3}-1)\big\}(1-e^{-(\theta+A^\lambda\beta)t_3})\Big]. \end{aligned} \tag{5.29}$$

Similar to holding cost equation, by considering the exponential terms up to power of 2, then we have

$$\begin{aligned} DC &= c_p\theta \times \Big[(K-A^\lambda bp^{-\epsilon})t_1t_2 - \frac{(K-A^\lambda bp^{-\epsilon})(\theta+A^\lambda\beta)t_1t_2^2}{2} \\ &- \frac{(K-A^\lambda bp^{-\epsilon})A^\lambda\beta t_1^2t_2}{2} + \frac{(K-A^\lambda bp^{-\epsilon})A^\lambda\beta(\theta+A^\lambda\beta)t_1^2t_2^2}{4} \\ &- \frac{A^\lambda bp^{-\epsilon}t_2^2}{2} - \frac{A^\lambda b(\alpha p)^{-\epsilon}(\theta+A^\lambda\beta)^2t_3^4}{4} + \frac{A^\lambda b(\alpha p)^{-\epsilon}t_3^2}{2}\Big]. \end{aligned} \tag{5.30}$$

5.4.6. Salvage value

Salvage value is an asset's estimated resale value at the end of its helpful life.

The salvage value is

$$\begin{aligned}
SV &= sc_p\theta \times \Big[\int_0^{t_2} \big(Q_1 e^{-(\theta+A^\lambda\beta)t} + \frac{A^\lambda bp^{-\epsilon}}{\theta+A^\lambda\beta}(e^{-(\theta+A^\lambda\beta)t}-1)\big)dt \\
&+ \int_0^{t_3} \big(Q_2 e^{-(\theta+A^\lambda\beta)t} + \frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta+A^\lambda\beta}(e^{-(\theta+A^\lambda\beta)t}-1)\big)dt\Big]. \\
&= sc_p\theta \times \Big[\frac{Q_1}{\theta+A^\lambda\beta}(1-e^{-(\theta+A^\lambda\beta)t_2}) + \frac{A^\lambda bp^{-\epsilon}}{\theta+A^\lambda\beta}\frac{(1-e^{(\theta+A^\lambda\beta)t_2})}{\theta+A^\lambda\beta} \\
&- \frac{A^\lambda bp^{-\epsilon}}{\theta+A^\lambda\beta}t_2 + \frac{Q_2}{\theta+A^\lambda\beta}(1-e^{-(\theta+A^\lambda\beta)t_3}) - \frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta+A^\lambda\beta}t_3 \\
&+ \frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta+A^\lambda\beta}\frac{(1-e^{-(\theta+A^\lambda\beta)t_3})}{\theta+A^\lambda\beta}\Big]. \qquad (5.31)
\end{aligned}$$

Similarly as before, substitute Q_1 and Q_2 into equation (5.31) we obtain

$$\begin{aligned}
SV &= sc_p\theta \times \Big[\frac{1}{\theta+A^\lambda\beta}\big\{\frac{K-A^\lambda bp^{-\epsilon}}{\beta}(1-e^{-A^\lambda\beta t_1})\big\}(1-e^{-(\theta+A^\lambda\beta)t_2}) \\
&+ \frac{A^\lambda bp^{-\epsilon}}{(\theta+A^\lambda\beta)^2}(1-e^{-(\theta+A^\lambda\beta)t_2}) - \frac{A^\lambda bp^{-\epsilon}}{\theta+A^\lambda\beta}t_2 \\
&+ \frac{A^\lambda b(\alpha p)^{-\epsilon}}{(\theta+A^\lambda\beta)^2}(1-e^{-(\theta+A^\lambda\beta)t_3}) - \frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta+A^\lambda\beta}t_3 \\
&+ \frac{1}{\theta+A^\lambda\beta}\big\{\frac{A^\lambda b(\alpha p)^{-\epsilon}}{\theta+A^\lambda\beta}(e^{(\theta+A^\lambda\beta)t_3}-1)\big\}(1-e^{-(\theta+A^\lambda\beta)t_3})\Big]. \qquad (5.32)
\end{aligned}$$

Likewise, as before we have

$$\begin{aligned}
SV &= sc_p\theta \times \Big[(K-A^\lambda bp^{-\epsilon})t_1t_2 - \frac{(K-A^\lambda bp^{-\epsilon})(\theta+A^\lambda\beta)t_1t_2^2}{2} \\
&- \frac{(K-A^\lambda bp^{-\epsilon})A^\lambda\beta t_1^2t_2}{2} + \frac{(K-A^\lambda bp^{-\epsilon})A^\lambda\beta(\theta+A^\lambda\beta)t_1^2t_2^2}{4} \\
&- \frac{A^\lambda bp^{-\epsilon}t_2^2}{2} - \frac{A^\lambda b(\alpha p)^{-\epsilon}(\theta+A^\lambda\beta)^2t_3^4}{4} + \frac{A^\lambda b(\alpha p)^{-\epsilon}t_3^2}{2}\Big]. \qquad (5.33)
\end{aligned}$$

5.4.7. Total revenue

Total revenue consists of pre-markdown revenue and revenue when the markdown is offered. Thus, the revenue equation can be obtained as

$$\begin{aligned}
TR &= p \times \Big[\int_0^{t_1} A^{\lambda}\big(bp^{-\epsilon} + \beta I(t)\big)dt + \int_0^{t_2} A^{\lambda}\big(bp^{-\epsilon} + \beta I(t)\big)dt \\
&+ \int_0^{t_3} A^{\lambda}\big(b(\alpha p)^{-\epsilon} + \beta I(t)\big)dt\Big]. \\
&= p \times \Big[A^{\lambda}bp^{-\epsilon}(t_1 + t_2) + A^{\lambda}b(\alpha p)^{-\epsilon}t_3 + A^{\lambda}\beta\big(\frac{K - A^{\lambda}bp^{-\epsilon}}{2}t_1^2 \\
&+ (K - A^{\lambda}bp^{-\epsilon})t_1t_2 - \frac{(K - A^{\lambda}bp^{-\epsilon})(\theta + A^{\lambda}\beta)t_1t_2^2}{2} \\
&- \frac{A^{\lambda}bp^{-\epsilon}t_2^2}{2} + \frac{A^{\lambda}b(\alpha p)^{-\epsilon}t_3^2}{2} + \frac{(K - A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta(\theta + A^{\lambda}\beta)t_1^2t_2^2}{4} \\
&- \frac{(K - A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta t_1^2t_2}{2} - \frac{A^{\lambda}b(\alpha p)^{-\epsilon}(\theta + A^{\lambda}\beta)^2t_3^4}{4}\big)\Big].
\end{aligned} \quad (5.34)$$

Annual profit, AP, is defined as the total revenue plus salvage value minus holding cost, deterioration cost, production cost, set up cost and advertisement cost. Hence,

$$
\begin{aligned}
AP &= p \times \Big[A^{\lambda}bp^{-\epsilon}(t_1+t_2) + A^{\lambda}b(\alpha p)^{-\epsilon}t_3 + A^{\lambda}\beta\big(\frac{K-A^{\lambda}bp^{-\epsilon}}{2}t_1^2 \\
&+ (K-A^{\lambda}bp^{-\epsilon})t_1t_2 - \frac{(K-A^{\lambda}bp^{-\epsilon})(\theta+A^{\lambda}\beta)t_1t_2^2}{2} \\
&- \frac{(K-A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta t_1^2t_2}{2} - \frac{A^{\lambda}bp^{-\epsilon}t_2^2}{2} + \frac{A^{\lambda}b(\alpha p)^{-\epsilon}t_3^2}{2} \\
&+ \frac{(K-A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta(\theta+A^{\lambda}\beta)t_1^2t_2^2}{4} - \frac{A^{\lambda}b(\alpha p)^{-\epsilon}(\theta+A^{\lambda}\beta)^2t_3^4}{4}\big)\Big] \\
&- c_h \times \Big[\frac{K-A^{\lambda}bp^{-\epsilon}}{2}t_1^2 + (K-A^{\lambda}bp^{-\epsilon})t_1t_2 \\
&- \frac{(K-A^{\lambda}bp^{-\epsilon})(\theta+A^{\lambda}\beta)t_1t_2^2}{2} - \frac{(K-A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta t_1^2t_2}{2} \\
&+ \frac{(K-A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta(\theta+A^{\lambda}\beta)t_1^2t_2^2}{4} - \frac{A^{\lambda}bp^{-\epsilon}t_2^2}{2} \\
&- \frac{A^{\lambda}b(\alpha p)^{-\epsilon}(\theta+A^{\lambda}\beta)^2t_3^4}{4} + \frac{A^{\lambda}b(\alpha p)^{-\epsilon}}{2}t_3^2\Big] \\
&- c_p(1-s)\theta \times \Big[(K-A^{\lambda}bp^{-\epsilon})t_1t_2 - \frac{(K-A^{\lambda}bp^{-\epsilon})(\theta+A^{\lambda}\beta)t_1t_2^2}{2} \\
&- \frac{(K-A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta t_1^2t_2}{2} + \frac{(K-A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta(\theta+A^{\lambda}\beta)t_1^2t_2^2}{4} \\
&- \frac{A^{\lambda}bp^{-\epsilon}t_2^2}{2} - \frac{A^{\lambda}b(\alpha p)^{-\epsilon}(\theta+A^{\lambda}\beta)^2t_3^4}{4} + \frac{A^{\lambda}b(\alpha p)^{-\epsilon}t_3^2}{2}\Big] \\
&- \frac{Kc_pt_1}{T} - \frac{c_0}{T} - \frac{AG}{T}. \qquad (5.35)
\end{aligned}
$$

Simplify,

$$
\begin{aligned}
AP &= \frac{p}{T} \times \Big[A^{\lambda}bp^{-\epsilon}(t_1+t_2) + A^{\lambda}b(\alpha p)^{-\epsilon}t_3\Big] \\
&+ \frac{(pA^{\lambda}\beta - c_h)}{T} \times \Big[\frac{K-A^{\lambda}bp^{-\epsilon}}{2}t_1^2\Big] \\
&+ \frac{(pA^{\lambda}\beta - c_h - c_p(1-s)\theta)}{T} \times \Big[(K-A^{\lambda}bp^{-\epsilon})t_1t_2 \\
&- \frac{(K-A^{\lambda}bp^{-\epsilon})(\theta+A^{\lambda}\beta)t_1t_2^2}{2} - \frac{(K-A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta t_1^2t_2}{2} \\
&+ \frac{(K-A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta(\theta+A^{\lambda}\beta)t_1^2t_2^2}{4} - \frac{A^{\lambda}bp^{-\epsilon}t_2^2}{2} \\
&- \frac{A^{\lambda}b(\alpha p)^{-\epsilon}(\theta+A^{\lambda}\beta)^2t_3^4}{4} + \frac{A^{\lambda}b(\alpha p)^{-\epsilon}t_3^2}{2}\Big] \\
&- \frac{Kc_pt_1}{T} - \frac{c_0}{T} - \frac{AG}{T}. \qquad (5.36)
\end{aligned}
$$

Note that AP is a function of t_1, t_2 and t_3. To reduce the complexity and to simplify the solution procedure, we optimize the AP function by following (Srivastava & Gupta, 2013) procedure where we rewrite,

$$\begin{aligned} t_1 &= \delta T, \\ t_2 &= \gamma(T - t_1) = \gamma(1-\delta)T, \\ t_3 &= T - (t_1 + t_2) = (1-\gamma)(1-\delta)T. \end{aligned}$$

Substitute t_1, t_2 and t_3 into equation (5.36), we get

$$\begin{aligned} AP &= p \times \Big[A^\lambda b p^{-\epsilon}\big(\delta + \gamma(1-\delta)\big) + A^\lambda b(\alpha p)^{-\epsilon}(1-\gamma)(1-\delta)\Big] \\ &+ (pA^\lambda\beta - c_h) \times \Big[\frac{(K - A^\lambda b p^{-\epsilon})\delta^2 T}{2}\Big] \\ &+ \big(pA^\lambda\beta - c_h - c_p(1-\lambda)\theta\big) \times \Big[(K - A^\lambda b p^{-\epsilon})\delta\gamma(1-\delta)T \\ &- \frac{A^\lambda b p^{-\epsilon}\gamma^2(1-\delta)^2 T}{2} - \frac{(K - A^\lambda b p^{-\epsilon})A^\lambda\beta\delta^2\gamma(1-\delta)T^2}{2} \\ &- \frac{(K - A^\lambda b p^{-\epsilon})(\theta + A^\lambda\beta)\delta\gamma^2(1-\delta)^2 T^2}{2} \\ &+ \frac{(K - A^\lambda b p^{-\epsilon})A^\lambda\beta(\theta + A^\lambda\beta)\delta^2\gamma^2(1-\delta)^2 T^3}{4} \\ &+ \frac{A^\lambda b(\alpha p)^{-\epsilon}\big((1-\delta)(1-\gamma)\big)^2 T}{2} \\ &- \frac{A^\lambda b(\alpha p)^{-\epsilon}(\theta + A^\lambda\beta)^2\big((1-\delta)(1-\gamma)\big)^4 T^3}{4}\Big] \\ &- Kc_p\delta - \frac{c_0}{T} - \frac{AG}{T}. \end{aligned} \tag{5.37}$$

It follows that the necessary conditions for AP, a function of two variables to be maximized are $\frac{\delta AP}{\delta T} = 0$ and $\frac{\delta AP}{\delta \gamma} = 0$, where

$$
\begin{aligned}
\frac{\delta AP}{\delta T} &= (pA^{\lambda}\beta - c_h) \times \Big[\frac{(K - A^{\lambda}bp^{-\epsilon})\delta^2}{2}\Big] \\
&+ \big(pA^{\lambda}\beta - c_h - c_p(1-s)\theta\big) \times \Big[(K - A^{\lambda}bp^{-\epsilon})\delta\gamma(1-\delta) \\
&- (K - A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta\delta^2\gamma(1-\delta) - \frac{A^{\lambda}bp^{-\epsilon}\big(\gamma^2(1-\delta)^2\big)}{2} \\
&+ \frac{3(K - A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta(\theta + A^{\lambda}\beta)\delta^2\gamma^2(1-\delta)^2T^2}{4} \\
&- (K - A^{\lambda}bp^{\epsilon})(\theta + A^{\lambda}\beta)\delta\gamma^2(1-\delta)^2T \\
&- \frac{3A^{\lambda}b(\alpha p)^{-\epsilon}(\theta + A^{\lambda}\beta)^2\big((1-\gamma)(1-\delta)\big)^4T^2}{4} \\
&+ \frac{A^{\lambda}b(\alpha p)^{-\epsilon}\big((1-\gamma)(1-\delta)\big)^2}{2}\Big] + \frac{c_0}{T^2} + \frac{AG}{T^2}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\delta AP}{\delta \gamma} &= p \times \Big[A^{\lambda}bp^{-\epsilon}(1-\delta) + A^{\lambda}b(\alpha p)^{-\epsilon}(-1+\delta)\Big] \\
&+ (pA^{\lambda}\beta - c_h) \times \Big[(K - A^{\lambda}bp^{-\epsilon})\delta(1-\delta)T\Big] \\
&+ \big(pA^{\lambda}\beta - c_h - c_p(1-s)\theta\big) \times \\
\Big[&- \frac{(K - A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta\delta^2(1-\delta)T^2}{2} \\
&- (K - A^{\lambda}bp^{-\epsilon})(\theta + A^{\lambda}\beta)\delta\gamma(1-\delta)^2T^2 - A^{\lambda}bp^{-\epsilon}\gamma(1-\delta)^2T \\
&+ \frac{(K - A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta(\theta + A^{\lambda}\beta)\delta^2(1-\delta)^2\gamma T^3}{2} \\
&+ A^{\lambda}b(\alpha p)^{-\epsilon}\big((1-\gamma)(1-\delta)\big)(-1+\delta)T \\
&- A^{\lambda}b(\alpha p)^{-\epsilon}(\theta + A^{\lambda}\beta)^2\big((1-\gamma)(1-\delta)\big)^3(-1+\delta)T^3\Big].
\end{aligned}
$$

We test the optimal condition by using the function,

$$
\frac{\delta^2 AP}{\delta T^2}\frac{\delta^2 AP}{\delta \gamma^2} - \left(\frac{\delta^2 AP}{\delta T \delta \gamma}\right)^2 \quad > \quad 0
$$

where,

$$
\begin{aligned}
\frac{\delta^2 AP}{\delta T^2} &= \big(pA^{\lambda}\beta - c_h - c_p(1-s)\theta\big) \times \Big[(K - A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta\delta^2\gamma(1-\delta) \\
&- (K - A^{\lambda}bp^{-\epsilon})(\theta + A^{\lambda}\beta)\delta\gamma^2(1-\delta)^2 \\
&- \frac{3}{2}A^{\lambda}b(\alpha p)^{-\epsilon}(\theta + A^{\lambda}\beta)^2\big((1-\gamma)(1-\delta)\big)^4 T \\
&+ \frac{3}{2}(K - A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta(\theta + A^{\lambda}\beta)\delta^2\gamma^2(1-\delta)^2 T\Big] - \frac{2c_0}{T^3}.
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\delta^2 AP}{\delta \gamma^2} &= \big(pA^{\lambda}\beta - c_h - c_p(1-s)\theta\big) \times \Big[A^{\lambda}bp^{-\epsilon}(1-\delta)^2 T \\
&- (K - A^{\lambda}bp^{-\epsilon})(\theta + A^{\lambda}\beta)\delta(1-\delta)^2 T^2 \\
&+ \frac{(K - A^{\lambda}bp^{-\epsilon})A^{\lambda}\beta(\theta + A^{\lambda}\beta)\delta^2(1-\delta)^2 T^3}{2} \\
&- 3A^{\lambda}b(\alpha p)^{-\epsilon}(\theta + A^{\lambda}\beta)^2\big((1-\gamma)(1-\delta)\big)^2(-1+\delta)(-1+\delta)T^3 \\
&+ A^{\lambda}b(\alpha p)^{-\epsilon}(-1+\delta)(-1+\delta)T\Big].
\end{aligned}
$$

Both functions, $\frac{\delta^2 AP}{\delta T^2}$ and $\frac{\delta^2 AP}{\delta \gamma^2}$ must be less than zero in order to maximize the annual profit.

5.5. Numerical example

In this section, we obtain the optimal solutions to illustrate the model by using numerical examples with the following parameter values.

The value of markdown rate, α is set from 0.5 to 0.9. The value of frequency of advertisement in the cycle, A is limited to less than or equal to 5, otherwise the advertisement is not beneficial to the inventory model anymore. By using equation(5.37) we are able to find the best markdown time γ and the best total cycle length T provided that all the optimality conditions are satisfied by using Excel Solver.

Example 1: Let c_0 = 100, p = 250, c_h = 0.5, c_p = 10, K = 200, β = 0.1, θ = 0.07, ϵ = 1.8, b = 10000, s = 0.6, A = 3, λ = 1.8, G = 100 and δ = 0.1.

Table 5.1: Experimental result for Example 1

α	T^*	γ^*	Q_1^*	Q_2^*	AP^*
0.5	3.038	0.420	53.6	38.6	3342.5
0.7	3.095	0.432	54.5	21.0	3139.1
0.9	3.130	0.436	55.0	13.4	3019.1

Example 2: Suppose c_0 = 100, p = 250, c_h = 0.5, c_p = 10, K = 200, β = 0.1, θ = 0.07, ϵ = 1.8, b = 10000, s = 0.6, A = 3, λ = 1.8, G = 100 however, in this second example we increase the production rate parameter, δ to 0.2 instead of 0.1 in Example 5.1. Using the same steps as Example 5.1, the result can be seen in Table 5.2.

Table 5.2: Experimental result for Example 2

α	T^*	γ^*	Q_1^*	Q_2^*	AP^*
0.5	3.989	0.424	119.2	50.4	6478.7
0.7	4.162	0.410	122.9	31.3	6333.5
0.9	4.327	0.395	126.4	22.7	6257.8

Table 5.1 and 5.2 show experimental results for measuring the effectiveness of the model with respect to the parameters α and δ on optimum cycle time, optimum markdown offering time, optimum quantities and optimum profit.

Observe that when α = 0.5, for δ = 0.1, the maximum total profit is 3342.5. By using the value of γ obtained from Table 5.1, the markdown time $t_2 = \gamma(1-\delta)T = 1.276$ and the cycle time is 3.038 with Q_1= 53.6 and Q_2 =38.6. Whereas, for δ = 0.2, the maximum total profit is 6478.7. Similarly, by using the value of γ obtained from Table 5.2 the markdown time $t_2 = \gamma(1-\delta)T = 1.691$ and the cycle time is 3.989 with Q_1= 119.2 and Q_2 = 50.4.

When α = 0.7, for δ = 0.1, the maximum total profit is 3139.1. By using the value of γ obtained from Table 5.1 the markdown time $t_2 = \gamma(1-\delta)T = 1.337$ and the cycle time is 3.095 with Q_1 = 54.5 and Q_2 = 21.0. Whereas, for δ = 0.2, the maximum total profit is 6333.5. Similarly by using the value of γ obtained from Table 5.2 the markdown time $t_2 = \gamma(1-\delta)T = 1.706$ and the cycle time is 4.162 with Q_1= 122.9 and Q_2 = 31.3.

Finally, when α = 0.9, for δ = 0.1, the maximum total profit is 3019.1, the markdown time $t_2 = \gamma(1-\delta)T = 1.365$ and the cycle time is 3.130 with Q_1 = 55.0 and Q_2 = 13.4. Whereas, for δ = 0.2, the maximum total profit is 6257.8, the markdown time $t_2 = \gamma(1-\delta)T = 1.709$ and the cycle time is 4.327 with Q_1= 126.4 and Q_2 = 22.7.

Notice that for the same value of α, a higher production percentage δ gives a higher annual profit. Moreover, the markdown time and total cycle length increase with the increase in production percentage δ.

Also, it can be seen from the tables above for example, by comparing using the same value of δ = 0.1, when α = 0.5 the annual profit is 3342.5 with Q_1 = 53.6 and Q_2 = 38.6 and the markdown time is 1.276. When α = 0.7 the annual profit is 3139.1 with Q_1 = 54.5 and Q_2 = 21.0 and the markdown time is 1.337. Whereas, when α = 0.9 the annual profit 3019.1 with Q_1 = 55.0 and Q_2 = 13.4 and the markdown time is 1.365.

As per the above numerical result, note that when the value of α increases (lower reduction in price), the higher the markdown percentage which means it is better to apply markdown at a later time in order to maximize the annual profit since the impact is lesser compare to when the value of α is lower which indicates higher reduction in price. Moreover, the quantity Q_1 at t_1 is higher since more quantity is needed in order to maximize the annual profit due to the smaller impact of markdown for this case. The quantity for markdown Q_2 is lower because again, the small reduction in price gives a small impact in maximizing the annual profit hence, it is more profitable to have less quantity during this interval.

From Table 5.1 and 5.2, notice that if we increase the production percentage δ by

100%, the annual profit is increased by 94%, 102% and 107% for the fixed value of markdown rate. Hence, we cay say that in order to maximize the total profit, it is better to increase the production cycle time .

Apart from that, with the increase in markdown rate α by 40% to 80%, there is a slight decrease in the annual profit by around 4% to 6% when δ = 0.1 and a further decrease in the annual profit by roughly 1% to 2% when δ = 0.2.

The effect of markdown price in order to optimize annual profit is case dependent. Notice that when the price is reduced to 50 % of its initial price, it dominants other markdown prices in order to maximize the profit. This research result, however contradicts the previous researches such as (Widyadana & Wee, 2007) as well as (Srivastava & Gupta, 2013) that considered the demand to be dependent solely on price or demand to be dependent on price and time without taking into consideration the effect of other factors such as inventory level and frequency of advertisement. A key element in markdown optimization is to observe how items react to price changes. Moreover, from these two examples, we can say that the higher the production rate, the higher the optimum total cycle time, the optimum markdown time, the optimum quantities and the optimum annual profit.

Figure 5.2 shows the comparison of annual profit for the case when δ = 0.1 and δ = 0.2 for every value of α.

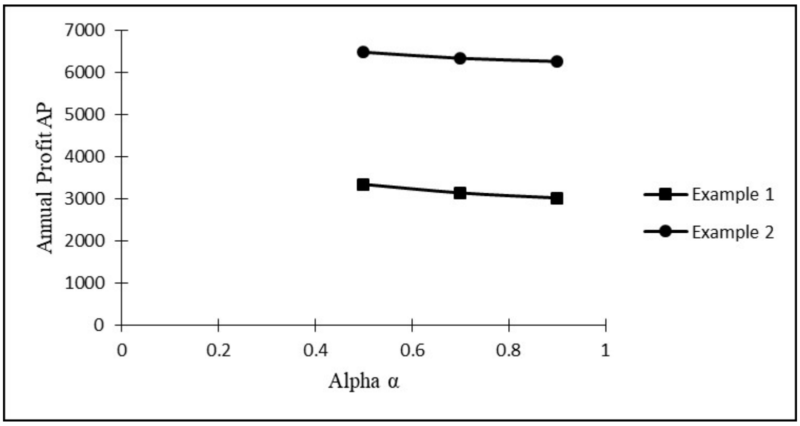

Figure 5.2: Comparison of annual profit for Example 1 and Example 2

5.5.1. Sensitivity Analysis

Using the same data as those in Example 2 and by fixing $\alpha = 0.5$, the sensitivity analysis of the optimal solution with respect to each parameters is obtained and shown in Table 5.3 to Table 5.15.

The sensitivity analysis for every parameter is tested by using the optimum value as well as the plus and minus 50 % from its optimum value to observe the effect of changes clearer.

Table 5.3: Effect of changes in b

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
b = 5000	4.429	0.401	129.7	33.5	6088.4
b = 10000	3.989	0.424	119.2	50.4	6478.7
b = 15000	3.780	0.428	113.4	67.5	6939.0

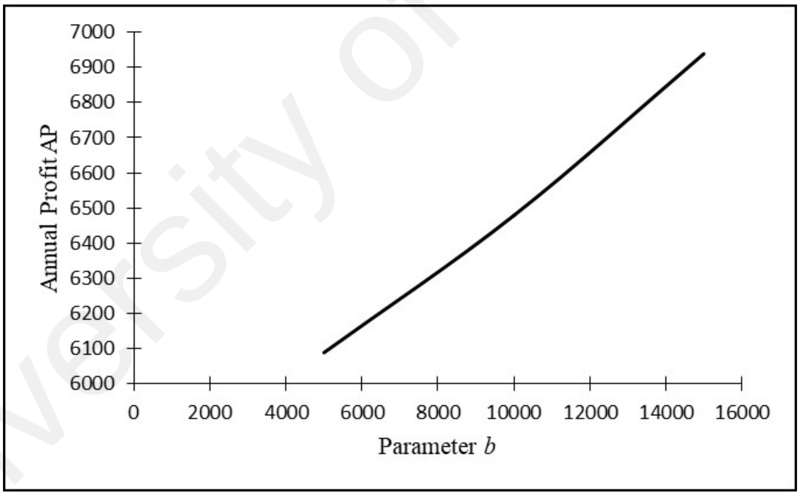

Figure 5.3: Effect of changes in b on the optimal annual profit

Table 5.4: Effect of changes in β

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
β = 0.05	7.393	0.422	255.1	95.0	6130.7
β = 0.10	3.989	0.424	119.2	50.4	6478.7
β = 0.15	2.739	0.425	81.2	34.5	6575.7

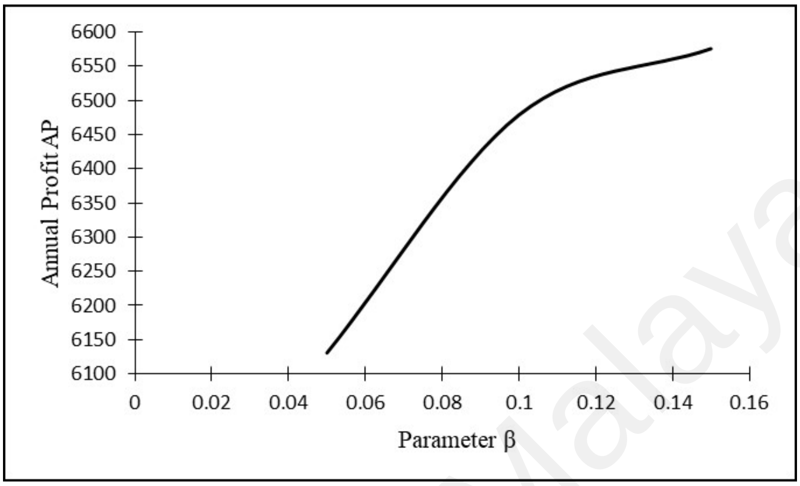

Figure 5.4: Effect of changes in β on the optimal annual profit

Table 5.5: Effect of changes in s

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
s = 0.3	3.991	0.424	119.2	50.4	6475.3
s = 0.6	3.989	0.424	119.2	50.4	6478.7
s = 0.9	3.987	0.424	119.1	50.3	6482.1

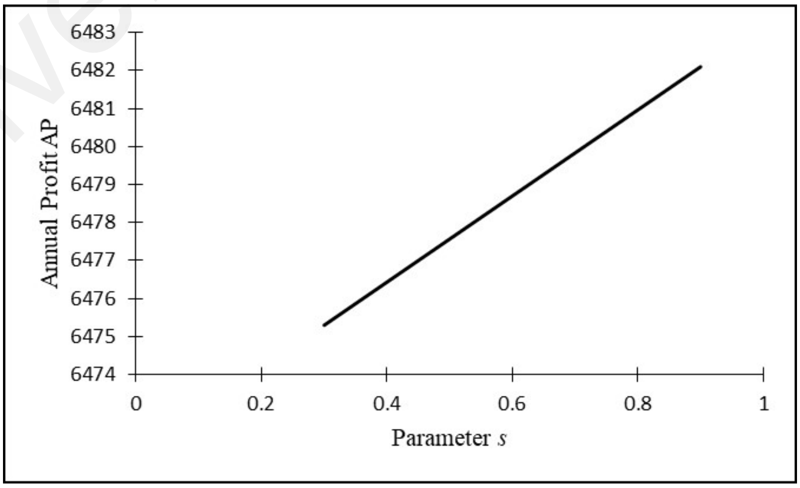

Figure 5.5: Effect of changes in s on the optimal annual profit

Table 5.6: Effect of changes in A

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
A = 1.5	14.520	0.362	434.1	85.8	5169.6
A = 3.0	3.989	0.424	119.2	50.4	6478.7
A = 4.5	1.841	0.428	54.5	44.2	7528.6

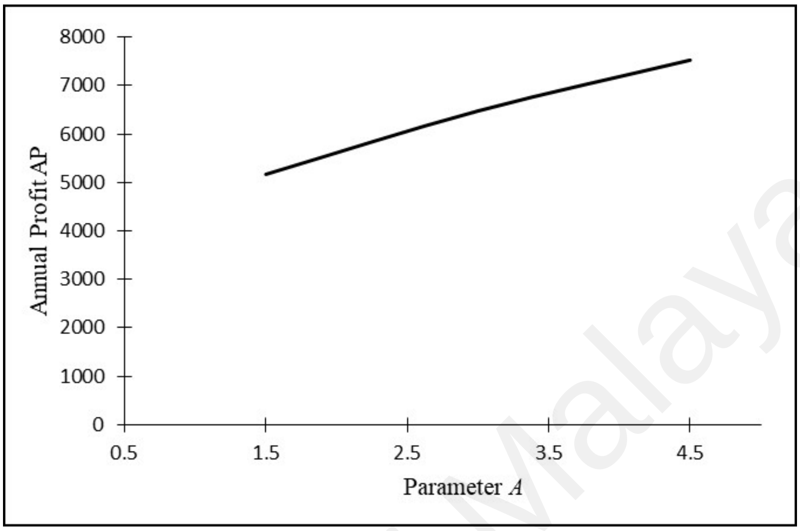

Figure 5.6: Effect of changes in A on the optimal annual profit

Table 5.7: Effect of changes in λ

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
λ = 0.9	11.160	0.380	333.5	73.6	5433.0
λ = 1.8	3.989	0.424	119.2	50.4	6478.7
λ = 2.7	1.460	0.421	43.2	42.6	8065.3

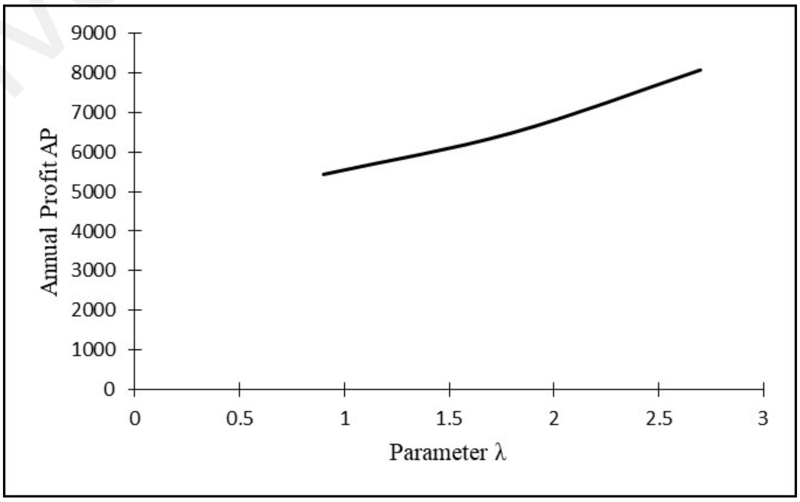

Figure 5.7: Effect of changes in λ on the optimal annual profit

Table 5.8: Effect of changes in ϵ

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
ϵ = 0.9	3.603	0.424	107.9	86.1	7538.5
ϵ = 1.8	3.989	0.424	119.2	50.4	6478.7
ϵ = 2.7	9.568	0.247	207.3	63.1	6772.5

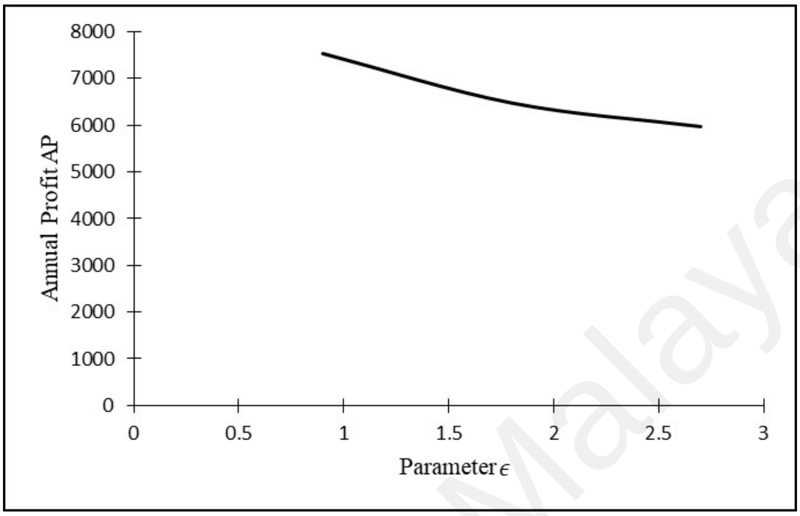

Figure 5.8: Effect of changes in ϵ on the optimal annual profit

Table 5.9: Effect of changes in c_0

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
c_0 = 50	2.984	0.424	119.0	50.2	6491.2
c_0 = 100	3.989	0.424	119.2	50.4	6478.7
c_0 = 150	3.994	0.424	119.3	50.5	6466.2

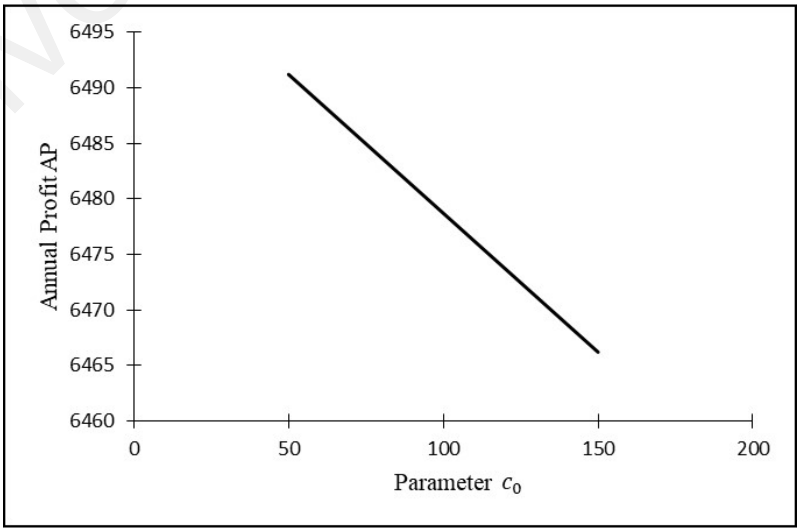

Figure 5.9: Effect of changes in c_0 on the optimal annual profit

Table 5.10: Effect of changes in c_h

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
c_h = 0.25	3.988	0.424	119.1	50.3	6486.7
c_h = 0.50	3.989	0.424	119.2	50.4	6478.7
c_h = 0.75	3.990	0.424	119.2	50.5	6470.7

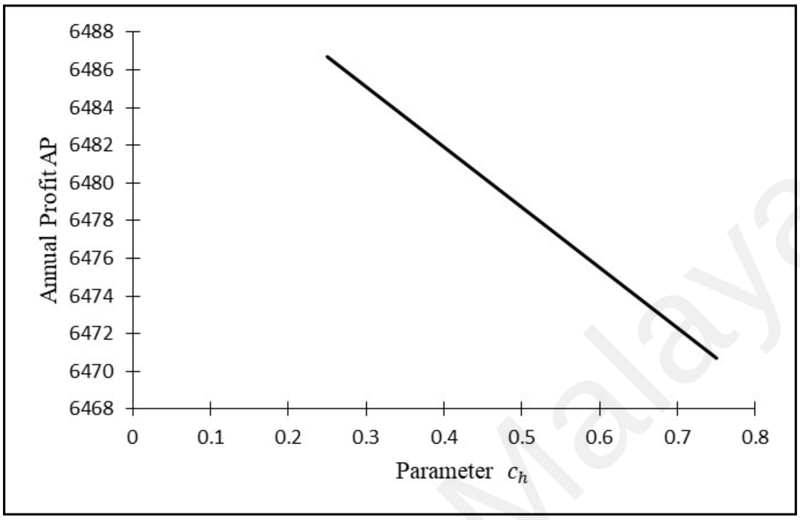

Figure 5.10: Effect of changes in c_h on the optimal annual profit

Table 5.11: Effect of changes in c_p

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
c_p = 5	3.988	0.424	119.1	50.3	6681.0
c_p = 10	3.989	0.424	119.2	50.4	6478.7
c_p = 15	3.990	0.424	119.2	50.5	6276.4

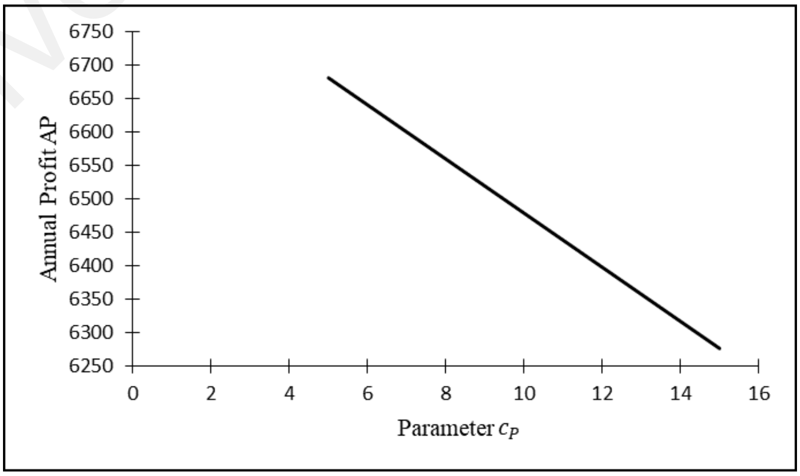

Figure 5.11: Effect of changes in c_p on the optimal annual profit

Table 5.12: Effect of changes in G

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
G = 50	3.974	0.424	118.8	50.0	6516.3
G = 100	3.989	0.424	119.2	50.4	6478.7
G = 150	4.004	0.424	119.5	50.7	6441.2

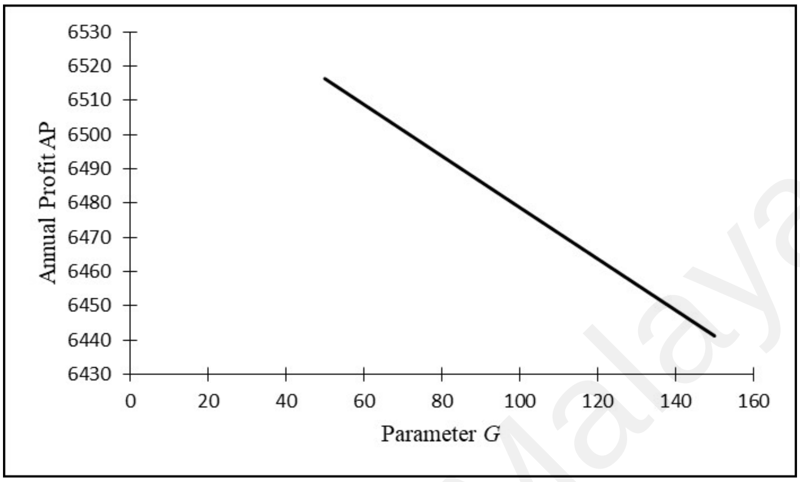

Figure 5.12: Effect of changes in G on the optimal annual profit

Table 5.13: Effect of changes in K

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
K = 100	3.666	0.426	54.9	42.8	3656.8
K = 200	3.989	0.424	119.2	50.4	6478.7
K = 300	4.212	0.413	187.1	58.1	9351.1

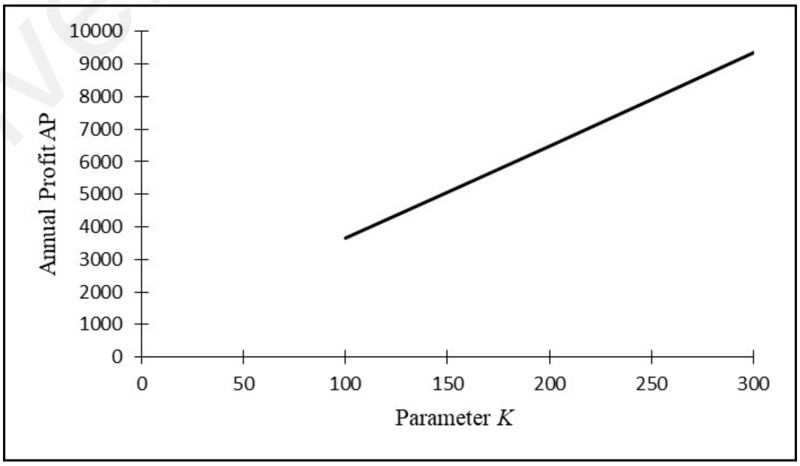

Figure 5.13: Effect of changes in K on the optimal annual profit

Table 5.14: Effect of changes in p

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
$p = 125$	3.528	0.419	105.7	102.5	4543.4
$p = 250$	3.989	0.424	119.2	50.4	6478.7
$p = 375$	4.439	0.400	130.0	32.6	9371.3

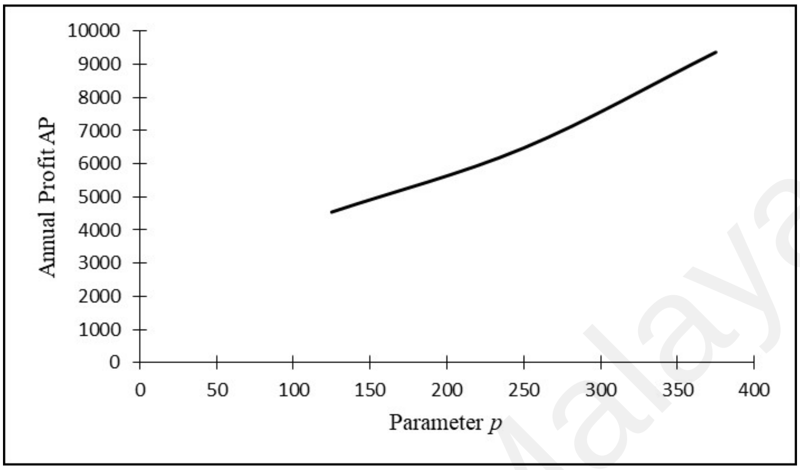

Figure 5.14: Effect of changes in p on the optimal annual profit

Table 5.15: Effect of changes in θ

Parameter	T^*	γ^*	Q_1^*	Q_2^*	AP^*
$\theta = 0.035$	4.141	0.426	122.5	51.7	6691.5
$\theta = 0.070$	3.989	0.424	119.2	50.4	6478.7
$\theta = 0.105$	3.848	0.423	116.0	49.1	6279.2

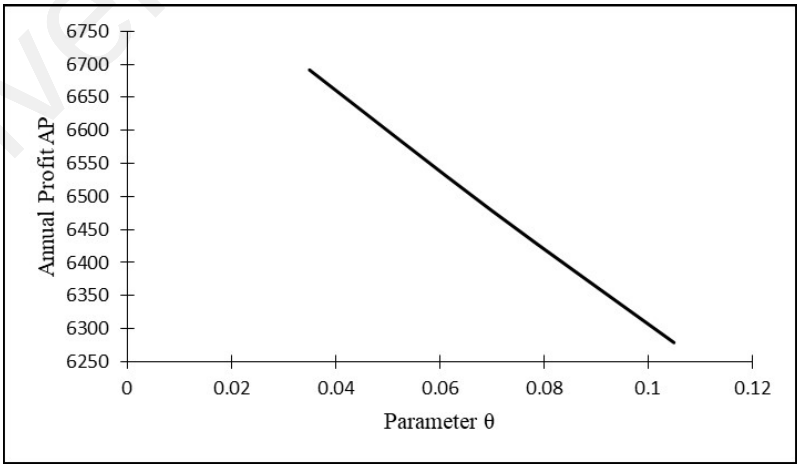

Figure 5.15: Effect of changes in θ on the optimal annual profit

Based on Table 5.3 to Table 5.15, we notice that :

1. When the rate of positive constant b increases, there is a decrease in the total cycle time and markdown time but an increase in the annual profit.

2. Increasing the value of β, the total cycle time and markdown time decrease whereas annual profit increases.

3. There is a slight increase in total cycle time, markdown time and annual profit when the salvage value for the deteriorated units, s increases.

4. When the value of frequency of advertisement A increases, total cycle time and markdown time decrease while annual profit increases.

5. Increasing the rate of change of frequency of advertisement λ, decrease the total cycle time and markdown time and increases the annual profit.

6. Increasing the index of price elasticity ϵ, increase the total cycle time and markdown time and decreases the annual profit.

7. When all the cost c_0, c_h, c_p and G increase, there is an increase in total cycle time, markdown time and decreases in annual profit.

8. Changes in the production rate K result in a positive change in the total cycle time, markdown time and annual profit.

9. Increasing the value of price p, increase the total cycle time, markdown time and annual profit.

10. Lastly, when the rate of deterioration θ increases, the total cycle time, markdown time and annual profit decrease.

In the first case of sensitivity analysis, only the value of b is allowed to vary while the other parameters remain constant. Increasing the value of b can increase the demand rate. Thus, there is an increase in annual profit and consequently, the retailer will reduce the cycle time and markdown time.

Likewise, for the second parameter, the annual profit increases as the value of β increases due to a higher demand as well. Since the demand is higher, a shorter cycle time and an earlier markdown time are needed.

Similarly, the impact of the changes in the third parameter shows that the increase in salvage value parameter s increases the annual profit and slightly increase in cycle time and markdown time. The reason is because the total inventory cost can be minimized if the salvage value is incorporated into the deteriorating items and the annual profit is maximized.

For the fourth and fifth parameters, with the increase in the rate of change of frequency of advertisement λ and frequency of advertisement in the cycle A, there is an increase in annual profit. This is due to more advertisement simply implies more demand. Therefore, decreasing the cycle time and markdown time. Realistically, minimum advertisement cost can maximize the annual profit of the retailer but more advertisement cost implies more demand.

Contrarily, for sixth parameter, the annual profit decreases if the value of ϵ increases. A higher value of ϵ means, an increase in price rate which lowers the demand. Thus, the retailer will lengthen the cycle time and markdown time. The patterns in the seventh, eighth, ninth and tenth case are similar to each other. The increase in cost whether c_p, c_0 c_h or G causes the annual profit to decrease as the total cost increases. Therefore it is sensible that the retailer lengthens the cycle time and markdown time and the retailer can boost the selling price marginally.

Meanwhile the effect of changes in eleventh and twelfth parameters show that annual profit, cycle time and markdown time increase as p or K increase. Increase in price p will not only cause the annual profit to increase, but also increase in the cycle time and markdown time since it is more time consuming to sell an item with a more expensive price. A higher production rate gives higher annual profit, total cycle time and also higher markdown time. The higher volume helps retailer to gain a higher profit from that and the retailer will need to lengthen the cycle time and markdown time.

Conversely, for the last parameter, as the deterioration rate θ increases, annual profit and also both total cycle time and markdown time decrease. A higher rate of deterioration causes loss to retailers. Due to that, annual profit, total cycle time and markdown time are decreasing with the higher value of θ. If the retailer is able to efficiently lower the deteriorating rate of the item by inventing a new preservation technology or by improving storage therefore, the annual profit can be increased.

5.5.2. Special cases

The important special cases that influence the optimal annual profit are described as follows :

1. If we take A^{λ} = 1 in demand rate and without considering demand to be depending on inventory level, $\beta I(t)$ = 0 in demand rate, our demand rate reduces to $b(\alpha p)^{-\epsilon}$ which is similar to the model of (Srivastava & Gupta, 2013).
2. When A^{λ} = 1, α = 1 thus, the demand rate becomes $bp^{-\epsilon} + \beta I(t)$ which is similar demand rate with the model of (Teng & Chang, 2005).

5.6. Conclusion

Advertisement helps in boosting demand rate by attracting potential customers and acts as a marketing promotion tool. However, as much as the advertisement is beneficial to an inventory model, there is a limitation to it in which at some point, advertisement is no longer beneficial to the inventory model. Therefore, it is important to study the effect of the frequency of advertisement in an inventory model.

This model is an extension of previous models by (Teng & Chang, 2005), (Srivastava & Gupta, 2013) as well as (Shah & Pandey, 2009). In this study, we developed a more realistic inventory model for delayed deteriorating items with price, frequency of advertisement and inventory level dependent demand. We have incorporated several realistic features in our model. First of all, by applying markdown policy which helps to clear out end-of-life or end-of-season inventory, reduce wastage as well as to give room for more desirable and up to date products. Moreover, markdown is proven to help in augmenting the total profit of the inventory model. Realistically, demand is diminished with time and due to this, markdown is expected to incite back the demand. It is important that the optimum markdown time must not be either too early or too late otherwise, it will not be able to help in maximizing the total profit. We obtained the optimum markdown time in this model which is demonstrated in Table 5.1 and Table 5.2.

Secondly, by incorporating salvage value with the deteriorated units in order to maximize annual profit. Thirdly, demand is considered not only to be depending on price and inventory level but also depending on advertisement since advertisement plays an important role and it affects the demand rate. Finally, to make this model more sensible, we consider delayed deterioration because realistically, items do no deteriorate at the instant of their arrival.

Necessary and sufficient conditions for the existence and uniqueness of optimal solution have been derived in this model. Moreover, optimum cycle time, optimum markdown time, optimum quantities and optimum annual profit have also been derived.

We can conclude that in this model, the higher the markdown rate (lesser reduction in price), the lower the annual profit. However, it contradicts previous studies such as (Widyadana & Wee, 2007) and (Srivastava & Gupta, 2013) that we refer to since they only considered demand rate to be dependent solely on price or price and time. The end result shows that the markdown rate gives remarkable contribution in optimizing the total profit and it is important for a policymaker to be cautious in setting the markdown rate. This is because the optimum policy is dissimilar for different cases as it is case dependent.

Sensitivity analysis is provided to help retailers in decision making. Notice that from the sensitive analysis in Table 5.3 until Table 5.15, the annual profit is more sensitive to b, β, A, λ, ϵ, c_p, p, K and θ. In order to obtain the best policy, it is vital for retailers to focus more on these parameters. We can also conclude that this model is a more generalized form of these models (Teng & Chang, 2005) and (Srivastava & Gupta, 2013).

CHAPTER 6: CONCLUSION AND SUGGESTIONS FOR FUTURE RESEARCH

In this final chapter, we discuss the conclusion of this research and some suggestions for future studies are given to extend this research.

6.1. Conclusion

In this thesis, we have proposed three different models for perishable items under markdown policy. The first model considers delayed deteriorating items with price and inventory level dependent demand under markdown policy. Whereas, the second model considers an inventory model for a fresh product when demand depends on freshness, price, inventory level and expiration date under markdown policy. Lastly, the third model considers delayed deteriorating items when demand depends on advertisement, price and inventory level under markdown policy.

All these three models are appropriate and applicable for those perishable items with low demand in the market compared to other items and they undergo a deterioration process which is continuous over time since we introduce markdown policy. We presented numerical examples and performed sensitivity analysis to show the efficacy of these models. Moreover, the optimum cycle time, optimum annual profit, optimum quantities and optimum markdown offering time have been derived.

We can conclude that for all three models, the higher the markdown rate (lesser reduction in price), the lesser the annual profit. However, it contradicts several previous studies such as (Widyadana & Wee, 2007) and (Srivastava & Gupta, 2013) since they only considered demand rate to be dependent solely on price or price and time. Therefore, a policymaker must be very careful in setting the markdown rate. This is because the optimum policy is different for different cases and it is case dependent.

Sensitivity analysis is a great help to retailers to make a wiser decision. Retailers should focus more on parameters that gives a huge impact on the inventory model such as b, β, initial price p and index of price elasticity ϵ. Other than that, salvage value is incorporated with the deteriorated units in order to maximize annual profit.

Markdown can benefit retailers by helping to increase sales, improve margins and better management on product life cycles. Retailers have historically viewed markdowns as an appropriate strategy designed to help sell old or slow-moving inventory. Following from there, markdown helps to reduce wastage and it is necessary especially in fast-evolving industries. Markdown is important and it should be practiced in industry.

6.2. Suggestions for future research

Inventory management is a field that has matured relatively over the past several decades and an applied mathematical model implemented is always a simplification of the complex real-life problem. To enhance the applicability, this research can be extended in several forms:

1. It is undeniable that shortage happens in real life thus, this problem may be considered in order to make the model more realistic.

2. Multiple markdowns may be considered since in reality, there are supermarkets that usually offer more than one-time markdown.

3. Different types of deteriorating function such as exponential distribution deterioration can be considered instead of constant deterioration.

4. Proposed models can be extended by allowing trade credit and investment in preservation technology.

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LIST OF PUBLICATIONS AND PAPERS PRESENTED

PAPER PRESENTED

1. **Kamaruzaman, N. A.,** & Omar, M. (2018). *An EPQ model of delayed deteriorating items with price and invntory level dependent demand under markdown policy*. Paper presented at the 4th International Postgraduate Conference on Physics and Mathematics (IPCPMS18), IPN Education Group, 3-5 July 2018, Langkawi, Malaysia.