CHAPTER 6 : RESULTS AND DISCUSSIONS

6.1 Static Analysis

Solutions of simultaneous linear equations that arise in a static analysis are solved and deformations and stresses can be evaluated at any time independent of the displacement history [10].

6.1.1 Relationship Between Force and Deformation

Using the stress-strain relationship $S = E\epsilon$, we can determine deformation if the force is known :

$$S = E\epsilon, \text{ where } \epsilon = e/L \qquad (6.1)$$

$$\text{therefore}, \quad S = E\,e/L \qquad (6.2)$$

$$e = SL/E \text{ where } S = P/A$$

$$\text{therefore,} \quad e = PL/AE \qquad (6.3)$$

where,   P = force

L = length

A = area

E = Young's Modulus

In this Static Analysis,   P (punch force)   =   103,950 N

L (punch length)   = 191.96×10^{-3} m

A (punch area)   = $[\pi \times (112.32 \times 10^{-3})^2]/4$

E (Young's Modulus) = 2.0×10^{11} N/m^2

Therefore, punch deformation during the deep drawing process is as calculated below :

$$e = PL / AE$$

$$e = (103{,}950 \times 191.96 \times 10^{-3}) / \{ [\pi \times (112.32 \times 10^{-3})^2]/4] \times 2 \times 10^{11} \}$$

$$e = 1.01 \times 10^{-5} \text{ m or } 0.0101 \text{ mm}$$

The maximum deformation from FEM analysis is 0.00044 inches at node ID 212 which is equivalent to 0.0111 mm. Please refer to following graphs:

a. Figure 6.1 : Total Translation in Z axis

b. Figure 6.2 : Total Translation vs Node ID
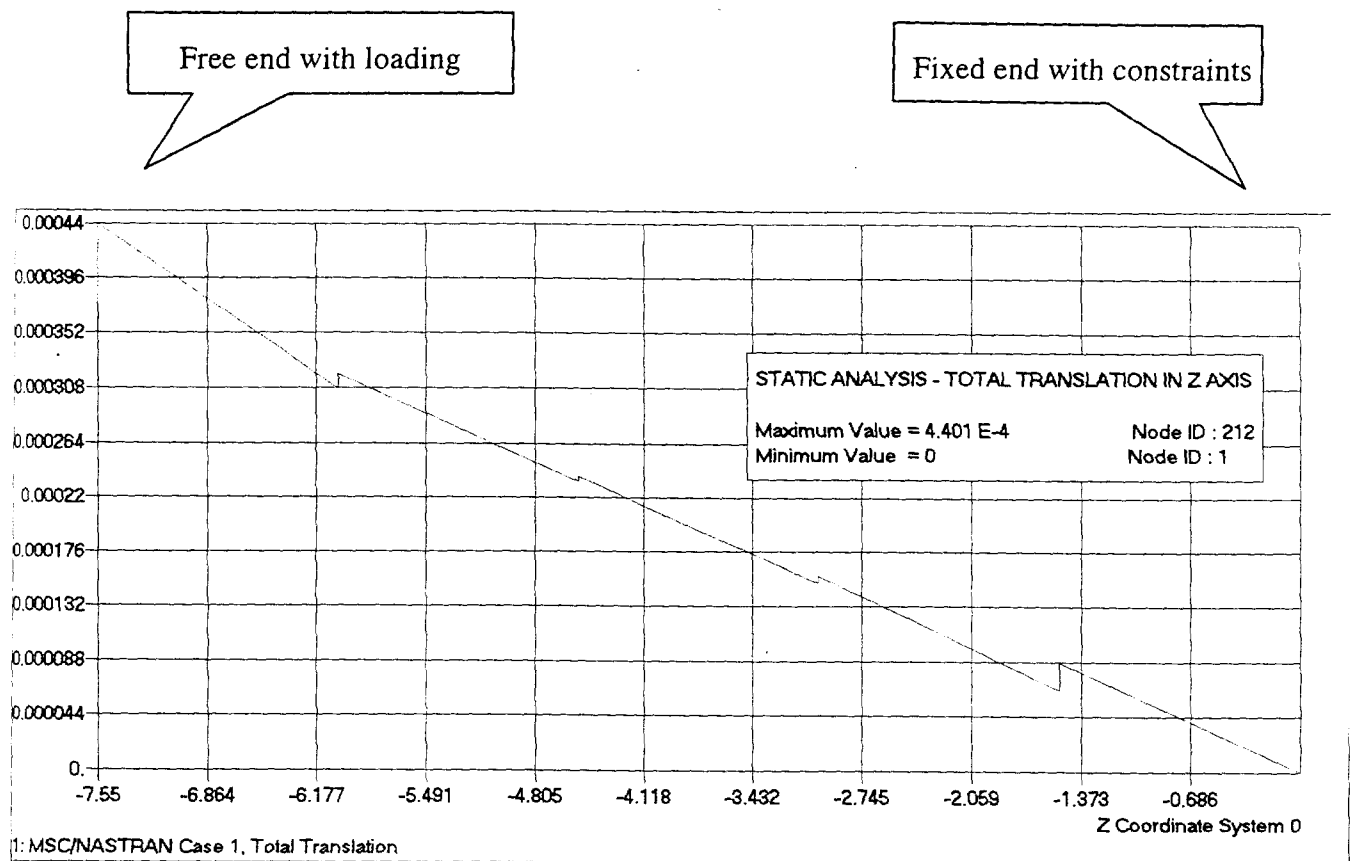
c. Figure 6.3 : Total Rotation vs Element ID
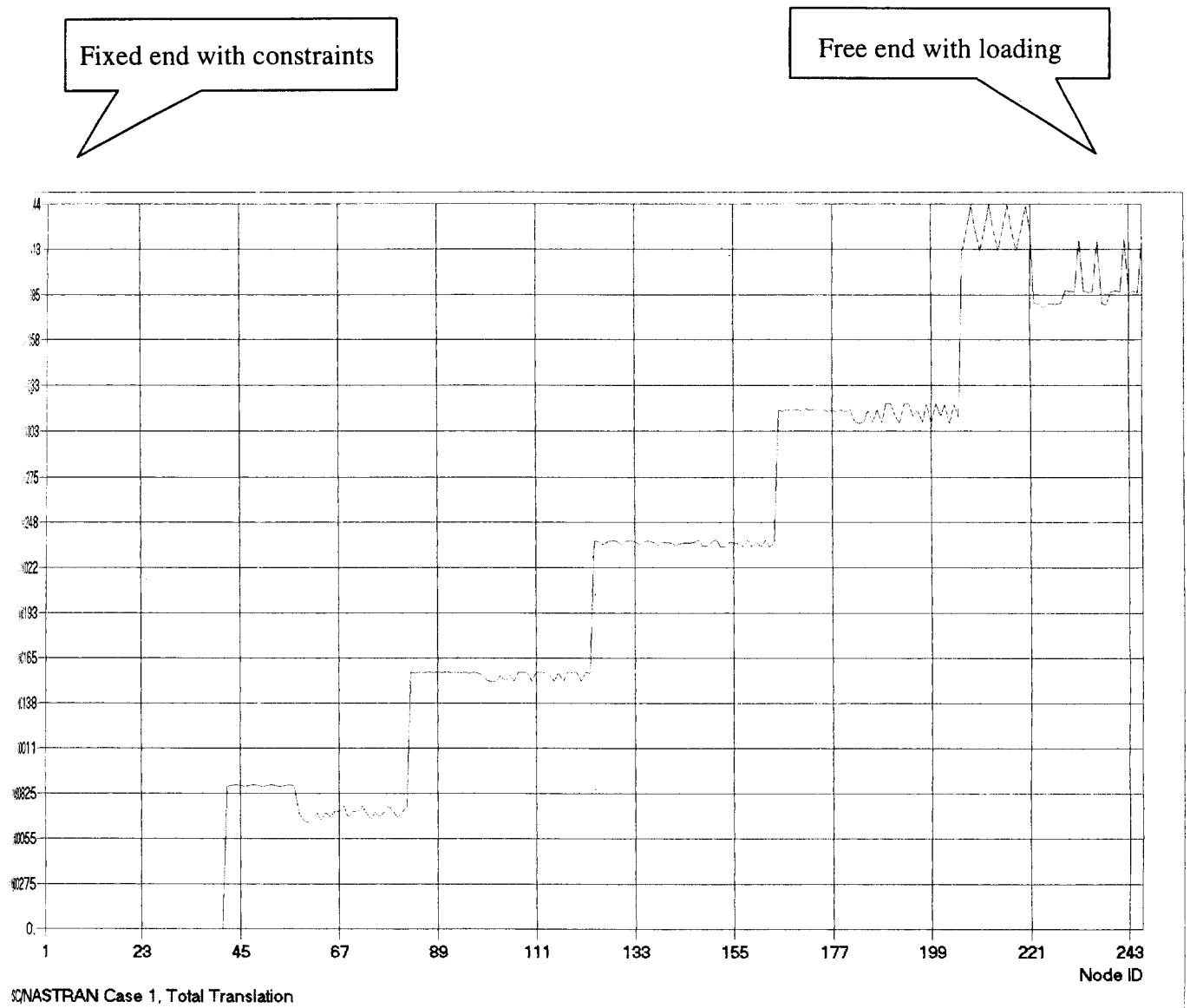
Figure 6.1 : Total Translation in Z axis

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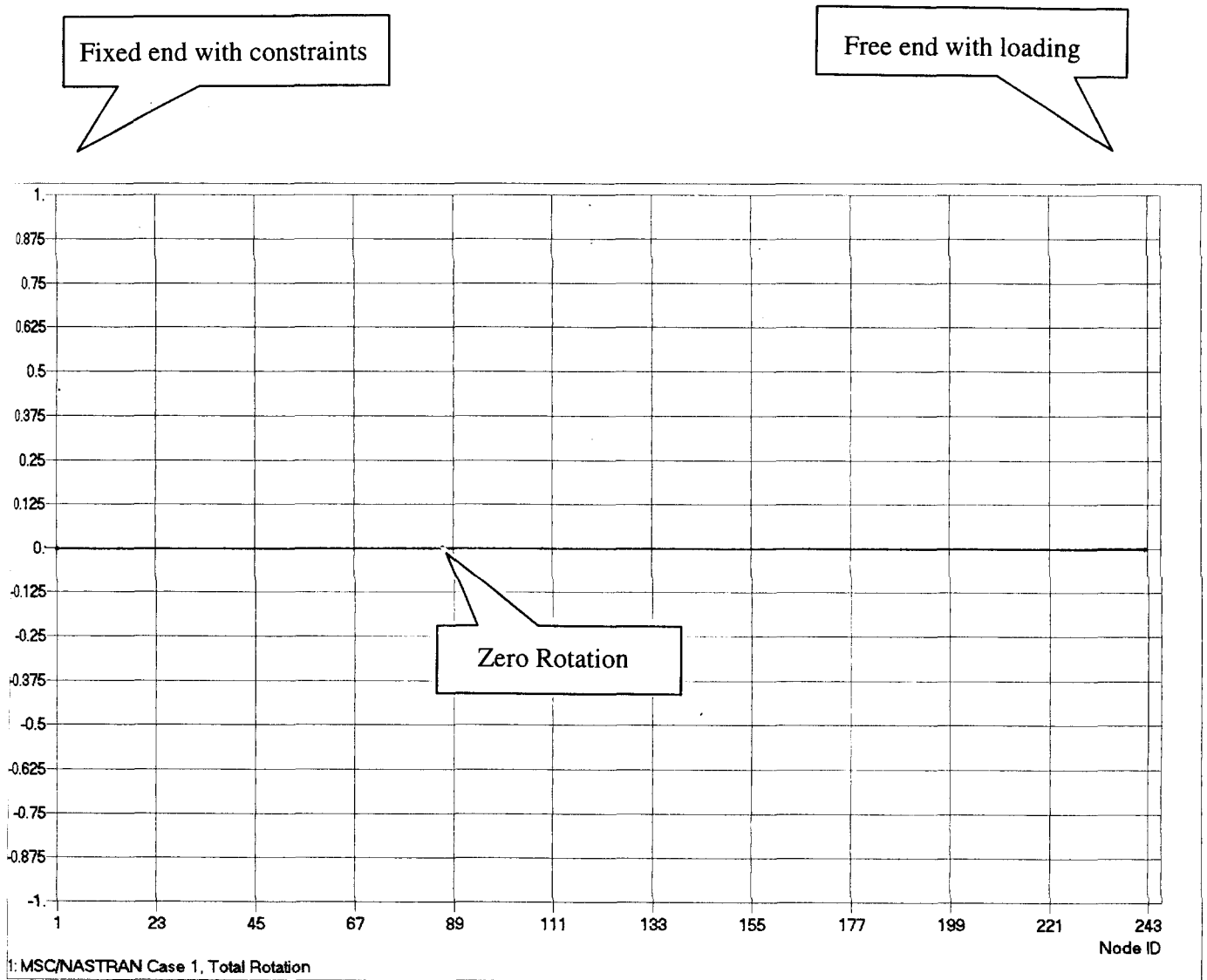**Figure 6.2 : Total Translation vs Node ID**

Figure 6.3 : Total Rotation vs Element ID

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It is clearly evident that the maximum translation or deformation occurs at element layer no.5 which has the free nodes that experience the highest punch reaction force of 2,535 N per node. The deformation values drops gradually to zero as it moves upwards towards element layer no.1 where all the nodes are fixed to the top die plate.

6.1.2 Failure Theories

In designing tools or components to resist failure, we usually assure ourselves that the internal stresses do not exceed the strength of material. If the material to be used is ductile, then it is the yield strength that must be used because a permanent deformation will constitute a failure [9]. For brittle materials which do not have a yield point, the ultimate strength is used as a criterion for failure.

Maximum Normal Stress Theory

The maximum normal stress theory states that failure occurs whenever the largest principal stress equals to the strength [9].

Suppose, we arrange the three principal stresses as shown below

$$\sigma_1 > \sigma_2 > \sigma_3$$

Then if yielding is the criterion for failure, this theory predicts that failure will occur whenever,

$$\sigma_1 = S_{yt} \qquad \text{or} \qquad \sigma_3 = -S_{yc} \qquad \qquad (6.4)$$

where S_{yt} and S_{yc} are the tensile yield strength and compressive yield strength respectively.

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In this Static Analysis, $\sigma_1 = 132.4279$ psi / 913,064 N/m^2

$\sigma_2 = 132.4068$ psi / 912,918 N/m^2

$\sigma_3 = -1964.035$ psi / -13,541,629 N/m^2

The highest normal compresive stress is $\sigma_3 = -13.5 \times 10^6$ N/m^2

Note : Since the punch is in compressive stress, equation $\sigma_3 = -S_{yc}$ is used to predict the static failure.

Please refer to Figure 6.4 : Solid Z Normal Stress vs Element ID
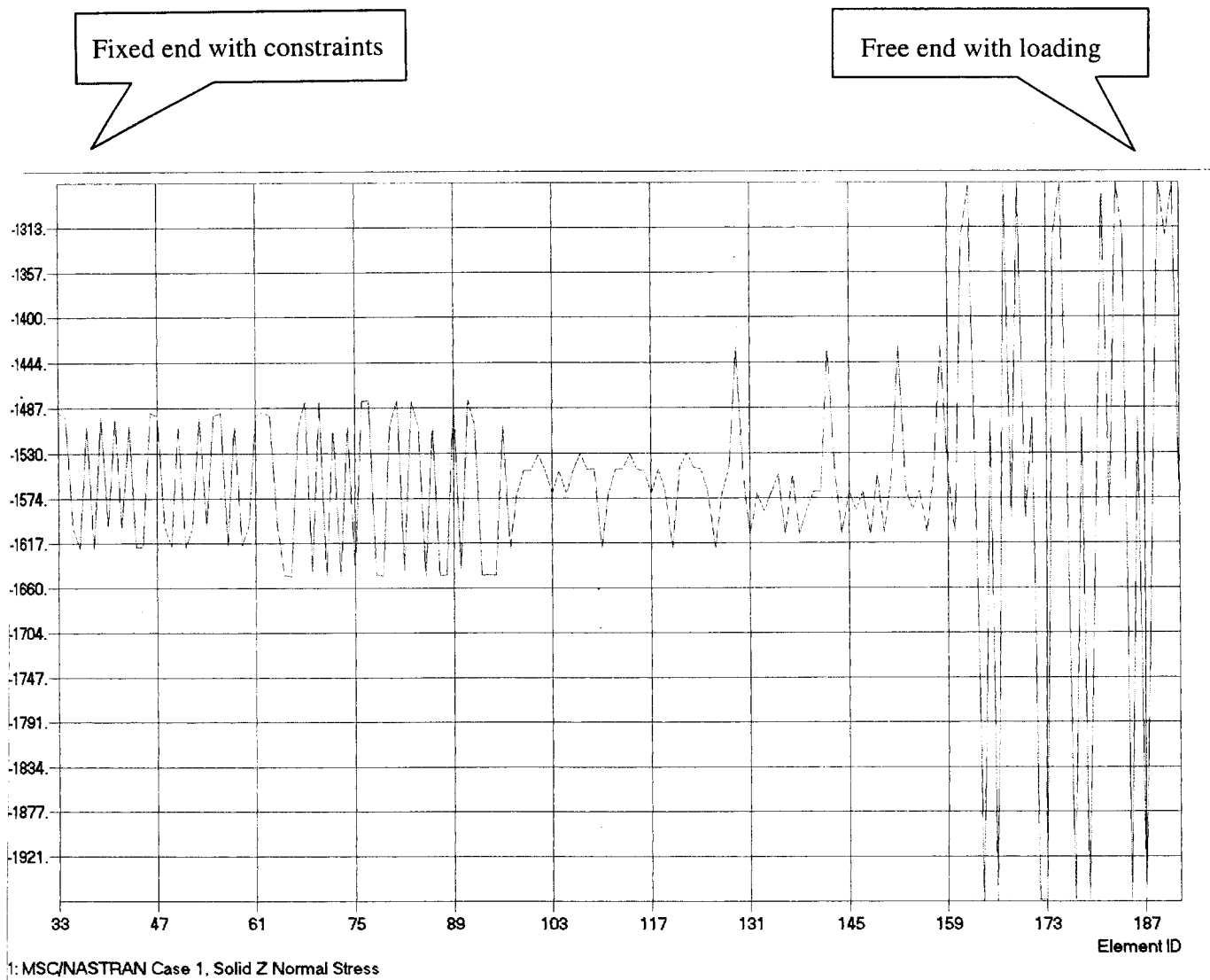
Figure 6.5 : Solid Z Normal Stress in Z axis

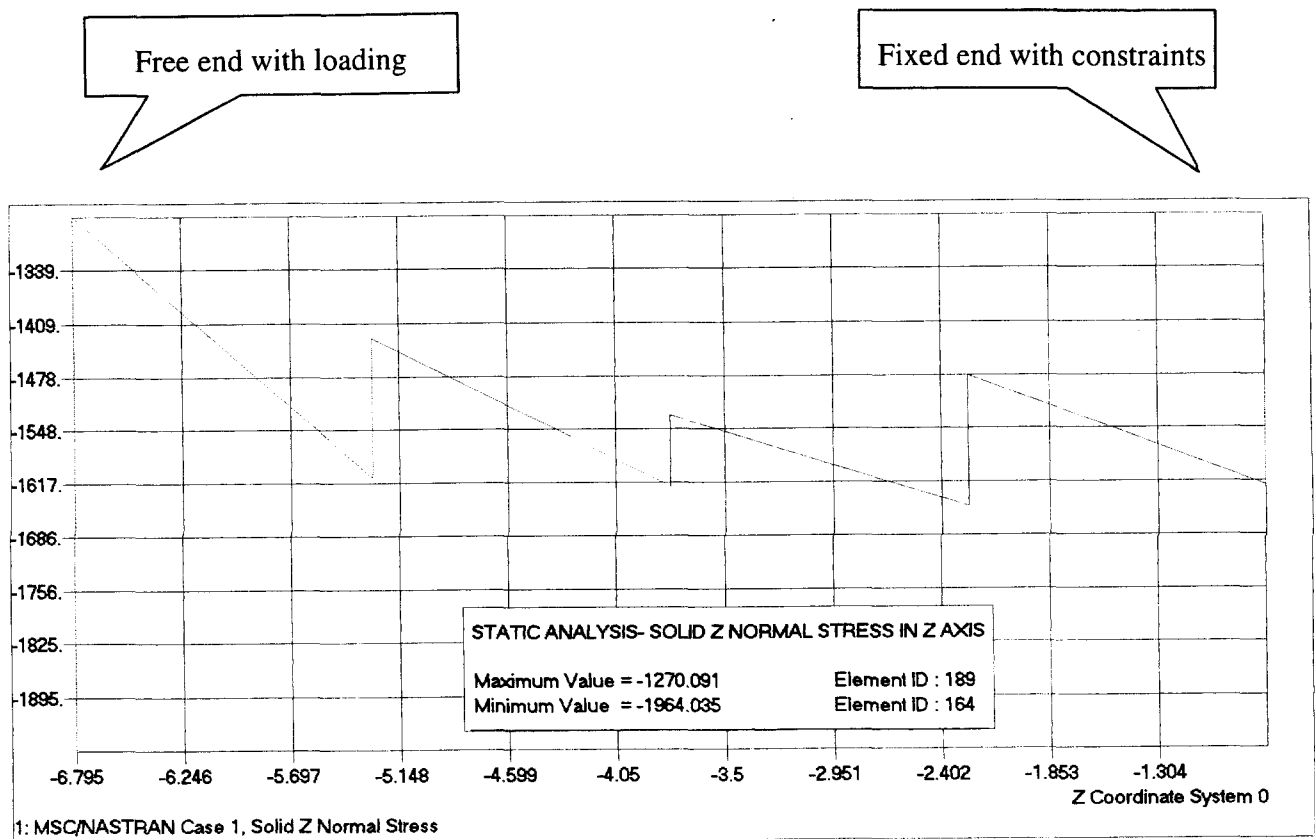**Figure 6.4 : Solid Z Normal Stress vs Element ID**

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Figure 6.5 : Solid Z Normal Stress in Z axis

It is clearly evident that the maximum compressive stress occurs at element layer no.5 which has the free nodes that experience the highest punch reaction force of 2,535 N per node. The compressive stress values drops gradually as it moves upwards towards element layer no.1 .

Compressive yield strength S_{yc} for XW-42 tool steel is - 3 x 10^9 N/m^2. As such, this design is very reliable because the maximum principal stress in the draw punch is only 0.45 % of the XW-42 compressive yield strength.

. Based on this failure *Maximum Normal Stress Theory*, the draw punch will not fail under static loading during the drawing action.

Maximum Shear Stress Theory

This theory is used to predict yielding. The maximum shear stress theory states that yielding begins whenever the maximum shear stress in any mechanical element becomes equal to the maximum shear stress in a tension test specimen of the same material when that specimen begins to yield [9].

This theory predicts that failure will occur whenever the following condition is attained :

$$\tau_{max} = S_y / 2 \tag{6.5}$$

In this Static Analysis, $S_{yc} = -3 \times 10^9$ N/m^2. Therefore, $S_{yc}/2 = 1.5 \times 10^9$ N/m^2.

From FEM analysis, Maximum Shear Stress is 997.8317 psi or an equivalent of 6.88 x 10^6 N/m^2 in element ID 164. Please refer to Figure 6.6 : Solid Maximum Shear Stress vs Element ID
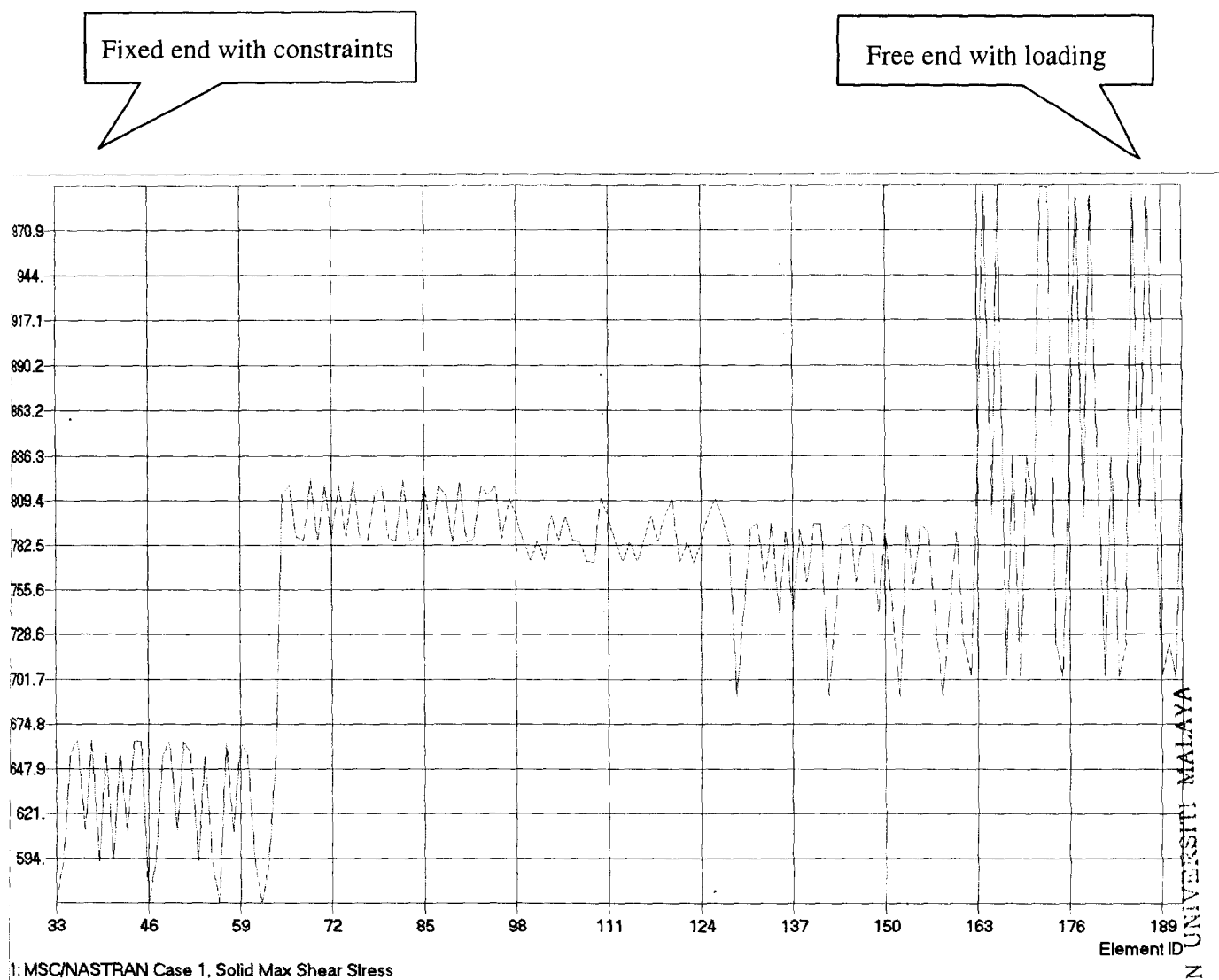
Figure 6.6 : Solid Maximum Shear Stress vs Element ID

It is evident that the maximum shear stress occurs at element layer no.5 which has the free nodes that experience the highest punch reaction force of 2,535 N per node. The shear stress values drops gradually as it moves upwards towards element layer no.1 where all the nodes are fixed to the top die plate.

Maximum shear stress is only 0.46 % of the XW-42 tool steel compressive yield strength. Based on this *Maximum Shear Stress Theory*, this punch draw design is reliable and will not fail under static loading during the deep drawing action. These results are consistent with the results of *Maximum Normal Stress Theory*.

Distortion Energy Theory

This failure theory is also called the shear-energy theory and the Von Mises-Hencky theory. It is the best theory to use for ductile materials. Like the maximum shear stress theory, it is employed to define only the beginning of yield [9].

The distortion -energy theory originated because of the observation that ductile materials stressed hydrostatically(equal tension and compression) and had yield strengths greatly in excess of the values given by the simple tension test. It was agreed that yielding was not a simple tensile or compressive phenomenon at all but rather , that it was related to the angular distortion of the stressed element.

The equation for Von Mises stress (σ') in a triaxial state is as stated below [9] :

$$\sigma' = \sqrt{[(\ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\ )\ /\ 2]} \qquad (6.6)$$

In this Static Analysis,　$\sigma_1 = 132.4279$ psi / 913,064 N/m^2

$$\sigma_2 = 132.4068 \text{ psi} / 912,918 \text{ N/m}^2$$

$$\sigma_3 = -1964.035 \text{ psi} / -13,541,629 \text{ N/m}^2$$

There fore Von Mises stress is calculated as shown below :

$$\sigma' = \sqrt{[((\ 913,064 - 912,918)^2 + (912,918 + 13.54 \times 10^6)^2 + (-13.54 \times 10^6 - 913,064)^2)\ /\ 2]}$$

$$= 14.45 \times 10^6 \text{ N/m}^2$$

From FEM analysis, the Von Mises Stress is 1953.093 psi or equivalent to 13.46 x 10^6 N/m^2. Please refer to following graphs :

a) Figure 6.7　:　Solid Von Mises Stress in Z axis
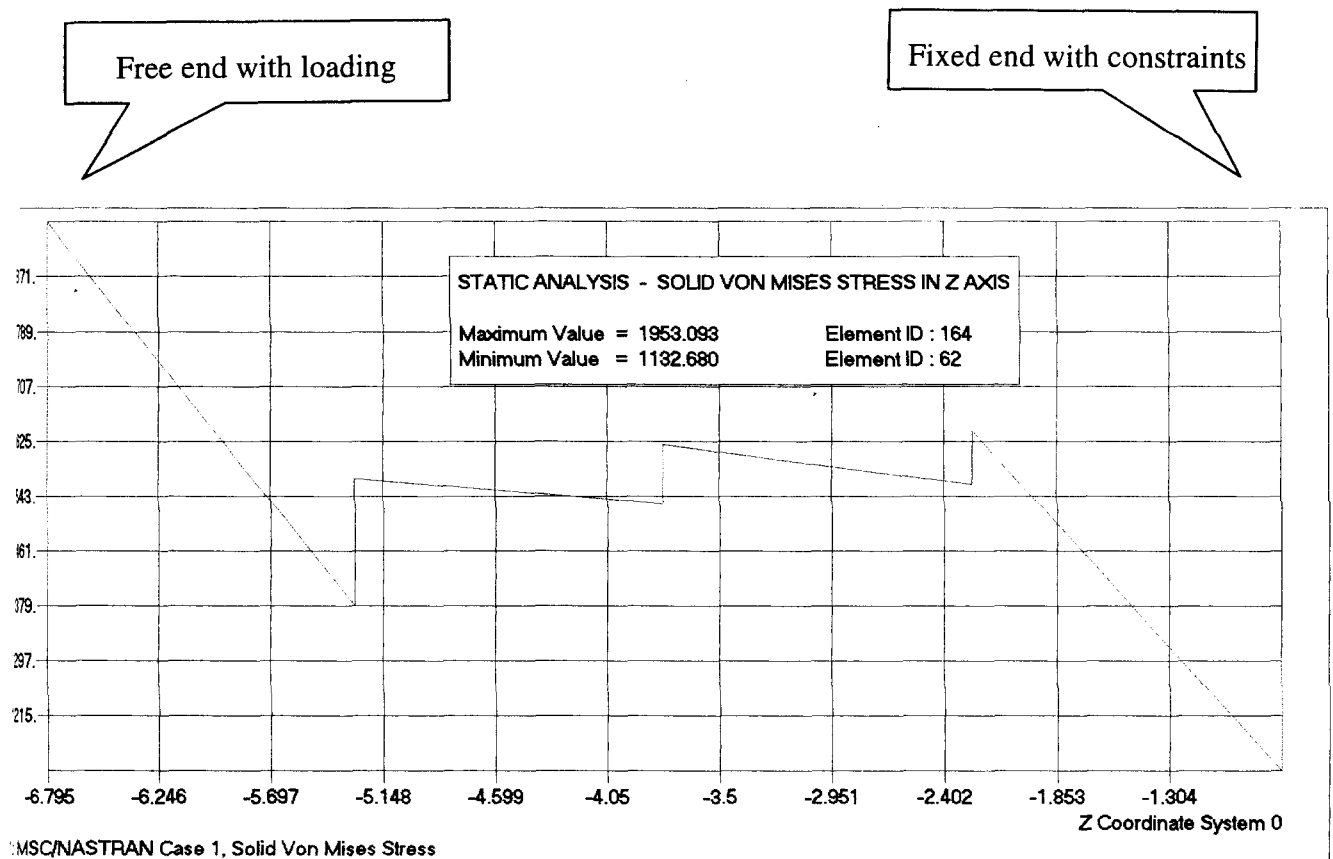
b) Figure 6.8　:　Solid Von Mises Stress vs Element ID

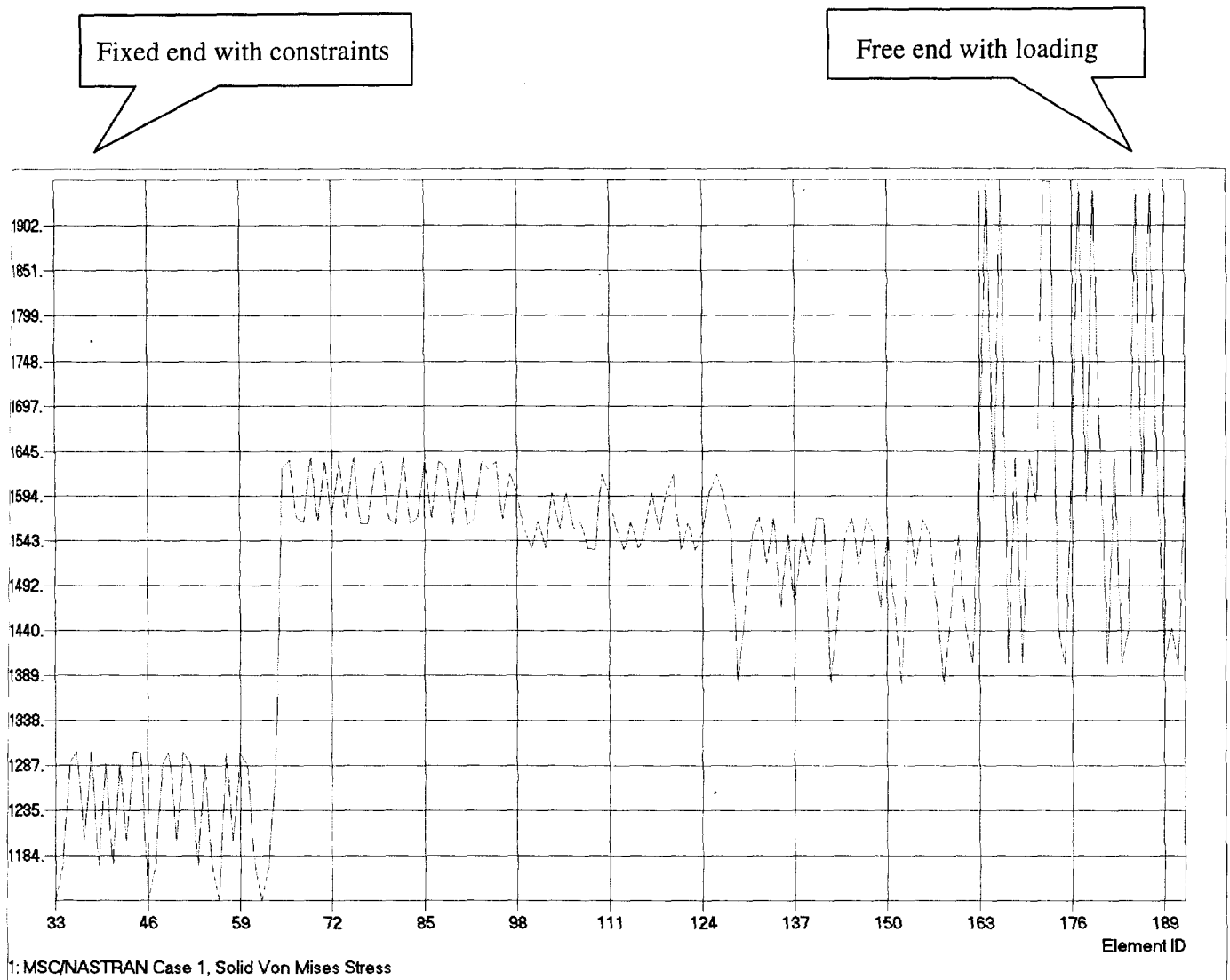**Figure 6.7 : Solid Von Mises Stress in Z axis**

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Figure 6.8 : Solid Von Mises Stress vs Element ID

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It is evident that the maximum Von Mises stress occurs at element layer no.5 which has the free nodes that experience the highest punch reaction force of 2,535 N per node. The Von Mises stress values drops gradually as it moves upwards towards element layer no.1 where all the nodes are fixed to the top die plate.

The maximum allowable stress under this failure theory is as stated below [5]:

$$S_{sy} = 0.577\, S_y = 1.73 \times 10^9\, N/m^2 \qquad\qquad (6.7)$$

where S_{sy} is the yield strength in shear and S_y is the compressive yield strength

The Von Mises stress is only 0.78 % of the maximum allowable stress. As such, based on this *Distortion Energy Theory*, this punch draw design is reliable and will not fail under static loading during the drawing action.

6.1.3 Error

In the static analysis , the theoretical deformation is 0.0101 mm as compared to the FEM value of 0.0111 mm. The difference of 0.001 mm is termed as error and it consitutes an error of 9.9 % of the calculated value.

I t should be apparent that discretization involves approximation. Consequently, what we obtain is not the exact solution but an approximation to the solution. The process of discretization is essentially an exercise of engineering judgement. The shapes, sizes, numbers and configuration of elements have to be chosen carefully such that the original body or domain is simulated as closely as possible.

The size of the elements influences the convergence of the solution directly and hence, it has to be chosen with care. If the size of the elements is small, the final solution is expected to be more accurate. However, the use of smaller elements will require longer computational time.

Another characteristic related to the size of elements is the aspect ratio of the elements. Aspect ratio describes the shape of the element in the assemblage of elements. Elements with an aspect ratio of nearly unity generally yield the best results.

The number of elements is another key factor in determining the accuracy of the FEM solution. Although an increase in the number of elements generally means a more accurate set of results, there will be a certain of number of elements beyond which the accuracy cannot be improved significantly.

Static Analysis Summary

Items	Descriptions	Max. Displacement
1	Deformation : Theory	0.00039 in
2	Deformation : From FEM Analysis	0.00044 in (ID212)

Items	Failure Theory	Max. Stress in psi	Max. Allowable Stress(psi)	Results
1	Maximum Normal Stress	ID 189 : 1964(-)	436,800	No static failure
2	Maximum Shear Stress	ID 164 : 998	218,000	No static failure
3	Distortion Energy Theory	ID 164 : 1953	252,033	No static failure

6.2 Dynamic Analysis

6.2.1 Fatigue Failure

In static analysis, we were concerned with the design and analysis of parts subjected to static loads. It is an entirely different matter when parts are subjected to time varying or non static loads.

In obtaining properties of materials relating to the stress and strain diagram, the load is applied gradually, giving sufficient time for the strain to develop. With the usual conditions, the specimen is tested to destruction while the stresses are applied only once. This condition is known as static condition.

The condition usually encountered in the stamping/presswork industry is a varying or fluctuations between the load values. These kinds of loads occurring in machine component or tools produce stresses which are called repeated, alternating or fluctuation of stresses.

Machine parts and tools are often found to have failed under the actions of repeated or fluctuating stresses and yet the most careful analysis reveals that the actual maximum stress were below the ultimate strength of the material and quite frequently even below the yield strength. The most distinguishing characteristic of these failures has been that the stresses have been repeated for a very large number of times. Hence, this failure is called a fatigue failure [9].

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A fatigue failure begins with a small crack. The initial crack is so minute that it cannot be detected by naked eyes or even x-ray inspection. This crack will develop at a point of discontinuity in the material such as a change in cross section, a keyway or a hole. Less obvious points at which fatigue failures are likely to begin are internal cracks or irregularities caused by machining.

A fatigue failure is characterized by two distinct areas of failure. The first of these is that due to the progressive development of the crack and the second is due to sudden fracture.

When a machine or tool component fails statically, they usually develop a very large deflection because the stress has exceeded the yield strength and the part is replaced before fracture really occurs. But a fatigue failure gives no warning, it is sudden, total and hence very dangerous.

However the fatigue phenomenon is complicated and only partially understood. Normally, the safety factors associated with the current machine components or tool designs are high in order to safeguard these parts from fatigue failures but such designs will not be able to compete in today's global market.

S-N Diagram

To determine the strength of materials under the action of fatigue loads, specimens are subjected to repeated or varying forces of specified magnitude while the cycles or stress reversals are counted to destruction. The most widely used fatigue testing device is the R.R Moore high speed rotating beam machine.

To establish the fatigue strength of a material, quite a number of tests are necessary because of the statistical nature of fatigue. All the results of these tests are plotted in the S-N diagram as shown in Figure 6.9.
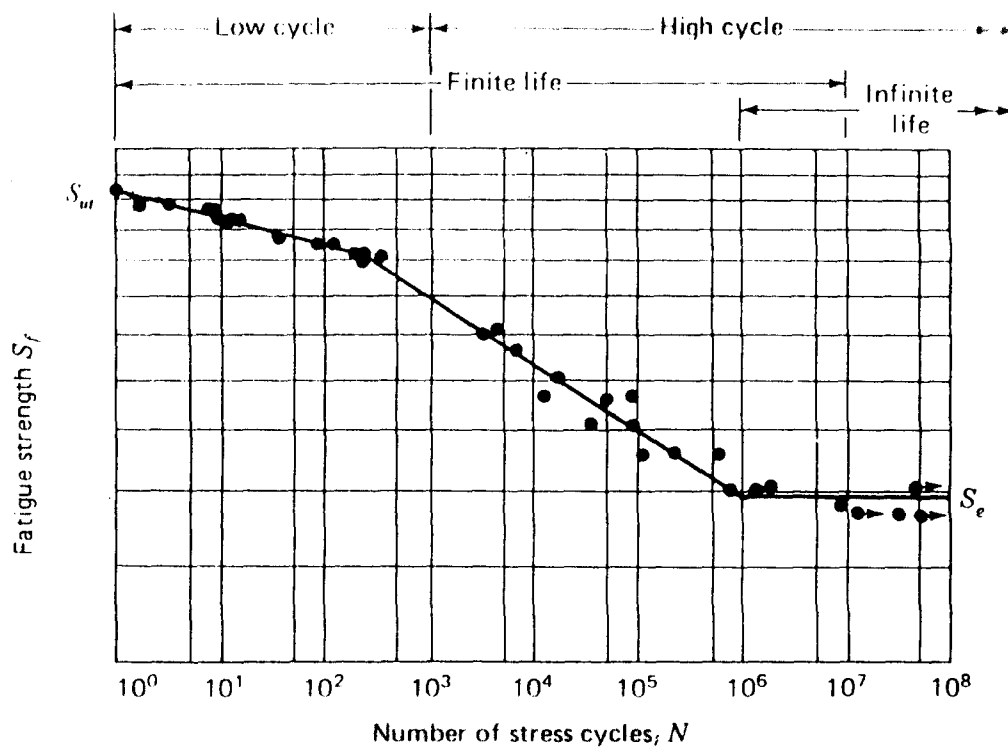

Figure 6.9 : S-N Diagram

In the case of steels, a knee occurs and beyond this knee fatigue failure will not occur. The fatigue strength corresponding to this knee is called the endurance limit S_e or the fatigue limit. From Figure 6.9 , a finite life and infinite life region can be established and generally for all steels, it lies between 10^6 and 10^7 cycles [9].

Axial Loading

A very extensive collection of data has been made by R.W Landgraf on axial fatigue. The outcome of these tests and data verification is the relation stated below [9] :

$$S'_e = [\ 0.566 - 9.68\ (10^{-5})\ S_{uc}\ ]\ S_{uc} \quad \text{for } S_{uc} > 400\,\text{MPa} \quad \text{·} \qquad \text{(6.8)}$$

where S_{uc} = ultimate compressive strength

$\quad\quad S'_e$ = axial endurance limit

For this dynamic analysis on draw punch, the ultimate compressive strength of the material XW-42 is 2950 MPa. Therefore the axial endurance limit S'_e for this draw punch is as calculated below :

$$S'_e = [\ 0.566 - 9.68\ (10^{-5})\ 3 \times 10^9]\ 3 \times 10^9$$

$$= 827.298\ \text{MPa} \quad \text{or} \quad 0.827 \times 10^9\,\text{N/m}^2$$

Endurance Limit Modifying Factors

A variety of endurance limit modifying factors are used to provide a more realistic endurance limit as compared to the values obtained during testings in laboratory [9]. However, in this study these factors have been omitted.

High Cycle Fatigue

High cycle fatigue is the region beyond the N= 10^3 cycles of strokes. A study of large quantities of data on fatigue strength of steels at 1000 cycles indicates that the mean S'_f= 0.80 S_u. The data also indicates that the dispersion of the strengths is constant over the life range. It is therefore recommended that a line on the log S- log N chart joining 0.8 S_{ut} at 10^3 cycles and S'_e at 10^6 cycles be used to define the mean fatigue strength S'_f corresponding to any life N between 10^3 and 10^6 cycles [9].

The equation of the S-N line is [9] :

$$\log S'f = b \log N + c \tag{6.9}$$

This line must intersect 10^6 cycles at S'_e and 10^3 cycles at 0.8 Sut. When these are substituted into the above equation, the resulting equations can be solved for b and c.

$$b = -1/3 \, \log(0.8 \, S_{ut}/S'_e) = -0.1517$$

$$c = \log[(0.8 \, S_{ut})^2/S'_e] = 3.8282$$

Now having solved the equations for the two constants, we can find S'f when N is given or vice versa by using the equation stated below :

$$S'_f = 10^c \, N^b \tag{6.10}$$

Using endurance limit S'_e as the fatigue limit S'_f in Eq 5.10 , we can now determine the allowable number of N cycles :

$$N = 994,676 \text{ cycles}$$

A stress cycle (N=1) constitutes a single application and removal of a load and then another application and removal in the opposite direction. Thus N=0.5 means that the load is applied once and removed. As such, this draw punch will not experience fatigue failure for 1,989,352 strokes in the actual presswork environment.

6.2.2 Free Vibration Analysis

If we disturb an component in a appropriate manner initially at time t=0 ie. by imposing properly selected initial displacements and then releasing these constraints, the component can be made to oscillate harmonically. This oscillatory motion is a characteristic property of the component and it depends on the distribution of mass and stiffness in the component. If the damping is absent, the oscillatory motion will continue indefinitely with the amplitudes of oscillations depending on the initially imposed disturbances or displacements. This oscillatory motion occurs at certain frequencies known as natural frequencies and it follows well defined deformation patterns known as mode shapes. The study of such free vibrations is very important in determining the dynamic responses of a component under dynamic loads [12].

6.2.3 Response To Harmonic Loading

Complementary Solution

It will now be assumed that a component subjected to a harmonically varying load p(t) of amplitude p_o and circular frequency w. In this case, the differential equation of motion becomes [11] :

$$m \, d^2v/dt^2 + c \, dv/dt + kv = p_o \sin wt \qquad (6.11)$$

The equation of motion of an undamped system to harmonic loading becomes

$$m \, d^2v/dt^2 + kv = p_o \sin wt \qquad (6.12)$$

The complementary solution of this equation is the free vibration response.

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Particular Solution

The particular solution is the specific behavior generated by the form of the dynamic loading. The response to the harmonic loading can be assumed to be harmonic and in phase with the loading. The amplitude of the responses can be determined if the natural free vibration frequency is known [11].

General Solution

The general solution to the harmonic excitation of the undamped system is then given by the combination of the complementary solution and particular solution [11].

6.2.4 Pro Mechanica Time Response Analysis

Pro Mechanica was used to create and mesh the punch model for this dynamic time response analysis. An isometric view of this model which has been loaded and constrained is shown in Figure 6.10.

The summary of the Von Mises stress is shown in Figure 6.11 and the Von Mises stress with respect to time in graph forms are as shown in Figure 6.12 . The draw punch is divided into 8 regions A,B,C, D,E,F,G and H to identify the stresses selectively and are as shown in Appendix B. The total time is 50,000,000 seconds which approximately represents 578 days of drawing operation.
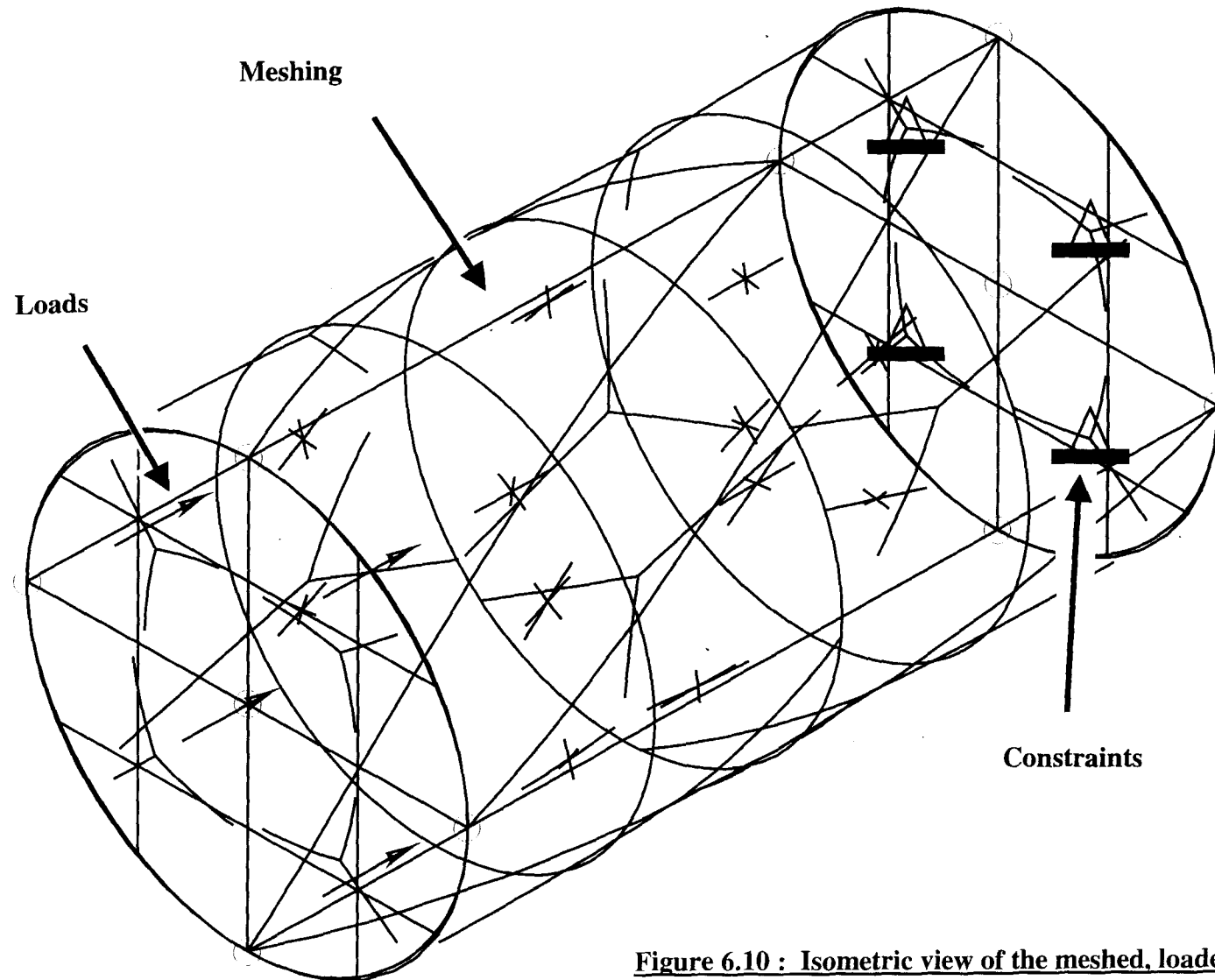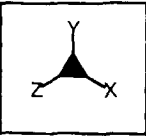
Meshing

Loads

Constraints

Figure 6.10 : Isometric view of the meshed, loaded and constrained draw punch.

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TIME RESPONSE : VON MISES STRESS

Time Steps	A	B	C	D	E	F	G	H	Maximum
Step 1	62060000	54430000	46810000	39180000	31560000	23930000	16310000	8680000	69685000
Step 2	51780000	45420000	39060000	32700000	26330000	19970000	13610000	7245000	58147000
Step 3	15420000	13520000	11630000	9733000	7839000	5945000	4051000	2157000	17309000
Step 4	26930000	23620000	20310000	17000000	13690000	10390000	7076000	3768000	30239000
Step 5	63020000	55270000	47530000	39790000	32050000	24300000	16560000	8816000	70761000
Step 6	30200000	26490000	22780000	19070000	15360000	11650000	7935000	4225000	33907000
Step 7	11950000	10480000	9013000	7545000	6077000	4608000	3140000	1672000	13418000
Step 8	70450000	61800000	53140000	44480000	35830000	27170000	18510000	9856000	79109000
Step 9	43730000	38360000	32980000	27610000	22240000	16860000	11490000	6118000	49104000
Step 10	3512000	3080000	2649000	2217000	1786000	1354000	922800	491300	3943100
Step 11	74990000	65780000	56560000	47350000	38130000	28920000	19710000	10490000	84205000
Step 12	55450000	48670000	41850000	35030000	28210000	21400000	14580000	7762000	62301000

Note : Please refer to Appendix B
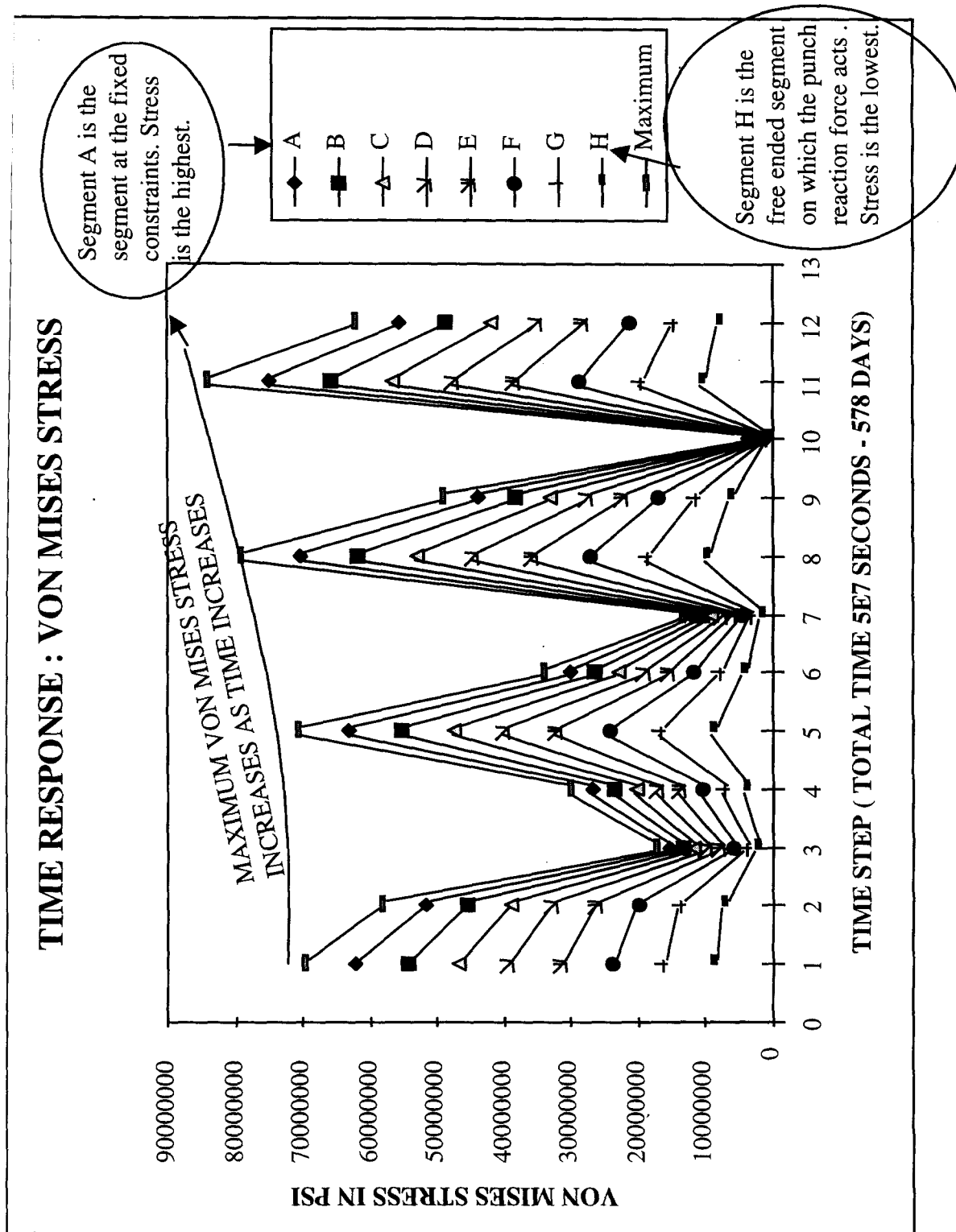
Figure 6.11 : Von Mises Stress

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Figure 6.12 : Von Mises Stress vs Time Graph

The Von Mises stress which is a combination stress will be used to determine whether this draw punch will experience a fatigue failure. It has been determined earlier that the endurance limit S'_e for this punch is 827.298 MPa which is equivalent to 119,958 psi. Based on the Von Mises stress distribution as indicated in Figure 6.11, it is clear that this punch does not have a infinite life as the global maximum Von Mises stress in each of the eight segments exceeds the endurance limit S'_e. The draw punch will experience a fatigue failure after 1,989,352 strokes.