A STUDY ON CHANNEL CODING FOR 5G

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FACULTY OF ENGINEERING
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ABSTRACT

Due to the interference and fading of mobile communication, errors will occur during the transmission. Therefore, Correction and error detection are very important processes in digital communication systems. They are used to enhance the anti-inference ability of data and improve system reliability. Channel coding is the process of error detection and correction. At present, 3GPP has standardized specifications for channel coding in the 5G NR eMBB scenario, LDPC code is adopted as a channel coding method for data information, and Polar code is adopted for control information encoding. This paper focused on Polar code, specifically described the development and principles of Polar code, and studied Polar code schemes for 5G eMBB specification. In addition, the performances of Polar code under different decoding algorithms and different construction method are simulated and compared. The simulation results show that adding the outer code CRC allows the Polar code to perform error detection during the decoding process and optimize the performance of the polar code. In addition, the SCL decoding algorithm can improve the decoding performance by increasing L. But with the increase of L, the performance improvement becomes smaller and the decoding complexity will be increased. Polar codes generated by different coding construction all can achieve good performance results if combined with the design-SNR construction method. There is no significant performance difference between different coding construction.

ABSTRAK

Oleh kerana gangguan dan pemudaran komunikasi mudah alih, kesilapan akan berlaku semasa penghantaran. Oleh itu, pembetulan dan pengesanan ralat adalah proses yang sangat penting dalam sistem komunikasi digital. Ianya digunakan untuk meningkatkan kebolehan anti inferens data dan mempertingkatkan kebolehpercayaan sistem. Saluran coding merupakan proses mengesan ralat dan pembetulan. Currently, 3GPP has a specification standard for encoding channels in the eMBB NR 5G scenario, LDPC digunakan untuk pengekodan maklumat data, dan Polar kod digunakan untuk pengekodan maklumat kawalan. Kertas ini memberi tumpuan kepada Kod Polar, khusus menyifatkan pembangunan dan prinsip-prinsip Kod Polar dan belajar Polar kod skim untuk spesifikasi eMBB 5G. Di samping itu, prestasi Polar kod di bawah algoritma penyahkodan berbeza dan kaedah pembinaan yang berbeza simulasi dan dibandingkan. Keputusan simulasi menunjukkan bahawa menambah kod luar CRC membolehkan kod Polar untuk melakukan pengesanan ralat semasa menyahkod, seterusnya meningkatkan lagi prestasi. Di samping itu, algoritma penyahkodan SCL yang boleh meningkatkan prestasi penyahkodan dengan meningkatkan L. Tetapi dengan peningkatan l, peningkatan prestasi menjadi lebih kecil dan kerumitan penyahkodan akan dinaikkan. Kod kutub yang dihasilkan oleh pembinaan pengkodan yang berbeza semua boleh mencapai prestasi yang baik jika digabungkan dengan kaedah pembinaan Reka bentuk-SNR. Terdapat tiada perbezaan ketara antara pembinaan pengkodan yang berbeza.

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LIST OF SYMBOLS AND ABBREVIATIONS

1G	:	First generation
2G	:	Second generation
3G	:	Third generation
3GPP	:	The 3rd Generation Partnership Project
4G	:	Fourth generation
5G	:	Fifth generation
ACK	:	Acknowledge Character
AWGN	:	Additive White Gaussian Noise
B-DMC	:	Binary Discrete Memoryless Channel
BEC	:	Binary Erasure Channel
BER	:	Bit Error Rate
BLER	:	Block error rate
BP	:	Belief Propagation
BPSK	:	Binary Phase Shift Keying
CRC	:	Cyclic Redundancy Check
DCI	:	Downlink control information
DE	:	Density Evolution
eMBB	:	Enhanced-mobile broadband
FDMA	:	Frequency Division Multiple Access
FER	:	Frame Error Rate
GA	:	Gaussian Approximation
GSM	:	Global System for Mobile communications
LDPC	:	Low Density Parity Check code
LLR	:	Log Likelihood Ratio

LP	:	Linear Programming
ML	:	Maximum Likelihood
mMTC	:	Massive Machine Type Communications
NACK	:	Negative Acknowledgment
NR	:	New Radio
OFDM	:	Orthogonal Frequency Division Multiplexing
OFDMA	:	Orthogonal Frequency Division Multiple Access
PC	:	Parity Check
PCCC	:	Parallel Concatenated Convolutional Code
QPSK	:	Quadrature Phase Shift Keying
RM	:	Reed-Muller code
RSC	:	Recursive System Convolutional code
SC	:	Successive Cancellation
SCL	:	Successive-Cancellation List
SCS	:	Successive-Cancellation Stack
SNR	:	Signal to Noise Ratio
UCI	:	Uplink control information
UFMC	:	Universal filter multi carrier
URLLC	:	Ultra-Reliable Low Latency Communications

CHAPTER 1: INTRODUCTION

1.1 Evolution of mobile communications

With the rapid development of science and economy, communication technology becomes more and more important in all fields of society(Liang et al., 2016). Mobile communication is one of the most important branches of communications. The mobile communication system has undergone four generations of evolution since it came out, and the fifth-generation communication system is about to come out. The first generation of mobile communication systems (1G) was born in 1978, using Frequency Division Multiple Access (FDMA) technology and analog modulation, mainly supporting voice services. Since the analog signal is not compressed and there is no error correction, the first-generation mobile communication system has low resource utilization, small capacity and poor communication quality.

From the second-generation mobile communication system (2G), it adopts digital modulation, and the most widely used 2G system is the Global System of Mobile Communication (GSM). In GSM, multiple access techniques based on time division multiple access are used, and digital signals are channel coded for error reduction. Although these techniques improve transmission efficiency and system capacity, the channel coding mainly used in GSM is the block code and convolutional code, the performance is not good. Another reason for the increase in 3G capacity is the use of Turbo codes. The main technologies of the fourth-generation mobile communication system (4G) are orthogonal frequency division multiplexing (OFDM) and orthogonal frequency division multiple access (OFDMA), and it continues to use the channel coding technology used in 3G-Turbo codes. The upcoming fifth-generation communication system will have a wider application environment, which is mainly divided into three major scenarios: Enhanced Mobile Broadband (eMBB), Ultra-Reliable Low Latency Communications (URLLC) and Massive Machine Type Communications (mMTC). This will mean that 5G will have higher requirements for communication capacity, delay, coverage, reliability and efficiency.

The aim of the communication process is to transmit information from the source to the receiver's sink efficiently, reliably, and securely. Code validity means that transmit more information bit with as few bits as possible. The main methods of validity coding are source coding and data compression. Reliability coding mainly refers to channel coding. The channel coding transforms the digital information bit sequence to be transmitted into a code sequence containing more information bits (by adding redundant bits), and the encoded sequence is modulated into the wireless channel, and the sink can detect or correct errors that occurred during transmission based on the mapping rule, thus, the information can be received correctly.

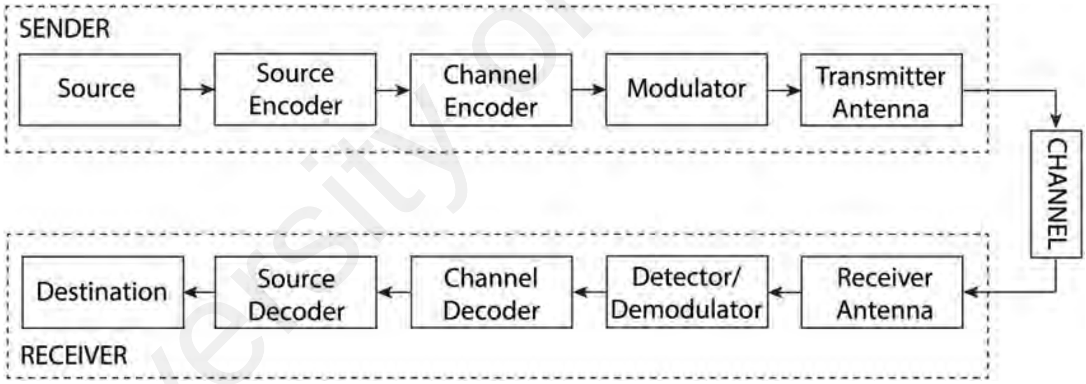

Figure 1.1 Block diagram for a digital communication system.

1.2 Problem statement

With the efforts of many scholars, the research on polar codes has achieved many achievements in recent years. Polar code with medium or short length codes will be mainly used in practical engineering applications in the future. In particular, the length of information transmitted by the control channel is generally short, but the minimum Hamming distance of Polar code is relatively small, the channels cannot be fully polarized in the case of a short code length, which will seriously affect the decoding performance.

In this case, it is necessary to select the channels with good reliability to transmit the information bits. Channel selection is critical to polar code. Therefore, many works of literature have published in which different construction methods of polar code have been proposed but it still needs to be further improved in performance. The performance of polar code can be improved by adding outer codes, but this also increases the complexity. Since the polar code has not been formally applied to the communication system, the current research is mainly theoretical research and simulation, and engineering implementation will be the focus of polar code research in the future.

1.3 Objectives

The objective of this study is to further understand coding theory and study the new potential 5G channel coding schemes (Turbo code, LDPC code and polar code). They have their own advantages and disadvantages based on different requirements, such as LDPC codes have the advantage of performance in the high-speed transmission scenarios; the design simplicity and the good performance of Turbo code. Although Polar code started late, it has a good theoretical basis. It has attained great attention in the coding field since it was proposed.

We focus on Polar code in this paper. We set the objective of this study as

1. To analyze the BLER performance of Polar code using in 5G-New Radio (NR) scenario.
2. To perform simulation on Polar code using different construction and analyze the BER performance.
3. To compare and analyze the FER and BER performance of Polar code using different decoding algorithm.

1.4 Organization

This report is divided into 5 chapters. This current chapter is to briefly introduce the evolution of mobile communications and the next generation of the mobile network. In chapter 2, we will go through the most potential channel coding schemes nowadays and the theory of polar code. Chapter 3 is the research methodology. This chapter explains the software being used and the implementation of the polar code, including an introduction for different decoding algorithm and different polar code construction. Chapter 4 presents the results collected and the observation. Conclusion and some possible future work are presented in chapter 5.

CHAPTER 2: LITERATURE REVIEW

This section will mainly introduce several coding schemes currently used: Low Density Check Code (LDPC), Polar Code and Turbo Code. The Polar code will be highlighted, including its origin, principle and encode and decode method.

2.1 Turbo Code

The appearance of Turbo codes is a revolution in the field of coding. It was the first time that coding performance was so close to the Shannon limit. Thanks to its simple coding structure, the Turbo code is widely used in 3G and 4G communication systems.

The Turbo code is also called Parallel Concatenated Convolutional Code (PCCC)(Ten Brink, 2000). The classic turbo code encoder is cascaded in parallel by two component encoders. They are Recursive system convolutional code (RSC) encoders, which are connected by an inner inter-leaver. The two sub-encoders are convolutional codes, and their polynomials are the same.

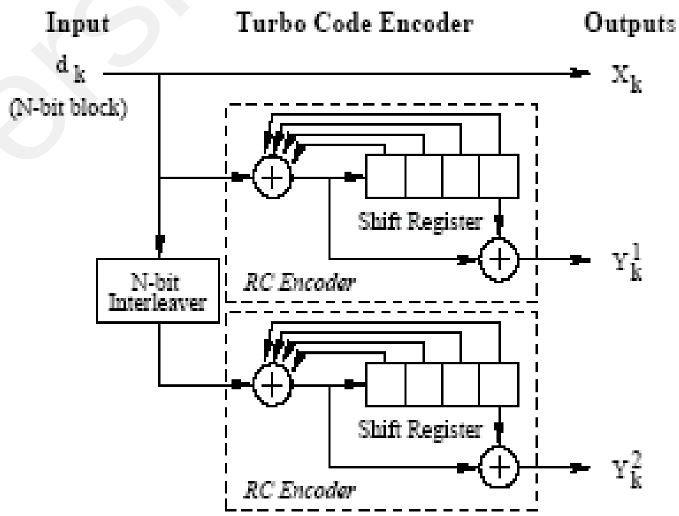

Figure 2.1: Turbo Encoder for Code Rate =1/3.

2.2 LDPC code

LDPC codes are used currently in systems such as 4G WiMax, Wi-Fi, and broadcasting. LDPC codes can be decoded fast because the decoder structure can be processed in parallel and decoding performance approaches Shannon limit. Therefore, it is the preferred solution in a high data rate and large-capacity communication systems.

LDPC code is a linear block code with a sparse check matrix, Gallager invented in 1963. However, due to the limitation of the computational capabilities at that time, it has been neglected for 30 years, D.Mackay and M.Neal re-discovered and studied LDPC code, LDPC code has gradually become widely applied. The irregular LDPC code proposed by Mckay and Luby, whose performance is not only better than the regular LDPC code but also better than that of the Turbo code, is the code closest to the Shannon limit in the current known code. (Ryan & Lin, 2009)

2.3 Polar code

The concept of channel polarization was first proposed in 2008 at IEEE ISIT (International Symposium on Information Theory) conference by Erdal Arıkan (Professor of the University of Birken in Turkey).

In 2009, the article (Arikan, 2009) by Arikan described the polarization phenomenon and proposed a channel coding method that is rigorously proven to achieve the channel capacity, named polar code.

Polarization phenomenon is, after combining and separating N binary input channels W who have the capacity of I(W), if the code length N is large enough, the new channels will present two extremes, some of them will get the channel capacity of 1, while the others will have the channel capacity of 0. The ratio of those channels with capacity 1 to the total number of new channels is exactly the capacity of the symmetric binary discrete memoryless channel (B-DMS) I(W).

In his article, Arikan gives a code scheme that can achieve the channel capacity I(W) of B-DMC channel W. Polar coding uses K=N×I(W) noise-free channels (the channel capacity is 1) to transmit message bits while transmitting frozen bits (Known at transmitting terminal and usually set to be 0) in reminding N-K full-noise channels (the channel capacity is 0). Therefore, code rate R will be R=(N×I(W))/N=I(W), channel capacity can be achieved.

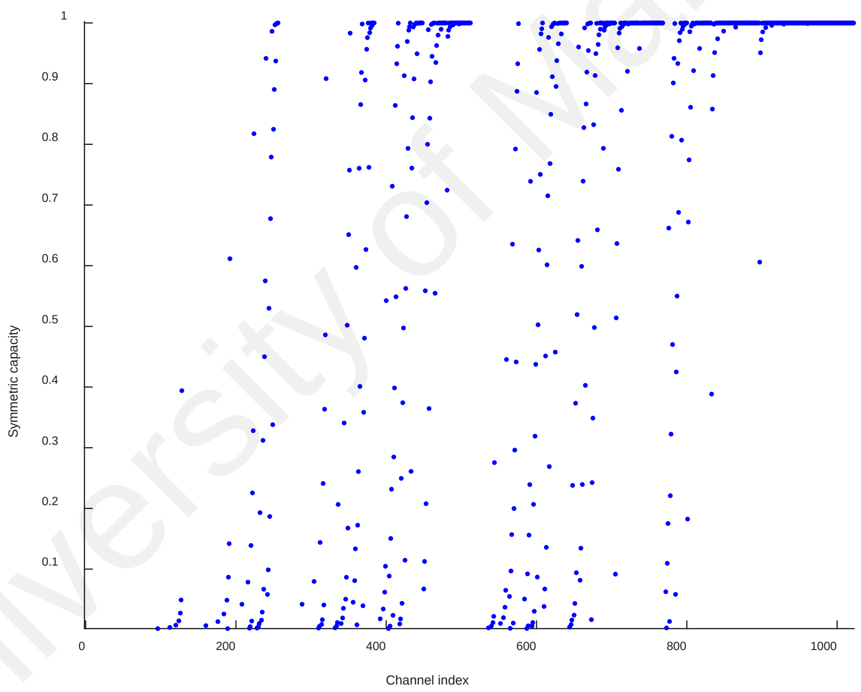

Figure 2.2 Channel polarization

The coding construction for BEC channel is given in Arikan's article, but no effective coding generation method is given for other channels. The literature gives the coding method under any binary memoryless channel (BMS) (Mori & Tanaka, 2009b).

Although polar code can theoretically achieve channel capacity when the code length is infinitely long, the performance is not ideal under the condition of limited code length.

The reason is not only the incomplete polarization but also the characteristic of error propagation using Successive Cancellation (SC) decoding algorithm (A decoding error in a certain information bit will cause an error in the next bit). Therefore, scholars from various countries have tried to apply the decoding algorithm that has performed well in other coding schemes to the decoding of polar code, like Belief Propagation (BP) decoding algorithm, Linear Programming (LP) decoding algorithm, and BCJR algorithm etc. Although these works have achieved performance improvements, they still have limitations in complexity or application.

BP decoding algorithm is proposed in the literature (Arikan, 2008). It showed the decoding principle through the Tanner graph and compared its performance with that of Reed-Muller (RM) code to prove the advantages of the polar code. At the meantime, it shows that the performance of the BP decoding algorithm is significantly improved compared with the SC decoding algorithm(Liang et al., 2016), but the BP decoding algorithm obtains good performance results through iteration, so its complexity is very high(Niu & Chen, 2012b).

(Goela, Korada, & Gastpar, 2010)LP decoding makes the performance of the polar code in the BEC channel become as good as that using BP decoding, and it achieved the channel capacity, but for other channels, performance is not very well. In this article, the reasons for performance degradation on other channels were analyzed.

To further improve performance and reduce complexity, Ido Tal and Alexander Vardy proposed an improved decoding algorithm based on SC decoding algorithm(Tal & Vardy, 2011b). For the conventional SC decoder, as its name, when it decodes a message bit, it will directly make a hard decision based on the corresponding log likelihood ratio (LLR), then uses the decision result to decode the next bit. This will cause the error transmission mentioned above.

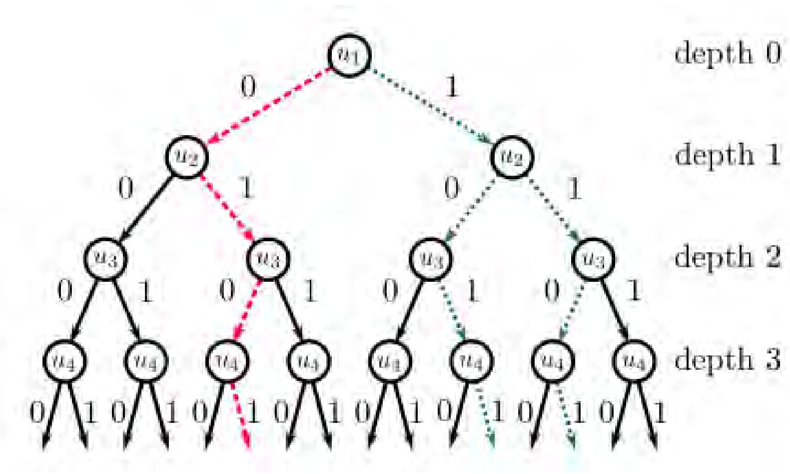

Figure 2.3 Decoding tree for N = 4

However, the improved decoding algorithm they proposed will retain two possible values for the message bit (0 and 1 for binary input), both paths will be stored and calculate the reliability. When the number of reserved paths exceeds a certain threshold value L, the path is trimmed, leaving only L the most reliable paths, then the subsequent bit will be decoded until the N bits are all decoded. This decoding algorithm was named as Successive-Cancellation List (SCL) decoding. Easy to find that it is equivalent to SC decoding when L=1. Since it does not make a hard decision immediately but reserved two possible values, thus reducing the probability of errors and increasing the performance (K Chen, Niu, & Lin, 2012).

Another improved decoding method which is like the SCL decoding scheme is Successive-Cancellation Stack (SCS) decoding (Niu & Chen, 2012b). The SCS algorithm implements storage of a certain number of paths by an ordered stack and searches for the best path in the stack to find the global optimal estimation. It was proved SCS decoding can get the same performance as SCL decoding. SCS decoding reduced the computational complexity (also called time complexity) but sacrificed the space complexity at the same time.

In order to combine the advantages of SCL decoding and SCS decoding, a decoding algorithm named Successive Cancellation Hybrid (SCH) decoding were proposed (Kai Chen, Niu, & Lin, 2013), which realizes the compromise between time complexity and space complexity.

Based on the SCL and SCS algorithms, the principle of CRC-Aided SCL/SCS decoding is that: Cyclic Redundancy Check (CRC), as Figure 2.4 shown, is added to the message bits before coding, then using SCL decoding to obtain L candidate paths and the path selection is performed known that the correct message bits can pass the CRC check. Through this way, better performance than the soft decision of maximum likelihood (ML) decoding with lower complexity can be implemented. (Niu & Chen, 2012a) (Li, Shen, & Tse, 2012)

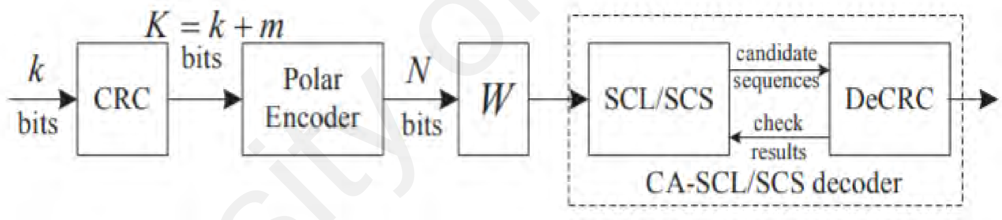

Figure 2.4 Polar code with CRC-aided decoding schemes

2.4 Theory of polar code

2.4.1 Basic knowledge

The B-DMC channel W：X→Y is shown below：

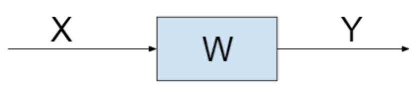

Figure 2.5 B-DMC Channel schematic

where X is a set of input variables: X = {0, 1}, Y is a set of output variables, who can be any value. So, the transition probability is $W(y|x), x \in X, y \in Y$, and can be calculated by

$$C(W) = I(X;Y) = \sum_{y \in Y} \sum_{x \in X} \frac{1}{2} W(y|x) \log \frac{W(y|x)}{\frac{1}{2}W(y|0) + \frac{1}{2}W(y|1)}$$

2.4.2 Channel polarization

Channel polarization is a method that aggregate and redistributes capacity after channel combing and channel splitting, as it is shown in Figure 2.6.

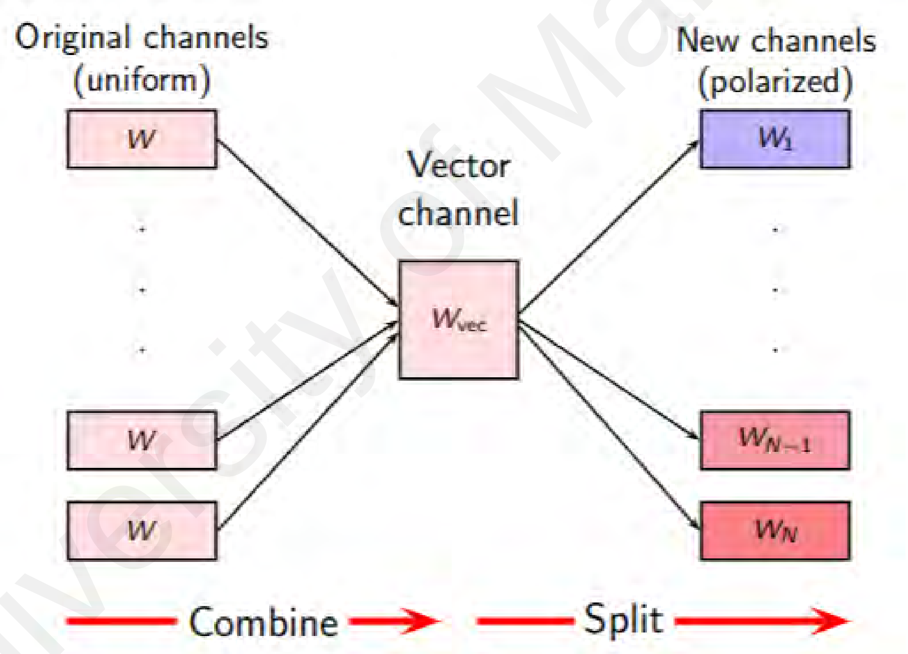

Figure 2.6 Channel combining and splitting

It takes a set of N copies of a known B-DMC channel W, combining in pairs to get W_2, each channel is independent, as shown in Figure 2.7. u_1, u_2 are the message bit that needs to be transmitted, y_1, y_2 are received bits after transmitting through channel W. Then, the independent W_2 are combined to get W_4, as it is shown in Figure 2.8. Until W_N is combined with two independent copies of $W_{N/2}$, as is shown in Figure 2.9.

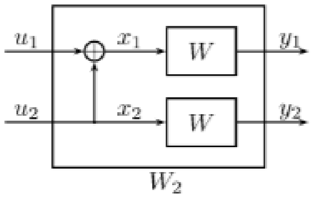

Figure 2.7 Construction of W_2 from two independent copies of $W_1 = W$.

The transition probability of W_2 is

$$W_2(y_1, y_2|u_1, u_2) = W(y_1|u_1 \oplus u_2)W(y_2|u_2)$$

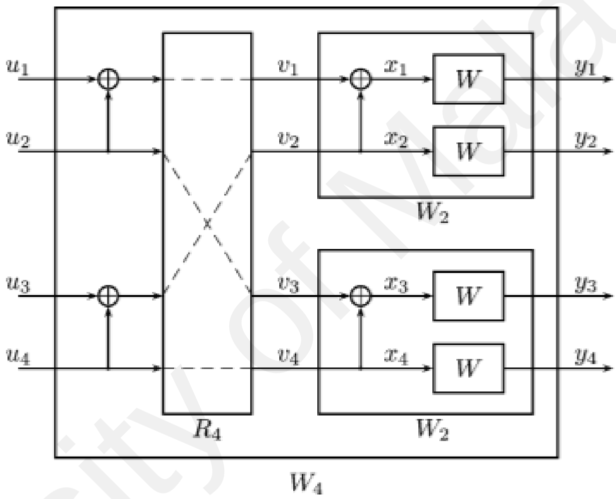

Figure 2.8 Construction of W_4 from two independent copies of W_2

The transition probability of W_4 is

$$W_4(y_1^4|u_1^4) = W_2(y_1^2|u_1 \oplus u_2, u_3 \oplus u_4)W_2(y_3^4|u_2, u_4)$$

R_4 is bit reversing, after which (s_1, s_2, s_3, s_4) becomes $v_1^4 = (s_1, s_3, s_2, s_4)$.

$$x_1^4 = u_1^4 G_4$$

Where $G_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

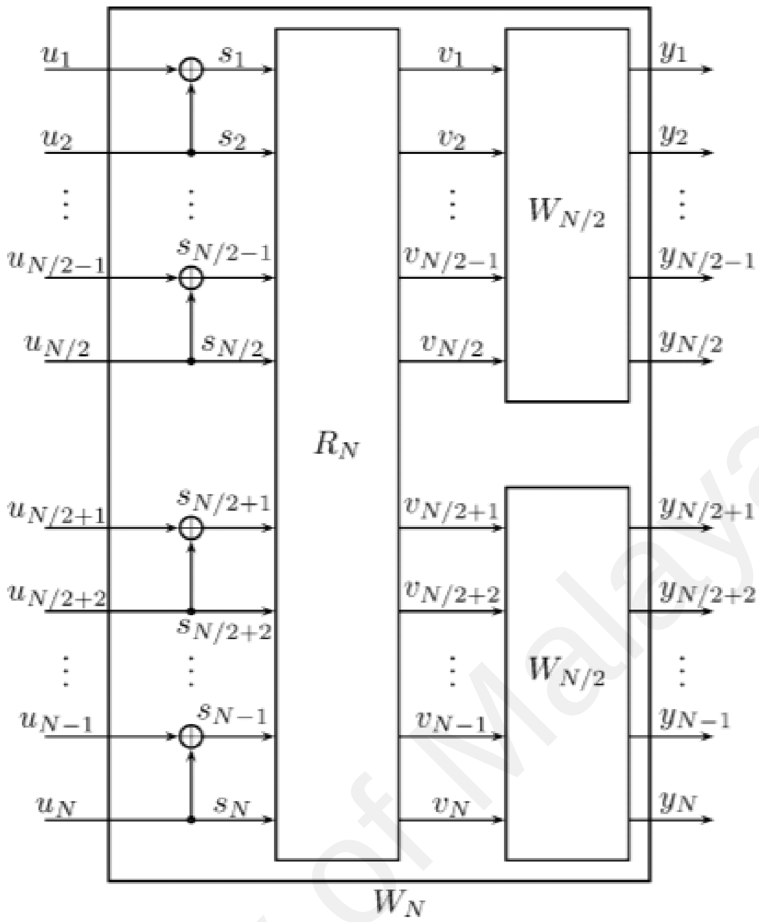

Figure 2.9 Construction of W_N from two independent copies of $W_{N/2}$

The transition probability of W_N is

$$W_N\ (y_1^N|u_1^N) = W^N(y_1^N|u_1^N G_N)$$

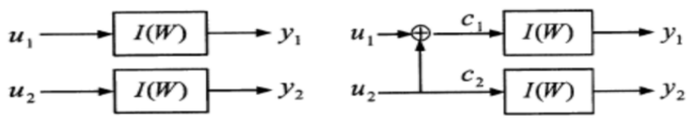

Figure 2.10 Traditional transmission mechanism and Polar code transmission mechanism

According to the chain rule of mutual information, and since the two uses of the channel W are independent of each other, the mutual information under the traditional mechanism is a superposition of mutual information of two independent channels.

$$
\begin{aligned}
I(u_1, u_2; y_1, y_2) &= I(u_1; y_1, y_2) + I(u_2; y_1, y_2|u_1) \\
&= I(u_1; y_1) + I(u_2; y_2) \\
&= I(W) + I(W) \\
&= 2I(W)
\end{aligned}
$$

The transmission mechanism of the polar code is that u_1, u_2 first pass through a binary domain operation $c_1 = u_1 \oplus u_2$ and $c_2 = u_2$ before being sent to the channel W.

$$
\begin{aligned}
I(u_1, u_2; y_1, y_2) &= I(c_1, c_{\,2}; y_1, y_2) \\
&= I(u_1; y_1, y_2) + I(u_2; y_1, y_2|u_1) \\
&= I\left(W_2^{(1)}\right) + I(W_2^{(2)}) \\
&= 2I(W)
\end{aligned}
$$

Let $I\left(W_2^{(1)}\right) = I(u_1; y_1, y_2)$, $I\left(W_2^{(2)}\right) = I(u_2; y_1, y_2|u_1)$, then

$$
\begin{aligned}
I\left(W_2^{(2)}\right) &= I(u_2; y_1, y_2|u_1) = I(u_2; y_1, y_2, u_1) \\
&= I(u_2; y_2) + I(u_2; y_1, u_1|y_2) \\
&= I(W) + I(u_2; y_1, u_1|y_2) \\
&\geq I(W)
\end{aligned}
$$

In contrast， $I\left(W_2^{(1)}\right) \leq I(W)$.

In fact, $I\left(W_2^{(1)}\right)$, $I\left(W_2^{(2)}\right)$ are the mutual information of two subchannels.

According to the mathematical calculation above, unlike the conventional transmission mechanism, the channel combination of the polar code makes the mutual information of the subchannel changed, but the total channel capacity does not change (the total channel capacity is conserved). The channel capacity of the subchannel carrying u_1 is lower than the original independent channel capacity, that is, the channel is worse than the original channel. The channel capacity of the subchannel carrying u_2 is higher than the original independent channel capacity, that is, the channel is better than the original channel.

As $\mathrm{W}_{N/2}$ merges further into W_N, such phenomenon still occurs, and as N approaches infinity, the channel capacity of the subchannel will tend to two extremes: noiseless channels (channel capacity is 1) and full noise channels (channel capacity is 0).

After channel splitting, the N new channels are different with the N copies original channels. The transition probability of the new channel is represented as

$$\mathrm{W}_{\mathrm{N}}^{(\mathrm{i})}\left(y_1^N, u_1^{i-1} \middle| u_i\right) \triangleq \sum_{u_{i+1}^N \in X^{N-i}} \frac{1}{2^{N-1}} \mathrm{W}_{\mathrm{N}}(y_1^N, u_1^N)$$

Which can be obtained by two recursive formula,

$$\begin{aligned}
&\mathrm{W}_{\mathrm{N}}^{(2\mathrm{i}-1)}\left(y_1^N, u_1^{2i-2} \middle| u_{2i-1}\right) \\
&\qquad = \sum_{u_{2i}} \frac{1}{2} \mathrm{W}_{N/2}^{i}\left(y_1^{N/2}, u_{1,odd}^{2i-2} \oplus u_{1,even}^{2i-2} | u_{2i-1} \oplus u_{2i}\right) \\
&\qquad \cdot \mathrm{W}_{\mathrm{N}}^{(2\mathrm{i}-1)}\left(y_{\frac{N}{2}+1}^{N}, u_{1,e}^{2i-2} \middle| u_{2i}\right)
\end{aligned}$$

$$\begin{aligned}
&\mathrm{W}_{\mathrm{N}}^{(2\mathrm{i})}\left(y_1^N, u_1^{2i-2} \middle| u_{2i-1}\right) \\
&\qquad = \frac{1}{2} \mathrm{W}_{N/2}^{i}\left(y_1^{N/2}, u_{1,odd}^{2i-2} \oplus u_{1,even}^{2i-2} | u_{2i-1} \oplus u_{2i}\right) \cdot \mathrm{W}_{\mathrm{N}}^{(\mathrm{i})}\left(y_{\frac{N}{2}+1}^{N}, u_{1,e}^{2i-2} \middle| u_{2i}\right)
\end{aligned}$$

2.5 Polar decoding algorithm

2.5.1 Successive cancellation decoding algorithm

Define Log-Likelihood Ratio as

$$\mathrm{L_N^{(i)}}(y_1^N, \hat{u}_1^{i-1}) \triangleq \ln \frac{W_N^{(i)}(y_1^N, \hat{u}_1^{i-1})|0}{W_N^{(i)}(y_1^N, \hat{u}_1^{i-1})|1}$$

The above formula can also be recursively calculated by

$$\mathrm{L_N^{(2i-1)}}(y_1^N, \hat{u}_1^{2i-2}) = \frac{L_{\frac{N}{2}}^{(i)}\left(y_1^{\frac{N}{2}}, \hat{u}_{1,odd}^{2i-2} \oplus \hat{u}_{1,even}^{2i-2}\right) \cdot L_{\frac{N}{2}}^{(i)}\left(y_{\frac{N}{2}+1}^{N}, \hat{u}_{1,even}^{2i-2}\right) + 1}{L_{\frac{N}{2}}^{(i)}\left(y_1^{\frac{N}{2}}, \hat{u}_{1,odd}^{2i-2} \oplus \hat{u}_{1,even}^{2i-2}\right) + L_{\frac{N}{2}}^{(i)}\left(y_{\frac{N}{2}+1}^{N}, \hat{u}_{1,even}^{2i-2}\right)}$$

$$\mathrm{L_N^{(2i)}}(y_1^N, \hat{u}_1^{2i-1}) = \left[L_{\frac{N}{2}}^{(i)}\left(y_1^{\frac{N}{2}}, \hat{u}_{1,odd}^{2i-2} \oplus \hat{u}_{1,even}^{2i-2}\right)\right]^{1-2\hat{u}_{2i-1}} \cdot L_{\frac{N}{2}}^{(i)}\left(y_{\frac{N}{2}+1}^{N}, \hat{u}_{1,even}^{2i-2}\right)$$

For the received bit sequence, $y_1^N = [y_1, y_2, \dots, y_i, \dots y_N]$, $\mathrm{LLR(y_i)} = \mathrm{L_1^1}(y_i) = \frac{W(y_i|0)}{W(y_i|1)}$.

To simplify the calculation, the above two formulas can be represented by the ***f*** and ***g*** functions.

$$f(\mathrm{a,b}) = \ln\left(\frac{1 + e^{a+b}}{e^a + e^b}\right)$$

$$g(\mathrm{a,b,u_s}) = (-1)^{u_s} a + b$$

$$\mathrm{L_N^{(2i-1)}}(y_1^N, \hat{u}_1^{2i-2}) = f\left(L_{\frac{N}{2}}^{(i)}\left(y_1^{\frac{N}{2}}, \hat{u}_{1,odd}^{2i-2} \oplus \hat{u}_{1,even}^{2i-2}\right), L_{\frac{N}{2}}^{(i)}\left(y_{\frac{N}{2}+1}^{N}, \hat{u}_{1,even}^{2i-2}\right)\right)$$

$$\mathrm{L_N^{(2i)}}(y_1^N, \hat{u}_1^{2i-1}) = g\left(L_{\frac{N}{2}}^{(i)}\left(y_1^{\frac{N}{2}}, \hat{u}_{1,odd}^{2i-2} \oplus \hat{u}_{1,even}^{2i-2}\right), L_{\frac{N}{2}}^{(i)}\left(y_{\frac{N}{2}+1}^{N}, \hat{u}_{1,even}^{2i-2}\right), \hat{u}_{2i-1}\right)$$

According to the transmission mechanism of the Polar code, it can be found that it is a modulo 2 addition operation. For this mechanism, the decoding operation only needs to consider the lower input bit as noise to decode the upper bit. The process of noise removal is performing the f function operation. When the decision value of the upper bit is obtained, the lower bit can be decoded according to the upper bit, that is, performing the g function.

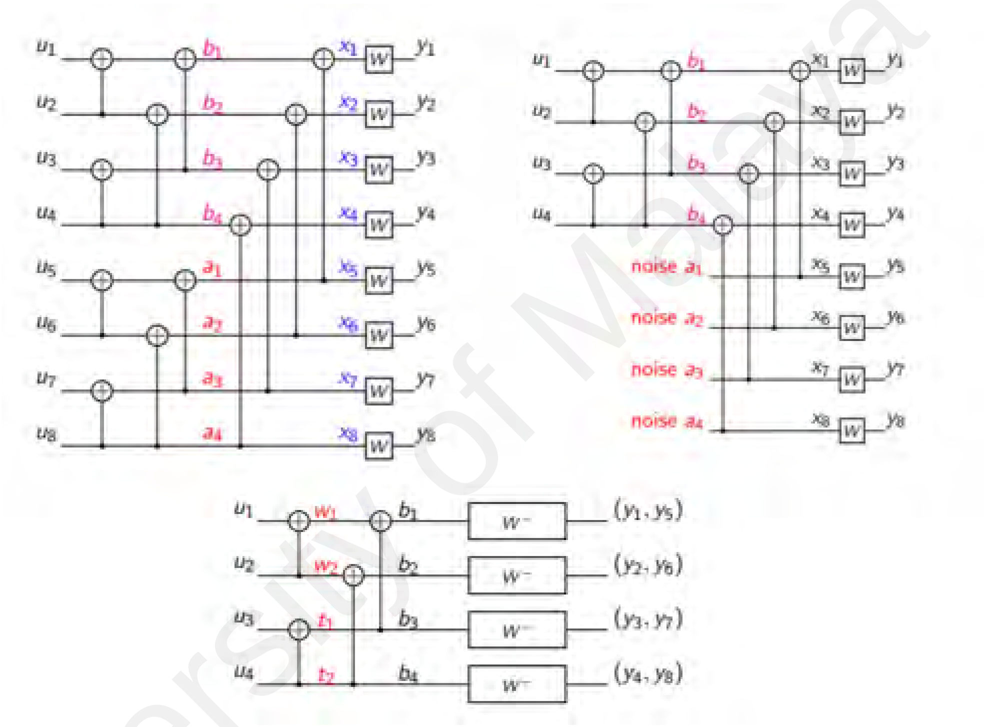

Figure 2.11 Successive Cancellation Decoding for N=8

As the example in Figure 2.11, first treat $x_5 \sim x_8$ as noise, $b_1 \sim b_4$ can be calculated by f function. After the estimated value of $b_1 \sim b_4$ is obtained. Performing further recursion can finally calculate the LLR of u_1. The decision function is

$$\hat{u}_i = \begin{cases} 0, & if\, u_i \; is \; not \; frozen \; bit \; and \; \mathrm{L}_{\mathrm{N}}^{(\mathrm{i})}\left(y_1^N, \hat{\mathrm{u}}_1^{i-1}\right) \geq 0 \\ 1, & if \; u_i \; is \; not \; frozen \; bit \; and \; \mathrm{L}_{\mathrm{N}}^{(\mathrm{i})}\left(y_1^N, \hat{\mathrm{u}}_1^{i-1}\right) < 0 \\ \hat{u}_i, & if \; u_i \; is \; frozen \; bit \end{cases}$$

According to the estimated value of u_1 , the LLR of u_2 can be calculated, then perform decision of u_2 . After obtaining the estimated value of u_1, u_2, modulo 2 addition performed on them, then we can get the estimated value of w_1, while $w_2 = u_2$. According to the estimated value of $w_{1,}$ w_2, the LLR of t_1, t_2 can be calculated by g function. The estimated value of u_3 can be calculated based on the LLR of t_1, t_2 by f function and get the estimated value of u_4. Estimated value of t_1, t_2, they have $t_1 = u_3 \oplus u_4$, $t_2 = u_2$, estimated value of $b_1 \sim b_4$ can be calculated by the same way. Then the LLR of $a_1 \sim a_4$ can be calculated by g function. The decoding of $u_5 \sim u_8$ is same as that of $u_1 \sim u_4$.

2.5.2 Successive-Cancellation List decoding algorithm

Based on the SC algorithm, the SCL algorithm performs the decoding of the next layer by saving the L path who have the largest a posteriori probability, and finally selects a path with the largest a posteriori probability.

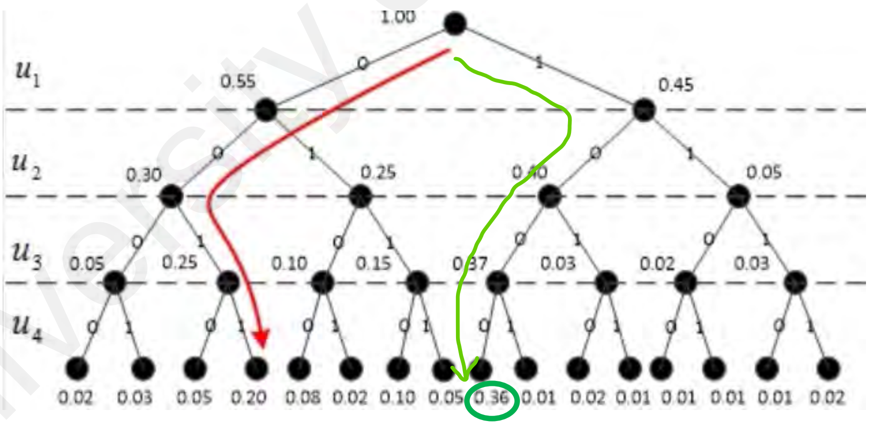

Figure 2.12 N=4 decoding tree

Take the code tree with mother code length N=4 as an example. Assume list size L=2. On the first layer (u_1), the SCL algorithm will find the left path expressed as {1, x, x, x} and the right path expressed as {0, x, x, x} based on the maximum a posteriori probability (APP), L=2 paths in total. The two paths will be stored in a list with list size L=2. Then, the SCL algorithm find the leftmost path and the third path on the second layer (u_2) based on APP, {0, 0, x, x} and {1, 0, x, x} respectively. On the third layer (u_3), it will find {0, 0, 1, x} and {1, 0, 0, x} respectively. On the fourth layer (u_3), it will find {0, 0, 1, 1} and {1, 0, 0, 0} respectively. Finally, as shown in Figure 2.12, the APP of the red path {0, 0, 1, 1} is larger than the APP of the green path {1, 0, 0, 0}. Therefore, the green path is selected to output the decoded bits {1, 0, 0, 0}, and the decoding is completed.

The SC decoder determines $\hat{u}_i$ *by* $\hat{u}_\mathrm{i} = \delta\left(\mathrm{LLR}_\mathrm{n}^{(\mathrm{i})}\right)$ based on the LLR. Where $\delta(x) = \frac{1}{2}\left(1 - sign(x)\right)$. The posterior probability of the l path for the ***i***-th bit can be measured by Path Metrics (PM).

$$\mathrm{PM}_\mathrm{l}^{(\mathrm{i})} \triangleq \sum_{j=0}^{i} ln\left(1 + e^{-(1-2\hat{u}_j[l])*LLR_n^{(j)}[l]}\right)$$

$$LLR_n^{(j)}[l] = \ln\frac{W_N^{(j)}\left(y_1^N, \hat{u}_1^{j-1}[l]\middle|0\right)}{W_N^{(j)}\left(y_1^N, \hat{u}_1^{j-1}[l]\middle|1\right)}$$

If $\mathrm{PM}_\mathrm{l}^{(\mathrm{i})}$ is a big value, the path is worse. The above formula can be represented by ψ function.

$$\mathrm{PM}_\mathrm{l}^{(i)} = \psi\left(\mathrm{PM}_\mathrm{l}^{(i-1)}, LLR_n^{(i)}[l], \hat{u}_i[l]\right)$$

$$\psi(\mu, \lambda, \mathrm{u}) = \mu + \ln(1 + e^{-(1-2u)*\lambda})$$

If $\mathrm{x} \geq 0$， for $\ln(1 + \mathrm{e}^\mathrm{x})$, it has $\ln(1 + \mathrm{e}^\mathrm{x}) \approx \mathrm{x}$. So, it can be further simplified. Determined $\mathrm{PM}_\mathrm{l}^{(0)} = 0$.

$$\mathrm{PM}_\mathrm{l}^{(\mathrm{i})} = \begin{cases} \mathrm{PM}_\mathrm{l}^{(\mathrm{i}-1)}, & if\ u_i\ is\ informationorfrozenbit\ and\ \hat{u}_i[l] = \delta\left(LLR_N^{(i)}[l]\right) \\ \mathrm{PM}_\mathrm{l}^{(\mathrm{i}-1)} + \left|L_N^{(i)}[l]\right|, & if\ u_i\ is\ informationorfrozenbit\ and\ \hat{u}_i[l] \neq \delta\left(LLR_N^{(i)}[l]\right) \\ +\infty, & if\ u_i\ is\ frozen\ bit\ and\ incorrect\ value \end{cases}$$

CHAPTER 3: RESEARCH METHODOLOGY

3.1 Comparative study on Polar Code Constructions and Polar decoding

3.1.1 Comparison of Polar Code Constructions for the AWGN Channel

For polar encoding, it is necessary to select highly reliable bit channels to transmit message bits, so how to measure the reliability of each bit channel is very important. The design of the bit channel reliability sequence is also called "good channel selection" and is used to indicate the order of channel reliability.

Since the calculation of channel capacity is somewhat complicated, Bhattacharyya Parameters which was first proposed by Arikan are used to determine the bound of channel capacity.

$$Z(W) = \sum_{y \in Y} \sqrt{W(y|0)W(y|1)}$$

The relationship between channel capacity $C(W)$ and Bhattacharyya Parameters $Z(W)$ is shown below:

$$I(W) \geq \log \frac{2}{1 + Z(W)}$$

$$I(W) \leq \sqrt{1 - Z(W)^2}$$

And the Bhattacharyya parameters of the channel can be recursively calculated according to the principle of channel splitting.

$$Z\left(W_N^{(2i-1)}\right) = 2Z\left(W_{N/2}^{(i)}\right) - Z\left(W_{\frac{N}{2}}^{(i)}\right)^2$$

$$Z\left(W_N^{(2i)}\right) = Z\left(W_{N/2}^{(i)}\right)^2$$

$Z\left(W_1^{(1)}\right)$ is the Bhattacharyya parameter of the binary input AWGN channel. In the BEC channel, it has $Z\left(W_1^{(1)}\right) = \varepsilon$, ε is erasure probability, and

$$I(W_n^{(i)}) = 1 - Z\left(W_N^{(i)}\right)$$

It can be easily known that initial value $Z\left(W_1^{(1)}\right)$ is non-universal. Its value is different under different channels and varies with SNR and code rate. The SNR of the actual wireless channel is not fixed. That makes the output of channel selection not fixed. The solution is to set only one SNR value that can be applied to a range of varying SNR. This SNR value is called best design-SNR.

However, for the AWGN channel, the calculation of $Z(W)$ cannot be achieved because the recursive formula is inequality in this situation. E.Arikan also proposed a Monte Carlo construction method for Polar codes for AWGN channels in his paper. It assumes that the code rate is 1 and transmits the all-zero code multiple times so that the previous bit is 0 can be considered as a known condition during decoding, and the error probability of each bit is counted after the decoding is finished to measure the reliability of each bit channel. For a B-AWGN channel with a noise variance of σ^2, if the modulation method is BPSK, and determining the received signal obtained from the channel is y.

$$y = (1 - 2x) + z, x \in \{0,1\}, z \sim N(0, \sigma^2).$$

If the transmitted bit is an all-zero sequence, the corresponding LLR value is

$$LLR(y) = ln\frac{p(y|x = 0)}{p(y|x = 1)} = \frac{2y}{\sigma^2}, y \sim N(1, \sigma^2)$$

Therefore, the LLR conforms to a Gaussian distribution with a mean of $\frac{2}{\sigma^2}$ and a variance of $\frac{4}{\sigma^2}$. Because the SC decoding is a recursive process, it can be inferred that

$LLR_N^{(i)}$ will be also a random variable that obeys the Gaussian distribution and the variance twice the mean. By calculating the mean of the LLR $\mathrm{E}[LLR_N^{(i)}]$, the reliability of the bit channel can be measured.

$$\mathrm{E}\left[LLR_N^{(2i-1)}\right] = \varphi^{-1}\left(1 - \left(1 - \varphi\left(E\left[LLR_{\frac{N}{2}}^{(i)}\right]\right)^2\right)\right)$$

$$\mathrm{E}\left[LLR_N^{(2i)}\right] = 2\mathrm{E}\left[LLR_{\frac{N}{2}}^{(i)}\right]$$

Where $\boldsymbol{\varphi}(\boldsymbol{x}) = \begin{cases} \sqrt{\frac{\pi}{x}}\left(1 - \frac{10}{7x}\right) e^{\left(-\frac{x}{4}\right)}, x \geq 10 \\ e^{\left(-0.4527x^{-0.86}+0.0218\right)}, 0 < x < 10 \end{cases}$

In this section, the above construction methods of polar codes will be simulated. The code resources are supported by H. Vangala et al from Monash University, this simulation adds MC coding structure and GA coding structure based on the original Matlab code, and the construction algorithm is referenced to another paper by H. Vangala (Vangala, Viterbo, & Hong, 2015). The simulation uses the AWGN channel, BPSK modulation, and the decoding algorithm are all SC decoding.

3.1.2 Comparison of Polar decoding

In this section, we compared the SC decoding algorithm with the SCL algorithm and the CRC-added SCL algorithm. The error detection principle of CRC and implementation of CRC and SCL algorithm will be specifically introduced in the next section in 5G scenario simulation.

3.2 Simulation of polar coding for 5G eMBB scenario

All the simulation in this part will run by Matlab, using the example from Matlab 5G toolbox. This simulation will model CRC-Aided Polar coding, which is one of code construction method defined by 3GPP and has been chosen for 5G -NR communications system.

According to the 3GPP's regulation for channel coding in the 5G-NR scenario, the polar code is applied to the 5G-NR control channels to encode the control information. For 5G-NR uplink control information (UCI), 1-bit UCI uses repeated coding, and 2-bit UCI uses simplex coding (using a simple mapping relationship, such as $C_2 = C_0 \oplus C_1$). UCI with a length of 3 to 11 bits uses Reed-Muller coding. For UCI with a bit length greater than or equal to 12, use the polar code for encoding.

Related parameter settings and representations are shown in Table 3.1.

Table 3.1 Related parameters representations in 5G-NR

A	The number of information bits
crcLen	The length of CRC bits
K	The number of information bits including CRC bits, $\mathrm{K} = \mathrm{A} + \mathrm{crcLen}$
N	The number of codeword bits and mother polar code size ($\mathrm{N} = 2^n$)
E	The rate matching output sequence length and the number of bits that a physical resource can carry
$N_{\max}$	Maximum mother polar code size, 512 for DCI, 1024 for UCI

Given K and E, $\mathrm{N} = 2^n$ can be determined by

If $\mathrm{E} \leq \frac{9}{8} \times 2^{[\log_2 E]-1}$ and $\frac{\mathrm{K}}{\mathrm{E}} < \frac{9}{16}$

$$n_1 = [\log_2 E] - 1$$

else

$$n_1 = [\log_2 E]$$

end if

$$R_{min} = \frac{1}{8}; n_2 = \left\lceil \log_2 \frac{K}{R_{min}} \right\rceil; \text{n} = \max\{\min\{n_1, n_2, n_{max}\}, n_{min}\}$$

Where $n_{min} = 5, n_{max} = \log_2 N_{\max}$.

The above calculation process indicates that if the code rate $\mathrm{R} = \mathrm{K/E}$ is small and the resource E is small, the smaller value is selected to be the mother code length.

Above coefficient $\frac{9}{8}$, $\frac{9}{16}$ and the following $\frac{7}{16}$ are more suitable values obtained through many computational simulations.

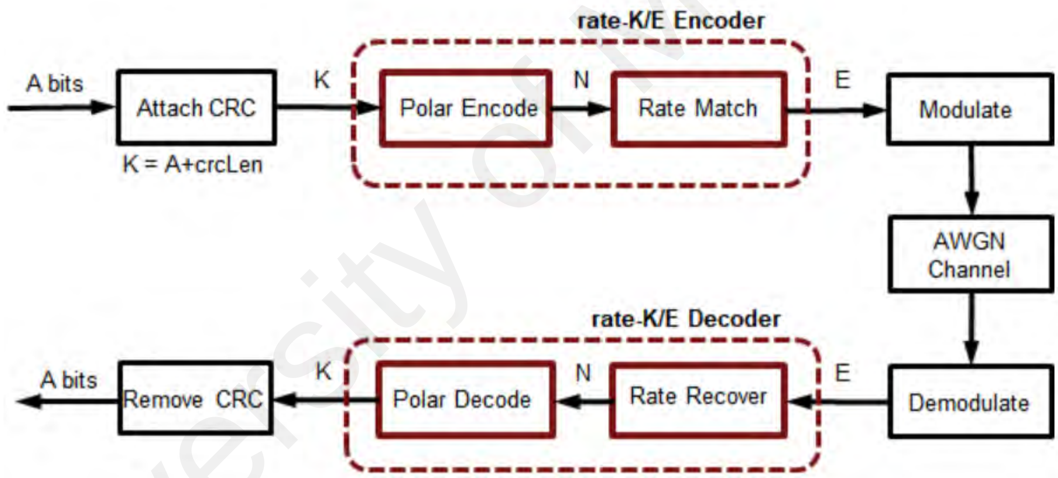

Figure 3.1 The simulation link

Figure 3.1 showed all the components for polar coding used in this simulation. In the next session, the settings of each component will be described in order. The Matlab function randomly generates A bits binary sequence as a message sequence for transmitting.

3.2.1 Segmentation

Segmentation is dividing the original information block to be encoded into two or more sub-blocks. Then encoding each sub-block separately. Segmentation is not required process, and segmentation is performed only when the information block to be encoded exceeds a certain length and the physical resources are sufficient. For example, the maximum information block lengths of the NR-PDCCH and the NR-PBCH are 140 bits and 56 bits, respectively, thus no segmentation is required. In the uplink direction, UCI (including ACK/NACK) may exceed 500 bits, and the 3GPP protocol specifies that the segmentation will be performed if the UCI information length A is greater than or equal to 360 bits and the number of bits after rate matching E is greater than or equal to 1088 bits. If necessary, add a bit "0" as a frozen bit at the beginning of the first segment. It can be divided into up to two segments and calculated the CRC separately.

3.2.2 Error detection

A given information bit sequence $\mathbf{m} = \{m_0, m_1, \cdots, m_{A-1}\}$ is encoded by CRC code, and a CRC encoded bit sequence $\mathbf{b} = \{b_0, b_1, \cdots, b_{K-1}\}$ is obtained.

Cyclic redundancy check (CRC) codes are used to assist coding in techniques commonly used in communication systems. The result of the calculation of the CRC is added behind the information bits waiting for encoding. Since the CRC introduces overhead, the overhead will be more obvious for control channel coding with a smaller code length. So, we hope that this overhead is as small as possible.

The sender and the receiver need to agree on a divisor in advance, that is, generator polynomial, which is generally denoted as $g(\mathrm{D})$.

$$g_{\mathrm{CRC24A}}(D) = [D^{24} + D^{23} + D^{18} + D^{17} + D^{14} + D^{11} + D^{10} + D^{7} + D^{6} + D^{5} + D^{4} + D^{3} + D + 1]$$

$$g_{\mathrm{CRC24B}}(D) = [D^{24} + D^{23} + D^{6} + D^{5} + D + 1]$$

$$g_{\mathrm{CRC24C}}(D) = [D^{24} + D^{23} + D^{21} + D^{20} + D^{17} + D^{15} + D^{13} + D^{12} + D^{8} + D^{4} + D^{2} + D + 1]$$

$$g_{\mathrm{CRC16}}(D) = [D^{16} + D^{12} + D^{5} + 1]$$

$$g_{\mathrm{CRC11}}(D) = [D^{11} + D^{10} + D^{9} + D^{5} + 1]$$

$$g_{\mathrm{CRC6}}(D) = [D^{5} + D^{4} + 1]$$

In the Physical Downlink Control Channel (PDCCH) of LTE, CRC length was set to be 16 bits, but in 5G-NR-PDCCH, it is 24 bits. In this simulation, CRC is set to be 24 bits for the downlink using version C. In order to prevent the problem that the length of the downlink control information carried on the NR-PDCCH is uncertain, the CRC shift register should be initialized to all ones. Only UCI greater than or equal to 12 bits uses Polar coding, and UCI with the length of 12 to 19 bits uses 6-bit CA-Parity Check (PC)-Polar code and UCI greater than or equal to 20 bits uses 11-bit CA-Polar code, PC can be seen as degeneration of CRC. The CA-Polar code will be treated as information bits and will be decoded, while PC Polar code calculates the value of the PC bits based on the decoded information bits and parity equations. Whether it is a CRC-Polar code or a PC-Polar code, it can be regarded as a Polar code cascading with a high-rate system linear block code. Distributed-CA-Polar code (Dist-CA-Polar code) inserts the CRC bits into the original information. The CRC bits are independent of the construction of the Polar code, and the process of breaking the original information block and the CRC bits is the interleaving process before encoding, called as Pre-Interleaving.

3.2.3 Pre-Interleaving

Interleaving is a process of reducing the correlation between bits by shuffling the bit order in the information block, thereby reducing the influence of burst interference. The UCI does not perform pre-interleaving, while the DCI and PBCH have pre-interleaving.

Since the pre-interleaving will disrupt the continuous placement of information in the PBCH, de-interleaving is also required for the PBCH channel. Both pre-interleaved and de-interleaved patterns can be found in the references (3GPP TS 38.212).

3.2.4 Channel selection

The construction of the polar code in the 5G specification uses the β-expansion (or polarization weigh) algorithm. It achieves the same performance as the GA algorithm, but it has a very low complexity compared to the GA algorithm. The index of the channel can be expressed as $B = (b_{n-1}, \dots, b_1, b_0)$, and the polarization weight of the $\boldsymbol{i}$th subchannel is defined as

$$f_i^{PW} = \sum_{j=0}^{n-1} b_j \times \beta^j$$

$\beta = 2^{\frac{1}{4}}$ as suggested in the literature (Huawei, 2016).

Observing the recursive manner of the polar code, it can be found that if there are ‘1’s in the channel index, this means that the subchannel will undergo modulo-2 addition operation on other subchannels, which makes the subchannel reliability increase, and ‘0’ means that the channel will be "polluted" by other subchannels and this will reduce its reliability. For easy operation, the construction of polar code is expected to be nested. Nestedness means that a sequence of length N/2 can be obtained by deleting the latter part of the element of the sequence of length N. In 5G specification, it designed a reliability sequence $Q^{N_{max}}$ with a length of $\mathrm{N_{max}} = 1024$. For any code block with a code length of N, only need to select an element with a value less than N from $Q^{N_{max}}$ to obtain the corresponding reliability sequence.

3.2.5 Sub-block interleaving

The first operation of rate matching is sub-block interleaving. The N bits are divided into 32 sub-blocks, then interleave them according to the pattern shown in Figure 3.2.

i	*P*(*i*)	*i*	*P*(*i*)	*i*	*P*(*i*)	*i*	*P*(*i*)	*i*	*P*(*i*)	*i*	*P*(*i*)	*i*	*P*(*i*)	*i*	*P*(*i*)
0	0	4	3	8	8	12	10	16	12	20	14	24	24	28	27
1	1	5	5	9	16	13	18	17	20	21	22	25	25	29	29
2	2	6	6	10	9	14	11	18	13	22	15	26	26	30	30
3	4	7	7	11	17	15	19	19	21	23	23	27	28	31	31

Figure 3.2 Sub-block interleaver pattern P(i)

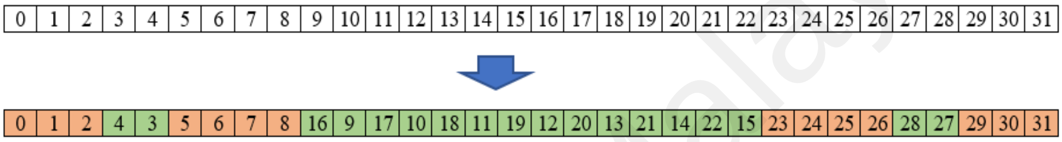

Figure 3.3 Schematic diagram of Sub-block interleaving

3.2.6 Pre-freezing before rate matching

When N > E, it has an insufficient number of bits allowed to be transmitted, N-E bits will not be sent, they need to be frozen bits.

If E ≥ 3×N/4, the first bit to the ceil(3×N/4-E/2) +1 bit will set to be frozen bits.

If E < 3×N/4, the first bit to the ceil(9×N/16-E/4) +1 bit will set to be frozen bits.

3.2.7 Polar Encoding

The polar encoding will compute the reliability of each channel. K bits with good channel reliability will be used to transmit information bits. And the remaining bits of the N bits will have bad performance according to the channel polarization theory, which will be frozen.

Then, the total N bits (stored in vector **u**) will be encoded with:

$$\mathbf{x} = \mathbf{u}G_N.$$

G_N is the generator matrix. It can be calculated by the Kronecker product $\otimes$.

$$\mathrm{G}_N = \mathrm{G}_2{}^{\otimes \mathrm{n}}$$

$$\mathrm{G}_2 = F = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

In order to reduce the decoding complexity, in the 5G-NR, the maximum value of N in the downlink is not more than 512, and the uplink direction is not more than 1024. Therefore, the N value available for the downlink channel are generally 32,64,128,256 and 512, for the uplink, they are 32,64,128,256, 512 and 1024.

3.2.8 Rate matching

After channel coding, data will be transmitted through wireless resources, but it often does not match between resource and the amount of data. Rate matching is the processing to match it, there are generally three ways:

1) Punching, throwing away some data bits, taken from the N-E+1th bit until the last bit
2) Repeating, copying data bits. In addition to N bits, need to continue to take the next one to the Mod [(E-N), N] bit.
3) Shorten. Take the first bit to the ***E***th bit.

The specific process of rate matching is shown in Figure 3.4 below.

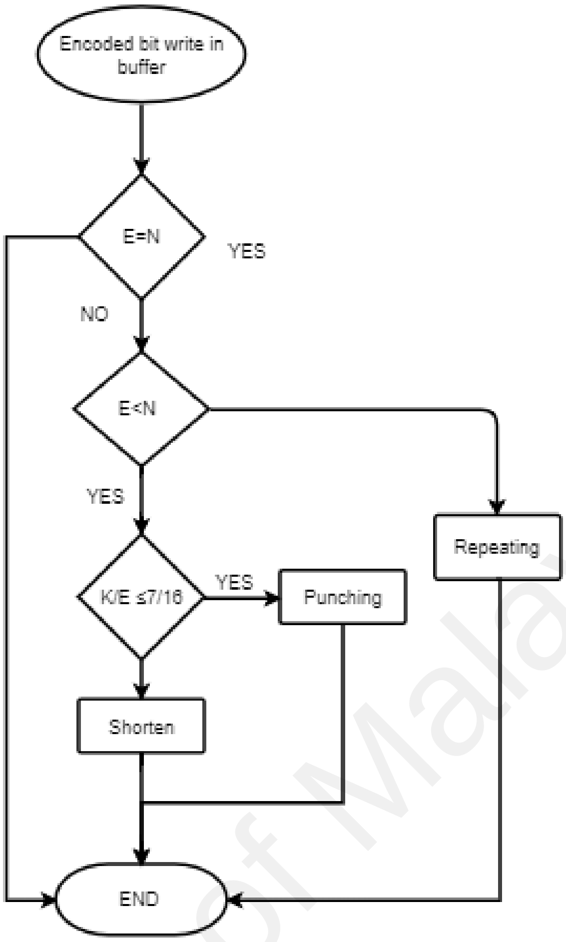

Figure 3.4 Flow chart of rate matching

3.2.9 Polar decoding

SCL decoding algorithm used in 3GPP specifications for Polar decoding. In this simulation, List length can be set to be 1, 2, 4 and 8. The specific decoding step is:

Algorithm 1 The main function of the SC decoding algorithm

Input: Received bit sequence LLR: $\mathbf{L_1^{(1)}(y_1), L_1^{(1)}(y_2), \ldots\ L_1^{(1)}(y_{N-1})}$
$\boldsymbol{m = log_2(N);}$
for $\boldsymbol{\varphi = 1: N}$
  recursivelyCalcP (m + 1, phase)
  if φ is frozen bit **then**
    $\boldsymbol{C_{m+1}^{\varphi} \leftarrow 0}$
    $\boldsymbol{\hat{u}_{\varphi}[l]) \leftarrow 0,\ PM_l^{(\varphi)} \leftarrow \psi\left(PM_l^{(\varphi-1)}, LLR_n^{(\varphi)}[l], \hat{u}_{\varphi}[l] = 0\right)}$
  else
    contPathsUnfrozenBit $\boldsymbol{(\varphi, LLR_{m+1}^{(\varphi)})}$
  if $\boldsymbol{\varphi}$ **mod 2 = 1 then**
    recursivelyUpdateC(m+1, φ);
  end if

end for
return C

Algorithm 2 recursivelyCalcP

Input: layer λ, phase φ

if $\lambda = 1$
  return
end if
$\phi = [(\varphi - 1)/2]$;
if $\varphi \bmod 2 = 0$ **then**
  recursivelyCalcP (layer-1, phase)
end if
for $\beta = 0 : 2^{(m-\lambda+1)} - 1$
  if $\varphi \ mod\ 2 = 0$ **then**
    $L_{\lambda}^{(\varphi+2^{\lambda}\cdot\beta)} = f\left(L_{\lambda-1}^{(\phi+2^{\lambda-1}\cdot 2\beta)}, L_{\lambda-1}^{\left(\phi+2^{\lambda-1}\cdot(2\beta+1)\right)}\right)$
  else
    $L_{\lambda}^{(\varphi+2^{\lambda}\cdot\beta)} = g\left(L_{\lambda-1}^{(\phi+2^{\lambda-1}\cdot 2\beta)}, L_{\lambda-1}^{\left(\phi+2^{\lambda-1}\cdot(2\beta+1)\right)}, C_{\lambda}^{(\varphi-1+2^{\lambda}\cdot\beta)}\right)$
  end if
end for

Algorithm 3 recursivelyUpdateC

Input: layer λ, phase φ
$\phi = [(\varphi - 1)/2]$
for $\beta = 0 : 2^{(m-\lambda+1)} - 1$
  if $\varphi \ mod\ 2 = 0$ **then**
    $C_{\lambda-1}^{(\phi+2^{\lambda-1}\cdot 2\beta)} = C_{\lambda}^{(\varphi-1+2^{\lambda}\cdot\beta)} \oplus C_{\lambda}^{(\varphi+2^{\lambda}\cdot\beta)}$
  else
    $C_{\lambda-1}^{\left(\phi+2^{\lambda-1}\cdot(2\beta+1)\right)} = C_{\lambda}^{(\varphi+2^{\lambda}\cdot\beta)}$
  end if
end for
if $\phi = 0$ **then**
  recursivelyUpdateC($\lambda - 1,\ \phi + 1$)
end if

Algorithm 4 contPathsUnfrozenBit

Input: phase φ, $LLR_{m+1}^{(\varphi)}$

$\mathrm{PM}_{l}^{(\varphi)} \leftarrow \psi\left(\mathrm{PM}_{l}^{(\varphi-1)}, LLR_{n}^{(\varphi)}[l], \hat{u}_{\varphi}[l] = 0\right)$
$\mathrm{PM}_{l+1}^{(\varphi)} \leftarrow \psi\left(\mathrm{PM}_{l}^{(\varphi-1)}, LLR_{n}^{(\varphi)}[l], \hat{u}_{\varphi}[l] = 1\right)$

If $l < L$ **then**
  Clone path $l,\ l+1$, from the first bit $\hat{u}_1$ to the $\hat{u}_{\varphi}$ bit
else
  Sort $\mathrm{PM}_{\mathrm{l}}^{(\varphi)}$. There are 2L paths currently. Then, select L paths corresponding to the smallest $\mathrm{PM}_{\mathrm{l}}^{(\varphi)}$ values, delete other paths,
  If both forks $\mathrm{PM}_{\mathrm{l}}^{(\varphi)}$, $\mathrm{PM}_{\mathrm{l+1}}^{(\varphi)}$ are selected **then**
    Clone path $l,\ l+1$, from the first bit $\hat{u}_1$ to the $\hat{u}_{\varphi}$ bit
  else
    continue;
  end if
end if

After SCL decoding, 'pruning' is performed with the CRC check, leaving only one path. The output after SCL decoding is L decoded bit sequences. Perform CRC encoding on the information bit sequences and compare the result with L decoded bit sequences respectively. If there is a decoded bit sequence that is the same, return it. Conversely, return the information bit sequences with the smallest $\mathrm{PM}_{\mathrm{l}}^{(\varphi)}$.

3.2.10 Frame Processing Loop

In this simulation, each code rate will be calculated based on different SNR values. The specific simulation process is shown in Figure 3.5. First, set the minimum number of frames, which is the number of times the codec operation is performed for each code rate at each SNR. After generating the random information sequence and frame loop, the BLER value can be calculated by counting the error bits in each transmission.

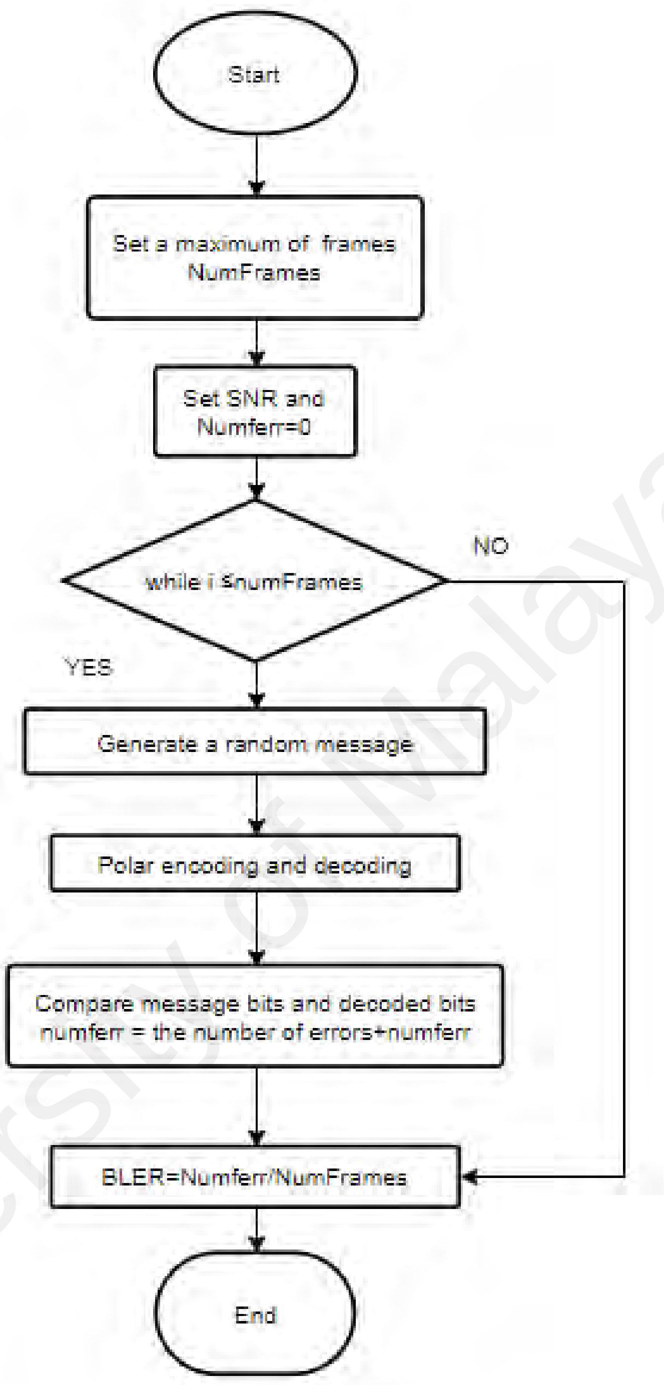

Figure 3.5 Flow chart of frame processing loop

3.2.11 Performance Parameters

Signal-to-noise ratio (SNR) is the ratio of signal power to the noise power, while $\frac{E_b}{N_0}$ is its normalized form, that is the energy per bit to noise power spectral density ratio.

$$\mathrm{SNR} = \frac{\mathrm{S}}{N} = \frac{\mathrm{E}_s \cdot \mathrm{R}_s}{\mathrm{N}_0 \cdot \mathrm{B}_w} = \frac{\mathrm{E}_b}{\mathrm{N}_0} \cdot \frac{\mathrm{R}_b}{\mathrm{B}_w} = \frac{\mathrm{E}_b}{\mathrm{N}_0} \cdot \mathrm{k} \cdot \mathrm{R}_s$$

$\mathrm{k} = \log_2 M$, $\mathrm{M} = 4$ for QPSK. In this paper, it varies from different range in different decoding parameter setting, the range given clearer performance results were chosen for different code rate, as it is shown in Table 3.2 and Table 3.3.

Table 3.2 Parameters setting for uplink simulation

Parameters setting	Uplink				
K	59	43	67	163	307
E	512	184	138	240	360
R	0.1152	0.2337	0.5234	0.6792	0.8528
$\frac{\mathrm{E}_b}{\mathrm{N}_0}$	-3.5~2.5	-5~3.5	-4~3.5	-3.5~3.5	-3.5~4.5

Table 3.3 Parameters setting for downlink simulation

Parameters setting	Downlink				
K	56	43	54	164	164
E	864	180	124	240	184
R	0.064815	0.23889	0.43548	0.68333	0.8913
$\frac{\mathrm{E}_b}{\mathrm{N}_0}$	-3~4.5	-3.5~2.5	-3.5~3	-3.5~3.5	0~5.5

CHAPTER 4: RESULTS AND DISCUSSION

4.1 Results of different polar code constructions for the AWGN Channel

The ratio of the number of errors bit in the received bits within per unit period to the total number of transferred bits is called the "bit error rate". Frame error rate (FER) is the probability of frame error during data transmission.

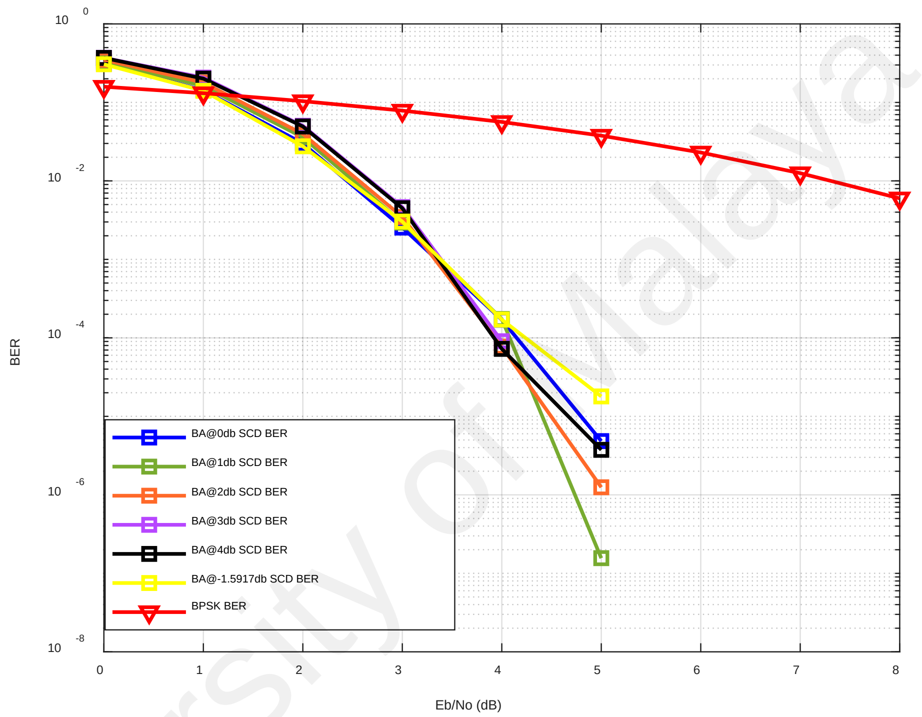

Figure 4.1 E_b/N_0 versus SNR for the Bhattacharyya bounds construction method at different design-SNR with N = 256 and R = 0.5

Figure 4.1 to Figure 4.3 show the BER simulation results of the polar code produced by Bhattacharyya bounds (BA), Monte-Carlo estimation (MC), Gaussian approximation (GA) construction method at N = 256 and R = 0.5 for AWGN channels. Curves with different colors represent the coding performance of the coding construction at different design-SNR. −1.5917dB is a special value, which is the corresponding SNR in the BA construction method that makes $Z(W)$ is equipped to 0.5. This is also the worst case, because $Z(W) = 0.5$ means that the probability of correct decoding is 0.5 even if the received bit has an error.

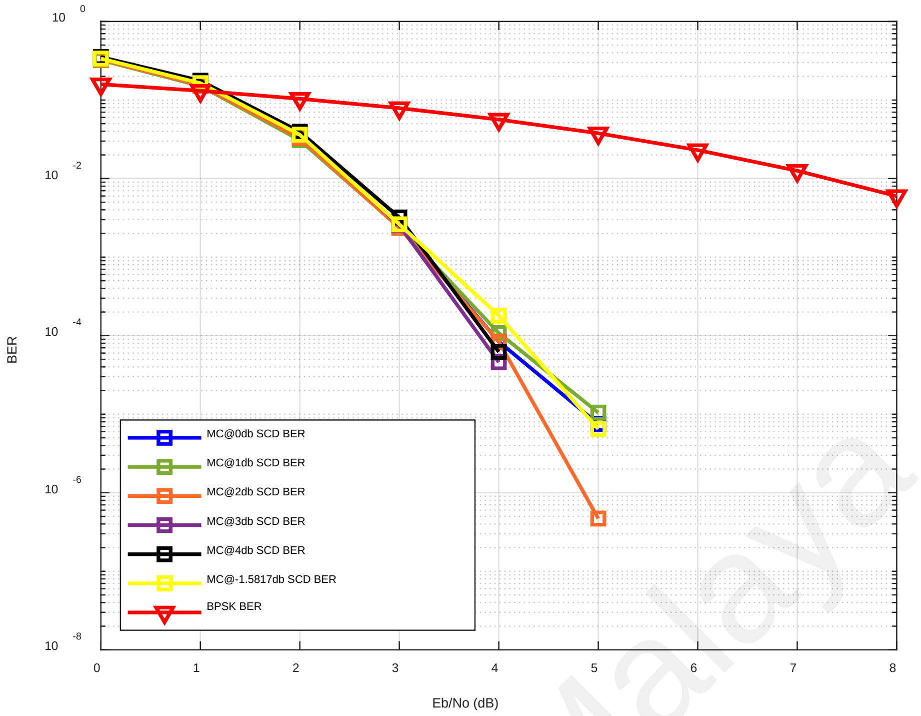

Figure 4.2 E_b/N_0 versus SNR for the Monte-Carlo estimation construction at different design-SNR with N = 256 and R = 0.5

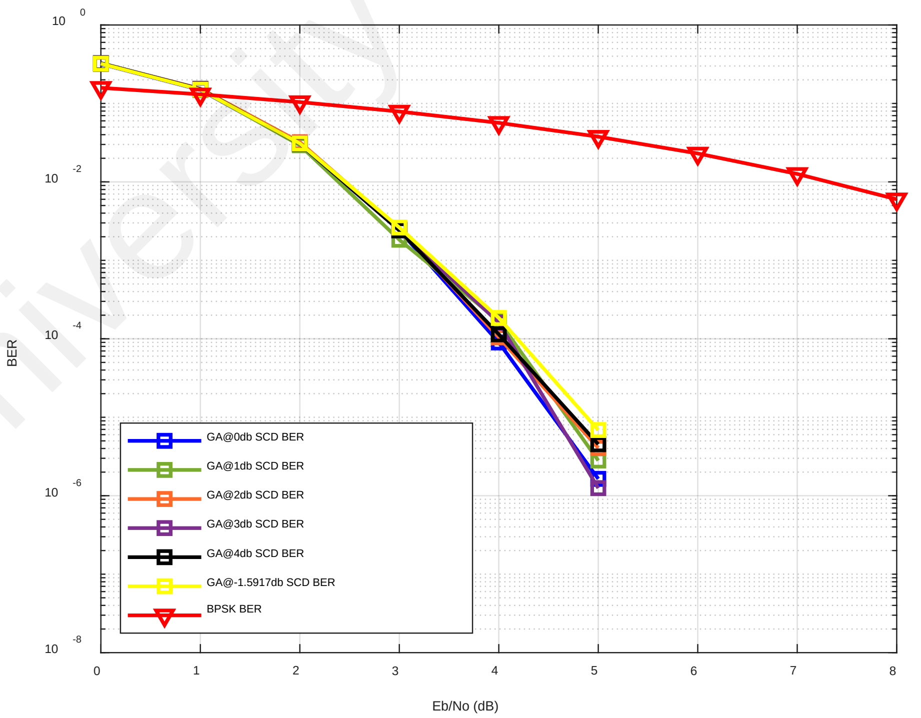

Figure 4.3 E_b/N_0 versus SNR for the Gaussian approximation construction method at different design-SNR with N = 256 and R = 0.5

Different from the result of the literature (Vangala et al., 2015), design-SNR has little effect on performance results when N is a small value. As the SNR increases, the impact of design-SNR on performance gradually emerges. In addition, it can be seen that among the three construction methods, design-SNR has the greatest impact on the performance of Bhattacharyya bounds than the other two construction methods, Gaussian approximation has the smallest difference in performance between different design-SNR. It can be seen from the figure that when the SNR is less than 1, the channel coding is useless, because the bit error rate without channel coding (red curve in Figure 4.1 to Figure 4.3) is smaller than the bit error rate of any one of the polar codes.

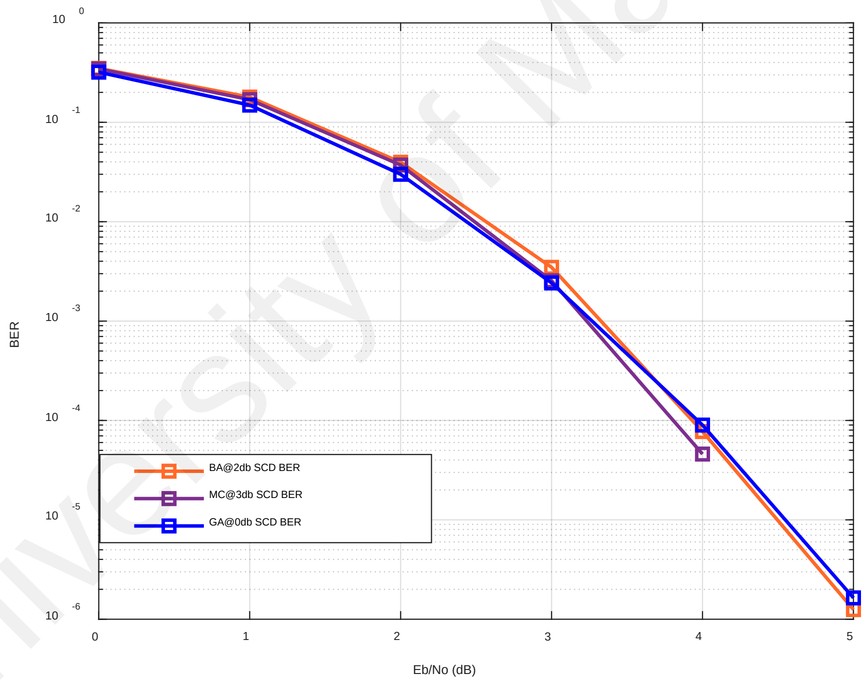

Figure 4.4 Comparison of all the constructions at their best design-SNRs with N=256 and R=0.5

If comparing all the constructions at their best design-SNRs, as it presented in Figure 4.4, it can be observed that the performances of different constructions are almost the same when N and the SNR are both small. For example, to get $\mathrm{BER} = 10^{-4}$, the performance difference between using MC construction and BA and construction is only about 0.005 dB, while the GA construction can bring about a performance improvement of about 0.05 dB.

4.2 Results of different polar decoding with same code rate R

The above simulation shows that the polar code structure has little effect on performance when N is small, so we use the simplest BA construction for encoding in this section. In addition, since there is no point in using the Polar code when the SNR is less than 1, the SNR is set to start from 1. Figure 4.5 shows the BER and FER simulation results for different decoding algorithms.

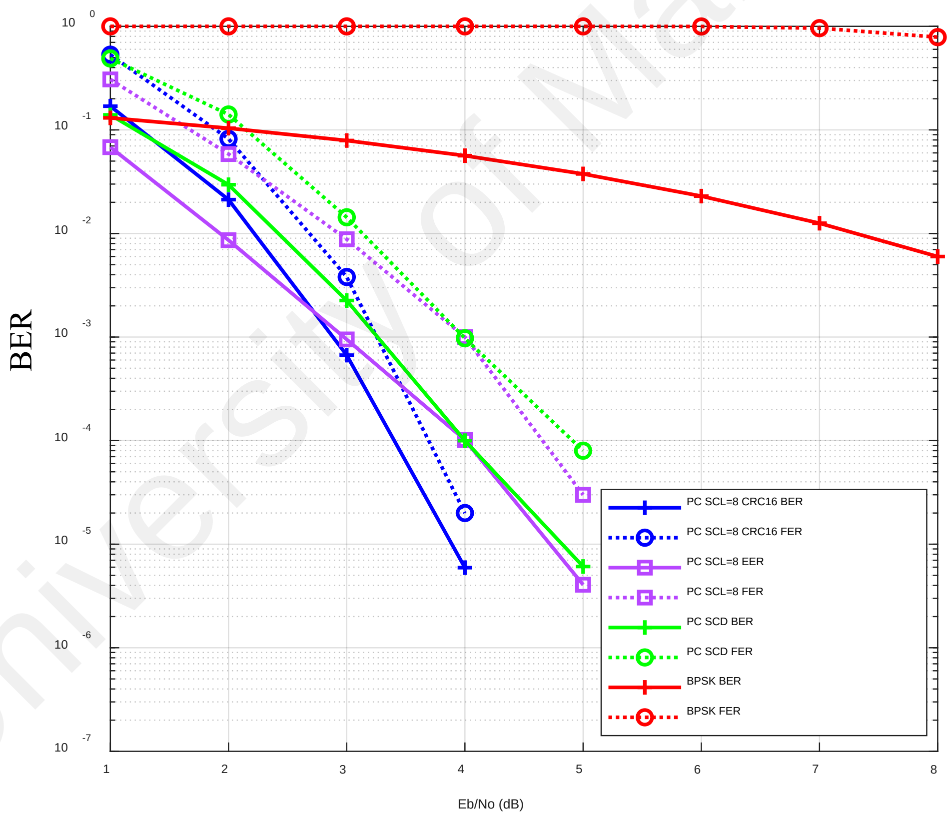

Figure 4.5 BER and FER venues three different decoding algorithms with N=256 and R=0.5

It can be seen that the polar code has the best performance when using the CA-SCL decoding algorithm, and compared with the uncoded curve (the red curve in Figure 4.5), the bit error rate drops from 10^{-1} to 10^{-3}, and the frame error rate drops from 1 to 0.5 ×

10^{-2}. The simulation result shows that CRC-assisted decoding performance is improved compared with that of the Polar code without CRC. And SCL decoding performance is significantly better than SC decoding performance. For example, to achieve a bit error rate of 10^{-5}, the CRC-assisted polar code needs $\mathrm{Eb/No} = 3.7\mathrm{dB}$, polar code without CRC requires 4.6dB, CRC brings performance gain about 0.9dB. Compared with the SC decoding algorithm, the improved performance of the SCL decoding algorithm is not obvious when the SNR is large. At low SNR, such as $\mathrm{Eb/No} = 3\mathrm{dB}$, the bit error rate is reduced from 0.5×10^{-2} to 10^{-3} by 50%. It can be inferred that the performance gain of the SCL decoding algorithm relative to that of the SCL decoding algorithm is getting smaller as the SNR increases, which means that when the SNR of the communication system is large, it is not necessary to use the more complex SCL decoding algorithm, the SC decoding algorithm is simpler and can also achieve as better performance as SCL decoding algorithm does.

The performance ascendancy of CA-SCL decoding algorithm gradually emerges as the SNR increases. Take $\mathrm{Eb/No} = 4\mathrm{dB}$ as an example, the BER performance at $\mathrm{Eb/No} = 4\mathrm{dB}$ is 0.4×10^{-5}, there is a difference of 9.6×10^{-5} relative to SC of SC decoding algorithm, which is much larger than that at Eb/No=3dB.

4.3 Results of polar code performance for the 5G eMBB scenario

The block error rate (BLER) is the ratio of the number of information blocks that has not successfully decoded at the receiving end to the total number of information blocks. The performance of the coding scheme is usually measured by BLER.

The simulation was done using the Matlab 5G Toolbox as it was discussed in Chapter 3.2.

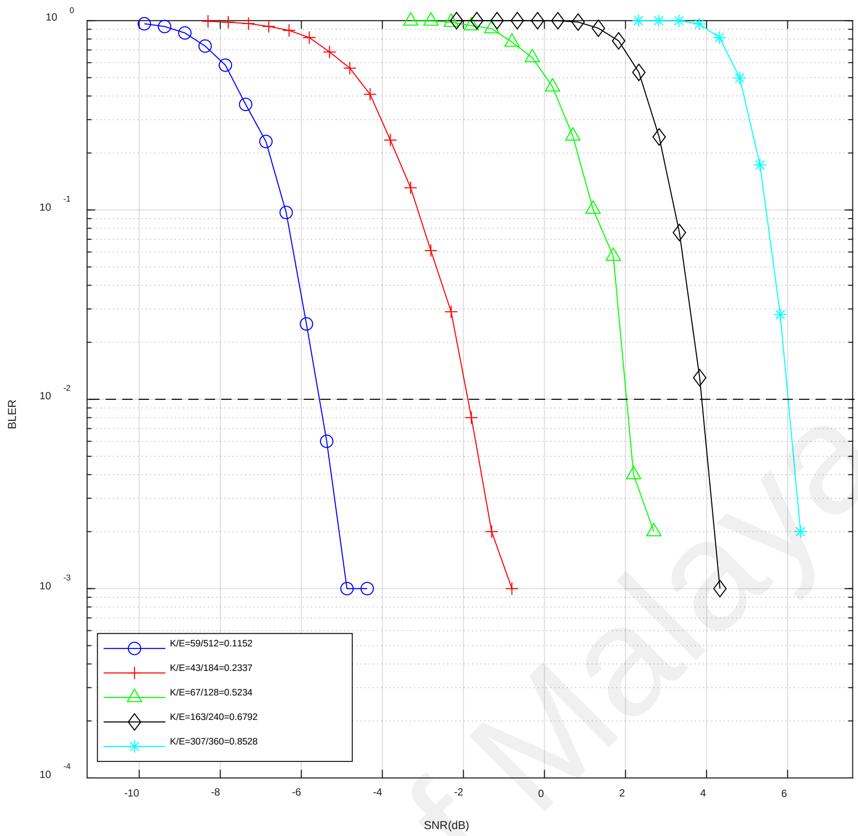

Figure 4.6 BLER versus SNR on the Uplink CRC-11 with L=8

Figure 4.6 shows the BLER of polar code at the five code rates (0.1152, 0.2337, 0.5234, 0.6792, 0.8528) operating in the 5G uplink environment. It can be seen that the BLER of polar codes with different code rates will all rapidly decline with the SNR gradually increases. This is because large SNR means a clearer signal (greater signal strength or less noise), and obviously, the probability of errors will be lower if the signal is better.

At the same time, in the case of the same block error rate, the required SNR will gradually decrease as the code rate increases. For example, when BLER $= 10^{-2}$, the SNR is $-5.6\text{dB}, -1.9\text{dB}, 2\text{dB}, 3.8\text{dB}$ and 6dB respectively. That is, when the code rate is lower, the signal strength required to achieve an acceptable BLER value is smaller. This is because when the code rate is increased, the information bits will be encoded with only a few redundant bits so the ability of channel coding to detect and correct the errors will be declined; on the contrary, the smaller code rate means inserted more redundant bits for error detection. The more redundant bits, the more error bits that can be detected and corrected. Usually, the communication system specifies a maximum BLER. At the same BLER, it is desirable that the SNR value is as small as possible, because this means low signal requirements. At $\text{BLER} = 10^{-2}$, code rate 0.6792 compared to code rate 0.8528 obtained coding gain of about 2dB. Obviously, a high code rate will result in poor performance. Therefore, the compromise between coding efficiency and performance becomes especially important.

Figure 4.7 shows the performance curves of the five code rates when the length of the SCL decoding algorithm is 1, 2, 4, and 8, respectively, where L = 1, it is equivalent to SC decoding. It can be seen that as the L grows, the decoded performance gradually become better. At the same SNR, the larger L, the smaller the BLER. However, it also can be seen that at the same BLER, the performance improved by L varying from 4 to 8 is significantly smaller than the performance improved by L varying from 1 to 2. For example, when the code rate is 59/512, when L varies from 4 to 8, the SNR required to achieve $\text{BLER} = 10^{-2}$ decreases about 0.4dB, but when L varies from L=1 to L=2, the SNR required decreases about 0.8dB. It indicates that if the list length is already large enough, continuing to increase the list length does not make much sense for performance. The infinite increase of L cannot continuously improve performance. And as it can be observed that as the code rate becoming higher, the performance improved due to the increases of the list length become smaller (Curves with a higher code rate is more compact).

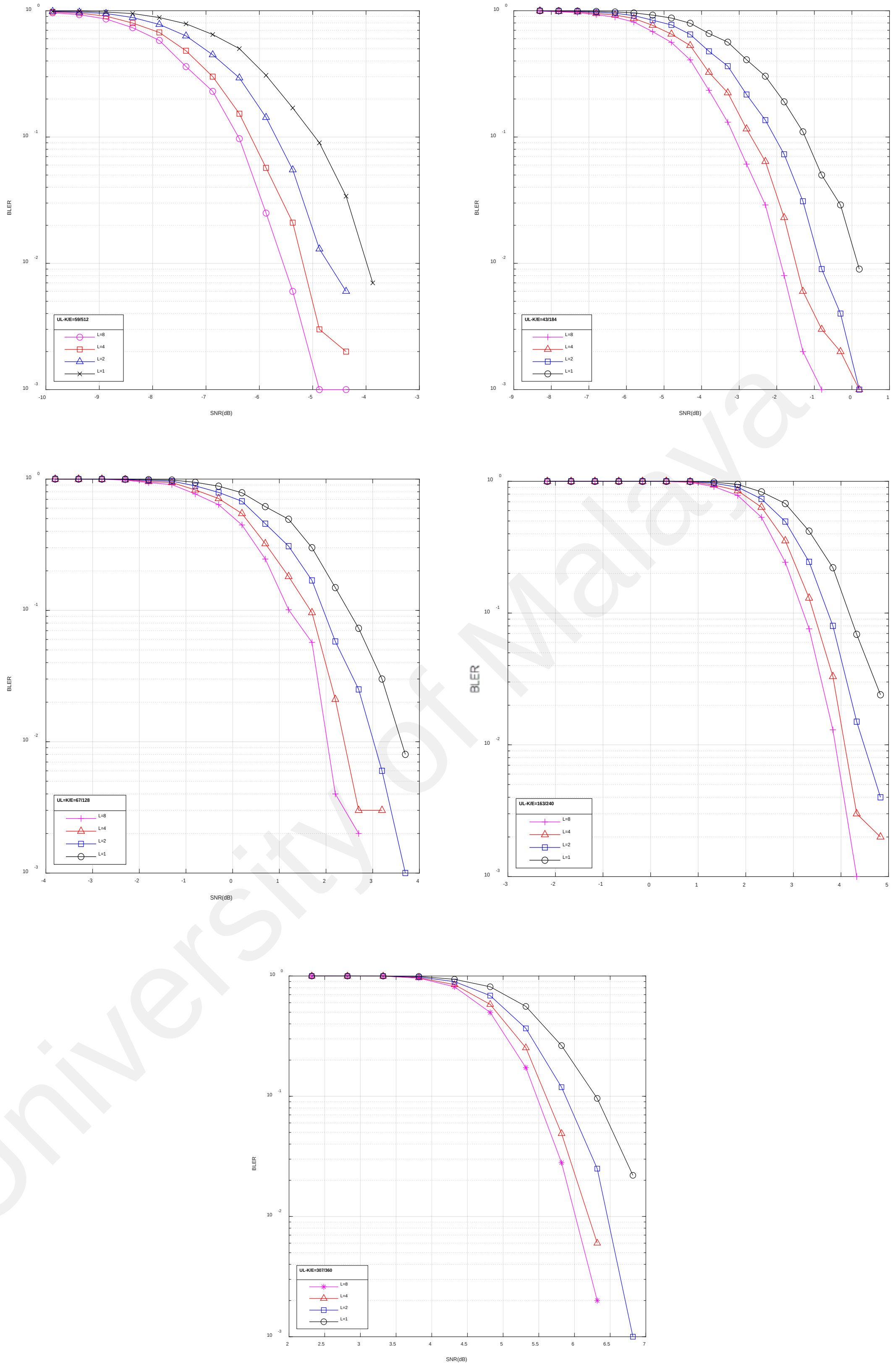

Figure 4.7 The performance on the uplink of five different code rates with L=1, L=2, L=4, L=8

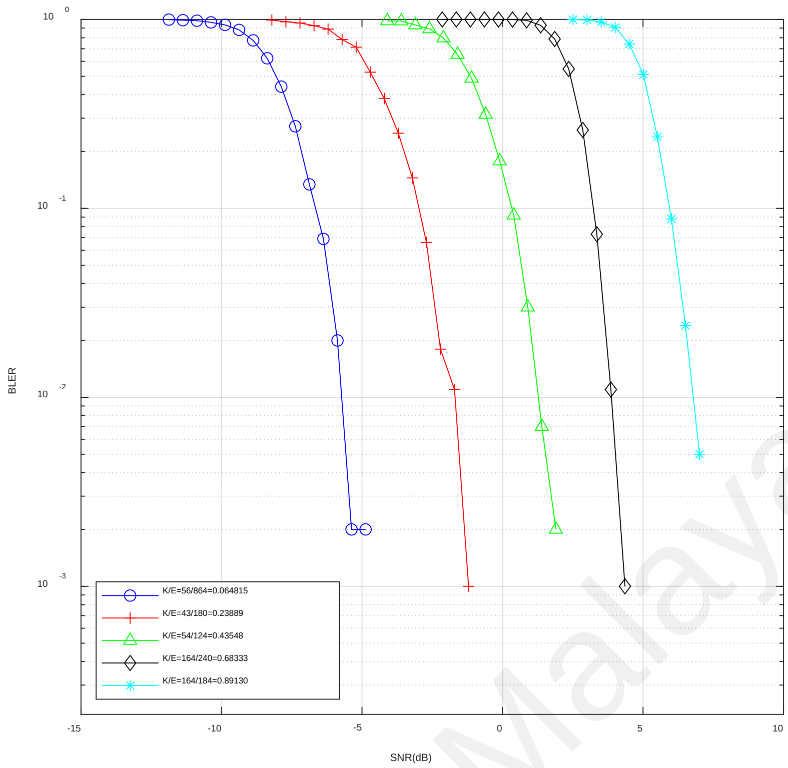

Figure 4.8 BLER versus SNR on the Downlink CRC-24 with L=8

Same as the uplink, the overall performance trend of the downlink is that when the code rate is fixed, the BLER decreases as the SNR increases. Moreover, as the code rate increases, a larger SNR is required to reach the specified BLER value.

From Figure 4.9, the same result can be observed. As the L increases, the SNR required to achieve a specified BLER will reduce. And for the same SNR, the larger L, the smaller the BLER. However, at the same BLER, the coding performance gain brought by the change of L is getting smaller and smaller as L becomes larger.

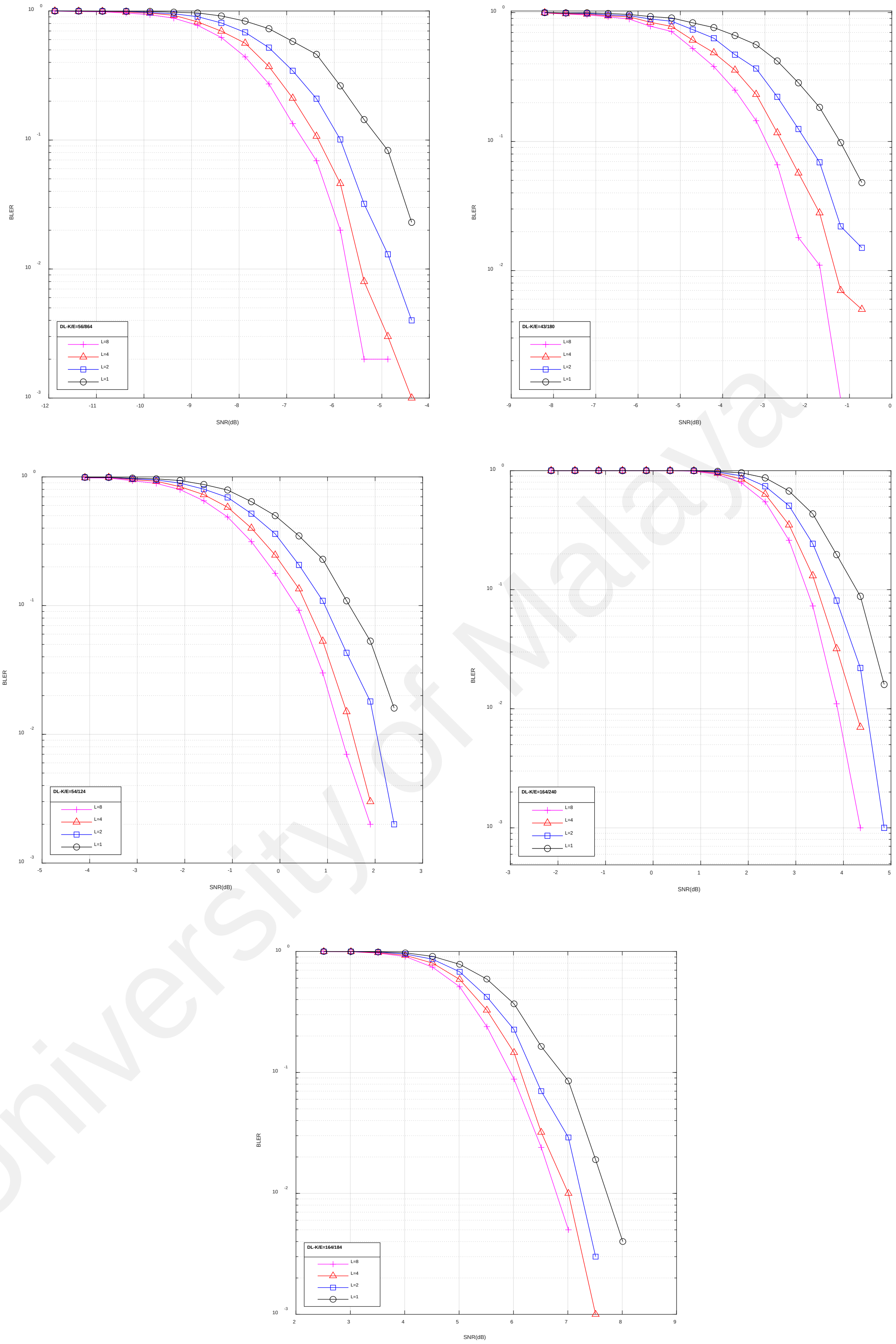

Figure 4.9 The performance on the downlink of five different code rates with L=1, L=2, L=4, L=8

CHAPTER 5: CONCLUSION AND FUTURE WORK

According to the simulation results, the following five conclusions can be summarized:

1. Design-SNR has different effects on different polar code constructions, where BA construction is most affected

2. Design-SNR has different effects on the performance of polar codes with different code lengths. When the code length is small, the impact is not obvious.

3. The performance of the construction method at the best design-SNR is almost the same, If the best design-SNR is known, using the simplest coding construction can simplify the coding complexity without affecting the performance.

4. SCL decoding can improve the performance of the polar code, and the larger the L, the better the performance, but the performance gain decreases as L increases, which means that the performance of the polar code cannot be improved indefinitely by increasing L. Instead, it needs to strike a balance between performance and complexity.

5. By adding an outer code, such as CRC or PC, can improve the performance of the polar code, but the added complexity caused by different outer codes is different. So, it needs to be carefully selected.

It can be seen from the simulation results in this paper that the BLER is sometimes large, which does not agree with the expectation. It is because of the limited time and limitations of equipment that the number of simulations cannot be set too large. Therefore, the statistical results of BLER may not be accurate.

In this paper, only the polar code encoding and decoding process for binary input were studied, the generation matrix is only the 2×2 matrix. However, it has been proved that using the high-dimensional generation matrix, the polarization rate is fast, the performance can be greatly improved. Therefore, the study of high-dimensional polarization core will be the future research direction of polar codes. In addition, due to the recursive nature of the polar code, the implementation of parallel decoding is very difficult, and the design of the parallel decoding algorithm with low complexity will also be a future research direction.

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