APPENDIX A

Gaussian, Error and Complementary Error function

The gaussian function, error function and complementary error function are frequently used in probability theory since the normalized gaussian curve represents the probability distribution with standard deviation σ relative to the average of a random distribution. The error function represents the probability that the parameter of interest is within the range between $-x/\sigma\sqrt{2}$ and $x/\sigma\sqrt{2}$, while the complementary error function provides probability that the parameter is outside that range. All these functions are shown below.

The Gaussian function

The Gaussian function (also referred to as bell-shaped or "bell" curve) is of the following form:

$$G(x) = Ae^{-\frac{x^2}{2\sigma^2}}$$

where σ is referred to as the spread or standard deviation and A is a constant. The function can be normalized so that the integral from minus infinity to plus infinity equals one yielding the normalized Gaussian:

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

by using the following definite integral:

$$\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

The gaussian function goes to zero at plus and minus infinity while all the derivatives of any order evaluated at x = 0 are zero.

The error function

The error function equals twice the integral of a normalized gaussian function between 0 and 2/σ♦2:

$$erfx = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

The relation between the normalized gaussian distribution and error function equals:

$$\int_{-x}^{x} G(x)dx = Erf\left(\frac{x}{\sigma\sqrt{2}}\right)$$

A series approximation for small value of x of this function is given by:

$$erfx = \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{3 \bullet 1!} + \frac{x^5}{5 \bullet 2!} + \frac{x^7}{7 \bullet 3!} + \cdots\right)$$

while an approximate expression for large values of x can be obtained from:

$$erfx \cong 1 - \frac{e^{-x^2}}{\sqrt{\pi x}}\left(1 - \frac{1}{2x^2} + \frac{1 \bullet 3}{\left(2x^2\right)^2} + \frac{1 \bullet 3 \bullet 5}{\left(2x^2\right)^2} + \cdots\right)$$

The complementary Error function:

The complementary error function equals one minus the error function yielding:

$$erfcx = 1 - \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-u^2} du$$

which, combined with the series expansion of the error function listed above, provides approximate expressions for small and large value of x, which are given below:

$$erfcx = 1 - \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3 \bullet 1!} + \frac{x^5}{5 \bullet 2!} + \frac{x^7}{7 \bullet 3!} + \cdots \right)$$

$$erfx \cong \frac{e^{-x^2}}{\sqrt{\pi x}} \left(1 - \frac{1}{2x^2} + \frac{1 \bullet 3}{(2x^2)^2} + \frac{1 \bullet 3 \bullet 5}{(2x^2)^2} + \cdots \right)$$

APPENDIX B

Beam characteristics of Semiconductor Laser Diodes

Semiconductor laser diodes have many advantages over other types of lasers. However, laser diodes also have certain short comings. Of these, the elliptical cross-section of the laser beam and the diode's intrinsic astigmatism are often the most likely to cause the problems in an application.

Circularity

The elliptical cross section of the beam is a result of the rectangular shape of the beam emission facet of the laser diode. This characteristic prevents the beam from being entirely collimated, allowing for qusi-collimation only. Wave optics theory tells us that a beam output from a small aperture has in one certain direction a full divergent angle θ given by

$$\theta = 4\lambda/\pi d$$

where λ is the wavelength and d is the size of the aperture in this direction. The difference in θx and θy causes the beam of laser diodes to have an elliptical cross section, as shown in Fig. 1. A universal characterization of this problem is made impossible by the different index-guided and gain-guided diodes as well as by the individualistic nature of the laser diode. To make matter worse, the difference between θx and θy is not consistent even between two laser diodes of the same type. So the shape of the elliptical cross section varies from diode to diode.

Astigmatism

Astigmatism is, in fact, another result of the rectangular facet of the laser diode. As illustrated in Fig. 2, the beam emitted from a small facet is equivalent to the beam emitted by an imaginary point source P, whose position can be located by the tracing the beam backwards. It can be seen immediately that P_x is located behind P_y because θx is smaller than θy. When dx is much larger than dy, θx is much smaller than θy and the distance between P_x and P_y is therefore much larger as well, as shown in Fig. 2. This phenomenon is called astigmatism, and the distance between P_x and P_y is the numerical description of astigmatism.

The existence of astigmatism means that when using a single, standard aspheric lens the beam can be collimated only in one direction, either in X direction or in the Y direction, because P_X and P_y can not be simultaneously converged at the focal point of the collimating lens.

For an index-guided, low power laser diode the rectangular emission facet, is shown in Fig.1, has an approximate size of $dx = 3\mu m$ and $dy = 1\mu m$. From equation we find that θy is approximately three times larger than θx. For a gain-guided, wide-stripe, high power laser diode dx can be tens or even hundred microns while dy is still approximately three micron. Therefore, unlike an indix-duided laser diode, the difference between θx and θy is unpredictable and can be much larger than three to one.

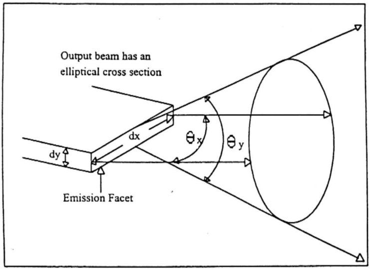

Fig. 1. Laser diode usually have rectangular emission facet which causes the output beam to have elliptical cross section.

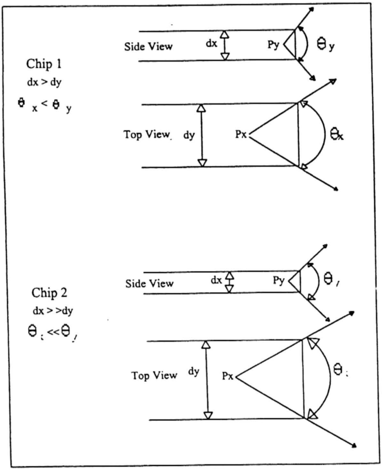

Fig. 2. Two laser viewed from the side and the top. Illustrating the radiation between lasers and the cause of stigmatism.