CHAPTER 2

2.0 THEORETICAL ASPECTS OF AN OPTICAL COMMUNICATION SYSTEM

In an optical communication system there are three things that are very important, which are optical source, optical fiber link, and optical detector. When comparing the relative performance of different systems, the detector is the most important element. Different types of optical detectors are used for different types of the optical communication systems. Efficiency of an optical communication system is highly dependent on the efficiency of optical detector. In this chapter detailed theoretical study of different detectors is done. In the last part of this chapter the relative efficiency of different types of detectors for different optical systems is also given.

2.1 Detectors

The detector is an essential part of an optical fiber communication system and it is one of the critial elements, which dictates the overall system performance. Its function is to convert the received optical signal into an electrical signal.

Energy of photon is given by

$$E = hf \tag{2.1}$$

If the incident optical power is P_o and γ_p is number of incident photons per second

where γ_p is

$$\gamma_p = \frac{P_o}{hf} \tag{2.2}$$

Quantum efficiency η is defined as $\eta = \dfrac{\text{number of electrons collected}}{\text{number of incident photons}}$

$$\eta = \frac{\gamma_e}{\gamma_P} \tag{2.3}$$

so

$$\gamma_c = \eta\,\gamma_p = \eta \frac{P_o}{hf} \tag{2.4}$$

Responsivity of a detector is defined as

$$R = \frac{I_P}{P_o} \quad (AW^{-1}) \tag{2.5}$$

Where I_P is the output photo-current in amperes and P_o is incident power in watt.

Since γ_c is the number of electrons collected per second then

Where e is the charge of an electron

$$\gamma_c\, e = I_P \quad \text{(is the output photo-current)} \tag{2.6}$$

so

$$R = \frac{\gamma_c e}{P_o} \tag{2.7}$$

or

$$R = \frac{\eta e}{hf} \tag{2.8}$$

or

$$R = \frac{\eta e \lambda}{hc} \tag{2.9}$$

because $f = \frac{c}{\lambda}$

or

$$I_P = \gamma_c e = \frac{\eta e P_o}{hf} = \frac{\eta e P_o \lambda}{hc} \tag{2.10}$$

For the direct detection photo-current is proportional to optical signal power, and quantum efficiency at a given wavelength.

$$I_P \propto P_o$$

2.1.1 Coherent Detection

Basic principle for coherent detection is briefly discussed and the results are used in forth-coming experiments. The low level, incoming signal field E_S is combined with a second much larger signal field E_L derived from a local oscillator [1]. This technique of mixing signals and extracting them is known as heterodyning. Incoming signal is denoted as e_S, which is defined as

$$e_S = \hat{E}_S \cos(\omega_S t + \phi) \quad (2.11)$$

where $\phi = (\phi_S - \phi_L)$

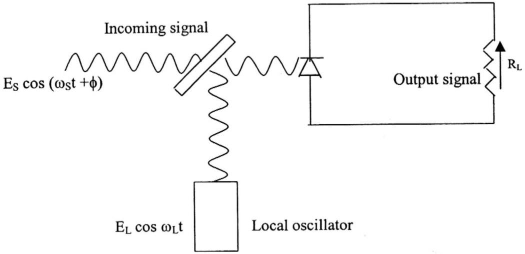

Fig. 2.1 Basic coherent receiver model.

where ϕ_S is the signal phase and ϕ_L is the local oscillator phase at the same point in time.

Local oscillator $e_S = E_L \cos \omega_L t$

E_S is the peak incoming signal field

E_L is the peak local oscillator field

ω_S is incoming signal angular frequency

ω_L is local oscillator angular frequency

The angle $\phi_{(t)}$ representing the phase relationship between the two fields contains the transmitted information in case of frequency shift keying (FSK) or phase shift keying (PSK). But with amplitude shift keying (ASK) $\phi_{(t)}$ is constant and is simply written as ϕ.

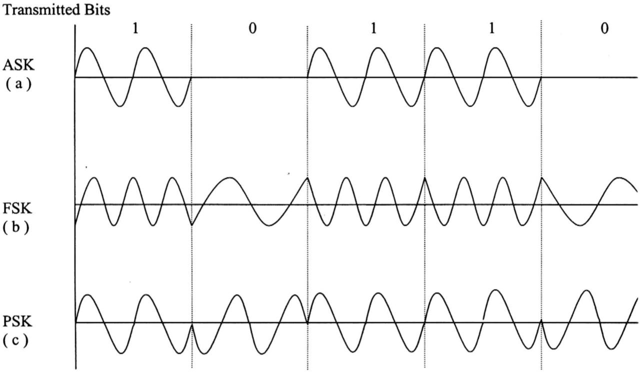

Fig. 2.2 Waveforms used for binary data transmission: (a) amplitude shift keying (ASK); (b) frequency shift keying (FSK); (c) phase shift keying (PSK).

For heterodyne detection, the local oscillator frequency ω_L is offset from the incoming signal frequency ω_S by an intermediate frequency (IF) [2].

$$\omega_{IF} = \omega_S - \omega_L \qquad (2.12)$$

In homodyning $\omega_{IF} = 0$ and the combined signal is recovered in the base band itself. In the case of coherent detection the two wave-fronts from the incoming signal and local oscillator laser must be perfectly matched at the surface of the detector. The optical detector produces a signal photo-current I_P, which is proportional to the optical intensity, so

$$I_P \propto (e_S + e_L)^2 \qquad (2.13)$$

But $(e_S + e_L)^2 = (E_S \cos(\omega_S t + \phi) + E_L \cos \omega_L t)^2$

Assuming perfect optical mixing, expansion of the right hand side of the equation (2.13)

$$[E_S \cos(\omega_S t + \phi) + E_L \cos \omega_L t]^2 = E_S^2 \cos^2(\omega_S t + \phi) + E_L^2 \cos^2 \omega_L t + 2E_S E_L \cos(\omega_S t + \phi) \cos \omega_L t \quad (2.14)$$

while

$$E_S^2 \cos^2(\omega_S t + \phi) + E_L^2 \cos^2 \omega_L t + 2E_S E_L \cos(\omega_S t + \phi) \cos \omega_L t = [\frac{1}{2} E_S^2 + \frac{1}{2} E_L^2 + \frac{1}{2} E_S \cos(2\omega_S t + \phi) + \frac{1}{2} E_L \cos(2\omega_L t + \phi) + E_S E_L \cos(\omega_S t + \phi - \omega_L t) + E_S E_L \cos(\omega_S t + \phi + \omega_L t)]$$

We can remove the higher frequency factor because the detector may not respond to very high frequencies (which can be filtered too) so removing $2\omega_L$ and $2\omega_S$, we have

$$I_P \propto \frac{1}{2} E_S^2 + \frac{1}{2} E_L^2 + 2 E_S E_L \cos(\omega_S t + \phi - \omega_L t)$$

and ignoring Since $I_P \propto E^2$ we can write

$$I_P \propto P_S + P_L + 2\sqrt{P_L P_S} \cos(\omega_S t + \phi - \omega_L t)$$

Where P_S and P_L are the optical power of the in incoming signal and local oscillator signal respectively. The relationship between the optical power and photo-current can be written as

$$I_P = \frac{\eta e P_o}{hf} \qquad (2.16)$$

where η is the quantum efficiency of the photo-detector, e is the charge of the electron, h is Plank's constant. A general expression of the photo-current is given in the equation 2.17

$$I_P = \frac{\eta e}{hf} [P_S + P_L + 2\sqrt{P_L P_S} \cos(\omega_S t + \phi - \omega_L t)] \qquad (2.17)$$

If the local oscillator signal is much larger than incoming signal then the third, a.c. term may be distinguished from the first two d.c. terms in the equation 2.17 and I_P can be replaced by the approximate I_S where [3]

$$I_S = 2\frac{\eta e}{hf} \sqrt{P_L P_S} \cos(\omega_S t + \phi - \omega_L t) \qquad (2.18)$$

This equation allows two coherent detection strategies to be considered.

For heterodyne detection $\omega_S \neq \omega_L$

$$\omega_{IF} = \omega_S - \omega_L$$

$$I_S = 2\frac{\eta e}{hf} \sqrt{P_L P_S} \cos(\omega_{IF} t + \phi) \qquad (2.19)$$

The output from the photo-detector is centered on an intermediate frequency. This varying , IF current is easily separated from the d.c current by appropriate filtering with prior electrical amplification and demodulation schemes.

For the special case of homodyne detection, where $\omega_S = \omega_L$

$$I_S = 2\,\frac{\eta e}{hf}\,\sqrt{P_L P_S}\,\cos\phi \qquad (2.20)$$

$$I_S = 2\,R\sqrt{P_L P_S}\,\cos\phi \qquad (2.21)$$

Since in a coherent system $I_S \propto \sqrt{P_S}\sqrt{P_L}$

and in a direct detection system $I_P \propto P_o$

If $P_S \ll P_L$ then $\sqrt{P_L}$ is considered as a large amplification factor. This local oscillator gain factor $\sqrt{P_L}$ has the effect of increasing the optical signal level without affecting the receiver preamplifier thermal noise or the photodiode-dark current noise. That is why a coherent detection scheme provides improved receiver sensitivities over direct detection.

2.1.2 Comparison Of Sensitivities

For amplitude shift keying heterodyne detection or IM/DD as is used in optical fiber system, the output current from the photo-detector $I_S(t)$ can be written as

$$I_S(t) = \begin{cases} I_{SH}\cos(\omega_{IF} + \phi) & \text{for a bit 1} \\ 0 & \text{for a bit 0} \end{cases}$$

where $I_{SH} = 2\,\frac{\eta e}{hf}\,\sqrt{P_L P_S}$

and $\omega_{IF} = \omega_S - \omega_L$

The probability of error $P_{(e)}$ for ASK heterodyne detection is given by

$$P_{(e)} = \frac{1}{2} \operatorname{erfc}\left(\frac{\eta P_S}{4hfB_T}\right) \tag{2.22}$$

where B_T is the transmission bit-rate.

Full derivation from equation 2.22 to 2.23 is given in a appendix A.

$$P_{(e)} \cong \frac{1}{2} \exp\text{-}\left(\frac{I_{SH}{}^2}{8i_{SL}{}^2}\right) \tag{2.23}$$

$$\text{Where } i_{SL}{}^2 = \frac{2e^2\eta P_L B_T}{hf}$$

For FSK Frequency heterodyne detection

$$I_S(t) = \begin{cases} I_{SH} \cos(\omega_1 + \phi) & \text{for a 1 bit} \\ I_{SH} \cos(\omega_2 + \phi) & \text{for a 0 bit} \end{cases}$$

where $I_{SH} = 2\dfrac{\eta e}{hf}\sqrt{P_L P_S}$

and $\omega_1 \neq \omega_2$

Since the signal frequency is changed, giving different IF's. The error probability is now

$$P_{(e)} = \frac{1}{2} \operatorname{erfc}\text{-}\left(\frac{I_{SH}}{2\left(i^2{}_{SH}\right)^{\frac{1}{2}}}\right)$$

$$P_{(e)} = \frac{1}{2}\,\text{erfc}\left(\frac{\eta P_S}{2hfB_T}\right)^{\frac{1}{2}}$$

which can be written as

$$P_{(e)} \cong \frac{1}{2}\exp\text{-}\left(\frac{I_{SH}^{\ 2}}{4i_{SL}^{\ 2}}\right)$$

$$P_{(e)} \cong \frac{1}{2}\exp\text{ -}\left(\frac{\eta P_S}{2hfB_T}\right)$$

To calculate the number of received photons per bit in order to maintain the bit error rate (BER) of 10^{-9} for FSK heterodyne,

then $$\eta N_P = \frac{\eta P_S}{hfB_T} \tag{2.24}$$

$$10^{-9} = \frac{1}{2}\exp\left(\frac{\eta N_P}{2}\right)$$ we get $N_P = 40$ photons

Phase shift keying heterodyne detection

In phase modulation, the employed variation is normally π radians so that

$$I_{S(t)} = \begin{cases} I_{SH}\cos(\omega_{IF}t + \phi) & \text{for a bit 1} \\ I_{SH}\cos(\omega_{IF}t + \pi + \phi) & \text{for a bit 0} \\ -I_{SH}\cos(\omega_{IF}t + \phi) & \text{for a bit 0} \end{cases}$$

Assuming that the output voltage from the receiver for a binary one is V_1 and for binary zero is V_2. Then the error probability is

$$P_{(e)} = \frac{1}{2}\int_{\infty}^{0}\frac{1}{\left(i_{SL}^{2}\right)\sqrt{2\pi}}\exp -\left(\frac{\left(I_{SH}-V_{1}^{2}\right)^{2}}{2i_{SL}^{2}}\right) dV_1$$

$$+\frac{1}{2}\int_{0}^{\infty}\frac{1}{\left(i_{SL}^{2}\right)\sqrt{2\pi}}\exp -\left(\frac{\left(I_{SH}-V^{2}{}_{2}\right)^{2}}{2i_{SL}^{2}}\right) dV_2 \qquad (2.25)$$

$$=\frac{1}{2}\,\text{erfc}\left(\frac{I_{SH}}{\left(i_{SL}^{2}\right)^{1/2}\sqrt{2}}\right) \qquad (2.26)$$

$i_{SL}^{2} = 2\,e\,B\,I_{PL}$ where $I_{PL} = \dfrac{\eta e P_o}{hf}$

and $I_{SH} = \dfrac{2\eta e}{hf}\sqrt{P_S P_L}$

The above can be rewritten as

$$P_{(e)} = \frac{1}{2}\,\text{erfc}\,-\left(\frac{\eta P_S}{hfB_T}\right)^{1/2} \qquad (2.27)$$

or

$$P_{(e)} = \frac{1}{2}\exp -\left(\frac{I_{SH}^{2}}{2i_{SL}^{2}}\right)$$

which can be written as $P_{(e)} = \dfrac{1}{2}\exp -\left(\dfrac{\eta P_S}{hfB_T}\right)$

If we were to calculate the number of received photons per bit in order to maintain a BER of 10^{-9} for PSK heterodyne detection . Then

$10^{-9} = ½ \exp -(\eta N_P)$ where quantum efficiency $\eta = 1$

we arrive at $N_p = 20$

Amplitude shift keying homodyne or IM/DD optical fiber system with local modulator.

The output current from the photo-detector $I_S(t)$ can be written as

$$I_S(t) = \begin{cases} I_{SH} \cos(\omega_{IF} + \phi) & \text{for a 1 bit} \\ 0 & \text{for a 0 bit} \end{cases}$$

where $I_{SH} = 2 \frac{\eta e}{hf} \sqrt{P_L P_S}$

$\omega_S = \omega_L$

The probability of error $P_{(e)}$ for ASK homodyne detection is given by

$$P_{(e)} = \frac{1}{2} \operatorname{erfc}\left(\frac{I_{SH}}{2\left(i_{SL}^{\,2}/2\right)^{1/2} \sqrt{2}} \right) \tag{2.28}$$

$$= \frac{1}{2} \operatorname{erfc}\left(\frac{I_{SH}}{2\left(i_{SL}^{\,2}\right)^{1/2}} \right) \tag{2.29}$$

$P_{(e)}$ can be written as

$$P_{(e)} \cong \frac{1}{2} \exp\text{-}\left(\frac{\eta P_S}{2hfB_T} \right) \tag{2.30}$$

This is a 3dB improvement compared to ASK heterodyning.

If we calculate the number of received photons per bit in order to maintain BER of 10^{-9} for amplitude shift keying for homodyne detection. Then

$$\left(\frac{S}{N}\right)_{het} = \frac{\eta P_S}{hfB_T} = \eta N_P \qquad (2.31)$$

$$\left(\frac{S}{N}\right)_{homo} = \frac{\eta P_S}{hfB_T/2} = 2\eta N_P \qquad (2.32)$$

$$P_{(e)} = 10^{-9} = \frac{1}{2}\exp\left(\frac{\eta N_P}{2}\right) \qquad (2.33)$$

we get the minimum needed of photons,

$$N_P = 20 \qquad (2.34)$$

For the Phase shift keying homodyne detection

$$I_{S(t)} = \begin{cases} I_{SH}\cos(\omega_{IF}t + \phi) & \text{for a bit 1} \\ I_{SH}\cos(\omega_{IF}t + \pi + \phi) & \text{for a bit 0} \\ -I_{SH}\cos(\omega_{IF}t + \phi) & \text{for a bit 0} \end{cases}$$

$$P_{(e)} = \frac{1}{2}\operatorname{erfc}\left(\frac{I_{SH}}{\left(i_{SL}^{2}\right)^{1/2}}\right) \qquad (2.35)$$

The above can be written as

$$P_{(e)} = \frac{1}{2}\,\text{erfc} - \left(\frac{2\eta P_S}{hfB_T}\right)^{1/2} \tag{2.36}$$

approximating it to

$$P_{(e)} = \frac{1}{2}\,\text{exp} - \left(\frac{2\eta P_S}{hfB_T}\right)^{1/2} \tag{2.37}$$

The above result obtained represents the lowest error probability hence the highest receiver sensitivity of all coherent detection schemes.

If we calculate the number of received photons per bit in order to maintain BER = 10^{-9} for PSK homodyne detection . Then

$$P_e = ½\,\text{erfc}\,(2N_P)^{1/2} \tag{2.38}$$

$$(2N_P)^{1/2} = 4.24$$

$$N_P = 18/2 = 9 \tag{2.39}$$

2.2 Qualitative Analysis

There are four main sources of error, which affect the performance of an optical detector.

1. Thermal noise, which is very small.
2. Dark current noise.
3. Amplitude or intensity response of detector.
4. Frequency response of detector.

In the case of ASK, coherent detection is more efficient than intensity modulation / direct detection (IM/DD). This because in IM/DD the dark current level is higher than in coherent detection. In ASK coherent detection homodyning is more efficient than heterodyning. It is so because the detected signal is in the base band itself rather than in the side-band as in the case of ASK heterodyne.

FSK detection is superior than ASK detection, as the amplitude in FSK does not change. The noise due to frequency shift is less than noise due to amplitude change especially for very small signals. Similarly a comparison can be done between ASK heterodyne to FSK heterodyne and ASK homodyne to FSK homodyne.

Phase shift keying modulation is superior than ASK and FSK. It is simply because in PSK the amplitude and frequency remain unchanged when bit-1 or bit-0 is detected. Homodyne PSK is the most superior detection system among all other detection system. Since it enables the BER to be maintained at the same level as with the other detection schemes, but with a very-very low number of detection photons.

2.3 Noise

Noise is defined as any undesired disturbance that masks the received signal in a communication system. In optical fiber communication system generally it arises due to spontaneous fluctuations.

There are three main types of noise due to spontaneous fluctuations.

i) Thermal noise

ii) Dark current noise

iii) Quantum noise

iv) Phase noise

2.3.1 Thermal Noise

It is the spontaneous fluctuation due to thermal interaction between, say, free electrons and the vibrating ions in a conducting medium and it specially exists in the resistors at room temperature. The thermal noise current I_t in a resistor R may be expressed by its mean square value [9] is given by:

$$i_t^2 = \frac{4kTB}{R} \qquad (2.40)$$

where k is Boltzmann's constant, T is the absolute temperature, and B is the post detection bandwidth for the resistance.

2.3.2 Dark Current Noise

Dark current is the reverse leakage current, which flows from the device terminals, even when there is no optical power incident on the photo-detector. Dark current contributes noise due to a random fluctuation in the photo-current.

It is given by

$$i_s^2 = 2\, e\, B\, I_d \tag{2.41}$$

where K is Boltzmann's constant, T is the absolute temperature and B is the post-detection (electrical) bandwidth of the system (assuming the resistor is in the optical receiver).

2.3.3 Quantum Noise

Due to quantum nature of light there is a quantum variation. Since hf > kT quantum fluctuations dominate over thermal fluctuations. In detecting photons, quantum limit is a result of the quantum nature of very weak optical signal. In the limit, the received signal may consist of only a few photons. It is found that the probability $P_{(Z)}$ of detecting z number of photons in time period T obeys the Poission distribution

$$P_{(Z)} = \frac{Z_M{}^Z \exp(-Z_M)}{Z!} \tag{2.42}$$

Where Z_M is the average number of photons received in interval T, Z is exact number of photons. So above equation tells us the probability of receiving exactly Z number of photons in time interval T if there are on the average Z_M number of photons in time T.

If we let the bit error rate (BER) be 10^{-9} or an average probability of error not exceeding 10^{-9}, we get 20 photons per pulse.

2.3.3.1 Digital Signaling Quantum Noise

For digital optical fiber communication it is possible to calculate a fundamental lower limit to the energy that a pulse of light must contain in order to be detected with a given probability of error. Assuming we have an ideal detector, which can detect a single photon and has zero dark current. Its function is to detect only if, there is a pulse is there or not. Only way an error can occur is if a light pulse is present and no electron-hole pair is generated in the detector. The probability of no pair being generated when a light pulse is present may be obtained by

$$P_{(Z)} = \frac{Z_M{}^2 \exp(-Z_M)}{Z!} \qquad (2.43)$$

$$P(0/1) = \exp(-Z_M)$$

Thus in the receiver described P(0/1) represents the system error possibility, $P_{(e)}$

$$P_{(e)} = \exp(-Z_M) \qquad (2.44)$$

For BER = 10^{-9}, then $Z_M = 20.7$

Although the quantum limit is 20 but it is not a realistic measure of the sensitivity detector,

i. when no signal is transmitted, receiver detects zero.

ii. when a signal is transmitted, the only output is due to signal photons from the transmitter.

For the purpose of studying the probability of error in binary decisions, noise is characterized with a Gaussian probability density function

Which is $$f_{(x)} = \frac{1}{\left[2\pi\sigma^2\right]^{1/2}} \exp\left[\frac{-(x-m)^2}{2\sigma^2}\right] \tag{2.45}$$

where m is the mean value of x and σ is the standard deviation of the distribution. When $f_{(x)}$ describes the probability of detecting a noise current or voltage, σ corresponds to the rms value of that current or voltage.

To find the probability that x lies between two specified values, say x_1 and x_2; we must integrate $f_{(x)}dx$ between limits x_1 and x_2. Although the statistics of quantum noise follow a Possion distribution, other important sources of noise within practical receivers are characterized by a Gaussian probability distribution. The Guassian approximation is sufficiently accurate for design purposes and is far easier to evaluate. The receiver sensitivity calculated by using Gaussian approximation is generally within 1dB of those calculated by other methods. Though the transmitted signal have two well defined light levels, in the presence of noise the signal at the receiver is not well defined.

2.3.4 Phase Noise

The term phase noise is used for describing short term random frequency fluctuations of a signal.

There are two types of fluctuations. The discreate signal fluctuation called spurious that appear distinct component in spectral density plot and the second one is random phase fluctuation. The source of phase noise in a oscillator is due to thermal and fliker or 1/f noise. In our experiment, since we have used same optical source as local oscillator so phase noise does not affect at all.

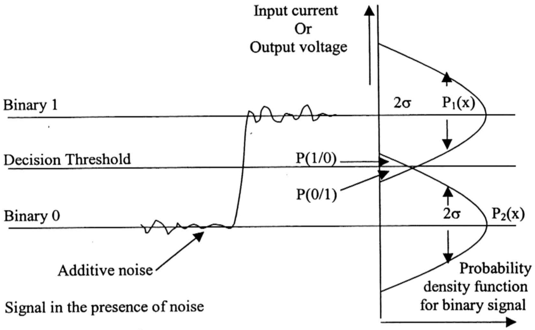

Fig. 2.3 Probability density functions for ASK heterodyne synchronous detection.

The probability density function (PDF) describes the probability that the input current (or output voltage) has a value *i* (or *v*) within the incremental range *di* (or *dv*). The expected values of the signal in the two transmitted states. namely 0 and 1, are indicated by $P_o(x)$ and $P_1(x)$ respectively.

The Gaussian probability function, which is continuous, is defined by

$$P_{(x)} = \frac{1}{\left[2\pi\sigma^2\right]^{1/2}} \exp\left[\frac{-(x-m)^2}{2\sigma^2}\right] \tag{2.46}$$

Where σ the standard deviation of distribution and m is the mean value of x

If a decision threshold D is set between the two signal states, as indicated in Fig. 2.3 signal levels greater than D are registered as a one and those less than D are registered as a zero. However if the noise current (or voltage) is sufficiently large it can either decrease a binary one to a zero or increase a binary zero to a one. These error probabilities are given by the integral of the signal probabilities outside the decision region. Hence the probability that a signal transmitted as a 1 is received as a 0, P(0/1), is proportional to the area indicated. The probability that a signal transmitted as a 0 is received as a 1, P(1/0), is similarly proportional to the other area indicated. The total probability of error P(e) may be defined as

$$P(e) = P(1)P(0/1) + P(0)P(1/0) \tag{2.47}$$

Let us consider a signal current i_{sig} together with an additive noise current i_N and a decision threshold set a $D = i_D$. If at any time a binary 1 is transmitted such that resulting current $i_{sig} + i_N$ is less than i_D then an error will occur.

$$i_{sig} + i_N < i_D \tag{2.48}$$

The corresponding probability of the transmitter as a 0 may be written as

$$P(0/1) = \int_{-\infty}^{i_D} P(i, i_{sig})\, di \tag{2.49}$$

$$P_1(x) = P(i, i_{sig}) = \frac{1}{\left(i_N^{\,2}\right)^{1/2}\sqrt{2\pi}} \exp - \left[\frac{\left(i - i_{sig}\right)^2}{2 i_N^{\,2}}\right]^{1/2} \tag{2.50}$$

Where i is the actual current, i_{sig} is the peak signal current during a binary 1 (this corresponds to peak photocurrent I_P when only a signal component is present), and i_N^2 is the mean square noise current.

$$P_1(x) = G_{sn}\,[i,\ i_{sig},\ (i_N^2)^{1/2}] \qquad (2.51)$$

$$P(0/1) = \int_{-\infty}^{i_D} G_{sn}\left[i, i_{sig}, \left(i_N^{\,2}\right)^{1/2}\right] di \qquad (2.52)$$

Similarly

$$P(1/0) = \int_{i_D}^{\infty} p(i,0) \qquad (2.53)$$

$$P_0(x) = P(i,0) = \frac{1}{\left(i_N^{\,2}\right)^{1/2}\sqrt{2\pi}} \exp - \left[\frac{(i-0)^2}{2i_N^{\,2}}\right]^{1/2} \qquad (2.54)$$

$$= G_{sn}\,[i,\ 0,\ (i_N^2)^{1/2}] \qquad (2.55)$$

$$P(1/0) = G_{sn}\,[i,\ 0,\ (i_N^2)^{1/2}] \qquad (2.56)$$

The error function is an integral of the Gaussian probability density function from which the probability of error can be found. It is a well-known tabulated function as shown in the Appendix A, defined as

$$\text{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-Z^2)dz \qquad (2.57)$$

and its complementary error function is

$$\text{erfc}(u) = 1 - \text{erf}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} \exp(-Z^2)dz \qquad (2.58)$$

$$P(0/1) = \frac{1}{2}\left[1 - erf\left(\frac{i_{sig} - i_D}{\left(\overline{i_N^{\,2}}\right)^{1/2}\sqrt{2}}\right)\right] \tag{2.59}$$

$$= \frac{1}{2}\left[erfc\left(\frac{i_{sig} - i_D}{\left(\overline{i_N^{\,2}}\right)^{1/2}\sqrt{2}}\right)\right] \tag{2.60}$$

and

$$P(1/0) = \frac{1}{2}\left[erfc\left(\frac{0 - i_D}{\left(\overline{i_N^{\,2}}\right)^{1/2}\sqrt{2}}\right)\right] = \frac{1}{2} erfc\left(\frac{\left|-i_D\right|}{\left(i_N^{\,2}\right)^{1/2}\sqrt{2}}\right) \tag{2.61}$$

If we assume that a binary code is chosen such that the number of transmitted ones and zeros are equal, then P(0) = P(1) = 1/2 , and net the probability of error is one half the sum of the indicated area.

$$P(e) = ½ [P(0/1) + P(1/0)] \tag{2.62}$$

by substitution

$$P(e) = ½[\frac{1}{2} erfc\left(\frac{i_{sig} - i_D}{\left(i_N^{\,2}\right)^{1/2}\sqrt{2}}\right) + \frac{1}{2} erfc\left(\frac{\left|-i_D\right|}{\left(i_N^{\,2}\right)^{1/2}\sqrt{2}}\right)] \tag{2.63}$$

Equation 2.63 may be simplified by setting the threshold decision level at the mid-point between zero current and peak current, such as $i_D = i_{sig}/2$.

By substitution

$$P(e) = \frac{1}{2}\left[\frac{1}{2}erfc\left(\frac{i_{sig}/2}{\left(\overline{i_N^{\,2}}\right)^{1/2}\sqrt{2}}\right) + \frac{1}{2}erfc\frac{\left|-i_{sig}\right|/2}{\left(\overline{i_N^{\,2}}\right)^{1/2}\sqrt{2}}\right] \qquad (2.64)$$

$$= \frac{1}{2}erfc\left(\frac{i_{sig}}{2\left(\overline{i_N^{\,2}}\right)\sqrt{2}}\right) \qquad (2.65)$$

The electrical signal to noise ratio (SNR) at the detector may be written in terms of the peak signal power to rms noise power as

$$\left(\frac{S}{N}\right) = \frac{i_{sig}^{\,2}}{\overline{i_N^{\,2}}} \qquad (2.66)$$

$$P(e) = \frac{1}{2}erfc\left(\frac{(S/N)^{1/2}}{2\sqrt{2}}\right) \qquad (2.67)$$

2.3.5 Summary Of Optical Receiver Sensitivity

	Average number of photon per bit		
Modulation	Homodyne Detection	Heterodyne Detection	Direct Detection
ASK	18	36	21
FSK	……..	36	…….
PSK	9	18	……..

2.4 References

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