CHAPTER 3

3.0 THEORETICAL FRAMEWORK

This chapter explains two theories underlying the research area, primarily it elaborates upon the hedging mechanism and, subsequently, discusses the Markowitz portfolio selection concept. The hedging theory begins with the historical information on the establishment of traditional derivative markets. Furthermore, it elaborates the purpose of hedging strategy, market participants and, finally, spot and futures price behaviour. For the portfolio theory, the chapter starts with the basic concept of portfolio selection within the mean variance framework, discussing the risk and return trade off measurement. Then, the respective sections relate the concept of hedging within the portfolio theory, and demonstrate the hedging portfolio risk and return measurement. The chapter further justifies the rationality of the minimum variance framework within the hedging context. Next, are brief explanations on hedging performance measurement and, ultimately, summaries of both theories in fulfilling the research gap identified in Chapter 2 (Literature Review).

3.1 HEDGING THEORY

3.1.1 Historical Background

Primitive futures' trading was started in 1850 but there was no proper exchange established during that period. The Chicago Board of Trading (CBOT) was the first futures exchange being introduced some 24 years later. Since its inception, the market has only been involved in commodity futures trading activities. Commodities futures can be categorised into two groups include storable and non-storable commodities. The storable commodities involve any type of commodities that can be stored at a certain period for example CPO, crude oil, soybean, coffee bean etc. While, the non-storable commodities involve any commodity that unable to be kept up to a certain period for example cattle and electricity. Later the CBOT introduced the Government National Mortgage Association (interest rate futures) as the first financial futures instrument in1975. There are contrasting views concerning the main driver for the futures market – speculators or hedgers. However, in 1920 the futures market was driven by speculators rather than hedgers. Nevertheless, later on there is statistical proof that hedgers played an important role as the main engine to move the futures market.[8] Based on the commodity futures trading mechanism, Working defines hedging as *"the purchase or sale of futures in conjunction with another commitment, usually in expectation of a favorable change in the relation between spot and futures prices"*.

The hedging theory was further discussed in Johnson's (1960) paper. He made a distinction between the roles of the hedgers and the speculators in the commodity futures market. Based on the interview survey results, Working (1953) claims that hedging for profit maximizing rather than reducing risk did not represent the true behaviour of hedgers present in such markets. Hedgers ensure the security of holding futures contracts by consistently measuring the benefits gained on mitigating the price risk of such commodity prices. As the buyer, unfavourable movement of commodity

[8](Working, 1953a) pp.326.

prices will result in unpredictable costs of purchasing such commodities. Consequently, commodity buyers will tend to protect their position by engaging in more futures contracts. Less futures contracts are involved from the commodity sellers' point of view. However, for speculators, there are two types of speculators – indirect speculators and direct speculators. A direct speculator will take advantage of the change in price on the same market (within spot or within futures market), whereas the indirect speculators will take advantage of the price changes in the other markets (basis) (Johnson, 1960).

3.1.2 Purpose of hedging

There are many reasons why people hedge, however, primarily, the decision is based on the market participant's anticipation of both spot and futures price movements that indicate the necessity to hedge or not. Such a scenario is synonymous with the price discovery channel where market participants use the futures price movement as the anticipated future movement of spot prices. One of the important roles of futures market is as the price discovery channel, therefore market participants are able to buy or sell at a fair price[9]. Secondly, a hedging strategy does not absolutely eliminate the price risk but is rather a risk minimization strategy. In addition, traders also aim for a return maximization motive, together with a risk minimization objective (Working, 1953b). The study will not intend to measure the effectiveness of futures market via price discovery context. Nevertheless, focus on measuring the effectiveness of hedging strategy within risk minimization and risk and return trade off context.

[9] Avoiding the overpriced or underpriced situation

For firms, the decision of hedging is not as simple as for individual investors. Bessembinder (1991) suggests that the reasons for firms to hedge are quite dissimilar to individual investors. Firms with higher growth opportunities, high financial constraints, foreign exchange rate exposure and economies of scales tend to use derivative instruments as a hedging tool rather than a speculating tool (Geczy, Minton and Schrand,1997). In an imperfect market, there are various risks (interest rate risk, commodity price volatility, etc.) that potentially affect a company's financial and investment decisions. As such, firms will implement hedging as one of their risk management programmes to mitigate their risk exposure. Since the market is imperfect, we can conjecture that hedging will have implications on the firm's market value. In contrast, the firm's capital structure is irrelevant and a hedging strategy will not increase a firm's value when the market is perfect (Modigliani and Miller, 1958 and 1961). At the firm level, first, when a firm has a tax convexity function the firm's[10] income volatility is quite costly and hedging will minimize the income fluctuation, making it less volatile and more certain (Smith and Stulz;1985). A firm with a highly volatile income stream tends to have high tax expenses (Stulz,1996; Froot *et al*., 1993; Nance, Smith and Smithson, 1993; and Graham and Rogers,2000), therefore, hedging will minimize the risk and also the tax expenses. However, Graham and Rogers (2002) fail to find any significant evidence that firms tend to hedge due to tax convexity, as the benefit is relatively small.

Second, hedging strategy is empirically proven to alleviate a firm's gearing capacity and further translates into tax shield benefits. Leland (1998) claims that when

[10] A tax convex function is where firms having a current net operating loss carry forward.

firms hedge they tend to hold more debt and when more debt is held the firm will have a higher tax shield and, ultimately, increase the firm value. Similar results were reported in Graham and Rogers (2000 and 2002). In addition, another hedging benefit relates to a firm's potential financial distress. A hedge user firm tends to have a stable and certain income stream and a very slim chance of facing bankruptcy (Working,1953 c; Smith and Stulz,1985; Stulz,1996, Haushalter,2000; Graham and Rogers 2000 and 2002).

In addition, hedging is able to strengthen a firm's future earnings and may further reduce the chances of a firm to let go potential profitable project opportunities (Myers and Majluf,1984). If the firm is highly dependent on the internal source of financing, this hedging strategy is considered to be one of the cheapest strategies to protect the firm's project risk. When the internal capital is costly to be raised, the firm is not able to raise external sources of funding to finance its potential project. In addition, if the firm is actively involved in hedging, it will raise the source of internal funding and further reduce the need to seek external funding and associated costs in financing the potentially profitable project (Bessembinder,1991; Stulz,1996; Nance, Smith and Smithson, 1993; Graham and Rogers,2000).Moreover, managerial risk aversion characteristic is another factor that encourages a firm to hedge. For managers in a firm having a concave utility function, the high fluctuation in a firms' potential income tends to effect the managerial compensation distribution (Smith and Stulz, 1985). Hence, if the firm's managers are risk averse, they will support the firm to hedge, as it will reduce the uncertain movement of future income stream. However, Geczy et al. (1997) and Haushalter (2000) contended this matter.

Additionally, through hedging, a firm can improvise its gearing capability and contract term, and, further, will reduce the firm's existing agency cost (Bessembinder,1991). This risk management technique is able to protect the stakeholder's investments, as it will increase the stakeholders' confidence in the firm's sustainability. Therefore, it will further improve the firm's contract term and, ultimately, the firm's value (Stulz,1996). Additionally, firm's have higher research and development expenses and offer higher dividends when they actively adopt hedging vis-à-vis non-hedger firms (Nance, Smith and Smithson, 1993).

Generally, based on this empirical evidence, it supports that a firm will hedge in order to reduce the uncertain movement in a firm's earning, then, with the certain earning stream the firm will further increase the gearing capability to finance its profitable future investment opportunities. A more certain earning stream will also improve the stakeholders contact term and confidence in the firm's financial standing and, additionally, reduce the chances of bankruptcy cost. With a stronger financial standing, more debt acquired, it will translate into a tax shield benefit and, finally, the firm's value will increase. However, when the interest rate, exchange rate and commodity prices change simultaneously, most empirical evidence greatly overemphasizes the risk protection served by hedging. In such a scenario, the derivative instruments held by firms tend to offer minimal risk reduction against the overall level of risk exposed by the companies, however, the benefit is more than the cost of implementing the strategy (Guay and Kothari 2003).

3.1.3 Hedging Performance Measurement

Since the establishment of the derivative market, Ederington (1979) is among the first to study the hedging performance of the Government National Mortgage Association and T-Bill Futures Markets. He rationalizes the minimum variance framework (introduced by Markowitz) in measuring the degree of risk minimizing via the hedging position. The Markowitz portfolio theory defines the hedging performance through calculating the variance of both the unhedged and hedge position. Referring to the classical theory of hedging the spot return or unhedged returns (U) can be calculated based on the spot price difference between two periods while the hedge returns (H) can be derived from the difference gained from the spot and futures market.

i) Based on price changes

Unhedged position (U)

$$U = X\left\lfloor P_s^{\,2} - P_s^{\,1} \right\rfloor \qquad \textbf{(1)}$$

where U represents the absolute Unhedged return, X is the quantity of the spot contract, while P_s^{1} is the Spot price at period 1 and P_s^{2} for Spot price at period 2.

$$H = X\left\{\left\lfloor P_s^{\,2} - P_s^{\,1} \right\rfloor - \left\lfloor P_f^{\,2} - P_f^{\,1} \right\rfloor\right\} \qquad \textbf{(2)}$$

where H is the absolute hedge return, X is the quantity of spot contract. P_f^1 and P_f^2 refer to the futures price at period 1 and at period 2, respectively. Since Stein (1961) posits that the spot price tends to move together with the futures price, logically the absolute value for H is less than the unhedged return. Therefore, we can assume that the variance for H is smaller than U.

ii) Based on basis changes

$$Basis = \left\{ \left(P_f^{\,2} - P_s^{\,2} \right) - \left(P_f^{\,1} - P_s^{\,1} \right) \right\} \qquad \textbf{(3)}$$

So

$$= \left\{ \left(P_f^{\,2} - P_f^{\,1} \right) - \left(P_s^{\,2} - P_s^{\,1} \right) \right\}$$

Similarly, with the returns based on price changes, we can agree that the value of hedged returns (basis changes) is also less than the amount gained from the unhedged position. This follows the classical hedging theory that assumes that the variance with hedging will be less than the variance for the unhedged position (σh < σu). Additionally, Paroush and Wolf (1989) highlight that the basis risk consist three elements include quality, location and timing. Meanwhile the volatility of basis risk in financial market can be determined by risk premia and business cycle factors. Besides, the production and firm default factors tend to influence the variance of bases in commodity market (Bailey and Chan, 1993). Paroush and Wolf (1989) infer that the basis risk has an adverse relationship with production and level of hedging decision. When basis risk increase, hedgers tend to reduce their hedging position (vice versa).

Castelino (1992) also acknowledge that a higher basis risk may results into a reduction in hedging ratio (hedging decision). Besides, Ederington (1979) claims that a perfect hedging can be achieved when the basis difference is equal to zero. However, Samuelson's pricing theory (1965) proves that basis will unfailingly not equal zero but will merge to zero when it is close to the maturity period. Hence, hedgers may only bring down the price risk with the condition that the spot risk must be larger than the basis risk. In contrast, Nelson and Collins (1985) infer a greater basis risk than the spot price risk in grain elevator market, however the hedgers able to minimize their total risk exposure compared to the non hedgers. The literature infers that basis risk exposed by hedgers is much smaller compared to the risk in either the spot or futures' market (Lien, 1992). Thus, although hedgers are exposed to the basis risk, they can still achieve the risk minimization goal via hedging.

Apart of basis risk, Penning and Meulenbergs (1997) among the pioneer to relate the liquidity risk into hedging effectiveness measurement. They claim that thin commodity futures markets may affect the hedging performance results. They tested the potato futures market that has an average of 200,000 trading volume per year. Based on their evidence, the results infer that the omission of the liquidity proxy (such as transaction size) give a slightly different risk reduction results compared to conservative Ederington measurement however, the magnitude is low. Based on two justifications, the study does not wish to include the liquidity risk into hedging performance measurement. Primarily, it is because of trivial hedging performance improvement displayed when we include the liquidity risk into hedging performance measurement and secondly, CPO market is among the most actively traded CPO market, globally, as such the study assumes the non presence of thin market problem in this tested market. For example, the annual volume traded for CPO-3mth contract recorded a huge increment from 563,050 in year 1995 to 1,605,411 in year 2007 (source: Bursa Malaysia Bhd).

The futures market is well known to be a price discovery channel for the spot market, hence, futures prices tend to adjust to the new information earlier than in the spot market (see Abhyankar, 1995). Thus, we can conjecture that the future price changes should not have the same magnitude in changes as in spot prices. Empirically, the future and spot prices tend to move together but not in an exact size price change. Ederington (1979) extended the adaptive expectation model to demonstrate the expected spot price changes using the following model:

$$E_n^{\,2} - E_n^{\,1} = \alpha\left[P_s^{\,2} - E_2^{\,1}\right] + u \qquad \textbf{(4)}$$

where E_n^2 -E_n^1 represents spot prices expected to prevail in period 'n' as of between period 2 and 1 and E_2^1 denotes as a price at period 1 but prevails at period 2. Based on equation 4, the expected spot price will base determine on $E_n^{\,1} + \alpha\left[P_s^{\,2} - E_2^{\,1}\right] + u$.

Now substituting $P_f^{\,2} = E_n^{\,2}$ and $P_f^{\,1} = E_n^{\,1}$ to demonstrate the price changes in futures market, so $P_f^{\,2} - P_f^{\,1} = \alpha\left[P_s^{\,2} - P_s^{\,1}\right] - \alpha\left[E_2^{\,1} - P_s^{\,1}\right]$. When there is no movement in spot prices between two periods, where $(E_2^{\;1} = P_s^{\;1})$ and $\alpha \neq 1$, so the changes of futures price is determined by $\alpha[P_s^{\;2} - P_s^{\;1}]$.

$$P_f^{\;2} - P_f^{\;1} = \alpha[P_s^{\;2} - P_s^{\;1}] - \alpha[P_s^{\;1} - P_s^{\;1}]$$

$$P_f^{\;2} - P_f^{\;1} = \alpha[P_s^{\;2} - P_s^{\;1}]$$

Instead, if P_s^{2} equal to E_2^{1} there will be no movement in futures price.

$$P_f^{\;2} - P_f^{\;1} = \alpha[P_s^{\;2} - P_s^{\;1}] - \alpha[E_2^{\;1} - P_s^{\;1}]$$

$$P_f^{\;2} - P_f^{\;1} = \alpha[E_2^{\;1} - P_s^{\;1}] - \alpha[E_2^{\;1} - P_s^{\;1}]$$

$$P_f^{\;2} - P_f^{\;1} = 0$$

The above mathematical illustrations validate two scenarios, first if there is no movement in spot prices between two periods, where $(E_2^{\;1} = P_s^{\;1})$ and $\alpha \neq 1$, so the changes of futures price is determined by $\alpha[P_s^{\;2} - P_s^{\;1}]$. And second, if the expected spot is equal to the actual spot prices in period 2, so there will be no changes in futures prices ($P_f^{\;2} - P_f^{\;1} = 0$) . These illustrations confirm that there will be unequal movement between futures price changes and spot price changes. Such pattern will clearly explain that the market participants can naturalize their loss by engaging in a hedging strategy. The loss can be minimized by the gain in one market (spot market) being offset by the losses suffered in another market (futures market). Therefore, for measuring hedging effectiveness and simplicity, the research posits that futures price is an unbiased prediction of the expected spot price (or martingale process[11]). Kofi (1973) highlight that discontinuous inventory (eg potato) futures market is a less reliable predictor for its spot price but not for continuous inventory futures market (eg Soybean and Corn).

3.2 PORTFOLIO THEORY

The modern portfolio theory establishes the concept of diversification (constructing a portfolio) in a market participant's investment strategy. The theory further emphasizes the crucial role of optimizing the investors' portfolio and further illustrates the risky assets pricing measurement (Markowitz, 1952). He suggests that a portfolio selection will only be made based on the investors' own experience and judgment of the expected portfolio performance in the future. The selection of the portfolio will be made based on the combination of the expected returns that the investor has potentially gained while considering the uncertainty that such expected returns can be achieved (mean variance framework). The expected return refers to an average return that the investor gains. Considering that there are two assets (asset m and asset n) in a portfolio, the portfolio's expected return can be computed as follows:

$$E(R_p) = [(w_m)R_m + (w_n)R_n] \tag{5}$$

[11] The martingale process refers to a probability of a fair game theory (Musiela and Rutkwoski, 2005). Intuitively, martingale process is a stochastic process such that the conditional expected value of an observation at to some time t, given all the observations up to the earlier time s, is equal to the observation at that earlier time s (www.wikipedia.org).

where $E(R_p)$ denotes as expected return of a portfolio, while w_m and w_n as weight for assets m and n. And, R_n and R_m are returns for asset n and m.

Meanwhile, Stein (1961) defines the risk element as the uncertain scenario faced by the investor that the intended expected return may or may not be achieved. In general, risk is always related to the variance or standard deviation (Tobin, 1958). By relating the risk with the portfolio selection decision process, another component that is also important is portfolio variance. The variance represents the deviation of the expected portfolio return and best refers to the variance or standard error/standard deviation. These parameters estimate the level of uncertainty of the actual portfolio return deviate or difference from the expected portfolio return.

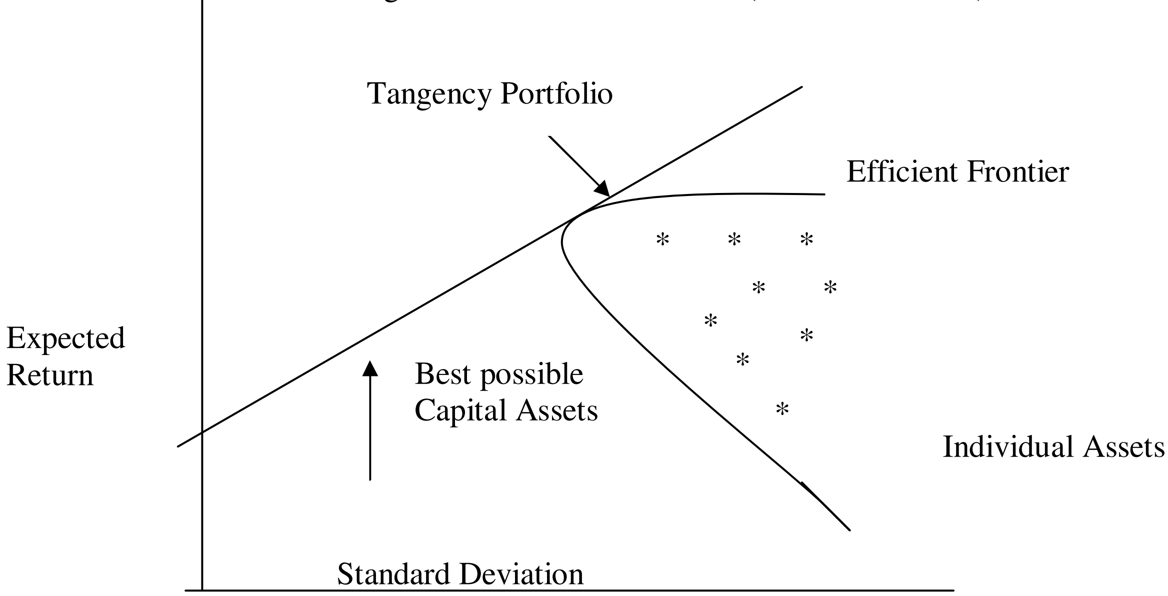
Using the previous example, where a portfolio consists of asset m and asset n, the portfolio variance can be computed base on:

$$VAR(Port) = w_m^{\ 2}\sigma_m^{\ 2} + w_n^{\ 2}\sigma_n^{\ 2} + 2w_m w_n \sigma_{mn} \qquad \textbf{(6)}$$

where VAR (Port) is relates to the Portfolio Variance, σ_n^2 , σ_m^2 and σ_{mn} refers to the variance for assets n, m and the covariance for both assets, respectively. The portfolio's standard deviation can be computed by taking the square root of the portfolio variance. Both the variance and standard deviation synonymy relates to the risk of the portfolio.

Figure 3.1 illustrates the portfolio efficient frontier developed by Markowitz. The optimal efficient portfolio lies on the efficient frontier[12] and investors will choose the combination of the lowest risk at a given level of expected return. Markowitz outlines another condition, where investors tend to choose the portfolio that gives the highest return but at a given level of risk.

Figure 3.1: Efficient Frontier (Markowitz, 1952)



Source: Reilly and Brown (2003: pp229)

Now in respect of the hedging context, we can combine both the hedging and portfolio theory together. The concept of hedging involves opposite trading contracts taken simultaneously in the spot and futures market. Now, no asset *m* or *n* are involved but just the return or loss position in both the spot market and futures market. To get the unhedged return or return in spot market (E(U));

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[12] The efficient frontier can be defined as the combination of various optimal portfolios that require either condition i or ii:

i) The lowest risk at a given level of return
ii) The highest return at a given level of risk

$$E(U) = X_s E\left(P_s^2 - P_s^1\right) \quad \textbf{(7)}$$

where X_s denotes the number of spot contracts.

Next, we combine the return gained in the spot market with the return achieved in the futures market (referring to E(R) = Expected return with hedge position).

$$E(R) = X_s E\left[P_s^2 - P_s^1\right] + X_f E\left[P_f^2 - P_f^1\right] - K\left(X_f\right) \quad \textbf{(8)}$$

where X_f is the number of futures contracts while, $K(X_f)$ is the transaction cost involved in futures trading. Commonly, hedging in the futures market does not involve substantial transaction cost scale economies (Nance, Smiths and Smithson, 1993). In the commodity futures markets, researchers acknowledge the existence of carrying cost[13], however it is not an easy task to generate a standard model with this cost into the hedging performance model (Alexander and Barbosa, 2007). In addition, if the transaction cost is introduced in the hedging ratio estimation model, empirically the investors or firm investment utility function is affected (presented in Kroner and Sultan, 1993 for currency market; Park and Switzer, 1995 and Yeh and Gannon, 2000 for stock index market). These five researchers found that the transaction cost will only take effect when hedgers rebalance their position and that hedgers are said to rebalance their current futures position when the realized gain is sufficient to cover the transaction cost. Additionally, the transaction cost will further bring down the expected investor utility gain, however, the dynamic estimation model tends to outperform the hedging performance of the static one, although the transaction cost is included in the model.

[13] Storage cost, and (risk free rate -convenient yield).

Subsequently, market participants update their hedging strategies timely but without any storage effect (refer to Mathews and Holthausen, 1991; Howard and D. Antonio, 1991 and Lence, Kimle and Hayenga, 1993). In general, since the transaction cost for hedging is very minimal, we can assume the insignificant impact of the cost element towards the hedging performance measurement.

"Why do you prefer, out of two funds with the same average compound return for the time horizon you say is crucial to you, the fund with the lower intra-horizon variance of the annual return?" … " We naturally feel that the fund with the steady yield through thick and thin has an uncanny ability to come up with better guesses about the true (unknown) probabilities we are going to be running up against in the future. It will likely to be better at avoiding losers".

Samuelson and Merton (1974), pp37-38

In the mean variance framework, one of the common conditions for an efficient portfolio is investors aiming at higher expected returns but with a given level of risk. However, in reality a higher return is associated with higher risk. In another condition, an efficient portfolio can be achieved when the portfolio with the lowest risk has a given level of expected return. Under the second condition, the investors are assumed to be risk averters (or less risk tolerant). Higher risk averters (risk aversion) can be defined as individuals that are highly sensitive to the level of risk they face and the level of risk plays an integral factor in their investment decision making (Merton, 1969). Samuelson and Merton (1974) suggest that investors tend to be more risk adverse when they focus on a certain lower gaining situation rather than an uncertain higher gaining situation. Holthausen (1979) outlines the characteristics of risk adverse firms, where he assumes that a firm is highly risk adverse when the price fluctuation is more volatile and the firm tends to hedge more. He defined risk adverse firms as firms that are willing to pay an additional amount to get low but more certain income. Furthermore, similar findings can be only achieved when a firm's risk aversion elasticity is less than one (Broll and Wahl, 2004).

Therefore, a portfolio with a lower risk and return (vice versa), is known to be an inefficient portfolio. This framework further highlights that a portfolio with strong positive correlated combined asset returns fails to reduce the risk via diversification. The efficiency of the portfolio in reducing risk can only be achieved when the asset returns are negatively correlated. In addition, when we relate with hedgers, the essence of the efficient portfolio, involving maximizing the Gagnon *et al.* (1998) investors utility function(Ω):

$$Max\Omega = E_t(R) - \frac{1}{2}\tau VAR(H) \tag{9}$$

where VAR(H) denotes the variance for the hedging portfolio and τ refers to the risk aversion parameter.

To compute the variance of the hedging portfolio, we need to get the unhedged variance (representing the risk of actual spot return deviate from its mean).

$$VAR(U) = X_s^{\,2}\sigma_s^{\,2} \tag{10}$$

where *VAR(U)* is the variance for the unhedged position, and σ_s^2 denotes the variance for the spot return.

The variance for the hedging portfolio can be achieved by combining the risk factor presence in both the spot and futures market:

$$VAR(H) = X_s^2 \sigma_s^2 + X_f^2 \sigma_f^2 - 2X_s X_f \sigma_{sf} \qquad \textbf{(11)}$$

where σ_s^2 and σ_{sf} represent the variance for future return and the covariance between the spot and future return.

Assuming that X_s is equal to 1 contract and let $b = -\frac{X_f}{X_s}$ represent the proportion of the spot position, which is the hedged or hedging ratio. The negative sign (- X_f) represents an opposite transaction taken (either short in spot and going long in futures vice versa) by hedges in futures contracts.

Assuming the hedger enters short in a futures market, b will be:

$$b = -\frac{X_f}{X_s}$$

$$b = -\left[\frac{-X_f}{X_s}\right]$$

$$b = +\frac{X_f}{X_s}$$

Since X_s and X_f have contrary signs (either positive or negative) b will always carry a positive definite value. Now substituting $b = -\frac{X_f}{X_s}$ into the expected return for the hedging position.

The return on hedging position (refer equation 8 on page 62) will be:

$$E(R) = X_s E\left(P_s^{\,2} - P_s^{\,1}\right) + X_f E\left(P_f^{\,2} - P_f^{\,1}\right) - K\left(X_f\right)$$

$$= X_s \left[E\left(P_s^{\,2} - P_s^{\,1}\right) - bE\left(P_f^{\,2} - P_f^{\,1}\right) - K\left(X_{s},_{b}\right)\right]$$

$$= X_s \left[(1-b)E\left(P_s^{\,2} - P_s^{\,1}\right) + bE\left(P_s^{\,2} - P_s^{\,1}\right) - bE\left(P_f^{\,2} - P_f^{\,1}\right) - k\left(X_{s},_{b}\right)\right]$$

Let $E(\Delta b) = E\left\{\left(P_f^{\,2} - P_s^{\,2}\right) - \left(P_f^{\,1} - P_s^{\,1}\right)\right\}$ represent the changes in basis. So $E(H)$ will be equal to:

$$E(H) = X_s\left[(1-b)E(S) - bE(\Delta B) - k\left(X_{s},_{b}\right)\right]$$

Now, setting the condition if $\Delta B = 0$ (basis equal to zero), $b = 1$ (represent naïve hedging).

$$E(H) = X_s\left[(1-b)E(S) - b(0) - K\left(X_s, 1\right)\right]$$

$$E(H) = X_s\left[(1-1)E(S) - 1(0) - K\left(X_s, 1\right)\right]$$

$$E(H) = X_s\left[E(S)\right]$$

When the basis risk is zero, and $b = 1$, the changes in expected profit or loss are driven by the expected return from the unhedged portfolio. Now substituting $-\frac{X_f}{X_s} = b$ into Var(H) equation (refer to equation 11 on page 65).

$$
\begin{aligned}
Var(H) &= X_s^{\;2}\sigma_s^{\;2} + X_f^{\;2}\sigma_f^{\;2} + 2b\,\mathrm{cov}_{sf} \\
&= X_s^{\;2}\sigma_s^{\;2} + X_s^{\;2}\left(-\frac{X_f^{\;2}}{X_s^{\;2}}\right)\sigma_f^{\;2} + 2X_s^{\;2}\left(-\frac{X_f}{X_s}\right)\mathrm{cov}_{sf} \\
&= X_s^{\;2}\left(\sigma_s^{\;2} + b^2\sigma_f^{\;2} - 2b\,\mathrm{cov}_{sf}\right)
\end{aligned}
$$

The mean variance framework explains that an efficient portfolio is achievable when the portfolio is able to maximize the investors' utility function. However, this research is not interested in seeking the optimization of the investment utility function but is more attracted to investigating the significant effect on the utility function using minimum variance hedging ratio (concentrate on reducing risk element) at a given range of risk aversion (τ) parameter (similar concept demonstrated in Yang and Allen, 2004).

The hedging ratio is the proportion of the futures contract against the spot contract (Ederington, 1979). The hedging ratio can be estimated either within the mean variance or the minimum variance context. According to Sephton (1993) the mean variance hedging ratio can be estimated as follows:

$$MEANVHR = \frac{Cov_{sf}}{\sigma_f^2} - \frac{E(f_t - f_{t-1})}{2\tau(\sigma_f^2)} \qquad \textbf{(12)}$$

where $E(f_t)$ represents the expected futures price.

Ford *et al.* (2005) found that the mean variance hedging ratio is much similar to the MVHR estimation results. Further, many earlier researchers suggest the minimum variance hedging ratio as a more practical method compared to the mean variance hedging ratio. This preference is because of two reasons; first, the framework defines the hedging ratio as consisting of the hedging and speculative elements. The hedging element refers to a simpler minimum variance hedging ratio (MVHR) while the speculative element is a more complex task to estimate, and second, speculators have differing risk aversion levels. Therefore, varying risk aversion levels will lead to a different combination of risk and return and it is difficult to cater for all these possible combinations in a model (detail refer to Sephton, 1993).

Additionally, Bond and Thompson, (1985) infer that the risk aversion (refer to τ) is irrelevant in the hedging ratio estimation process, if there is no transaction cost (e.g. storage cost and other related cost). In order words, with the presence of transaction costs, the risk aversion parameter does influence the hedging ratio estimation. Both authors highlight that very few studies attempt to explore and confirm the severity of the risk aversion effect on the hedging ratio estimation. With very limited evidence on risk attitude measurement, it is almost impossible to study the effect of acceptable risk tolerance towards the hedging ratio. Furthermore, in earlier evidence suggests that the hedging ratio is independent of the investors level of risk aversion (see Telser, 1955 & 1956; Johnson, 1960; Ward and Fletcher, 1971; Heifner, 1972; and Ederington, 1979).

Similarly, Karp (1983) conceptually infer that hedgers constantly revise their hedging decisions but their risk aversion parameters remain static. Later, Lien (2001) introduces the hedging concept under disappointment aversion. The conceptual paper infers that either in the normal backwardation or cotango scenario, market participants will hedge up to full spot position when they have a high disappointment aversion level. Since most literature indicates that firms tend to be more risk adverse, they tend to hedge more when the market is more volatile. However, Brorsen (1995) demonstrates that when the risk aversion assumption is dropped firms tend to have a risk neutral behaviour. Nevertheless, they will hedge more when the market is more volatile (assuming there is no relationship between the value of capital and output prices) and when the firm is a highly geared firm (assuming existence of transaction cost). Based on the above evidence, we generally agree that the risk adverse parameter will not influence the level of hedging position implemented by the hedger from time to time. As such, we consider that the hedging decision (hedging ratio) encompasses the minimum variance framework.

As the hedger's ultimate goal is to reduce their risk exposure, the MVHR is a more reasonable concept in hedging performance measurement. However, a MVHR will fulfil the condition of such utility function in the event that futures follow the fair game theorem or the highest τ parameter (Kahl, 1993; Sephton, 1993; and Moschini and Myers, 2002). In the minimum variance framework, MVHR represents the ratio (b) of futures contract against the spot contract. Since X_s^2 is assumed to be constant (1), thus, $Var(H^*) = \left(\sigma_s^{\,2} + b^2\sigma_f^{\,2} - 2b\,\mathrm{cov}_{sf}\right)$.

Thus, to get MVHR we need to differentiate b:

$$\frac{dVar(H)}{db} = X_s^{\ 2}\left(\sigma_s^{\ 2} + b^2\sigma_f^{\ 2} - 2b\,\mathrm{cov}_{sf}\right)$$

$$= X_s^{\ 2}\left(2b\sigma_f^{\ 2} - 2\,\mathrm{cov}_{sf}\right)$$

$$X_s^{\ 2}\left(2b\sigma_f^{\ 2} - 2\,\mathrm{cov}_{sf}\right) = 0$$

$$2b\sigma_f^{\ 2} - 2\,\mathrm{cov}_{sf} = 0$$

$$2b\sigma_f^{\ 2} = +2\,\mathrm{cov}_{sf}$$

$$b = \frac{+2\,\mathrm{cov}_{sf}}{2\sigma_f^{\ 2}}$$

so $b = \dfrac{\mathrm{cov}_{sf}}{\sigma_f^{\ 2}}$ (MVHR)

$$Var(H^*) = X_s^{\ 2}\left(\sigma_s^{\ 2} + b^2\sigma_f^{\ 2} - 2b\,\mathrm{cov}_{sf}\right)$$

Now, substitute $b = \dfrac{\mathrm{cov}_{sf}}{\sigma^2{}_f}$ into *Var(H*)*equation :

$$Var(H^*) = X_s^{\ 2}\left[\sigma_s^{\ 2} + \left(\frac{\mathrm{cov}_{sf}^{\ 2}}{\left(\sigma^2{}_f\right)^2}\right)\left(\sigma_f^{\ 2}\right) - 2\left(\frac{\mathrm{cov}_{sf}}{\sigma^2{}_f}\right)\mathrm{cov}_{sf}\right]$$

$$= X_s^{\ 2}\left[\sigma_s^{\ 2} + \left(\frac{\mathrm{cov}_{sf}^{\ 2}}{\sigma_f^{\ 2}}\right) - 2\left(\frac{\mathrm{cov}_{sf}^{\ 2}}{\sigma_f^{\ 2}}\right)\right]$$

$$= X_s^{\ 2}\left(\sigma_s^{\ 2} - \frac{\mathrm{cov}_{sf}^{\ 2}}{\sigma_f^{\ 2}}\right)$$

"The effectiveness of hedging is measured by considering the gain and loss due to the price changes incurred in an unhedge position relative to that incurred in a hedge position."

Johnson (1960), pp144

Johnson (1960) defines hedging effectiveness or hedging performance measurement as the degree of risk reduction achievable by hedgers vis-à-vis to non hedgers. Similarly, Lence, Kimle and Hayenga (1993:pp131) define hedging effectiveness as the *"percentage reduction attributable to hedging in the ex ante variance of terminal wealth"*. The hedging strategy tends to perform successfully when a higher degree of risk reduction is achieved. If the price or return movement are equal for the futures and spot market, hedgers will not minimize risk but will further eliminate the price risk. The hedging effectiveness or hedging performance also refers to the squared correlation coefficient between the spot and futures returns (refer to equation 14). This framework specifies a slightly different efficient frontier compared to the mean-variance framework. Under the minimum variance framework, hedgers will aim to reduce the risk at point M rather than aim at the higher return at point C (refer to Figure3.2). In summary, the theoretical framework presented in Figure 3.3 considers the existing gap presented in the literature review section and the theories underpinning this research area.

Mathematical proof equation:

$$HE = \frac{Var(U) - Var(H^*)}{Var(U)} \qquad \textbf{(13)}$$

$$HE = \frac{Var(U)}{Var(U)} - \frac{Var(H^*)}{Var(U)}$$

So $HE = 1 - \frac{Var(H^*)}{Var(U)}$

Now, substitute Var(H*) == $X_s^{\ 2}\left(\sigma_s^{\ 2} - \frac{\mathrm{cov}_{sf}^{\ 2}}{\sigma_f^{\ 2}}\right)$ into $HE = 1 - \frac{Var(H^*)}{Var(U)}$

$$HE = 1 - \frac{Var(H^*)}{Var(U)}$$

$$= 1 - \left[\frac{X_s^{\ 2}\left(\sigma_s^{\ 2} - \frac{\mathrm{cov}_{sf}^{\ 2}}{\sigma_f^{\ 2}}\right)}{X_s^{\ 2}\left(\sigma_s^{\ 2}\right)} \right]$$

$$= 1 - \left[\left(\sigma_s^{\ 2} - \frac{\mathrm{cov}_{sf}^{\ 2}}{\sigma_f^{\ 2}}\right) \times \left(\frac{1}{\sigma_s^{\ 2}}\right) \right]$$

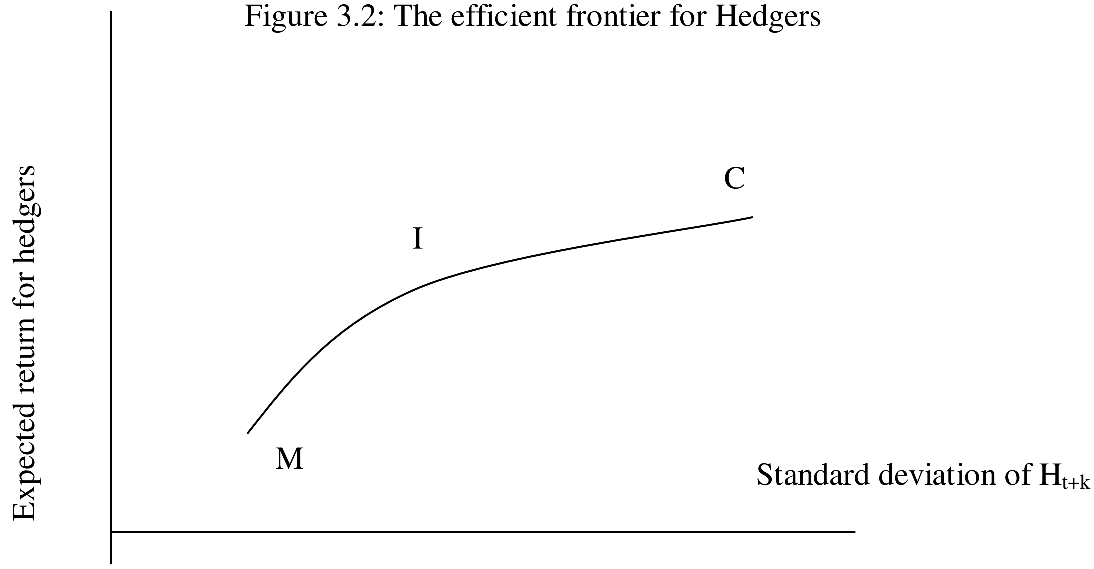
$$= 1 - \left(\frac{\sigma_s^{\,2}}{\sigma_s^{\,2}} - \frac{\mathrm{cov}_{sf}^{\,2}}{\sigma_s^{\,2}\sigma_f^{\,2}} \right)$$

$$= 1 - \left(1 - \frac{\mathrm{cov}_{sf}^{\,2}}{\sigma_s^{\,2}\sigma_f^{\,2}} \right)$$

$$= 1 - 1 + \frac{\mathrm{cov}_{sf}^{\,2}}{\sigma_s^{\,2}\sigma_f^{\,2}}$$

$$= \frac{\mathrm{cov}_{sf}^{\,2}}{\sigma_s^{\,2}\sigma_f^{\,2}} = \rho^2 \text{ (Correlation Coefficient for spot and futures)} \qquad \textbf{(14)}$$

Figure 3.2: The efficient frontier for Hedgers



Source: Sutcliffe C. M. S (1993), Stock Index Futures: Theories and International Evidence, Chapman & Hall, p239.

FIGURE 3.3: THEORETICAL FRAMEWORK