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ON RIGHT FINITE DIMENSIONAL AND
NONSINGULAR GROUP RINGS

by

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ABSTRACT

Let R be an associative ring with identity $1 \neq 0$. R is said to be *right finite dimensional* if it contains no infinite direct sum of nonzero right ideals. A right ideal I of R is *essential* if it has nonzero intersection with the nonzero right ideals of R . The *right singular ideal* $Z_r(R)$ of R is defined as consisting of all elements x in R such that the right annihilator of x is essential in R . R is said to be *right nonsingular* if $Z_r(R) = \{0\}$. The main aim of this thesis is to study necessary and sufficient conditions for a group ring to be right finite dimensional or right nonsingular. Along the way we study some characteristics of essential right ideals and right singular ideals. In a related thread, we shall also study relations between the singular ideals and the prime and Jacobson radicals of various group rings.

ABSTRAK

Biar R suatu gelanggang kalis sekutuan dengan unsur identiti $1 \neq 0$. R dikatakan *bermatra terhingga kanan* jika ia tidak mengandungi sebarang hasil tambah langsung yang tak terhingga bagi semua unggulan kanan bukan sifar. Suatu unggulan kanan I bagi R dikatakan *penting* jika ia mempunyai persilangan bukan sifar dengan semua unggulan kanan bukan sifar bagi R . *Unggulan singular kanan* $Z_r(R)$ bagi R ditakrifkan sebagai suatu unggulan yang mengandungi semua unsur x dalam R supaya pemusnah habis kanan bagi x adalah penting dalam R . R dikatakan *tak singular kanan* jika $Z_r(R) = \{0\}$. Tumpuan kajian disertasi ini adalah terhadap syarat-syarat perlu dan cukup bagi gelanggang-kumpulan bermatra terhingga kanan dan juga gelanggang-kumpulan tak singular kanan. Pada masa yang sama kita akan mengkaji beberapa ciri bagi unggulan kanan penting and unggulan singular kanan. Selaras dengan penyelidikan seterusnya, kita juga mengkaji hubungan di antara unggulan singular, radikal perdana dan radikal Jacobson bagi pelbagai jenis gelanggang-kumpulan.

INTRODUCTION

In this thesis we shall study some necessary and sufficient conditions for right finite dimensional and right nonsingular group rings. Along the way we shall study some characteristics of essential right ideals and right singular ideals. In a related thread, we shall also study relations between the singular ideals and the prime and Jacobson radicals of various group rings.

We begin with the mandatory preliminaries in Chapter 1. Some basic results on groups and rings shall be reviewed. For convenience, we shall also fix notations and terminologies for later use.

In Chapter 2 we shall study some characteristic of essential right ideals and right singular ideals as well as some relations between these ideals. We shall also look at uniform and dense right ideals. Among other things, we shall see that a ring R is nonsingular (that is, its singular ideal is trivial) if and only if every essential right ideal of R is dense.

In Chapter 3 we shall focus on right finite dimensional rings. We are particularly interested in necessary and sufficient conditions for a group ring to be right finite dimensional. The conditions that we shall discuss in this chapter are based mainly on Wilkerson's work. However, the proofs given here provide more details and differ slightly from those of Wilkerson [Wi].

Finally, in Chapter 4 we shall be mainly concerned with necessary and sufficient conditions for a group ring to be nonsingular. We shall also look at various conditions under which the singular ideal of a group ring coincides with its prime and Jacobson radicals.

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