ON CHROMATIC POLYNOMIALS OF GRAPHS

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ABSTRAK

Satu pewarnaan bagi suatu graf adalah pemberian warna kepada bucunya supaya tiada dua bucu berhubungan mendapat satu warna yang sama. Suatu set bucu yang mendapat warna yang sama dikenali sebagai satu *kelas warna*. Suatu *n-pewarnaan* membahagikan bucu-bucu graf kedalam n kelas warna. Bilangan λ-pewarnaan bagi suatu graf G, ditandakan sebagai $P(G;\lambda)$, dipanggil polinomial kromat bagi G. Dua graf dikatakan *bersetara secara kromat* jika mereka mempunyai polinomial kromat yang sama. Jika terdapat hanya satu graf G dengan polinomial kromat yang tertentu, maka G dikatakan *unik secara kromat.*

Bab 1 memberikan perkenalan dan juga takrif bagi graf. Kebanyakan istilah yang digunakan boleh didapati dari buku Harary [35] dan Biggs [2]. Polinomial kromat untuk beberapa graf terkemuka juga diberikan.

Bab 2 adalah berkenaan dengan sifat-sifat pekali polinomial kromat. Perbincangan mengenai sifat berunimod dan sifat kecekungan logaritma kuat juga diselitkan. Ungkapan bagi beberapa pekali pertama polinomial kormat yang diungkapkan dalam sebutan bilangan subgraf bagi G diberikan. Ungkapan-ungkapan tersebut adalah penting dalam penghitungan polinomial kromat. Beberapa kaedah yang diketahui dalam penghitungan polinomial kromat juga diberikan.

Dalam Bab 3, kita memberi beberapa keputusan yang diketahui berkenaan dengan kesetaraan kromat bagi graf dan juga syarat perlu bagi dua graf untuk bersetara secara kromat. Beberapa kelas graf yang unik secara kromat ditinjau secara am.

Akhirnya, dalam Bab 4, kita membuktikan dengan menggunakan kaedah yang berlainan bahawa graf $K_{2,s}$ yang bertindan dengan C_m pada satu tepi adalah unik secara kromat.

ABSTRACT

A *colouring* of a graph is an assignment of colours to its vertices so that no two adjacent vertices have the same colour. The set of all vertices with the same colour is called a *colour class.* An *n-colouring* of a graph partitions its vertex set into n colour classes. The number of λ-colourings of a graph G is called the *chromatic polynomial* of G, denoted $P(G;\lambda)$. All graphs sharing the same chromatic polynomial are said to be *chromatically equivalent.* If there is only one graph G with a given chromatic polynomial, then G is said to be *chromatically unique.*

Chapter 1 provides introduction and definitions on graphs. Most of the terms used can be found in Harary [35] and Biggs [2]. The chromatic polynomials of some well-known graphs are also given.

Chapter 2 deals with coefficients of chromatic polynomials and their behaviour. In particular, some discussion on the properties on unimodality and strong logarithmic concavity were included. Explicit expression for the first few coefficients in terms of the number of certain subgraphs of G are given. These expressions are useful in the computation of chromatic polynomials of graphs. Some known methods for computing chromatic polynomials are presented.

In Chapter 3, we provide some known results on the chromatic equivalence of graphs and also some necessary conditions for two graphs to be chromatically equivalent. Several classes of chromatically unique graphs are briefly surveyed.

Finally, in Chapter 4, we give an alternative proof that the edge-gluing of $K_{2,s}$ and C_m is chromatically unique.