CHAPTER III

PRACTICAL DETERMINATION OF THE MIXING HEIGHT

3.1 INTRODUCTION

To fully characterise the ABL, it is necessary to determine not only the surface fluxes (sensible heat, latent heat, and momentum (i.e. friction velocity) but also the vertical extent of the boundary layer. The height of the ABL is not only an important parameter in air pollution dispersion modelling, but also a fundamental parameter characterising the structure of the lower troposphere. Its determination under the tropical atmospheric boundary layer is one of the objectives of this work.

In the literature related to the dispersion of air pollutants, this vertical height is referred to as the mixing height and is defined generally as the level where vertical dispersion becomes negligible. For the daytime situation, the vertical dispersion is limited by the temperature inversion that caps the convective boundary layer. Appropriately, the daytime mixing height is referred to as the inversion height. For night-time, on the other hand, the vertical extent of dispersion is limited by the sharp decrease of turbulent energy with height. The mixing height in this case is referred to as the boundary layer height $h$ and marks the level where turbulent energy drops to a negligible value. Each height reflects a different mechanism limiting the vertical dispersion in each case.
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Two basic possibilities for the practical determination of the mixing height are its derivation from profile data (measurements or numerical model output) and its parameterisation using simple equations or models (which need only a few measured input values). Different methods suggested in the literature are reviewed below.

3.1.1 Methods for the Determination of the Mixing Height

This chapter contains a review of practical, robust methods for the determination of mixing height. First, the concept of the mixing layer is introduced in Section 3.2. After briefly describing methods based on profile measurements in Section 3.3, Section 3.4 discusses determinations based on parameterisations and simple models, which require only operationally available input data. A detailed derivation of the selected model will be presented with its implementation using the data set of this study in Chapter VI.

Only the studies most pertinent to the determination of the mixing height will be covered in this review. A critical review of mixing height determination was conducted within the COST Action 710 report (European Co-operation in the Field of Scientific and Technology Research). The discussion contained in Sections 3.3 and 3.4 will be a summary based on the comprehensive reviews in Kustas (1986), Brown (1996), Seibert et al. (1998), and Seibert et al. 2000.

3.2 MIXING LAYER: CONCEPTS AND DEFINITION OF ITS HEIGHT

Because of the lack of agreement on the proper definition of the mixing height, its practical determination is not trivial. This is reflected in the numerous definitions
found in the literature (see Stull, 1988; Garratt, 1992; Seibert et al., 1998). The mixing height

1) is defined as the height above the surface through which relatively vigorous vertical mixing occurs. (Holzworth, 1972).

2) refers to the height above the ground of the layer of the atmosphere adjacent to the surface where vigorous mixing occurs as a result of thermal and mechanical turbulence. (Norton & Hoidale, 1976).

3) is defined as the height up to which significant turbulence transfer of heat, mass, and momentum between the local earth surface and the atmosphere occur when averaged over a period of the order of one hour (Arya, 1981).

4) is the level of a potential barrier to the dispersion of pollutants at the interface between stable and less stable air (Maughan et al., 1982).

5) based on temperature structure, can be defined as the height at which a ground-based unstable to neutral vertical temperature profile becomes stable (Baxter, 1991).

6) is defined as the height to which the pollutants would mix over a relatively short period of time, 1-2 hours (Baxter, 1991).

7) defines the vertical extent of vigorous thermal turbulence during daytime heating and thus sets a limit to upward mixing of pollutants (Myrick et al., 1994).

The above definitions represent quite different ideas concerning what constitutes the mixing height. It may also be the case that the mixing heights as defined by different authors have to be viewed in the context of the data available to them.
The following definition of the mixing height is adopted for the purpose of this study (after Seibert et al. 1998; 2000): "The mixing height is the height of the layer adjacent to the ground over which pollutants or any constituents emitted within this layer or entrained into it become vertically dispersed by convection or mechanical turbulence within a time scale of about an hour".

When proceeding from this general definition to practical realisations, it is necessary to consider separately the structures of the stable boundary layer (SBL) and the convective boundary layer (CBL) (COST 710). For the CBL, an important feature is the entrainment layer (Gryning & Batchvarova, 1994), a zone that is not well mixed and where turbulence intensity declines towards its top. The above definition corresponds to the top of the entrainment layer. The most widespread definition however is the value $z_i$, defined as the height where the heat flux gradient reverses its sign. It is usually applied for scaling purposes and it is the definition closest to the thermodynamical CBL height definition in a zero-order jump model (i.e., where the entrainment layer thickness is neglected). We will use this definition here, but one should be aware, as for example, in the specification of turbulence parameterisations for dispersion models, that turbulence extends beyond $z_i$.

The SBL can be divided into two layers: a layer of continuous turbulence and an outer layer of sporadic or intermittent turbulence. Under very stable conditions the layer of sporadic turbulence may extend to the ground. Since it is notoriously difficult to measure sporadic turbulence, and even more difficult to develop a related scaling theory, the scaling height, $h$, used for the SBL is generally the layer of continuous turbulence. As in the convective case, however, this does not mean that
turbulence is strictly confined to the region below \( h \). The asymptotic case with the heat flux approaching zero from either stable or unstable stratification is often termed the neutral boundary layer. However, it must be kept in mind that even in this case stable stratification will prevail above the ABL, which limits the validity of idealised concepts based on an infinitely deep neutral boundary layer. In this situation, like in the stable boundary layer, wind shear is the main source of turbulence and therefore in this chapter it is subsumed within the SBL. It is clear from this short summary that researchers should pay attention to which definitions of the mixing height or ABL height their work is based upon, and to specify it clearly.

3.3 DETERMINATION OF THE MIXING HEIGHT FROM MEASUREMENTS

Generally, the underlying characteristics of the atmospheric boundary layer evolution may be studied by in situ data of profiles of potential temperature, specific humidity and/or winds (using radiosonde and/or tethered balloon). Another way in which the structure of the boundary layer can be analysed is by measurements realised at a distance (remote sensing), or SODAR (sound detection and ranging), or LIDAR (light detection and ranging). In the case of SODAR, the physical principle is the sound signal reflection by the thermal inversion, which is particularly useful in the determination of the nocturnal boundary layer and at the beginning of the convective boundary layer formation. In the LIDAR case, the aerosols, liquid water drops, and air molecules reflect and diffuse the luminous signal, from which inferences can be made concerning the boundary layer structure as a whole (Stull, 1988; Fisch, 1996).
Recently, Seibert et al. (1998, 2000) presented a comprehensive review on the methods of determination of mixing height from measurement. A summary of the most relevant information for the purposes of this study is presented in the following sections.

3.3.1 Radiosondings

Radiosondings are the most common source of data for operational determination of the mixing height. They are widely distributed, and the data are continuously controlled for quality. On the other hand, at most stations the data are taken only twice daily at specified synoptic times (00 UTC, 12 UTC). Consequently, the soundings can often be used as a reference for comparison with modelled mixing heights only at around midnight and noon. Other limitations of radiosondings are the poor vertical resolution of standard aerological data with respect to boundary layer studies, the smoothing due to the sensor lag constant, the high ascent rate of the sonde, and the fact that a sounding gives only a “snap shot” view of the ABL structure.

Mixing height estimations based on (standard) radiosonde data may result in quite high uncertainty (e.g., Russell et al., 1974; Hanna et al., 1985; Martin et al., 1988). Specific problems occur in the stable (nocturnal) boundary layer since no universal relationship seems to exist between the profiles of temperature, humidity or wind, and turbulence parameters (heat or momentum fluxes, turbulent kinetic energy). The interpretation of profiles thus is not straightforward and several criteria have been used (see Table 1 in Seibert et al., 1998).
Mixing height estimation from profiles obtained with tethered balloons or aircraft, which may include turbulence and/or trace gas concentration profiles, is in principle not very different from the analysis of radiosonde data. The operation of both systems, however, is very expensive and therefore not suited for routine applications. Temperature (and trace gas) profiles taken by commercial aircraft during take-off and landing may become a useful data source in the future.

3.3.1.1 Subjective Methods

Radiosonde temperature and wind profiles in the lower part of the atmosphere are often used for a subjective estimation of the mixing height. Under convective conditions, the mixing height is often identified with the base of an elevated inversion or stable layer, or as the height of a significant reduction in air moisture, often accompanied by wind shear. Some authors recommend taking the inversion base altitude increased by half of the depth of the inversion layer as the characteristic CBL height (Stull, 1988).

3.3.1.2 Objective Methods

Holzworth (1964, 1967, 1972) and others have developed objective methods to simplify and homogenise the estimation of the mixing height under convective conditions. The basic idea of the “Holzworth” or “parcel” method is to follow the dry adiabate, starting at the surface with the measured or expected (maximum) temperature, then up to its intersection with the temperature profile from the most recent radiosounding. It determines the mixing height as the equilibrium level of a
hypothetical rising parcel of air representing a thermal. However, this method depends heavily on the surface temperature, and a high uncertainty in the estimated mixing height value may result in situations lacking a pronounced inversion at the CBL top. Some authors have noticed that the mixing height determined by the "Holzworth method" is not strongly correlated with observed trace gas concentrations (e.g., Aron, 1983; Jones 1985).

Different refinements of this simple scheme have been suggested, to account for temperature advection, subsidence, and other effects (e.g., Miller, 1967; Garrett, 1981). They differ in how the temperature of this air parcel is found, and in the thermodynamical variable used to define the equilibrium level. An advanced parcel method has been proposed by Beljaars & Betts (1992) and has been also applied (with slightly different values of the constants) by Wotawa et al. (1996).

Another popular approach is the bulk Richardson number methods (e.g., Troen & Mahrt, 1986; Vogezezang & Holtslag, 1996). They differ mainly in the choice of the level of the near-surface temperature and wind, the parameterisation of shear production of turbulence in the surface layer, and the consideration of an excess surface temperature under convective conditions. Parcel methods can be understood as a simplification of the Ri-number methods where the shear contribution is neglected. Thus they are only suited for unstable conditions.

Methods based on conserved variables (e.g., mixing ratio, equivalent-potential temperature, etc.) permit analyses of air mass structures and vertical mixing. Betts & Albrecht (1987) proposed different criteria on the basis of averaged
and smoothed profiles. For individual profiles, a refinement of these criteria is necessary to differentiate the main features from secondary stratification (Seibert et al., 1998).

3.3.2 Remote Sounding Systems

Remote sounding systems (lidars, sodars, RASS, wind profiling radars; see Clifford et al., 1994 for an overview) are becoming popular and being incorporated into operational applications. They provide an interesting alternative for mixing height estimations. The basic advantages of remote sounding systems are that they operate continuously and they do not cause any modification of the investigated flow.

The sodar is one of the simpler and less expensive remote sounding systems, making it well suited for routine operation. Sodar signals are scattered by temperature inhomogeneities characterised by the structure parameter of the acoustic refractive index, $C_n^2$. Vertical profiles of $C_n^2$ (and thus the sodar backscatter intensity) show typical features under stable and convective conditions (e.g., a strong decrease above a region of less variable $C_n^2$ in a SBL with significant shear-produced turbulence, or an elevated maximum at the top of a CBL), which can be used to derive the mixing height (for more details, see Beyrich, 1997). Sodars have problems in neutral conditions when the temperature inhomogeneities in the atmosphere become negligible. In addition, sodar systems with Doppler capability allow determination of the mean wind and vertical velocity variance profiles which may also be employed for mixing height determination. Methods and algorithms to derive the mixing height from sodar data are compared in Beyrich (1997) and Seibert
et al. (1998). However, the vertical range of most sodars is limited to a maximum of about 1-km, and often to only a few hundred metres. The lowest range gate of typical sodars is around 40 m. This is lower than for other remote sounding systems, but it can still be too high for very shallow SBLs. In this case, minisodars (Asimakopoulos et al., 1996) are an attractive option.

Lidars allow the measurement of aerosol or trace gas concentration profiles and may therefore be considered to provide direct measurements of the mixing height. Since the top of a convectively mixed layer is often associated with strong gradients of the aerosol content, a simple aerosol backscatter lidar seems suited to determine the convective mixing height. However, interpreting data from aerosol lidars is often not straightforward, because the detected aerosol layers are not always the result of ongoing vertical mixing, but may originate from advective transport or past accumulation processes (e.g., Russell et al., 1974; Coulter, 1979; Baxter, 1991; Batchvarova et al., 1999). Under stable conditions, problems in estimating the mixing height from lidar data can arise from the weak vertical gradients in the aerosol content. Moreover, in the evening, it usually takes some time until a sufficiently clear discontinuity in the backscatter intensity profile develops at the top of the SBL, within the previously well-mixed layer (e.g., Russell et al., 1974).

The boundary layer wind profiler seems to be a very promising device for direct and continuous measurement of the mixing height in a deep CBL (Angevine et al., 1994a,b; Gaynor et al., 1994; Dye et al., 1995) but less suited for shallow boundary layers due to its large range gate. The backscatter intensity of the electromagnetic signal is proportional to the structure parameter of the
electromagnetic refractive index $C_n^2$ which depends on small-scale fluctuations of the temperature and especially the moisture fields. Vertical profiles of $C_n^2$ usually show a maximum at the top of a well-developed CBL. However, the moisture profile is often not as well mixed as that of temperature, which may result in some ambiguity of the derived mixing heights. Additional problems occur in the presence of cumulus clouds, even if it is only shallow cloud in the upper part of the ABL.

Radio-acoustic sounding systems (RASS) are extensions of either sodars or radar wind profilers. In addition to wind and $C_n^2$ profiles, they provide virtual potential temperature profiles. Therefore, they appear to be well suited for the determination of the mixing height with Richardson number methods, provided that temperature is retrieved with sufficient accuracy (Seibert et al., 1998) and that the range and resolution are adequate.

The combination of different remote sounding systems (e.g., sodar + wind profiler, or sodar + lidar) offers a promising way towards the direct and continuous monitoring of the evolution of the mixing height throughout the complete diurnal cycle (e.g., Beyrich & Görsdorf, 1995). However, the interpretation of data measured with remote sounding systems is not always straightforward. Nevertheless, this holds true also for the direct measuring systems and may (at least partially) be attributed to the general problem of mixing height definition as discussed in Section 3.2. Table 3.1 reports advantages and disadvantages of the various methods.
3.3.3 General Assessment of Measurement-based Methods

The advantages and shortcomings of the various empirical methods used to estimate mixing height are summarised in Table 3.1 (from Seibert et al. 1998, 2000). According to the mixing height discussion given in section 3.2, the mixing height could be determined by investigating the dispersion process of non-reactive tracer gases through the analysis of concentration profiles. However, vertical mixing is not the only process determining such profiles and additional measurements would be needed for a safe interpretation. The second-best choice would be turbulence profile measurements. Both types of measurements are difficult and expensive and are therefore not operational. Thus, mixing height determination is based in most cases on profile measurements of mean meteorological variables such as wind temperature, humidity, and refractive index. These profiles should satisfy the following conditions:

- They should cover the layer between Earth’s surface and about 2-3 km above ground, considering the typical height range over which the mixing height varies during its annual and diurnal cycles in defined climatic regions.
- The profile measurements should be available with a time resolution of about 1 hour or less in order to properly describe the evolution of the mixing height, especially during the morning and evening transition phases.
- The measured profiles must have a vertical resolution of about 10-30 m to avoid relative uncertainties of more than 10-20 %, especially for low mixing heights (<250 m).
The measured parameters should be linked physically to the vertical mixing of pollutants.

Table 3.2 indicates which of the above-mentioned requirements are fulfilled by the different sounding systems. It is evident that none of the systems meets all the requirements, i.e., the perfect "mixing height-meter" does not exist. Reliable mixing height determination under all conditions is therefore still an unsolved problem (Seibert et al., 1998). The best approach is to use a combination of systems.

Table 3.1: Measuring platforms and their qualification for mixing height determination (from Seibert et al. 1998, 2000).

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
<th>Shortcomings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Measuring techniques/sensor platforms</td>
<td><strong>Routine ascents for many years all over the world, therefore especially suited for climatological studies</strong></td>
<td><strong>Crossing the ABL along a slanted path within a few minutes, provides a &quot;snapshot&quot;-like profile.</strong></td>
</tr>
<tr>
<td>Radiosonde</td>
<td><strong>Measured data transmitted via international communication networks with very short time delay, therefore well suited for operational use</strong></td>
<td><strong>Limited height resolution of routine ascents</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Compatibility with measurements in the free atmosphere</strong></td>
<td><strong>Operationally only 2-4 soundings per day at fixed times, even during field campaigns; 1.5-3 h as closest interval</strong></td>
</tr>
<tr>
<td>Tethered balloon</td>
<td><strong>Ascent velocity can be chosen according to the desired vertical resolution</strong></td>
<td><strong>Tracking problems at low levels (site dependent) may affect wind profiles</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Turbulence and trace gas concentration measurements possible.</strong></td>
<td><strong>Limited to field campaigns, no unmanned operation</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Synchronous profile measurement difficult</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Limited measurement range, usually below 500 m</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Not possible in cases of high wind speed or strong convection</strong></td>
</tr>
</tbody>
</table>

44
<table>
<thead>
<tr>
<th>Mast</th>
<th>Aircraft and remote sensing techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Installation of a large number of different sensor types possible including detailed turbulence measurements</td>
<td>- Very high installation/operation costs, increasing with height</td>
</tr>
<tr>
<td>- Continuous operation</td>
<td>- Limited range: 50 to at most 300 m</td>
</tr>
<tr>
<td>- Good resolution of the lowest layers</td>
<td>- High vertical resolution requires a high number of sensors (increasing costs)</td>
</tr>
</tbody>
</table>

**Aircraft**

- Possibility to operate many different sensors, including mean meteorology, chemistry, and turbulence sensors, as well as remote sensing systems
- Provides spatial information, well suited for mesoscale studies
- High costs, only for field campaigns
- Operation mostly limited to daylight hours
- Lowest flight level subject to restrictions (security)

**Doppler weather radar/wind profiler**

- Ground based and aircraft based operation possible (for radar only)
- High sampling rate and continuous operation
- Lowest range normally not below 200 m
- Limited vertical resolution (50-250 m)
- Expensive
- Weather radars do not work well in clear air
- Interpretation not always straightforward.

**Lidar**

- Ground-based and aircraft-based operation possible
- High sampling rate
- Return signals originate directly from aerosols ("pollution")
- Expensive
- Unattended operation often not possible for safety reasons
- Limited range resolution and lowest range gate
- Tracer necessary (gas, aerosol)
- Interpretation sometimes ambiguous

**Sodar**

- Relatively simple, not very expensive: suited for unmanned long-term operation
- High temporal and vertical
- Limited sounding range (500-1000 m)
- Sensitivity to environmental noise
- Noise contamination to
resolution
- Minisodars allow probing of shallow SBL

the environment
- Interpretation requires experience, sometimes ambiguous

Table 3.2: Critical assessment of different methods to determine the mixing height (from Seibert et al., 2000).

<table>
<thead>
<tr>
<th>Continuous data output</th>
<th>Range covered well</th>
<th>Determination of turbulence parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10-100m (low SBL)</td>
<td>100-500m (SBL/CBL)</td>
</tr>
<tr>
<td><strong>In-situ measurements</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radiosonde</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Tethered</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>balloon</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Mast</td>
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<td>•</td>
</tr>
<tr>
<td>Aircraft</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td><strong>Remote Sounding</strong></td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Mini-sodar</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Sodar</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Radar&lt;sup&gt;a&lt;/sup&gt;</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>RASS&lt;sup&gt;b&lt;/sup&gt;</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Lidar</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td><strong>Numerical models</strong></td>
<td>•</td>
<td>•</td>
</tr>
</tbody>
</table>

Note: * means fulfilled; *** partly fulfilled and • not fulfilled
<sup>a</sup> Electromagnetic boundary layer wind profiler.
<sup>b</sup> As an extension of the electromagnetic boundary layer wind profiler.

3.3.3.1 Comparative Studies of Empirical Mixing Height Determination

Complete agreement between mixing height values derived from different sounding systems cannot be expected <i>a priori</i> due to several reasons, the most important ones being that different sounding systems measure different atmospheric variables (mean
temperature, humidity, wind, turbulent fluxes or structure parameters) with varying height resolution and accuracy.

Vertical profiles of these properties are influenced in different ways by the processes occurring on Earth's surface. In addition, a host of turbulent and non-turbulent processes (heating and cooling, convection and subsidence, radiation processes, baroclinity, advection, gravity waves, phase changes of water) interact with each other within the ABL to influence the vertical profiles. It is nearly impossible to separate the various contributions to the observed ABL structure.

Often it is difficult to identify a clear upper boundary of the mixing layer or ABL because vertical profiles of turbulent parameters are smooth without any clear signatures and decrease asymptotically toward values close to zero which are typical for the residual layer. This occurs especially under stable conditions with weak turbulence and near-neutral conditions.

Convective Boundary Layer

Field studies that compare mixing height values derived from different measurement systems (radiosonde, sodar, radar, lidar, aircraft) under convective conditions have been described, for example, by Russell et al. (1974), Noonkester (1976), Coulter (1979), Kaimal et al. (1982), Baxter (1991), and Marsik et al. (1995). These studies show that the relative differences are mostly less than 10 %, provided that the elevated inversion capping the well-mixed CBL is not too weak and has a well-defined base. Conclusions on possible systematic deviations between different
estimates of the mixing height are not consistent (except for the lidar; under certain conditions, pollutants trapped within the stable capping inversion or free atmosphere can cause a systematic overestimation of mixing heights from lidar observations (McElroy and Smith, 1991)). This should be attributed to the different criteria applied to analyse the profiles, as well as to the often limited number of observations, and in some cases also to spatial differences between the sites where the different systems were used.

*Stable Boundary Layer*

For methods that utilise wind and temperature profiles, the comparison of mixing heights derived from different observing systems under stable conditions is much more difficult. This is due to certain features of the structure and evolution of the SBL such as the intermittent and weak turbulence, gravity waves, radiative cooling, drainage flows, and inertial oscillations. Time scales of most of the relevant processes are much longer than in the CBL, so that the SBL is often far from stationarity. Different SBL height scales derived from temperature and wind profiles are compared, e.g., in Yu (1978), Mahrt and Heald (1979), Mahrt et al. (1979, 1982), Arya (1981), and Wetzel (1982). No significant relationship exists between the height scales based on the temperature profile and the height of the low-level wind maximum. This is basically due to the different time evolutions of the temperature and wind profiles during the night. The structure and the evolution stage of the SBL should be considered when deriving the stable mixing height from temperature or wind profiles, or when comparing mixing height values derived from different observing systems under stable conditions.
Concerning methods that utilise turbulence data, the comparison of mixing height estimates for the SBL derived from turbulence profiles with those derived from mean temperature and wind profiles is difficult due to the scarcity of data and to the variety of definitions for the SBL height. It is particularly important to stress, as noted by Caughey (1982), that "there is no simple relationship between the SBL depth and the depth of the surface inversion layer. As this layer deepens and becomes more intense, significant turbulence exchange becomes confined to a shallow layer close to the ground." Garratt (1982a) and Smedman (1991) confirmed this notion. Kurzeja et al. (1991) found a good correlation between the top of a strong surface inversion and the SBL height derived from profile measurements of the wind direction standard deviation at the beginning of the night, and in general between the latter height and the height of the wind maximum. Model calculations often show an increase of the SBL height defined by the turbulence profile or different temporal behaviour during different phases of the SBL evolution (e.g., Nieuwstadt & Driedonks, 1979).

Concerning methods that utilise remote sounding instruments and data, only acoustic sounders seem capable of providing mixing height data under stable conditions. Radar profilers and lidars have a range resolution that does not allow them to resolve the SBL in detail. In addition, their first usable range gate is often at or above the ABL top, a fact which sometimes even limits the application of a conventional sodar (Garratt, 1982b; Smedman, 1988; Baxter, 1991). For very shallow stable boundary layers (below 50-100 m), minisodars with a lowest range gate around 10 m can overcome this deficit. However, the interpretation of sodar data for mixing height determination under stable conditions is controversial (Hanna,

It seems clear that the question as to which of the suggested SBL height scales is best suited to characterise the vertical mixing of pollutants under stable conditions has not yet been answered.

3.4 THE DETERMINATION OF MIXING HEIGHT FROM PARAMETERISATION AND MODELS

As continuous profile measurements for the operational determination of the mixing height are not generally available, simple parameterisations based on standard surface observations and single profile data, as well as numerical models, are widely used in the practice of meteorological and environmental services. Simple diagnostic or prognostic parameterisation equations for the mixing height are still very attractive for operational purposes because of their simplicity and the limited number of required input data. They are also used within comprehensive parameterisation schemes for the treatment of the ABL in some numerical weather prediction and climate models.

3.4.1 Modelling and Parameterisation of the Mixing Height Under Stable Conditions

Many parameterisation expressions for the height of the turbulent SBL have been suggested in the literature (e.g., Hanna, 1969; Zilitinkevich 1972; Arya, 1981; Mahrt,
1981; Nieuwstadt, 1984; Koracin & Berkowicz, 1988). Both diagnostic and prognostic relationships have been proposed and there has been a controversial debate on which type is the most suitable (Nieuwstadt, 1981, 1984; Garratt, 1982a,b).

It has generally been assumed that the structure of the stable ABL depends on external parameters such as the Coriolis parameter \( f \) and the surface roughness length \( z_0 \), and on internal turbulent parameters such as the friction velocity \( u^* \) and the surface heat flux \( Q_0 = \langle w' \Theta' \rangle \). Then, the stable (and neural) ABL height is assumed to be a function of the Ekman and Monin-Obukhov length scales \( L_E = u^*/f \) and \( L_* = - \frac{u^3}{\beta k Q_0} \), respectively. Kitaigorodskii & Joffre (1988) have extended these similarity theories to include the effect of the background stratification of the atmosphere through the length scale \( L_N = u^*/N_{BV} \), with the Brunt-Väisälä frequency \( N_{BV} = \sqrt{\gamma_0} \), where \( \gamma_0 \) is the potential temperature gradient above the mixing height.

The most popular diagnostic equations (based on scaling arguments) are:

\[
h = a_1 L_E = a_1 u^*/f \quad (3.1)
\]

and

\[
h = a_2 (L_x L_y)^{1/2} = \frac{a_2 u^2}{\sqrt{-\beta k Q_0 f}} \quad (3.2)
\]

with the empirical coefficients \( a_1 = 0.07-0.3 \) and \( a_2 = 0.3-0.7 \). Nieuwstadt (1981), proposed a combination of (3.1) and (3.2), namely:
\[ h = \frac{L_\ast}{3.8} \left( -1 + \sqrt{1 + 2.28 \frac{u_\ast}{fL_\ast}} \right) \]  \hspace{1cm} (3.3)

Most of the verification studies do not seem to favour the application of more elaborated parameterisations. However, one should be cautious in using these equations because Equation 3.1 assumes a neutral, stationary boundary layer, and because \( 1/f \) is generally too long to be a relevant time scale. The subordinate role of the Coriolis force for the turbulent fluxes is also supported by large-eddy simulation (see Fig 11 in Andren, 1995).

As an alternative to \( 1/f \) as time scale, some authors, e.g., Kitaigorodskii & Joffre (1988), have suggested using \( 1/N_{BV} \), and

\[ h = a_3 L_N = a_3 \frac{u_\ast}{N_{BV}} \]  \hspace{1cm} (3.4)

with the empirical constant \( a_3 = 4 - 14 \). This model has been corroborated by measurements in the Arctic (Overland & Davidson, 1992), by lidar measurements in the Netherlands (van Pul et al., 1994), and by LES-computations (Vogezezang & Holtslag, 1996).

Zilitinkevich & Mironov (1996) have proposed a multi-asymptotic expression that combines all these different scales. A comprehensive survey of the various formulations can be found in Seibert et al. (1998).
3.4.2 Modelling and Parameterisation of the Mixing Height Under Convective Conditions

Diagnostic relations based on similarity theory have sometimes been suggested to be able to parameterise the CBL depth (e.g., Tennekes, 1970; Zilitinkevich, 1972; San Jose and Casanova, 1988). However, these are valid only under certain conditions (e.g., free convection) and are not of much practical relevance. The Ri-method has been used by Vogelezang & Holtslag (1996) also for unstable situations by adding an excess temperature to the near surface temperature, as suggested by Troen & Mahrt (1986).

Currently, the numerical integration of mixed-layer slab models is a well-established way to simulate the evolution of the convective mixing height. These models use surface fluxes and an initial temperature profile as basic input parameters. The initial temperature profile represents a general problem for these models since normally the network of radiosounding stations is not dense enough for boundary layer studies.

Prognostic equations describing the growth of the CBL are normally derived from a parameterisation of the TKE budget equation which is either averaged over the whole mixed layer or specified at the mixed layer top. The equations proposed by various authors differ mainly in the terms that are neglected in the TKE budget, and how the remaining terms are parameterised. The spectrum ranges from simply considering surface heating as the only relevant driving force (Betts, 1973; Tennekes 1973) to additional consideration of: mechanical turbulence production due to surface friction (Driedonks, 1981, 1982b); local changes of TKE in the mixed layer.
(the so-called “spin-up” effect) (Zilitinkevich, 1975; Gryning & Batchvarova, 1990; Batchvarova & Gryning 1991); wind shear across the entrainment layer (Stull, 1976a); Driedonks, 1981; Manins, 1982; Rayner & Watson 1991); explicit parameterisation of TKE dissipation (Zeman & Tennekes, 1977); and finally, rather complex equations taking into account energy losses in connection with gravity waves (Stull, 1976b) or the influences of moisture and advection (Steyn, 1990).

A survey of relationships suggested in the literature to describe the ML growth during daytime is given in Appendix A (page 217). Comparisons of some of them with observational data can be found in Driedonks (1981, 1982b), Arya & Byun (1987), and Batchvarova & Gryning (1991, 1994).

3.4.2.1 One-dimensional Integral Models

One-dimensional, integral models of the CBL provide a more theoretically sound and robust framework for inversion-height modelling than the simple profile-intersection methods discussed earlier. This class of models evolves the inversion height from its initial early-morning value using time histories of the surface turbulence fluxes $\overline{u'\omega'}$ and $(\omega'\theta')$. The simplest variety of integral models is known as the “encroachment” model that simply integrates the surface sensible heat transfer to “fill” the potential-temperature profile. Encroachment models fail to adequately model inversion-height growth in many instances, especially when mechanical turbulence is significant (i.e.-$z_r/L<10$). To alleviate these problem, more advanced treatments consider various entrainment mechanisms at the inversion itself.
3.4.3 Modelling of the Inversion and Boundary-layer Structure

Before the turbulence in the boundary layer and at the inversion is described, the conceptual models for both the boundary layer and the inversion must be outlined. In most integral models, the inversion is modelled as an infinitesimally thin region across which potential temperature and sensible heat transfer change discontinuously. This is referred to as a "zero-order-jump" inversion model and forms the framework for the discussion in this section. Betts (1973) and Deardorff (1979) outline "second-order-jump" and "third-order-jump" models, respectively, but the limited improvement achieved with these more complicated representations does not generally warrant their use in light of the overall uncertainties in the analysis.

If we neglect radiative heating, which is negligible in comparison with other energy-budget components at the inversion (Tennekes, 1973), the inversion heat transfer is related to the growth rate of the boundary layer by equating the enthalpy loss of the air entrained into the boundary layer with the sensible heat transfer at the inversion. This balance leads to the relationship (Lilly, 1968)

\[-(\overline{\theta'\omega'})_h = \Delta \theta (\frac{dh}{dt} - W_h)\]  \hspace{1cm} (3.5)

where $\Delta \theta_h$ is the temperature jump across the inversion and $W_h$ is the mean vertical velocity at the inversion. By (a) assuming a constant potential temperature throughout the boundary layer, which forces the sensible heat transfer to vary linearly from $(\overline{\omega'\theta'})_s$ at the surface to $-(\overline{\omega'\theta'})_h$ at the inversion, and (b) applying an
energy balance across the boundary layer, the rate of change of inversion strength with time is given by

$$\frac{d\Delta \theta}{dt} = \gamma \left( \frac{dh}{dt} - W_h \right) - \left( \frac{\langle \omega' \theta' \rangle_h}{h} - \frac{\langle \omega' \theta' \rangle_h}{h} \right)_{bl}$$

(3.6)

where $\gamma$ is the lapse rate immediately above the inversion and the subscripts "s" and "h" refer to surface and inversion, respectively. The governing equation for the inversion strength is then formulated by combining Equations 3.5 and 3.6:

$$\frac{d\Delta \theta}{dt} = \left( \frac{dh}{dt} - W_h \right) - \left[ \gamma - \frac{\Delta \theta}{h} \right] - \frac{\langle \omega' \theta' \rangle_h}{h}$$

(3.7)

Idealized profiles of both potential temperature and sensible heat transfer used in the integral model, along with their more realistically observed counterparts, are shown in Figures 3.1a and 3.1b. Most likely, a sharp interface exists instantaneously as convective thermals encounter stable air, but the erratic nature of the rising thermals acts to spatially smooth these discontinuities upon averaging.

Given that a morning temperature profile and the surface sensible heat transfer history after sunrise are available, the additional quantities necessary to solve Equations 3.5-3.7 are $W_h$, $\langle \omega' \theta' \rangle_h$, and the initial values of $h$ and $\Delta \theta$. In the absence of appropriate synoptic data, $W_h$ can usually be neglected with little error since $W_h$ is rarely greater than 20% of $dh/dt$ in the early morning. The initial values of $h$ and $\Delta \theta$ are easily specified. Specification of the remaining quantity $\langle \omega' \theta' \rangle_h$ represents a classic turbulence closure problem and is discussed in the following section.
Figure 3.1: The daytime CBL showing: (a) the instantaneous structure together with realistic potential temperature and heat flux profiles; and (b) the idealised representation based on perfectly mixed CBL and infinitesimally thin inversion used in most integral inversion height models.
3.4.4 Basic Entrainment Parameterisations for the Inversion flux

The inversion heat transfer \( (\omega' \theta')_h \) can be related to other known variables through parameterisation of the turbulent kinetic energy budget at the inversion. In this analysis, it is assumed that air entrained at the inversion is dry and horizontally homogeneous, thus eliminating latent heat transfer and non-vertical derivatives. Additionally, radioactive effects and shear production of turbulence across the inversion are neglected. Under these constraints, the turbulent energy budget at the inversion is

\[
\frac{\partial E}{\partial z} = \frac{g}{\theta} (\omega' \theta')_h - \frac{\partial}{\partial z} \left[ (\epsilon \omega') + \frac{1}{\rho} (\rho' \omega') \right] - \varepsilon \quad (3.8)
\]

Where \( E \) and \( \epsilon \) are the mean and fluctuating turbulent kinetic energy, respectively, \( g \) is the gravitational acceleration, \( \rho' \) is the fluctuating pressure, and \( \varepsilon \) is the dissipation rate.

The first efforts in parameterising Equation 3.8 to yield an expression for \( (\omega' \theta')_h \) were presented by Tennekes (1973). In his analysis, Tennekes neglected the time derivative and dissipation terms and, on the basis that the large convection cells contain most of the energy, argued that the turbulent energy \( E \) at the inversion scales with the variance of the vertical velocity \( \sigma^2_w \) throughout the boundary layer. These assumptions led to the relationship.
\[ \frac{g}{\theta} \left( \omega' \theta' \right)_h = \frac{\partial}{\partial z} \left[ \left( e \omega' \right) + \frac{1}{\rho} \left( p' \omega' \right) \right] = A \sigma_w^3 \frac{h}{h} \] \tag{3.9}

where \( \sigma_w \) include both the effect of convective and mechanical turbulence, and \( A \) is a constant identified by Tennekes to be approximately 0.2 (Tennekes did not explicitly include the pressure-flux term \( p' \omega' \), but the scaling he employed is valid for the entire transport term (Tennekes, 1975).

Zilitinkevich (1975) extended Tennekes' analysis to include the time derivative term in Equation 3.8. By considering a co-ordinate frame moving with the inversion and employing the same scaling relationships proposed by Tennekes, Zilitinkevich estimated the temporal of turbulent kinetic energy budget as

\[ \frac{\partial E}{\partial t} = \frac{\partial E}{\partial z} \frac{\partial z}{\partial t} = - \frac{\partial E}{\partial z} \frac{\partial h}{\partial t} = C \sigma_w^3 \frac{dh}{h} \] \tag{3.10}

where \( C \) is a constant. From Equation 3.10, it is clear that the inclusion of Zilitinkevich's correction reduces the entrainment rate. This reduction is most pronounced during early morning periods when \( h \) is small and \( dh/dt \) is large.

Due to the absence of an obvious length scale and the lack of necessary validation data, the dissipation term is not as well characterized as other terms in Equation 3.8. Nevertheless, several parameterisations for this term have been proposed (e.g., Stull, 1973; Mahrt et al., 1979). Zeman & Tennekes (1977) recommended the scaling relationship proposed by Stull (1973) who related the
dissipation rate to the distance by which rising parcels of air penetrate the stable layer above the inversion. If the inversion strength is not large, a simple expression for this depth can be obtained by equating the kinetic energy of the rising parcels to the deceleration associated with the stable layer. This formulation leads to

$$\frac{g}{\theta} \rho d^2 \propto \sigma_{wh}^2$$

where $\sigma_{wh}^2$ is specifically $\sigma_w$ at the base of the inversion and $d$ is given by

$$d \propto \frac{\sigma_{wh}}{N} , \quad N = \left( \frac{g}{\theta} \gamma \right)^{1/2}$$

where the $N$ is the Brunt-Vaisala frequency. Using these formulations, Zeman & Tennekes (1977) expressed the dissipation rate as

$$\varepsilon = C_d \overline{\sigma_w^2} N$$

where $C_d$ is the dissipation constant and $\overline{\sigma_w}$ is the velocity scale introduced earlier which is substituted for $\sigma_{wh}$ under the assumption of mixed layer similarity. Since the Brunt-Vaisala frequency is associated with inertial disturbances, this particular form has intuitive appeal. However, a theoretical drawback exists in that there is no limit to $\gamma$; thus, negative entrainment rates ($\frac{d h}{d t} < 0$) can occur. Fortunately, the large magnitudes of $\gamma$ required for this to occur are exceedingly rare in the lower atmosphere.
The velocity scale \( \overline{\sigma_{wh}} \) used in the above analysis is estimated directly from surface turbulence fluxes. In convection conditions, \( \overline{\sigma_{wh}} \) is simply taken as the convective velocity scale \( w^* \)

\[
w^* = \left[ \frac{gh}{\theta} \left( \overline{\theta'} \overline{\theta'} \right) \right]^{1/3}
\]

In situations where appreciable shear exists across the boundary layer, entrainment from mechanical turbulence can become significant. The most straightforward method of including mechanical effects involves parameterising \( \overline{\sigma_{wh}} \) in terms of both \( w^* \) and \( u^* \). Here, most workers choose to interpolate between the two limiting cases of \( w^* \to 0 \) (neutral) and \( u^* \to 0 \) (free convection) using the interpolation relation

\[
\overline{\sigma_w^a} = w^a + b^a u^a \quad (3.11)
\]

where \( a \) and \( b \) are constants. This interpolation approach was first used by Tennekes (1973) who suggested values of \( a = 2 \) and \( b = 2 \). In a re-examination of the data originally used by Tennekes (1973) and Zeman & Tennekes (1977), Driedonks (1982a) suggested a cubic formula; namely, Equation 6.10 with \( a=3 \) and \( b=2 \). Although the interpolation equation is supported by laboratory observations (Kato & Phillips, 1969; Brown, 1996), it is at best a rough approximation of the turbulent state of the CBL. For instance, in the free convection ABL, the friction velocity has no effect on the inversion; whereas in the near-neutral ABL, the vertical velocity variance is a somewhat more complicated function of the friction velocity.
Fortunately, the effect of $u_*$ in Equation 3.11 quickly diminishes as strongly convective conditions are approached since $w^* \rightarrow w^3$ for such cases. With regard to the latter limiting case, Zeman & Tennekes (1977) noted that simple interpolation formulae such as Equation 3.11 perform well so long as $h/L > 1$.

### 3.4.4.1 Relationships for the Inversion Growth Rate

Using the entrainment parameterisations of Tennekes (1973), Zilitinkevith (1975), and Stull (1973), the inversion turbulent kinetic energy budget (Equation 3.8) becomes

$$\frac{-C}{h} \overline{\sigma_w^2 (\omega' \theta')_h} = g \left( \frac{\omega' \theta}{\theta} \right)_h + A \frac{\sigma_w^2}{h} - C_d \overline{\sigma_w^2} N$$

(3.12)

which upon grouping terms, simplifying, and employing Equation 3.5, gives the growth rate of the inversion height as

$$\frac{dh}{dt} = \frac{A}{gh \Delta \theta} \left[ \frac{h}{\sigma_w} - \frac{C_d N}{C \sigma_w^2} \right]$$

(3.13)

Inversion models as developed and discussed by Tennekes (1973), Zeman & Tennekes (1977), and Driedonks (1982b) may all be expressed by Equations 3.7, 3.11, and 3.12 with variations in the constants $A$, $C$, $C_d$, and $b$. Values for these constants as proposed by those authors are summarised in Table 3.3.
Table 3.3: Published values for the constants in Equations 3.11 and 3.13 (from Brown, 1996).

<table>
<thead>
<tr>
<th></th>
<th>Tennikes</th>
<th>Zeman &amp; Tenneks (1977)</th>
<th>Driedonks (1982b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0.2</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>$C$</td>
<td>0</td>
<td>3.55</td>
<td>4.3</td>
</tr>
<tr>
<td>$C_d$</td>
<td>0</td>
<td>0.024</td>
<td>0.03</td>
</tr>
<tr>
<td>$a$</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$b$</td>
<td>2.32</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Based on the entrainment parameterisations previously discussed, other relationships governing the inversion-height growth rate have been proposed. For example, Thomson (1992) discussed an analytical solution to Equation 3.13 based on the model proposed by Tennekes (1973). However, Thomson assumed a constant potential-temperature gradient, which is overly restrictive. An additional approach that is worth noting is that of Bachvarova & Gryning (1991) who reduced the coupled system given by Equations 3.7 and 3.13 to a single differential equation for $h$. In their analysis, they developed a differential equation for the inversion strength as a function of inversion height by first rearranging Equation 3.7 and then employing the chain rule to obtain

$$
\left[ h \frac{d\Delta \theta}{dz} + \Delta \theta - \gamma h \right] \frac{dh}{dt} = -\left( \omega^2 \theta \right)_h
$$

(3.14)

Then, by neglecting the time-dependent and dissipation terms, they used the turbulent kinetic energy budget at the inversion, Equation 3.8, to derive an analytically tractable differential equation for inversion strength. With substitutions, Equation 3.14 becomes.
\[
\frac{d\Delta \theta}{dh} = -\Delta \theta \left[ \frac{1}{h} + \frac{1}{Ah - BkL} \right] + \gamma \tag{3.15}
\]

where $B$ is equivalent to $A^b$. Gryning & Batchvarova (1990) proposed an analytical solution to Equation 3.15 where the time-dependent nature of the surface turbulence fluxes is neglected by implicitly assuming $\partial h/\partial t = 0$. The exact solution to Equation 3.15 is quite cumbersome. However, a simplified solution with the correct asymptotic limits for neutral and convective conditions is given by Gryning & Batchvarova (1990) as

\[
\frac{\Delta \theta}{\gamma h} = \frac{Ah - BkL}{h(1 + 2A) - 2BkL} \tag{3.16}
\]

To obtain the equation governing the inversion-height growth rate, Batchvarova and Gryning (1991) first substitute Equation 3.15 and Equation 3.16 into Equation 3.14, and then use an entrainment relationship similar to Equation 3.12 to relate the heat transfer at the inversion-height to that at the surface. This leads to the expression

\[
\left\{ \left[ \frac{h^2}{(1 + 2A)h - 2BkL} \right] + \frac{Cu^2 T}{\gamma g((1 + A)h - BkL)} \right\} \frac{dh}{dt} = \frac{(w' \theta')_L}{\gamma} \tag{3.17}
\]

where Batchvarova & Gryning choose $A$, $B$ and $C$ to be 0.2, 2.5 and 8.0, respectively.

In a critical review of the suggested equations for the growth of the convective boundary layer, Seibert et al. (2000) reported that "These studies as well
as sensitivity experiments carried out by Beyrich (1994) have shown that the observed variability of the mixing height during daytime can in general be well described if surface heating and mechanical turbulence production due to surface friction are taken into account, with the entrainment heat flux parameterised in terms of the surface heat flux. These effects are properly parameterised in the following two equations from Driedonks (1982a) and Batchvarova and Gryning (1991), respectively:

\[
\frac{dh}{dt} = A \frac{\left(\omega' \theta^h\right)_{p}}{\Delta \theta} + B \frac{u^3}{\beta h \Delta \theta} = \frac{Aw^3}{\beta h \Delta \theta} + Bu^3 \tag{3.18}
\]

\[
\frac{dh}{dt} = (1 + 2A) \frac{\left(\omega' \theta^h\right)_{o}}{\gamma_o h} + 2B \frac{u^3}{\gamma_o \beta h^2} \tag{3.19}
\]

More recently, Nagar et al. (2001) used a one-dimensional prognostic model to determine the mixing height over a tropical station in India. The performance of the model was found to be sensitive to the potential temperature gradient above the inversion base ($\gamma$). The model predicted mixing heights that agreed well with observed heights when $\gamma$ is specified as a function of time.

### 3.4.4.2 Models with Inversion Layer Structure

For the slab/jump model, the inversion flux of sensible heat is parameterised. This closes the system of equations needed to predict the development of the mixed layer $h$, and the mean value of $\theta$ or $\theta_v$ over time. When the TKE equation is used to parameterise $(w'\theta')_h$, the inversion layer is given some structure when terms such as
dissipation and mechanical production are considered (e.g., Zeman & Tennekes, 1977). Nevertheless, all closure models employ the jump approximation to acquire the prediction of $dh/dt$. The following discussion is based on a review by Kustas (1986) and Batchvarova & Gryning (1994).

Betts (1974) was one of the first to account for a finite inversion thickness in the inversion layer, $\Delta h$, when integrating the enthalpy equation over $\Delta h$. Betts showed that integration of the enthalpy equation without the source and advective terms) over the interfacial layer leads to:

$$
\left( \omega' \theta' \right)_h = \frac{dh}{dt} (\Delta \theta - \gamma \Delta h)
$$

(3.20)

so long as $\Delta h$ remains constant. According to Betts, Equation 3.20 explains why in the capping inversion the gradient in $\theta$ is always larger than in the free atmosphere if there is to be entrainment of warmer air into the mixing layer. But, as pointed out by Deardorff (1979), this assumes that the gradient of $\theta$ is approximately linear through the interfacial layer.

Deardorff (1979) went further with the Betts model by showing that the heat flux at the inversion is a function of the entrainment at the top and bottom of $\Delta h$. Deardorff's principal finding was that a model giving structure to the interfacial layer implies an entrainment relation whose closure require an assumption about $\Delta h$, instead of the layer's negative buoyancy flux. Deardorff developed a model using the profile of $\theta v$ in the interfacial layer resembling a 3rd order polynomial (Figure 3.2).
His model did well in predicting the inversion height, $h$, and mean mixed layer temperature of laboratory data; but the height where $(w' \theta') = 0$ (known as the cross-over point) was needed to simplify his analysis. This cross-over point is difficult to determine with radiosonde data.

![Diagram](image)

**Figure 3.2:** Profiles of the potential temperature (left) and sensible heat flux (right) generally observed in the ABL.

Turbulence measurements with aircraft have shown that the cross-over point lies mainly between 0.6 $h$ (Mahrt & Paumier, 1984) and a value that is essentially $h$ (Yamamoto _et al._, 1977; Kaimal _et al._, 1976; Caughey & Palmer, 1979). More recently, Mahrt & Paumier (1984) derived semi-empirical equation for the ratio $h'/h$, where $h'$ is the height of the cross-over point, as a function of $h/L$. They argued that because the majority of buoyantly generated turbulence is consumed by viscous dissipation, a purely convective ABL has its cross-over point just below the inversion. However, with substantial shear (e.g., caused by wind shear near the inversion) the entrainment of warmer air is expected to extend to lower levels due to
the fact that \( \frac{dh}{dt} \) is partly maintained by shear generation of TKE and greater buoyant destruction (that is dampening) of turbulent energy. Mahrt & Paumier's Figure 5 is a plot of \( h'/h \) versus \( h/L \) reveals that \( h'/h \) decreases with decreasing \( (h/L) \), though there is considerable scatter. For near-neutral conditions (i.e., \( -h/L \sim 0 \) (1)) their figure shows \( h'/h \approx 0.5 \) while \( h'/h \approx 0.8 \) for strongly convective situations (i.e., \( -h/L \sim 0 \)).

Deardorff himself acknowledges other limitations of the application of his formulation to atmospheric data. There is the uncertainty in the entrainment rate which can be of the same order of magnitude as the subsidence. The data obtained from radiosondes are instantaneous, and soundings taken every several hours to acquire the mean values of \( \Delta h \), \( \theta \Delta \) and \( h \), for example, is probably unsatisfactory. Instead, several radiosondes released over the same time interval would give more accurate estimates, since it would help in reducing the effects of convective plumes and turbulent eddies creating the often contorted interfacial layer. Until better data are available for the atmosphere, refinements such as Betts' (1974) or Deardorff's (1979) to the jump approximation for the inversion layer (i.e., Equation 3.5) in the convective boundary layer would appear to be the most attractive prospect in improving the prediction of the inversion buoyancy flux.

In most of the existing models for the daytime mixing-layer height, the mixing-layer is capped by a sharp potential temperature jump. This assumption is not in accordance with measurements. The entrainment zone typically constitutes about 30 \% of the mixing-layer and it can even reach depths comparable to the mixing layer itself (Batchvarova Gryning, 1994). Gryning & Batchvarova (1994) proposed the
parameterisation of the entrainment zone (EZ) depth. Based on the EZ parameterisation and on the zero-order mixing layer height model of Batchvarova & Gryning (1991), Batchvarova & Gryning (1994) derived a combined (two-layer) model for the height of the mixing layer and the depth of the entrainment layer zone under near-neutral and unstable atmospheric conditions. Further discussion of this model is presented in Chapter VII.

3.5 SUMMARY

Two general possibilities exist for the operational mixing height determination. One is the analysis of profile measurements, and the second is the application of parameterisations or models based on operationally available data. Analysis of profile measurements is preferred if suitable data are available. Since none of the methods and models is perfect, care must be taken in estimating the mixing height.

For convective boundary layers, the numerical slab model is appropriate. Its integration should use the actual initial temperature profile, and not a predetermined value of the gradient of the potential temperature above the mixing height. Rate equations in such models should include also the mechanical contribution to mixing layer growth, parameterised by the friction velocity (u*).

The preceding chapters have presented the statement of the problem, a general description of the Atmospheric Boundary Layer, and a review of the methods used in the determination of the mixing height. An important suggestion for future research in the COST 710 report was the establishment of a long term monitoring
programmes for the ABL in different climatic regions, including measurements of
the surface energy budget components, turbulence parameters at several levels, at
least two profiling systems, and radiosondings. As a part of the world-wide research
effort in this subject, this study will contribute to that goal by collecting data and
using it in the determination of mixing height in a near-equatorial area from
measurements (Chapter IV) and from parameterisations using a slab model (Chapter
VI) employing the necessary surface parameters (Chapter V). Improvements to the
slab model and a discussion on the constants and parameters specific to this
particular climatic region are considered (Chapter VII). This will generate
recommended values of these constants and parameters to use when applying air
pollution dispersion models in this region.