HEAT FLUX PERFORMANCE OF HETEROGENEOUS NANOFLUID BOUNDARY LAYER FLOWS OVER AN INCLINED CYLINDER USING VARIANT GRAPHENE AND CARBON-BASED NANOPARTICLES

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Field of Study: **APPLIED MATHEMATICS**

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ABSTRACT

The heat transfer processes, boundary layer, heat exchangers, nanofluid models are briefly explained in the initial part of study. Then, the laconic preface on magnetohydrodynamic (MHD) and stagnation point flow are also included. The equations of boundary layer assumptions for continuity, momentum and energy are describe generally followed by the selected cylindrical coordinates. The governing partial differential equations (PDEs) are molded according to Tiwari-Das model and reformulated into nonlinear ordinary differential equations (ODEs) by using similarity expressions. A shooting technique is opted to reformulate the ensuing equations into boundary value problems which are then solved numerically by using a finite difference code that executes the three-stage Lobatto IIIa formula in Matlab. Thus, variant graphene-based nanoparticles; graphenes, graphene nanoplatelets (GNPs), graphene oxides (GOs), Single Walled Carbon Nanotubes (SWCNTs) and Multiple Walled Carbon Nanotubes (MWCNTs) in a water-base fluid is the center of interest for the present study. The comparisons between the present and previous published results are presented for accuracy and veracity of the numerical results. The effects of constructive parameters toward the model on dimensionless velocity and temperature disseminations, reduced skin friction coefficient and reduced Nusselt number are presented graphically and discussed in details. Research outcomes show graphenes-water nanofluid has the highest heat flux performance compared to other selected nanofluids across many emerging parameters considered in this study.

Keywords: Hetereogeneous, Tiwari-Das, nanofluids, graphenes, GNPs, GOs.

PRESTASI FLUKS HABA TERHADAP ALIRAN LAPISAN KEPELBAGAIAN BENDALIR NANO KE ATAS SEBUAH SILINDER CONDONG DENGAN MENGGUNAKAN VARIASI NANO PARTIKEL GRAPHENE DAN NANO PARTIKEL ASAS KARBON

ABSTRAK

Proses pemindahan haba, lapisan sempadan, penukar haba, model bendalir nano dijelaskan secara ringkas dalam bahagian awal kajian. kemudian, pendahuluan ringkas mengenai magnetohidrodinamik (MHD) dan aliran titik genangan juga disertakan. Persamaan andaian lapisan sempadan untuk kelangsungan, momentum dan tenaga dijelaskan secara umum diikuti oleh koordinat silinder yang dipilih. Persamaan pembezaan separa terkawal (PDE) dibentuk mengikut model Tiwari-Das dan dirumuskan semula menjadi persamaan pembezaan biasa tak linear (ODE) dengan menggunakan ungkapan kesamaan. Teknik tembakan dipilih untuk merumuskan semula persamaan yang seterusnya menjadi masalah nilai sempadan yang kemudian diselesaikan secara berangka dengan menggunakan kod perbezaan hingga yang melaksanakan formula tiga tahap Lobatto IIIa di Matlab. Oleh itu, variasi nano partikel berasaskan graphene; graphene, graphene nano platelet (GNPs), graphene oksida (GOs), nanotiub karbon satu dinding (SWCNTs) dan nanotiub karbon multi dinding (MWCNTs) dalam cecair asas iaitu air adalah tumpuan utama kajian ini. Perbandingan antara hasil yang diterbitkan sekarang dan sebelumnya dibentangkan untuk ketepatan dan kebenaran keputusan berangka. Kesan parameter konstruktif terhadap model pada penyebaran halaju dan suhu tanpa dimensi, pekali geseran dan nombor Nusselt ditunjukkan secara grafik dan dibincangkan secara terperinci. Hasil penyelidikan menunjukkan gabungan bendalir nano graphene dan air mempunyai prestasi fluks haba tertinggi berbanding dengan bendalir nano terpilih yang lain terhadap semua parameter dalam kajian ini.

Kata kunci: Kepelbagaian, Tiwari-Das, bendalir nano, graphene, GNPs, GOs.

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LIST OF SYMBOLS AND ABBREVIATIONS

T_∞	:	Ambient temperature
ω	:	Angle of inclination parameter
L	:	Characteristic length
γ	:	Curvature parameter
ρ_f	:	Density of fluid
ρ_{nf}	:	Density of nanofluid
ρ_s	:	Density of solid nanoparticle
θ	:	Dimensionless temperature of nanofluid
F	:	Dimensionless velocity of nanofluid
σ	:	Electrical conductivity
Gr	:	Grashof number
G	:	Gravity
υ_f	:	Kinematic viscosity of fluid
υ_{nf}	:	Kinematic viscosity of nanofluid
M	:	Magnetic parameter
U_∞	:	Mainstream velocity
λ	:	Mixed convection parameter
Nu	:	Nusselt number
Pr	:	Prandtl number
R	:	Radius of cylinder
Re	:	Reynolds number
η	:	Similarity variable
C_f	:	Skin friction coefficient
ϕ	:	Solid nanoparticle volume fraction

C_{pf} : Specific heat capacity of fluid

$C_{p\,nf}$: Specific heat capacity of nanofluid

$C_{p\,s}$: Specific heat capacity of solid nanoparticle

κ : Stagnation parameter

ψ : Stream function

T : Temperature of nanofluid

k_f : Thermal conductivity of fluid

k_{nf} : Thermal conductivity of nanofluid

k_s : Thermal conductivity of solid nanoparticle

α_f : Thermal diffusivity of fluid

α_{nf} : Thermal diffusivity of nanofluid

β_f : Thermal expansion coefficient of fluid

β_s : Thermal expansion coefficient of solid nanoparticle

B_0 : Uniform magnetic field of strength

V^*_w : Uniform surface mass flux

V : Velocity component along r-axis

U : Velocity component along x-axis

μ_f : Viscosity of fluid

q_w : Wall heat flux

v_w : Wall permeability parameter

τ_w : Wall shear stress

T_w : Wall temperature

AGM : Adomian Galerkin Method

CFD : Computational Fluid Dynamic

CM : Collocation Method

CNTs : Carbon Nanotubes

CVFEM	:	Central Volume Finite Element Method
DTM	:	Differential Transform Method
FDM	:	Finite Difference Method
FEM	:	Finite Element Method
FHD	:	Ferrohydrodynamic
FVM	:	Finite Volume Method
GM	:	Galerkin Method
GNPs	:	Graphene Nanoplatelets
GOs	:	Graphene Oxides
HAM	:	Homotopy Analysis Method
HPM	:	Homotopy Perturbation Method
KKL	:	Koo–Kleinstreuer–Li
LBM	:	Lattice Boltzmann Method
MCHS	:	Microchannel Heat Sinks
MFD	:	Magnetic Field Dependent
MHD	:	Magnetohydrodynamic
MHEXs	:	Multi-stream Heat Exchangers
MWCNTs	:	Multiple Wall Carbon Nanotubes
MWLS	:	Meshless Weighted Least-Squares
ODEs	:	Ordinary Differential Equations
OHAM	:	Optimal Homotopy Analysis Method
PCME	:	Phase Change Material Emulsion
PDEs	:	Partial Differential Equations
PGHE	:	Pile Geothermal Heat Exchanger
PHPs	:	Pulsating Heat Pipes
PSCM	:	Pseudospectral Collocation Method

RKF	:	Runge-Kutta Method
RSM	:	Response Surface Methodology
SWCNTs	:	Single Wall Carbon Nanotubes
VIM	:	Variational Iteration Method

LIST OF APPENDICES

CHAPTER 1: INTRODUCTION

1.1 Background of Study

Fluid flows exist in every domain of our biological and technological system and every human being discerns and assesses the benefits of the wide-ranging fluid flows properties. With nonexistence of fluid flows in life, science and technology are impossible to run and might not even complete their processes (Hauke, 2008). For this reason, fluid flows are therefore indispensable in survival. Such flows might be viscous or inviscid, unsteady, or steady, three dimensional or two dimensional, reverse or forward, asymmetric or symmetric, immiscible or homogeneous and oblique or normal (Hayat et al., 2017; Yamaguchi, 2008). Therefore, fluid flows act as a first-rate duty for heat transfer in science and industrial processes such in transportation, cooling and heating exchanger, microelectronics and microfluidics, chemical reactions, hybrid engines, wire drawing, fuel cells, glass fiber, biomedical and pharmaceutical fields, food manufacturing, water purification, polymer and metal extrusion and drug delivery. A great deal of works has been led in this field to date, considering the many applications in engineering, science, and innovation.

Fluid mechanic is defined as an application of the laws of force and motion to the fluids. There are two branches of fluid mechanics which are fluid statics (hydrostatic) and fluid dynamics (hydrodynamics). In the 19th century, this hydrodynamics had split into two different directions which are experimental and theoretical hydrodynamics. However, since the results give the contradictions between each other, Ludwig Prandtl made a great achievement in which he managed to unify the diverging direction between theoretical and experimental hydrodynamics of fluid mechanics. Thereby, in 1904, the imperatives of the boundary-layer theory were presented by Prandtl in his paper of breakthrough fluid mechanics (Yamaguchi, 2008). By using theoretical consideration together with some simple experiments, Prandtl showed that the flow past a body can be divided into two regions which are boundary layer flow and the remaining region outside the layer. As we already know, boundary layer is a thin layer of viscous fluid close to the solid surface of a wall in contact with a moving stream in which the flow velocity varies from zero at the wall (where the flow sticks to the wall because of its viscosity) up to free stream velocity at the boundary. Consequently, the concept of a boundary layer is one of the most important ideas in understanding transport processes.

There are two well-known flow patterns which are laminar and turbulent flows and these flow patterns can be defined effectively in terms of Reynolds number. These flow patterns are well illustrated in Fig 3.2 in Chapter 3. Actually, before laminar flow begins to develop, there is a natural flow at the first point of flow. This flow is not very popular among other flows because of lack of significant applications in boundary layer fields due to its low Reynolds number. As the flow speed is further increased, a laminar boundary layer begins to develop. It is defined as the parallel or smooth flow of fluid in one direction at a constant speed between multiple layers with no disturbance and the range of its Reynolds number is less than 2000 (Menon, 2015). After some distance, small chaotic oscillations begin to develop in the boundary layer and the flow begins to transitions. In simple words, transition flow oscillates between laminar and turbulent flow. The flow is turned to turbulence in which recirculation, eddies and apparent randomness occurred. The Reynolds number is greater than 4000 for this flow (Menon, 2015). These boundary layers include various forms of flow which are laminar, transient or turbulent and the distinction between these flows is very crucial toward the development of transport at interface based on their features and specialties.

According to Hauke (2008), many of the science engineering procedures such as friction analysis, heat and fluid mass transfer are conducted at the area around the interface referred to as boundary layer. In fluid mechanics, there are three methods used to address boundary layer flow issues which are theoretical, numerical, and experimental solutions. There are several researchers who studied the flow and heat transfer over boundary layer problems in diverse geometries and different aspects in which are listed in the next Chapter 2 (2.3).

The conventional heat transfer fluids which are used frequently in various industrial processes namely water, oil and ethylene glycol have low thermal conductivity which lead to limitation of heat transfer performance and efficiency of industrial equipment. The remarkable improvements in heat transfer rate were analyzed when nano-scaled solid particles (nanoparticles) with higher thermal conductivity are introduced into these ordinary fluids (Halim et al., 2017). The suspension of nanoparticles in regular host fluid which subsequently termed as nanofluid was initially proposed by Choi in 1995. Additionally, studies on nanoparticles in terms of size, shape and type are also discussed by the researchers to explore the best heat transfer performance of the nanofluids. Apart from industry broadly-used SWCNTs and MWCNTs which are formed from single and multiple rolls of graphene sheet in tube-like nanostructure, research on graphenes, GNPs and GOs as potential nanomaterials in fluid mechanics analysis is still sporadic despite of their special combination of first-rate electrical and mechanical properties.

Nevertheless, this study is the first attempt that examined graphenes, GNPs, GOs, SWCNTs and MWCNTs altogether in the heat transfer enhancement of heterogenous nanofluids out of the long list of existing common literatures. The current research is also unique as it ventures on reversed stagnation-point flow phenomenon which is rare with varying angle of inclined permeable cylinder as the surface medium for heat exchanger. A finite difference code in MATLAB environment is employed to solve the transformed boundary value problems from the initial set of governing partial differential equations (PDEs) defining the current variant graphene magnetohydrodynamic (MHD) nanofluids model. The enumerated results are validated and compared with previous published works of slightly different model of others by adjusting or dropping out some of the parameters presently foreseen. Emerging effects of constructive parameters toward the nanofluid flow in terms of non-dimensional velocity and temperature disseminations, skin friction coefficient and Nusselt number are analyzed and discussed extensively. These findings are then presented in numerical tables and graphical disseminations for concluding remarks at the end.

1.2 Guiding Research Questions

i. How can the mathematical models for heterogeneous nanofluid boundary layer flows be developed?

ii. How can the new heterogeneous nanofluid boundary layer flows be solved?

iii. What the heat transfer rate performance for the new heterogeneous nanofluid boundary layer flows using variant graphene-based nanoparticles could be?

The above guiding research questions are successfully explained in subsection 1.3 until 1.6 below.

1.3 Scope of Research

Analyze heat transfer performance of reversed stagnation-point flow of variant graphene-based nanoparticles past an inclined cylinder. Newtonian fluid is considered which is water-based nanofluids. Tiwari-Das nanofluid model is employed towards the governing problems and numerically solved by using finite difference method and code in MATLAB bvp4c package.

1.4 Research Objectives

i. To extend mathematical models in the form of system of PDEs and ODEs for heterogeneous nanofluid boundary layer flows.

ii. To find solutions of the presently extended heterogeneous nanofluid boundary layer models using selected numerical/approximate methods.

iii. To determine the heat transfer rate performance of the heterogeneous nanofluid mixtures using variant graphene-based nanoparticles.

1.5 Problem Statement

A limited number of studies are found in the literature for investigation of heat transfer performance of variant graphene-based nanoparticles. Due to insufficient information on specific graphene properties needed to conduct the necessary theoretical and numerical simulations, thereby, this research is very important to fill the gap of the literatures on graphene-based nanoparticles in heterogeneous liquid boundary layer flows especially due to their cheaper price than the carbon nanotubes (CNTs) and magnetite nanoparticles.

1.6 Significance of Research

According to Naseer et al. (2014) addition of a very small amount of nanoparticles into base fluids enhanced the thermal conductivity of the fluids up to two times better than conventional fluids. This significant improvement of heat transfer performance is the reason for the great usage of the nanofluids in the last decade. According to Mabood et al. (2017) suspending a small amount of nanoparticles dispersed in base fluids remarkably solved the issue concerning low thermal conductivity of these fluids. Thus, due to limited financial resources and the growing need for energy, it seems that the use of nanofluids in thermal systems can be responsive to the need of various industries such as solar collectors, thermal engineering, heat exchangers, cancer treatment, aerospace, and etc. (Mabood et al., 2017). Moreover, thermal conductivity and viscosity of nanofluids changed across different layers of the liquid due to heterogeneous concentration. Abo-Eldahab and Salem (2005) revealed that the results of numerical analysis considering heterogeneous concentration distribution are in reasonable agreement with experimental data as compared to homogeneous distribution. Many of the researchers from 1995 to 2020 also concerned on the heterogeneous reactions in boundary layer flow. However, there are still limited published studies considering the effect of the heterogeneous reaction of nanofluids boundary layer flow over a reversed stagnation-point flow with deviating angle of inclined permeable cylinder using variant graphene-based nanoparticles. Hence, the scarcity of the study will be improved in the current proposed research.

1.7 Thesis Organization

This thesis consists of five chapters.

Chapter 1 provides a general overview of this thesis and discusses the background of the study, objectives, problem statement and significance of research. **Chapter 2** presents the literature review of boundary layer flow, heat transfer, potential nanofluids and methods of solution. **Chapter 3** is mathematical formulation and methodology used in the study. **Chapter 4** presents the results and discusses the effect of emerging parameter and heat transfer performance. **Chapter 5** highlights the problem encountered in the study, the present contribution, the conclusion and suggestion for future research.

CHAPTER 2: LITERATURE REVIEW

2.1 Fluid Mechanics

Fluid mechanics is a science that explains of the motion of fluids using the essential laws of mechanics and thermodynamics (Durst, 2008). There are two major branches in fluid mechanics which are fluid statics and fluid dynamics. For fluid static, it is additionally called a study of stationary fluids where there is no motion of fluid particles occur, no shear strain and shear stress disappear. While fluid dynamics is defined as fluid in motion and the related forces can be expressed in vector equations.

Up to date, the field of fluid mechanic is extremely important in engineering, science and technology sectors such as in the application of heat exchanger, electronics devices, cooling and drying technology, thin-film, aerodynamics, pharmaceutical, medical and environmental area. Moreover, the usage of the components of flow devices such as pipes and joints, valves and nozzles are also related to fluid mechanics.

2.2 Heat Transfer Processes

There are three main processes considered in the study of heat transfer.

2.2.1 Conduction

Thermal conduction consists of kinetic energy transfer from one molecule to an adjacent molecule (Annaratone, 2010). Conduction occurs more naturally in solids and liquids, where the particles are close together compared to gases, where particles are further apart. Conductive transports are not only important and interesting for their applications, but they are also fundamental for our understanding of energy in models of physical processes (Fuchs, 2010).

There are many studies conducted in this field until now. Kumar et al. (2008) numerically developed a conduction heat transfer model utilizing a blown-powder laser cladding process. Hachem et al. (2010) presented an alternative approach via finite elements method (FEM) to treat the thermal shocks in transient conduction heat transfer numerically. Apart from that, Erdoğdu and Turhan (2006) analyzed the dimensional ratios of regular geometries for infinite geometry assumptions in unsteady-state conduction heat transfer processes. In this study, the certainty of the infinite body assumption was investigated for circular and square rods and slab group geometries. Yang (2008) developed a distributed transfer function method for problems of conduction heat transfer in one-dimensional multilayer composite bodies in which this proposed method is claimed to be numerically efficient as it only requires simple operations of two-by-two matrices. Also, Liu et al. (2005) studied the meshless weighted least-squares (MWLS) method for steady and unsteady states of conduction heat transfer problems while Saedodin et al. (2011) analyzed a nonlinear non-Fourier conduction heat transfer model in which variable specific heat coefficient were imposed into the heat equation. Loureiro et al. (2009) described an efficient time-Laplace domain approach to numerically analyze heat conduction problems. An efficient recurrence relationship for the temperature in the time-domain, based on the Green's functions of the model was presented in this study. Sadat et al. (2006) decided to utilize a diffused approximation-based meshless method to find the best conclusion for their study by considering conduction heat transfer.

2.2.2 Convection

Heat transfer of fluids by convection typically occurs after mixing one part of fluid with another fluid or the wall licked by the fluid itself with a temperature difference between one area and the other (Annaratone, 2010). Convection heat transfer processes occurs in three situations: free convection, forced convection and mixed convections which will be discussed in the next subsections.

2.2.2.1 Free Convection

The first heat transfer situation occurs with free convection. Free convection which is also called as natural convection in which caused by the fluid different temperature and locations (Khan & Pop, 2010).

Sheikholeslami et al. (2014) investigated the MHD effect on natural convection heat transfer in an enclosure filled with nanofluid and the governing equations are solved via Control Volume based Finite Element Method (CVFEM). Aside from that, Sun and Pop (2011) conducted a steady-state free convection heat transfer behavior of Tiwari-Das nanofluids inside a right-angle triangular enclosure filled with a porous medium with flush mounted heater on the wall. Hamad et al. (2011) discussed the similarity reductions for problems of a free convection nanofluid flow past a semi-infinite vertical flat plate considering the magnetic field effects. Sheikholeslami and Chamkha (2016) determined the electrohydrodynamic free convection heat transfer of a Fe3O4-ethylene glycol nanofluid by utilizing the CVFEM. Besides, Chen (2007) numerically studied the effects of laminar free convection heat transfer of a Newtonian fluid between an inner sphere and an outer vertical cylinder with isothermal boundary conditions. Sheikholeslami et al. (2016) analyzed the effect of magnetic field dependent (MFD) viscosity on free convection heat transfer of single phase nanofluid in an enclosure while considering the constant flux heating element at the bottom wall. Thus, CVFEM is also applied to simulate this problem. On the other hand, Bhargava et al. (2009) examined the effects of Soret-Dufour MHD free convection heat and mass transfer in a Darcian porous medium past a semi-infinite vertical plate with the presence of significant thermal radiation, viscous heating and first order homogenous chemical reaction. In the meanwhile, Chamkha and Aly (2010) focused on the numerical solution of a natural convection of nanofluid flow across a permeable vertical plate by considering the important emerging parameters such as magnetic field, heat generation or absorption, and suction or injection effects. Additionally, an MHD free convection heat flux of fluid mixture was presented by means of lattice Boltzmann method (LBM) in a paper by Sheikholeslami et al. (2018). Esmaeilpour and Abdollahzadeh (2012) numerically determined the heat flux performance of Cu-water nanofluid inside a two-dimensional vertical wavy enclosure under natural convection. Then, in the study of free convective flow model, Aghamajidi et al. (2018) considered an MHD laminar nanofluid flow adjacent to a spinning down-pointing vertical cone.

2.2.2.2 Forced Convection

Forced convection is defined as the flow of heat caused by some external applied forces (Khan & Pop, 2010). According to Wetzel and Boeckh (2012), forced convection is the heat is transferred between two fluids separated by a wall of exchanger.

For this reason, we list some of the research related to forced convection heat transfer. In early of 2007, Bharti et al. (2007) explored the steady cross-flow of incompressible fluids over a heated cylinder under forced convection condition and they chose to solve the case by finite volume method (FVM). Sheikholeslami et al. (2015) described on force convection heat transfer of ferrofluid in a lid driven semi annulus enclosure with the presence of non-uniform magnetic field by applying the similar numerical method. In the next year of 2016, Sheikholeslami et al. extended their research study on similar type of convection utilizing ferrofluid (Fe3O4-water) in a semi annulus lid with the Ferrohydrodynamic (FHD) and MHD effects. Bharti et al. (2008) examined numerically the steady, laminar cross-flow regime of forced convection heat transfer fluids flow past a heated elliptical cylinder. In another paper, Demir et al. (2011) considered the numerical investigation on the single phase forced convection heat transfer characteristics of TiO_2-water in a double-tube counter flow of horizontal tube heat exchanger. In addition, the study of MHD Al_2O_3-water flow past a horizontal stretching flat plate is investigated using Homotopy Analysis Method (HAM) and fourth order Runge–Kutta (RK4) by Hatami et al. (2013) under forced convection. Sundar et al. (2012) experimentally described the effect of forced convection and friction factor in a circular tube with Fe3O4 nanofluid flow. Hussein et al. (2014) conducted both experiment and numerical method for solving the friction factor and forced convection heat transfer of SiO_2-water in a car radiator while considering differences in concentration. Sheikholeslami and Bhatti (2017) reported on heat transfer in a porous semi-annulus considering the Brownian motion and different shapes of nanoparticles. Moreover, Bianco et al. (2009) indicated the development of laminar forced convection flow of a single- and two-phase water-Al_2O_3 nanofluid model in a circular tube.

2.2.2.3 Mixed convection

Mixed convection is known as the combination of free convection and forced convection. Thus, this mixed convection can be specifically defined as the ratio of buoyancy to viscous forces in the boundary layer (Khan & Pop, 2010). There are three physical cases in mixed convection which are assisting flow, opposing flow, and forced convection flow. The mixed convection heat transfer concept is essentially applied in various industrial operations such as solar power collector, thermal designing for buildings, cooling nuclear reactors, and many more (Oztop & Abu Nada, 2008; Gholinia et al., 2018).

For the sake of having wide applications in many engineering and industrial sectors, many researchers have given their attention toward this kind of heat transfer process. Sui et al. (2015) figured out the effect of shear flow and power law viscosity on the temperature field in mixed convection boundary layer heat transfer through a moving conveyor of an inclined plate by applying HAM. Later on, an optimization of mixed convection heat transfer with entropy generation in a wavy surface of square lid-driven cavity filled with Cu–water nanofluid by utilizing of Taguchi approach, FVM and SIMPLE algorithm was conducted by Mamourian et al. (2016). Besides that, Karimipour et al. (2012) carried out a research on the gravity effects of mixed convection heat flux in a microchannel using LBM with consideration of modified hydrodynamic boundary condition equations. Ghorbani et al. (2010) experimentally designed a coil-in-shell heat exchanger with various tube-to-coil diameter ratios and different dimensionless coil pitch of mixed convection heat transfer in both laminar and turbulent flow regimes. Turkyilmazoglu (2013) considered the Dufour-Soret effects of mixed convection for the MHD viscoelastic fluid flow past a permeable stretching vertical surface in a porous medium. Esfe et al. (2015) presented the research on mixed convection flow of alumina-water nanofluids in a horizontal channel where two hot obstacles are mounted on the bottom wall and conducted numerically using FVM and SIMPLER algorithm. Moreover, in the study of mixed convection, Hussain and Hussein (2011) measured numerically the steady laminar of fluid flow in a heated square enclosure with a conductive rotating circular cylinder. Al-Salem et al. (2012) probed on the effects of moving lid-direction on MHD mixed convection in a cavity with the bottom wall being linearly heated using FVM. Next, Dinarvand et al. (2017) investigated the steady axisymmetric mixed convective stagnation-point flow of an incompressible electrically conducting nanofluid over a vertical permeable circular cylinder in the presence of magnetic field. Then, Gholinia et al. (2018) extended the study by considering concentration equation and chemical species.

2.2.3 Radiation

Heat transfer by thermal radiation is conveyed by electromagnetic waves. Heat transfer by radiation happens in vacuum, or in materials (glasses and gases) which permits the transmission of electromagnetic waves (Wetzel & Boeckh, 2012).

Sakurai et al. (2010) implemented both the radiation element method (REM) and LBM to solve a combined mode transient conduction and radiation heat transfer problem with heat generation in a participating medium. In 2014, Mushtaq et al. evaluated numerically an MHD boundary layer flow and heat transfer over a linearly stretching sheet by utilizing a nonlinear radiative heat flux in Rosseland approximation. In the next year, Mustafa et al. (2015) analyzed the non-linear radiation heat transfer problem for stagnation-point flow of a non-Newtonian fluid. Moreover, they notified that the heat transfer from the plate decreases and temperature increases as they increase the radiation parameter. According to Bao et al. (2011), constructed solid oxide fuel cells (SOFC) mathematical model for analyzing the impact of radiation heat transfer on the density and temperature of the cells. Reiss (2012) researched on the impact of stability against quench of a superconductor radiation heat transfer in which the outcomes demonstrated that radiative heat transfer cannot be neglected if periodic disturbances have to be considered. An investigation on MHD flow and radiation heat transfer of three types of nanoparticles Cu, Al_2O_3 and Ag mixed with water along a flat plate in porous medium was conducted by Zhang et al. (2015) by using efficient analytical method. Next, Zheng et al. (2013) indicated the radiation heat transfer of a nanofluid flow over a stretching surface with velocity slip and temperature under the Rosseland's approximation. Qayyum et al. (2017) considered the nonlinear thermal radiation and homogeneous-heterogeneous heat and mass reactions of MHD flow based on silver-water and copper-water nanoparticles. Solutions of the problem are presented via a numerical technique namely Euler's Explicit Method (EEM). A paper by Li et al. (2016) who performed an investigation on an unsteady MHD flow and radiation heat transfer of a nanofluid in a finite thin film over a stretching surface in which the effects of heat generation, thermophoresis and Brownian was also examined.

2.3 Boundary Layer

Boundary layer is one of the most significant concepts in understanding transport processes. The essentials of the boundary-layer theory was introduced in 1904 by Prandtl in his paper on breakthrough fluid mechanics (Gholinia et al., 2018; Yamaguchi, 2008). Prandtl's theory was groundbreaking because it offered a theoretical explanation of the critical role of viscous effects in assessing motions of fluid. (Gholinia et al., 2018). Boundary layers are described as thin areas within the fluid in which most of the transport processes of momentum, heat, and mass take place there. (Hauke, 2008). Boundary layers can be differentiated into laminar, transient, or turbulent flow schemes. Reynolds number is used to quantify the transport coefficients for each kind of flow schemes.

There are several researchers who studied the flow and heat transfer over boundary layer problems in diverse geometries and different aspects. Salleh et al. (2010) explored the steady boundary layer flow and heat transfer over a stretching sheet with Newtonian heating and solved the governing problems by finite difference method (FDM). In the next year, Grosan and Pop (2011) addressed the steady axisymmetric boundary layer flow past a vertical cylinder and reported that the enhancement of particle concentration gives significant changes in both skin friction coefficient and local Nusselt number. Later, in the following year, Bachok et al. (2012) considered the unsteady boundary layer flow of a nanofluid over a permeable stretching and shrinking sheet theoretically. Ibrahim and Makinde (2013) used double stratification on an induced boundary layer flow and heat transfer of a nanofluid over a vertical plate and the result was performed by using Keller-box method. Subsequently, Shateyi and Prakash (2014) carried out a numerical simulation of MHD nanofluid flow past a moving surface with thermal radiation existence. Next, the effects of radiation and chemical reaction on the steady melting heat transfer boundary layer flow of MHD Williamson fluid through porous medium toward a stretching sheet are investigated numerically by Krishnamurthy et al. (2016). Kasmani et al. (2016) aimed on determining the influence of first-order chemical reaction on convective heat transfer of boundary layer flow of nanofluid past a wedge by considering heat generation and absorption and suction effects. Dinarvand et al. (2015) conducted a steady mixed convective heat transfer nanofluid flow past a vertically circular cylinder with specified external flow and surface temperature. On the other hand, Khan and Hashim (2015) worked on boundary layer flow and heat transfer of Carreau fluid over a nonlinear stretching sheet and solved the model by Runge-Kutta Fehlberg (RKF) method along with shooting technique. Further, Abbasi et al. (2015) prepared the Cattaneo–Christov heat flux model for laminar incompressible boundary layer flow of an Oldroyd-B fluid and the velocity and temperature analysis are obtained through optimal homotopy analysis method (OHAM). MHD boundary layer flow of Cu-water and Ag-water nanofluid over a rotating disk through a porous medium with chemical reaction, partial slip and thermal radiation were solved by Reddy et al. (2017) in the following years.

2.4 Heat Exchangers

Heat exchanger is defined as a device which let heat passes from one side to another. Heat exchangers basically are considered when different fluids are utilized at different stages of operation in any thermal equipment. There are many designated heat exchangers which differ in their geometrical arrangements, fluids usage and type of heat flow processes (Fuchs, 2010). These geometrical designs of heat exchanger also perform a vital role in improving heat and mass transfer of the flow.

Recently, modification and designing of heat exchangers are widely studied and applied to heat transfer in fluid mechanics. Farajollahii et al. (2010) measured the heat transfer characteristics of γ-Al_2O_3-water and TiO_2/water nanofluids in a shell and tube heat exchanger under turbulent flow condition. A study by Sui et al. (2010) numerically examined a laminar liquid–water flow and its heat transfer in three-dimensional wavy microchannels with a rectangular cross section. They notified that heat transfer performance of the present wavy microchannels is much better than that of straight microchannels because of the chaotic advection of the secondary flow called as Dean vortices. Later on, Mohammed et al. (2011) examined the laminar heat transfer flow of trapezoidal microchannel heat sinks (MCHS) using various types of base nanofluids and various MCHS substrate materials. Zamzamian et al. (2011) experimentally prepared a study of the performance of Al_2O_3/EG and CuO/EG in a double pipe and plate heat exchangers under turbulent flow regime. Cui et al. (2011) designed a pile of geothermal heat exchanger (PGHE) with buried spiral ring-coils in order to investigate the transient heat conduction of heat pump systems. They discovered that the upgraded finite ring-coil source model had better heat transfer accuracy compared to cylindrical source models. Ibrahim et al. (2013) analyzed the effect of magnetic field on stagnation point flow and heat transfer due to a nanofluid towards a stretching sheet while Pătrulescu et al. (2014) determined the steady mixed convection boundary layer flow from a vertical frustrum cone in a water based nanofluid (copper-water, alumina-water and titanium oxide-water). In addition, the MHD flow and heat transfer problem of water functionalized SWCNTs and MWCNTs over a static/moving wedge was carried out by Khan et al. (2015) to determine the impact of these properties on the thermofluidic performance. Mythili and Sivaraj (2016) probed the effect of higher order chemical reaction and non-uniform surface of heat source/sink on an unsteady Casson fluid flow over a vertical cone and a flat plate with the influence of thermal radiation. According to Tsay et al. (2017), multiple chemical processes rely on multi-stream heat exchangers (MHEXs) for heat integration, particularly at cryogenic temperatures. Thus, they tend to present a novel framework in designing a detailed spiral wound MHEX with the simultaneous optimization of the process flowsheet incorporating multi-stream heat exchanger models. On other cases, Sheikholeslami et al. (2019) operated on heat transfer behavior and energy storage efficiency of copper oxide nanoparticle through an enclosure with V shaped fins. Hence, the important parameters such as angle of V-shaped and length of fins are considered in this study.

2.5 Type of Fluids

Fluids are mainly categorized into two different types of fluids which are Newtonian and non-Newtonian fluids.

2.5.1 Newtonian Fluids

Newtonian fluids obey Newton's law of viscosity. Gases and small molecules of liquids such as air, water, alcohol and basic oils are said to be Newtonian fluids. Newtonian fluid has a linear relationship between shear stress and shear rate which hold the Stokes' hypothesis (Kleinstreuer, 2018). These types of fluids commonly encountered in fluid engineering processes.

Hussain and Ahmad (2015) discussed an unsteady MHD flow and heat transfer for Newtonian fluids over an exponentially stretching sheet in the presence of a uniform magnetic field which is applied perpendicular to the sheet. Meanwhile, Kishore and Ramteke (2016) studied a steady axisymmetric forced convective heat transfer of spherical particles in a Newtonian fluid flow regime with a velocity slip at fluid–solid interface. The problem was solved by using a computational fluid dynamics (CFD) based in-house solver. Patil (2017) experimentally investigated on heat transfer of Newtonian fluids through spiral coils. Next, Sheikholeslami et al. (2017) explained the flow and heat transfer in a CuO-water with the effect of Lorentz forces in a permeable enclosure with constant heat flux using CVFEM. In order to predict properties of nanofluid, Koo–Kleinstreuer–Li (KKL) model has been utilized in this case. Bondareva et al. (2018) presented a transient natural convection of water-based nanofluid in a partially open trapezoidal cavity under the influence of Brownian diffusion and thermophoresis.

Furthermore, Usman et al. (2018) considered MHD heat and fluid flow of water based and ethylene-glycol based Cu-nanoparticles between two parallel squeezing porous disks with suction/injection effects and the model was evaluated by least square Galerkin (LSGM) approach. It was observed that local Nusselt number achieved from Cu-water remains lesser than Cu-ethylene glycol. Haq et al. (2017) introduced an MHD pulsatile flow of Newtonian engine oil based CNTs between two concentric cylinders to control the random motion of the nanoparticles for both turbulent and laminar regimes. While in 2018, Wang et al. (2018) examined a direct absorption solar collectors of ZnO-Au mixed with oil-based Newtonian nanofluids and Al-Waeli et al. (2018) numerically conducted the effect of operating Newtonian nanofluids fluids (water, glycerin, and ethylene glycol) of photovoltaic thermal system (PV/T) on the convective heat transfer. They claimed that glycerin showed the maximum pressure drop while water indicated the minimal value in the problem study.

2.5.2 Non-Newtonian Fluids

Non-Newtonian fluids are defined as the fluids that do not obey Newton's law of viscosity (Yamaguchi, 2008). These fluids are usually highly viscous and very elastic fluids. Non-Newtonian fluids are macromolecule fluids, for example, latex paints, lubricants, polymeric liquids, multi-fluid blends, and particle suspensions and paste, pseudoplastic fluids, micropolar fluids, Eyring-Powell fluid, Walters-B fluid, Casson fluid and etc. (Grosan & Pop, 2011; Hayat et al., 2016; Kleinstreuer, 2018). The vast application of non-Newtonian fluids in the industry and commerce has led to develop research in this field.

Ellahi et al. (2016) proposed a generalized Couette flow of Eyring-Powell fluid in order to discuss numerically the diverse issues befell for the heat transfer, magnetohydrodynamics and slip condition. Das et al. (2015) consulted an investigation on melting heat transfer and thermal radiation effect of non-Newtonian Jeffrey fluid over a moving surface considering partial slip. They discovered that the fluid temperature of Jeffrey fluid is higher compared to Newtonian fluid. Alloui and Vasseur (2015) reported a numerical study of natural convection in a vertical enclosure filled with a Carreau–Yasuda non-Newtonian fluid model. This non-Newtonian fluid is used to characterize the behavior of the shear thinning fluids and the results revealed the strong influence of the pseudoplastic behavior of the selected non-Newtonian fluid on its natural convection heat transfer within the enclosure. On another paper by Hsiao (2017), he studied an electrical MHD ohmic dissipation for integrated forced and natural convection on a stagnation point heat and mass transfer energy conversion problem using an incompressible non-Newtonian Maxwell fluid. Ali et al. (2016) investigated the MHD flow and heat transfer of a couple stress in viscous fluid over an oscillatory stretching sheet embedded in a porous medium in the presence of heat source/sink. It is found that the presence of couple stress in viscous fluid increases the amplitude of oscillations in velocity and skin friction coefficient. Also, Malik et al. (2016) examined the MHD Sisko fluid heat transfer model over a stretching cylinder. This paper observed that Sisko fluid parameter enhances fluid velocity and enlarges skin friction coefficient as well. Sandeep and Sulochana et al. (2018) indicated in their study that the Jeffrey nanofluid has better enhancement of heat transfer than the Maxwell and Oldroyd-B nanofluids. Later on, Siddiqa et al. (2017) prepared an analysis on a boundary-layer behavior of a two-phase dusty non-Newtonian fluid flow along a vertical surface by using a modified power-law viscosity model. Other than that, the heat transfer analysis of steady flow of an incompressible Burgers' fluid over a stretching surface during a melting process is investigated by Awais et al. (2015). Further to this, in the study of non-Newtonian heat and mass transfer, Raju and Sandeep (2016) claimed that the heat transfer performance of Sisko nanofluid is higher than the Sisko ferro fluid.

2.6 Nanofluids

The conventional heat transfer fluids which are used frequently in various industrial processes namely water, oil and ethylene glycol have low thermal conductivity which lead to limitation of heat transfer performance and the efficiency and compactness of many engineering and industrial equipment. To encounter with the above thermal conductivity issue, nano-scaled solid particles (nanoparticles with average sizes below 10nm) having higher thermal conductivity are integrated within these ordinary fluids which provides remarkable improvement in heat transfer rate (Halim et al., 2017). This fundamental idea of usage of nanoparticles can be traced back to Maxwell in 19th century. Pioneering work on the novel concept of this new class of nanotechnology-based heat transfer fluid was proposed by Choi in the spring of 1993 to generate better heat transfer and energy power. In fact, these nanofluids contribute high thermal conductivity and heat transfer coefficient compared to regular fluid (Hayat et al., 2017).

Nanoparticles stay suspended much longer than microparticles and, if below a threshold level they remain in suspension almost indefinitely. These nanoparticles are fairly close in size to the molecules of the base fluid and, thus, can enable extremely stable suspensions and do not encompass sedimentation and erosion problems (Abbas et al., 2015; Khan et al., 2015). Additionally, the surface area per unit volume of nanoparticles is said to be significantly bigger than microparticles and thus support the stability of the suspensions, enhanced the flow, heat-transfer efficiency, and other characteristics (Haq et al., 2015). Although the increase in thermal conductivity is a very important interest, there is also a boost up in the average temperature of nanofluids compared to that of base fluid (Pandey & Kumar, 2016).

2.6.1 Performance and Research on Nanofluids

Nanoparticles have various shapes for example sheet, platelet, rod-like and spherical. They are made from different materials such as metal (copper, silver, iron, gold, titanium), oxide ceramics (alumina, titania, silica, copper oxide), metal nitride (silicon nitride, aluminium nitride) carbide ceramics (silicon carbide, titanium carbide), carbon (diamond, graphite, carbon nanotubes, fullerene) and functionalized nanoparticles.

As a result, the research topic of nanofluids has been receiving increased attention worldwide. For example, Ben Hamida and Charrada (2015) considered a numerical approach on a natural convection heat transfer in a square enclosure filled with an ethylene glycol-copper nanofluid under magnetic fields. Jafari et al. (2015) discussed on Lattice Boltzmann simulation of natural convection heat transfer of SWCNTs and copper nanofluids with different volume fractions of nanoparticles (0–1%) in an open enclosure. A comparison between SWCNT and Cu-nanoparticles showed that the SWCNT-nanoparticle has better performance to enhance the convection rate. Haq et al. (2015) analyzed MHD squeezed flow of metallic nanofluid (copper, alumina, and titanium dioxide) with the ordinary fluid (water) over a sensor surface. They concluded that Cu–water gives better heat flux performance as compared with the rest of mixtures in squeezing flow phenomena. In addition to that, Malvandi et al. (2015) issued theoretically an MHD mixed convection of Al_2O_3-water nanofluid within a vertical direction of annular pipe by considering Brownian motion and thermophoretic diffusivities for nanoparticle migration. However, the study indicated that the heat performance of Al_2O_3-water nanofluids is reduced in the presence of a magnetic field. Rahimi et al. (2016) simulated the study on thermal and flow of an unsteady squeezing nanofluid. They also investigated the effect on important heat transfer parameters in presence of magnetic field and their results presented that the performance of CuO is better than Al_2O_3's. To highlight this matter, they reported that the increment in nanofluid volume fraction, Nusselt number, Hartman number and the decrement in skin friction coefficient, result in an increase in velocity and temperature profiles. In another case, the study of heat transfer in a Microchannel Heat Sinks (MCHS) under the influence of magnetic field through Koo–Kleinstreuer–Li (KKL) model was described in a paper by Hosseini et al. (2018). They chose Al_2O_3-water nanofluid as the best nanofluid and it is used as a coolant fluid in the solution where the KKL correlation is utilized for calculation of the effective thermal conductivity and viscosity of nanofluid. Heydari et al. (2018) handled the study on the impact of attack angle of triangular ribs in laminar heat transfer of cooling nanofluids (Ag-water) in a two-dimensional microchannel by utilizing finite volume method. Aman et al. (2018) addressed an impact of cylindrical shape of gold nanoparticles in kerosene oil on mixed convection Poiseuille flow of a nanofluid passing through a porous medium. Some comparisons were made to clarify their work by selecting four other types of nanoparticles (silver, copper, alumina and magnetite). Moreover, Kandasamy et al. (2016) examined theoretically the effect of copper and SWCNTs in the water flow over a porous wedge with variable stream condition due to thermal radiation energy. Hayat et al. (2016) worked on water-based silver and copper nanofluids under nonlinear thermal radiation and mixed convection. They found that the skin friction is dominant for silver while local Nusselt number is dominant for copper. In other research done by Haq et al. (2015), they mentioned that engine oil has greater skin friction coefficient and heat transfer performance as compared to water and ethylene glycol when combining with CNTs nanoparticles.

2.6.2 Application of Nanofluids

The utilization of nanofluids becomes more vital in various areas such as in industrial, manufacturing, science and engineering, transportation, and medical fields. Below are the lists of examples for each field;

1) Industrial and manufacturing fields - paper production, chimneys and food stuff processing, microchips in computers, batteries, fuel cell, lubricating and engine oil, electronic equipment and weapons.
2) Science and engineering fields - polymer and metal extrusion, hot rolling, rotary dan spinning of metals, fiber and wire coating, glass bridge cables and columns, solid state lightening, ventilation, air-conditioning, heating and cooling heat process, the initial refrigerant in pressurized water reactors, fast safety systems, clutch, hydraulic braking system, combustion system, power generation processes, energy extraction, hybrids engines, heat exchanger tubes, petroleum, chemical reaction towers, nuclear reactors, electric coolers and solar collector.
3) Transportation fields - War vehicles, submarines, and aerospace.
4) Medical fields - Drug transfer, pharmaceutical manufacture, thermal therapy for cancer treatment and blood flow.

2.7 Heterogeneous Nanofluid Boundary Layer Flow

There are two different nanofluid models reported in the literature for boundary layer flow and its heat transfer which are homogeneous and heterogeneous types. Hayat et al. (2016) explained that the distinction is close to the fact that catalyst operates respectively in the same phase (gaseous phase) where the reaction takes place is said to be homogeneous or in different phase (solid phase) is said to be heterogeneous. These homogeneous and heterogeneous reaction parameters have opposite behavior for different base fluids (Hayat et al., 2016). In 2006, Buongiorno and Tiwari-Das came up with their mathematical models of homogeneous and heterogeneous nanofluids boundary layer flows respectively in order to solve heat transfer problems. Boungiorno had considered seven slip mechanisms (gravity, inertia, Brownian diffusion, thermophoresis, fluid drainage, diffusiophoresis, Magnus diffusion) that can form relative velocity between the regular fluid and the nanoparticles. Thus, after some validations, Boungiorno finalized that there are only two significant slip mechanisms consider in homogeneous nanofluids in which Brownian diffusion and thermophoresis (Boungiorno, 2006). In the meanwhile, Tiwari and Das developed a model of the problem to analyse the behavior of nanofluids by considering the solid nanoparticles volume fraction (Tiwari & Das, 2007). The correlation of the heterogeneity concept can be clearly seen via this model because it considers the thermophysical properties of regular base fluid and nanoparticles itself.

Consequently, a voluminous body of information on heterogeneous nanofluid boundary layer flow phenomena accumulated over the years. Several excellent research on the related heterogeneous cases are now available in literatures. In early 2011, Oueslati and Bennacer developed a study on natural convective heat transfer using different types of heterogeneous nanofluid mixture with weak solutal diffusivity and possible Soret separation. Apart from that, Sheremet et al. (2015) investigated a free convection in a differentially heated porous square cavity filled with a heterogeneous nanofluid considering the realistic empirical correlations for the nanofluid physical properties has been operated in numerical approach in the proposed study. Dinarvand et al. (2016) worked analytically on Homotopy Analysis Method (HAM) for solving an unsteady mixed convective stagnation-point flow of an electrically conducting nanofluid based on Tiwari-Das nanofluid model over a vertical moving sheet. By following the proposed heterogeneous nanofluid model, the skin friction coefficient and the local Nusselt number are reported to be the highest for copper-water nanofluid as compared to other selected nanomaterials. Yet, Chen et al. (2017) deal with heterogeneous nanofluid flow in a porous channel with suction and chemical reaction by introducing the conservation equation of nanoparticle volume fraction into Tiwari and Das nanofluid model. Moreover, an entropy generation rates of a biological fluid flow considering heterogeneous particle distribution in a mini double-pipe heat exchanger were explained by Bahiraei et al. (2017). They used the synthesized silver nanoparticles by utilizing plant extract method from green tea leaves for completing their study. Mabood et al. (2017) conducted the Tiwari-Das nanofluid model for copper and alumina combining with water under the mixed convection and unsteady fluid flow condition. Continuing to that, Selimefendigil and Chamkha (2019) presented a numerical research on a mixed convection of power law nanofluid-filled in a lid-driven triangular cavity with an opening using Tiwari and Das nanofluid model under the effect of an inclined magnetic field.

2.8 Magnetohydrodynamic (MHD)

Magnetohydrodynamic (MHD) refers to the electrically conducting liquid flow in the of magnetic fields existence. Lorentz force is a drag force in opposite direction of the flow generated by the magnetic field, which reduces the fluid flow velocity. Joule Heating improves heat transfer process by using electric current movement, which reduces the dynamic viscosity and increases electrical conductivity (Grosan & Pop, 2011).

The MHD systems have been found to have several fascinating applications in power generators, accelerators, electrostatic filters, pumps, magneto-optical wavelength filter, the cooling of reactors, optical modulators and grating, drug delivery and also in cancer treatment (Oztop & Abu Nada, 2008).

In this manner, several studies for nanofluid with magnetic properties were undertaken. Alsaedi et al. (2017) studied on mass and heat flux rates of an MHD stratified bioconvective stretched flow of a nanofluid due to gyrotactic microorganisms. Magneto nanofluid in presence of an inclined magnetic field is adopted through this study. Sheikholeslami and Shehzad (2017) explored on MHD CuO-water nanofluid convection in a porous semi enclosure considering constant heat flux boundary condition by utilizing CVFEM. They found that by adding nanoparticle augments, influences in increasing Lorentz forces. In the following month, the main author of the previous paper, Sheikholeslami (2017) solved another problem of MHD nanofluid in a three-dimensional porous cavity using Lattice Boltzmann method by considering similar Brownian motion effect. He figured out that Lorentz force causes convection heat transfer to decrease for the case of study. Apart from that, Sandeep et al. (2017) analyzed heat transfer enhancement of an unsteady MHD nanofluid flow embedded with water-based aluminum alloy nanoparticles over the vicinity of a thin elastic sheet by considering the variable heat source/sink. They explained that the external magnetic fields are proficient to determine the physical, thermal and other flow properties of ferrofluids. Sandeep et al. (2017) examined numerically the MHD liquid-film flow and heat transfer processes of homogeneous magnetite (Fe_3O_4) nanoparticles dispersed in water-based non-Newtonian (Jeffrey and Oldroyd-B) fluids past a stretched plate. In spite of that, Sucharitha et al. (2017) carried out a numerical investigation on the impact of Joule heating and wall flexibility on the peristaltic flow of MHD nanofluid in a uniform/non-uniform porous channel. Later on, Chamkha et al. (2018) solved numerically an MHD nanofluid natural convection in an enclosure under thermal radiation and shape factor of nanoparticles impacts. Meanwhile, Souayeh et al. (2019) made a comparative study of non-linear radiative heat transfer in an MHD Casson nanofluid along a fixed/moving thin needle in which the mathematical model of this case considered the thermo-diffuso and diffuso-thermo effects. In the corresponding year, Derakhshan et al. (2019) designed a hydrothermal framework of a steady MHD nanofluid flow between two parallel plates using Akbari-Ganji's Method (AGM) in the presence of uniform magnetic field. Then they compared their results with RK4 method in order to verify the correctness of the AGM.

2.9 Stagnation Point Flow

Stagnation flow is defined as the motion of fluid near the stagnation region where it meets the highest pressure, heat transfer rate and mass decomposition. This related flow utilized in many applications especially in transportation industries on designing the rockets, aircrafts, submarines, and oil ships. Moreover, these flows are also noticeable in polymer extrusion, food processing, paper production, continuous casting, and glass fiber production.

At present, the researchers are curious to investigate the stagnation point flows. Thereby, many reasonable literatures are available on stagnation point flow. Hayat et al. (2015) examined the nanofluid stagnation point flow over an impermeable moving cylinder with mass transfer and slip effects. Continuing this research, Hayat et al. (2016) studied the impact of homogeneous and heterogeneous reactions in a boundary layer flow which is caused by a stretching cylinder with melting heat transfer. Halim et al. (2017) numerically addressed a steady stagnation point flow with active and passive controls of the Williamson nanofluid over a horizontally linear stretching/shrinking surface. Their results depicted that the temperature and nanoparticle volume fraction decreased as stagnation parameter was reduced. Else, Hayat et al. (2017) operated on numerical simulation for simultaneous melting heat transfer and the radiation effects in stagnation point flow of carbon–water (SWCNTs and MWCNTs) nanofluids over a stretching cylinder. Not only that, Alizadeh et al. (2019) introduced a MHD CuO-water nanofluid stagnation-point flow over a cylinder enclosed in porous media under mixed convection heat transfer and thermodynamic irreversibility. Nadeem and Abbas (2019) reported on both MHD and thermal slip effects in a micropolar hybrid nanofluid past a circular cylinder under three-dimensional stagnation point flow. This study was carried out to describe the heat transfer enhancement of micropolar solid nanoparticles on the surface of cylinder having a sinusoidal variation. In another study, a KKL-model of nonlinear thermal radiation of CuO-water nanofluid flow over a stagnation point moving sheet were numerically exposed by Mohammadein et al. (2018). Rostami et al. (2018) made a detail explanation on dual solutions for MHD laminar mixed convective boundary layer flow near a stagnation point of water-based silica–alumina hybrid nanofluids past a vertical permeable plate. Tlili et al. (2019) explored on a steady forced convective MHD water-based nanofluid flow across a horizontal circular cylinder by considering the Brownian motion and thermophoresis effects by means of implicit finite-difference method. Later on, Khan et al. (2019) indicated the generalized mass diffusion effects by following Cattaneo–Christov model on upper convected stagnation point of Maxwell nanofluid flow over a stretchable surface. At the same year, Khan et al. (2019) extended their study by finding the changes in internal energy of thermal diffusion stagnation point in an upper convected Maxwell nanofluid flow with viscous dissipation, temperature-based viscosity, solar radiation and thermal conductivity over a linear stretching sheet.

2.10 Inclined Cylinder Heat Exchanger

Most early studies were concerned with overall heat transfer to different fluids flowing across cylinders in vertical and horizontal positions. However, fluid flow and heat transfer over an inclined cylinder have received notable interest among the various researchers nowadays due to fast growing applications in engineering and industrial processes.

There are some related literary works on heat transfer of nanofluid over an inclined cylinder. Dhanai et al. (2016) investigated the Buongiorno's nanofluid flow model over an inclined cylinder due to the impact of velocity and thermal slip. They claimed that their problem model contributed to many applications in food processing specially drying of cylinder-shaped bodies in which takes shrinkage into account at different air-drying temperatures. Moreover, the mutual effects of thermal and solutal stratification on Jeffrey magneto-nanofluid along an inclined stretching cylinder with thermal radiation and heat generation/absorption were analytically investigated by Ramzan et al. (2017). Usman et al. (2018) and Khan et al. (2019) utilized non-Newtonian Casson nanofluid and Williamson nanofluid respectively to study the effect of fluid flow and heat transfer over an inclined cylinder. The theoretical studies on nanofluid heat transfer over an inclined cylinder have gained much research attention in year of 2020 (Bilal et al., 2020; Ibrahim and Gadisa, 2020; Walelign et al., 2020). Ibrahim and Gadisa (2020) reported that the velocity field is retarded when the angle of inclination enhances and the heat transfer rate is reduced when increasing the curvature of the cylinder. In the latest study of Abbas et al. (2021), they extended the Xue model and Yamada-Ota model for hybrid nanofluid flow over nonlinear stretching cylinder and focusing on the stagnation point flow.

2.11 Method Used in Solving Fluid Problems

There are three conventional methods employed for addressing flow issues in fluid mechanics which are theoretical, experimental, and numerical solution methods.

2.11.1 Analytical Method

Analytical method is an exact solution and most differential equations are limited to be solved exactly (analytic or closed solution). Additionally, this method needs more time and sometimes impossible to be solved. Thus, nowadays, known semi analytical methods like Variational Iteration Method (VIM), Homotopy Analysis Method (HAM), Optimal Homotopy Analysis Method (OHAM), Homotopy Perturbation Method (HPM), Differential Transform Method (DTM) and the rest have been proposed by the researchers to conduct flow problem analyses.

On top of that, many scientists, who showed that applied mathematics can make important contributions to the analytical solution of flow problems, could be mentioned here. Hu et al. (2016) proposed a new analytical Fourier-series-based solution to the conduction–convection equation in order to calculate soil surface temperature, determine soil thermal parameters and the parameterization of land surface processes for modeling permafrost changes under global warming conditions or to estimate water flux. Hayat et al. (2017) addressed an analytical approach for three-dimensional convective flow of electrically conducting Oldroyd-B nanofluid induced by a stretching sheet with heat generation/absorption considering the effects of Brownian motion and thermophoresis. The nonlinear differential equations of this problem were solved analytically via OHAM. Abbasi et al. (2017) presented a paper on theoretical model for both convection-conduction heat transfer during cooled water injection into a fully penetrating well in fractured geothermal reservoirs. In order to calculate the total amount of heat extracted from a geothermal reservoir, they proposed two methods and a conceptual definition for the thermal recovery efficiency. On the other hand, Ijaz et al. (2018) reported an analytical study on heat and mass transfer of a non-Newtonian fluid flow interaction in the presence of heat and mass transfers through a finite symmetric wavy channel by considering electro-osmotic and MHD effects. The series solutions for the governing problem were presented and solved by Homotopy Pertubation Method (HPM) up to second order. Next, Mohamed et al. (2018) solved analytically the nonlinear governing equations an MHD nanofluid flow between non-parallel plates using Differential Transform Method (DTM). The obtained DTM data match perfectly with the numerical RK4 approach. El-Masry et al. (2020) studied the impacts of free convection heat transfer and varying magnetic field on a non-Newtonian Eyring–Powell fluid flow with peristalsis through variational iteration method (VIM) solution. They claimed that the previous attempts of peristaltic flow problems for a non-Newtonian fluid have been solved commonly by using the perturbation technique specifically limited for a small non-Newtonian parameter. Thus, they tend to solve the model by using VIM to avoid such limitation. Besides, a new analytical solution for single-phase convective heat transfer process from the combination of equivalent inner heat source efficiency field and effective thermal conductivity field was proposed by Chui (2020).

2.11.2 Experimental Method

A new chapter in fluid mechanic was opened by Osborne Reynolds in 1832-1912. He conducted the novel experiments in fluid mechanics, essentially for turbulent flow investigations. Experimental methods focus on how to analyze and solve the thermal fluid, fluid low, heat and complex engineering heat problems.

Much experimental works was carried out on fluid mechanic heat transfer for example, Fotukian and Esfahany (2010) issued a model on convective heat transfer enhancement and pressure drop of dilute CuO/water nanofluid inside a circular tube under turbulent flow and this model was solved experimentally using a measurement method. Furthermore, Aberoumand et al. (2016) established an experimental study of rheological interactions on thermal conductivity and viscosity of heat transfer silver-heat oil nanofluids and the results noted that thermal conductivity of oil based nanofluids increases with rising temperature. Other than that, Esfahani and Toghraie (2017) developed a new model for the thermal conductivity of silica/water-Ethylene glycol (40%–60%) nanofluid at different temperatures and solid nanoparticle volume fractions by means of an experimental measurement method. Based on their measurements, the thermal conductivity showed an increment with increasing temperature and volume fraction. Plus, in the research of Nazari et al. (2018), they conducted an experimental solution on heat transfer performance of four different concentrations of graphene oxide-water nanofluid in pulsating heat pipes (PHPs). The results revealed that by adding graphene oxide sheets, increased the thermal conductivity and viscidity of a water-based fluid while decreased the thermal resistance of PHP up to 42%. Furthermore, an experimental investigation of laminar forced convective heat transfer behavior and thermophysical properties of a Phase Change Material Emulsion (PCME) which is also known as paraffin-water emulsion used in cooling applications was conducted by Vasile et al. (2018). Tawfik et al. (2020) analyzed heat transfer and hydrodynamics of a particles mixture in a swirling fluidized bed reactor and then Akbarzadeh and Valipour (2020) carried out an experiment on a non-uniform thermal performance and heat transfer enhancement of a parabolic trough collector (PTC) in helically corrugated absorber tubes under the turbulent regimes. Their experimental results showed that the thermal performance and heat transfer are considerably increased in the corrugated absorber tubes. Above that, Patkavak and Demir (2020) run an experiment for evaluating the effect of monofilament cooling process on convective heat transfer coefficients around small diameter circular wires and air velocities of wind tunnel.

2.11.3 Numerical Method

There are some of most commonly numerical methods used in fluid mechanics such as Runge Kutta Fehlberg method (RKF), Fourth and Fifth Order Runge Kutta method (RK4 and RK5), Finite difference method (FDM), Finite Element Method (FEM) and Finite Volume Method (FVM). These numerical methods work on basic fluid mechanics equations for related flow problems. Moreover, the advancement of numerical works was also pushed ahead along with splendid development of high-speed computers and computer programs.

Numerical fluid mechanics become a vital sub-domain of the entire fluid mechanics sector and therefore, a large number of numerical studies have been carried out to analyzed the heat transfer efficiency of nanofluids flow over a boundary layer. Sajjad et al. (2018) examined a laminar forced convective heat transfer of graphene oxide nanosheets with water and ethylene glycol in a circular horizontal tube numerically by using FVM. Next, Pandey and Kumar (2016) had employed RKF method after reducing the differential equations of a model of MHD nanofluid flow over a wedge with porous medium and slip. In one paper of them, they explored the convergent and divergent stretching and shrinking channels with heat generation and absorption by using copper water based nanofluid and solved them under similar numerical technique (Pandey & Kumar, 2018). Other than that, Ahmed et al. (2016) scrutinized the heat transfer of CNTs-water nanofluid by solving the nonlinear equations of the problem using two numerical approaches which are Galerkin method (GM) and RKF method. Armaghani et al. (2016) conducted a numerical FVM for solving a case of natural convection heat transfer of a water-alumina mixture with entropy generation in a baffled L-shaped cavity. Shirvan et al. (2017) also solved the proposed equations by using FVM with SIMPLE algorithm and discretization on the Response Surface Methodology (RSM) for the two-phase mixture model. Additionally, Badruddin (2019) measured the heat transfer in irregular permeable cavity filled with porous medium subjected to various boundary conditions by means of Finite Element Method (FEM) while Vo et al. (2019) simulated the CVFEM on MHD water-based nanomaterial convective migration and heat transfer within a sinusoidal porous space in the presence of the radiation and magnetic effects. Oguntala et al. (2019) presented the thermal performance of a porous fin with heat sink using Pseudospectral Collocation method (PSCM) by considering the influence of an inclination parameter of the porous fin. The proposed PSCM results of the study were compared with RK method. Not only that, another approach was implied by means of bvp4c technique onto governing mathematical equations to describe the behavior of an unsteady MHD flow of micropolar time-dependent rotating hybrid nanofluid in a porous medium by Subhani and Nadeem (2019). Beyond this, Torkaman et al. (2020) established a numerical barycentric rational interpolation method for a steady two-dimensional incompressible viscous MHD nanofluid flow and heat transfer while considering the effect of sink/source and thermal radiation in nonparallel plates. Their research found that the proposed CM gives high accuracy when it is compared to the previously existing method plus this is fast with shorter run time procedure.

2.12 Variant Graphene Based Nanoparticles

Graphene is identified as a two-dimensional single-atom thick planar sheet of sp^2-bond carbon atoms sorted in a honeycomb crystalline lattice structure. It is the thinnest known material in the world and conceptually it postulates as a basic build block for constructing many other carbon materials (Yang et al., 2013). Novoselov et al. (2012) discovered graphene which has attracted much attention due to its unique and great thermal and electrical conductivity, chemical, mechanical and optical properties. Graphene being lighter in weight, flexible and strongest material is getting more popular day by day.

New study shows that graphene nanofluids could provide higher thermal conductivity enhancement in comparison to that of other examined nanofluids. According to Mishra et al. (2016) the thermal conductivity of suspensions containing a dispersed single layer of graphene at the room temperature is estimated experimentally within the range of 4800–5300 W/m K which is far higher than those of CNTs and other solid additives. The excellent thermophysical properties and heat transfer rate of graphene has made it an excellent candidate for use in synthesis of nanofluids. In addition, Fan et al. (2011) claimed that, synthesizing graphene nanoparticles is relatively an eco-friendly and cost effective. Graphene nanoparticles have gained much more interest, thanks to its high thermal conductivity, Young's modulus, fracture strength, etc. Indeed, graphene is truly an outstanding nanomaterial with notably high theoretical surface area, great quality electron transport at room temperature and finest durability under atmospheric conditions (Hayat et al., 2015; Perreault et al., 2015). There are many approaches of graphene-based nanomaterials such as single layer of graphene, functionalized graphene, graphene oxide (GO), reduced graphene oxide (rGO) and graphene nanoplatelets (GNPs).

Recently, it is important to highlight that a number of studies have been conducted on the use of variant graphene-based nanostructures to prepare nanofluids. Even though CNTs are widely utilized as nanoparticles for high storage and heat transfer performance as compared to metallic nanoparticles, Sadeghinezhad et al. (2016) experimentally discovered that graphene has higher thermal conductivity than any other carbon-based nanoparticles. Additionally, Sadeghinezhad et al. (2016) stated that graphene nanoparticles are more stable and have 1000 times larger surface area as compared to other nanoparticles. According to Ghozatloo et al. (2014), thermal performance of graphene nanofluid is evidenced with the convection heat transfer coefficient has increased by 0.1 wt.%. While Jeremy et al. Robinson et al. (2008) claimed that graphene oxide is an ideal material for balancing the defect density sites in SWCNTs because it contains a diverse range surface sites whose density is easily controlled. Subsequently, Liu et al. (2015) held both experimental and numerical analysis on a one-dimensional transient heat transfer of graphene/ionic liquid nanofluid based high temperature direct absorption solar collector. Their research provided an important description of the graphene/ionic liquid nanofluids for use in direct solar thermal collectors under concentrated solar incident radiation based on the proposed heat transfer model. Ijam et al. (2015) experimentally analyzed the stability and other important properties of GO-deionized water/ethylene glycol based nanofluid. They measured the stability of the GO-deionized water/ethylene glycol mixture with sedimentation time and found that the nanofluid mixture managed to stable for more than 2 months. Mehrali et al. (2015) presented the heat transfer behavior and entropy generation for the flow of GNPs in a horizontal stainless-steel tube and indicated that GNPs nanofluid with a concentration between 0.075 wt.% and 0.1 wt.% is more energy efficient than other concentrations. Moreover, Azimi and Riazi (2016) issued a paper on a heat transfer analysis of MHD viscous flow by utilizing GO-water nanofluid with convergent-divergent condition between two non-parallel walls while Yarmand et al. (2016) provided an experimental analysis on functionalized GNPs mixture in a square heated pipe under turbulent flow. The results showed remarkable thermal conductivity and overall heat transfer enhancement for all nanofluid samples compared to water in their study. Meanwhile, in another research, the angle effect on a pin-fin heat sink channel for convective heat transfer performance using water-based GNPs nanofluids was experimentally addressed by Ali and Arshad (2017). Also, Zou et al. (2018) considered a composite phase change materials (PCM) by using graphene and CNTs nanomaterials to investigate the thermal performance enhancement of lithium-ion power battery. Ghadikolaei et al. (2018) examined an incompressible MHD radiative flow of a micropolar dusty fluid with the suspension of GO-engine oil over a moving sheet with joule heating effect.

Because of its special abilities and properties, the research on graphene have resulted in numerous exciting applications such as in pulsed laser stimulation and in various photothermal-based cancer therapies (Mehrali et al., 2016; Perreault et al., 2015). According to Kumar et al. (2015), graphene as one of the carbon-based materials has also attracted a significant number of researchers to conduct scientific studies of its special thermal, electrical, optical and mechanical properties. In these years, graphene term has covered the complete industry and it is considered to be the most important material for whole industrial applications. Besides, graphene-based nanoparticles have been used for environmental decontamination, as building blocks for next generation water treatment membranes, and as electrode materials for contaminant monitoring or removal (Perreault et al., 2015). Therefore, exploring the use of graphene-based materials in heat transfer applications is a great challenge for nowadays research, due to the excellent properties of graphene.

CHAPTER 3: MATHEMATICAL FORMULATION AND METHODOLOGY

3.1 Research Methodology

Figure 3.1 below shows the methodology chart of this current research. Note that, the methodology steps of 7 until 9 will be explained in the next chapter.

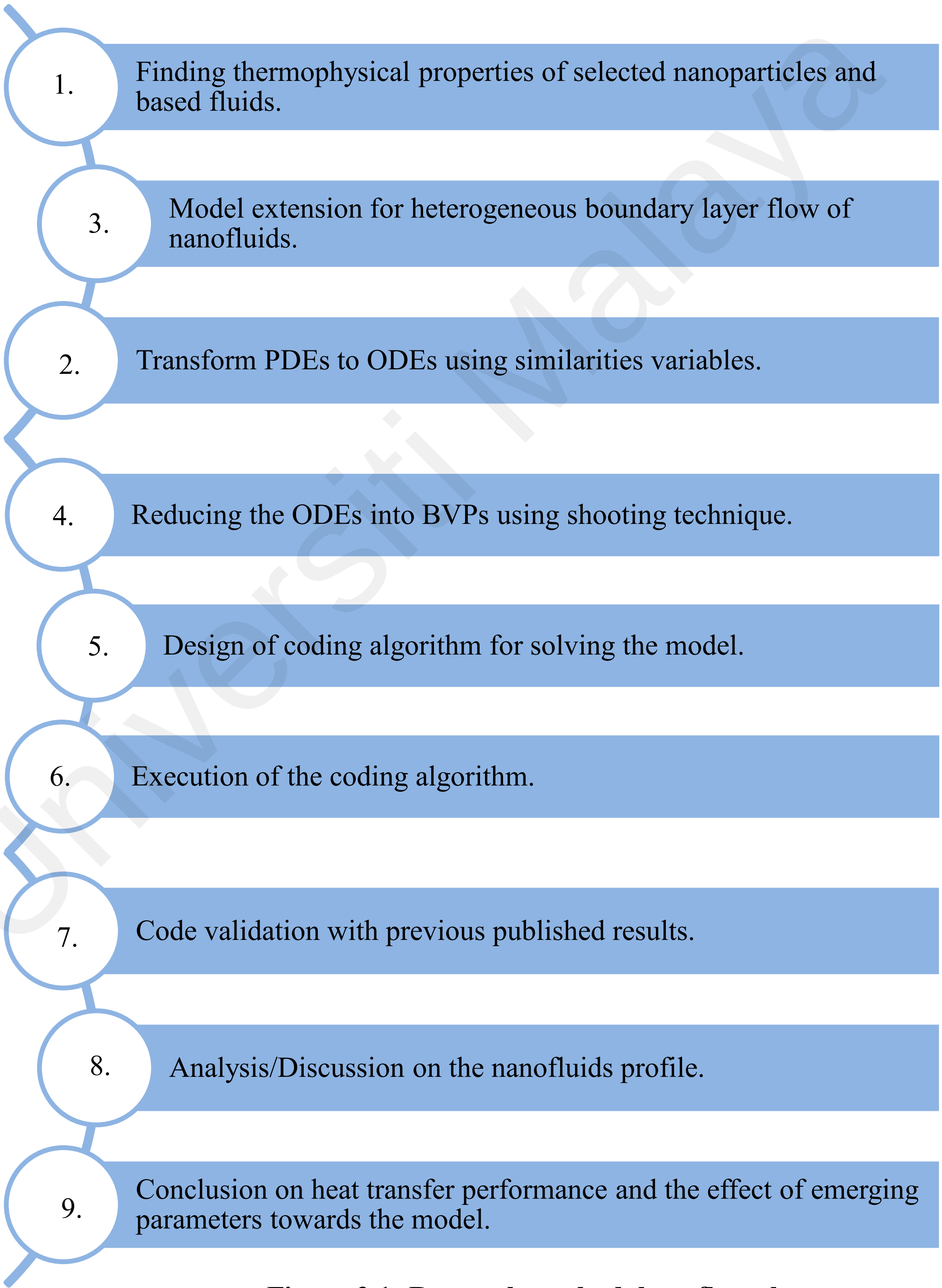

Figure 3.1: Research methodology flow chart.

3.2 Thermophysical Properties of Fluids

Thermophysical properties of fluids occupy a very important place in the physical and chemical science procedures and these thermophysical properties include the thermal conductivity coefficient, specific heat capacity, density, viscosity, ionic conductivity, electrical conductivity, speed of sound, liquid-gas surface tension, thermodynamic derivatives, etc. (Oster et al., 2018). Above all, we focus on three types of thermophysical properties which are density, specific heat capacity and thermal conductivity of the selected based fluid and nanoparticles for this current study as shown in Table 3.1 and 3.2.

3.2.1 Thermophysical Properties of Conventional Fluids

Table 3.1: List of thermophysical properties of conventional fluids (Haq et al., 2015).

Types of conventional fluids	Density (kg/m^3)	Specific heat capacity (J/kgK)	Thermal conductivity (W/mK)
Pure water	997.1	4179	0.613
Ethylene Glycol	1115	2430	0.253
Engine Oil	884	1910	0.144

3.2.2 Thermophysical Properties of Various Types of Nanoparticles

Table 3.2: List of thermophysical properties of nanoparticles (Amin et al., 2017; Azimi et al., 2014; Haq et al., 2015; Kandasamy & Muhammad, 2016; Tippa et al., 2016; Upadhya et al., 2018).

Types of nanoparticles	Density (kg/m^3)	Specific heat capacity (J/kgK)	Thermal conductivity (W/mK)
Copper	8933	385	401
Silver (Ag)	10500	235	429
Alumina (Al_2O_3)	3970	765	40
Titanium Oxide (TiO_2)	4250	6862	8.954
Copper Oxide (CuO)	6320	531.8	76.5
SWCNTs	2600	425	6600
MWCNTs	1600	796	3000
Graphene	2250	2100	2500
GNPs	1800	717	5000
GOs	2200	538.2	3000

3.3 Conservation Laws for Continuum Mechanics

One of the main focus of the present study is to learn how to develop mathematical models for the actual transport phenomena. Most of the fluid transport systems are governed by continuum mechanics laws. In continuum mechanics, they are basically written in the three parts of conservation equations which are mass, momentum and energy equations (Kleinstreuer, 2018).

3.3.1 Conservation of Mass

The principle of mass conservation establishes that the mass of a fluid particle volume is constant (Hauke, 2008). This means that the cumulative time rate for mass change in a fixed region is equivalently zero. Thus, specifically, the total of the fluid convected into and out of the fixed region is equal to the rate of change of fluid density of in motion (Li, 2006). Thus, the differential equation of mass conservation, called the continuity equation is obtained as,

$$\frac{D\rho}{Dt}(\rho\nabla\cdot u) = 0\,, \tag{3.1}$$

where ρ is denoted as fluid density, u is the vector of velocity and D/Dt is the derivative for the medium. For particular case of an incompressible flow, the density is constant, and the equation of continuity is simplified to

$$\nabla\cdot u = 0\,. \tag{3.2}$$

Therefore, the above equation means that the divergence of the velocity field of an incompressible fluid flow is zero (divergence free) (Li, 2006).

3.3.2 Conservation of Momentum

The principle of momentum conservation follows Newton's second law of motion. In fluid mechanics, the momentum conservation is defined as the total time rate of change of fluid linear momentum is equal to the total of external forces on a rigid area. Newton's second law is expressed mathematically as (Li, 2006),

$$\rho \frac{Du}{Dt} = \nabla \cdot \sigma + f_b \,, \tag{3.3}$$

where σ and f_b are indicated as the Cauchy stress tensor and the force of the body per unit volume respectively.

Thus, the momentum equation for a Newtonian incompressible fluid with constant viscosity becomes (Hauke, 2008),

$$\rho \frac{Dw}{Dt} + \nabla u \cdot (\rho w) - \nabla u \cdot \rho \mathrm{w} = \nabla \times (\nabla \cdot \tau) \,, \tag{3.4}$$

where w is the vorticity vector and τ is the deviatoric stress tensor.

3.3.3 Conservation of Energy

While the energy conservation law can be considered from the first law of thermodynamics, which implies the conservation of thermal energy and work. For an incompressible fluid cases, the energy conservation law expressed in the form of (Li, 2006),

$$\rho C_p \frac{DT}{Dt} = -\nabla \cdot q + Q + \Phi \,. \tag{3.5}$$

where Q is a lumped sum of all source inputs and called as the internal energy generation,

$$Q = Q_s + Q_r + Q_R + Q_c \,, \tag{3.6}$$

In which Q_s refers to any applied heat sources, Q_r and Q_R is the heat source deteriorated occurs throughout chemical reaction and the heat source of interior radiation respectively,

$Q_c = -\rho\nabla \cdot u$ is the energy source from mechanical work, which is zero for incompressible fluids.

3.4 Boundary Layer Equations of Newtonian Fluid Flow Over A Flat Surface

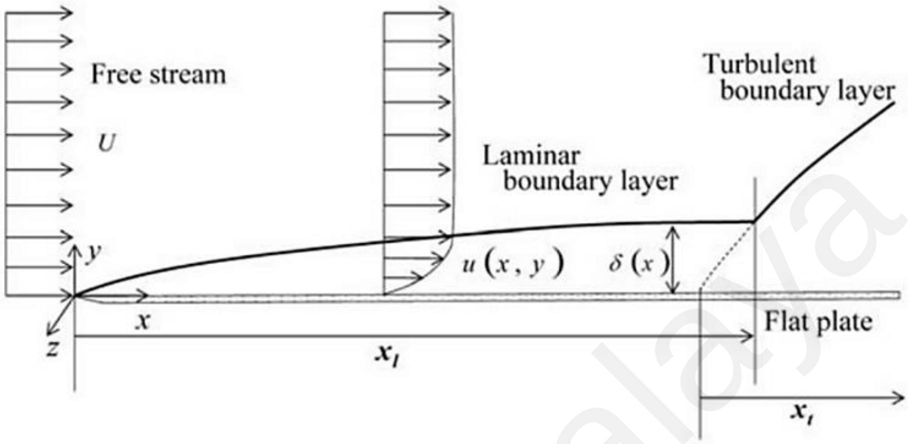

Figure 3.2: Boundary layer flow past a flat surface (Yamaguchi, 2008)

Generally, fluid dynamics and heat transfer problems are classified as boundary value problems and the study of fluid flow past a flat surface or sheet is one of the most popular study of boundary layer system. Thus, as the initial of the discussion for a flat surface boundary layer flow, the continuity and Navier-Stokes equations were obtained.

A basic boundary layer model of fluid flow past a flat surface was shown in Figure 3.2. In this case, we consider an incompressible, two-dimensional, steady, laminar Newtonian fluid flow while choosing Cartesian coordinates as the flow moves along a flat surface, plane or sheet (Hauke, 2008). Thus, the set of simplified Navier-Stokes equations in a two-dimensional form of laminar boundary layer equations were obtained as follows:

$$
\left.\begin{aligned}
&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,,\\
&u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\left(\frac{\partial^2 u}{\partial y^2}\right),\\
&0=-\frac{1}{\rho}\frac{\partial p}{\partial y},\\
&u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right).
\end{aligned}\right\}\quad \textbf{(3.7)}
$$

3.5 Mathematical Formulation

In this section, we analyzed and extend the model from the previous study of Dinarvand et al. (2017) and transforming governing partial differential equations (PDEs) to dimensionless nonlinear ordinary differential equations (ODEs) by using shooting technique.

3.5.1 Previous Model for Heterogeneous Nanofluid Flow

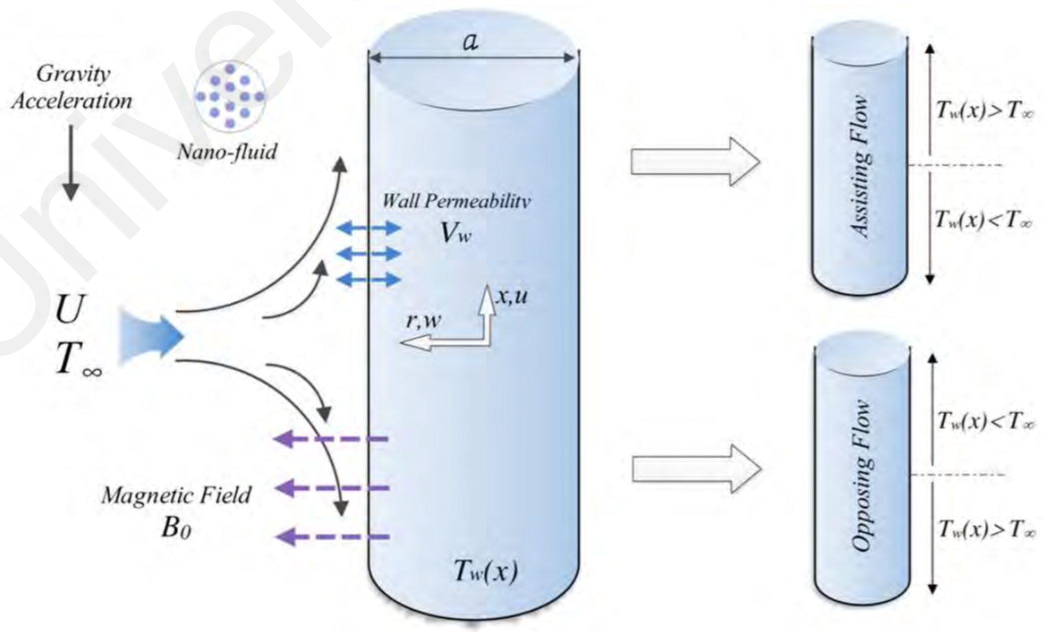

Figure 3.3: Schematic of the problem with geometrical coordinates of previous study (Dinarvand et al., 2017).

As mentioned in the methodology of the current research, we extend the model of heterogeneous nanofluid boundary layer flow of Dinarvand et al. (2017). They investigated the steady axisymmetric mixed convective stagnation-point flow of an incompressible electrically conducting water utilizing three different types of nanoparticles; copper, alumina, and titania over a vertical permeable circular cylinder considering transverse magnetic field existence as shown in Figure 3.3. Their proposed mathematical model has been formulated based on Tiwari-Das nanofluid model and following are the governing mathematical boundary layer flow equations (Dinarvand et al., 2017):

$$\left.\begin{aligned}
&\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rw) = 0\,, \\
&u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial r} = U\frac{\partial U}{\partial x} + \upsilon_{nf}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + \frac{\sigma {B_0}^2}{\rho_{nf}}u \\
&\qquad + \frac{\phi\rho_s\beta_s + (1-\phi)\rho_f\beta_f}{\rho_{nf}} g(T - T_\infty), \\
&u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \alpha_{nf}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right).
\end{aligned}\right\} \quad \textbf{(3.8)}$$

With boundary conditions,

$$\begin{aligned}
&u = 0, \qquad w = {V^*}_w, \qquad T = T_w(x) = T_\infty + \Delta T\left(\frac{x}{l}\right) \quad \text{at} \quad r = R\,, \\
&u \to U(x) \to U_\infty\left(\frac{x}{l}\right), \qquad T \to T_\infty, \qquad \text{as} \quad r \to \infty\,,
\end{aligned} \qquad \textbf{(3.9)}$$

they chose the cylindrical coordinates (*x, r*) such that *x* and *r*-axes are along the cylinder surface (vertically) and radial directions, respectively.

3.5.2 Model Extension for Heterogeneous Nanofluid Flow

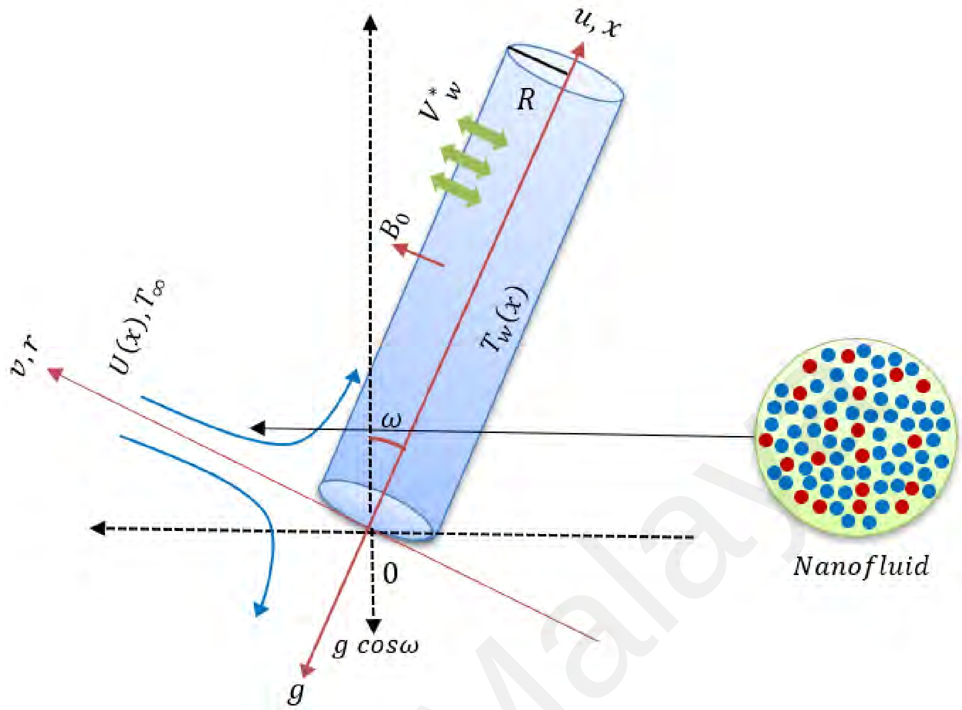

Figure 3.4: Schematic of the problem with geometrical coordinates of the current study.

For our current research, we consider a steady, laminar, incompressible, mixed convective, reverse stagnation-point flow of water based nanofluids containing variant graphene-based nanoparticles with the presence of transverse magnetic field over an inclined permeable cylinder, ω with radius, R as shown in Figure 3.4. We let the cylinder surface temperature, T_w and ambient temperature, T_∞ respectively, while assuming $U(x)=U_\infty(x/l)$ as the mainstream velocity. Uniform magnetic field of strength, B_0 and electrical conductivity, σ are applied. Here, the cylindrical coordinates are also chosen in such a manner that x-axis is taken along the axial direction of cylinder while r-axis is normal to it. Under these assumptions and following the proposed nanofluid model by Tiwari and Das (2007), the relevant boundary layer equations including the continuity, momentum and energy equations can be defined as:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0\,, \tag{3.10a}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = U\frac{\partial U}{\partial x} + \upsilon_{nf}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + \frac{\sigma {B_0}^2}{\rho_{nf}}(U-u)$$

(3.10b)

$$+\frac{\phi\rho_s\beta_s + (1-\phi)\rho_f\beta_f}{\rho_{nf}} g(T-T_\infty)cos\omega,$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \alpha_{nf}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right). \tag{3.10c}$$

The boundary conditions for the present problem are given as follows:

$$u = 0, \qquad v = V^*{}_w, \qquad T = T_w(x) = T_\infty + \Delta T\left(\frac{x}{l}\right) \quad \text{at} \quad r = R\,,$$

(3.11)

$$u \to \kappa U(x), \qquad T \to T_\infty, \qquad \text{as} \quad r \to \infty\,,$$

where V^*_w is the uniform surface mass flux ($V^*_w > 0$ corresponds to suction and $V^*_w < 0$ corresponds to injection), $\kappa < 0$ is for reverse stagnation flow and $\kappa > 0$ is for positive stagnation flow, u and v are the velocity components along the x and r directions, T is the temperature of nanofluid, ϕ is the nanoparticle volume fraction, β_f, β_s, ρ_f and ρ_s are the coefficients of thermal expansion and the densities of the fluid and of the solid nanoparticles, respectively. Here, ν_{nf} is the kinematic viscosity of nanofluid and α_{nf} is the thermal diffusivity of nanofluid, which are given by Oztop and Abu-Nada (2008):

$$\upsilon_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}\left[(1-\phi)\rho_f + \phi\rho_s\right]}, \tag{3.12}$$

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s\,, \tag{3.13}$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \tag{3.14}$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s, \tag{3.15}$$

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}, \tag{3.16}$$

in which k_{nf} denotes the thermal conductivity of the nanofluid, k_f and k_s are the thermal conductivities of the fluid and of the solid nanoparticles, respectively while $c_{p_{nf}}$ denotes the heat capacity of the nanofluid, c_{p_f} and c_{p_s} are the heat capacities of the fluid and of the nanoparticles, respectively.

3.5.3 Transforming PDEs To ODEs Using Similarity Variables

By introducing the following similarity transformations:

$$\eta = \frac{r^2 - R^2}{2v_f l}\sqrt{\frac{U_\infty v_f l}{R^2}}, \quad \psi = \sqrt{\frac{U_\infty v_f R^2}{l}}\, x f(\eta),$$
$$T = T_\infty + \Delta T\left(\frac{x}{l}\right)\theta(\eta), \tag{3.17}$$

the stream function, ψ defined as follow:

$$u = r^{-1}\frac{\partial \psi}{\partial r} \qquad \text{and} \qquad v = r^{-1}\frac{\partial \psi}{\partial r} \tag{3.18}$$

in which identically satisfy the Eq. (3.10a). By substituting Eq. (3.10a) into Eqs. (3.10b) and (3.10c), we acquire a system of dimensionless nonlinear ordinary differential equations as follow:

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)}[(1+2\gamma\eta)f''' + 2\gamma f'']$$

$$+\frac{M}{1-\phi+\phi\frac{\rho_s}{\rho_f}}(1-f') - f'^2 + ff'' + 1 \qquad \textbf{(3.19)}$$

$$+\frac{1-\phi+\phi\frac{\rho_s\beta_s}{\rho_f\beta_f}}{1-\phi+\phi\frac{\rho_s}{\rho_f}}\lambda\theta cos\omega = 0\,,$$

$$\frac{\left(\frac{k_{nf}}{k_f}\right)}{1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}}[(1+2\gamma\eta)\theta'' + 2\gamma\theta'] + Pr[f\theta' - f'\theta] = 0\,, \qquad \textbf{(3.20)}$$

subject to boundary conditions:

$$f(0) = v_w\,, \qquad f'(0) = 0, \qquad f'(\infty) = \kappa\,,$$

$$\theta(0) = 1, \qquad \theta(\infty) = 0\,, \qquad \textbf{(3.21)}$$

where the primes denote differentiation with respect to η, f is the function related to the velocity field, and θ is the dimensionless temperature of the nanofluid (the details on the derivation are shown in Appendix A), Pr is the Prandtl number, λ is the mixed convection parameter, Gr is the Grashof number, Re is the Reynolds number, γ is the curvature parameter, v_w is the permeability parameter and M is the magnetic parameter which are all defined as:

$$Pr = \frac{\mu_f c_p}{k_f}, \qquad \lambda = \frac{Gr}{Re^2}, \qquad Gr = \frac{g\beta_f \Delta T l^3}{v_f{}^2}, \qquad Re = \frac{U_\infty l}{v_f{}^2},$$

$$\gamma = \sqrt{\frac{v_f l}{U_\infty}}, \qquad v_w = -\frac{r}{R}\sqrt{\frac{l}{U_\infty v_f}} V^*{}_w, \qquad M = \frac{\sigma B_0{}^2}{\rho_f \left(\frac{U_\infty}{l}\right)}. \tag{3.22}$$

Nonetheless, $\lambda < 0$ corresponds to opposing flow and $\lambda > 0$ corresponds to assisting flow and $\lambda = 0$ corresponds to forced convection flow. The two important physical properties of present problem are the skin friction coefficients, C_f and the local Nusselt number, *Nu*, which are defined in mathematical expressions as follows:

$$C_f = \frac{\tau_w}{\rho_f U_\infty{}^2}, \qquad Nu = \frac{l q_w}{k_f \Delta T}. \tag{3.23}$$

In above equations, τ_w is the shear stress at the surface of the cylinder and q_w is the heat flux from the surface of the cylinder, which are denoted as follow:

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial r}\right)_{r=R}, \qquad q_w = -k_f\left(\frac{\partial T}{\partial r}\right)_{r=R}. \tag{3.24}$$

By invoking Eqs. (3.17), (3.23) and (3.24), the dimensionless forms are obtained as below:

$$Re^{\frac{1}{2}} C_f = \frac{\left(\frac{x}{l}\right)}{(1-\phi)^{2.5}} f''(0), \qquad Re^{-\frac{1}{2}} Nu = -\frac{k_{nf}}{k_f}\left(\frac{x}{l}\right)\theta'(0). \tag{3.25}$$

3.6 Solution Approach

A theoretical or numerical solution for solving the problem equations leads to a deeper understanding, allow some predictions and helping to improved engineering procedures or devices development.

3.6.1 Reducing the Ode into BVPs Using A Shooting Technique

For this section, the nonlinear ordinary differential equations (3.19) and (3.20) according to boundary condition (3.21) are transformed by using a shooting technique. Then, the differential equations are decreased to first order BVPs in this form:

$$f'_{(1)} = f_{(2)} \, , \tag{3.26}$$

$$f'_{(2)} = f_{(3)} \, , \tag{3.27}$$

$$f'_{(3)} = -\frac{1}{1+2\gamma\eta}\left((1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\left[f_{(1)}f_{(3)} + 1 - \left({f_{(2)}}^2\right) + \frac{1-\phi+\phi\frac{\rho_s\beta_s}{\rho_f\beta_f}}{1-\phi+\phi\frac{\rho_s}{\rho_f}}\lambda f_{(4)} cos\omega + \frac{M}{1-\phi+\phi\frac{\rho_s}{\rho_f}}\left(1-f_{(2)}\right)\right] + 2\gamma f_{(3)}\right), \tag{3.28}$$

$$f'_{(4)} = f'_{(5)} \, , \tag{3.29}$$

$$f'_{(5)} = -\frac{1}{1+2\gamma\eta}\left(\left(\frac{1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}}{\left(\frac{k_{nf}}{k_f}\right)}\right)\left[Prf_{(1)}f_{(5)} - f_{(2)}f_{(4)}\right] + 2\gamma f_{(5)}\right), \tag{3.30}$$

with the boundary conditions:

$$f_{(1)}(0) = v_w, \quad f_{(2)}(0) = 0, \quad f_{(4)}(0) = 1, \quad f_{(2)}(\infty) = \kappa,$$
$$f_{(4)}(\infty) = 0, \tag{3.31}$$

by applying the following variables:

$$f = f_{(1)}, \quad f' = f_{(2)} = f'_{(1)}, \quad f'' = f_{(3)} = f'_{(2)},$$
$$f''' = f'_{(3)}, \tag{3.32}$$

The details on derivation of Eqs. (3.10) - (3.31) are shown in Appendix A. The selected limit for η is 14 and the initial values of $f_{(3)}$ and $f_{(5)}$ are assumed to be 0. The boundary value problem is solved with an accepted residual tolerance of 10^{-6}.

3.6.2 Design of Coding Algorithm for Solving the Model

Continuing to this, the present problem of partial differential equation requires an appropriate solution to meet the accuracy of the outcomes. Thus, the present problem is performed by using the MATLAB package (bvp4c). Bvp4c is a finite difference code that implements the three-stage Lobatto IIIa implicit Runge–Kutta formula (Shampine & Kierzenka, 2001; Shampine et al., 2004). This is a collocation formula and the collocation polynomial provides a C^1-continuous solution with fourth order accuracy. The central type finite difference is utilized due to the fact that it produces better result performance. This finite difference code does not require users to provide analytical partial derivatives plus it is high in computational speed despite using low-cost procedure. This bvp4c algorithm has been proven to be effective for solving a vast of mathematical and engineering problems with nonlinear aspects. Moreover, bvp4c code also divides the range of integration in case of multipoint boundary value problems. The collocation technique uses a mesh for dividing the interval of integral to subintervals. Thus, mesh selection and error control are referred to the residual of the continuous solution. In our study, we consider about 100 mesh points as shown in part of the algorithm (in Appendix B).

3.6.3 Execution of The Coding Algorithm

A finite difference method based on 3-stage Lobatto IIIa formula is considered to conduct this research. The algorithm coded in MATLAB bvp4c package is utilized in order to generate results of the model studied. See Appendix B for details on the coding algorithm.

CHAPTER 4: RESULTS AND DISCUSSION

In this section, our results of the boundary layer properties namely dimensionless velocity and temperature disseminations, local skin friction coefficient and local Nusselt number, are carefully analyzed for varying parameters. We consider five types of graphene-based nanoparticles: graphenes, GNPs, GOs, SWCNTs and MWCNTs. Thermophysical properties of these nanoparticles and the base fluid (water) are provided in Table 3.1 and 3.2 of Chapter 3. In addition, the value of nanoparticle volume fraction parameter, ϕ ranged from 0 to 0.4.

4.1 Code Validation

In order to verify the precision of the present study, we compare our results calculated via the present bvp4c code for water-based copper nanofluids (Cu-water) with the previously published results obtained by Dinarvand et al. (2017) using Homotopy Analysis Method (HAM) and RK4. The comparisons depicted in Table 4.1 show a very good agreement (5^{th} decimal places or less than 10^{-4}) between our results and theirs'.

Table 4.1: The influence of the nanoparticle volume fraction and mixed convection parameter on the skin friction coefficient and Nusselt number for copper-water nanofluid when *Pr*=6.2, *γ*=1, *M*=0 and v_w=0.

		$\left(\frac{l}{x}\right) Re^{\frac{1}{2}} C_f$			$\left(\frac{l}{x}\right) Re^{-\frac{1}{2}} Nu$		
		Dinarvand et al. (2017)		Present study	Dinarvand et al. (2017)		Present study
λ	ϕ	HAM	RK4	bvp4c	HAM	RK4	bvp4c
0	0	1.70764	1.70762	1.70766	2.15177	2.15173	2.15175
	0.1	2.51216	2.51214	2.51213	2.69664	2.69666	2.69667
	0.2	3.46826	3.46828	3.46829	3.26935	3.26939	3.26938

Table 4.1, continued.

λ	ϕ	$\left(\frac{l}{x}\right) Re^{\frac{1}{2}} C_f$			$\left(\frac{l}{x}\right) Re^{-\frac{1}{2}} Nu$		
		Dinarvand et al. (2017)		Present study	Dinarvand et al. (2017)		Present study
		HAM	RK4	bvp4c	HAM	RK4	bvp4c
1	0	1.99210	1.99215	1.99218	2.21366	2.21362	2.21363
	0.1	2.79276	2.79273	2.79271	2.74838	2.74831	2.74831
	0.2	3.75691	3.75698	3.75700	3.31539	3.31532	3.31533
5	0	3.02007	3.02004	3.02005	2.40937	2.40936	2.40935
	0.1	3.83616	3.83608	3.83609	2.92289	2.92285	2.92286
	0.2	4.84849	4.84840	4.84841	3.47660	3.47668	3.47668

4.2 Dimensionless Velocity and Temperature

Figures 4.1 to 4.13 illustrate the effect of stagnation parameter κ, inclination angle ω, mixed convection parameter λ, Prandtl number Pr, curvature parameter γ, solid nanoparticle volume fraction ϕ, magnetic parameter M and wall permeability parameter v_w of the selected nanoparticles on the model's dimensionless velocity and temperature disseminations.

Firstly, the behavior of stagnation parameter κ is depicted in Figure 4.1 to 4.3. Figure 4.1 and 4.2 explain that the velocity dissemination starts to rise near the surface of the cylinder and begins to fall off apart from the surface at $\eta \approx 0.75$. The corresponding figure 4.1 show that MWCNTs-water has the highest fluid velocity and the results for temperature dissemination almost similar for all nanofluids when κ = -3. For κ = 0, there is no stagnation-point occurs in the streamline while the fluid velocity is strongly enhanced for all nanofluids when there is no stagnation-point and positive stagnation κ = 3 is imposed to the flow where the impact of SWCNTs on velocity profile is more prominent in this case (see Figure 4.2 and 4.3). Graphenes-water contributes to the least temperature dissemination in both corresponding stagnation problem. In general, the fluid temperature decreases for all cases of stagnation parameter κ. However, it is worth to specify that the thermal boundary layer thickness and temperature dissemination of GOs-water is more prominent at both $\kappa = 0$ and $\kappa = 3$.

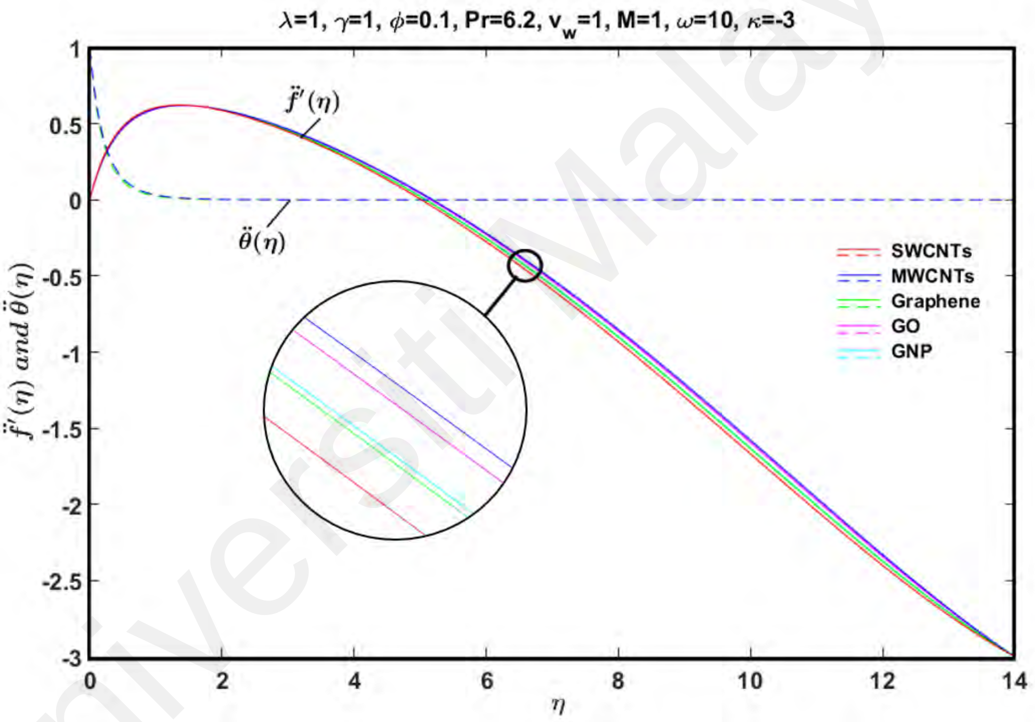

Figure 4.1: Influence of stagnation parameter (when κ=-3) on velocity and temperature disseminations for the nanofluids.

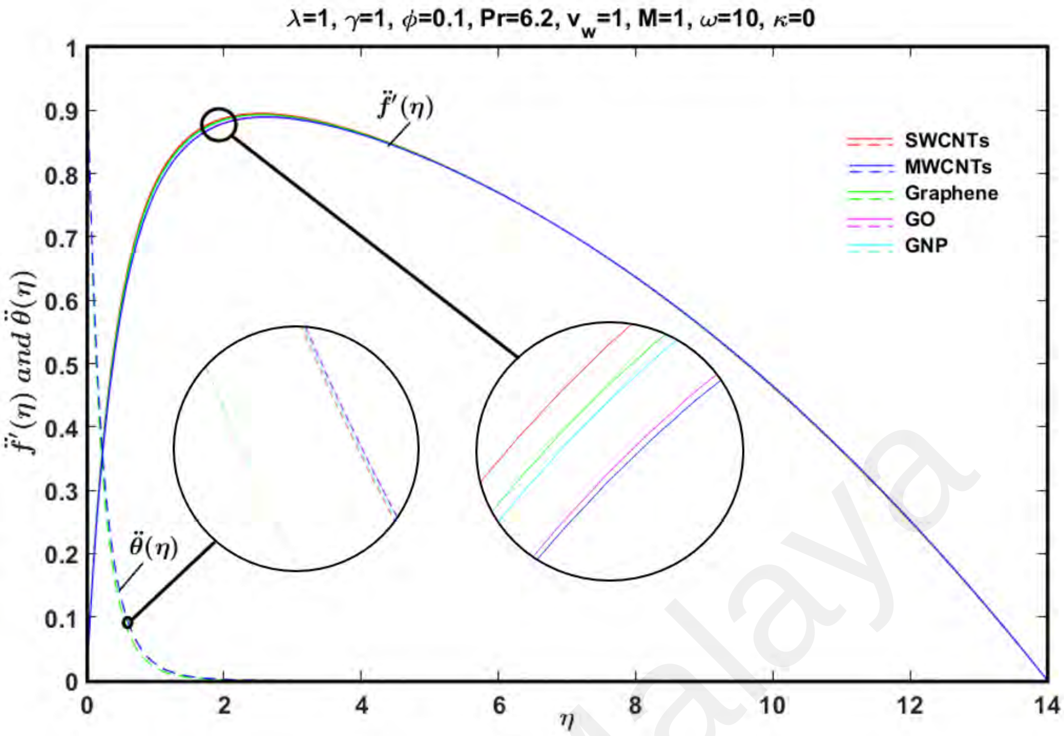

Figure 4.2: Influence of stagnation parameter (when κ=0) on velocity and temperature disseminations for the nanofluids.

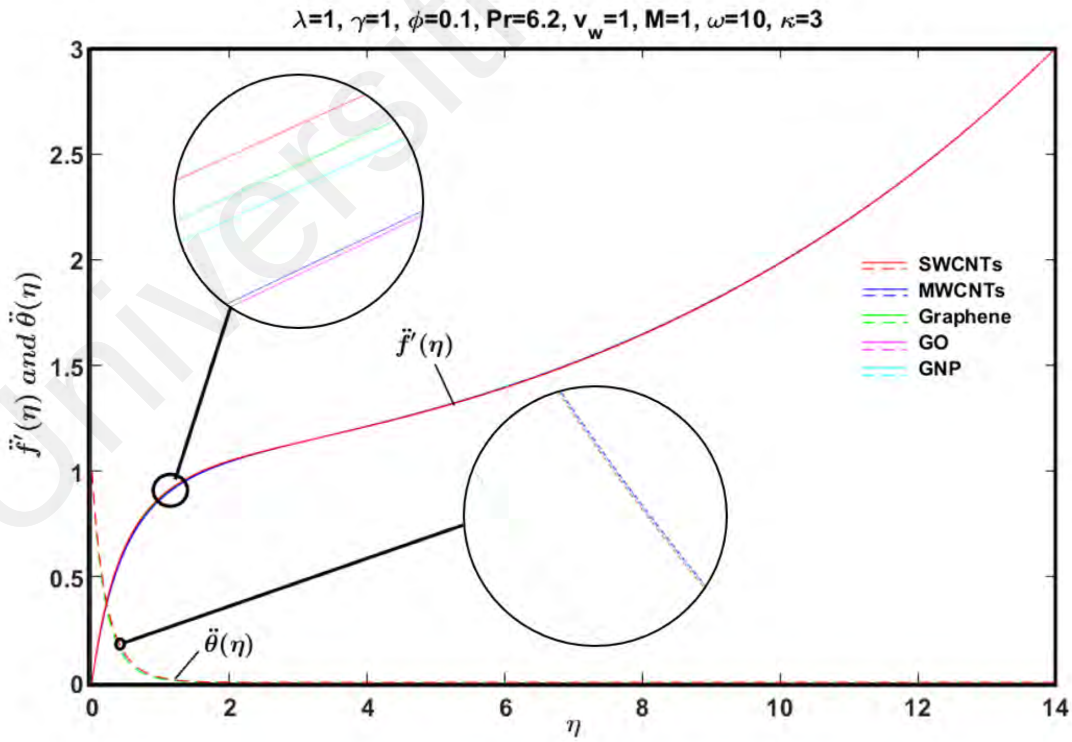

Figure 4.3: Influence of stagnation parameter (when κ=3) on velocity and temperature disseminations for the nanofluids.

The characteristics nature of inclination angle ω on velocity dissemination is visualized in Figure 4.4 and 4.5 respectively. Fluid velocity slightly reduces with the increase of ω. In fact, gravity effect is minimized for inclined cylinder ($0 < \omega \leq 90$) and it results in decreasing velocity dissemination (Hauke, 2008). As can be seen, the influences of SWCNTs-water contribute the highest velocity dissemination than other nanofluids when the inclination angle, ω is increased from 0 up to 40 degrees as positive stagnation value (κ= 1) are imposed to the cases (see Figure 4.4(a) and 4.5(a)). Oppositely, MWCNTs-water consistently contributes towards the lowest velocity dissemination but the highest temperature dissemination when $0 < \omega \leq 40$. However, the results show the opposite trends for negative value of stagnation (κ= -3) (see Figure 4.4(b) and 4.5(b)). The corresponding figures show that MWCNTs-water have the highest velocity disseminations and SWCNTs-water turned to be the lowest velocity disseminations. Nevertheless, graphenes-water show the lowest temperature dissemination for all trends in Figure 4.4 and 4.5.

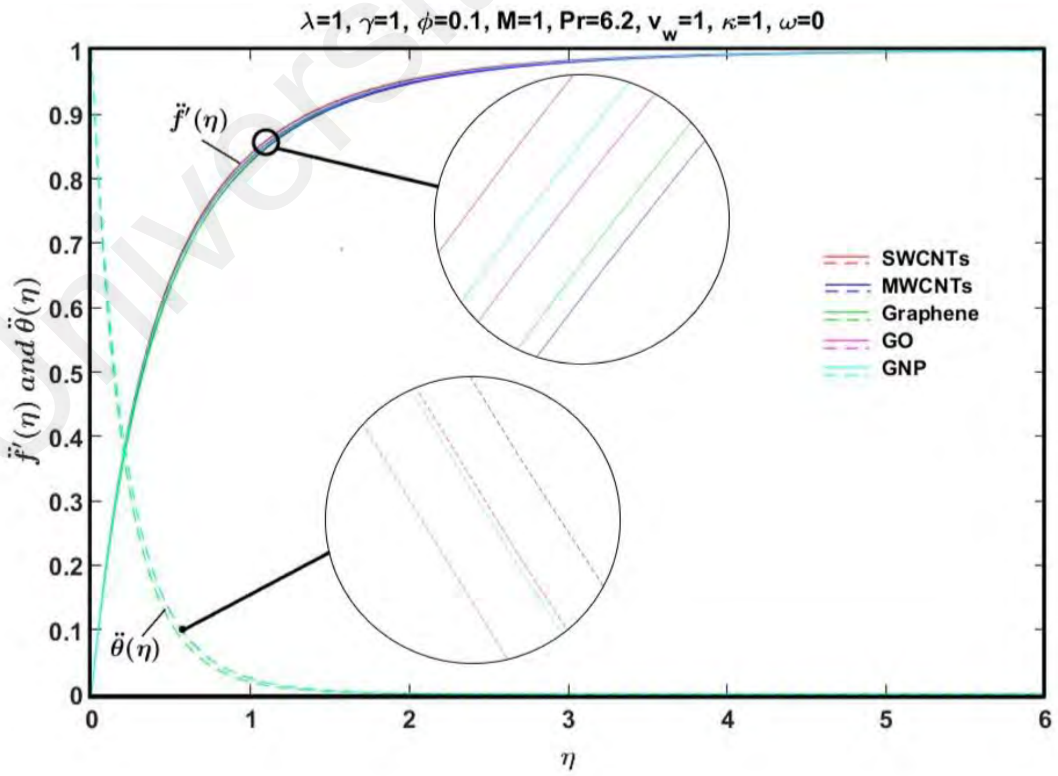

Figure 4.4(a): Influence of inclination angle parameter on positive stagnation value (when ω=0 and κ=1) on velocity and temperature disseminations for the nanofluids.

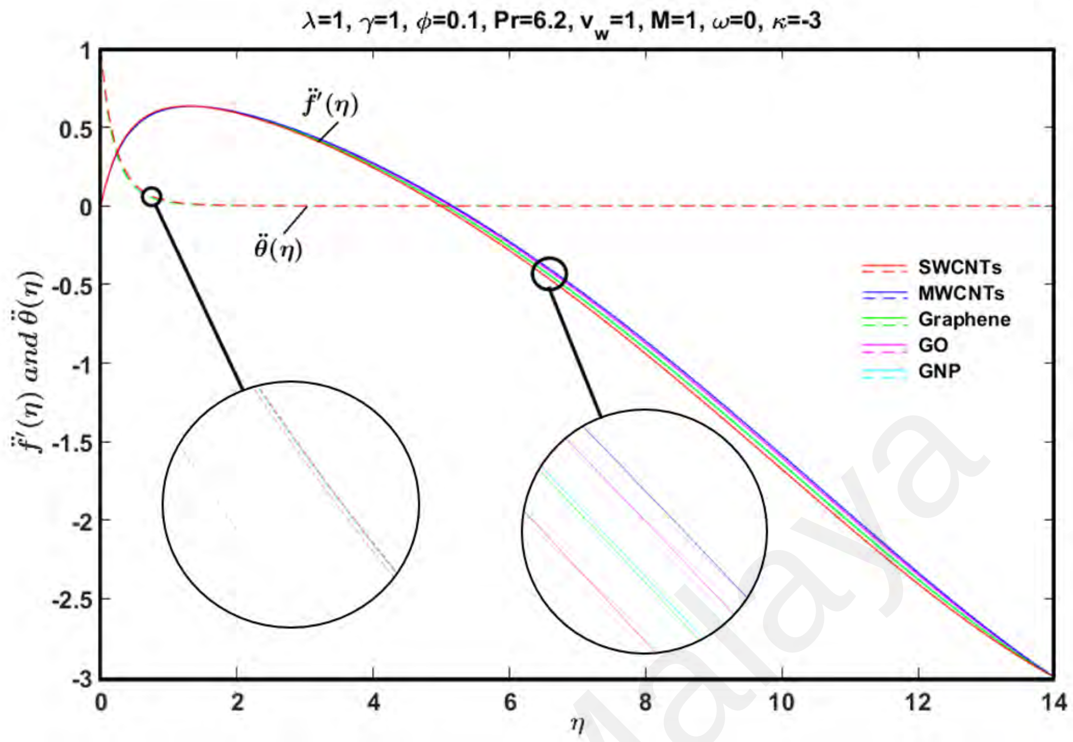

Figure 4.4(b): Influence of inclination angle parameter on negative stagnation value (when ω=0 and κ=-3) on velocity and temperature disseminations for the nanofluids.

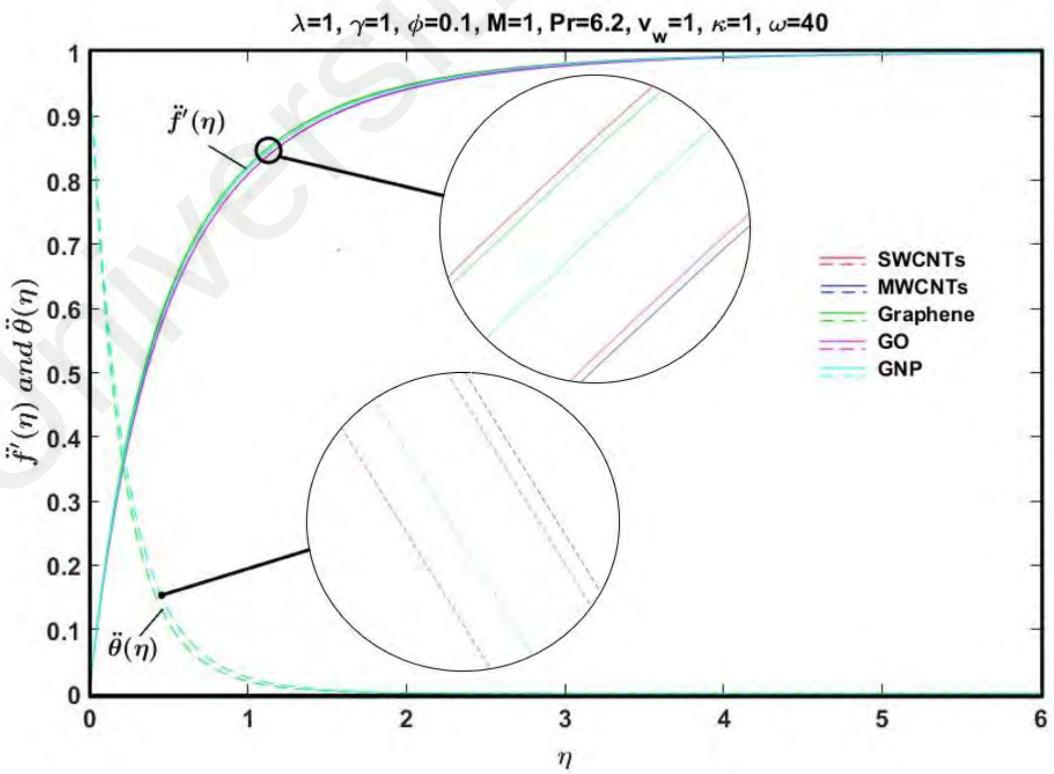

Figure 4.5(a): Influence of inclination angle parameter on positive stagnation value (when ω=40 and κ=1) on velocity and temperature dissemination for the nanofluids.

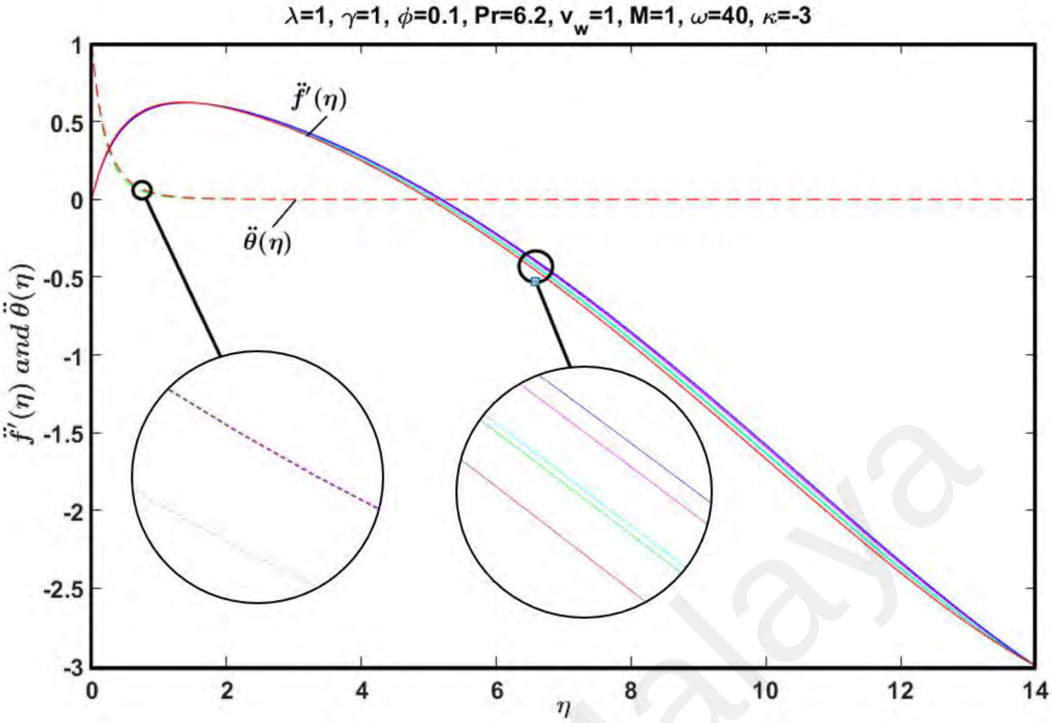

Figure 4.5(b): Influence of inclination angle parameter on negative stagnation value (when *ω*=40 and *κ*=-3) on velocity and temperature dissemination for the nanofluids.

The changes in the mixed convection parameter consisting assisting flow ($\lambda > 0$) and opposing flow ($\lambda < 0$) on velocity disseminations are portrayed in Figure 4.6 and 4.7. To be specific, both considered figures are sketched to show the motion trend of the nanofluids when λ = -3 and λ = 3. It is noted that with the rise in λ, the fluid velocity decreases. The result shows that graphenes-water dominates over the velocity dissemination for assisting flow when λ is increased to 3. Graphenes-water also produces the lowest temperature dissemination as compared to other nanofluids regardless the impact of assisting (λ = 3) or opposing flow (λ = -3).

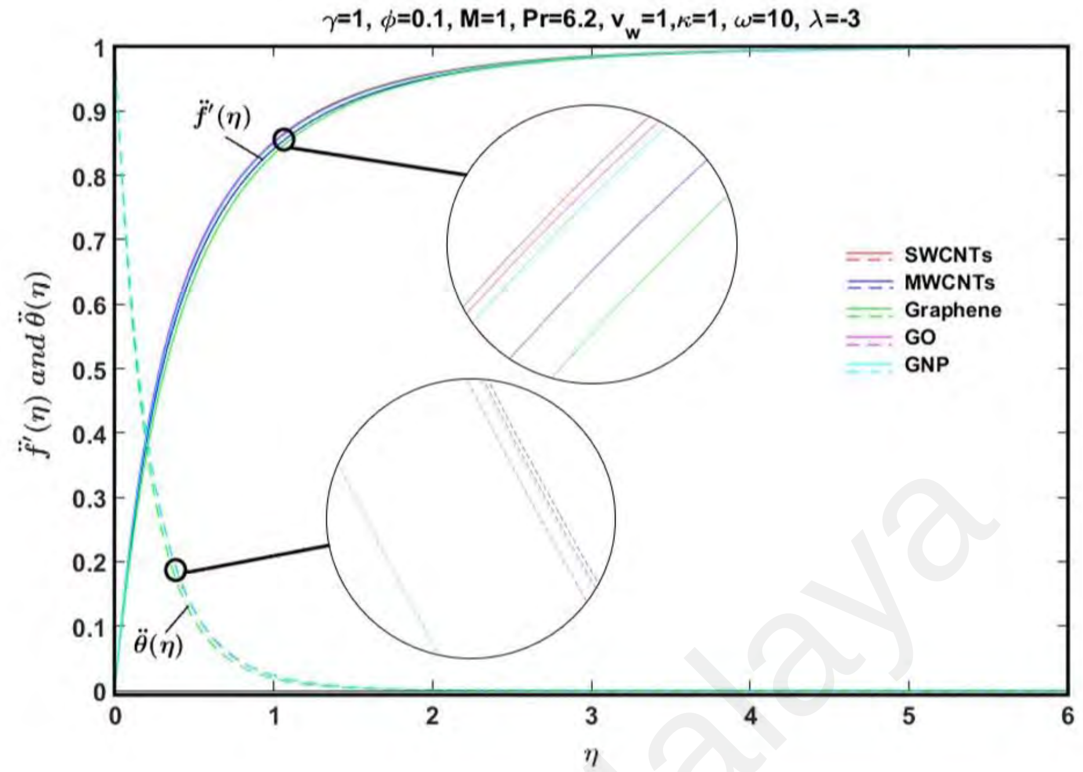

Figure 4.6: Influence of mixed convection parameter (when *λ*=-3) on velocity and temperature disseminations for the nanofluids.

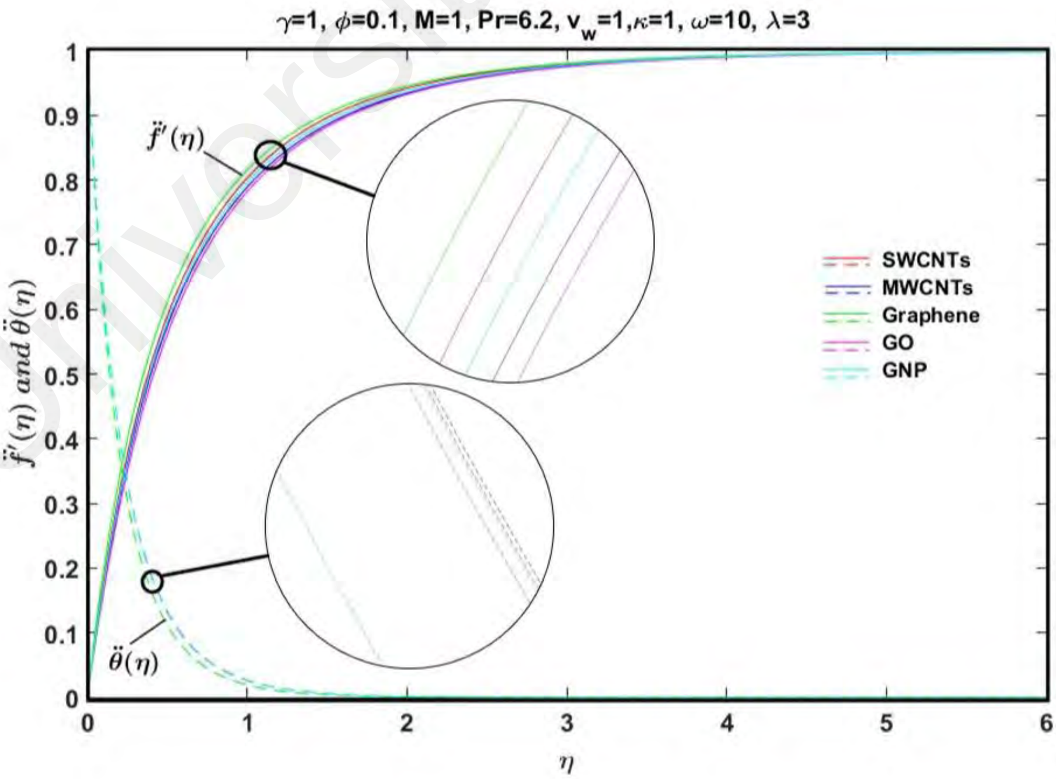

Figure 4.7: Influence of mixed convection parameter (when *λ*=3) on velocity and temperature disseminations for the nanofluids.

Figure 4.8 and 4.9 capture the trend of the fluid velocity for all selected nanofluids with variation of Prandtl number *Pr*. The velocity dissemination remains almost stagnant because *Pr* only appears in the energy equation (3.20). As observed in Figure 4.8, it is important to mention that graphenes-water under low Prandtl number offers a positive impact on the flow speed as compared to other nanofluids. Again graphenes-water gives the lowest temperature dissemination here than other nanofluids at any value of *Pr*.

Figures 4.4 to 4.9 show that the fluid temperature declines faster for larger inclination angle ω, mixed convection λ and *Pr* values. In all these considered figures, graphenes-water emerges as the best nanofluid since it has the highest heat flux performance to maintain the flow at the minimum temperature as compared to other nanofluids. Additionally, it is worth to highlight that GOs-water contributes to the lowest flow velocity at a smaller value of Prandtl, *Pr* but at a greater value of mixed convection λ.

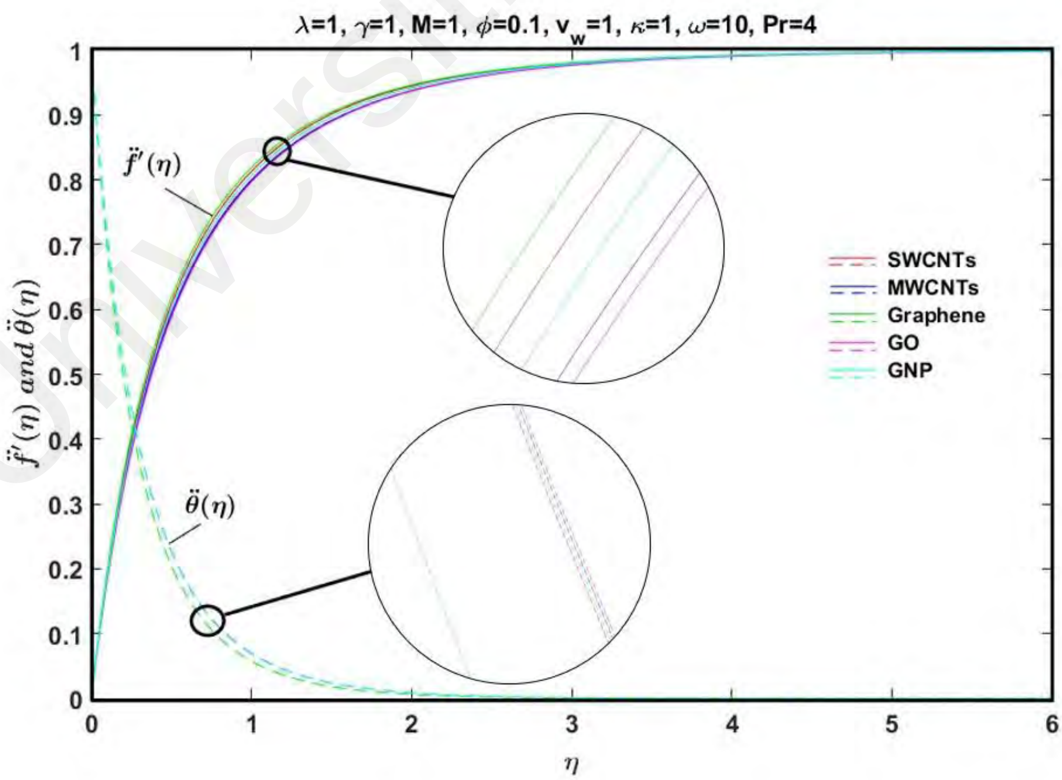

Figure 4.8: Influence of Prandtl number (when *Pr*=4) on velocity and temperature disseminations for the nanofluids.

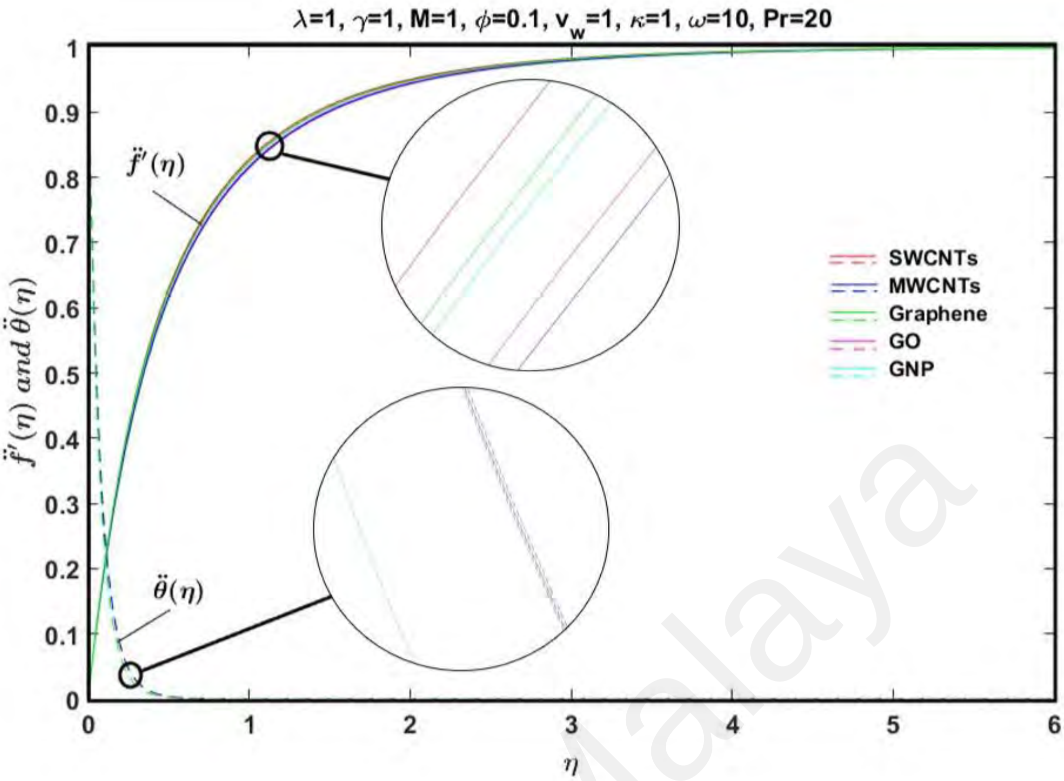

Figure 4.9: Influence of Prandtl number (when *Pr*=20) on velocity and temperature disseminations for the nanofluids.

The effects of curvature parameter γ, wall permeability parameter v_w, Hartmann number M and nanoparticle volume fraction ϕ on velocity dissemination are presented in Figure 4.10 to 4.13. It is found that an increment in γ and ϕ results in the reduction of fluid velocity while increasing the fluid temperature. The rise of ϕ results in increasing nanofluid viscosity which then slows down the fluid motion. Furthermore, thermal conductivity and the thermal boundary layer thickness are enhanced as ϕ increases. Fluid velocity rises more rapidly for all selected nanofluids when M and v_w increase. However, increasing the value of M and v_w causes the temperature dissemination to decay for all nanofluids. Given that, increasing M creates more resistance force in the current and depresses the hydrodynamic boundary layer thickness. Then this feature slow down the flow, consequently decreases the fluid temperature (Choi & Eastman, 1995; Haq et al., 2015). In the current research, the suction case ($v_w > 0$) and the injection case ($v_w < 0$) are implemented. From all the considered figures, it is discovered that the velocity dissemination decreases close to the surface of the cylinder while it increases away from the cylinder. The opposite trend is noticed for temperature dissemination. A small gap of fluid velocity and temperature appears between all selected nanofluids due to difference in thermophysical properties of the nanofluids. The results in Figure 4.10 and 4.11 present that the couple graphenes-water and SWCNTs-water, both dominate over the velocity dissemination at $\gamma = 0.8$ and $v_w = 1$. On the other hand, Figure 4.12 and 4.13 illustrate that graphenes-water still has the highest velocity dissemination after increasing the value of M and ϕ up to 8.0 and 0.4 respectively. Additionally, it is examined that GOs-water contributes the most outstanding development of fluid temperature when increasing the value of v_w, γ and ϕ as compared to other selected nanofluids. This signifies that GOs-water has the lowest heat flux performance.

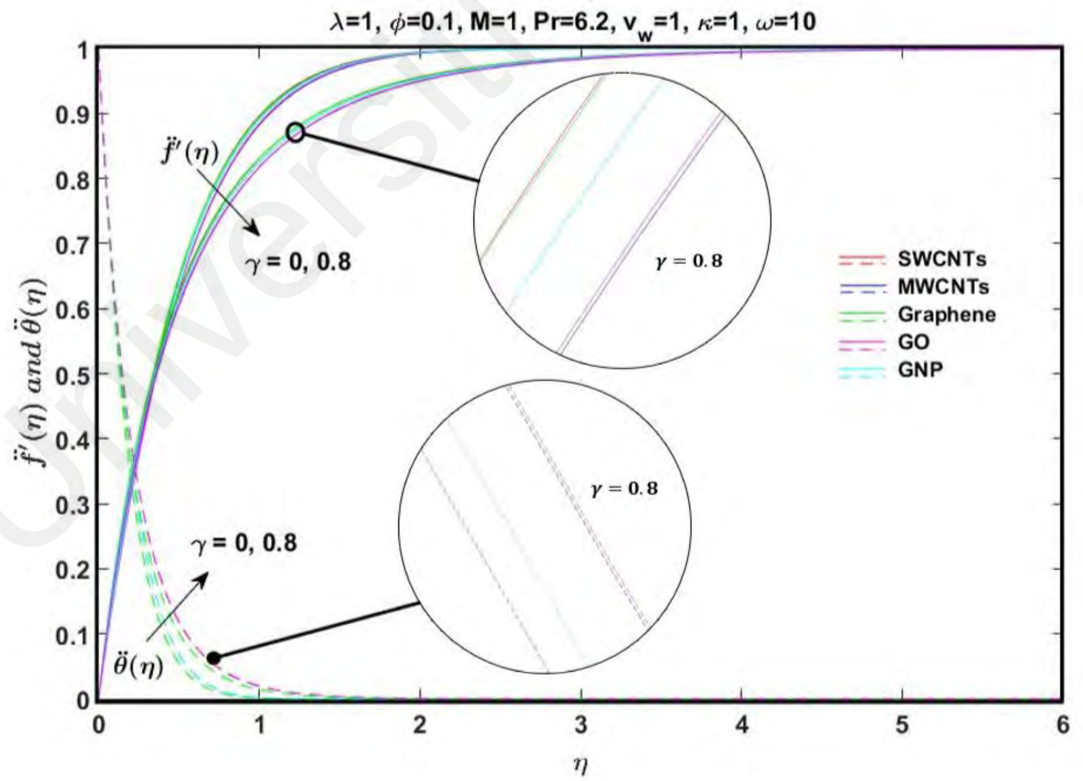

Figure 4.10: Influence of curvature parameter (when γ=0,0.8) on velocity and temperature disseminations for the nanofluids.

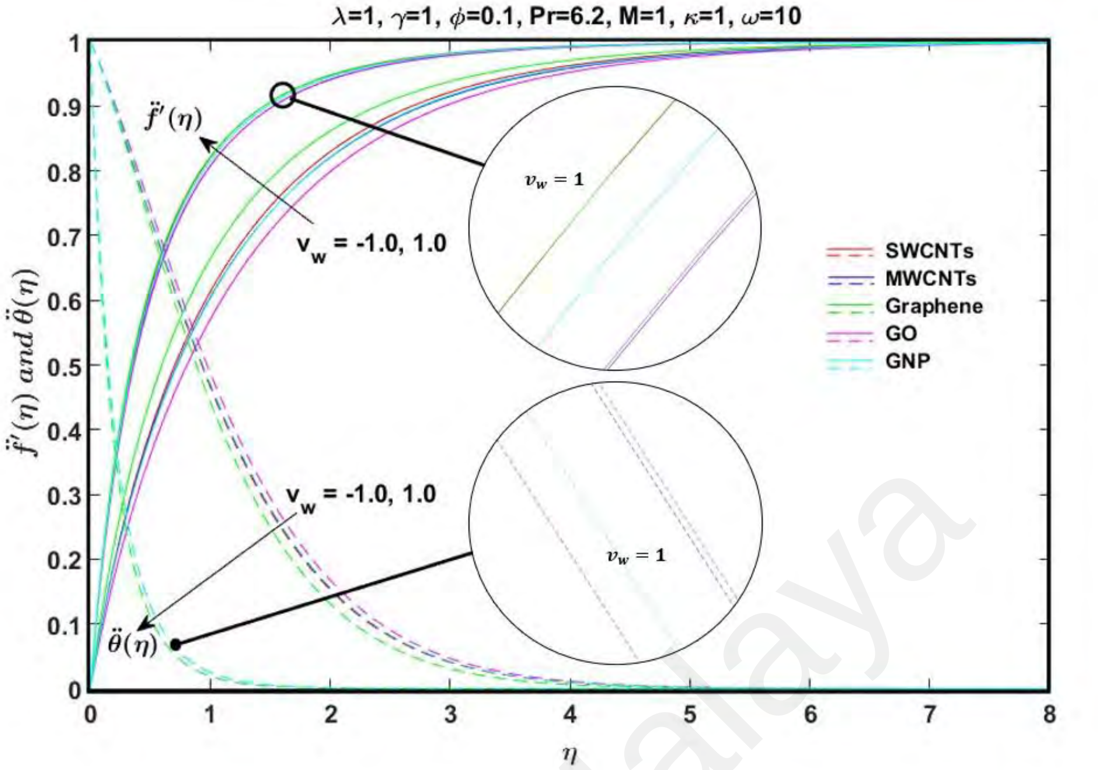

Figure 4.51: Influence of wall permeability parameter (when v_w=-1.0,1.0) on velocity and temperature disseminations for the nanofluids.

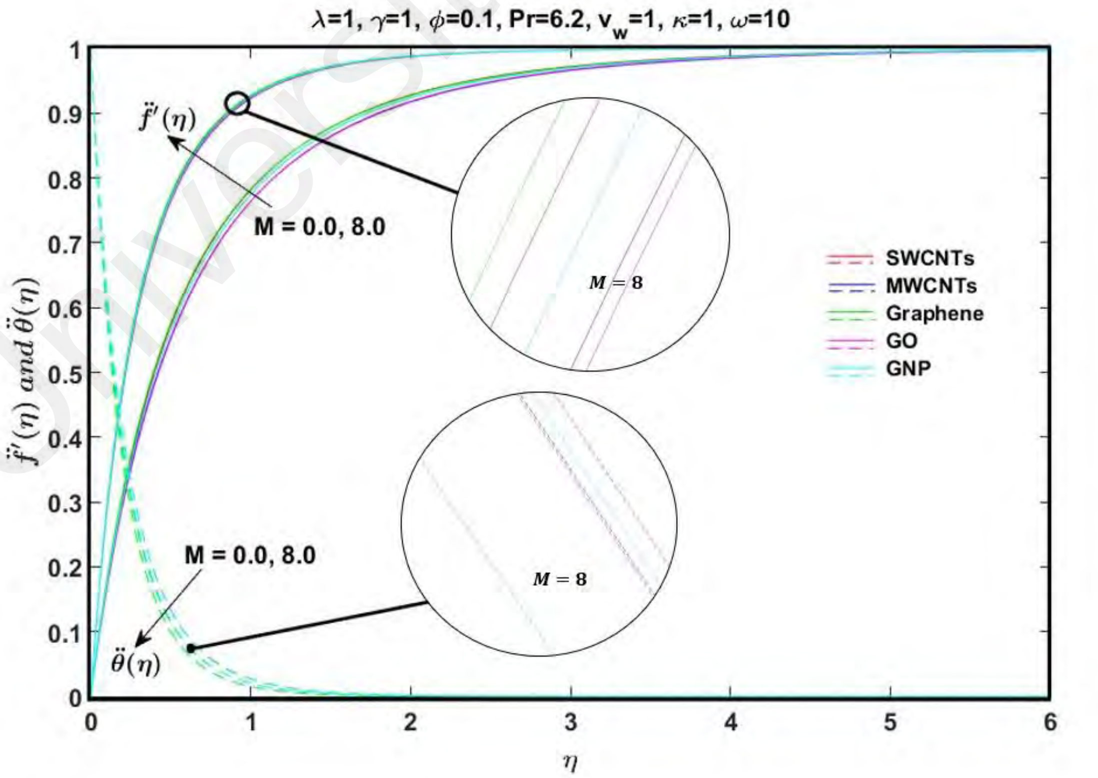

Figure 4.62: Influence of magnetic parameter (when M=0,8.0) on velocity and temperature disseminations for the nanofluids.

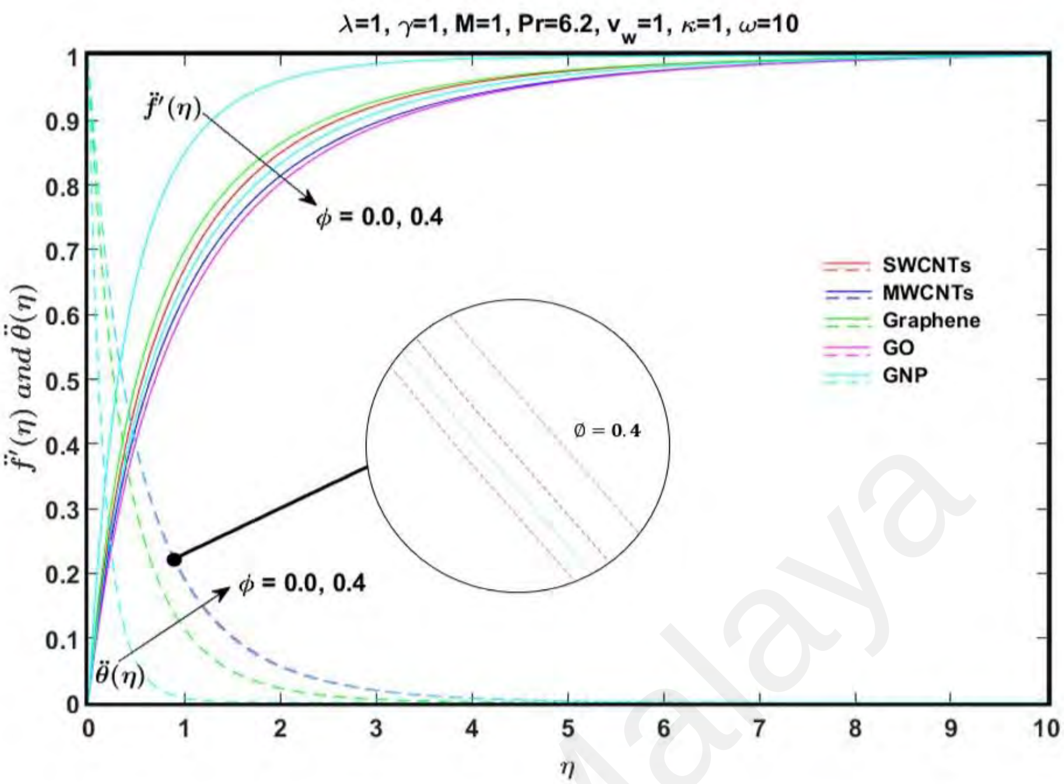

Figure 4.73: Influence of solid nanoparticle volume fraction parameter (when ϕ=0,0.4) on velocity and temperature disseminations for the nanofluids.

4.3 Skin Friction Coefficient and Nusselt Number

Figures 4.14 to 4.29 represent the effect of all emerging parameters on skin friction coefficient and Nusselt number respectively. It should be emphasized that the heat transfer rate and surface tension enhance owing to their correlation with skin friction coefficient and Nusselt number (Choi & Eastman, 1995).

An increment in the curvature parameter γ, Hartmann number M, Prandtl number Pr and wall permeability parameter v_w results in enhancement of reduced skin friction coefficient (Figures 4.14-4.17) and Nusselt number (Figures 4.22-4.25). However, increasing the value of mixed convection parameter λ constantly decreases the skin friction coefficient and Nusselt number for all selected nanofluids (see Figure 4.18 and 4.26). Eventually, graphenes-water emerges as the most significant nanofluid with the highest rates of reduced skin friction coefficient (Figures 4.14-4.17 for γ, M, Pr and v_w) and reduced Nusselt number (Figures 4.22-4.24, 4.26 and 4.29 for γ, M, Pr, λ and ω) against SWCNTs-water, MWCNTs-water, GNPs-water and GOs-water.

The effects of ϕ on skin friction coefficient and Nusselt number for all selected nanofluids are illustrated in Figures 4.19 and 4.27. The skin friction coefficient is gradually increasing as ϕ increases. As can be viewed in Figure 4.27, it is pointed out that except for graphenes-water, the trend of reduced Nusselt number for the rest of the nanofluids decreases when ϕ increases up to 0.2. The trend however started to rise again from 0.2 to 0.4. It is important to emphasize that graphenes-water excellently dominates over the skin friction and Nusselt number when the nanoparticles volume concentration ϕ is increasing. Furthermore, Figures 4.20 and 4.28 indicate changes in skin friction coefficient when the value of stagnation parameter κ is increased. The studied figures show that GNPs-water and SWCNTs are more sensitive (highly increased) towards skin friction coefficient and Nusselt number when $\kappa < 0$ (reversed stagnation) while graphene-water progressively dominates skin friction coefficient and Nusselt number when $\kappa > 0$ (positive stagnation). It is also notable that there is only a minor change in Nusselt number for all the nanofluids when $\kappa > 0$ (see Figure 4.28). Apart from that, Figure 4.21 and 4.29 display the relationship between skin friction coefficient and Nusselt number against the inclination angle ω. There is no particular pattern can be defined in reduced skin friction coefficient with respect to the inclination angle ω however the contrary result can be observed in Figure 4.29 in which graphenes-water strongly dominates the reduced Nusselt number as compared to other nanofluids. Basically, this indicates that particles with higher density provides higher friction at the surface of the cylinder. Besides, the specific heat capacity and thermal conductivity of nanofluids provide an influential contribution for heat transfer coefficient.

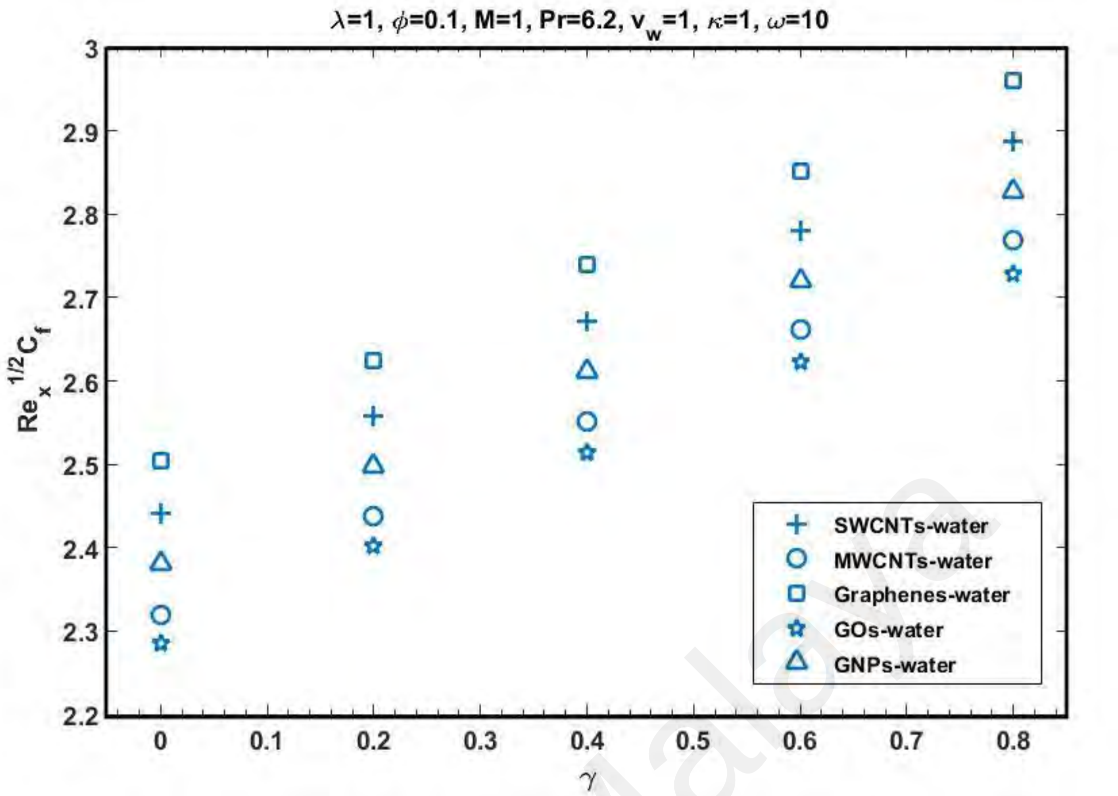

Figure 4.84: Effect of curvature parameter (γ) on reduced skin friction coefficient for the nanofluids.

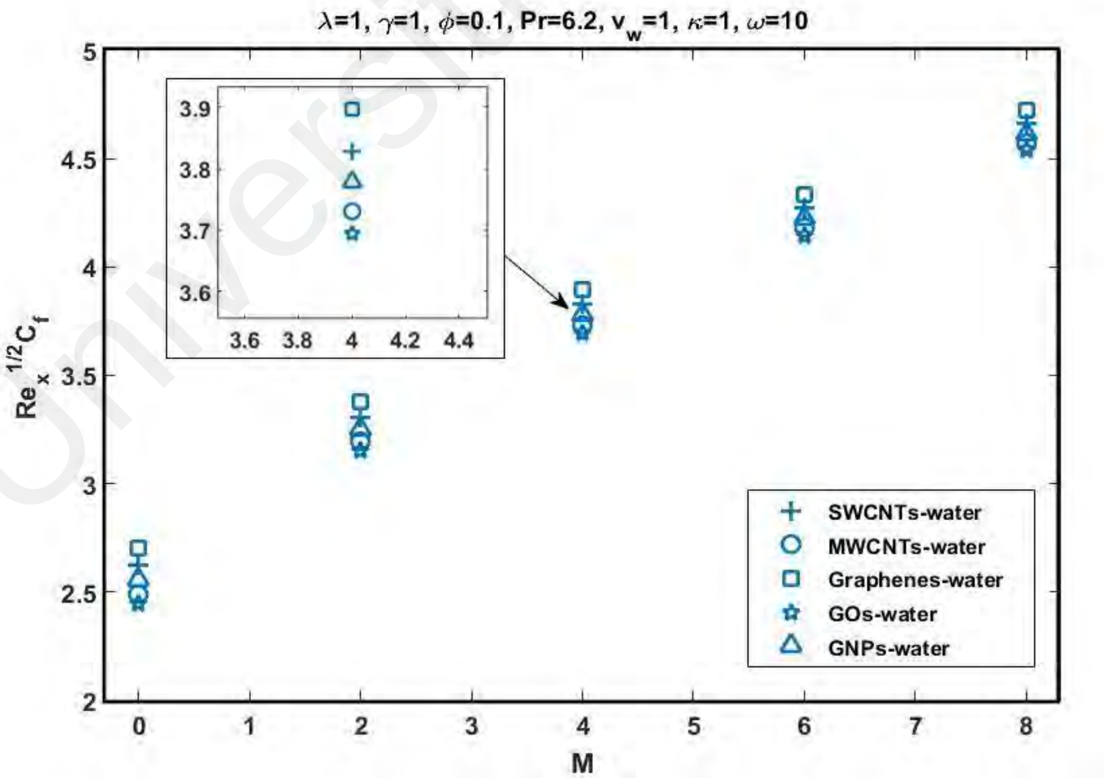

Figure 4.95: Effect of magnetic parameter (M) on reduced skin friction coefficient for the nanofluids.

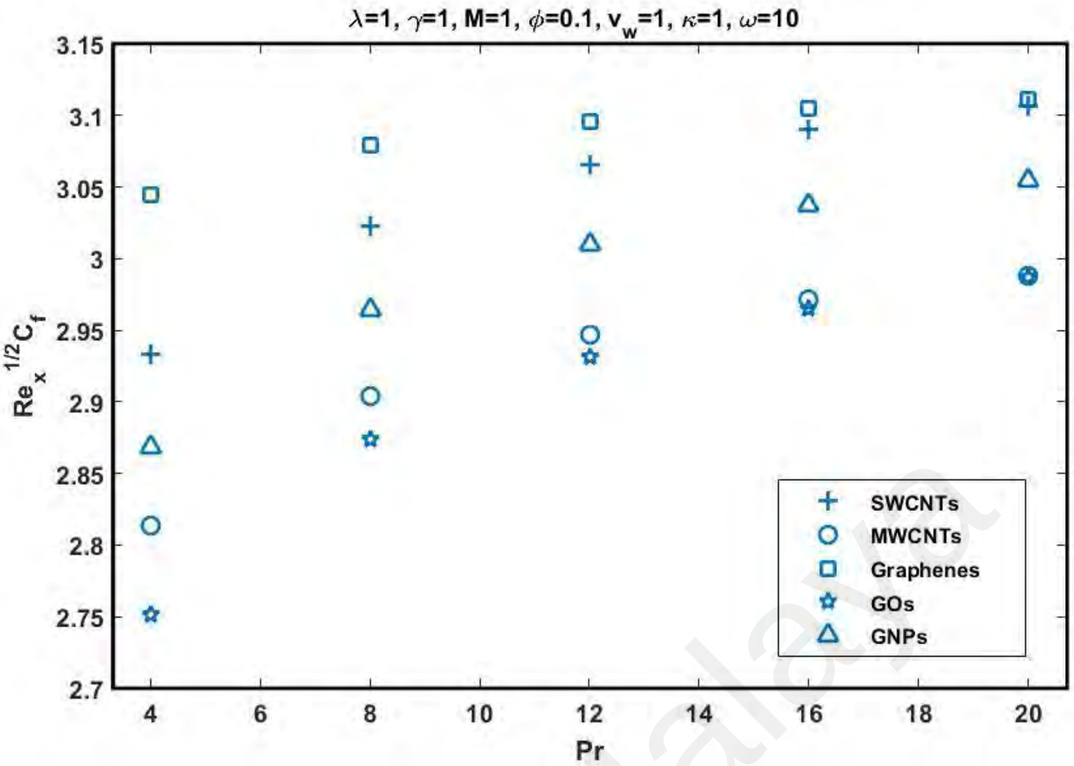

Figure 4.106: Effect of Prandtl number (*Pr*) on reduced skin friction coefficient for the nanofluids.

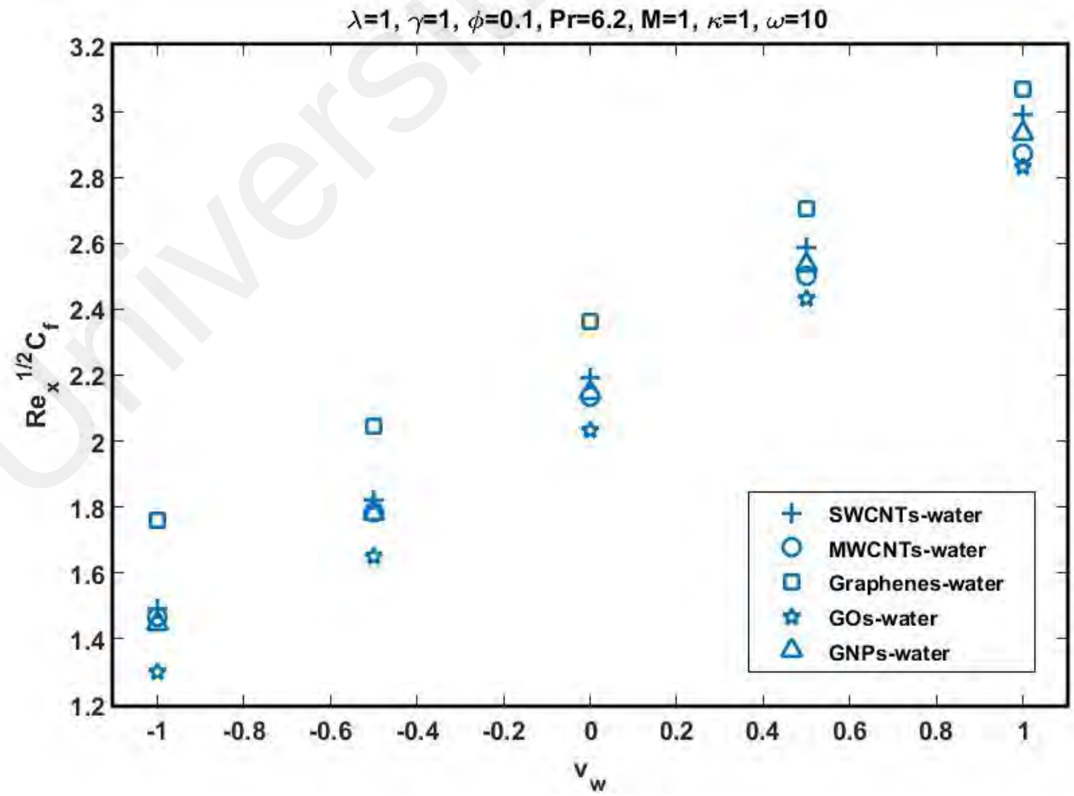

Figure 4.117: Effect of wall permeability parameter (v_w) on reduced skin friction coefficient for the nanofluids.

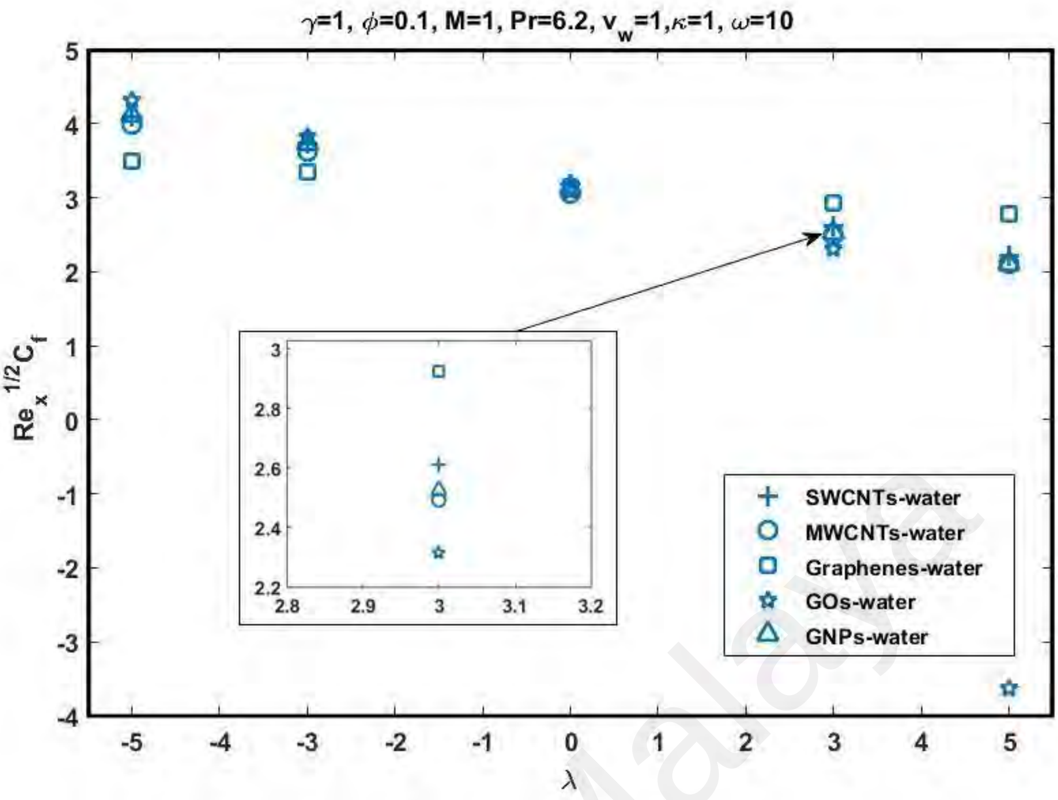

Figure 4.18: Effect of mixed convection parameter (λ) on reduced skin friction coefficient for the nanofluids.

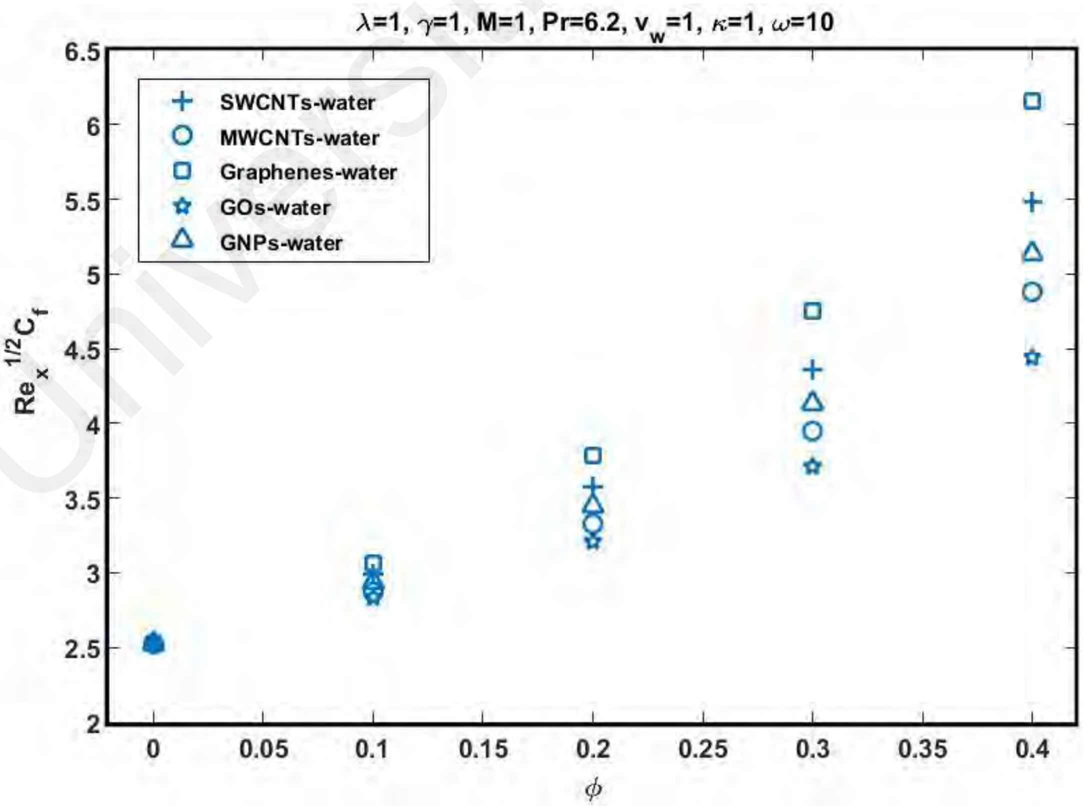

Figure 4.19: Effect of solid nanoparticle volume fraction parameter (ϕ) on reduced skin friction coefficient for the nanofluids.

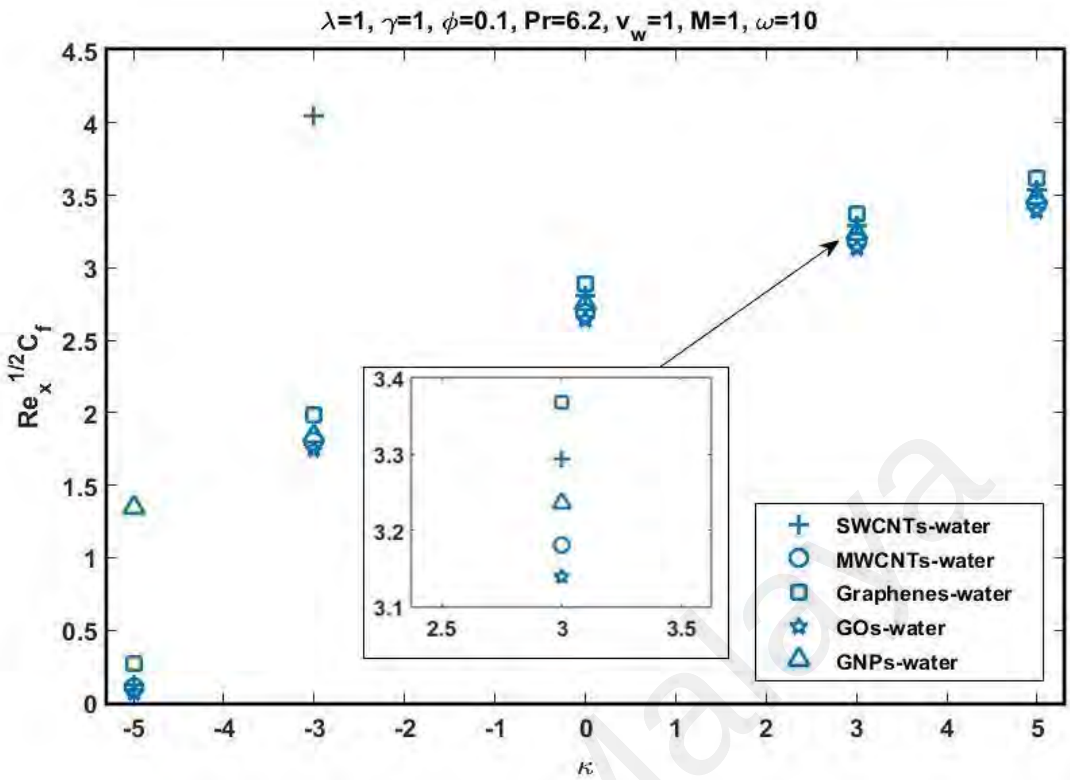

Figure 4.120: Effect of stagnation parameter (*κ*) on reduced skin friction coefficient for the nanofluids.

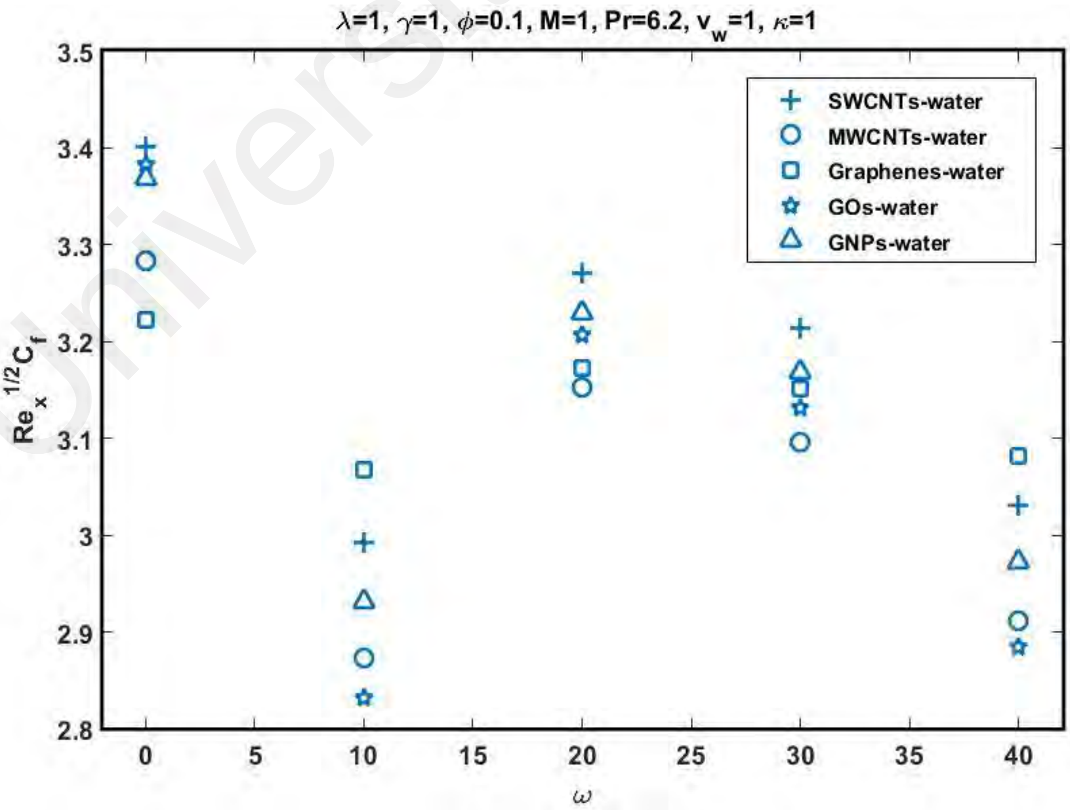

Figure 4.131: Effect of inclination angle parameter (*ω*) on reduced skin friction coefficient for the nanofluids.

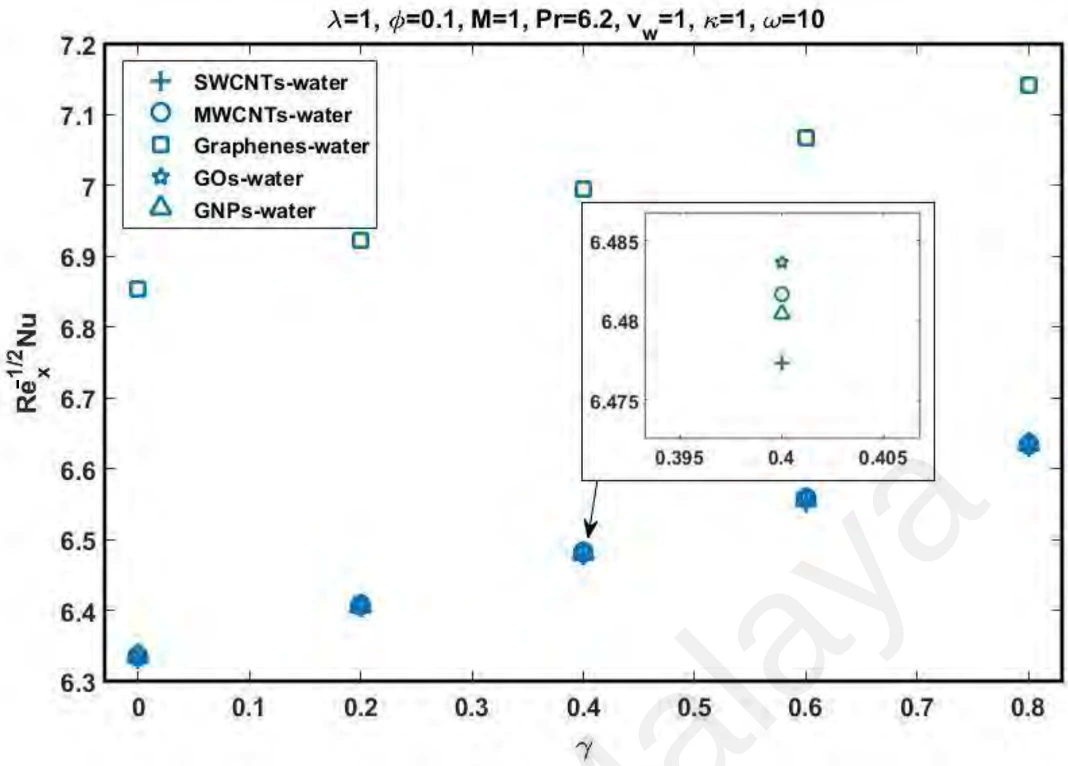

Figure 4.142: Effect of curvature parameter (γ) on reduced Nusselt number for the nanofluids.

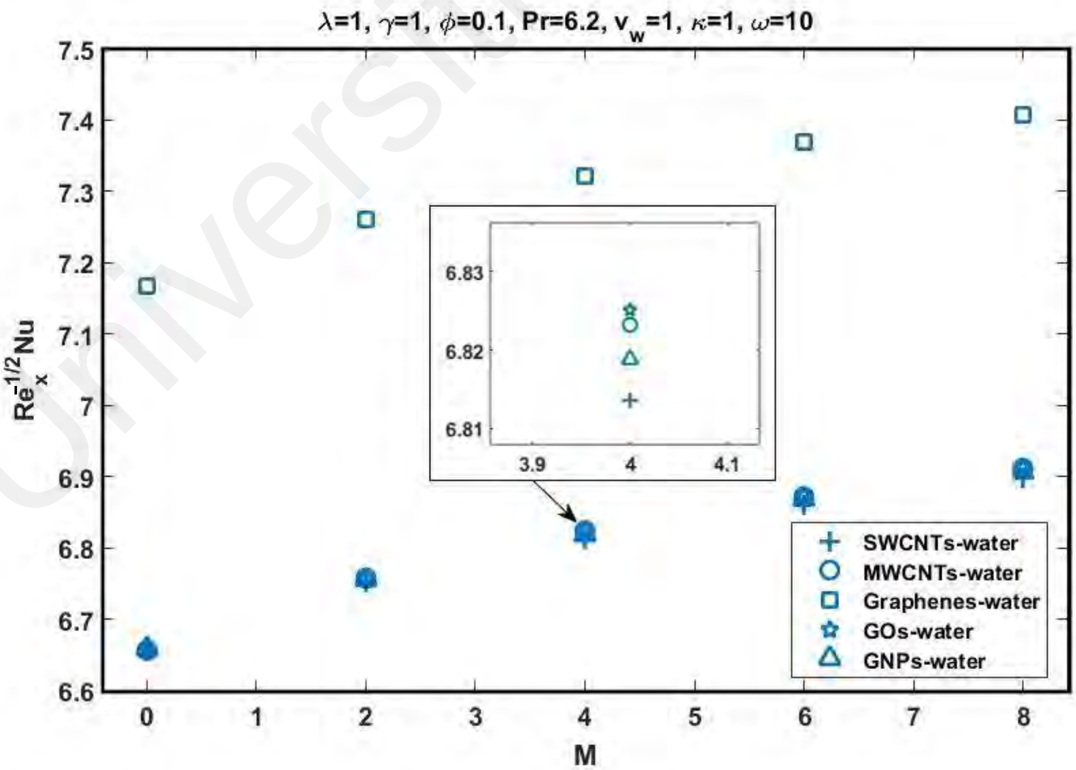

Figure 4.153: Effect of magnetic parameter (*M*) on reduced Nusselt number for the nanofluids.

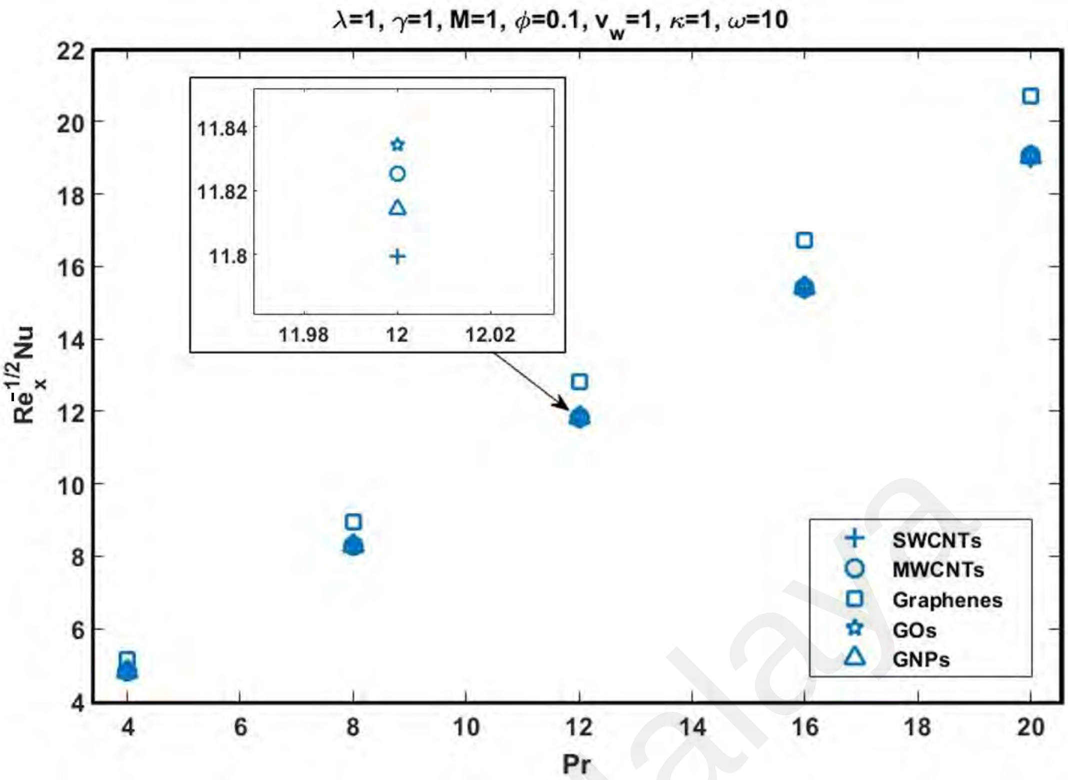

Figure 4.164: Effect of Prandtl number (*Pr*) on reduced Nusselt number for the nanofluids.

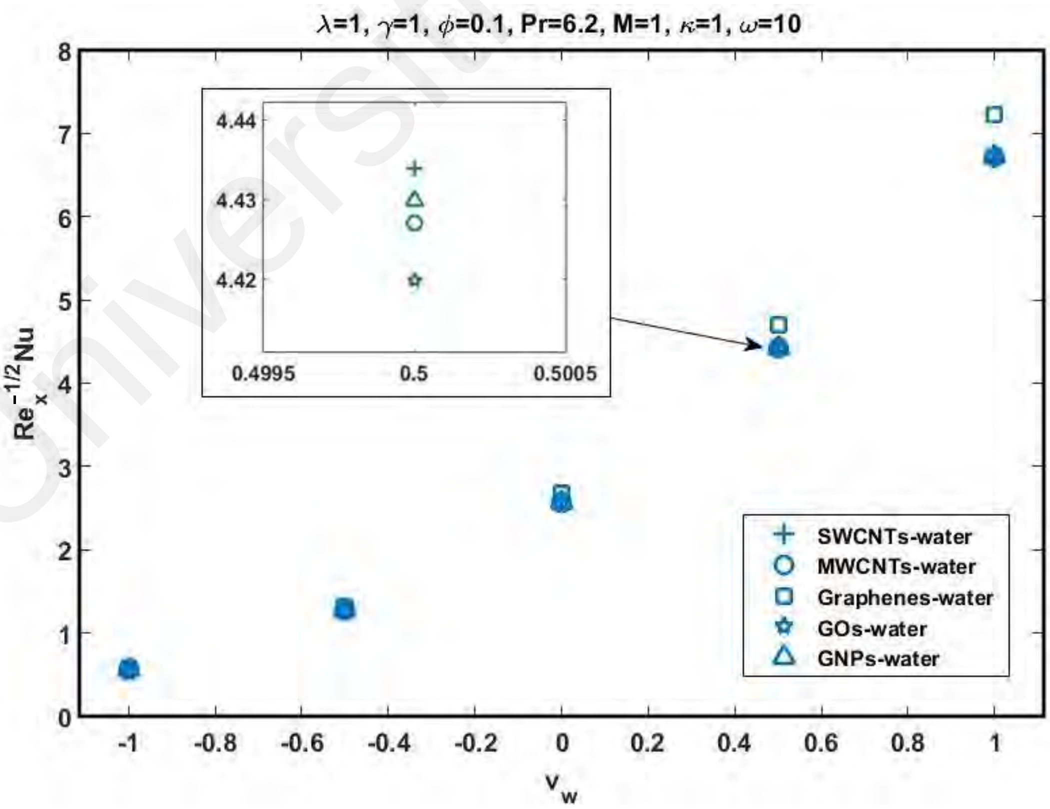

Figure 4.175: Effect of wall permeability parameter (v_w) on reduced Nusselt number for the nanofluids.

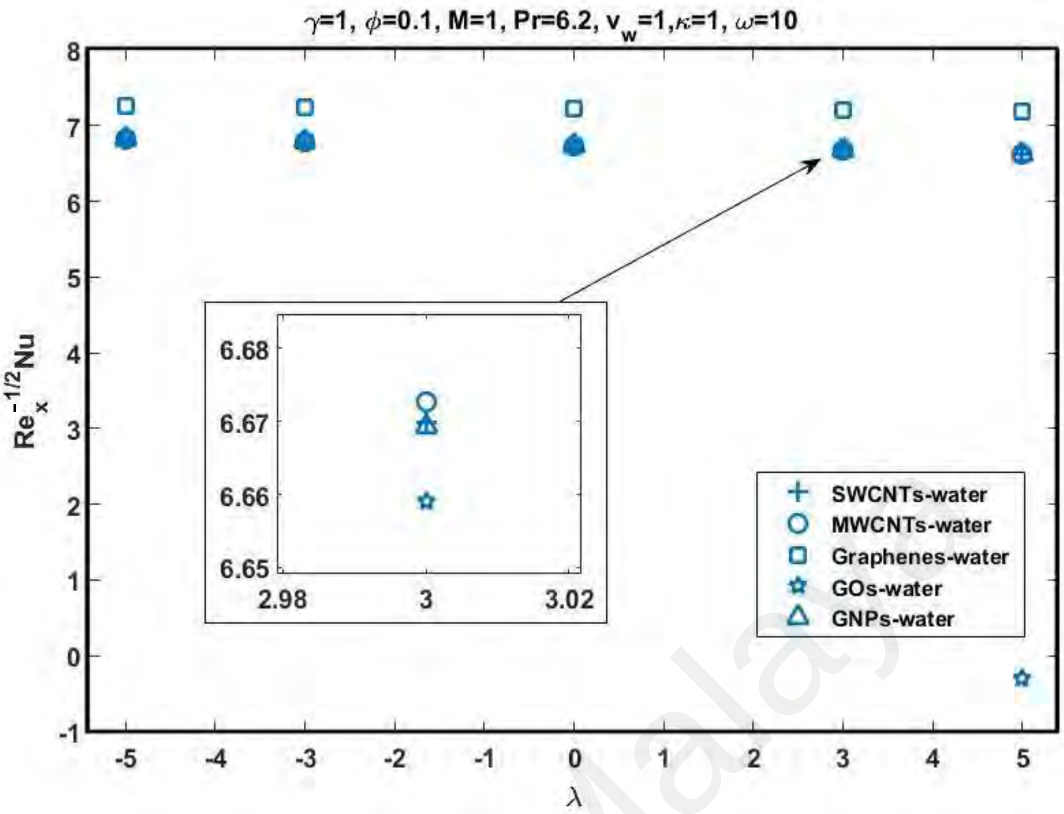

Figure 4.186: Effect of mixed convection parameter (λ) on reduced Nusselt number for the nanofluids.

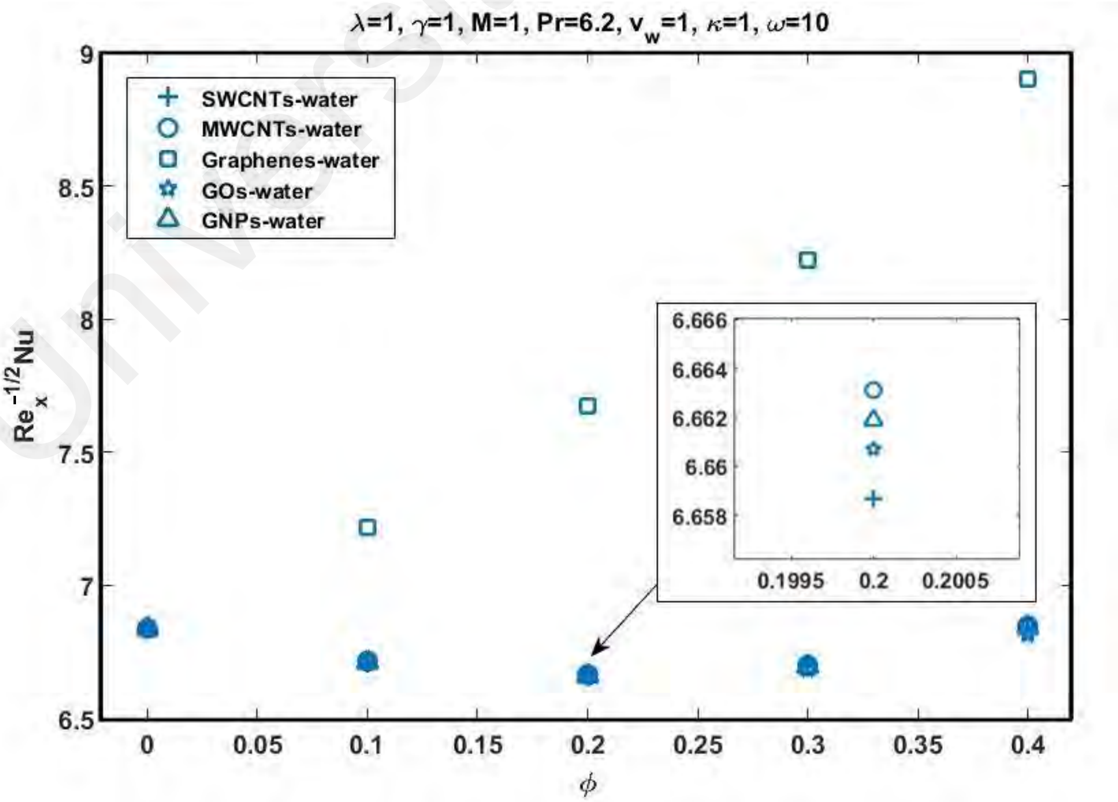

Figure 4.197: Effect of solid nanoparticle volume fraction parameter (ϕ) on reduced Nusselt number for the nanofluids.

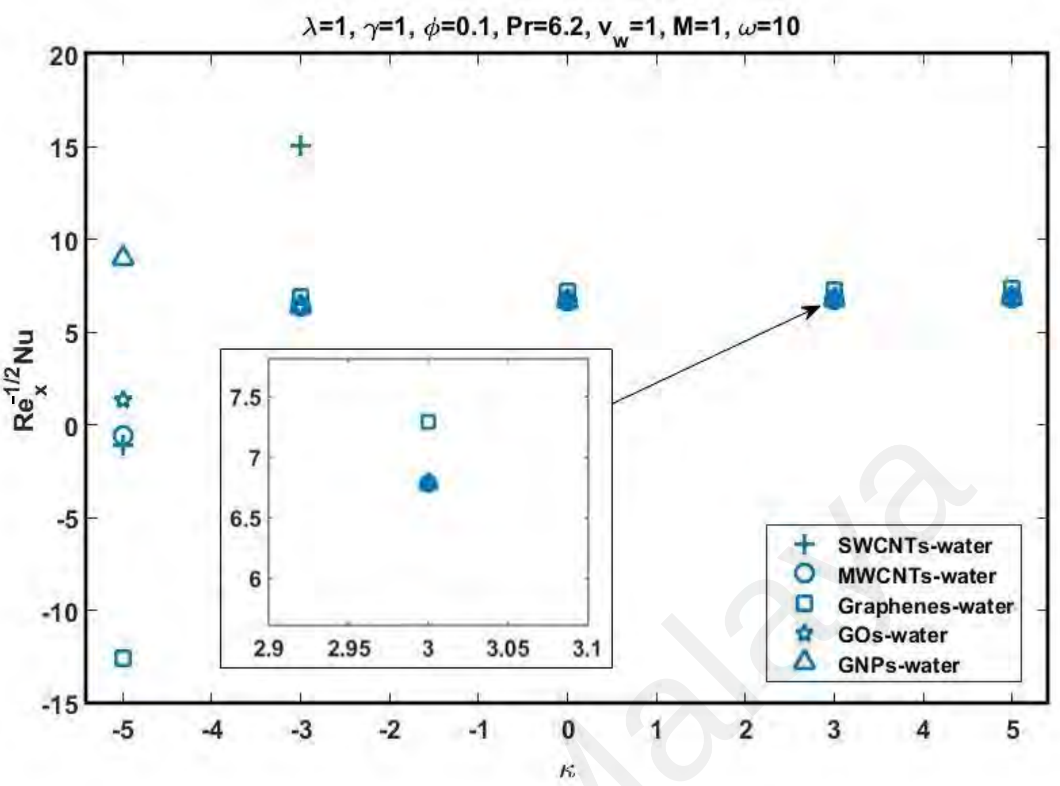

Figure 4.28: Effect of stagnation parameter (*κ*) on reduced Nusselt number for the nanofluids.

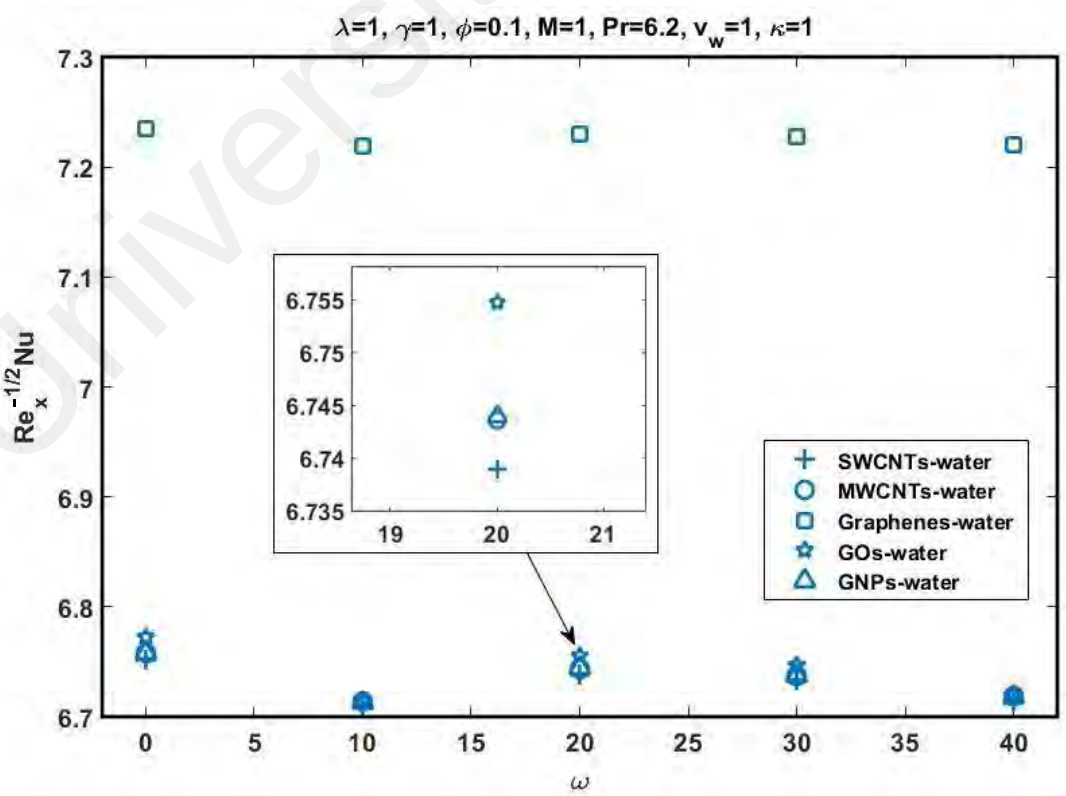

Figure 4.29: Effect of inclination angle parameter (*ω*) on reduced Nusselt number for the nanofluids.

On top of all these, it is also remarkable how graphene-water easily beats off the performance of SWCNTs-water and other nanofluids across many emerging parameters considered in the current work. A closer look on the numerical values against each physical parameter for both reduced skin friction coefficient and reduced Nusselt number to support the latter statement is possible as outlined in Appendix C. It is obvious that graphenes-water has the highest heat flux performance as compared to other nanofluids considered based on its value of reduced Nusselt number.

CHAPTER 5: CONCLUSION AND FUTURE RECOMMENDATIONS

5.1 CONCLUSION

In this study, steady, laminar, incompressible, mixed convective, reversed stagnation-point flow of heterogeneous variant graphene-based nanofluids flows over an inclined permeable cylinder in the presence of transverse magnetic field have been examined. The governing partial differential equations were transformed to nonlinear ordinary differential equations by using similarity variables before they were solved numerically using bvp4c package in MATLAB. The effects of emerging parameters on the flow for different nanoparticles: graphenes, GOs, GNPs, SWCNTs and MWCNTs with water as the base fluid are also analyzed and discussed with aid of tables and figures. The following outcomes are deduced:

1. Velocity dissemination decreases close to the surface of the cylinder and increases as the flow moves away from the cylinder for all nanofluids. The opposite trend is noticed for temperature dissemination.
2. GOs-water nanofluid has the hottest temperature for zero and positive stagnation (when κ value are 0 and 3).
3. SWCNTs-water and MWCNTs-water nanofluid have the highest velocity dissemination for positive and negative value of stagnation respectively while graphene-water has the lowest temperature as ω increases from 0 to 40 degrees.
4. Graphenes-water nanofluid dominates over the velocity dissemination for assisting flow under low Prandtl number, for greater Hartmann number and nanoparticle volume fraction as compared to other nanofluids.
5. Graphenes-water nanofluid also produces the lowest temperature dissemination as compared to other nanofluids regardless of the impact of mixed convection parameter and Prandtl number.

6. GOs-water nanofluid contributes the lowest flow velocity at a smaller value of Prandtl, but at a greater value of mixed convection parameter.
7. GOs-water nanofluid has the lowest heat flux performance under increasing values of wall permeability parameter, curvature parameter and nanoparticle volume fraction as compared to other nanofluids.
8. An increment in the curvature parameter, Hartmann number, Prandtl number and wall permeability parameter results in enhancement of reduced skin friction coefficient and reduced Nusselt number. However, increasing the value of mixed convection parameter constantly decreases the reduced skin friction coefficient and Nusselt number for all nanofluids.
9. Graphenes-water nanofluid consistently has the highest rate of reduced skin friction coefficient (for various curvature parameter, Hartmann number, Prandtl number and wall permeability parameter) and the highest rate of reduced Nusselt number (for various curvature parameter, Hartmann number, Prandtl number, wall permeability parameter, mixed convection parameter and inclination angle).
10. Graphenes-water nanofluid excellently dominates over the skin friction and Nusselt number when the nanoparticles volume concentration is increasing and for positive stagnation flow.
11. GNPs-water and SWCNTs-water nanofluids are more sensitive (highly increased) towards skin friction coefficient and Nusselt Number in reversed stagnation flow.
12. Graphenes-water nanofluid has the highest heat flux performance compared to other nanofluids across many emerging parameters considered in this work.

5.2 FUTURE RECOMMENDATIONS

In defiance of rapid developments in the fluid mechanics, the conundrums of heat transfer of nanofluid flows are not fully uncovered, thus, offering new opportunities, ideas and challenges for the scientists and engineers. Future research on heat flux performance of nanofluids can be vitally important for the sector of engineering, science, and technology. Therefore, we list some of recommendations on future research that can be extended through this present study:

1. To consider melting heat transfer and radiation effects of variant graphene-based nanofluids over an inclined stretching/shrinking or a divergent/convergent channel.
2. To add the variation of ordinary fluid such as ethylene glycol and engine oil to find the highest heat flux performance when combining with the selected graphene-based nanoparticles.
3. To conduct both experiment and numerical analysis for solving new upgraded models.

REFERENCES

Abbas, N., Nadeem, S., Saleem, A., Malik, M. Y., Issakhov, A., & Alharbi, F. M. (2021). Models base study of inclined MHD of hybrid nanofluid flow over nonlinear stretching cylinder. *Chinese Journal of Physics*, *69*, 109-117.

Abbas, Z., Rasool, S., & Rashidi, M. M. (2015). Heat transfer analysis due to an unsteady stretching/shrinking cylinder with partial slip condition and suction. *Ain Shams Engineering Journal*, *6*(3), 939-945.

Abbasi, F. M., Mustafa, M., Shehzad, S. A., Alhuthali, M. S., & Hayat, T. (2015). Analytical study of Cattaneo–Christov heat flux model for a boundary layer flow of Oldroyd-B fluid. *Chinese Physics B, 25*(1), 014701.

Abbasi, M., Khazali, N., & Sharifi, M. (2017). Analytical model for convection-conduction heat transfer during water injection in fractured geothermal reservoirs with variable rock matrix block size. *Geothermics, 69*, 1-14.

Aberoumand, S., Jafarimoghaddam, A., Moravej, M., Aberoumand, H., & Javaherdeh, K. (2016). Experimental study on the rheological behavior of silver-heat transfer oil nanofluid and suggesting two empirical based correlations for thermal conductivity and viscosity of oil based nanofluids. *Applied Thermal Engineering, 101*, 362-372.

Abo-Eldahab, E. M., & Salem, A. M. (2005). MHD flow and heat transfer of non-Newtonian power-law fluid with diffusion and chemical reaction on a moving cylinder. *Heat and Mass Transfer*, *41*(8), 703-708.

Aghamajidi, M., Yazdi, M., Dinarvand, S., & Pop, I. (2018). Tiwari-Das nanofluid model for magnetohydrodynamics (MHD) natural-convective flow of a nanofluid adjacent to a spinning down-pointing vertical cone. *Propulsion and Power Research*, *7*(1), 78-90.

Ahmed, N., Mohyud-Din, S. T., & Hassan, S. M. (2016). Flow and heat transfer of nanofluid in an asymmetric channel with expanding and contracting walls suspended by carbon nanotubes: a numerical investigation. *Aerospace Science and Technology, 48*, 53-60.

Akbarzadeh, S., & Valipour, M. S. (2020). Experimental study on the heat transfer enhancement in helically corrugated tubes under the non-uniform heat flux. *Journal of Thermal Analysis and Calorimetry*, 1-13.

Ali, H. M., & Arshad, W. (2017). Effect of channel angle of pin-fin heat sink on heat transfer performance using water-based graphene nanoplatelets nanofluids. *International Journal of Heat and Mass Transfer, 106*, 465-472.

Ali, N., Khan, S. U., Sajid, M., & Abbas, Z. (2016). MHD flow and heat transfer of couple stress fluid over an oscillatory stretching sheet with heat source/sink in porous medium. *Alexandria Engineering Journal, 55*(2), 915-924.

Alizadeh, R., Karimi, N., Arjmandzadeh, R., & Mehdizadeh, A. (2019). Mixed convection and thermodynamic irreversibilities in MHD nanofluid stagnation-point flows over a cylinder embedded in porous media. *Journal of Thermal Analysis and Calorimetry, 135*(1), 489-506.

Alloui, Z., & Vasseur, P. (2015). Natural convection of Carreau–Yasuda non-Newtonian fluids in a vertical cavity heated from the sides. *International Journal of Heat and Mass Transfer, 84*, 912-924.

Alsaedi, A., Khan, M. I., Farooq, M., Gull, N., & Hayat, T. (2017). Magnetohydrodynamic (MHD) stratified bioconvective flow of nanofluid due to gyrotactic microorganisms. *Advanced Powder Technology, 28*(1), 288-298.

Al-Salem, K., Öztop, H. F., Pop, I., & Varol, Y. (2012). Effects of moving lid direction on MHD mixed convection in a linearly heated cavity. *International Journal of Heat and Mass Transfer, 55*(4), 1103-1112.

Al-Waeli, A. H., Chaichan, M. T., Kazem, H. A., Sopian, K., & Safaei, J. (2018). Numerical study on the effect of operating nanofluids of photovoltaic thermal system (PV/T) on the convective heat transfer. *Case Studies in Thermal Engineering, 12*, 405-413.

Aman, S., Khan, I., Ismail, Z., & Salleh, M. Z. (2018). Impacts of gold nanoparticles on MHD mixed convection Poiseuille flow of nanofluid passing through a porous medium in the presence of thermal radiation, thermal diffusion and chemical reaction. *Neural Computing and Applications, 30*(3), 789-797.

Amin, M., Putra, N., Kosasih, E. A., Prawiro, E., Luanto, R. A., & Mahlia, T. M. I. (2017). Thermal properties of beeswax/graphene phase change material as energy storage for building applications. *Applied Thermal Engineering*, *112*, 273-280.

Annaratone, D. (2010). *Engineering heat transfer.* Springer Science & Business Media.

Armaghani, T., Kasaeipoor, A., Alavi, N., & Rashidi, M. M. (2016). Numerical investigation of water-alumina nanofluid natural convection heat transfer and entropy generation in a baffled L-shaped cavity. *Journal of Molecular Liquids, 223*, 243-251.

Awais, M., Hayat, T., & Alsaedi, A. (2015). Investigation of heat transfer in flow of Burgers' fluid during a melting process. *Journal of the Egyptian Mathematical Society, 23*(2), 410-415.

Azimi, M., & Riazi, R. (2016). Heat Transfer Analysis of Magnetohydrodynamics Graphene Oxide-Water Nanofluid Flow through Convergent-Divergent Channels. *Journal of Computational and Theoretical Nanoscience, 13*(1), 659-665.

Azimi, M., Azimi, A., & Mirzaei, M. (2014). Investigation of the unsteady graphene oxide nanofluid flow between two moving plates. *Journal of Computational and Theoretical Nanoscience*, *11*(10), 2104-2108.

Bachok, N., Ishak, A., & Pop, I. (2012). Unsteady boundary-layer flow and heat transfer of a nanofluid over a permeable stretching/shrinking sheet. *International Journal of Heat and Mass Transfer, 55*(7-8), 2102-2109.

Badruddin, I. A. (2019). Investigation of heat transfer in irregular porous cavity subjected to various boundary conditions. *International Journal of Numerical Methods for Heat & Fluid Flow, 29*(1), 418-447.

Bahiraei, M., Jamshidmofid, M., & Heshmatian, S. (2017). Entropy generation in a heat exchanger working with a biological nanofluid considering heterogeneous particle distribution. *Advanced Powder Technology, 28*(9), 2380-2392.

Bao, C., Cai, N., & Croiset, E. (2011). An analytical model of view factors for radiation heat transfer in planar and tubular solid oxide fuel cells. *Journal of Power Sources, 196*(6), 3223-3232.

Ben Hamida, M. B., & Charrada, K. (2015). Natural convection heat transfer in an enclosure filled with an ethylene glycol—copper nanofluid under magnetic fields. *Numerical Heat Transfer, Part A: Applications, 67*(8), 902-920.

Bhargava, R., Sharma, R., & Bég, O. A. (2009). Oscillatory chemically-reacting MHD free convection heat and mass transfer in a porous medium with Soret and Dufour effects: finite element modeling. *International Journal of Applied Mathematics and Mechanics, 5*(6), 15-37.

Bharti, R. P., Chhabra, R. P., & Eswaran, V. (2007). Steady forced convection heat transfer from a heated circular cylinder to power-law fluids. *International Journal of Heat and Mass Transfer, 50*(5-6), 977-990.

Bharti, R. P., Sivakumar, P., & Chhabra, R. P. (2008). Forced convection heat transfer from an elliptical cylinder to power-law fluids. *International Journal of Heat and Mass Transfer, 51*(7-8), 1838-1853.

Bianco, V., Chiacchio, F., Manca, O., & Nardini, S. (2009). Numerical investigation of nanofluids forced convection in circular tubes. *Applied Thermal Engineering, 29*(17-18), 3632-3642.

Bilal, S., Majeed, A. H., Mahmood, R., Khan, I., Seikh, A. H., & Sherif, E. S. M. (2020). Heat and mass transfer in hydromagnetic second-grade fluid past a porous inclined cylinder under the effects of thermal dissipation, diffusion and radiative heat flux. *Energies*, *13*(1), 278.

Bondareva, N. S., Sheremet, M. A., Öztop, H. F., & Abu-Hamdeh, N. (2018). Transient natural convection in a partially open trapezoidal cavity filled with a water-based nanofluid under the effects of Brownian diffusion and thermophoresis. *International Journal of Numerical Methods for Heat & Fluid Flow, 28*(3), 606-623.

Buongiorno, J. (2006). Convective transport in nanofluids. *Journal of Heat Transfer*, *128*(3), 240-250.

Chamkha, A. J., & Aly, A. M. (2010). MHD free convection flow of a nanofluid past a vertical plate in the presence of heat generation or absorption effects. *Chemical Engineering Communications, 198*(3), 425-441.

Chamkha, A. J., Dogonchi, A. S., & Ganji, D. D. (2018). Magnetohydrodynamic nanofluid natural convection in a cavity under thermal radiation and shape factor of nanoparticles impacts: a numerical study using CVFEM. *Applied Sciences, 8*(12), 1-16.

Chen, H., Xiao, T., & Shen, M. (2017). Nanofluid flow in a porous channel with suction and chemical reaction using Tiwari and Das's nanofluid model. *Heat Transfer—Asian Research, 46*(7), 1041-1052.

Chen, W. R. (2007). A numerical study of laminar free convection heat transfer between inner sphere and outer vertical cylinder. *International Journal Of Heat and Mass Transfer, 50*(13-14), 2656-2666.

Choi, S. U., & Eastman, J. A. (1995). Enhancing thermal conductivity of fluids with nanoparticles (No. ANL/MSD/CP-84938; CONF-951135-29). Argonne National Lab., IL (United States).

Cui, P., Li, X., Man, Y., & Fang, Z. (2011). Heat transfer analysis of pile geothermal heat exchangers with spiral coils. *Applied Energy, 88*(11), 4113-4119.

Cui, Y. (2020). New analytical method for single-phase convective heat transfer and unified mechanism analyses on buoyancy-induced supercritical convective heat transfer deterioration. *International Journal of Heat and Mass Transfer, 146*, 1-13.

Das, K., Acharya, N., & Kundu, P. K. (2015). Radiative flow of MHD Jeffrey fluid past a stretching sheet with surface slip and melting heat transfer. *Alexandria Engineering Journal, 54*(4), 815-821.

Demir, H., Dalkilic, A. S., Kürekci, N. A., Duangthongsuk, W., & Wongwises, S. (2011). Numerical investigation on the single phase forced convection heat transfer characteristics of TiO_2 nanofluids in a double-tube counter flow heat exchanger. *International Communications in Heat and Mass Transfer, 38*(2), 218-228.

Derakhshan, R., Shojaei, A., Hosseinzadeh, K., Nimafar, M., & Ganji, D. D. (2019). Hydrothermal analysis of magneto hydrodynamic nanofluid flow between two parallel by AGM. *Case Studies in Thermal Engineering, 14*, 1-11.

Dhanai, R., Rana, P., & Kumar, L. (2016). MHD mixed convection nanofluid flow and heat transfer over an inclined cylinder due to velocity and thermal slip effects: Buongiorno's model. *Powder Technology*, *288*, 140-150.

Dinarvand, S., Abbassi, A., Hosseini, R., & Pop, I. (2015). Homotopy analysis method for mixed convective boundary layer flow of a nanofluid over a vertical circular cylinder. *Thermal Science, 19*(2), 549-561.

Dinarvand, S., Hosseini, R., & Pop, I. (2016). Homotopy analysis method for unsteady mixed convective stagnation-point flow of a nanofluid using Tiwari-Das nanofluid model. *International Journal of Numerical Methods for Heat & Fluid Flow, 26*(1), 40-62.

Dinarvand, S., Hosseini, R., & Pop, I. (2017). Axisymmetric mixed convective stagnation-point flow of a nanofluid over a vertical permeable cylinder by Tiwari-Das nanofluid model. *Powder Technology*, 311, 147-156.

Durst, F. (2008). *Fluid mechanics: an introduction to the theory of fluid flows.* Springer Science & Business Media.

Ellahi, R., Shivanian, E., Abbasbandy, S., & Hayat, T. (2016). Numerical study of magnetohydrodynamics generalized Couette flow of Eyring-Powell fluid with heat transfer and slip condition. *International Journal of Numerical Methods for Heat & Fluid Flow, 26(5)*, 1433-1445.

El-Masry, Y. A. S., Abd Elmaboud, Y., & Abdel-Sattar, M. A. (2020). The impacts of varying magnetic field and free convection heat transfer on an Eyring–Powell fluid flow with peristalsis: VIM solution. *Journal of Taibah University for Science, 14*(1), 19-30.

Erdoğdu, F., & Turhan, M. (2006). Analysis of dimensional ratios of regular geometries for infinite geometry assumptions in conduction heat transfer problems. *Journal of Food Engineering, 77*(4), 818-824.

Esfahani, M. A., & Toghraie, D. (2017). Experimental investigation for developing a new model for the thermal conductivity of silica/water-ethylene glycol (40%–60%) nanofluid at different temperatures and solid volume fractions. *Journal of Molecular Liquids, 232*, 105-112.

Esfe, M. H., Arani, A. A. A., Niroumand, A. H., Yan, W. M., & Karimipour, A. (2015). Mixed convection heat transfer from surface-mounted block heat sources in a horizontal channel with nanofluids. *International Journal of Heat and Mass Transfer, 89*, 783-791.

Esmaeilpour, M., & Abdollahzadeh, M. (2012). Free convection and entropy generation of nanofluid inside an enclosure with different patterns of vertical wavy walls. *International Journal of Thermal Sciences, 52*, 127-136.

Fan, Z. J., Kai, W., Yan, J., Wei, T., Zhi, L. J., Feng, J., … Wei, F. (2011). Facile synthesis of graphene nanosheets via Fe reduction of exfoliated graphite oxide. *ACS Nano, 5*(1), 191-198.

Farajollahi, B., Etemad, S. G., & Hojjat, M. (2010). Heat transfer of nanofluids in a shell and tube heat exchanger. *International Journal of Heat and Mass Transfer, 53*(1-3), 12-17.

Fotukian, S. M., & Esfahany, M. N. (2010). Experimental study of turbulent convective heat transfer and pressure drop of dilute CuO/water nanofluid inside a circular tube. *International Communications in Heat qnd Mass Transfer, 37*(2), 214-219.

Fuchs, H. U. (2010). *The dynamics of heat: a unified approach to thermodynamics and heat transfer.* Springer Science & Business Media.

Ghadikolaei, S. S., Hosseinzadeh, K., & Ganji, D. D. (2018). MHD raviative boundary layer analysis of micropolar dusty fluid with graphene oxide (Go)-engine oil nanoparticles in a porous medium over a stretching sheet with joule heating effect. *Powder Technology, 338*, 425-437.

Gholinia, M., Gholinia, S., Hosseinzadeh, K., & Ganji, D. D. (2018). Investigation on ethylene glycol nano fluid flow over a vertical permeable circular cylinder under effect of magnetic field. *Results in Physics*, *9*, 1525-1533.

Ghorbani, N., Taherian, H., Gorji, M., & Mirgolbabaei, H. (2010). Experimental study of mixed convection heat transfer in vertical helically coiled tube heat exchangers. *Experimental Thermal and Fluid Science, 34*(7), 900-905.

Ghozatloo, A., Rashidi, A., & Shariaty-Niassar, M. (2014). Convective heat transfer enhancement of graphene nanofluids in shell and tube heat exchanger. *Experimental Thermal and Fluid Science*, *53*, 136-141.

Grosan, T., & Pop, I. (2011). Axisymmetric mixed convection boundary layer flow past a vertical cylinder in a nanofluid. *International Journal of Heat and Mass Transfer*, 54(15-16), 3139-3145.

Hachem, E., Digonnet, H., Kosseifi, N., Massoni, E., & Coupez, T. (2010). Enriched finite element spaces for transient conduction heat transfer. *Applied Mathematics and Computation, 217*(8), 3929-3943.

Halim, N. A., Sivasankaran, S., & Noor, N. M. (2017). Active and passive controls of the Williamson stagnation nanofluid flow over a stretching/shrinking surface. *Neural Computing and Applications, 28*(1), 1023-1033.

Hamad, M. A. A., Pop, I., & Ismail, A. M. (2011). Magnetic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate. *Nonlinear Analysis: Real World Applications, 12*(3), 1338-1346.

Haq, R. U., Nadeem, S., Khan, Z. H., & Noor, N. F. M. (2015). Convective heat transfer in MHD slip flow over a stretching surface in the presence of carbon nanotubes. *Physica B: Condensed Matter*, *457*, 40-47.

Haq, R. U., Nadeem, S., Khan, Z. H., & Noor, N. F. M. (2015). MHD squeezed flow of water functionalized metallic nanoparticles over a sensor surface. *Physica E: Low-Dimensional Systems and Nanostructures*, *73*, 45-53.

Haq, R. U., Shahzad, F., & Al-Mdallal, Q. M. (2017). MHD pulsatile flow of engine oil based carbon nanotubes between two concentric cylinders. *Results in Physics, 7*, 57-68.

Hatami, M., Nouri, R., & Ganji, D. D. (2013). Forced convection analysis for MHD Al_2O_3–water nanofluid flow over a horizontal plate. *Journal of Molecular Liquids, 187*, 294-301.

Hauke, G. (2008). *An introduction to fluid mechanics and transport phenomena (Vol. 86).* New York: Springer.

Hayat, T., Farooq, M., & Alsaedi, A. (2015). Stagnation point flow of carbon nanotubes over stretching cylinder with slip conditions. *Open Physics*, *13*(1), 188-197.

Hayat, T., Hussain, Z., Farooq, M., & Alsaedi, A. (2016). Effects of homogeneous and heterogeneous reactions and melting heat in the viscoelastic fluid flow. *Journal of Molecular Liquids*, *215*, 749-755.

Hayat, T., Hussain, Z., Muhammad, T., &Alsaedi, A. (2016). Effects of homogeneous and heterogeneous reactions in flow of nanofluids over a nonlinear stretching surface with variable surface thickness. *Journal of Molecular Liquids*, *221*, 1121-1127.

Hayat, T., Khan, M. I., Waqas, M., Alsaedi, A., & Farooq, M. (2017). Numerical simulation for melting heat transfer and radiation effects in stagnation point flow of carbon–water nanofluid. *Computer Methods in Applied Mechanics and Engineering, 315*, 1011-1024.

Hayat, T., Muhammad, T., Shehzad, S. A., & Alsaedi, A. (2017). An analytical solution for magnetohydrodynamic Oldroyd-B nanofluid flow induced by a stretching sheet with heat generation/absorption. *International Journal of Thermal Sciences, 111*, 274-288.

Hayat, T., Qayyum, S., Imtiaz, M., & Alsaedi, A. (2016). Comparative study of silver and copper water nanofluids with mixed convection and nonlinear thermal radiation. *International Journal of Heat and Mass Transfer*, *102*, 723-732.

Heydari, A., Akbari, O. A., Safaei, M. R., Derakhshani, M., Alrashed, A. A., Mashayekhi, R., ... & Nguyen, T. K. (2018). The effect of attack angle of triangular ribs on heat transfer of nanofluids in a microchannel. *Journal of Thermal Analysis and Calorimetry, 131*(3), 2893-2912.

Hosseini, S. R., Sheikholeslami, M., Ghasemian, M., & Ganji, D. D. (2018). Nanofluid heat transfer analysis in a microchannel heat sink (MCHS) under the effect of magnetic field by means of KKL model. *Powder Technology, 324*, 36-47.

Hsiao, K. L. (2017). Combined electrical MHD heat transfer thermal extrusion system using Maxwell fluid with radiative and viscous dissipation effects. *Applied Thermal Engineering, 112*, 1281-1288.

Hu, G., Zhao, L., Wu, X., Li, R., Wu, T., Xie, C., ... Cheng, G. (2016). New Fourier-series-based analytical solution to the conduction–convection equation to calculate soil temperature, determine soil thermal properties, or estimate water flux. *International Journal of Heat and Mass Transfer, 95*, 815-823.

Hussain, S. H., & Hussein, A. K. (2011). Mixed convection heat transfer in a differentially heated square enclosure with a conductive rotating circular cylinder at different vertical locations. *International Communications in Heat and Mass Transfer, 38*(2), 263-274.

Hussain, S., & Ahmad, F. (2015). Unsteady MHD flow and heat transfer for Newtonian fluids over an exponentially stretching sheet. *Science International (Lahore), 27*(2), 853-857.

Hussein, A. M., Bakar, R. A., & Kadirgama, K. (2014). Study of forced convection nanofluid heat transfer in the automotive cooling system. *Case Studies in Thermal Engineering, 2*, 50-61.

Ibrahim, W., & Gadisa, G. (2020). Double Stratified Mixed Convective Flow of Couple Stress Nanofluid past Inclined Stretching Cylinder Using Cattaneo-Christov Heat and Mass Flux Model. *Advances in Mathematical Physics*, *2020*.

Ibrahim, W., & Makinde, O. D. (2013). The effect of double stratification on boundary-layer flow and heat transfer of nanofluid over a vertical plate. *Computers & Fluids, 86*, 433-441.

Ibrahim, W., Shankar, B., & Nandeppanavar, M. M. (2013). MHD stagnation point flow and heat transfer due to nanofluid towards a stretching sheet. *International Journal of Heat and Mass Transfer, 56*(1-2), 1-9.

Ijam, A., Saidur, R., Ganesan, P., & Golsheikh, A. M. (2015). Stability, thermo-physical properties, and electrical conductivity of graphene oxide-deionized water/ethylene glycol based nanofluid. *International Journal of Heat and Mass Transfer, 87*, 92-103.

Ijaz, N., Zeeshan, A., Bhatti, M. M., & Ellahi, R. (2018). Analytical study on liquid-solid particles interaction in the presence of heat and mass transfer through a wavy channel. *Journal of Molecular Liquids, 250*, 80-87.

Jafari, M., Farhadi, M., Akbarzade, S., & Ebrahimi, M. (2015). Lattice Boltzmann simulation of natural convection heat transfer of SWCNT-nanofluid in an open enclosure. *Ain Shams Engineering Journal, 6*(3), 913-927.

Kandasamy, R., & Muhammad, R. (2016). Thermal radiation energy on squeezed MHD flow of Cu, Al_2O_3 and CNTs-nanofluid over a sensor surface. *Alexandria Engineering Journal, 55*(3), 2405-2421.

Kandasamy, R., Mohammad, R., & Muhaimin, I. (2016). Carbon nanotubes on unsteady MHD non-Darcy flow over porous wedge in presence of thermal radiation energy. *Applied Mathematics and Mechanics*, *37*(8), 1031-1040.

Karimipour, A., Nezhad, A. H., D'Orazio, A., & Shirani, E. (2012). Investigation of the gravity effects on the mixed convection heat transfer in a microchannel using lattice Boltzmann method. *International Journal of Thermal Sciences, 54*, 142-152.

Kasmani, R. M., Sivasankaran, S., Bhuvaneswari, M., & Siri, Z. (2016). Effect of chemical reaction on convective heat transfer of boundary layer flow in nanofluid over a wedge with heat generation/absorption and suction. *Journal of Applied Fluid Mechanics, 9*(1), 379-388.

Khan, M., & Hamid, A. (2019). Effects of Thermal Radiation and Slip Mechanism on Mixed Convection Flow of Williamson Nanofluid Over an Inclined Stretching Cylinder. *Communications in Theoretical Physics*, *71*(12), 1405.

Khan, M., & Hashim. (2015). Boundary layer flow and heat transfer to Carreau fluid over a nonlinear stretching sheet. *AIP Advances, 5*(10), 107203.

Khan, M., Malik, M. Y., Salahuddin, T., & Khan, F. (2019). Generalized diffusion effects on Maxwell nanofluid stagnation point flow over a stretchable sheet with slip conditions and chemical reaction. *Journal of the Brazilian Society of Mechanical Sciences and Engineering, 41*(3), 1-9.

Khan, M., Salahuddin, T., Tanveer, A., Malik, M. Y., & Hussain, A. (2019). Change in internal energy of thermal diffusion stagnation point Maxwell nanofluid flow along with solar radiation and thermal conductivity. *Chinese Journal of Chemical Engineering, 27*(10), 2352-2358.

Khan, W. A., & Pop, I. (2010). Boundary-layer flow of a nanofluid past a stretching sheet. *International Journal of Heat and Mass Transfer*, *53*(11-12), 2477-2483.

Khan, W. A., Culham, R., & Haq, R. U. (2015). Heat transfer analysis of MHD water functionalized carbon nanotube flow over a static/moving wedge. *Journal of Nanomaterials, 2015*, 1-13.

Kishore, N., & Ramteke, R. R. (2016). Forced convective heat transfer from spheres to Newtonian fluids in steady axisymmetric flow regime with velocity slip at fluid–solid interface. *International Journal of Thermal Sciences, 105*, 206-217.

Kleinstreuer, C., 2018. *Modern fluid dynamics.* CRC Press.

Krishnamurthy, M. R., Prasannakumara, B. C., Gireesha, B. J., & Gorla, R. S. R. (2016). Effect of chemical reaction on MHD boundary layer flow and melting heat transfer of Williamson nanofluid in porous medium. *Engineering Science and Technology, an International Journal, 19*(1), 53-61.

Kumar, S., Roy, S., Paul, C. P., & Nath, A. K. (2008). Three-dimensional conduction heat transfer model for laser cladding process. *Numerical Heat Transfer, Part B: Fundamentals, 53*(3), 271-287.

Kumar, V., Tiwari, A. K., & Ghosh, S. K. (2015). Application of nanofluids in plate heat exchanger: a review. *Energy Conversion and Management*, *105*, 1017-1036.

Li, B. Q. (2006). *Discontinuous finite elements in fluid dynamics and heat transfer.* Springer Science & Business Media.

Li, J., Liu, L., Zheng, L., & Bin-Mohsin, B. (2016). Unsteady MHD flow and radiation heat transfer of nanofluid in a finite thin film with heat generation and thermophoresis. *Journal of the Taiwan Institute of Chemical Engineers, 67*, 226-234.

Liu, J., Ye, Z., Zhang, L., Fang, X., & Zhang, Z. (2015). A combined numerical and experimental study on graphene/ionic liquid nanofluid based direct absorption solar collector. *Solar Energy Materials and Solar Cells, 136*, 177-186.

Liu, Y., Zhang, X., & Lu, M. W. (2005). A meshless method based on least-squares approach for steady-and unsteady-state heat conduction problems. *Numerical Heat Transfer, Part B, 47*(3), 257-275.

Loureiro, F. S., Mansur, W. J., & Vasconcellos, C. A. B. (2009). A hybrid time/Laplace integration method based on numerical Green's functions in conduction heat transfer. *Computer Methods in Applied Mechanics and Engineering, 198*(33-36), 2662-2672.

Mabood, F., Ibrahim, S. M., Kumar, P. V., & Khan, W. A. (2017). Viscous dissipation effects on unsteady mixed convective stagnation point flow using Tiwari-Das nanofluid model. *Results in Physics, 7*, 280-287.

Mabood, F., Khan, W. A., & Rashidi, M. M. (2017). The new analytical study for boundary-layer slip flow and heat transfer of nanofluid over a stretching sheet. *Thermal Science*, *21*(1), 280-301.

Malik, M. Y., Hussain, A., Salahuddin, T., & Awais, M. (2016). Numerical solution of MHD Sisko fluid over a stretching cylinder and heat transfer analysis. *International Journal of Numerical Methods for Heat & Fluid Flow, 26*(6), 1787-1801.

Malvandi, A., Safaei, M. R., Kaffash, M. H., & Ganji, D. D. (2015). MHD mixed convection in a vertical annulus filled with Al2O3–water nanofluid considering nanoparticle migration. *Journal of Magnetism and Magnetic Materials, 382*, 296-306.

Mamourian, M., Shirvan, K. M., Ellahi, R., & Rahimi, A. B. (2016). Optimization of mixed convection heat transfer with entropy generation in a wavy surface square lid-driven cavity by means of Taguchi approach. *International Journal of Heat and Mass Transfer, 102*, 544-554.

Mehrali, M., Sadeghinezhad, E., Akhiani, A. R., Latibari, S. T., Talebian, S., Dolatshahi-Pirouz, A., ... Mehrali, M. (2016). An ecofriendly graphene-based nanofluid for heat transfer applications. *Journal of Cleaner Production*, *137*, 555-566.

Mehrali, M., Sadeghinezhad, E., Rosen, M. A., Akhiani, A. R., Latibari, S. T., Mehrali, M., & Metselaar, H. S. C. (2015). Heat transfer and entropy generation for laminar forced convection flow of graphene nanoplatelets nanofluids in a horizontal tube. *International Communications in Heat and Mass Transfer, 66*, 23-31.

Menon, E. S. (2015). Fluid flow in pipes. Transmission Pipeline Calculations and Simulations Manual, 149-234.

Mishra, K. K., Ghosh, S., Ravindran, T. R., Amirthapandian, S., & Kamruddin, M. (2016). Thermal conductivity and pressure-dependent Raman studies of vertical graphene nanosheets. *The Journal of Physical Chemistry C, 120*(43), 25092-25100.

Mohamed, K., Mohamed Rafik, S., Rabah, B. O. U. R. E. N. A. N. E., Rashidi, M. M., & Ammar, H. A. I. A. H. E. M. (2018). Heat transfer in hydro-magnetic nano-fluid flow between non-parallel plates using DTM. *Journal of Applied and Computational Mechanics, 4*(4), 352-364.

Mohammadein, S. A., Raslan, K., Abdel-Wahed, M. S., & Abedel-Aal, E. M. (2018). KKL-model of MHD CuO-nanofluid flow over a stagnation point stretching sheet with nonlinear thermal radiation and suction/injection. *Results in Physics, 10*, 194-199.

Mohammed, H. A., Gunnasegaran, P., & Shuaib, N. H. (2011). Influence of various base nanofluids and substrate materials on heat transfer in trapezoidal microchannel heat sinks. *International Communications in Heat and Mass Transfer, 38*(2), 194-201.

Mushtaq, A., Mustafa, M., Hayat, T., & Alsaedi, A. (2014). On the numerical solution of the nonlinear radiation heat transfer problem in a three-dimensional flow. *Zeitschrift für Naturforschung A, 69*(12), 705-713.

Mustafa, M., Mushtaq, A., Hayat, T., & Alsaedi, A. (2015). Model to study the non-linear radiation heat transfer in the stagnation-point flow of power-law fluid. *International Journal of Numerical Methods for Heat & Fluid Flow, 25*(5), 1107-1119.

Mythili, D., & Sivaraj, R. (2016). Influence of higher order chemical reaction and non-uniform heat source/sink on Casson fluid flow over a vertical cone and flat plate. *Journal of Molecular Liquids, 216*, 466-475.

Nadeem, S., & Abbas, N. (2019). On both MHD and slip effect in Micropolar Hybrid nanofluid past a circular cylinder under stagnation point region. *Canadian Journal of Physics, 97*(4), 392-399.

Naseer, M., Malik, M. Y., Nadeem, S., & Rehman, A. (2014). The boundary layer flow of hyperbolic tangent fluid over a vertical exponentially stretching cylinder. *Alexandria Engineering Journal*, *53*(3), 747-750.

Nazari, M. A., Ghasempour, R., Ahmadi, M. H., Heydarian, G., & Shafii, M. B. (2018). Experimental investigation of graphene oxide nanofluid on heat transfer enhancement of pulsating heat pipe. *International Communications in Heat and Mass Transfer, 91*, 90-94.

Novoselov, K. S., Fal, V. I., Colombo, P. R., Schwab, M. G., & Kim, K. (2012). A roadmap for graphene. *Nature, 490*(7419), 192-200.

Oguntala, G., Sobamowo, G., Abd-Alhameed, R., & Noras, J. (2019). Numerical Investigation of Inclination on the Thermal Performance of Porous Fin Heatsink using Pseudospectral Collocation Method. *Karbala International Journal of Modern Science, 5*(1), 19-26.

Oster, K., Goodrich, P., Jacquemin, J., Hardacre, C., Ribeiro, A. P. C., & Elsinawi, A. (2018). A new insight into pure and water-saturated quaternary phosphonium-based carboxylate ionic liquids: Density, heat capacity, ionic conductivity, thermogravimetric analysis, thermal conductivity and viscosity. *The Journal of Chemical Thermodynamics, 121*, 97-111.

Oueslati, F. S., & Bennacer, R. (2011). Heterogeneous nanofluids: natural convection heat transfer enhancement. *Nanoscale Research Letters, 6*(1), 1-11.

Oztop, H. F., & Abu-Nada, E. (2008). Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *International Journal of Heat and Fluid Flow*, *29*(5), 1326-1336.

Pandey, A. K., & Kumar, M. (2016). Effect of viscous dissipation and suction/injection on MHD nanofluid flow over a wedge with porous medium and slip. *Alexandria Engineering Journal*, *55*(4), 3115-3123.

Pandey, A. K., & Kumar, M. (2018). MHD flow inside a stretching/shrinking convergent/divergent channel with heat generation/absorption and viscous-ohmic dissipation utilizing Cu−water nanofluid. *Computational Thermal Sciences: An International Journal*, *10*(05), 457-471.

Patil, R. H. (2017). Experimental studies on heat transfer to Newtonian fluids through spiral coils. *Experimental Thermal and Fluid Science, 84*, 144-155.

Patkavak, M., & Demir, H. (2020). An experimental study for determination of convective heat transfer coefficients on cooling monofilament. *International Journal of Heat and Mass Transfer, 149*, 1-9.

Pătrulescu, F. O., Groşan, T., & Pop, I. (2014). Mixed convection boundary layer flow from a vertical truncated cone in a nanofluid. *International Journal of Numerical Methods for Heat & Fluid Flow, 24*(5), 1175-1990.

Perreault, F., De Faria, A. F., & Elimelech, M. (2015). Environmental applications of graphene-based nanomaterials. *Chemical Society Reviews*, *44*(16), 5861-5896.

Qayyum, S., Khan, M. I., Hayat, T., & Alsaedi, A. (2017). A framework for nonlinear thermal radiation and homogeneous-heterogeneous reactions flow based on silver-water and copper-water nanoparticles: A numerical model for probable error. *Results in Physics, 7*, 1907-1914.

Rahimi-Gorji, M., Pourmehran, O., Gorji-Bandpy, M., & Ganji, D. D. (2016). Unsteady squeezing nanofluid simulation and investigation of its effect on important heat transfer parameters in presence of magnetic field. *Journal of the Taiwan Institute of Chemical Engineers, 67*, 467-475.

Raju, C. S., & Sandeep, N. (2016). Heat and mass transfer in 3D non-Newtonian nano and ferro fluids over a bidirectional stretching surface. *In International Journal of Engineering Research in Africa* (Vol. 21, pp. 33-51). Trans Tech Publications Ltd.

Ramzan, M., Bilal, M., & Chung, J. D. (2017). Effects of thermal and solutal stratification on Jeffrey magneto-nanofluid along an inclined stretching cylinder with thermal radiation and heat generation/absorption. *International Journal of Mechanical Sciences*, *131*, 317-324.

Reddy, P. S., Sreedevi, P., & Chamkha, A. J. (2017). MHD boundary layer flow, heat and mass transfer analysis over a rotating disk through porous medium saturated by Cu-water and Ag-water nanofluid with chemical reaction. *Powder Technology, 307*, 46-55.

Reiss, H. (2012). Radiation heat transfer and its impact on stability against quench of a superconductor. *Journal of Superconductivity and Novel Magnetism, 25*(2), 339-350.

Robinson, J. T., Perkins, F. K., Snow, E. S., Wei, Z., & Sheehan, P. E. (2008). Reduced graphene oxide molecular sensors. *Nano Letters*, *8*(10), 3137-3140.

Rostami, M. N., Dinarvand, S., & Pop, I. (2018). Dual solutions for mixed convective stagnation-point flow of an aqueous silica–alumina hybrid nanofluid. *Chinese Journal of Physics, 56*(5), 2465-2478.

Sadat, H., Dubus, N., Gbahoue, L., & Sophy, T. (2006). On the solution of heterogeneous heat conduction problems by a diffuse approximation meshless method. *Numerical Heat Transfer, Part B: Fundamentals, 50*(6), 491-498.

Sadeghinezhad, E., Mehrali, M., Saidur, R., Mehrali, M., Latibari, S. T., Akhiani, A. R., & Metselaar, H. S. C. (2016). A comprehensive review on graphene nanofluids: recent research, development and applications. *Energy Conversion and Management*, *111*, 466-487.

Saedodin, S., Yaghoobi, H., & Torabi, M. (2011). Application of the variational iteration method to nonlinear non-Fourier conduction heat transfer equation with variable coefficient. *Heat Transfer—Asian Research, 40*(6), 513-523.

Sajjad, M., Kamran, M. S., Shaukat, R., & Zeinelabdeen, M. I. M. (2018). Numerical investigation of laminar convective heat transfer of graphene oxide/ethylene glycol-water nanofluids in a horizontal tube. *Engineering Science and Technology, An International Journal*, *21*(4), 727-735.

Sakurai, A., Mishra, S. C., & Maruyama, S. (2010). Radiation element method coupled with the lattice Boltzmann method applied to the analysis of transient conduction and radiation heat transfer problem with heat generation in a participating medium. *Numerical Heat Transfer, Part A: Applications, 57*(5), 346-368.

Salleh, M. Z., Nazar, R., & Pop, I. (2010). Boundary layer flow and heat transfer over a stretching sheet with Newtonian heating. *Journal of the Taiwan Institute of Chemical Engineers, 41*(6), 651-655.

Sandeep, N., & Sulochana, C. (2018). Momentum and heat transfer behaviour of Jeffrey, Maxwell and Oldroyd-B nanofluids past a stretching surface with non-uniform heat source/sink. *Ain Shams Engineering Journal, 9*(4), 517-524.

Sandeep, N., Chamkha, A. J., & Animasaun, I. L. (2017). Numerical exploration of magnetohydrodynamic nanofluid flow suspended with magnetite nanoparticles. *Journal of the Brazilian Society of Mechanical Sciences and Engineering, 39*(9), 3635-3644.

Sandeep, N., Sharma, R. P., & Ferdows, M. (2017). Enhanced heat transfer in unsteady magnetohydrodynamic nanofluid flow embedded with aluminum alloy nanoparticles. *Journal of Molecular Liquids, 234*, 437-443.

Selimefendigil, F., & Chamkha, A. J. (2019). Magnetohydrodynamics mixed convection in a power law nanofluid-filled triangular cavity with an opening using Tiwari and Das' nanofluid model. *Journal of Thermal Analysis and Calorimetry, 135*(1), 419-436.

Shampine, L.F., & J. Kierzenka. "A BVP Solver based on residual control and the MATLAB PSE." *ACM Trans. Math. Softw.* Vol. 27, Number 3, 2001, pp. 299–316.

Shampine, L.F., M.W. Reichelt, & J. Kierzenka. "Solving Boundary Value Problems for Ordinary Differential Equations in MATLAB with bvp4c." *MATLAB File Exchange*, 2004.

Shateyi, S., & Prakash, J. (2014). A new numerical approach for MHD laminar boundary layer flow and heat transfer of nanofluids over a moving surface in the presence of thermal radiation. *Boundary value problems, 2014*(1), 2.

Sheikholeslami, M. (2017). Magnetohydrodynamic nanofluid forced convection in a porous lid driven cubic cavity using Lattice Boltzmann method. *Journal of Molecular Liquids, 231*, 555-565.

Sheikholeslami, M., & Bhatti, M. M. (2017). Forced convection of nanofluid in presence of constant magnetic field considering shape effects of nanoparticles. *International Journal of Heat and Mass Transfer, 111*, 1039-1049.

Sheikholeslami, M., & Chamkha, A. J. (2016). Electrohydrodynamic free convection heat transfer of a nanofluid in a semi-annulus enclosure with a sinusoidal wall. *Numerical Heat Transfer, Part A: Applications, 69*(7), 781-793.

Sheikholeslami, M., & Shehzad, S. A. (2017). Magnetohydrodynamic nanofluid convection in a porous enclosure considering heat flux boundary condition. *International Journal of Heat and Mass Transfer, 106*, 1261-1269.

Sheikholeslami, M., & Zeeshan, A. J. C. M. I. A. M. (2017). Analysis of flow and heat transfer in water based nanofluid due to magnetic field in a porous enclosure with constant heat flux using CVFEM. *Computer Methods in Applied Mechanics and Engineering, 320*, 68-81.

Sheikholeslami, M., Gorji-Bandpy, M., Ganji, D. D., Rana, P., & Soleimani, S. (2014). Magnetohydrodynamic free convection of Al_2O_3–water nanofluid considering thermophoresis and brownian motion effects. *Computers & Fluids, 94*, 147-160.

Sheikholeslami, M., Haq, R. U., Shafee, A., & Li, Z. (2019). Heat transfer behavior of nanoparticle enhanced PCM solidification through an enclosure with V shaped fins. *International Journal of Heat and Mass Transfer, 130*, 1322-1342.

Sheikholeslami, M., Rashidi, M. M., & Ganji, D. D. (2015). Effect of non-uniform magnetic field on forced convection heat transfer of Fe_3O_4–water nanofluid. *Computer Methods in Applied Mechanics and Engineering, 294*, 299-312.

Sheikholeslami, M., Rashidi, M. M., Hayat, T., & Ganji, D. D. (2016). Free convection of magnetic nanofluid considering MFD viscosity effect. *Journal of Molecular Liquids, 218,* 393-399.

Sheikholeslami, M., Shehzad, S. A., & Li, Z. (2018). Water based nanofluid free convection heat transfer in a three-dimensional porous cavity with hot sphere obstacle in existence of Lorenz forces. *International Journal of Heat and Mass Transfer, 125,* 375-386.

Sheikholeslami, M., Vajravelu, K., & Rashidi, M. M. (2016). Forced convection heat transfer in a semi annulus under the influence of a variable magnetic field. *International Journal of Heat and Mass Transfer, 92*, 339-348.

Sheremet, M. A., Grosan, T., & Pop, I. (2015). Free convection in a square cavity filled with a porous medium saturated by nanofluid using Tiwari and Das' nanofluid model. *Transport in Porous Media, 106*(3), 595-610.

Shirvan, K. M., Mamourian, M., Mirzakhanlari, S., & Ellahi, R. (2017). Numerical investigation of heat exchanger effectiveness in a double pipe heat exchanger filled with nanofluid: a sensitivity analysis by response surface methodology. *Powder Technology, 313*, 99-111.

Siddiqa, S., Begum, N., Hossain, M. A., & Gorla, R. S. R. (2017). Natural convection flow of a two-phase dusty non-Newtonian fluid along a vertical surface. *International Journal of Heat and Mass Transfer, 113*, 482-489.

Souayeh, B., Reddy, M. G., Sreenivasulu, P., Poornima, T., Rahimi-Gorji, M., & Alarifi, I. M. (2019). Comparative analysis on non-linear radiative heat transfer on MHD Casson nanofluid past a thin needle. *Journal of Molecular Liquids, 284*, 163-174.

Subhani, M., & Nadeem, S. (2019). Numerical investigation into unsteady magnetohydrodynamics flow of micropolar hybrid nanofluid in porous medium. *Physica Scripta, 94*(10), 1-11.

Sucharitha, G., Lakshminarayana, P., & Sandeep, N. (2017). Joule heating and wall flexibility effects on the peristaltic flow of magnetohydrodynamic nanofluid. *International Journal of Mechanical Sciences, 131*, 52-62.

Sui, J., Zheng, L., Zhang, X., & Chen, G. (2015). Mixed convection heat transfer in power law fluids over a moving conveyor along an inclined plate. *International Journal of Heat and Mass Transfer, 85*, 1023-1033.

Sui, Y., Teo, C. J., Lee, P. S., Chew, Y. T., & Shu, C. (2010). Fluid flow and heat transfer in wavy microchannels. *International Journal of Heat and Mass Transfer, 53*(13-14), 2760-2772.

Sun, Q., & Pop, I. (2011). Free convection in a triangle cavity filled with a porous medium saturated with nanofluids with flush mounted heater on the wall. *International Journal of Thermal Sciences, 50*(11), 2141-2153.

Sundar, L. S., Naik, M. T., Sharma, K. V., Singh, M. K., & Reddy, T. C. S. (2012). Experimental investigation of forced convection heat transfer and friction factor in a tube with Fe_3O_4 magnetic nanofluid. *Experimental Thermal and Fluid Science, 37*, 65-71.

Tawfik, M. H. M., Diab, M. R., & Abdelmotalib, H. M. (2020). Heat transfer and hydrodynamics of particles mixture in swirling fluidized bed. *International Journal of Thermal Sciences, 147*, 1-8.

Tippa, S., Narahari, M., & Pendyala, R. (2016). Unsteady natural convection flow of nanofluids past a semi-infinite isothermal vertical plate. *In AIP Conference Proceedings* (Vol. 1787, No. 1, p. 020014). AIP Publishing LLC.

Tiwari, R. K., & Das, M. K. (2007). Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *International Journal of heat and Mass transfer*, *50*(9-10), 2002-2018.

Tlili, I., Khan, W. A., & Ramadan, K. (2019). MHD flow of nanofluid flow across horizontal circular cylinder: steady forced convection. *Journal of Nanofluids, 8*(1), 179-186.

Torkaman, S., Heydari, M., Loghmani, G. B., & Ganji, D. D. (2020). Barycentric rational interpolation method for numerical investigation of magnetohydrodynamics nanofluid flow and heat transfer in nonparallel plates with thermal radiation. *Heat Transfer—Asian Research, 49*(1), 565-590.

Tsay, C., Pattison, R. C., & Baldea, M. (2017). Equation-oriented simulation and optimization of process flowsheets incorporating detailed spiral-wound multistream heat exchanger models. *AIChE Journal, 63*(9), 3778-3789.

Turkyilmazoglu, M. (2013). The analytical solution of mixed convection heat transfer and fluid flow of an MHD viscoelastic fluid over a permeable stretching surface. *International Journal of Mechanical Sciences, 77*, 263-268.

Upadhya, S. M., Raju, C. S. K., Saleem, S., & Alderremy, A. A. (2018). Modified Fourier heat flux on MHD flow over stretched cylinder filled with dust, graphene and silver nanoparticles. *Results in Physics*, *9*, 1377-1385.

Usman, M., Hamid, M., Haq, R. U., & Wang, W. (2018). Heat and fluid flow of water and ethylene-glycol based Cu-nanoparticles between two parallel squeezing porous disks: LSGM approach. *International Journal of Heat and Mass Transfer, 123*, 888-895.

Usman, M., Soomro, F. A., Haq, R. U., Wang, W., & Defterli, O. (2018). Thermal and velocity slip effects on Casson nanofluid flow over an inclined permeable stretching cylinder via collocation method. *International Journal of Heat and Mass Transfer*, *122*, 1255-1263.

Vasile, V., Necula, H., Badea, A., Revellin, R., Bonjour, J., & Haberschill, P. (2018). Experimental study of the heat transfer characteristics of a paraffin-in-water emulsion used as a secondary refrigerant. *International Journal of Refrigeration, 88*, 1-7.

Vo, D. D., Shah, Z., Sheikholeslami, M., Shafee, A., & Nguyen, T. K. (2019). Numerical investigation of MHD nanomaterial convective migration and heat transfer within a sinusoidal porous cavity. *Physica Scripta, 94*(11), 1-10.

Walelign, T., Haile, E., Kebede, T., & Walelgn, A. (2020). Analytical study of heat and mass transfer in MHD flow of chemically reactive and thermally radiative Casson nanofluid over an inclined stretching cylinder. *Journal of Physics Communications*, *4*(12), Article#125003.

Wang, X., He, Y., Chen, M., & Hu, Y. (2018). ZnO-Au composite hierarchical particles dispersed oil-based nanofluids for direct absorption solar collectors. *Solar Energy Materials and Solar Cells, 179*, 185-193.

Wetzel, T., & Boeckh, P. V. (2012). *Heat transfer:basics and practice.* Springer.

Yamaguchi, H. (2008). *Engineering fluid mechanics (Vol. 85).* Springer Science & Business Media.

Yang, B. (2008). A distributed transfer function method for heat conduction problems in multilayer composites. *Numerical Heat Transfer, Part B: Fundamentals, 54*(4), 314-337.

Yang, Y., Asiri, A. M., Tang, Z., Du, D., & Lin, Y. (2013). Graphene based materials for biomedical applications. *Materials Today, 16*(10), 365-373.

Yarmand, H., Gharehkhani, S., Shirazi, S. F. S., Amiri, A., Alehashem, M. S., Dahari, M., & Kazi, S. N. (2016). Experimental investigation of thermo-physical properties, convective heat transfer and pressure drop of functionalized graphene nanoplatelets aqueous nanofluid in a square heated pipe. *Energy Conversion and Management, 114*, 38-49.

Zamzamian, A., Oskouie, S. N., Doosthoseini, A., Joneidi, A., & Pazouki, M. (2011). Experimental investigation of forced convective heat transfer coefficient in nanofluids of Al_2O_3/EG and CuO/EG in a double pipe and plate heat exchangers under turbulent flow. *Experimental Thermal and Fluid Science, 35*(3), 495-502.

Zhang, C., Zheng, L., Zhang, X., & Chen, G. (2015). MHD flow and radiation heat transfer of nanofluids in porous media with variable surface heat flux and chemical reaction. *Applied Mathematical Modelling, 39*(1), 165-181.

Zheng, L., Zhang, C., Zhang, X., & Zhang, J. (2013). Flow and radiation heat transfer of a nanofluid over a stretching sheet with velocity slip and temperature jump in porous medium. *Journal of the Franklin Institute, 350*(5), 990-1007.

Zou, D., Ma, X., Liu, X., Zheng, P., & Hu, Y. (2018). Thermal performance enhancement of composite phase change materials (PCM) using graphene and carbon nanotubes as additives for the potential application in lithium-ion power battery. *International Journal of Heat and Mass Transfer, 120*, 33-41.