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
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UNDERSTANDING OF THE LIMIT OF FUNCTIONS
AMONG COLLEGE STUDENTS

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ABSTRACT

This study explored concept images and concept definitions of college students on limits of functions. Their views concerning the attainability of the limit and the extent to which they understood the stand taken by the formal view on this matter were also probed on.

The sample comprised 10 students who had completed the Calculus and Analytic Geometry 1 course in the School of American University Studies at a private college in the Klang Valley. A questionnaire on beliefs about limits was administered to them, and 4 students were then selected for a task-based interview on the attainability of the limit.

The students exhibited a variety of concept images on limits. Four students described limits in terms of the dynamic view and related limits to determining the slope of a tangent to a point on a curve. Only 1 student included the formal definition. This indicated that the formal definition was not a dominant aspect of the students' concept image on limits. Two students produced a correct concept definition on the limit of a function, implying that many of these students had not understood the formal definition. Majority of the students agreed to both the informal views and formal view on limits. During the interviews, 2 students had related attainability of the limit to continuous functions, and 2 other students believed that the limit was unattainable. For the latter 2, one student viewed constant functions as an exception to the rule. The other student insisted that constant functions were no exception because limits pertained to a point of interest. He also believed that although continuous functions attained their limits, the

process of approaching the limit was unrelated to the situation in which the function attained its limit. His view contradicted the concept of continuity.

The findings imply that these students were unaware of the inconsistent and conflicting aspects of their concept images which were at variance with the formal view on the limits of functions. Alternative pedagogical strategies were proposed by considering the opportunity for students to explore the meanings of words used in communicating the limit concept, exposing them to the historical development of the limit concept, and discussing the attainability of the limit within the concept of continuity.

PEMAHAMAN HAD FUNGSI DI KALANGAN PELAJAR KOLEJ

ABSTRAK

Kajian ini bertujuan menerokai imej konsep dan definisi konsep pelajar-pelajar kolej tentang had fungsi. Pelajar-pelajar juga ditemuduga tentang isu pencapaian had fungsi dan pemahaman mereka tentang pandangan formal terhadap isu ini.

Sampel kajian terdiri daripada 10 orang pelajar yang telah mengambil kursus Kalkulus dan Analitik Geometri 1. Mereka mengikuti program pemindahan kredit ke Amerika Syarikat di sebuah kolej swasta di Lembah Kelang. Semua pelajar menjawab soal-selidik tentang kepercayaan pelajar terhadap had fungsi manakala 4 orang pelajar dipilih untuk ditemuduga tentang isu pencapaian had.

Pelajar-pelajar mempunyai pelbagai imej konsep tentang had fungsi. Empat orang pelajar memerihalkan had fungsi dari segi pandangan dinamik dan penentuan kecerunan tangen pada sesuatu lengkungan. Seorang pelajar sahaja menerangkan had fungsi berasaskan pandangan formal. Ini menunjukkan bahawa pandangan formal tidak merupakan aspek dominan dalam imej konsep had fungsi sampel kajian. Hanya 2 orang pelajar mendefinisikan had fungsi secara betul. Ini menunjukkan bahawa sebahagian besar sampel kajian tidak memahami definisi ini. Majoriti pelajar bersetuju dengan pandangan-pandangan bukan formal serta pandangan formal tentang had fungsi. Semasa temuduga dijalankan, 2 orang pelajar mengaitkan pencapaian had fungsi dengan fungsi-fungsi selanjur, manakala 2 orang pelajar lain berpendapat bahawa had fungsi tidak boleh dicapai. Di antara 2 orang pelajar tersebut, seorang pelajar berpendapat hanya fungsi malar dapat mencapai hadnya. Seorang lagi berpendapat fungsi malar tidak dapat

mencapai hadnya kerana pencapaian had berkait dengan nilai x tertentu. Dia bersetuju bahawa fungsi selanjar dapat mencapai hadnya, tetapi proses dalam mana fungsi menghampiri had tidak berkaitan dengan situasi di mana fungsi mencapai hadnya. Pandangan ini berkontradiksi dengan konsep keselajaran.

Dapatan kajian mengimpikasikan bahawa pelajar-pelajar tidak sedar tentang kehadiran konflik dan ketidaksetaraan di dalam imej konsep mereka, serta kelainan imej konsep mereka dengan pandangan formal tentang had fungsi. Beberapa pendekatan alternatif dari segi pedagogi dicadangkan. Pelajar boleh diberi peluang untuk menerokai pengertian perkataan-perkataan tertentu yang digunakan untuk mengkomunikasikan konsep had fungsi, meneliti sejarah perkembangan konsep had fungsi, dan membincangkan isu pencapaian had fungsi dalam konteks keselajaran.

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