EFFECTIVENESS OF VIRTUAL MANIPULATIVE TEACHING METHOD IN ENHANCING YEAR FOUR PUPILS' UNDERSTANDING OF FRACTIONS

KAVITHA KARBAL SINGH

**FACULTY OF EDUCATION
UNIVERSITI MALAYA
KUALA LUMPUR**

2020

EFFECTIVENESS OF VIRTUAL MANIPULATIVE TEACHING METHOD IN ENHANCING YEAR FOUR PUPILS' UNDERSTANDING OF FRACTIONS

KAVITHA KARBAL SINGH

DISSERTATION SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF MATHEMATICS EDUCATION WITH INFORMATION COMMUNICATION AND TECHNOLOGY

FACULTY OF EDUCATION
UNIVERSITY OF MALAYA
KUALA LUMPUR

2020

UNIVERSITY OF MALAYA
ORIGINAL LITERARY WORK DECLARATION

Name of Candidate: Kavitha Karbal Singh
Matric No: PGG 140004
Name of Degree: Master of Education (Mixed Mode)
Title of Dissertation: Effectiveness of Virtual Manipulative Teaching Method In Enhancing Year Four Pupils' Understanding of Fractions

Field of Study: Master of Mathematics Education with Information Communication and Technology

I do solemnly and sincerely declare that:

(1) I am the sole author/writer of this Work;
(2) This Work is original;
(3) Any use of any work in which copyright exists was done by way of fair dealing and for permitted purposes and any excerpt or extract from, or reference to or reproduction of any copyright work has been disclosed expressly and sufficiently and the title of the Work and its authorship have been acknowledged in this Work;
(4) I do not have any actual knowledge nor do I ought reasonably to know that the making of this work constitutes an infringement of any copyright work;
(5) I hereby assign all and every rights in the copyright to this Work to the University of Malaya ("UM"), who henceforth shall be owner of the copyright in this Work and that any reproduction or use in any form or by any means whatsoever is prohibited without the written consent of UM having been first had and obtained;
(6) I am fully aware that if in the course of making this Work I have infringed any copyright whether intentionally or otherwise, I may be subject to legal action or any other action as may be determined by UM.

Candidate's Signature Date:

Subscribed and solemnly declared before,

Witness's Signature Date:

Name:

Designation:

ABSTRACT

Fractions are common numerical representations in mathematics. However, they seem to be a complicated mathematical concept to master, particularly among school pupils. Pupils have a hard time studying fraction due to insufficient understanding of the concept. An effective teaching method is therefore essential for the teaching and learning of this particular component. The purpose of this study is to determine the effectiveness of the virtual manipulative teaching method in improving the understanding of fraction among Primary Year Four pupils. Essentially a quasi-experimental research, the method used was a non-equivalent, control group pretest-posttest approach. A total of 80 pupils from a government primary school in Selangor were categorized into two groups in which the first group was the experimental group (N = 40) and second was the control group (N = 40). The experimental group was taught the concept of fraction using the virtual manipulative method, while the control group was taught the concept of fraction without using the method. Both groups were tested before and after the experiment using the Test for Understanding of Concept of Fraction. The result of the paired t-test of the experimental group for pre and posttest scores shows that the pupils' understanding of fraction increased significantly after the experiment i.e. $t(39) = 18.14$, $p < .0005$. Whereas, the independent posttest mean experimental score i.e. $M = 18.83$, $SD = 2.872$ and control group i.e. $M = 14.18$, $SD=4.242$ shows that the pupils' understanding the concept of fraction is noticeably different from the control group i.e. $t(78) = -5.79$, $p < .05$. Thus, it has been concluded that the virtual manipulative method is successful in improving Year Four pupils' understanding of the concept of fraction.

KEBERKESANAN KAEDAH PENGAJARAN MANIPULATIF MAYA DALAM MENINGKATKAN PEMAHAMAN PECAHAN MURID TAHUN EMPAT

ABSTRAK

Pecahan adalah perwakilan numerik biasa dalam matematik. Walau bagaimanapun, konsep matematik ini dilihat sangat rumit untuk dikuasai terutama bagi murid sekolah rendah. Murid- murid mengalami kesukaran untuk mempelajari pecahan kerana tidak memahami konsep dan tidak dapat menguasai konsep pecahan tersebut. Oleh yang demikian, kaedah pengajaran yang berkesan sangat penting dalam melaksanakan sesi pengajaran dan pembelajaran konsep pecahan ini. Kajian ini bertujuan untuk menentukan keberkesanan kaedah pengajaran manipulatif maya ke arah peningkatan pemahaman pecahan murid tahun empat. Reka bentuk kuasi eksperimen dengan ujian pra dan pasca telah digunakan. Seramai 80 murid sekolah rendah kerajaan di Selangor telah mengambil bahagian dalam kajian ini. Mereka dikategorikan kepada dua kumpulan, di mana kumpulan pertama dikumpulkan sebagai kumpulan eksperimen (N = 40) dan yang lain sebagai kumpulan kawalan (N = 40). Kumpulan eksperimen telah ditugaskan untul menjalani rawatan dengan diajar tentang konsep pecahan menggunakan manipulatif maya dan kumpulan kawalan diajar tentang konsep pecahan tanpa menggunakan manipulatif maya. Kedua-dua kumpulan telah diuji sebelum dan selepas eksperimen menggunakan Ujian Pemahaman Konsep Pecahan. Hasil daripada ujian t-berpasangan kumpulan eksperimen untuk skor pra dan pasca menunjukkan bahawa pemahaman murid tentang konsep pecahan meningkat dengan cara signifikan selepas eksperimen, $t(39) = 18.14$, $p < .0005$. Di samping itu, hasil analisis ujian t-tak bersandar menunjukkan bahawa min skor ujian pasca bagi kumpulan eksperimen ialah $M = 18.83$, $SD =$

2.872 dan kumpulan kawalan pula M = 14.18, SD = 4.242, ini menunjukkan bahawa pemahaman konsep pecahan murid kumpulan eksperimen adalah signifikan berbeza berbanding kumpulan kawalan selepas eksperimen, $t (78) = -5.79$, $p < .05$. Dapat di simpulkan bahawa, penggunaan manipulatif maya dalam mengajar konsep pecahan berkesan dalam meningkatkan pemahaman konsep pecahan di kalangan murid tahun empat.

ACKNOWLEGEMENT

First of all, I would like to thank to God for giving me the blessing to complete this research. It gives me immense pleasure to acknowledge my parents, Mr. Karbal Singh and Mrs. Santanamary, husband Mr. Magendran and in-law Madam Letchumy whose love, persistence support and understanding has enhanced my determination to complete this journey. Without my family supports, the journey would have been more difficult for me. I'm extremely grateful to my supervisor, Mr. Norjoharuddeen Bin Mohr Nor for his guidance, encouragement and valuable discussions of my thesis. Besides my supervisor, I would like to thank Dr. Hutkemri and Prof Madya Datin Dr Sharifah Norul Akmar bt Syed Zamri for their suggestions, insightful comments, and encouragements. Their challenging questions brought this work towards a completion. Finally, I would like to thank my dear friends and course mates Devi, Nandini, Pavithra, Usha, Priya and Kelly in University Malaya for their moral support, and motivation in completing my thesis. It's difficult to carry out this research without the generous help I received from a number of kind people.

CONTENTS

Chapter 3: Methodology

Chapter 4 : Findings

Chapter 5 : Discussion and Conclusion

LIST OF FIGURES

LIST OF TABLES

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Fraction is one of the most important topics in school curriculum and, in particular, a central component of mathematics. It is not only the basic foundation for a broad range of mathematical principles, but also an important feature of other subjects. The use of fractions goes beyond the classroom, covering a variety of fields including medical services, skilled occupations, such as craftsmen, and elementary occupations (Davidson, 2012; Jordan, Resnick, Rodrigues, Hansen, & Dyson, 2017; Lortie-Forgues, Tian, & Siegler, 2015; Siegler et al., 2012). Fractions are also an integral component of daily routines such as cooking, carpentry, sports and sewing. According to a survey conducted by Handel (2016), almost 68% of American employees in the white and blue collar sectors, as well as service workers, utilized fractions and decimals in their work. Its broad range of applications in everyday life makes it necessary to learn fractions as early as elementary schools.

In general, the Mathematics curriculum for Malaysian primary schools has been regularly revised to make mathematics more challenging from one level to the next. This is evident when the Mathematics syllabus for the Primary Integrated Curriculum (KBSR) and Primary Standard Curriculum (KSSR) are compared. The contents of KSSR are more complex, requiring pupils to build and possess adequate conceptual knowledge and skills. The most obvious examples can be found in the topic of fractions, where the concepts of one half, one-quarter, two-quarter, and three-quarter were introduced to Year One pupils in the KSSR Mathematics curriculum (Kementerian Pendidikan Malaysia 2015). By comparison, these were incorporated in the Year Three Mathematics curriculum of the KBSR (Ministry of

Education Malaysia, 2003). Since fractions are considered central to the comprehension of basic mathematical concepts, such as decimal, percentage, algebra, ratio and proportion, it is undoubtedly an important topic for pupils to understand with sufficient depth (Lamon, 2013; Panel, 2008; Powell, Fuchs, & Fuchs, 2013; Reeder, 2017; Siegler, Thompson, & Schneider, 2011).

In order to learn fraction and the procedures involved, the most important task is to comprehend the concept of fraction (Hallett, Nunes, & Bryant, 2010; Hecht & Vagi, 2010; Seethaler, Fuchs, Star, & Bryant, 2011; Siegler et al., 2011; Vamvakoussi & Vosniadou, 2010). Pupils need to develop a strong understanding of the concept of fraction to be able to perform complex operations as they transition to higher level mathematics. A Braithwaite, Pyke, and Siegler (2017) and Bruce, Chang, Flynn, and Yearley (2013) believe that the limited understanding of the concept by pupils poses a major obstacle in learning advanced mathematics.

In fact, Hallett et al. (2010) suggests that pupils who understand the concepts of fraction and its arithmetic principles are more valuable than pupils who simply memorize procedures. Ultimately, a high level of understanding of the concept will enable pupils to perform well in their mathematics examination. An in-depth understanding of the concept of fraction will allow them to explain a solution and not simply perform computing. For example, pupils should be able to explain that the simplified form of $\frac{6}{8}$ is $\frac{3}{4}$ through diagrams, number line and equivalencies (Van de Walle, Karp, Bay-Williams, Wray, & Brown, 2016).

The KSSR Mathematics curriculum therefore emphasizes understanding-based learning in order to allow pupils comprehend the concepts, principles and procedures involved in performing fractions. According to Rittle-Johnson and Schneider (2014), the effectiveness of teaching conceptual understanding will help to improve pupils' computational skills in relation to fractions. The emphasis on an effective learning environment is incorporated into the education system to encourage pupils to acquire knowledge through comprehension. Known as constructivism, this approach ensures that pupils gain an in-depth understanding of the concepts of fraction.

Constructivism is a theory that describes the learning process through knowledge construction. Pupils may use the approach either through interaction with the environment or by dynamic participation in the learning process. Knowledge construction is considered to be an active rather than a passive process. According to J.Thiel-Burgess (2012), the theory is used to tactically test the level of understanding of pupils in their progress towards an advanced level. Deep understanding enables them to build networks of explanatory demonstrations, which are particularly vital for the understanding of mathematical concepts (Barmby, Harries, Higgins, & Suggate, 2007). In short, as Clement (2005) stated, constructivism emphasizes understanding through the effective use of thought-process.

Thus, there is a need to introduce new teaching approaches in order to inspire pupils to better understand the concepts of fractions. To achieve this, teaching and learning strategies should be diversified. It is equally important that pupils have space and opportunities for learning, while teachers play the role of facilitator.

In this regard, it is important to employ representative models and technology tools to help pupils comprehend fractions in an engaging way. Researchers suggest that illustrations such as graphs, figures and symbols, are important resources that greatly assist learning purposes (Agrawal & Morin, 2016; Bouck et al., 2017; Hughes, Riccomini, & Witzel, 2018). Through a technology-based teaching and learning environment, pupils will be able to develop and explore alternative methods. With the support of multimedia and visual tools, pupils who perform poorly in academics could easily keep track of the learning process (Jamalludin & Siti Nurulwahida, 2010; Shaharuddin & Khairi, 2013).

Recent advancements have made available innovative technology at practically no cost on the internet. Also known as virtual manipulatives, they are an “an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (P. S. Moyer-Packenham & Bolyard, 2016; Moyer, Bolyard, Johnna, Spikell, & Mark, 2002; Reiten, 2018). Specifically, the computer-based delivery of basic mathematics manipulative tools are a useful and user-friendly feature for an immersive classroom session (Dorward, 2002). Unlike mathematics software tools such as GeoGebra, Scratch and Geometry Sketch Pad, virtual manipulatives do not require pupils to possess exceptional computer skills to explore mathematics concepts (Chrysanthou, 2008).

One of the benefits of using virtual manipulatives is that they could enhance pupils’ ability to visualize and interlink words, images and symbols. This skill is very useful for them to understand fractions (Paivio & Clark, 2006). The efficacy of virtual manipulatives in this case depends on whether the visual images shown to pupils could stimulate their ability to perceive fractions, so to say. In view of the fact that web-based virtual manipulatives can allow pupils to understand the of fraction, this study aims to assess the extent to which virtual manipulatives can be successfully implemented among Year Four pupils**.**

1.2 Problem of Statement

In Malaysia, the learning of the concept of fraction commences at the lower primary level. By the time pupils reach Year Four, they are expected to have gained a proper understanding of fractions and mixed numbers. However, fraction remains a challenge for them to understand (Ghani & Maat, 2018; Tan See Teng 2018).

According to (Fuchs et al., 2014)), little is known as to why some pupils tend to grasp fractions quickly, while some struggle.

A number of works are available on the topic of the comprehension of fractions among primary school pupils. These studies mostly examine the concepts of rudimentary fraction as a major problem faced by pupils. Researchers argued that the difficulty of understanding the concept was, in fact, common (Bertolone-Smith, 2016; Lortie-Forgues et al., 2015; Roesslein & Codding, 2019; Vukovic et al., 2014; Wijaya, 2017). Malaysian researchers have also found a similar problem among Malaysian pupils (Abdullah, Abidin, & Ali, 2015; Kathir Veloo & Puteh, 2017; Razak, Noordin, Dollah, & Alias, 2017).

Lamon (2013) states that many pupils are not only less interested in Mathematics but also have some kind of fear about it. This seems to be the case, especially after fractions have been introduced. This statement is also backed by researchers (Alghazo & Alghazo, 2017; Noorbaizura & Leong, 2013) who accepted that the concept of fragmentation could not be easily understood by primary school pupils. The topic often triggers anxiety among pupils, causing them to lose confidence that they will not be able to perform well in Mathematics.

One of the main reasons pupils struggle to understand fractions is that they do not develop their understanding of the topic at the lower primary level. Teaching method plays an important role to develop understanding among pupils. Loong (2014) believed that misconceptions in fractions could be reduced if pupils were guided to understand the mathematical concept properly. The knowledge of fraction held by Mathematics teachers and the manner in which they describe fractional concepts are factors that decide whether or not pupils can understand them.

According to Noraini (2006), the majority of Mathematics teachers in Malaysia emphasize calculation skills in learning fractions. Unfortunately, the study also indicates that teaching activities concentrate more on rules and memorization than on understanding the collection of symbols and abstract mathematical concepts. This scenario was observed by Veloo and Puteh (2017), who suggest that the concept of fraction is simply introduced as symbols and abstract concepts. Teachers tend to overemphasize the importance of procedure rather than the understanding of the concept itself. Halimah and Poerwanti (2013) back this statement, as they believe that the main issue is the over-reliance on procedure-based teaching method and under-emphasis on learning-media usage. Orhun (2007) is therefore of the opinion that teaching fractions with the aid of visual models can improve their reasoning skills compared to rules-based teaching, which is only applicable in the short term.

Most of the teachers in Malaysia are also examination-oriented. As a result, they emphasize on algorithms, so pupils could easily calculate without the need for reasoning. According to Zakiah Salleh, Norhapidah Mohd Saad, Mohamad Nizam Arshad, Hazaka Yunus, and Zakaria (2013), pupils between 12 and 15 years of age solve mathematical problems by memorizing the techniques. They are unable to explain the concepts of addition and subtraction in fractions through the use of diagrams. Suaidi (2017) highlights that the poor mathematics problem-solving ability in Trends International Mathematics and Science Study (TIMSS) and Programme International Student Assessment (PISA) is due to the fact that the teaching method is too concerned with procedures and memorization rather than conceptual comprehension. As a result, pupils could not grasp the deep meaning of fractions, although they can easily solve fractional problems by memorizing the computation methods. They are not exposed to exploring formulas and alternative solutions.

Western education system modified its teaching approach by recognizing this problem in order to enable pupils to develop an in-depth logical understanding of the numbering system rather than the teaching of algorithms (Hàwera, Taylor, Young-Loveridge, & Sharma, 2007). Evidently, in the words of Wu (2001), "*...no matter how much "algebraic thinking" is introduced in the early grades, and no matter how worthwhile this might be, the failure rate in algebra will continue unless the teaching of fractions and decimals is radically revamped*".

It is possible to hypothesize that pupils may not develop their understanding of fractions due to the limited use of learning materials at the lower primary level. Dienes and Perner (1999) state that the lack of usage of manipulative learning materials has led to a lower understanding of fractions among lower primary level pupils. Moreover, Dienes and Perner (1999) suggest that every mathematical concept can be easily understood if it is introduced to pupils through concrete examples. They propose six learning stages, namely; free games, structured games, character search, representation of symbols, and; formalization. The researchers also suggest that descriptions of a concept, or 'multiple embodiments', are required to encourage pupils' understanding.

Furthermore, pupils construct solution in relation to fractions based on previous knowledge of whole numbers. However, when pupils compute fractions, it is not the same as counting whole numbers by fingers. Confusion arises when fractions are involved (Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013). Pupils tend to mistakenly interpret whole number and fraction numerators and denominators as the same values. This is because they are unable to visualize fractions in terms of representation and confuse their whole-number knowledge for fractions (McNamara & Shaughnessy, 2010). This is known as the whole number bias phenomenon (Ni & Zhou, 2005; Vamvakoussi, Van Dooren, & Verschaffel, 2012). For example, when comparing two proper fractions, namely 1/5 and 1/10, pupils assume 1/10 is bigger than 1/5, they assume the denominator value of 10 makes 1/10 bigger than 1/5. As a result, pupils are unable to visualize the partition of fraction.

Fraction has many meanings and constructs, including model or coding depictions (5/4, 1 1/4, 1.25, 125%) that tend to puzzle both pupils and teachers (Kilpatrick, Swafford, & Findell, 2001). Based on Table 1.1, according to Kieren (1976), fractions can be explained as (1) part-whole, (2) measure, (3) quotient/division, (4) operator, and (5) ratio. For instance, 2/5 can be regarded as two out of five equal parts (part-whole), or as a point on a number line (measure), or two divided by five (quotient), or two-fifth of a quantity (operator), or two parts to five parts (ratio) (Pantziara & Philippou, 2012). Despite having five different interpretations, the part-whole model is the most commonly used approach in school textbook. Previous studies have shown that pupils' understanding of fractions is better accomplished when taught beyond the different interpretations of fraction (Clarke, Roche, & Mitchell, 2008; Siebert & Gaskin, 2006). In this respect, generally, Bruner (1986) argued that conceptual understanding could be improved if pupils were not exposed to various representations of a concept.

Table 1.1

Different Fraction Interpretations for the fractions $\frac{2}{5}$

Interpretations	Examples
Part of whole	2 out of 5 equal parts of a whole or set of object or collection
Measure	$\frac{2}{5}$ means a distance of 2 (1/5 units) from 0 on the number line
Operator	$\frac{2}{5}$ of quantity
Quotient	2 divided by 5, 2/5 is the amount each person receives
Ratio	2 parts cement to 5 parts sand

There is clearly a need to design and adapt a teaching method that is effective in boosting the conceptual understanding of a mathematical concept, such as fraction. Whether fractional or other mathematical concepts, a strong understanding can only be attained when pupils participate dynamically in the learning process and manipulate objects in their surroundings. Opportunities must be made available for them to explore fraction via manipulative exercises, share their notions with peers, and effectively strengthen their knowledge of fractions.

1.3 Objective of Study

This study aimed to determine the effectiveness of using virtual manipulative teaching method in enhancing Year Four pupils' understanding of fractions. The objective of the study is:

a. To determine whether or not there is a difference in the mean score of virtual manipulative teaching method as compared to the traditional teaching method.

1.4 Research Questions

This study poses the following research questions:

1. Is there any significance difference between the pretest mean score of year four pupils taught using the virtual manipulative method compared to those taught using the traditional method?
2. Is there any significant difference between the pre and posttest mean score of year four pupils taught using the virtual manipulative method?
3. Is there any significance difference between the posttest mean score of year four pupils taught using the virtual manipulative method compared to those taught using the traditional method?
4. Is there any significant difference between the pre and posttest mean score of year four pupils taught using the traditional method?

1.5 Research Hypothesis

1. Ho 1: There is no significant difference between the pretest mean score of year four pupils in Klang taught using the virtual manipulative method compared to those taught using the traditional method.

   H_1 1: There is a significant difference between the pretest mean score of year four pupils in Klang taught using the virtual manipulative method compared to those taught using the traditional method.

2. Ho 2: There is no significant difference between the pre and post-test mean score of year four pupils taught using the virtual manipulative method.

   H_1 2: There is a significant difference between the pre and post-test mean score of year four pupils taught using the virtual manipulative method.

3. Ho 3: There is no significant difference between the posttest mean score of year four pupils taught using the virtual manipulative method compared to those taught using the traditional method.

   H_1 3: There is a significant difference between the posttest mean score of year four pupils taught using the virtual manipulative method compared to those taught using the traditional method.

4. Ho 4: There is no significant difference between the pre and post-test mean score of year four pupils taught using the traditional method.

   H_1 4: There is a significant difference between the pre and post-test mean score of year four pupils taught using the traditional method

1.6 Significance of the study

This study is useful to stakeholders in Mathematics education, such as Mathematic teachers, curriculum developers and teacher educators, as it will provide them with guidelines on how to effectively teach fractions. Firstly, the findings of this study are useful for mathematics teachers who want to implement modifications in their teaching methods by incorporating technology into their lessons. It also encourages teachers on the need to use web-based manipulative tools to make lessons effective and enjoyable. Secondly, this study is beneficial for teacher trainees/pre-service teachers to learn how to use web-based resources (manipulatives) in line with the widespread use of technology in the education system. Moreover, early exposure will further enhance their own understanding, and increase their ability to deliver web-based manipulative teaching to their pupils. Finally, this study is significant as developers may develop a new curriculum or syllabus that uses web-based tools or interactive computer software to facilitate teachers.

The findings of this study can be used for potential developments in the education system, in particular to reap the advantages of web-based manipulative tools to transform the learning environment of mathematics. It will also serve as a guide for researchers who plan to conduct similar studies in the future.

1.7 Definitions of terms

This study used a set of definitions of terms, four of them are Understanding, Fraction, Understanding of Fraction, Virtual Manipulative. The definitions of the terms are as follow:

A. Understanding

Understanding is defined as an ability of making connection of new knowledge to be fitted with the existing schema that students have. Beside that pupils conception can be identified from their thinking on a number of things such as mental picture, representation, meaning giving, comparison, and problem solving related to fractions (Leslie P Steffe & Olive, 2009).

Mental picture: Mental imagery is an image produced immediately by pupils without involving the use of their five senses (Thompson, 1996). Mental picture is interpreted as pupils apply knowledge of fractions, spontaneously.

Representative: Representative refers to the representation of experience, that is, the reconstruction of knowledge based on past experience. In this study, pupils were asked to re-represent fractions by sketching the representation using virtual manipulative.

Meaning: According to Von Glasersfeld (1987), meaning is an interpretation given by a pupil and occurs in the conscious state that the situation has more than one possible answer. In this study pupils able to explain the meaning of fraction beyond part of whole and include the three model of fractions.

Comparison: Comparison is the act of identifying similarities and differences between several things. In this study pupils can make comparisons or differences of fractions through diagrams or symbols in fractions

Problem solving: Problems refer to conflicts or disruptions experienced by pupils when they fail to assimilate a given task to achieve a specific goal. In this study, problem solving refers to the methods used by pupils to overcome by responding based on the knowledge they have.

B. Fractions

Fraction can be defined as the "part-whole" or is better understood by the translation of the word "part of the whole" (Kementerian Pendidikan Malaysia, 1998) Fraction also defined as any number that can be expressed as $\frac{a}{b}$ where a and b are integers, a is not multiple of b and b id not equal to zero (Borowski & Borwein, 2002). However, in this present study we only focus in the proper fraction where the value of the numerator is less than the value of the denominator.

C. Understanding of fraction

Understanding of fraction means understanding all the possible concepts that fractions can represent and able to explain using three type of models-area, length, set. One of the commonly used meanings of fraction is part-whole. Usually it can be represented by darkening a region (area model), part of a group of people (35 of the class went on the field trip) (set model), or part of a length (we walked 3 ½ miles) (length model). Researchers believe pupils would understand fractions better with extra importance across other meanings of fractions (Clarke et al., 2008; Lamon, 2012; Siebert & Gaskin, 2006). In this study, the operational definition of understanding of fraction measured by the total mean score of the Understanding of Fraction Assessment.

D. Virtual Manipulative

In this study Virtual Manipulative is as an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge (P. S. Moyer, Bolyard, & Spikell, 2002). Virtual manipulatives are exact pictorial of concrete manipulatives such as pattern blocks, geometric solids, base-ten blocks, Cuisenaire rods, or geoboards that can be accessed via Internet or computer software. These dynamic visual representations can be manipulated similar to the concrete manipulative.

1.8 Limitation and Delimitation of the Study

This research has several limitations and delimitations. The limitations relate to the research design, the sampling method and the time factor. As far as the research design is concerned, the quantitative-based quasi-experimental approach used may have some drawbacks compared to the true experiment approach. Weaknesses were resolved by validation in an effort to minimise the impact. As suggested by Creswell and David (2017), the variables were controlled by the use of test scores based on a set of standards and skills.

The second limitation relates to the sampling method. This study was conducted by taking a small sample of 80 pupils from 2 Year Four classes from a primary school (SK) in Klang, Selangor. As such, the findings are limited to this particular school only. As a result, the sample may not be representative of all primary school pupils across Malaysia. The researcher assigned one class as an experimental group, while the other class as a control group. Apart from the lack of randomization and reduced internal validity, the findings on the causality are less conclusive in quasi-experiments. The lack of random assignment into experimental groups leads to non-equivalent test groups that do not control all variables.

Therefore, the findings of the study cannot be attributed exclusively to any changes to the modifications made. In addition, statistical analyses may not be accurate due to lack of randomization and internal validity.

The third limitation is that of the time factor. The research was completed approximately within a month and took 1 hour 30 minutes a day or approximately 3 classes a week. This limited data collection, and therefore influenced the results of the study. Nevertheless, a small-scale study is sufficient for the purpose of improving the teaching and learning of fractions.

This study has delimitations in relation to the subject of mathematics, technology and research participants. The first delimitation relates to the particular topic selected. For the purpose of this study, the focus is on proper fraction. This topic will be particularly challenging if pupils quickly move to computing without grasping the concept. The second delimitation relates to the web-based virtual manipulative tools used in the study. While there a number of educational software for mathematics, such as Geometry Sketch Pad (GSP), Tinker Plot, GeoGebra, and Scratch, web-based tools that are developed for pupils were selected. Such tools can be accessed anywhere and help pupils to build up sufficient conceptual knowledge and procedural knowledge in fractions.

The third delimitation is that the researcher restricted the study to Year Four pupils in a primary school. The focus is on the topic of fraction planned for the Year Four syllabus of the national curriculum, which was approved by the Ministry of Education. Year Four pupils were specifically selected for the reason that they will move to learn operational procedures in fractions, such as addition and subtraction of proper fractions.

1.9 Theoretical Framework

This study is based on the theory of constructivism, which suggests that pupils construct new knowledge on specific topics based on their previous knowledge. It also emphasizes the interaction between teachers and pupils in the learning process, and highlights the educational needs of pupils (Molenda & Januszewski, 2008). This theory is consistent with the learning of the concept of fraction. Because it is a challenging process for pupils, the learning of fraction requires a stimulating classroom environment to support the pupils' ability to understand. Pupils would need to be motivated to focus on understanding concepts rather than memorizing procedures, as well as learning collaboratively through the use of technology. Specifically, this study concentrated on the learning ability of the pupils of the concept of proper fraction through virtual manipulatives.

Constructivist theory suggests that each pupil has a stock of conception and skills that enables them to construct knowledge and find solution to a problem. Teachers, on the other hand, play the role of setting up the environment, giving tests and providing rewards to inspire pupils to understand deeply the idea of fraction. In particular, it is important for pupils to construct knowledge by trying to make sense of the situation in which they are encouraged to use the methods of inquiry to form questions and to utilize the available sources to find answers. This ability of exploration would make it possible for students to draw conclusions. With respect to the learning of fraction, the ability to assess one's own questions can lead to more questions and eventually allows pupils to improve their conceptual skills. Based on the perspective of constructivism, a teacher should be conscious of what the pupils know and what they should do, how the pupils should reach a consensus by interacting with each other and with the teachers, and how pupils can test their knowledge and receive feedback on its adequacy.

The constructivist approach is fundamentally different from the cognitivist approach on the basis of the nature of knowledge. Constructivist elements include learning in an appropriate environment, collaborating to obtain multiple viewpoints and supporting a pupil-centered approach. Whereas, cognitivists assume that pupils simply transfer knowledge from the environment rather than construct their own knowledge through social interaction (Solso, Maclin, & Maclin, 2008). Therefore, constructivism is the more preferred approach than the cognitivist approach, since it acknowledges the value of data collection and data synthesis. Von Glasersfeld (2012) lists the following constructivist elements involved in the learning of fractions:

1. Pupils form their understanding of fractions from their prior knowledge.
2. Learning fraction requires prior experience and understanding.
3. Communication plays a role in the learning of fraction.
4. Technology helps to teach and learn fractions effectively.

These characteristics allowed the researcher to restrict the scope of this study in such a way that it can be controlled. Furthermore, they also assist the researcher in collecting and analyzing data to address the research questions.

1.10 Summary

There are eight important sections in this chapter, namely the background of the study, the problem statement, the theoretical framework, the purpose and research questions, the operational definitions, the limitations and delimitations, as well as the significance of the study and the conclusion. Issues related to the field of study are discussed in the background. One of the critical issues of the research problem was addressed in the problem statement. Justifications for selecting the critical issue were given. In addition, explanations were also given on the use of the theoretical framework, and the theoretical assumptions of the research. Subsequently, the object of the research and the research questions were stated, followed by the operational definitions of this study. While the delimitations were under the control of the researcher, limitations were beyond the control of the researcher. In addition, relevant factors that will not be discussed by the researcher in this study were stated. Significant studies, including the importance of clarifying the research questions and the quality of mathematics education, are discussed.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter has two broad purposes to address the theoretical contributions to the research and to review the evaluations of the relevant authority on existing practices of virtual manipulatives. It also provides a deeper understanding of the problem statement and the reasoning behind the research question. The chapter then explores the relationship between fractions, manipulative and virtual manipulatives in mathematics education. Following that, the constructivist theory and the conceptual framework are discussed. The final section of this chapter explains how educational technologies form the foundation of the research.

2.2 Understanding

In his study, Piaget (1973) explained the theory of constructivism with regard to the mental progress of pupils. He believed that pupils should be put in a classroom environment that consists of engaging activities to facilitate their comprehension of an idea. Understanding is gradually established through active participation in classroom activities (Hansen & Zambo, 2005; Piaget, 1973). Learner does not acquire knowledge by way of transferring, but constructs interpretations based on personal involvements and communications. Learners learn most effectively when they personally construct their own knowledge. Therefore, in order to understand the learning process of an individual, it is important to scrutinize their actual experiences.

Piaget (1973) remarked the following: “to understand is to discover or reconstruct by rediscovery and such condition must be complied with if in the future individual to be formed who are capable of production and creativity and not simply repetition" (pg20). The author also stated that learning and understanding are active processes, not determined by environmental forces or simply by shaping them. He stated that pupils play a dynamic role in their own learning process and adapts environmental measures into their own cognitive functioning. As a result, knowledge is built or, in the words of Piaget, "to understand is to invent".

Hence, learning environment and lesson should be more interesting and fun for pupils, then they able to concentrate and develop their understanding especially young learner or primary school pupils. Teacher need to in more efforts to create an active learning whereby pupils will engage and interested to learn.

2.3 Concept of Fraction

Fractions have different meanings depending on the context in which the concept is used. According to Ohlsson (1988) in order to understand the meaning of fraction, it is important to "pay attention to the mathematical theory in which fractions are embedded, to which fractions are applied, and to the referential mapping between the theory and those situations" (p. 54). Fractions form a part of a subset of rational numbers, which form a subset of a larger set of real numbers. Freudenthal (1986) viewed fractions as the phenomenological source of rational numbers. Kieren (1976) divided the concept of fraction into five major subconstructs: part-whole relationship, measure, operator, quotient, and ratio. Each subconstruct will be described briefly. The fraction ¾, where 3 is called the numerator and 4 the denominator, is used to provide a better understanding of the concept.

Part- Whole: Part-whole construct is an important initial step for the understanding of fractions (Kathleen Cramer, Monson, Whitney, Leavitt, & Wyberg, 2010; Van de Walle et al., 2016). It can be represented by shading a region, part of a group of people (35 of the class went on the field trip), or part of a length (we walked $3^{1}/_{2}$

miles). Figure 2.1 shows the fraction 3/4 representing three equal slices of a cake, cut into four equal pieces (continuous model) or three eggs from a carton containing 4 eggs (discrete model). In this regard, Lamon (2018) argued that even divisions can occur in a variety of contexts depending on the nature of a unit, such as number, length, area and so on. In Western schools where English is the medium of instruction, students often get confused as the word "whole" sounds close to the word "hole" (Haylock, 2007). In this regard, Lamon also suggested the use of the term “fractions of a unit” to mean “a part of a whole”.

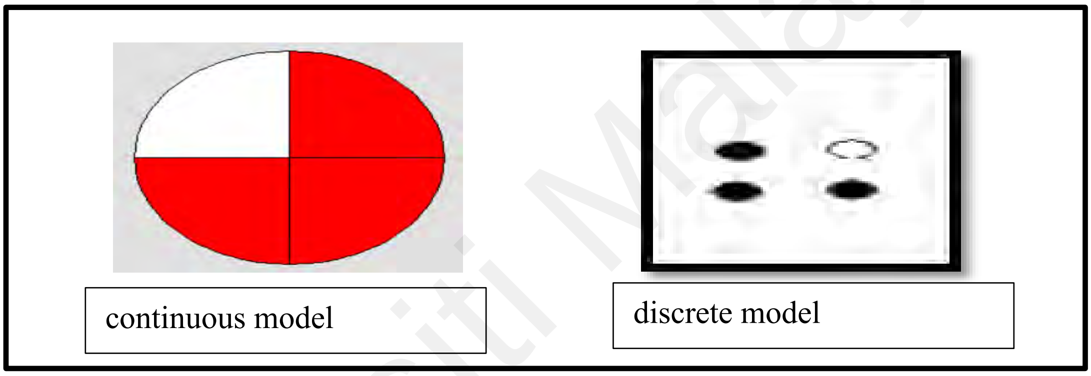


Figure 2.1. Represent Fraction in Continuous Model and Discrete Model

Measurement: Measure refers to the position of a number on the number line as shown in Figure 2.2. In that regard, Van de Walle et al. (2016) stated that the subconstruct measurement is the basis of iterating fractional parts. That is, two-fifths are constructed by iterating two one-fifths. This definition underscores how many numerator parts rather than how many denominator parts, as is the case in part-whole interpretations.

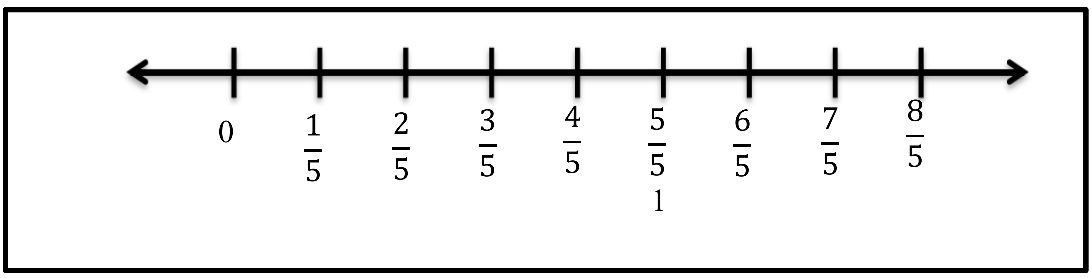


Figure 2.2. Concept Fraction as measurement

The quotient or division is a subconstruct of a fraction that focuses on the process. For example, $\frac{10}{4}$ can be interpreted as 10 divided by 4 or as sharing 10 cookies between 4 people. This is not a part-whole scenario, but it still means that each person will receive one-fourth (1/4) or $2\frac{1}{2}$ cookies. It can be written as $\frac{10}{4}$, $4\sqrt{10}$, $10\div 4$, $2\frac{2}{4}$, or $2\frac{1}{2}$. However, previous studies seem to have shown that division could not be correlated with fractions (Yanik, Samson, & Flores, 2006). Kilpatrick et al. (2001) argued that "*in some ways, equal sharing can play the role for rational numbers that counting does for whole numbers"* (p. 232).

Operator: Fraction could be practiced in enabling the procedure of calculation, as in 4/5 of 20 square is sixteen. Such a situation represents a fraction of whole number, making it easier to calculate mentally – known as 'mental math' – to find a solution (Johanning, 2008; Usiskin, 2007).

Ratio: The concept of ratio is another context in which fractions are used (Confrey & Carrejo, 2005; Streefland, 1985). For example, the fraction $\frac{1}{4}$ can mean that the probability of an event is one in four. Ratios can be part-part or part-whole. For example, the ratio $\frac{3}{4}$ could be the ratio of those wearing jackets (part) to those not wearing jackets (part), or it could be part-whole, meaning those wearing jackets (part) to those in the class (whole).

For the purpose of the research, the concept of fraction is specified in three significant designs for proper understanding. These are unit and unitizing, partitioning and iterating and equivalence (Barnett-Clarke, Fisher, Marks, & Ross, 2010; Van de Walle et al., 2016). Units and unitizing are foundational concepts According to Barnett-Clarke et al. (2010) units may be discrete or countable. Units can also be part of a whole or continuous and measurable, as in pizzas, brownies, ribbon and miles. The unit fraction is the size of the counting piece. Determining the unit is key to describing the quantity size with rational number. The first step is to determine what is the unit or whole (Behr, Lesh, Post, & Silver, 1983; Carraher, Schliemann, & Brizuela, 2000; Kieren, 1976; Lamon, 2007). The unit is used within all of these foundational concepts, since it is the most fundamental aspect of rational number. For example, when pupils understand that 'one' can be broken down into 1/b units, they will be able to count past 'one' with the unit fraction and be able to understand how many 1/b fractions make up the whole (Lamon, 2007).

The second and third design described partitioning as breaking or fracturing of the whole. It can also be described as dividing an object or objects into a number of disjoint and exhaustive parts. In addition, partitioning is described as non-overlapping parts. When the whole is partitioned, each of the parts is of equal area.. Mack (2001) described the conceptualization of the whole as essential for partitioning. The knowledge of piecing the whole back together is essential in deciding how many pieces to cut and how large or small the pieces will be. Lamon (2007), provides the following ground rules for partitioning and iterating:

1. Each unit is equal.
2. If a unit consists of more than one element, they must be in the same size.
3. Although a share is equal in amount, shares do not always have the same number of parts.
4. Equal shares do not have to be the same shape.

When pupils start partitioning, they are able to determine which fraction is larger. Pupils should be able to discover by how much more the largest fraction is. Iteration of fractions is related, but is the 'building up' of the unit piece. It is another way to make sense of fractions and improper fractions. When a unit is copied to create one or whole, the unit has been iterated (Lamon, 2007). Barnett-Clarke et al. (2010) suggested that a whole can be subdivided into units. For example, it can divide into four equal-sized pieces. Each of these pieces is considered as ¼. An example of iteration is using four ¼ pieces to create one. When given a number of one-fourth pieces, such as five ¼ pieces, it is notated as $^5/_4$ i.e. five copies of the unit fraction ¼. Experiences with both partitioning and iterating can help to clear up the confusion between the number of parts in the share and the name of the share (Van de Walle et al., 2016). When the numbers of counting pieces in the unit increases, the opposite can be said for the size of the counting pieces; they will decrease in size (Lamon, 1996; L. P Steffe, Cobb, & Von Glasersfeld, 1988).

Last but not least, the equivalence of fractions is important for understanding rational number ideas (J. P. Smith, 2002). Lamon (2007) defined equal in part-whole fractions as equal in number, length, and area. In other words, many different fractions can be of the same amount. Equivalence is an important concept in mathematical development and should not be overlooked during instruction. As pupils develop their vocabulary to explain their models and thought process, they should be able to understand the difference between parts and pieces. One part is not the same as one piece. A part may have more than one piece included within it. When pupils begin to have a firm understanding of equivalence, they may use this knowledge to determine which fraction is larger, how much larger, or if they are equal (Van de Walle et al., 2016).

2.4 Understanding of Fraction

In previous study by Pearn et al (2007; 2002), it was found that pupils demonstrated incorrect use of whole number thinking strategies to problems relating to fractions. They failed to realize that this thinking strategy was only applicable to fractions with the same denominators. Since fractions are an integral part of the Malaysia primary school curriculum, it is important to examine whether students have a strong understanding of the fractional concepts by the end of Year 6.

According to Van de Walle et al. (2016) once fractions are understood, all the other related concepts are understood as well. Understanding the concept of fractions is a prerequisite for dealing with other mathematical concepts. This is when pupils are able to connect concepts with procedures (National Research Council, 2001; Wong & Evans, 2007). Algebra, measurement, ratio and proportion require the application of fractions. Due to this, pupils who misconceptualize fraction will encounter considerable challenge (Seethaler et al., 2011). This could eventually lead to complications with fraction-involving computation, decimal and percentage concepts, and in other areas such as ratio and, in particular, algebra (Bailey, Hoard, Nugent, & Geary, 2012; Panel, 2008). When pupils are introduced to basic fractional concepts in their early school years, their understanding of fractions improves over the years (Saxe, Taylor, McIntosh, & Gearhart, 2005).

It is therefore important to helps pupils establish a connection between their constructed knowledge and instructed knowledge, while presenting fractions as an interesting topic. This study would specifically address how Year Four pupils use proper fractions after completing the learning process.

2.5 Manipulatives

Manipulatives are tools used to aid pupils' mathematical knowledge in a more productive way (Stein & Bovalino, 2001). In addition, they do mention that manipulatives are one of the ways to make mathematics learning much expressive or easy-to-read. According to Moyer (2001) manipulatives are the materials constructed to symbolize the abstract mathematical ideas explicitly and concretely.

Similarly, Gagnon and Maccini (2001) identified manipulatives as objects that students physically handle to symbolize mathematical concepts and relationships.

Manipulatives These are tangible materials that vary in size, shape, and color. They may also include physical models, such as fraction circles, paper folding, pizza pieces, dice, and coins that enable pupils to easily create mental pictures for fractions. Manipulatives are not rulers, projectors, or calculators. Manipulatives are recognized as tools that constantly aids pupils with a high frequency of success in mathematics (Maccini & Gagnon, 2000; Miller, Butler, & Kit-hung, 1998). According to Johnson (1993) computers can be considered a manipulative tool because they mimic solid materials. Using manipulatives allows pupils to correlate different mathematical topics and gain a better understanding of other subject areas (C. Y. Lee & Chen, 2010). Manipulatives are commonly accepted as the best practice in modern classrooms to allow pupils to develop conceptual understanding (NCTM, 2014)

Moreover, the usage of manipulatives in mathematics classroom could lead pupil to much positive attitude towards mathematical education in general. The early childhood of age 0-8 years old, are critical in terms of development (McGuire, Kinzie, & Berch, 2012), so it is important to explore instructional strategies that is aligned with and furnished appropriately to the young growing child's understanding of the world. In agreement to Smith (2009) quote, "a good manipulative bridges the gap between informal math and formal math". In order to accomplish this objective, the manipulative must fit the developmental level of the child.

Research conducted by Yusof and Lusin (2013a), with the purpose to determine whether teaching fractions using manipulatives could promote active learning and enhance the achievement among those pupils, was found that from the pre- and post-tests findings, there were significant positive effects on the performance of pupils. This small-scale study used both quantitative and qualitative data found from the pre and post-tests, interviews and classroom observation to determine the effectiveness of using virtual manipulative in teacher design the instructions. Overall result shows that active hands-on participation with mathematical experiences promotes understanding of concepts that will benefit pupil throughout their lives. Therefore, providing opportunities for pupil to explore and investigate mathematical ideas with concrete materials is one of the crucial aspects in the process of learning mathematics. The researchers mentioned that pupil did not get ample of opportunities to explore the use of fraction manipulative and some teachers lacked skills in using manipulative effectively. They infrequently used this manipulative for explanation dan demonstration purpose only.

However, it is not easy to incorporate manipulatives into the classroom norm (P. S. Moyer-Packenham & Bolyard, 2016; Moyer, 2001; Puchner, Taylor, O'Donnell, & Fick, 2008). In order to ensure that pupil benefits from the use of manipulatives, teachers must carefully plan and prepare meticulously for the lessons. They must take into account the materials and potential drawbacks of the manipulative method (Kathleen Cramer, Behr, Post, & Lesh, 2009; Sarama & Clements, 2016).

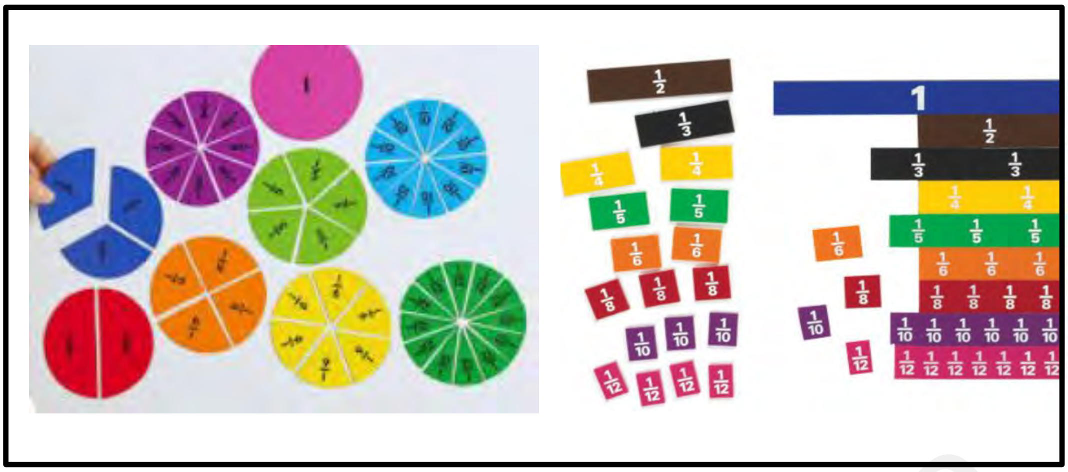


Figure 2.3. Physical Manipulative

2.6 Virtual Manipulative

Virtual manipulative technologies are cost-effective and freely accessible resources for pupils and teachers on the World Wide Web. A virtual manipulative is an interactive, web-based visual representation of a dynamic object that offers a prospect for building mathematical knowledge (Moyer et al., 2002). Meanwhile Dorward (2002) do mention virtual manipulative also known as the computer version of collective mathematics tools. Based on the author's research, visual images or graphics can be interpreted as virtual manipulatives, given that they can be viewed and slid, flipped and rotated in a three-dimensional space (Moyer et al., 2002). From the point of view of teaching, virtual tools can be "debugged, reconstructed, transformed, separated and combined together" (Healy & Hoyles, 1999) for later use. Virtual manipulatives tend to improve pupils' level of preparedness due to fear over feedback on their mistakes during class activities (Suh, Moyer, & Heo, 2005; Uribe-Flórez & Wilkins, 2017).

There are two types of virtual manipulatives – static and dynamics visual representation of physical manipulatives (Spicer, 2000). Static visual means a picture or an image in a computer that can only be viewed. Even though the visuals look like a concrete manipulative, they cannot be manipulated. Static visual illustrations are unreal virtual manipulatives. Whereas, dynamic visuals are objects, pictures, or images that resemble concrete manipulatives and can be changed.

There are many dynamic websites used in these sorts of research approach which provide pupils with free applets. One of the websites used in this study is The National Library Virtual Manipulative (NLVM). NLVM was created in 1999, it is a digital library of Java applets and activities for primary and secondary school pupils. This NLVM is graded and divided into five sections; Number and Operation, Algebra, Geometry, Measurement and Data Analysis and Probability (Utah State University 2009). The topic of interest which is fraction is categorized under the Number and Operation section. Figure 2.4 shows a sample of dynamic virtual manipulative NLVM.

Figure 2.4. National Library Virtual Manipulative

Meanwhile fraction circle and fraction bar are used to help pupils to observe the relationships between fractional parts of the same whole. Fraction circles contain set of nine circles of various colors which are broken into equal fractional parts and uses the same-sized whole. It helps pupils to see the relationship between fractional parts of the same whole. At the same time, pupils are able to compare and order fractions, understand the equivalent fractions, explore common denominators, as well as explore basic operations with fractions. Figure 2.5 shows Fraction Circle and Fraction Bar used in this study. Meanwhile Figure 2.6 shows sample questions posted during virtual manipulative of fraction circle activities.

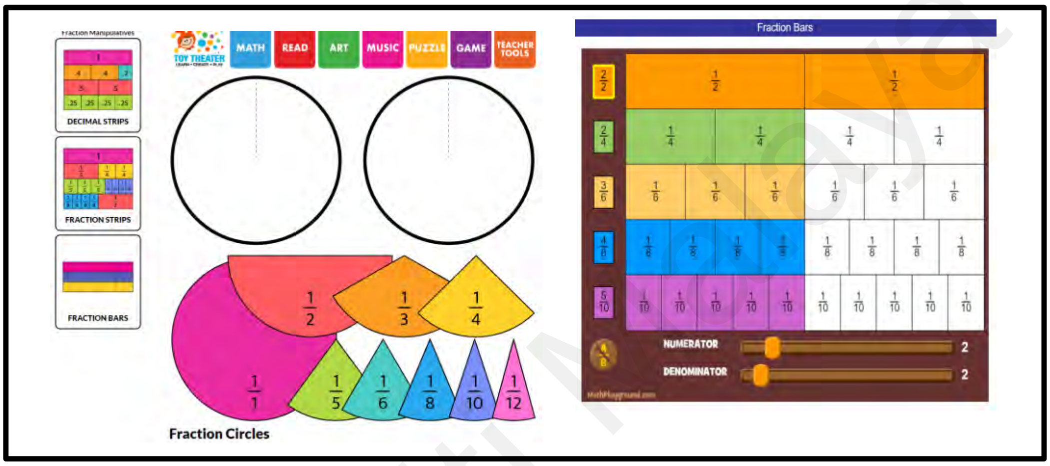


Figure 2.5. Fraction Circle and Fraction Bar

3. Sample questions teacher post.
   - Have students orally count the 'purple' pieces to create a whole. Have someone else count the 'blue' pieces after that. This allows students to practise saying the names of the parts that make up the whole.
   - How many different ways can you make a half? a quarter? a third? one and one sixth?
   - The more parts you share the smaller is the size of parts
   - When sharing the parts must equally share.
   - When teacher shows the diagram pupils able to use the fraction terminology (half, quarter).

Figure 2.6. Sample questions

Virtual manipulative is a more manageable tool than their physical counterparts, since certain built-ups are easier to perform with software than with physical manipulatives. Moreover, they are often user-friendly because pupils are able to change the original characteristics of the manipulative tool, such as size and shape. Using virtual manipulatives, unlike in a non-virtual environment, computers can also record, replay actions and interlink concrete ideas and symbols with feedback. In order to incorporate online virtual manipulatives into their teaching, teachers need to familiarize themselves with online platforms, assess the different activities available online, and determine which activities are suitable for their curriculum and the skills of their students.

Besides that, virtual manipulative is more efficient because less time is spent planning preparation and cleaning up the materials. In addition, pupils who utilize virtual manipulatives practices exercise more than those who do not (Johansen et al., 2007; P. S. Moyer, Niezgoda, & Stanley, 2005; Reimer & Moyer, 2005). One study also suggests that fractions may be especially suited to virtual manipulatives (Suh et al., 2005). A study conducted by Ngan Hoe Lee (2012), on primary school pupils showed a positive effect on the usage of virtual manipulative. In addition, the use of virtual manipulative also keeps the pupils focused in their learning process, moreover it has a deep effect on student achievement. Researchers also mentioned that the virtual manipulative encourages some pupils to put effort in finding the answers on their own.

In conclusion virtual manipulative is an appropriate representation that allows pupils to use concrete thinking skills to construct understanding of mathematic concept. This virtual manipulative is beneficial for pupils because it has multiple representation for the same concept; for example, fraction circle and fraction bar. Pupils should explain and demonstrate their solution after they engage in a task involving virtual manipulatives. By doing it this way, pupils can build their understanding and analyze error in constructivist manner.

2.6.1 Virtual Manipulative in Teaching and Learning Fractions

Fractions are difficult for pupils to understand, but it is an important topic in mathematics (Leslie P Steffe & Olive, 2009). There are various types and efforts made by the researchers to find solutions in fractional learning. A study conducted by Tan (2010) in Kota Samarahan, Sarawak involving one hundred and twelve standard five pupils from two primary schools. There are a few objectives in this study one of which is to investigate the changes in pupils' procedural knowledge and conceptual understanding of fractions using virtual manipulative. Researchers used both pre and posttest quasi-experimental and questionnaire to collect data. The results show that those pupils taught using virtual manipulative have better procedural knowledge and conceptual understanding compared to pupils taught using traditional instruction. Researchers conclude that the use of virtual manipulative in primary mathematic classes has the potential to improve the learning process. This statement was supported byN. H. Lee and Ferrucci (2012) whereby his research on primary school students, showed a positive effect on the use of virtual manipulative. Also, the use of virtual manipulative engages students in their learning process and it has a tremendous effect on student's achievement.

In addition, a research was done by Moyer (2012) on the low achievement of pupils on fractions. Low achievement pupils having a hard time visualizing the image in fraction concepts. After visual manipulative implementation in their fraction instructions, it assists pupils in developing visualization skills needed for the understanding concept of fractions. The result shows that the low achievement of pupils significantly increases when using virtual manipulative in fraction instructions. The researcher concluded that low achievement students developed a greater understanding and visualization skill of fractions and achieved better results on test questions.

Furthermore, research was also done specifically on pupil with disability, autism, or mild intellectual disability on learning fractions using virtual manipulative. Bouck et al. (2017) examined the virtual-abstract (VA) and virtual representational-abstract (VRA) instructional sequences, respectively, to teach students with disabilities-including students with mild intellectual disability to find equivalent fractions or add fractions with unlike denominators, respectively. The students are first taught to solve problems with a virtual manipulative, then a drawing, and finally with a numerical strategy. The Virtual abstract is a modification to the VRA, in which the representational (i.e., drawing) phase is not introduced. In all these of these studies, a functional relationship existed between the intervention and acquisition of solving the fraction problems in question. In the later study, Bouck, Shurr, Bassette, Park, and Whorley (2018) compared the efficacy of concrete and virtual manipulatives to support students with mild intellectual disability and learning disabilities to add fractions with unlike denominators. The results show both types of manipulatives as equally effective.

Study done by Finti, Shahrill, and Salleh (2016) on virtual manipulative in teaching and learning fraction but by using iPad shows that introducing the virtual manipulative method by using an iPad affects students' performance. Also, weak students show great improvement as well. This is because learning through modern gadgets increase their motivation to learn fraction. Learning using virtual manipulative can increase the students' understanding in depth of fractions. Overall, the research shows that the outcome of using virtual manipulative is highly positive and students can understand the concept of fractions.

Furthermore, study by Alshehri (2017) on physical and virtual manipulatives comparison of learning fraction shows that overall students' performance significantly improves after they have been introduced to physical and virtual manipulation. The students review statements revealed that learning through virtual manipulative was fun and easier to learn. Furthermore, the both the outcomes after using both manipulatives are almost the same. Both virtual and physical manipulatives are useful for students as they can get significantly better results after using it. However, majority of students stated that learning through physical manipulative eventually will become dull while learning through virtual manipulative will remain fun and exciting.

Study conducted by Mendiburo and Hasselbring (2010) on virtual and physical manipulative in learning fractions shows that the students score higher in virtual manipulative conditions compared to physical manipulative conditions. Besides that, using virtual manipulative also promises more successful outcome than physical manipulative. Not only that, learning through virtual manipulative will be more time-efficient than physical manipulative.

Morris (2014) conducted a study on the use of virtual manipulative to improve mathematics performance. The study shows that there were impressive changes in the students' performance in the virtual manipulative group. The pupil was able to understand the instructions and work in their level of capability. Besides that, by using virtual manipulative, it is time efficient as it is easier to set up.

2.7 Constructivism Theory

The theory of constructivism was adopted in this study to determine the effectiveness on pupil's improvement in fractions. Based on the theory, pupils construct their own information depending on the basis of prior lessons and experiences or through informal knowledge. From a constructivist point of view, information is built by interactions with the environment. In the traditional method, teacher plays an active role in the teaching and learning process, while pupils accept the content. As a result, constructivists believe that learning should be focused on the pupils (Major & Mangope, 2012). The idea of constructivism originated from Piaget's cognitive development theory and Vygotsky's zone of proximal development, both suggesting that teachers play the role of mentor and facilitate, while pupils play the main role of learning. It is in line with this study, where pupils learn or search their own input using virtual manipulative in order to build new knowledge using an informal learning method and their prior knowledge of fractions learned since grades 1- 3.

According to Jonassen (1999), The idea of constructivism originated from Piaget's cognitive development theory and Vygotsky's zone of proximal development, both suggesting that teachers play the role of mentor and facilitate, while pupils play the main role of learning. It is in line with this study, where pupils learn or search their own input using virtual manipulative in order to build new knowledge using an informal learning method and their prior knowledge of fractions learned since grades 1- 3. (Ali, 2001; Mohaiadin, 1999; Wilson & Lowry, 2000). The purpose of this research was to evaluate the effectiveness and relevance of learning through a teaching method designed based on the theory of constructivism. Moreover Wang (2009) pointed out from a constructivist perspective that the learning environment itself can also be considered as a technology-based platform providing for pupils to explore, experience, build, communicate and reflect what they have been learning through experience. Pupils who learn to use web-based techniques (virtual manipulatives) experience unique environmental conditions (i.e. sound effects and computer-generated feedback). This learning platform differs from the traditional learning method, in which pupils become inactive learners and are unable to explore, let alone experience, what they learn. Instruction in the conventional style was heavily teacher-centered. Using web-based learning tools, the approach changes to pupil-centered that allows pupils to collaborate in line with 21st century learning (Keser & Özdamli, 2012; Qing & Li, 2011)

Past researchers believe that every individual constructs information rather than receiving it from external factors. Constructivism theory states that pupils construct their worldview by synthesizing new experiences from what they have learned before (Brooks & Brooks, 1999). They form rules through reflection of objects and ideas. When they encounter meaningless objects, they will either interpret what they see to match the rules they have shaped or adjust their rules to better describe the new information. According to Mvududu and Thiel-Burgess (2012) They form rules through reflection of objects and ideas. When they encounter meaningless objects, they will either interpret what they see to match the rules they have shaped or adjust their rules to better describe the new information.

The basic concept of constructivism is that knowledge cannot be passed from one individual to another, and that each individual forms his or her own knowledge through unique experiences. Troelstra and Van Dalen (2014) mentioned that constructivism is nothing but an overview of how humans shape their own knowledge. This suggests that a person's knowledge is a product of the activity executed by the individual, not information passively obtained from external sources. In other words, the theory of constructivism plays an important role for teachers to understand the factors that influence and accelerate the learning process of the pupils

In conclusion, the constructivism theory is a learning theory which plays an important role for teachers because through these theories' teachers will understand the learning process that occurs in the pupils, understand the factors that influence and accelerate the learning process of the pupils.

2.8 The Function of Technology in Mathematics Education

Technological developments have provided many more alternatives to mathematics education. With the availability of technology in a classroom setting, pupils have more opportunities to visualize, analyze and investigate important mathematical concepts, as well as create links between mathematical ideas and real-life situations. Technology can empower teachers and students to learn mathematics by dynamically manipulating objects using computer software. National Council of Teachers of Mathematics (NCTM, 2000) points out that the technology used to study mathematics is especially useful. Although the use of computers involve multiple applications, more focus should be placed on the process of learning mathematics than algorithms and solutions (Noraini, 2006). Technology has been integrated into school curriculum, as a step towards instilling and fostering an interest and a positive attitude towards the learning process.

Abdullah and Zakaria (2013) investigated the effectiveness of Van Hiele's phases of geometry learning through Geometer's Sketchpad among Form Two students through a quasi-experimental research design. Their findings showed that the level of geometric thinking geometric thinking in the treatment group improved compared to the control group. The findings suggest that Van Hiele's phases of learning geometry significantly improved the geometrical thinking of the students. In addition, a combination of technological tools, such as the GSP, and graphing calculator for teaching quadratic function have contributed to teaching and learning Teoh and Fong (2005). The study found that visualization using technological tools could promote learning and increase students' understanding of quadratic functions.

On the other hand, visualization capacity using the conventional method is very minimal. Teachers are therefore encouraged to use the educational technology in the teaching and learning process.

Arganbright (2005) employed fundamental techniques using Excel to create animated graphical displays for teaching mathematics, and the result showed improvement after teaching. The three main criteria set by the researcher for his study are: (a) appropriate mathematics teaching and learning process; (b) fixed set of time for the efficient use of the software during the lesson; (c) the software should be effective in the long-term. Based on the results, the author stated that all the requirements had been met. There were two main benefits of using Excel as a teaching tool. First of all, Excel is an effective teaching tool since it is readily available and essential for future employment. Secondly, students may explore, illustrate and share their ideas, techniques, animations, and data table.

Dogan and İçel (2011) conducted an experiment investigating the effect of GeoGebra on triangle among eighth-grade students. The study conducted a two-week pre-post test for a twelve-hour duration. Based on the result, they concluded that a computer-based lesson is effective and suitable for use in the teaching and learning process. In addition, after the implementation of GeoGebra, students gained high-level thinking skills. Technology has thus created a positive impact by motivating pupils and improving their long-term memory.

In his research, Al-Mashaqbeh (2016) investigated the impact of using iPad to teach mathematics compared to the traditional teaching method among first-grade pupils in Jordan. The pupils had a personal iPad and each of them had access to mathematics software and websites. The results of the investigation indicate a positive impact on the pupils' performance compared to the traditional method of teaching. By using iPads, they had ample room of creativity and had a greater chance to read, write, watch, and listen. The researcher also encouraged teachers and pupils to use iPad to teach and learn mathematics.

Furinghetti, Morselli, and Paola (2005) investigated covariance occurrences using Cabri for drawing geometric figures, measuring and sketching graphs. Pupils participating in the study had difficulties comprehending and solving problems when they were not using Cabri. The researchers emphasized Cabri's need by stating that without it, the possibility of a pupil exploring the function of dependent and independent variables in the experiment would have been lost and would not have had a depth in linear dependence. Teaching and learning therefore became enjoyable, and students could easily solve problems. The pupils also believed that Cabri facilitated their learning; teachers too reviewed the software positively. In addition. Dahan (2010) found that teachers who used Cabri were able to solve all sorts of problems in geometrical shapes more quickly. In fact, teachers who initially considered the software difficult changed their view, stating that it was enjoyable and user-friendly. Koklu and Topcu (2012) examined the effect of the Cabri-assisted instruction on the misconception of graphs of quadratic functions by 10th graders. The findings revealed that the intervention group that took the Cabri-assisted instructions had a greater understanding than the control group that had received conventional instructions.

In order to improve the quality of academics in our country, the government has provided all the necessary equipment. It therefore promotes the use of technology in teaching and learning. Hardware and software technology will benefit pupils by increasing their understanding of a concept, providing visuals and simplifying complex calculations. The uses of tools that can help to interpret mathematical concepts will facilitate the learning of abstract concepts. The software can also allow students to explore their modeling problems more effectively (Pusat Perkembangan Kurikulum (PPK), 2010).

2.9 The Function of Technology in Learning Fractions

Fraction is a challenging topic in mathematics, particularly among pupils who explore fewer real-world experiences as regards fractions. According to the (National Research Council (2001) , teachers have to deliver appropriate 'experiences' to help pupils understand fractions and link procedure with conceptual understanding. However, learning fraction through traditional teaching only encourages each pupil to memorize facts and rules (Idris, 2007). On the other hand, the use of software and graphical elements will influence pupils to better explore the learning process (Jamalludin & Siti Nurulwahida, 2010; Shaharuddin & Khairi, 2013). In addition to the strategies used, the technologies are capable of enhancing the memory of pupils more up to the level of processed information. Text, graphics and audio combinations may influence pupils' enthusiasm for receiving information and bringing realistic elements by triggering their feelings and emotions (Harun & Tasir, 2003; Heo, Suh, & Moyer-Packenham, 2004).

In addition, studying fraction with technology helps pupils with disabilities to learn fractions. Norazrena and Khairul (2011) were investigating this with pupils using fragmented software developed using Macromedia Authorware 7.0. The pupils were sitting for the pre- and post-test. The results indicate that their post-test achievement was substantially improved compared to their pre-test achievement. In fact, some of them with special needs also managed to score full marks in the post-test. Ling (2008) designed a mathematics game to learn fractions quickly and in addition to assisting pupils with different mathematical abilities. Based on the findings, he stated that the game provided a chance for them to visualize the physical sizes of fractions, join symbols and representations of fractions and constructing a mental image of fractions. Such experiences are essential for pupils to understand the principal characteristics of fractions, such as order and equivalence.

Furthermore, Noorbaizura and Leong (2013) investigated the effect of pupils' performance in fractions using GeoGebra. This research was a quasi-experiment design conducted to compare the performance of two groups of Year Four primary school pupils. The findings revealed that the pupils in the experimental group performed better when using GeoGebra than the control group that was taught using the traditional learning method. Moreover, Pilli and Aksu (2013) conducted a four-month study to assess the effects of Frizbi Mathematics 4 on fourth graders. The result obtained from ANOVAs test revealed that there was a significant difference between the control group and experimental group. The evidence indicates that Frizbi Mathematics 4 is an effective tool for learning and teaching mathematics at the primary school level.

There are more and more options of technology every day. Students in this age are more likely to use educational hardware and software. It is therefore more important for educators to concentrate on how best to employ current technology to optimize the advantages of using technology. On the other hand, there are online applications such as Applets, Graphing Calculator and Microsoft Excel. For the purposes of this study, teaching technology refers to a well-planned approach using a web-based Virtual Manipulative for teaching and learning Fraction for Year 4 pupils.

2.10 The Function of Virtual Manipulative in Fractions

The use of manipulatives as physical demonstration in the instruction for mathematics has been debated over the years. Balka (1993) described the benefits of utilizing manipulatives in promoting pupils to establish connections between conceptual and procedural knowledge; in addition to recognizing relationship of diverse capacities of mathematics, to observe mathematics as an integrated whole, to discover problems using physical models and to relate procedures in an equivalent representation. According to Sessoms (2008), pupils who use virtual manipulatives experience different environmental factors than pupils who learn fractions in the traditional style. For example, they can receive computer-generated sound effects and obtain immediate feedback on their exercises. Pupils who learn fractions using the chalk-and-talk method experience the tactile nature of the manipulatives themselves, and receive delayed feedback from the teacher, usually in the form of corrected responses to practice exercises that are returned to the pupil one or more days after they were turned in to the teacher (Sessoms, 2008; Steen, Brooks, & Lyon, 2006).

Additionally, in their study, Pilli and Aksu (2013) indicated that programmed and computer-assisted instruction had an impact on performance of fourth grade pupils mathematics in fractions. Besides that, Yusof and Lusin (2013b) investigated whether the utilization of manipulative in the teaching process helps to promote active learning and improve the performance of Year Five pupils in the topic. Based on the pre and post-tests results, there were significant positive changes on the performance of pupils when manipulatives were used to teach fractions. The post-test of the intervention group was statistically important in indicating an improvement in the overall performance of the pupil’s ability to learn fractions. A research by Reimer and Moyer (2005) on the effects of employing a number of virtual manipulative applets for instruction reveals statistically significant changes in student post-test scores on the conceptual knowledge test and a strong correlation between student scores on post-test conceptual knowledge and procedural knowledge. Interviews with pupils and attitude surveys revealed that virtual manipulation enabled students in this class to learn more about fractions by obtaining immediate and detailed feedback, making it easier for them to study mathematics.

A further study was conducted by Bouck et al (2017) on the Virtual – Representational – Abstract (VRA) framework. The study was conducted to investigate the efficiency of the VRA method for identifying equivalent fractions for three middle-school pupils with disabilities. The result shows that the performance of all three pupils improved after the intervention. Similarly, research conducted by Tan (2010) on the impact of virtual manipulatives on the procedural knowledge and conceptual understanding of fractions among Grade 5 pupils in Sarawak. The researcher also investigated their attitudes towards virtual manipulatives and interest in learning mathematics. The results showed that pupils who were taught using virtual simulation had greater methodological awareness and conceptual comprehension. In addition, they had a favourable outlook towards educational methods and keener interest in studying mathematics.

In addition, Al-Mashaqbeh (2016) investigated the efficacy of Virtual Manipulative with the use of iPads in teaching equivalence and fraction addition. The target groups consisted of two year 7 classes of high-achieving of low-achieving students in one secondary school in Brunei. The results showed that the introduction of virtual manipulative through iPad had a significant influence on in-group activities and among students with low skills. The study also showed that their motivation to learn fractions increased as they considered the use of iPads to be enjoyable and interesting.

The academic community has generally accepted the influential role of virtual manipulatives in teaching fractions. It can be concluded from the literature review that the utilization of virtual manipulatives offers opportunity to pupils to learn and explore activities in mathematics, and to enable them to acquire, analyze, measure, and compare mathematical arguments in order to make conjectures and test a proposition.

2.11 Conceptual Framework

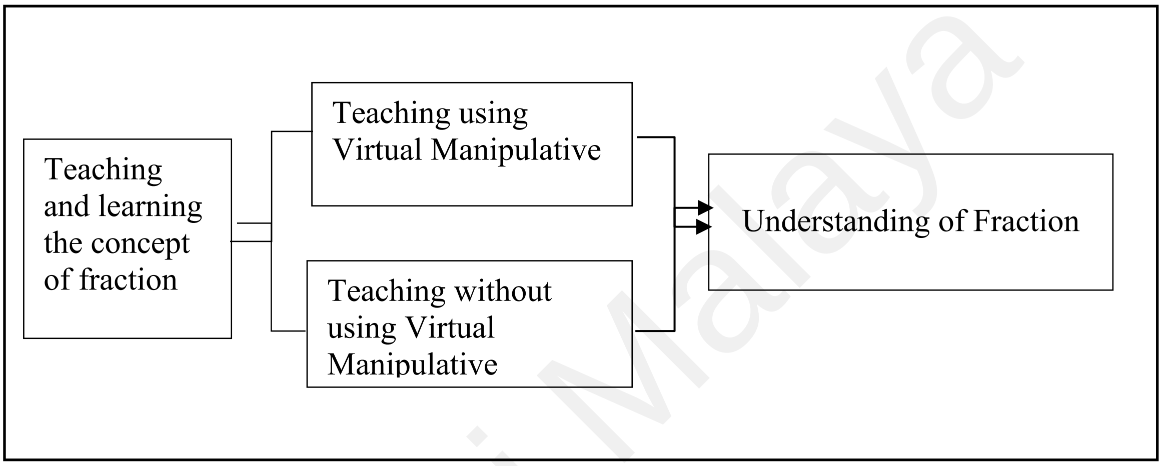


Figure 2.7 Adapted from Cross-sectional model by Richey et al. (2004). Development Research: Studies of Instructional Design and Development

Figure 2.3 shows the method used to define the research questions. It shows how pupils' understanding of fraction is influenced by virtual manipulatives and without the use of virtual manipulatives. The two variables are virtual manipulative (independent variable) and the understanding of the concept of fraction (dependent variable). Based on the figure, there are two different approaches to teaching, including virtual manipulative. This approach was extended to pupils in the experimental group. There were a total of seven lessons using virtual manipulatives. On the other hand, pupils assigned in the control group learned fraction only through textbook and exercises. The dependent variable of this study is a subject to be measured, which is the understanding of the concept of fraction among Year Four pupils. The pupils from both the control and experimental groups were tested in the post-test after intervention for the experimental group.

Based on the literature review and study needs, the conceptual framework was constructed using the constructivism theory in order to describe learning as an active process of interaction and collaboration between pupils and teachers. This method was chosen to better understand the effectiveness of the teaching and learning approach in fractions. Constructivism helped to describe pupils' needs to be engaged in activities with different pedagogical concepts (Richard, 2005). According to J. S. Bruner (1957): "generic coding systems that permit one to go beyond the data to new and possibly fruitful predictions" (p. 234).

In addition, constructivism also facilitates in the method of thinking and knowledge acquiring, moreover smoothens communication skills between the peer groups of pupils. In summary, a constructivists' academic method focuses more on methodologies to be much pupil-based, thought-provoking learning environments, and integrates pupils' past understanding in the learning process. Overall, this theory helped this study to plan, design and execute the intervention efficiently and effectively.

Additionally, manipulative been used in this study as it allow pupils to build up mental representations and acquire skills in using and modifying these representations and synthesizing new ones (Cai & Knuth, 2005; Capraro & Joffrion, 2006), manipulative have been described and found by many to be the best approach to resolve the difficulties inherent in learning arithmetic and concepts and processes (Gningue, 2006; Loveless, 2008). According to Brown and Crawford (2003), stated manipulative resources useful for pupils to learn in engaging and interesting method. In this study, the implementation of Virtual Manipulative to enhance pupils understanding in fraction, that creates opportunity for pupils to visualize and hands-on activities. Moreover, pupils able to engaged on task and motivated when use Virtual Manipulative (Drickey, 2006). As a result, dynamic visual representation using Virtual Manipulative are essential as it has the features to visualize images on the computer that rise up pupils' interest in learning fraction. The conceptual framework in this study serves as the proposed research model of the study to examine whether virtual manipulative instruction is able to improve understanding of fraction among year four pupils.

2.12 Summary

In conclusion, a compilation of studies discussed in this chapter support the needs of the use of technology in learning and teaching fractions. There are various studies that attempt to explain the use of technology in teaching and learning of mathematics and fraction to help pupil to develop their knowledge. Past studies provide some basic information as references for conducting this study. Among them is the theory underlying this study, the difficulty of pupils in the exploration of fraction and the use of technology in learning fractions. Previous studies have sparked the idea to focus on how pupils use knowledge in the process of fraction. The advancement of technology in the teaching of mathematics and statics around the word also reflect their needs in mathematics learning in Malaysia. Although the use of technology in the mathematics curriculum in Malaysia has introduces and developed since 2001. But its use is still at a minimum and fraction among pupils of primary school is still limited. Based on the review of the literature that has been made, there is need to study in order to identified use of Virtual Manipulatives by pupils in the process developing fraction. Thus, the studies how to identified the year 4 pupils understand the concept of fraction to solve arithmetic using virtual manipulative is a fair conduct to get more information from the perfective of the pupil themselves.

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter discusses the methodology for this study. The employed research design, population and sample, data collection method, instrument of the study, validity and reliability of instruments, pilot study, data analysis method and summary will be further conferred in this chapter. The first section of this chapter describes and justifies the relevance of the research design. Additionally, the description on the, population, location, sample and sampling method were discussed in section two. Third section scrutinizes on the types of instrumentation used, objective and content of the instrumentation. Next, validity and reliability of instruments and pilot study were explained in the fourth section and instructional activities with and without virtual manipulative were explicitly explained. The sixth section analyses about ethics followed description on data collection method that were exerted in this study. Last but not the least, data analysis method was described in section eight and summary of chapter three was mentioned in section nine.

3.2 Research Design

Research design is a route to map out the outline of a study Bogdan (2003). It is necessary to describe the design by identifying the research questions as the starting point of a study Yin (2003). The primary research question of the current study is to measure the effectiveness of virtual manipulative. In addition to that, it is also to test the hypotheses. In preparation to achieve that, a quantitative approach was chosen as the research design. Basias and Pollalis (2018) stressed that a quantitative methodology would not be influenced by researcher's personal feelings or opinion as it deals with data in numerical form.

Moreover, quantitative research simplifies the process of larger data and allows easier comparison of data. In addition, quantitative research samples are selected based on research objectives and questions that align with the purpose of the study. The instrument used in this study is formal which focuses on the concept of understanding the fraction among year four pupil and a rubric-marking scheme with measurement scale was employed for this study.

This design is pertinent to this study as it will support the researcher to examine the effect between two variables through statistical inferential as it simplifies the processing of larger number of data and allows easier comparison of data. The independent variable for this study is virtual manipulative teaching approach and the dependent variable is the comprehensiveness of the fraction concept.

A quasi-experimental research design will be carried out to determine the effectiveness of teaching using Virtual Manipulative Approach to intensify the comprehensiveness of the fraction concept among year four pupils. This quasi-experimental design is commonly employed to evaluate educational programs when random assignment of subjects is assigned to two groups which are either classified as impossible or impractical to comprehend a theory. Thus, in a quasi-experimental design, the research substitutes statistical method that controls the absence of physical dominance of the experimental situation. The quasi-experimental design is considered more suitable because it is usually appropriate to evaluate the effectiveness of a program or intervention when the respondent's responses cannot be disseminate randomly (Chua, 2006; Creswell & David, 2017).

The non-equivalent quasi-experimental design allows researcher to use present groups thus it is more relevant in comparison to true experimental design. The pupils will be chosen from two intact classrooms and pupils will be segregated accordingly as control group and treatment group in a school setting (Gall, Gall, & Borg, 2007; Shadish, Cook, & Campbell, 2002). According to Babbie (2011), a non-equivalent quasi-experimental design implicates the use of an existing control group that is similar to the experimental group instead of random selection of groups. The non-equivalent quasi-experimental design is considered more relevant with the research design when the true experimental does not fit in this study.

A non-equivalent quasi-experimental pre-test and post-test design will be carried out for seven classes using fraction concepted activities using virtual manipulative approach to determine the effectiveness among Year Four pupils. Quasi-experimental design is regularly used to measure the effectiveness of a program if the subject cannot be distributed randomly (Chua, 2012) which is appropriate in this study. In order to determine the influence of an intervention with the target population, experimental design was used. An experimental design is more appropriate research design for this study in comparable to the correlation design as this helps in the process of testing the significant differences between both the experimental and observed groups to answer research hypothesis in this study.

Meanwhile, a pre-test provides a measure on few aspects or specific characteristics assessed for pupils in both groups prior to receiving an intervention. A post-test is a measure on few aspects or specific characteristics that are assessed for pupils after an intervention. This study measured pupil's success in understanding the concept of fraction after using Virtual Manipulative approach. Previous study revealed that a pretest–posttest comparison of pupils' achievement in understanding concept of fraction provided an accurate data (Campbell & Stanley, 1963). The following Table 3.1 illustrates the research design for this study.

Table 3.1

Quasi-experimental pretest posttest design

Experimental Group	O_1	X_1	O_2
Control Group	O_1	X_2	O_2

Adapted from Campbell and Erlebacher (1970) Experimental and Quasi - Experimental Design for Research

O_1 : Pretest X_1: Taught using Virtual Manipulative Method

O_2 : Posttest X_2: Taught using Traditional Method

In this study, to test the pretest scoring the pupils in both control and treatment group were given a test prior to teaching the concept of fraction. It is vital to record the scoring as it will determine the effectiveness of virtual manipulative method's knowledge that the pupils have gained later. After the pretest, both the groups are provided with similar mathematical instructions and teaching activities for the fraction lessons. The control group was given teaching activities without any virtual manipulative tool whereas the treatment group was presented with virtual manipulative tool as their teaching material for this topic of interest. After the activities, both the control group and treatment group will be given a posttest for scoring again after experiencing the instruction program

3.3 Sample of the Study

The population of this study includes all year four pupils from Klang district was elected as the sample for this study. The sample of this study was selected using convenience sampling method; eighty pupils from government primary schools were from Sekolah Kebangsaan Methodist Pelabuhan Klang. Convenience sampling was employed as these students were easier to contact or to reach out too. The researcher used a nonequivalent groups design but ensured that both groups are as similar as possible.

Moreover, the convenience sampling is vital for this study as it fulfills certain criteria of choice; first of all this study was planned to generalize the findings to the population; thus, probability sampling could not be used in this study (Merriam, 2002). Secondly, sample and location of this study were chosen based on the needs of the research. According to Etikan, Musa, and Alkassim (2016) convenience sampling is suitable when the sample of a study is chosen to satisfies specific criteria and those selected have the required information to fulfil the research objectives. Besides that, this sampling method was employed on the basis postulation that the researcher can explore, comprehend and concentrate in depth on specific cases (Merriam, 2002). Moreover, convenience sampling method is more convenient to analyze and to collect data during pilot study as it assist in identify and addressing issues that are associated with limitations.

However, the convenience sampling method has drawbacks. Convenience sampling method's variability and biases cannot be measured or controlled (Acharya, Prakash, Saxena, & Nigam, 2013). The results from the data obtained cannot be generalized beyond the sample. These limitations can be shafted by providing judgements that are based on clearer criteria such as theoretical framework (Acharya et al., 2013). Moreover, the limitation can be accepted because if common problems are notified among lower primary pupils. On that purpose, Year four pupils were chosen and they represented other lower primary pupils with similar characteristics.

Furthermore, the school practices co-educational system where both female and male study in the same institution. This particular school is facilitated with computer laboratory with sufficient number of working desktops to ensure smooth teaching and learning activities. The computer lab equipped with 42 computers with the strong internet connection. In addition, the school principal is very cooperative and supports the use of technology to improve pupil's academic achievement. With these reasons considered, this school was chosen as it aids in assisting researchers to carry out the studies.

Forty pupils were in experimental group (taught using virtual manipulative method) meanwhile the remaining forty pupils were in control group. Since all the samples were mixed abilities pupils therefore the samples were randomly assigned into two groups. According to Fraenkel, Wallen, and Hyun (1993), a sample size needs to be more than thirty units because of the assumption that the normal distribution is usually met when the subject numbers more than thirty units. The sample selection is made on the recommendation of the school head of mathematics with the provided criteria such as willingness to participate in the study and will be actively involved. This year four pupils were not to be tested for public examinations. The pupils chosen for this study achieved averagely in mathematics. Students were taught mathematics as a subject based on 'Dokumen Standard Kurikulum dan Pentaksiran' (DSKP) prepared by the Curriculum Development Centre of the Ministry of Education Malaysia

3.4 Instrument of the Study

Instrument that measured the comprehensiveness of the fraction concept among year four was adapted from Boulet (1996) as cited in Stewart (2005) in his research "Making Sense of Students' Understanding of Fractions: An Exploratory Study of Sixth Graders' Construction of Fraction Concepts Through the Use of Physical Referents and Real-World Representations". The name of the instrument used in this study "Understanding the Concept of fraction Assessment". The instrument consisted of 10 multiple choice questions and five open-ended questions; a copy of the instrument is included as Appendix A. The main purpose of this test was to obtain a detailed diagnostic skill profile for each learner as it relates to the pupils' current understanding of fractions and to verify their understanding of fraction.

Prior to generating the current instrument, the draft instrument consisted of seven multiple choice and 18 free-response items however, the when the validity and reliability of the were performed the items were reduced to 10 multiple choice questions and five open-ended questions which were more relevant and in line with the hypothesis as well as the objectives of this study.

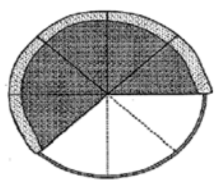
The items of the instruments gathered from a few resources as per Table 3.2 below, such as NAEP (sample test question), from previous studies related to fractions (Boulet, 1996; Mack, 1990) and from the written test databank of the Rational Number Project [RNP] (K Cramer, Behr, Post, & Lesh, 1997). It uses a paper and pencil procedure and pupils are given 60 minutes to complete it.

Pretest carried out to measure the existing comprehensiveness of the fraction concept that pupils have. Posttest was carried out to measure the effectiveness of the intervention on the understanding concept of fraction between the experimental group and control group. Pretest will be given before the intervention to determine the differences in understanding the concept of fractions between the control group and experimental group. Meanwhile, the Posttest will be given after the intervention to examine the impact of the intervention using virtual manipulative method for the experimental group. Score 1 is given for the correct answer for multiple choice questions and Score 1 till 3 is given for the correct answer for each answer according to rubric scale whereas score 0 is given for the incorrect answer. The total score of the test is 25 and the score will be converted into percentage. The instrument was built to ensure that pupil provide answer based on conceptual understanding on fraction.

In addition, back translation method was employed for this study. This re-translation was completed by two teachers in mathematics who were embroiled in this study and who have no prior knowledge of the goals and context of this research. Both teacher's proficiency level in English and Bahasa Melayu is advanced, they are well versed with vocabularies that are associated with Mathematics. The original document was used throughout the translation process to ensure that the consistency of the intended meaning of sentences were not affected. Besides that, back translation offers additional checks and different translations (Pym, 2009). The accuracy of translation is vital in this study as it enables pupils to comprehend the instrument given.

Table 3.2

Instrument Description

No	Questions	Resources
1	Rajah di bawah menunjukkan bahagian pizza yang telah dimakan oleh Ali. Berapakah bahagian pizza yang tinggal ? *The figure below shows the part of a pizza has been eaten by Ali. How many parts of the pizza are left uneaten?*  A. $\frac{3}{8}$ B. $\frac{3}{5}$ C. $\frac{5}{8}$ D. $\frac{5}{3}$	Sources : Boulet (1996) and Mack (1990) Original Question: The figure below shows that part of a pizza has been eaten. What part of the pizza is still there?

For this question, pupils should recognize the number of parts shaded or unshaded.

Table 3.2 (continued)

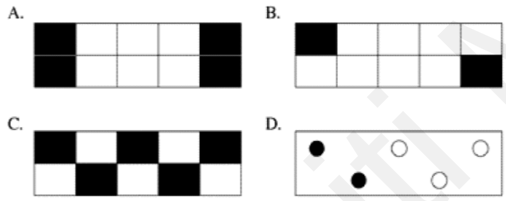
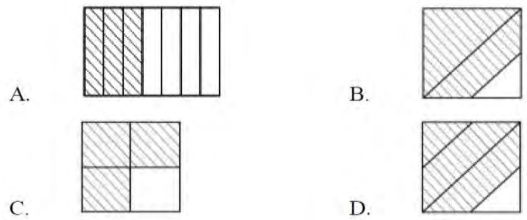
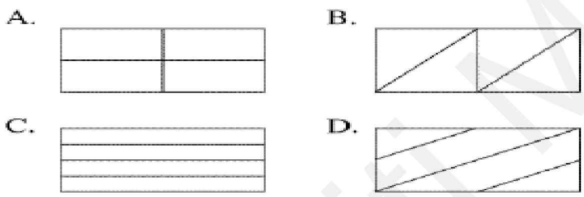
No	Questions	Resources
2	Apakah pecahan bahagian yang berlorek? *What fraction of the circle is shaded?*  A. Di antara 0 dan $\frac{1}{4}$ / *Between 0 dan* $\frac{1}{4}$ B. Di antara $\frac{1}{4}$ *dan* $\frac{1}{2}$ / *Between* $\frac{1}{4}$ *dan* $\frac{1}{2}$ C. Di antara $\frac{1}{2}$ *dan* $\frac{3}{4}$ / *Between* $\frac{1}{2}$ *dan* $\frac{3}{4}$ D. Di antara $\frac{3}{4}$ *dan* 1 / *Between* $\frac{3}{4}$ *dan* 1	Sources : Boulet (1996) and Mack (1990)
For this question, pupils should recognize the number of parts shaded or unshaded by estimate the fraction.		
3.	Antara rajah berikut yang mana menunjukkan $\frac{2}{5}$ besamaan dengan $\frac{4}{10}$. *Which picture shows that* $\frac{2}{5}$ *is equivalent to* $\frac{4}{10}$  	Sources : Boulet (1996) and Mack (1990)
This question, pupils should recognize the equivalent fractions.		
4.	Antara berikut yang manakah pecahan yang terkecil? *Which of these fractions is smallest?* A. $\frac{1}{6}$ B. $\frac{2}{3}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$	Sources : Boulet (1996) and Mack (1990)
This question, pupils should recognize the unit fraction and smallest fractions and able to arrange the fractions.		
5.	Murid, Pn Aminah di kehendaki menerangkan mengapa $\frac{4}{5}$ besar daripada $\frac{2}{3}$. Pernyataan siapakah paling tepat? *Pupils in Pn Aminah's class were asked to tell why* $\frac{4}{5}$ *is greater than* $\frac{2}{3}$. *Whose reason is the best?* A. Kelly kata, " kerana 4 lebih besar dari 2" *Kelly said, "Because 4 is greater than 2."* B. Khairul kata, " kerana 5 lebih besar daripada 3" *Khairul said, "Because 5 is larger than 3."* C. Kim kata, "kerana $\frac{4}{5}$ lebih dekat daripada $\frac{2}{3}$ ke 1." *Kim said, "Because* $\frac{4}{5}$ *is closer than* $\frac{2}{3}$ *to 1."* D. Devi kata, "kerana 4 +5 adalah lebih daripada 2+3." *Devi said, "Because 4 +5 is more than 2+3."*	Sources: Boulet (1996) and Mack (1990) Original question: Students in Mrs. Johnson's class were asked to tell why $\frac{4}{5}$ is greater than $\frac{2}{3}$. Whose reason is the best*?*
This question, pupils should able to compare two fractions and give the reasons.		

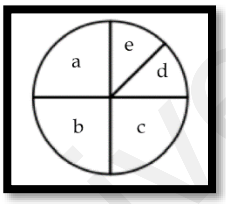
Table 3.2 (continued)

No	Questions	Resources
6.	Bulatkan rajah yang menunjukka pecahan $\frac{3}{4}$ *Circle the diagram that shows $\frac{3}{4}$.* 	Sources : NAEP (Sample Item) Original Question: Which shows 3/4 of the picture shaded?

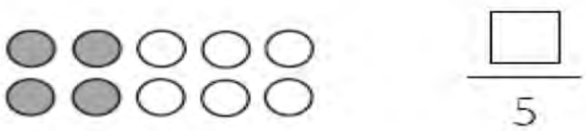
This question, pupils should recognize the inequality of the parts in similar wholes regardless of the equal number of parts shaded or unshaded.

No	Questions	Resources
7.	Bulatkan, segiempat yang tidak dibahagikan kepada 4 bahagian yang sama besar. *Circle the rectangle that not divided into four equal parts* 	Sources : NAEP (Sample Item) Original Question: Which rectangle is not divided into four equal parts?

This question, pupils should recognize the inequality of the parts of similar wholes that are partitioned differently.

No	Questions	Resources
8	 Apakah pecahan yang mewakili bahagian e? *What fraction of the circle is part e?* A. $\frac{3}{4}$ B. $\frac{1}{4}$ C. $\frac{1}{8}$ D. $\frac{1}{6}$	Source: Boulet (1996) and Mack (1990) Open ended question changes to multiple question.

This question investigates pupils' knowledge of the role of equal parts and naming the fraction. By fact that the parts constitute the whole and it must be equal in area.

No	Questions	Resources
9	Berapakah nilai 5 yang telah dilorekkan? *How many fifths are shaded?*  A. 2 B. 4 C. 5 D. 0	Source: Boulet (1996) and Mack (1990)

This question, pupils should recognize the whole from part of set

Table 3.2 (continued)

No	Questions	Resources
10	Antara pecahan berikut yang manakah lebih besar daripada $\frac{1}{2}$? *Which of these fractions is larger than* $\frac{1}{2}$*?* A. $\frac{3}{5}$ B. $\frac{3}{6}$ C. $\frac{3}{8}$ D. $\frac{3}{10}$	Source: Boulet (1996) and Mack (1990)

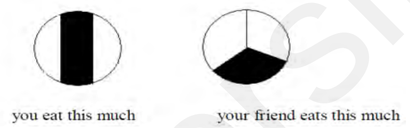
This question, pupils should recognize the unit fraction and able to compare fractions.

11	Lukiskan gambarajah yang menunjukkan pecahan lima per lapan dalam dua cara yang berbeza? *What does the fraction five-eight looks like? Using a diagram show this fraction in two different way.*	Sources : Boulet (1996) and (Mack, 1990)

This question requires pupil to generate two different representation of the fraction. Expecting pupils to produce a diagram (3 model – area, length and set) and a symbolic notation of fraction and a word definition for a fraction.

12	Terdapat berapa sukuan (perempat) dalam satu keseluruhan? Tunjukan jawapan anda dalam gambarajah dan simbol pecahan. *How many fourth are in a whole? Provide your answer in diagram with fraction symbol?*	Sources : Boulet (1996) and Mack (1990)

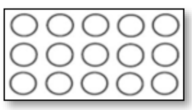
This question is to test the fact that the sum of the equal parts makes up the whole which is an essential point in the understanding of the fraction.

13	Rajah dibawah mewakili dua pizza yang sama saiz, satu untuk anda dan satu lagi untuk rakan anda. *The circles below represent two pies of the same size – one for you and one for your friend.*  Adakah anda makan pizza yang sama banyak seperti rakan anda. Jelaskan jawapan anda. *Did you eat as much pie as your friend? Explain your answer?*	Sources : Boulet (1993)

This question, pupils should recognize the inequality of the parts in similar wholes regardless of the equal number of parts shaded or unshaded.

14	6 orang berkongsi 3 pizza.Berapa bahagian setiap orand dapat. Tunjukkan jawapan dalam gambarajah. *6 persons shared 3 pizzas. How much was each person's share? Represent your answer in diagram.*	Source: Boulet (1996) and Mack (1990)

This question, pupils should recognize the equal share and partitioned.

15	Lorekkan $\frac{2}{3}$ dalam gambarajah yang diberi.Jelaskan jawapan anda. *Shade* $\frac{2}{3}$*.Explain your answer.* 	Source: Boulet (1996) and Mack (1990)

This question, pupils should recognize the whole from part of set

3.4.1 Rubric

Rubric is designed to score pupils' responses in pre and post-tests depending on how accurate and to what extent they managed to complete and respond to each question. Pupils may receive 1, 2, or 3 points for each open-ended question. In addition, pupils who failed to identify the key words, hidden condition of the problem, and goal of the problem will be scored 0. Meanwhile for 10 multiple choice questions, the given score is 1 for each correct answer and 0 for incorrect answer. Therefore, the total score from this pre and posttests is 25. This rubric was adapted from Stewart (2005), refer to the Appendix B for more details of the rubric.

3.5 Reliability and Validity

The instrument used to assess the effectiveness of virtual manipulative method were developed based on students' enhancement on the comprehensiveness of the fraction concept. The instrument was evaluated by two mathematics experts from same schools before it was administered to the pupils.

The criteria for the evaluation were based on the validation content that is relevant to the taught concept and pilot study. It is essential to check the validity, reliability and practicality of an instrument to draw warranted and conclusions from the sample that is admissible in this study. An instrument is valid when it is accurately measuring what is supposed to measure (Campbell & Stanley, 1963).

Validity is vital as it checks the instruments 'accuracy in measuring the intended context of the research. In determining the validity of the pretest and posttest, content validity was used. Content validity refers to the content of items, the scores from the instrument as it checks the test's content relates to what the test is intended to measure (Creswell, 2002). The researcher explored the literature from theories, previous instruments, frameworks and past research findings for the understanding concept of fraction instrument. To accomplish the validity, the test was given to two mathematics experts to evaluate and validate the content. One of the experts is a mathematics teacher with 17 years of experience and a panel head of mathematics' board. Meanwhile, the other expert has nine years of teaching experience and experienced in marking Ujian Pencapaian Sekolah Rendah (UPSR) examination mathematics paper for the past two years and prepares exam items in the school frequently. Both were informed on the purpose of this study and there were requested to review the concepts and skills focused on, the difficulty level of the concept, clarity of problems as well as the appropriateness of language and terms used.

Pilot study is a small-scale study that is implemented before the actual study. It is important to conduct a pilot study as the data collected may alert the researcher if there is any drawbacks in the actual study, the outline of the research proposal is less robust or the use of the research instrument is inappropriate. Hence it is crucial to conduct pilot test to ensure the instrument is valid and reliable (Chua, 2006). The pilot study was conducted with year four pupils who studies in a different school. These students also learned the concept of fraction as it is stated in their syllabus (Malaysia, 2015). These pupils represented the respondents of the study very closely. A group of pupils were administered to the understanding the concept of fraction assessment, with the help of a school teachers who were responsible in distributing and collecting the test papers. The consistency of the test was calculated using Cronbach's alpha coefficient. There were 30 subjects in the sample (N = 30) and the instrument obtained Cronbach alpha of .724; hence the coefficient indicated that the instrument was reliable. The reliability of the pre-test and post-test is accepted because the values are > 0.7.

Followed by, inter-rater reliability were conducted by two mathematics expert teachers to ensure the rubric scoring of two different examiners were consistent and precise. Inter-rater reliability helps to identify whether the rubric of the instrument considered relatively subjective and precise scoring (Creswell & David, 2017). The Pearson correlation was used in this study to measure the consistency of the raters were in marking the Understanding of Fraction Assessment. Correlation, coefficient was used in this study, as it is an excellent tool to measure the association between two independent raters. The rater was a school mathematics teacher with 17 years of experience and nine years' experience teaching primary school mathematics. Results of the Pearson correlation coefficient indicated a very strong and positive correlation between the two raters' scoring, r (30) = .578, $p < .01$. This indicated strong positive consistency between both raters in scoring, suggesting that the understanding of fraction assessment is reliable.

3.6 Instructional Activities

The researcher was aware of the importance of knowing the fundamentals of selecting the appropriate assessment tools and then constructively plan and conduct classroom environment to make pupils engage in raw data or primary sources, aiming to develop pupils' understanding in the concept of fraction. Constructivist theory is a great value in this study in efforts to help pupils engaged with their lesson.

The researcher prepared seven activities for the lessons on Fraction. Pupils from experimental group will be using computer lab during the lesson. The experimental group learning fraction will be using virtual manipulative teaching method. The activities were planned to help pupils to investigate and reflect on the concept of fraction. Pupils are exposed to online quizzes, worksheet and virtual manipulative. Before starting each lesson, pupils will be guided on how to use the particular visual manipulative software. These activities were adapted from the government mathematics textbook (year one until year four) used in the school, National Assessment of Educational Progress sample questions (NAEP), Rational Number Project, and Elementary and Middle School Mathematics Teaching Developmentally book. The objectives of each activity are listed in Table 3.3. and the lesson plan in Appendix C.

On the other hand, the control group will be taught using traditional method of teaching without any intervention or treatment. This group of pupils used only text book, notebook and work book or work sheet and the lesson conducted in classroom. . Even though, the lesson carried out using traditional method of teaching, but researcher used 21^{st} century learning approach. The researcher prepared the lesson based on the 21^{st} century learning approach, where the pupils familiar with the 21^{st} terms like shoulder partner, gallery walk and rally table. In 21^{st} century learning pupils actively involved in group discussion, sharing information, idea and answer and integration of other resources such as video. After each instructional activity, pupils will be given individual task and group activity as well as drilling practice to reinforce their understanding in the concept of fractions.

Furthermore, the objectives for traditional teaching method same as experimental group were listed in Table 3.3. and the lesson plan attached in Appendix D. Besides that, the same teacher who taught the two groups as to eliminate researcher's biasness during the teaching and learning process. After the intervention, both control and experimental group will be taking posttest. The content of instructional activities reviewed by two mathematics teachers who have 17 years of experience and nine years of experience respectively. They were requested to assess the suitability and the difficulty level of the activities for Year Four pupils. Not only that, the activities conducted should be in line with the KSSR Syllabus.

Table 3.3

Content of Instructional Activities

Date / Duration	Topic	Objectives *By the end of the lesson, pupils should be able to:*
1 Nov 2019	Pretest – Control group and experimental group	
Lesson 1 5/11/2019 1 hour 30 minutes	Introduction to concept of sharing	1. tell the concept sharing 2. recognize fraction terminology such as half or one of two equal or same size. 3. discover a need for fractions in daily life.
Lesson 2 6/11/2019 1 hour 30 minutes	Concept of Part of Whole	1. recognize that when a whole is divided into equal parts each of those parts represents a fraction 2. identify the numerator and denominator 3. name the faction from shading diagram and vice versa
Lesson 3 7/11/2019 1 hour 30 minutes	Concept of partitioning in area model	1. partitioning the area model to equal parts 2. identify the fraction from the diagram – same shape same size and different shape same size
Lesson 4 12/11/2019 1 hour 30 minutes	Concept of partitioning in length model	1. state the whole number as a fraction in number line 2. discover and show part of whole for Length model. 3. represent fractions on a number line.
Lesson 5 13/11/2019 1 hour 30 minutes	Concept of partitioning in set model	1. divide sets into equal parts and identify the parts using fractional names. 2. identify fractions within a set.
Lesson 6 14/11/2019 1 hour 30 minutes	Concept of equivalent fraction	1. explain that when the numerators are the same size the denominator can be used to determine which fraction is smaller or larger 2. compare two fractions with like denominators indicating which is smaller and which is larger. 3. Pupils will use the symbols for greater than, less than and equal to compare fractions.
Lesson 7 19/11/2019 1 hour 30 minutes	Concept of ordering and comparing fraction	1. Able to compare two fractions 2. Able to ordering the fractions
20/11/2019	Post test - Control group and experimental group	

3.7 Ethics

After conducting the pilot test for the instrument, the researcher conducted the actual study. The researcher requested permission to conduct the study in the respective schools. Before this study begins, each pupil was provided with consent letter to fill in to ensure they were willing to serve as a subject in this study. Their signature indicates willingness to participate in this study and was later collected by researcher as a proof. Besides that, researcher also distributed a hand-out of information on researcher and study being conducted such as purpose of the study, duration and procedures that will be carried out in a study to the pupils and personnel involved.

3.8 Data Collection Procedure

The test was administered in two classrooms in view of the class size, which is about forty pupils per class, treatment and control group. A special test arrangement were made to accommodate all the 40 pupils in the classroom. This arrangement aimed to prevent plagiarism among pupils during the test. Researcher with the help of class teachers assigned one hour as the test period for pupils to answer the instrument. Prior to the test, these pupils were ascertained that the outcome of the test will not affect their school grades. The written responses were collected immediately after pupils have completed the test.

3.9 Data Analysis

Data analysis is a process to organize the data in a significant way and represent it in a solid manner. Results obtained from this study were further analyzed using Statistical Package for Social Sciences (SPSS) software. Pupils' responses for pre-test and post-test were checked based on the rubric set. The data were analyzed based on research questions (refer to table 3.4).

Table 3.4

Data Analysis Method of Each Research Questions

Research Questions	**Statistical Analysis**
RQ1. Is there any significance difference between the pretest mean score of year four pupils taught using the virtual manipulative method compared to those taught using the traditional method?	Independent T-Test Two tailed
RQ2. Is there any significant difference between the pre and posttest mean score of year four pupils taught using the virtual manipulative method?	Paired T-Test One-tailed
RQ3. Is there any significance difference between the posttest mean score of year four pupils taught using the virtual manipulative method compared to those taught using the traditional method?	Independent T-Test One-tailed
RQ4. Is there any significant difference between the pre and posttest mean score of year four pupils taught using the traditional method?	Paired T-Test One-tailed

To answer first and third research questions, independent T-test was employed. This test was adopted as it aids in finding the significant differences between the two research questions. Firstly the data from two groups were independent to one another as it was distributed after testing using Shapiro-Wilk. First research question was to identify whether there is any significance difference between the pre-test mean score of virtual manipulative teaching method in comparable to the direct teaching method among year four pupils' in understanding the concept of fraction.

Meanwhile the third research question is to find out the significance difference between posttest mean score of year four pupils taught using the virtual manipulative method compared to those who taught using the traditional method. In addition, based on the research hypothesis, there is a predicted direction indicated that the mean of post-test score pupils taught using the virtual manipulative method will be higher than the mean of posttest scores in pupils taught using the traditional method group. Hence the type of the test will be a one- tailed test.

Besides that, to answer research question two and four, paired T-test was used. This test was chosen as the two groups must were dependent on one another and the had different variables (pretest and posttest) between two samples that were normally distributed. Research question two is to find out if there is any significant difference between the pre and posttest mean score of year four pupils taught using the virtual manipulative method. Furthermore, there is a predicted direction based on the research hypothesis indicated that the mean of post-test score will be higher than the mean of pre-test scores in pupils taught using the virtual manipulative method. Hence the type of the test will be a one- tailed test.

In addition, to analysis research question four to analyze if there is any significant difference between the pre and posttest mean score of year four pupils taught using the traditional method. The research hypothesis indicated that the mean of post-test score will be higher than the mean of pre-test scores in pupils taught using the traditional method. Hence the type of the test will be a one- tailed test.

3.10 Summary

Chapter three discussed on research design, population and sample, data collection method, instrumentation, validity and reliability of instruments, pilot study and data analysis method. This study used quasi-experimental design to identify the effect of using Virtual Manipulative Method to enhance the comprehensiveness of grasping the fraction concpet among year four pupils. Pre-test and post-test were used as data collection tool in this study. Inferential analysis was employed as the data analysis tool. The analysis and findings of the study were presented in chapter four. Interpretation of the data, conclusion, discussion and implication were presented in chapter five.

CHAPTER 4

FINDINGS

4.1 Introduction

This chapter presents the data analysis of the results based on the data collected. The data were obtained from the understanding concept of the fraction assessment using the Virtual Manipulative Method in learning concept of fraction from 80 pupils in year four. The findings of the study gave an insight to Virtual Manipulative Method in enhancing pupils' comprehensiveness to the concept of fraction.

This study aimed to evaluate the effectiveness of using the Virtual Manipulative teaching method in enhancing year four pupils understanding the of fraction. Understanding the Concept of Fraction Assessment was administered as a pretest before the intervention was carried out. Then, test was again administered at the end of the intervention as posttest. In order to analyze the data and answer the research questions, statistical tool SPSS was used to run independent *t*-test and paired *t*-test.

4.2 Finding of Analysis First Research Question

This section presents the inference and data analysis of four research questions from this study. Before answering the first research question, there are four main inference that needs to be address in order to carry out the independent t-test. The first inference that needs to be addressed is the dependent variable that should be measured on an interval or ratio measurement scale and the independent variable should consists of two or more categorical groups (Creswell & Creswell, 2017). Since, the data collected for this research is based on the scores of the respondent from two different groups namely; virtual manipulative method and traditional method groups hence the first inference is met. The next interference is normality test for data distributions. One of the basic conditions for inferential statistics such as t-test is that the data collected from the sample should be distributed normally (Chua, 2006). Therefore, the Skewness and Kurtosis was conducted to test the normality and the result are presented in Table 4.1 with consideration for the z- values that should be somewhere in the span of -1.96 to +1.96 for pretest. The z score for statistical skewness (.110) is .41 and kurtosis (-.326) is -.61 suggesting that the total score of pretests were normally distributed for the pupils in both virtual manipulative method and traditional method groups.

Table 4.1

Skewness & Kurtosis of Pretest Scores

	N	*Skewness*		*Kurtosis*	
	Statistics	*Statistics*	*SE*	*Statistics*	*SE*
Pretest	80	.110	.269	-.326	.532

According to Creswell and Creswell (2017) Shapiro-Wilk test for normality is the commonest test in most study which is related to quantitative method. Moreover, the sample size in this study was less than 50 hence Shapiro-Wilk test was more relevant. The Table 4.2 shows the result of Shapiro-Wilk is greater than significant value .05. with $p = .165$. Therefore, the data were normally distributed.

Table 4.2

Shapiro-Wilk Test of Pretest Scores

	Shapiro-Wilk		
	Statistics	*df*	*p*
Pretest	.977	80	.165

The last inference is to check the homogeneity of variance, the independent t-test must infer the variances between the two groups as it measures the population equally. If the variances are unequal, this can affect the Type I error rate. The assumption of homogeneity of variance tested using Levene's Test of Equality of Variances. The Table 4.3 shows that the homogeneity of variance assumption was $F= 1.95$, $p = .16$. As resulting $p >.05$ means that variances are infer equal. After the inference was met, an independent samples *t*- test conducted to answer the first research question as stated below,

Is there any significance difference between the pretest mean score of year pupils taught using the virtual manipulative method compared to those taught using the traditional method?

H_O : There is no significance difference between the pretest mean score of year 4 pupils taught using the virtual manipulative method compared to those taught using the traditional method.

H_1 1: There is a significance difference between the pretest mean score of year 4 pupils taught using the virtual manipulative method compared to those taught using the traditional method.

Based on Chua (2013) when the significant value is smaller than .05, then the result is significant. Table 4.3 shows the independent t-test significant results (2-tailed) shows the significant value is more than .05. Thus, there is no significant difference in the scores for virtual manipulative method ($M = 10.18$, $SD = 1.97$) and traditional method ($M = 10.15$, $SD=2.43$) conditions. A very small statistically significant mean increases of .025, 95% CI [-.96, 1.01], t (78) =. 05, $p = .96$ with a smaller effect size d = .014 and lower power value (0.6). Thus, this result failed to reject the null hypothesis as the p value is greater than the significant level of .05 and the effect size and power value indicates that use of Virtual Manipulative has smaller effect on students' achievement in learning the concept of fraction. Therefore, there is not significant difference in the results and shows that there is no sufficient evidence that the two groups are significantly not different.

Table 4.3

Result of the independent t-test on the pretest of both virtual manipulative and the traditional methods

	Levene's Test for Equality of Variance		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variance assumed	1.95	.166	.05	78	.960	.025	.49	-.96	1.01
Equal variance not assumed			.05	74.79	.960	.025	.49	-.96	1.01

4.3 Finding of Analysis Second Research Question

The next section focuses on the second research question. Before test is conducted, inference must be met hence the first and second inference are explained in the first research question findings hence the next inference is Skewness and Kurtosis as presented in Table 4.4 with consideration for the z- values that should between the span of -1.96 to +1.96 . The results obtained for experimental group on pretest is, statistical skewness (-.105) with a z-score of -.28 and kurtosis (-.623) with z-score of -.85. Meanwhile, for posttest, statistical skewness (-.623), z-score of .75 and kurtosis (-.280) whereby z-score -.38. The z-score for both the pre and posttest are within the range hence the total score of pre-posttests were normally distributed for the students in experimental group.

Table 4.4

Skewness & Kurtosis of Pre-Posttest Scores for Experimental Group

	N	*Skewness*		*Kurtosis*	
	Statistics	*Statistics*	*SE*	*Statistics*	*SE*
Pretest	40	-.105	.374	-.623	.733
Posttest	40	.280	.374	-.280	.733

Further analysis for normality inference was analyzed and presented in Table 4.5, Shapiro-Wilk test and the data were normally distributed for pupils learned using the virtual manipulative method, (pretest=.575 and posttest=.457).

Table 4.5

Shapiro-Wilk Test of Pre-Posttest Scores for Experimental Group

	Teaching approach	Shapiro-Wilk		
		Statistics	*df*	*p*
Pretest	virtual manipulative method	.977	40	.575
Posttest		.973	40	.457

Next assumption, which needs to be met, is outliers. Outliers can reduce the accuracy of paired t-test results and effect the statistical significance of the test. Therefore, it is important to detect possible outliers. This study used boxplot to identify the extreme values. Based on the Figure 4.1 the results of boxplot showed that there were no outliers hence the requirement to run paired t-test were fulfilled.

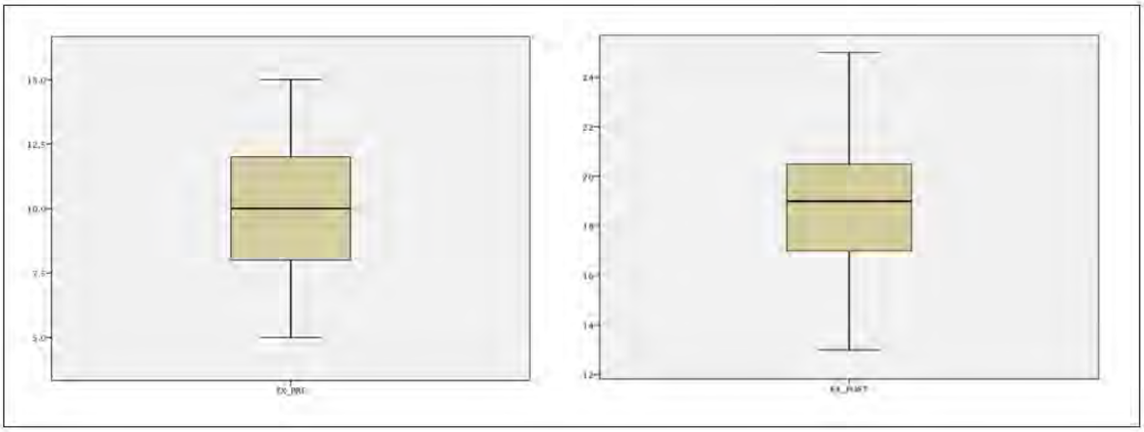


Figure 4.1. Assumption of no Outlier

After all the assumptions were met, the second research question was analyzed.

Is there any significant difference between the pre and posttest mean score of year 4 pupils taught using the virtual manipulative method?

H_O: There is no significant difference between the pre and posttest mean score of year 4 pupils taught using the virtual manipulative method.

H_1: There is a significant difference between the pre and posttest mean score of year 4 pupils taught using the virtual manipulative method.

A two-tailed paired-samples t-test was conducted to evaluate the statistically significant differences between the pre and posttest mean score of year 4 pupils taught using the virtual manipulative method. Based on Cohen (1992) the effects on the relationship should be included in this study in order to standardize the differences between the two variables in this research.

Therefore, this studies' effect the size as calculated using the mean differences and standard error, $(18.825 - 10.15) / 2.64 = 3.3$. As Table 4.6 shows, a statistically significant mean increases of 8.67, 95% CI [7.70, 9.64], $t\,(39) = 18.14$, $p < .05$ with a large effect size $d = 3.3$. As the p value is less than the significant level of .05 thus, the null hypothesis is rejected and the alternative hypothesis is accepted. Moreover, the power value of .6088 falls in medium level, rejecting the null hypothesis.

Therefore, the result is significantly different and there is no sufficient evidence to confer that the two groups are significantly similar after the intervention

Table 4.6

Paired Samples Test for Experimental Group

Teaching Approach	Test	M	SD	S.E	95% CL LL	95% CL UL	t	df	p
Using the virtual manipulative	Posttest Pretest	8.67	3.02	.47	7.70	9.64	18.14	39	.000

These results indicate that using virtual manipulative in learning concept of fraction do cause differences in pupils' achievement. The effect size indicates that use of virtual manipulative method has a large effect on pupil's achievement in understanding the concept of fraction. Thus, the data provide sufficient evidence to conclude that the Year Four pupils' abilities in the experimental group advanced after using virtual manipulative.

4.4 Finding of Analysis Third Research Question

Since the result of the analysis in first research question shows that there is no significant difference between the virtual manipulative method and the traditional method groups in pretest score therefore the independent t-test was conducted to answer the third research question. Table 4.7 shows the values of the skewness and kurtosis of posttest of virtual manipulative method and traditional method. The result showed that the traditional method posttest statistical skewness (.205) with z score of .55 and kurtosis (-.921) with z score of -1.26 meanwhile virtual manipulative teaching method's posttest statistical skewness (.280) with z score of .75 and kurtosis (-.280) with z score of -.38. Both the z score values were in between the span of -

1.96 to +1.96 that the total score of posttests were normally distributed for the students in both groups.

Table 4.7

Skewness & Kurtosis of Posttest Scores

	N	*Skewness*		*Kurtosis*	
	Statistics	*Statistics*	*SE*	*Statistics*	*SE*
Posttest	80	-.316	.269	-.529	.532

Table 4.8 shows that data were normally distributed for pupils from both the groups, as assessed by Shapiro-Wilk test (pretest= .109).

Table 4.8

Shapiro-Wilk Test of Posttest Scores

	Shapiro-Wilk		
	Statistics	*df*	*p*
Posttest	.974	80	.109

All the inference is met hence an independent samples *t*-test was conducted to determine the pretest score of year 4 pupils taught using the virtual manipulative method compared to those taught using the traditional method based on the research question as stated below.

Is there any significance difference between the posttest mean score of year 4 pupils taught using the virtual manipulative method compared to those taught using the traditional method?

H_O: There is no significance difference between the posttest mean score of year 4 pupils taught using the virtual manipulative method compared to those taught using the traditional method.

H_1: There is a significance difference between the posttest mean score of year 4 pupils taught using the virtual manipulative method compared to those taught using the traditional method.

Table 4.9, shows the F test and p value of Levene's Test for Equality of Variances was reviewed to determine if the equal variances assumptions have been met. According to Levene's Test, the homogeneity of variance assumption of was F= 5.689, p = .001. had been that there is a significant difference in the scores for virtual manipulative method (*M* = 18.83, *SD* = 2.872) and traditional method (*M* = 14.18, *SD*=4.242) conditions; 95% CI [-6.255, -3.045], t (67.97) = -5.79, $p < .05$ with a larger effect size of 1.28 and power is medium with a value of .609 that rejects the null hypothesis. Moreover, the significance (2-tailed) value is .000, which is less than .05 therefore fails to reject the alternative hypothesis. Thus, there is a statistically significant difference between the posttest score of year 4 pupils taught using the virtual manipulative method compared to those taught using the traditional method.

Table 4.9

Result of the independent t-test on the pretest of both virtual manipulative and the traditional methods

	Levene's Test for Equality of Variance		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variance assumed	7.35	.008	-5.79	78	.000	-4.650	.806	-6.255	-3.045
Equal variance not assumed			-5.76	67.94	.000	-4.650	.806	-6.258	-3.042

4.5 Finding of Analysis Fourth Research Question

The values of the Skewness and Kurtosis are shown in Table 4.10, for pretest statistical skewness (.524) with z score is 1.45 and kurtosis (.374) with a z score of .19 and for posttest, statistical skewness (.205) and z score is .55 and kurtosis (-.921) with z score of -1.26 suggested that the total overall score of pre-posttests were normally distributed for the students in traditional method group (control group). Overall the z score values were in between the span of -1.96 to +1.96 which indicates that the total score of pre-posttests were normally distributed for the pupils

Table 4.10

Skewness & Kurtosis of Pre-Posttest Scores for Control Group

	N	*Skewness*		*Kurtosis*	
	Statistics	*Statistics*	*SE*	*Statistics*	*SE*
Pretest	40	.524	.374	.137	.733
Posttest	40	.205	.374	-.921	.733

Table 4.11 shows that data were normally distributed for pupils taught using the traditional method group, as assessed by Shapiro-Wilk test (pretest=.077 and posttest=.119).

Table 4.11

Shapiro-Wilk Test of Pre-Posttest Scores for Control Group

	Teaching approach	Shapiro-Wilk		
		Statistics	*df*	*p*
Pretest	Traditional	.977	40	.077
Posttest	method	.973	40	.119

Based on the Figure 4.2 the result of boxplot showed that there were no outliers hence the assumptions are met.

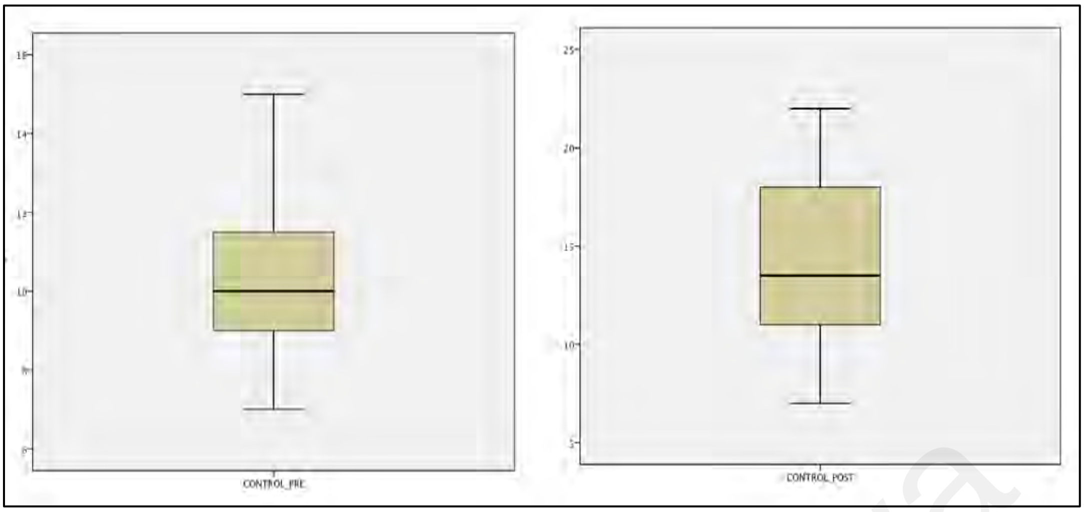


Figure 4.2. Assumption of no Outlier

After the inference was met, an independent samples *t*- test conducted to answer the last research question as stated below,

Is there any significant difference between the pre and posttest mean score of year 4 pupils taught using the traditional method?

H_O 4: There is no significant difference between the pre and posttest mean score of year 4 pupils taught using the traditional method.

H_1 4: There is a significant difference between the pre and posttest mean score of year 4 pupils taught using the traditional method.

A two-tailed paired-sample *t*-test was conducted to evaluate whether a significant mean difference between the pre and posttest mean score of year 4 pupils taught using the traditional method. The effect size was calculated using the mean differences and standard error, (14.175 – 10.175)/3.3 = 1.21. As Table 4.12 shows, a statistically significant mean increases of 4.00, 95% CI [2.88, 5.11], t (39) = 7.27, p < .05 with a large effect size d = 1.21. Since the p value is greater than the significant level of .05 thus, the null hypothesis failed to accept and the alternative hypothesis is accepted. Therefore, the result is significantly different and shows that there is no sufficient evidence that the two groups are significantly similar.

Table 4.12

Paired Samples Test for Control Group

Teaching Approach		M	SD	S.E	95% CL LL	95% CL UL	t	df	p
Using the traditional method	Posttest Pretest	4.00	3.47	.55	2.88	5.11	7.27	39	.000

These results indicate that using traditional method in learning concept of fraction do create changes in pupil's achievement. Thus, the data provide shows evidence to conclude that the Year Four pupils' abilities in the control group differ significantly after using learn using traditional method.

4.6 Summary

Results of the achievement test analyzed using independent t-test showed no significance difference between the pretest mean score of year 4 pupils taught using the virtual manipulative method compared to those taught using the traditional method. Meanwhile, independent t-test showed there is a significance difference between the posttest mean score of year 4 pupils taught using the virtual manipulative method compared to those taught using the traditional method. Pupil learned using Virtual Manipulative performed significantly better than students in the traditional method. Furthermore, the paired-samples *t*-test analysis reveals the experimental group performance enhanced after learning the concept of fraction using virtual manipulative. Even though, fourth research question revealed that pupil learned in traditional method showed improvement yet the pupil learned using virtual manipulative surpass the performance with higher mean score.

CHAPTER 5

DISCUSSIONS AND CONCLUSION

5.1 Introduction

In previous chapter, the findings of this study were analyzed using quasi-experimental design. To investigate the effect of using virtual manipulative teaching method towards enhancing the understanding of fraction concept among year four pupils. This chapter summarizes the major findings of the research based on the objectives of the study. The chapter also presents a comprehensive discussion from the major findings of the research and provides the conclusions. Finally, this chapter elaborates the implications of the study, and the recommendations for further research.

5.2 Summary of Findings

There are total of four research questions analyzed in Chapter four, this part presents the summary of the results.

5.2.1 Finding of First Research Question

Is there any significance difference between the pretest mean score of year 4 pupils taught using the virtual manipulative method compared to those taught using the traditional method?

The first research question was analyzed by independent *t*-test, the findings showed that there is no significant difference in year four pupils' pretest mean score between the virtual manipulative method and to those taught using the traditional method (M =. 025), t (78) = .050, $p > .05$. The significant value more than .05, which shows that there are no much differences between the two teaching methods. Thus, the data provide sufficient evidence to conclude that the year four pupil's knowledge before carrying out the research was at the same level.

5.2.2 Finding of Second Research Question

Is there any significant difference between the pre and posttest mean score of year 4 pupils taught using the virtual manipulative method?

The results of paired samples t-test shows that there was a significant difference between pre and posttest mean score of year 4 pupils after taught using the virtual manipulative method (M = 8.67, SD = 3.02), $t\ (39) = 18.14$, $p < .05$. This shows that by using virtual manipulative method, pupils'were to be able to do concept of fraction more effectively. Pupil who learned via virtual manipulative method enhanced more in posttest as they engaged during the lesson and had more time to reflect on their answers. Hence pupil in virtual manipulative group had performed better however the comparison is done between two groups in next research question to check the improvement in understanding fractions.

5.2.3 Finding of Third Research Question

Is there any significance difference between the posttest mean score of year 4 pupils taught using the virtual manipulative method compared to those taught using the traditional method?

Moving on to the next research question where an independent *t*-test was carried out to answer the second research question. The findings of pretest mean score between the virtual manipulative method and to those taught using the traditional method is (M = -4.650), $t\ (78) = -5.79$, $p < .05$ shows that there is a significant difference between the two methods. The mean difference value of -4.650 shows that there is a difference between the pupils learned using virtual manipulative method and to those learned using the traditional method. Pupils learned using virtual manipulative (mean score = 18.83) performed well compared to pupil learned using traditional method (mean score = 14.18). The findings revealed, even though pupil proficiency were at the same level during pretest, after the intervention pupil learned in traditional group had not performed well in comparable to the virtual manipulative groups scored. This is due to hands-on and implementation of new teaching style that probed the virtual manipulative group to be more engaged and participate in lesson. Meanwhile, traditional group continues with the same textbook with chalk-and-talk method which has been implemented in the past years.

5.2.4 Finding of Fourth Research Question

Is there any significant difference between the pre and posttest mean score of year 4 pupils taught using the traditional method?

The last research question was analyzed using paired t-test and the results shows that there was a significant difference between pre and posttest mean score of year 4 pupils after taught using the traditional method ($M = 4.00$, $SD = 3.47$), $t(39) = 7.27$, $p < .05$. This is evident that pupils were able to grasp and comprehend the fraction concept when traditional method was used to teach. Though the traditional group did not perform well as virtual manipulative group as analyzed in research question three but within the group, pre-posttest the pupils have showed difference, which means there is an improvement in their posttest score. After they complete the pretest, pupils somehow knew their scope and area of focused or concerned that needs to be addressed hence it could be one of reason to why there is difference in their scores. Besides that, grasping new knowledge with prior knowledge enable pupils' understanding as well as remembrances. Therefore, pupils should score higher than pretest.

5.3 Discussion of the Findings

This section discusses the findings of the four research questions. Before begin the discussion, the research questions are illustrated in figure 5.1 below for better understanding of the research outcomes.

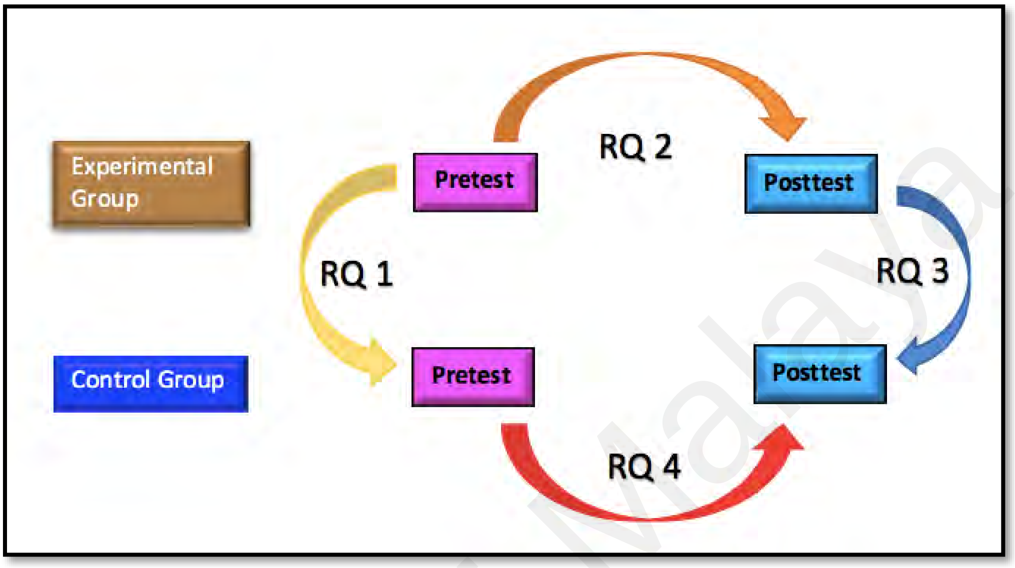


Figure 5.1. Summary of Research Questions

Firstly, the findings result confirms that the pupils from two groups have the same abilities and levels of proficiency in the subject. It is important to analyze the pretest scores to prevent biasness and to conduct appropriate statistical analysis. This might result in researcher selecting students from similar educational background as mentioned in chapter three. Therefore, the entire pupil from both groups have similar abilities and level of understanding in mathematics.

The experimental group pupils who were taught using virtual manipulative showed there is significant differences between pretest and posttest score. Based on the result of the pretest mean and posttest mean the pupils had improved their understanding in the concept of fraction after learning using virtual manipulative. Besides that, the comparison of posttest score between the groups which learned by traditional method and learned by virtual manipulative showed there are differences in their mean score. Pupils who learned using virtual manipulative showed higher mean value than those who learnt by traditional method. What is interesting in the mean value comparison is that, it shows pupils who learnt using virtual manipulative had greater improvement than pupil who learnt without any technology. Even though, pupil in traditional group showed there is a significant difference between pre and posttest yet, the pupil in virtual manipulative method had larger mean difference value in research question two. A clear effectiveness of virtual manipulative method in this study had enhanced the understanding of the fraction concept that will be identified in these findings.

Reflecting on problem statement stating the pupils had less interest in Mathematics but also have some sort of phobia against it, especially after being introduced to fractions (Lamon, 2013; Alghazo & Alghazo, 2017; Noorbaizura & Leong, 2013). Therefore, pupil lack in confidence and fear that they will disable to highlight their outstanding performance on the topic of fractions. Moreover, exam-oriented emphasized on algorithm, hence pupils are bound with calculation procedure without using reasoning skills to justify or state the reason for the particular result. Pupil could not visualize the fraction in terms of representation. These issues have caused pupils knowledge and their way of dealing with fractions were altered which resulted in poor performance, which in simple terms referred to as whole number bias phenomenon (Ni & Zhou, 2005; Vamvakoussi et al., 2012).

The present findings showed the issues addressed in problem statement is solved and the findings were consistent with other research which found elementary pupil learned fraction using multi-touch tabletop shows the effeteness was greater and enable pupils to easily collaborate with peers when practice their problem-solving skills (Hwang, Shadiev, Tseng, & Huang, 2015). In additional, as mentioned in literature review using constructivism theory for creating learning environment with technology that been a platform for pupils to explore, experience, construct converse and reflecting their answers clearly guided the teaching method as well as helped the pupils to understand the concept of fraction better. These results are supported by a study done by Markey, Power, and Booker (2003), whereby a constructivist approach used to plan a teaching and learning program to facilitate early notion of fractions understanding through games. Their study concluded that the program was successfully implemented and the gaming enable pupils to understand through using it and seeing it used. This study confirmed that constructivism along with manipulative method on fraction could have a positive effect on students' understanding on the fraction concept.

Further discussions can help to understand why and how the theory along with the technology had played an important role in enhancing pupil's understanding the concept of fractions. First of most, theory provided insight on role of a teacher when planning the lessons. In this study, the researcher has planned the instructional activities whereby putting the teacher as guider or facilitator rather than delivering the content of knowledge just to cover the syllabus. The urge to complete the syllabus was not been stressed on teacher hence the teaching is done in a holistic way. On that account, pupil was able to make connection between fraction and real-life situations by giving more importance to their future employee, intelligence and ability as more complex than the marks one receives on tests. A systematic learning method has made the pupil as an active participant in the instructional activities, their role as explainer, interpreter, questioner as well as reflector had made pupil to gauge in activities interestingly. In contrast, the traditional teaching method focused in the former perspective as the mathematics subject based on mastering a corpus of facts and procedures. Narli (2011) has supported that constructivism theory which does not support teacher-centered teaching method and encourages active learning had shown pupil's motivation as well as their understanding of the mathematics topics improved than the pupils learnt in traditional method. It is a widely held view that constructivism learning approaches had positive effects on student achievement and ideas (Narli & Baser, 2010; Osmanoglu & Dincer, 2018; Sanders, 2016; Zengin & Tatar, 2017)

5.3.1 Effectiveness of Virtual Manipulative Teaching Method compared to Traditional Teaching Method

The results showed that pupil's understanding the concept of fraction in virtual manipulative group significantly improved after the intervention. This might have resulted because of the capability of virtual manipulative and its unique features that allowed pupils to view the question in a colorful diagram, provided clear instruction on the right side of the screen, pupil can check their answers by themselves by clicking "Check" and generate more questions by clicking "New Fraction" (Figure 5.2).

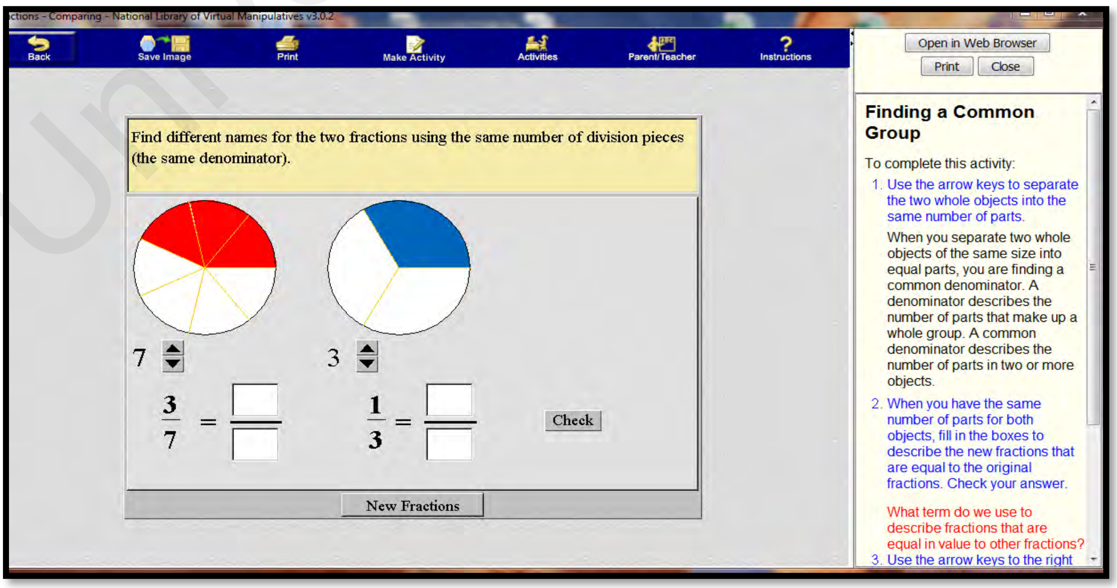


Figure 5.2. Features of Virtual Manipulative

This can be the reasons for pupils in virtual manipulative lesson was able to obtain greater score because in virtual manipulative pupil was able to check the answers immediately and generate a lot of questions in short time. A quick feedback along with different computations were able to keep pupil engaged and that aided in developing understanding, as they do not need to wait till teacher take time to check their answers or pose new questions or even provide feedback.

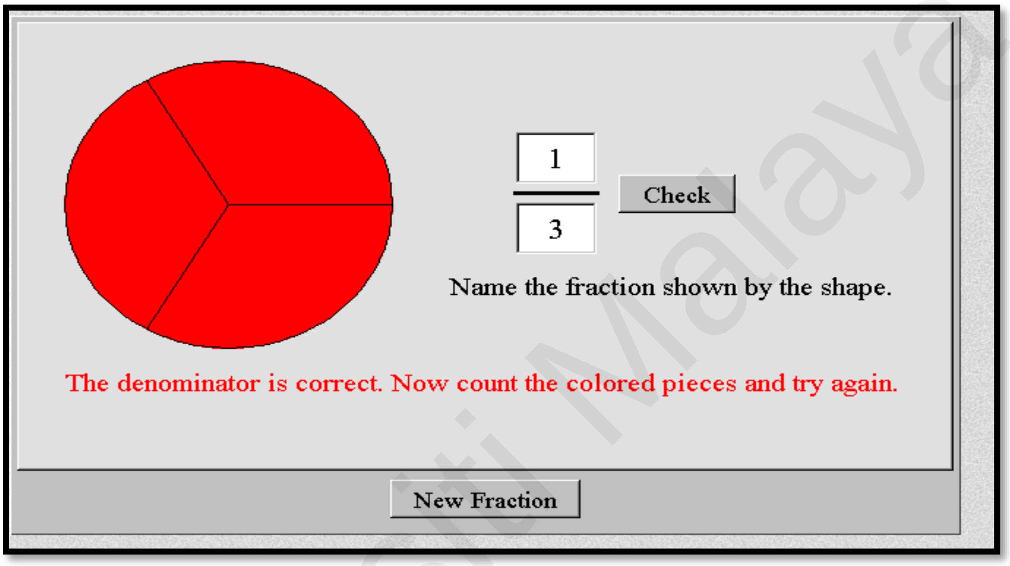


Figure 5.3. Example of feedback and instruction from Virtual Manipulative

This methodology of teaching emphasized to elicit prior knowledge by posting various applications of knowledge computations with immediate feedback and more time on reflecting on learning as suggested in constructivist learning activities had improved learning concept of fraction using virtual manipulative. Baviskar, Hartle, and Whitney (2009) agreed that, these constructivism learning activities process along with a suitable technology are the sequential steps that pupil can create new knowledge. Besides that, Lee, Luchini, Michael, Norris, and Soloway (2004) found in their pilot study with 39 second grade students using mathematic drill software Skills Arena that each student answered as many as 1296 questions in 19 days during pilot study, which was 285% more than they could have finished using printed worksheets or exercises. Furthermore, concluded that computer and software should be used in a classroom as it tributes and possibly improve upon traditional teaching tools.

Moreover, in virtual manipulative method lessons are prepared to promote active learning as mentioned in constructivism theory along with the technology. Meanwhile traditional method only focused on textbook and paper-pencil concept of teaching fraction. The lesson conducted using instructional activities with virtual manipulative in the classroom had impact on pupil's achievement in the posttest score. Whereby, pupils are required to engage in discussions, had shoulder partner tasks to justify their answers. According to Stone, Alfeld, Pearson, Lewis, and Jensen (2006) pupil learn in a meaningful way and extend their knowledge when they are actively engaged in the learning process. This is another reason to why both the pupil from traditional and virtual manipulative method have showed differences in their posttest mean scores.

Additional benefit been obtained by the pupil as they able to see the visualization of fractions. Hence, their understanding is stimulated as they could witness the changes, they made to the fraction concepts. This is not possible in traditional method class because the lesson fully dependent on teacher to draw or create the diagram as well as provide practices and checking on pupils' feedbacks. Moreover, time constrain did not able to allow more representations unlike in virtual manipulative lesson. Shaharuddin and Khairi (2013) agreed that weak pupil will learn more interestingly when multimedia and visual elements were added into learning concept. Aligned with this, another study done by Stohlmann, Yang, Huang, and Olson (2019) to investigate the ability to solve the fraction division problem by drawing a pictorial representation. The result showed statistically significant improvement after using the pictorial representations to solve their problems. Pupil was able to explore changing the fraction partitions for different equivalent fractions using different representations. This was supported by Yusof and Lusin (2013b) stating that pupils can have more chances of creating a number of equivalent fractions for particular fractions in much shorter time compared to the paper cutting and paper folding trial and errors. Pupil communicated with peers and teacher showed there is a possibility for their anxiety on fraction to be reduced as they showed more interest to discover the answers and enthusiastic on justifying the answers. Hence, using diagrams, images and representations are beneficial at the same time and it does save time if teachers use available technology as many researchers had agreed and supported that learning by seeing the manipulative that can develop pupils understanding (Amarin & Ghishan, 2013; Kathir Veloo & Puteh, 2017; P. Moyer-Packenham & Suh, 2012)

Figure 5.4 Output answer from virtual manipulative

Subsequently in virtual manipulative lesson teacher had more time to assist pupil hence there is an opportunity to ask more questions (figure 5.4), which made the pupil to think and reflect. The communication between pupil and teacher helped to identify pupil's uncertainty in the topic. Therefore, the lesson in virtual manipulative method leaned more towards student-centered rather than teacher-centered. In this way, pupil are active participants in their lesson, which means they contribute to class discussions not only by answering direct questions posed by the teacher, but also by answering questions posed by their peers. Pupil freely asked questions, and express their own understanding about the concept of fraction, among peers and teacher. Based on Rao, Slovin, Zenigami, and Black (2017) pupil should be given opportunities to freely communicate and let them explain their thinking, ask questions, and challenge answers without being negative.

3. "Let's suppose that there is a girl named Laili who is running one kilometer. So far she has run one-half of a kilometer ". How could we show the distance that she has run on the number line?".
4. "What if Lucy has run two-thirds of the kilometer? How could we show that?"
5. "What if Lucy has run two-thirds of the kilometer? How could we show that?"
6. **Stand N Share the answer (not mention the correct answer.)**

Figure 5.5. Example of question from lesson

Subsequently in virtual manipulative lesson teacher had more time to assist pupil hence there is an opportunity to ask more questions (figure 5.4), which made the pupil to think and reflect. The communication between pupil and teacher helped to identify pupil's uncertainty in the topic. Therefore, the lesson in virtual manipulative method leaned more towards student-centered rather than teacher-centered. In this way, pupil are active participants in their lesson, which means they contribute to class discussions not only by answering direct questions posed by the teacher, but also by answering questions posed by their peers. Pupil freely asked questions, and express their own understanding about the concept of fraction, among peers and teacher. Based on Rao et al. (2017) pupil should be given opportunities to freely communicate and let them explain their thinking, ask questions, and challenge answers without being negative (Alabdulaziz & Higgins, 2017).

5.4 Contribution of theory

This study used constructivism theory as it provides the structure and guide the choice of research design based on the goals of this study. From the theoretical aspect, the findings of the current study are consistent with constructivism theory as it conceptualizes that new knowledge are built based on prior knowledge by conducting lesson using effective technology aids (Dienes & Perner, 1999; Wilson & Lowry, 2000). This is supported by a study done by Alabdulaziz and Higgins (2017), investigating relationship between technology and constructivist strategies in learning mathematics. The findings of the study supported that technology not only can make the lesson interesting, increase the practice and motivation of pupils but also used to help students with their misconceptions with constructivist approaches. Hence, this theory proves that if pupils learn concept of fraction with virtual manipulative and constructivism strategies such as representations, iconic and symbolic, their understanding on the concept of fraction are being developed (Hughes et al., 2018). Instructional materials that are developed for learning and teaching the fraction lesson plays an important role in creating an effective and conducive classroom environment which incorporates constructivism theory.

In a traditional classroom, a teacher delivers information to pupil who passively listens and gets information. Meanwhile, constructivism theory-based classroom emphasize pupil to actively get involved in their learning which ensures that pupil gain new understanding. This study used constructivism theory as guideline to develop the instructional materials as well as teaching and learning process. Based on past researchers, proper learning materials, environment must be prepared and used for meaningful learning outcomes (Özalp, 2006; Ugurel & Morali, 2006; Balim, Inel, & Evrekli, 2008). This study had created learning environment where pupil construct their own understanding with the help of technology rather than teacher providing everything to the pupil as suggested by the theory. Pupil used virtual manipulative method to be able to answer question, investigate and reason after they solve the fraction since the feedback is given immediately. Pupil got the chance to think and reflect on how to answer the question and why their answers are correct or wrong. Besides that, constructivist approach focused towards student-centered as that was used as the based to prepare materials that encourages pupils' participations rather than normal lecturing method. In this study material, researcher illustrated the instructional materials with more thinking questions, reasoning and challenging practices so that pupils' will be able to visualize at the same time relate what is been taught. Teacher created situations in which the pupil questions their own answers and each other's assumptions.

Lesson plans and activities were incorporated based on constructivist learning environment whereby focused on active activities and teacher acted as facilitator this process. Pupils did their discussion and peer helping while solving questions using virtual manipulative method. Teacher played the role of a guide and helped pupils to relate their prior knowledge with new information. Past researchers, had agreed and suggested that in a constructivist learning environment, pupils play an active role by resolutely involving themselves in the learning process and constructing their knowledge by taking a part in activities (Kroasbergen & Van Luit, 2005; Metin & Özmen, 2009; Gray, 2007).

Therefore, these data give ideas that teachers should embrace more technology along with constructivism approaches because there is a sense of efficacy in doing so. Perhaps teachers can explore more role of manipulative in mathematics and encourage pupils to construct their own fractional representations using virtual manipulative. This study provided information based on teachers and pre-service teachers as they generate wide range and variety of concepts for thinking about teaching innovatively as well as finding proper solution to the problems of teaching and learning.

5.5 Implications

The quasi-experimental research was conducted to study the effectiveness of manipulative virtual-based activities by enhancing the understanding of the concept of fraction among year four pupil. Based on the findings, it is found that techniques using virtual manipulative activities have a positive impact on improving the understanding of fractional concepts. Moreover, the results of the study revealed that teaching and learning using virtual manipulative could improve pupil's understanding on the concept of fraction. These outcomes highlighted the features of virtual manipulative along with guidelines of constructivism theory to carry out the web-based instructional activities have showed remarkable improvement on year four pupil's understanding on the concept of fraction. Hence the effectiveness of this technique can influence the teaching and learning of fraction. Therefore, the findings of this study have listed number of important implications for future practice. The researchers have divided the implications of the study on three aspects, namely from the implications for students, teachers and to the ministry of education.

a. Conducting activities using the virtual manipulative in concept of fractions is very important to apply to pupil. The need to be exposed to this technique, as it improves the quality of learning and enhances their understanding on the concept of fractional. Encourages pupils to participate in hands-on activities from early stage and foster tech-savvy generation. Since, today world is driven by technology hence exposing technology by emerging into education able to improve pupils' skills in technology as well as mathematics.

b. Pupil able to get immediate feedback and able to correct their mistakes immediately rather than waiting for teacher to examine their work. Through this, pupil can practice more questions not only in classroom but after school hours. Since, virtual manipulative is more on hands-on and connected to gadgets hence pupils will be interested to learn using new technique rather than textbook or workbook.

c. From a teacher's perspective, teaching using virtual manipulative especially in fractions topic becomes an alternative technique to teaching the concept of fractions in order to improve students' achievement scores. Teachers can use this technique to prevent the boredom and anxiety that usually arises when a topic is taught. The concept of fractions is associated with visualization and concepts that give meaning to most students. Pupils were able to sees image or object transforming when virtual manipulative is used. Hence, their understanding deepens when they see the effect. Precisely speaking, this technique is very helpful in helping students who are advanced or beginners to learn and master the concept of fractions thoroughly.

d. The education ministry, in particular the curriculum developer, benefited from the study of the effectiveness of virtual manipulative as a technique of teaching the fraction. Preparing module or additional course book whereby teaching-using technology would be extra beneficial for teachers and students. Teachers be more confident when using technology in classroom as their reference resources are available.

5.6 Recommendation on Further Study

This research raised few questions which needs further investigation in integrating technology into understanding the concept of fraction in future. The following recommendations have been made for future research that would add to the general knowledge on the teaching and learning of concept of fraction. Many further studies could be conducted as a follow up to further investigate the effectiveness of virtual manipulative teaching method and the barriers in traditional teaching method in today's tech-savvy generation. Various web-based technologies can be used and be suggested to teachers as it aids in enabling the lesson to be more interesting and understandable for pupils. Future researchers can investigate the effectiveness of using virtual manipulative activities in other fraction subtopics such as addition, subtractions, multiplication and division. Next the researchers can extend the study to other domains in mathematics such as whole number, decimals, integers and more.

More specifically, researcher beliefs both quantitative and qualitatively (mix-method) study should be conducted in order to to compare experiences of pupils within the same context. More information on pupil's perception on using virtual manipulative teaching method would help to establish a greater degree of accuracy on this teaching method. Pupil's perceptions can be examined by an interview, observation or questionnaire to gain different perspective of answers for the same context. A qualitative research study adds more information regarding the scope of the study, which is beneficial for pupil and teachers.

Another possible area of future research would be to investigate in different demographic background. The students involved for this study were conveniently sampled from public schools in Klang. Hence, future studies can be conducted in different schools and states within Malaysia.

In addition, there is evidence that teachers' conceptual knowledge is weak (Rahman, Zamri, & Eu, 2018). There is also evidence in the literature that teachers view mathematics as a fixed science and a sequential knowledge in which it is best learned through repetition, repetitive algorithms and procedures (Nyaumwe, 2004). Similarly, a study done by Stacey et al. (2001) found that 20% of prospective teachers in their study did not have a good understanding of the topic related to decimal numbers. Thus, there is a need to study the level of conceptual knowledge of future teachers in particular on the topic.

5.7 Conclusion

This study was carried out to investigate the effect of Virtual Manipulative on year four pupil’s understanding on the concept of fraction. The pupil showed a remarkable improvement in these skills through this intervention. Although, the findings showed pupil taught in traditional method had improved but pupils taught using virtual manipulative had scored excelling with higher mean differences. The use of virtual manipulative had made the lesson more interesting and communications was prominent among the peers as well as the teacher. Meanwhile, in traditional method the communication was limited because teacher spent most of the time in drawing the diagrams on whiteboard and relating to the fractions. Moreover, pupils learnt not only for exam but also to comprehend the meaning and concepts of fractions. This can be viewed when pupil was able to represent the answers in different ways with a correct justification. Virtual manipulative features of visualizing fraction in the form of diagram have played a vital role in enhancing pupils understanding the concept of fraction, which was unable for the pupils who learned the traditional method.

This study has provided an alternative approach to enhance pupil's understanding and reflect on the concept of fraction. Therefore, mathematic educators may consider applying this method to encourage pupil to learn fraction in meaningful way.

REFERENCES

Abdullah, A. H., Abidin, N. L. Z., & Ali, M. (2015). Analysis of Students' Errors in Solving Higher Order Thinking Skills (HOTS) Problems for the Topic of Fraction. *Asian Social Science, 11*(21), 133.

Abdullah, A. H., & Zakaria, E. (2013). Enhancing students' level of geometric thinking through van hiele's phase-based learning. *Indian Journal of Science and Technology, 6*(5), 4432-4446.

Acharya, A. S., Prakash, A., Saxena, P., & Nigam, A. (2013). Sampling: Why and how of it. *Indian Journal of Medical Specialities, 4*(2), 330-333.

Agrawal, J., & Morin, L. L. (2016). Evidence ‐ based practices: Applications of concrete representational abstract framework across math concepts for students with mathematics disabilities. *Learning Disabilities Research & Practice, 31*(1), 34-44.

Al-Mashaqbeh, I. F. (2016). IPad in Elementary School Math Learning Setting. *International Journal of Emerging Technologies in Learning, 11*(2).

Alabdulaziz, M., & Higgins, S. (2017). Understanding technology use and constructivist strategies when addressing Saudi primary students' mathematics difficulties. *International journal of innovative research in science, engineering and technology., 6*(1), 105.

Alghazo, Y. M., & Alghazo, R. (2017). Exploring Common Misconceptions and Errors about Fractions among College Students in Saudi Arabia. *International Education Studies, 10*(4), 133-140.

Ali, M. F. (2001). *Distance Education: Constructivism and the Internet*: Universiti Teknologi Malaysia.

Alshehri, S. (2017). *The Comparison of Physical/Virtual Manipulative on Fifth-Grade Students' Understanding of Adding Fractions.* University of Cincinnati.

Amarin, N. Z., & Ghishan, R. I. (2013). Learning with technology from a constructivist point of view. *International Journal of Business, Humanities and Technology, 3*(1), 52-57.

Arganbright, D. (2005). Enhancing mathematical graphical displays in Excel through animation. *Spreadsheets in Education, 2*(1), 125-147.

Babbie, E. R. (2011). *Introduction to social research*: Wadsworth Cengage learning.

Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology, 113*(3), 447-455.

Balka, D. S. (1993). Making the connections in mathematics via manipulatives. *Contemporary Education, 65*(1), 19.

Barmby, P., Harries, T., Higgins, S., & Suggate, J. (2007). *How can we assess mathematical understanding.* Paper presented at the Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education.

Barnett-Clarke, C., Fisher, W., Marks, R., & Ross, S. (2010). *Developing essential understanding of rational numbers for teaching mathematics in grades 3-5*: National Council of Teachers of Mathematics.

Baviskar, S. N., Hartle, R. T., & Whitney, T. (2009). Essential Criteria to Characterize Constructivist Teaching: Derived from a review of the literature and applied to five constructivist‐teaching method articles. *International Journal of Science Education, 31*(4), 541-550.

Behr, M. J., Lesh, R., Post, T. R., & Silver, E. A. (1983). Rational number concepts. *Acquisition of mathematics concepts and processes*, 91-126.

Bertolone-Smith, C. M. (2016). *A Fourth Grade Teaching Experiment on Fraction Magnitude: Investigating Student Reasoning Through Mathematical Discourse and Design Research.*

Bogdan, R. C. (2003). Biklen., SK (2003). *Qualitative research for education: An introduction to theories and methods, 4*.

Borowski, E. J., & Borwein, J. M. (2002). *Collins dictionary of mathematics*: Collins.

Bouck, E. C., Bassette, L., Shurr, J., Park, J., Kerr, J., & Whorley, A. (2017). Teaching Equivalent Fractions to Secondary Students With Disabilities via the Virtual–Representational–Abstract Instructional Sequence. *Journal of Special Education Technology, 32*(4), 220-231.

Bouck, E. C., Shurr, J., Bassette, L., Park, J., & Whorley, A. (2018). Adding it up: Comparing concrete and app-based manipulatives to support students with disabilities with adding fractions. *Journal of Special Education Technology, 33*(3), 194-206.

Boulet, G. (1996). The construction of the unit fraction concept.

Braithwaite, D. W., Pyke, A. A., & Siegler, R. S. (2017). A computational model of fraction arithmetic. *Psychological review, 124*(5), 603.

Brooks, M. G., & Brooks, J. G. (1999). The courage to be constructivist.

Brown, E., & Crawford, C. (2003). Integrating internet-based mathematical manipulatives within a learning environment. *Journal of Computers in Mathematics and Science Teaching, 22*(2), 169-180.

Bruce, C., Chang, D., Flynn, T., & Yearley, S. (2013). Foundations to learning and teaching fractions: Addition and subtraction. *Retrieved July, 4*, 2014.

Bruner. (1986). 1986 Actual minds, possible worlds. Cambridge, MA: Harvard University Press.

Bruner, J. S. (1957). Going beyond the information given. *Contemporary approaches to cognition, 1*(1), 119-160.

Cai, J., & Knuth, E. J. (2005). Introduction: The development of students' algebraic thinking in earlier grades from curricular, instructional and learning perspectives. *ZDM–The International Journal on Mathematics Education, 37*(1), 1-4.

Campbell, D. T., & Erlebacher, A. (1970). How regression artifacts in quasi-experimental evaluations can mistakenly make compensatory education look harmful. *Compensatory education: A national debate, 3*, 185-210.

Campbell, D. T., & Stanley, J. C. (1963). Experimental and quasi-experimental designs for research. *Handbook of research on teaching*, 171-246.

Capraro, M. M., & Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? *Reading Psychology, 27*(2-3), 147-164.

Carraher, D., Schliemann, A. D., & Brizuela, B. M. (2000). *Early algebra, early arithmetic: Treating operations as functions.* Paper presented at the Presentado en the Twenty–second annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Tucson, Arizona.

Chrysanthou, I. (2008). The use of ICT in primary mathematics in Cyprus: The case of GeoGebra. *Master's thesis, University of Cambridge, UK*.

Chua, Y. P. (2006). Kaedah dan Statistik Penyelidikan. *Malaysia: Mc. Graw Hill Sdn Bhd*.

Chua, Y. P. (2012). Effects of computer-based testing on test performance and testing motivation. *Computers in Human Behavior, 28*(5), 1580-1586.

Clarke, D. M., Roche, A., & Mitchell, A. (2008). Ten Practical Tips for Making Fractions Come Alive and Make Sense. *Mathematics teaching in the middle school, 13*(7), 372-380.

Confrey, J., & Carrejo, D. (2005). Chapter 4: Ratio and fraction: the Difference between epistemological complementarity and conflict. *Journal for Research in Mathematics Education. Monograph*.

Cramer, K., Behr, M., Post, T., & Lesh, R. (1997). Rational Number Project: Fraction lessons for the middle grades-Level 1. Dubuque, IA: Kendall: Hunt Publishing Co.

Cramer, K., Behr, M., Post, T., & Lesh, R. (2009). Rational number project: Initial fraction ideas. *Retrieved on September, 9*, 2013.

Cramer, K., Monson, D., Whitney, S., Leavitt, S., & Wyberg, T. (2010). Dividing Fractions and Problem Solving. *Mathematics teaching in the middle school, 15*(6), 338-346.

Creswell, J. W. (2002). *Educational research : planning, conducting and evaluating quantitative and qualitative research* (Fourth. ed.): Pearson New International.

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*: Sage publications.

Creswell, J. W., & David, C. J. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*: Sage publications.

Davidson, A. (2012). Making it in America. *The Atlantic, 309*, 58-70.

Dienes, Z., & Perner, J. (1999). A theory of implicit and explicit knowledge. *Behavioral and brain sciences, 22*(5), 735-808.

Dogan, M., & İçel, R. (2011). The role of dynamic geometry software in the process of learning: GeoGebra example about triangles. *Journal of Human Sciences, 8*(1), 1441-1458.

Dorward, J. (2002). Intuition and research: Are they compatible? *Teaching Children Mathematics, 8*(6), 329.

Drickey, N. (2006). Learning technologies for enhancing student understanding of mathematics. *International Journal of learning, 13*(5).

Etikan, I., Musa, S. A., & Alkassim, R. S. (2016). Comparison of convenience sampling and purposive sampling. *American journal of theoretical and applied statistics, 5*(1), 1-4.

Finti, H. N. F. M. M., Shahrill, M., & Salleh, S. M. (2016). Integrating virtual manipulative with the use of iPad in the teaching and learning of fractions. *Knowledge Management & E-Learning: An International Journal, 8*(4), 581-601.

Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (1993). *How to design and evaluate research in education* (Vol. 7): McGraw-Hill New York.

Freudenthal, H. (1986). *Didactical phenomenology of mathematical structures* (Vol. 1): Springer Science & Business Media.

Fuchs, L. S., Schumacher, R. F., Sterba, S. K., Long, J., Namkung, J., Malone, A., . . . Siegler, R. S. (2014). Does working memory moderate the effects of fraction intervention? An aptitude–treatment interaction. *Journal of Educational Psychology, 106*(2), 499.

Furinghetti, F., Morselli, F., & Paola, D. (2005). Interaction of Modalities in Cabri: A Case Study. *International Group for the Psychology of Mathematics Education, 3*, 9-16.

Gagnon, J. C., & Maccini, P. (2001). Preparing students with disabilities for algebra. *Teaching Exceptional Children, 34*(1), 8-15.

Gall, M., Gall, J., & Borg, W. (2007). Nonexperimental research: Descriptive and causal-comparative designs. *Educational research: An introduction*, 298-330.

Ghani, S. N. A., & Maat, S. M. (2018). Misconception of fraction among middle grade Year Four pupils at primary school. *Research on Education and Psychology, 2*(1), 111-125.

Gningue, S. (2006). Students working within and between representations: An application of dienes's variability principles. *For the Learning of Mathematics, 26*(2), 41-47.

Halimah, I. N., & Poerwanti, J. I. (2013). Penggunaan Media Blok Pecahan Untuk Meningkatkan Kemampuan Penjumlahan Bilangan Pecahan Sederhana. *Jurnal Didaktika Dwija Indria, 1*(7).

Hallett, D., Nunes, T., & Bryant, P. (2010). Individual differences in conceptual and procedural knowledge when learning fractions. *Journal of Educational Psychology, 102*(2), 395.

Handel, M. J. (2016). What do people do at work? *Journal for Labour Market Research, 49*(2), 177-197.

Hansen, C. C., & Zambo, D. (2005). Piaget, meet Lilly: Understanding child development through picture book characters. *Early Childhood Education Journal, 33*(1), 39-45.

Harun, J., & Tasir, Z. (2003). *Multimedia dalam pendidikan*: PTS Publications.

Hàwera, N., Taylor, M., Young-Loveridge, J., & Sharma, S. (2007). "Who helps me learn mathematics, and how?": Màori Children's Perspectives. *Findings from the New Zealand Numeracy Development Projects 2006*, 54-66.

Healy, L., & Hoyles, C. (1999). Visual and symbolic reasoning in mathematics: Making connections with computers? *Mathematical Thinking and learning, 1*(1), 59-84.

Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology, 102*(4), 843.

Heo, H., Suh, J., & Moyer-Packenham, P. S. (2004). Impacting student confidence: The effects of using virtual manipulatives and increasing fraction understanding. *The Journal of Educational Research in Mathematics, 14*(2), 207-219.

Hughes, E. M., Riccomini, P. J., & Witzel, B. (2018). Using Concrete-Representational-Abstract Sequence to Teach Fractions to Middle School Students with Mathematics Difficulties. *Journal of Evidence-Based Practices for Schools, 16*, 171-190.

Hwang, W.-Y., Shadiev, R., Tseng, C.-W., & Huang, Y.-M. (2015). Exploring effects of multi-touch tabletop on collaborative fraction learning and the relationship of learning behavior and interaction with learning achievement. *Journal of Educational Technology & Society, 18*(4), 459-473.

Idris, N. (2007). Teaching and learning of Mathematics: Making Sense and Developing Cognitives: Kuala Lumpur: Utusan.

J.Thiel-Burgess, M. N. (2012). Constructivism in Practice: The Case for English Language Learners. . *International Journal of Education, 4(3)*, p108-p118.

Jamalludin, H., & Siti Nurulwahida, Z. (2010). Pembangunan Modul Pembelajaran Berbantukan Video Interaktif Bagi Pembelajaran Pengaturcaraan Visual Basic Berdasarkan Pendekatan Projek.

Johanning, D. I. (2008). Learning to use fractions: Examining middle school students' emerging fraction literacy. *Journal for research in mathematics education*, 281-310.

Johansen, A., Oishi, J. S., Mac Low, M.-M., Klahr, H., Henning, T., & Youdin, A. (2007). Rapid planetesimal formation in turbulent circumstellar disks. *Nature, 448*(7157), 1022.

Johnson, K. A. (1993). Manipulatives allow everyone to learn mathematics. *Contemporary Education, 65*(1), 10.

Jonassen, D. H. (1999, September 24, 2003). *Constructivist learning environments on the web: engaging students in meaningful learning.* Paper presented at the The Educational Technology Conference And Exhibition, Singapore.

Jordan, N. C., Resnick, I., Rodrigues, J., Hansen, N., & Dyson, N. (2017). Delaware longitudinal study of fraction learning: Implications for helping children with mathematics difficulties. *Journal of learning disabilities, 50*(6), 621-630.

Kathir Veloo, P., & Puteh, M. (2017). Mathematical Knowledge In Teaching Fraction Concepts. *Science International, 29*(1), 109-114.

Kementerian Pendidikan Malaysia (2015). Kurikulum Bersepadu Sekolah Rendah. Huraian Sukatan Pelajaran. Matematik. . *Pusat Perkembangan Kurikulum.*

Keser, H., & Özdamli, F. (2012). What are the Trends in Collaborative Learning Studies in 21st Century? *Procedia - Social and Behavioral Sciences, 46*, 157-161. doi: 10.1016/j.sbspro.2012.05.086

Kieren, T. E. (1976). *On the mathematical, cognitive and instructional.* Paper presented at the Number and measurement. Papers from a research workshop.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding It.

Koklu, O., & Topcu, A. (2012). Effect of Cabri-assisted instruction on secondary school students' misconceptions about graphs of quadratic functions. *International journal of mathematical education in science and technology, 43*(8), 999-1011.

Lamon, S. J. (1996). The development of unitizing: Its role in children's partitioning strategies. *Journal for research in mathematics education*, 170-193.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. *Second handbook of research on mathematics teaching and learning, 1*, 629-667.

Lamon, S. J. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*: Routledge.

Lamon, S. J. (2013). *MORE! Teaching Fractions and Ratios for Understanding: In-Depth Discussion and Reasoning Activities*: Routledge.

Lee, Luchini, K., Michael, B., Norris, C., & Soloway, E. (2004). *More than just fun and games: Assessing the value of educational video games in the classroom.* Paper presented at the CHI'04 extended abstracts on Human factors in computing systems.

Lee, C. Y., & Chen, M. P. (2010). Taiwanese junior high school students' mathematics attitudes and perceptions towards virtual manipulatives. *British Journal of Educational Technology, 41*(2).

Lee, N. H., & Ferrucci, B. J. (2012). Enhancing learning of fraction through the use of virtual manipulatives.

Ling, L. Y. (2008). Educational game themes of a fraction brick game. *Proceedings of Ascilite Melbourne*.

Loong, Y. (2014). Fostering mathematical understanding through physical and virtual manipulatives. *The Australian mathematics teacher, 70*(4), 3-10.

Lortie-Forgues, H., Tian, J., & Siegler, R. S. (2015). Why is learning fraction and decimal arithmetic so difficult? *Developmental Review, 38*, 201-221.

Loveless, T. (2008). The Misplaced Math Student: Lost in Eighth-Grade Algebra. The 2008 Brown Center Report on American Education. Special Release. *Brookings Institution*.

Maccini, P., & Gagnon, J. C. (2000). Best practices for teaching mathematics to secondary students with special needs. *Focus on Exceptional Children, 32*(5), 1.

Mack, N. K. (1990). Learning fractions with understanding: Building on informal knowledge. *Journal for research in mathematics education*, 16-32.

Mack, N. K. (2001). Building on informal knowledge through instruction in a complex content domain: Partitioning, units, and understanding multiplication of fractions. *Journal for research in mathematics education*, 267-295.

Major, T. E., & Mangope, B. (2012). The constructivist theory in Mathematics: The case of Botswana primary schools. *International Review of Social Sciences and Humanities, 3*(2), 139-147.

Malaysia, K. P. (2015). Primary Standard Curriculum (KSSR).

Markey, C., Power, D., & Booker, G. (2003). Using structured games to teach early fraction concepts to students who are deaf or hard of hearing. *American Annals of the Deaf, 148*(3), 251-258.

McGuire, P., Kinzie, M. B., & Berch, D. B. (2012). Developing number sense in pre-k with five-frames. *Early Childhood Education Journal, 40*(4), 213-222.

Mendiburo, M. A., & Hasselbring, T. (2010). *Virtual manipulatives and physical manipulatives: Technology's impact on fraction learning*: Vanderbilt University.

Merriam, S. B. (2002). Introduction to qualitative research. *Qualitative research in practice: Examples for discussion and analysis, 1*(1), 1-17.

Miller, S. P., Butler, F. M., & Kit-hung, L. (1998). Validated practices for teaching mathematcis to students with learning disabilities: A review of literature. *Focus on Exceptional Children, 31*(1), 1.

Ministry of Education Malaysia. (2003). Integrated Curriculum for Primary

Schools. Curriculum Specifications. Mathematics Year 3. *Curriculun Development Centre, Ministry of Education.*

Mohaiadin, J. (1999). Konstruktivisme: Aplikasinya dalam reka bentuk pembelajaran berasaskan web. *Teknologi instruksi dan pendidikan bestari: Persediaan dan cabaran dalam alaf baru*, 1-14.

Molenda, M., & Januszewski, A. (2008). Educational technology: A definition with commentary. *Association for Educational Communications and Technology (AECT)*.

Morris, J. (2014). *The use of virtual manipulatives in fourth grade to improve mathematic performance.*

Moyer-Packenham, P., & Suh, J. (2012). Learning mathematics with technology: The influence of virtual manipulatives on different achievement groups. *Journal of Computers in Mathematics and Science Teaching, 31*(1), 39-59.

Moyer-Packenham, P. S., & Bolyard, J. J. (2016). Revisiting the definition of a virtual manipulative *International perspectives on teaching and learning mathematics with virtual manipulatives* (pp. 3-23): Springer.

Moyer. (2001). Are We Having Fun Yet How Teachers Use Manipulatives to Teach Mathematics. *Educational Studies in Mathematics, Vol. 47*(No. 2), 175-197.

Moyer, Bolyard, Johnna, J., Spikell, & Mark, A. (2002). What Are Virtual Manipulative? *Teaching Children Mathematics, 8*(6), 372-377.

Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Children Mathematics, 8*(6), 372.

Moyer, P. S., Niezgoda, D., & Stanley, J. (2005). Young children's use of virtual manipulatives and other forms of mathematical representations. *Technology-supported mathematics learning environments, 67*, 17-34.

Mvududu, N., & Thiel-Burgess, J. (2012). Constructivism in practice: The case for English language learners. *International Journal of Education, 4*(3), 108.

Narli, S. (2011). Is Constructivist Learning Environment Really Effective on Learning and Long-Term Knowledge Retention in Mathematics? Example of the Infinity Concept. *Educational Research and reviews, 6*(1), 36-49.

Narli, S., & Baser, N. e. (2010). The Effects of Constructivist Learning Environment on Prospective Mathematics Teachers' Opinions. *Online Submission, 7*(1), 1-16.

National Research Council. (2001). *Grand challenges in environmental sciences*: National Academy Press.

NCTM. (2014). National Council of Teachers of Mathematics Principles to actions: Ensuring mathematical success for all: NCTM Reston, VA.

Ni, Y., & Zhou, Y.-D. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist, 40*(1), 27-52.

Noorbaizura, & Leong, E. K. (2013). Effect of students' achievement in fractions using GeoGebra. *SAINSAB, 16*, 97-106.

Noraini, I. (2006). Teaching and learning of mathematics: Making sense and developing cognitive abilities, *Utusan Publication & Distributors*.

Nyaumwe, L. (2004). The Impact of Full Time Student Teaching on Preservice Teachers' Conceptions of Mathematics Teaching and Learning. *Mathematics Teacher Education and Development, 6*, 19-32.

Obersteiner, A., Van Dooren, W., Van Hoof, J., & Verschaffel, L. (2013). The natural number bias and magnitude representation in fraction comparison by expert mathematicians. *Learning and Instruction, 28*, 64-72.

Ohlsson, S. (1988). Mathematical meaning and applicational meaning in the semantics of fractions and related concepts. *Number concepts and operations in the middle grades, 2*, 53-92.

Osmanoglu, A., & Dincer, E. O. (2018). Prospective Mathematics Teachers' Perceptions on and Adaptation of Student-Centred Approach to Teaching. *International Journal of Progressive Education, 14*(1), 75-87.

Paivio, A., & Clark, J. (2006). *Dual coding theory and education.* Paper presented at the Draft chapter presented at the conference on Pathways to Literacy Achievement for High Poverty Children at The University of Michigan School of Education.

Panel, N. M. A. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*: US Department of Education.

Pantziara, M., & Philippou, G. (2012). Levels of students'"conception" of fractions. *Educational Studies in Mathematics, 79*(1), 61-83.

Pearn, C., & Stephens, M. (2007). Whole number knowledge and number lines help develop fraction concepts. *Mathematics: Essential research, essential practice, 2*, 601-610.

Pearn, C., Stephens, M., & Lewis, G. (2002). Assessing rational number knowledge in the middle years of schooling. *Mathematics~ making waves*, 170.

Piaget, J. (1973). To understand is to invent: The future of education (GA Roberts, Trans.). *NY: Grossman Publishers*.

Pilli, O., & Aksu, M. (2013). The effects of computer-assisted instruction on the achievement, attitudes and retention of fourth grade mathematics students in North Cyprus. *Computers & Education, 62*, 62-71.

Powell, S. R., Fuchs, L. S., & Fuchs, D. (2013). Reaching the mountaintop: Addressing the common core standards in mathematics for students with mathematics difficulties. *Learning Disabilities Research & Practice, 28*(1), 38-48.

Puchner, L., Taylor, A., O'Donnell, B., & Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School science and mathematics, 108*(7), 313-325.

Qing, F. & Li, L. (2011). Web-based Collaborative Learning. *Procedia Environmental Sciences, 11*, 189-192.

Rahman, M. N. A., Zamri, S. N. A. S., & Eu, L. K. (2018). KAJIAN META-ANALISIS PENGETAHUAN GURU MATEMATIK DI MALAYSIA. *JuKu: Jurnal Kurikulum & Pengajaran Asia Pasifik, 6*(2), 11-22.

Rao, K., Slovin, H., Zenigami, F., & Black, R. (2017). Challenges and supports for struggling learners in a student-centered mathematics classroom. *Investigations in Mathematics Learning, 9*(2), 69-85.

Razak, F. A., Noordin, N., Dollah, R., & Alias, R. (2017). How Do 13-Year-Olds in Malaysia Compare Proper Fractions? *Journal of ASIAN Behavioural Studies, 2*(2), 27-35.

Reeder, S. (2017). A Deep Understanding of Fractions Supports Student Success in Algebra *And the Rest is Just Algebra* (pp. 79-93): Springer.

Reimer, K., & Moyer, P. S. (2005). Third-graders learn about fractions using virtual manipulatives: A classroom study. *Journal of Computers in Mathematics and Science Teaching, 24*(1), 5-25.

Reiten, L. (2018). Promoting Student Understanding through Virtual Manipulatives. *Mathematics Teacher, 111*(7), 545-548.

Richard, J.-F. (2005). Construction of Game Performance. *Teaching games for understanding: Theory, research, and practice*, 19.

Rittle-Johnson, B., & Schneider, M. (2014). Developing conceptual and procedural knowledge of mathematics. *Oxford handbook of numerical cognition*, 1102-1118.

Roesslein, R. I., & Codding, R. S. (2019). Fraction interventions for struggling elementary math learners: A review of the literature. *Psychology in the Schools, 56*(3), 413-432.

Sanders, S. (2016). Critical and creative thinkers in mathematics classrooms. *Journal of Student Engagement: Education Matters, 6*(1), 19-27.

Sarama, J., & Clements, D. H. (2016). Physical and virtual manipulatives: What is "concrete"? *International perspectives on teaching and learning mathematics with virtual manipulatives* (pp. 71-93): Springer.

Saxe, G. B., Taylor, E. V., McIntosh, C., & Gearhart, M. (2005). Representing fractions with standard notation: A developmental analysis. *Journal for research in mathematics education*, 137-157.

Seethaler, P. M., Fuchs, L. S., Star, J. R., & Bryant, J. (2011). The cognitive predictors of computational skill with whole versus rational numbers: An exploratory study. *Learning and Individual Differences, 21*(5), 536-542.

Sessoms, D. (2008). Interactive instruction: Creating interactive learning environments through tomorrow's teachers. *International Journal of Technology in Teaching and Learning, 4*(2), 86-96.

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference/William R. Shedish, Thomas D. Cook, Donald T. Campbell*: Boston: Houghton Mifflin.

Shaharuddin, M., & Khairi, A. (2013). MA (2011). *Pembangunan Web E-Pembelajaran Menggunakan Elemen Video Dalam Topik 'Work And Energy'Berasaskan Teori Konstruktivisme Sosial*.

Siebert, D., & Gaskin, N. (2006). Creating, Naming, and Justifying Fractions. *Teaching Children Mathematics, 12*(8), 394-400.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., . . . Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological science, 23*(7), 691-697.

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive psychology, 62*(4), 273-296.

Smith. (2009). Early Childhood Mathematics *(4th ed.)*.

Smith, J. P. (2002). The Development of Students' Knowledge of Fractions and Ratios.

Solso, R. L., Maclin, O. H., & Maclin, M. K. (2008). Psikologi kognitif. *Jakarta: Erlangga*.

Spicer, J. (2000). Virtual manipulatives: A new tool for hands-on math. *ENC Focus, 7*(4), 14-15.

Steen, K., Brooks, D., & Lyon, T. (2006). The impact of virtual manipulatives on first grade geometry instruction and learning. *Journal of Computers in Mathematics and Science Teaching, 25*(4), 373-391.

Steffe, L. P., Cobb, P., & Von Glasersfeld, E. (1988). Young children's construction of arithmetical meanings and strategies.

Steffe, L. P., & Olive, J. (2009). *Children's fractional knowledge*: Springer Science & Business Media.

Stein, M. K., & Bovalino, J. W. (2001). Manipulatives: One piece of the puzzle. *Mathematics teaching in the middle school, 6*(6), 356.

Stewart, V. (2005). *Making Sense of Students' Understanding of Fractions: An Exploratory Study of Sixth Graders' Construction of Fraction Concepts Through the Use of Physical Referents and Real World Representations.* Doctoral dissertation, Florida State University.

Stohlmann, M., Yang, Y., Huang, X., & Olson, T. (2019). Fourth to Sixth Grade Teachers' Invented Real World Problems and Pictorial Representations for Fraction Division. *International Electronic Journal of Mathematics Education, 15*(1), em0557.

Stone, J., Alfeld, C., Pearson, D., Lewis, M., & Jensen, S. (2006). Building academic skills in context: Testing the value of enhanced math learning in career and technical education. *St. Paul, MN: University of Minnesota, National Research Center for Career and Technical Education. Retrieved May, 25*, 2007.

Streefland, L. (1985). Search for the roots of ratio: Some thoughts on the long term learning process (towards... a theory). *Educational Studies in Mathematics, 16*(1), 75-94.

Suaidi, M. S. (2017). Pengetahuan pedagogi isi kandungan dalam kalangan guru novis dan guru berpengalaman di sekolah rendah daerah Hulu Langat.

Suh, J., Moyer, P. S., & Heo, H.-J. (2005). Examining technology uses in the classroom: Developing fraction sense using virtual manipulative concept tutorials. *Journal of Interactive Online Learning, 3*(4), 1-21.

Tan, G. G. S. (2010). *Effects of Virtual Manipulatives on Students' Procedural Knowledge and Conceptual Understanding of Fractions.* Universiti Malaysia Sarawak.

Tan See Teng , E. Z. (2018). Analisis Jenis Kesilapan Murid Dalam Operasi Penambahan Dan Penolakan Pecahan.

Teoh, B., & Fong, S. (2005). The Effects of Geometer's Sketchpad and Graphic Calculator in the Malaysian Mathematics Classroom. School of Educational Studies, Universiti Sains Malaysia. Retrieved online in March 2006.

Thompson, P. W. (1996). Imagery and the development of mathematical reasoning. *Theories of mathematical learning*, 267-283.

Troelstra, A. S., & Van Dalen, D. (2014). *Constructivism in mathematics* (Vol. 2): Elsevier.

Uribe-Flórez, L. J., & Wilkins, J. L. (2017). Manipulative use and elementary school students' mathematics learning. *International Journal of Science and Mathematics Education, 15*(8), 1541-1557.

Usiskin, Z. (2007). The arithmetic operations as mathematical models *Modelling and Applications in Mathematics Education* (pp. 257-264): Springer.

Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *The Journal of Mathematical Behavior, 31*(3), 344-355. doi: https://doi.org/10.1016/j.jmathb.2012.02.001

Vamvakoussi, X., & Vosniadou, S. (2010). How many decimals are there between two fractions? Aspects of secondary school students' understanding of rational numbers and their notation. *Cognition and instruction, 28*(2), 181-209.

Van de Walle, J. A., Karp, K. S., Bay-Williams, J. M., Wray, J. A., & Brown, E. T. (2016). Elementary and middle school mathematics: Teaching developmentally.

Veloo, P. K., & Puteh, M. (2017). *Mathematical knowledge in teaching of fraction concepts using diagrammatical approach.* Paper presented at the AIP Conference Proceedings.

Von Glasersfeld, E. (2012). A constructivist approach to teaching *Constructivism in education* (pp. 21-34): Routledge.

Vukovic, R. K., Fuchs, L. S., Geary, D. C., Jordan, N. C., Gersten, R., & Siegler, R. S. (2014). Sources of individual differences in children's understanding of fractions. *Child development, 85*(4), 1461-1476.

Wang, Q. (2009). Designing a web-based constructivist learning environment. *Interactive Learning Environments, 17*(1), 1-13.

Wijaya, A. (2017). The Relationships between Indonesian Fourth Graders' Difficulties in Fractions and the Opportunity to Learn Fractions: A Snapshot of TIMSS Results. *International Journal of Instruction, 10*(4), 221-236.

Wilson, B., & Lowry, M. (2000). Constructivist learning on the Web. *New Directions for Adult and Continuing Education, 2000*(88), 79-88.

Wong, M., & Evans, D. (2007). Students' conceptual understanding of equivalent fractions. *J. Watson & K. Beswick Mathematics: Essential Research, Essential Practice, 2*, 824-833.

Wu, H. (2001). How to prepare students for algebra. *American Educator, 25*(2), 10-17.

Yanik, H. B., Samson, J., & Flores, A. (2006). Quotient and measurement interpretations of rational numbers. *Teaching Children Mathematics, 13*(1), 34-39.

Yin, R. K. (2003). Case study research: design and methods (ed.). *Thousand Oaks*.

Yusof, J., & Lusin, S. (2013a). The Role of Manipulatives in Enhancing Pupils' Understanding on Fraction Concepts. *International Journal for Infonomics (IJI), 6*(3), 4.

Yusof, J., & Lusin, S. (2013b). The Role of Manipulatives in Enhancing Pupils' Understanding on Fraction Concepts. *International Journal for Infomics (IJI). September/December*, 2-3.

Zakiah Salleh, Norhapidah Mohd Saad, Mohamad Nizam Arshad, Hazaka Yunus, & Zakaria, E. (2013). ANALISIS JENIS KESILAPAN DALAM OPERASI.

Zengin, Y., & Tatar, E. (2017). Integrating dynamic mathematics software into cooperative learning environments in mathematics. *Journal of Educational Technology & Society, 20*(2), 74-88.