# EFFECT OF PHASE-BASED INSTRUCTION IN PLANE GEOMETRY USING GEOGEBRA ON THE GEOMETRIC THINKING OF GRADE SEVEN STUDENTS IN CHINA

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## ORIGINAL LITERARY WORK DECLARATION

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**ABSTRACT**

One of the issues in geometric teaching in China is the lack of focus on geometric thinking and the usage of technology. The purpose of this study is to determine the effect of phase-based instruction in plane geometry using Geogebra on the geometric thinking of Grade Seven students in China. The population of study is two classes of Grade Seven students in the national type public Chinese schools in China. In this study, the participants were Grade Seven pupils from two intact mixed-ability classrooms. There were 58 students in the experimental group and 54 students in the control group. This study adopted a quasi-experimental non-equivalent pretest-posttest design. The results of Mann-Whitney *U* test for pre Van Hiele Achievement Test showed that there is no difference in van Hiele geometric thinking levels between the control group and the experimental group (Mdn=1), $U$ ($n_1$=54, $n_2$=58)= 1476.00, $z$=0.70, $p$=0.49 at the significant level of 0.05.

The finding of this study showed that the difference in van Hiele levels of geometric thinking between the pupils in the control group (Mdn=3) and the pupils in the experimental group (Mdn=3) was significant, $U$ ($n_1$=54, $n_2$=58)=1250.00, $z$=2.53, $p$=0.01. The result showed that the performance of pupils in the experimental group was higher in mean rank of van Hiele geometric thinking levels than the pupils in control group after being intervened. Thus, more researches should be done to provide mathematical educators to use phase-based instruction with Geogebra and to provide students with more meaningful mathematics teaching and learning experiences.

**KESAN PENGAJARAN BERASASKAN FASA DALAM SATAH GEOMETRI MENGGUNAKAN GEOGEBRA TERHADAP PERMIKARAN GEOMETRI MURID GRED TUJUH DI CHINA**

## ABSTRAK

Salah satu isu dalam pengajaran geometrik di China adalah kurangnya tumpuan kepada pemikiran geometri dan penggunaan teknologi. Tujuan kajian ini adalah untuk menentukan kesan pengajaran berasaskan fasa dalam satah geometri menggunakan geogebra terhadap permikaran geometri murid gred tujuh di China. Populasi kajian adalah dua kelas pelajar kelas tujuh di sekolah-sekolah Cina jenis kebangsaan di China. Dalam kajian ini, para peserta adalah murid Gred Seven dari dua bilik kelas bercampur yang utuh. Terdapat 58 pelajar dalam kumpulan eksperimen dan 54 pelajar dalam kumpulan kawalan. Kajian ini menggunakan reka bentuk pretest posttest yang tidak bersamaan kuasi eksperimen. Keputusan ujian Mann-Whitney *U* menunjukkan bahawa, pada aras keyakinan 0.05 daripada kumpulan kawalan dan kumpulan eksperimen (Mdn=1), $U$ ($n_1$=54, $n_2$=58)= 1476,00, $z$=0.70, $p$=0.49 ada perbezaan Hiele tahap pemikiran geometri. Dapatan kajian menunjukkan bahawa perbezaan tahap van Hiele dalam pemikiran geometri antara pelajar dalam kumpulan kawalan (Mdn=3) dan pelajar dalam kumpulan eksperimen (Mdn=3) adalah signifikan, ($n_1$=54, $n_2$=58)=1250.00, $z$=2.53, $p$=0.01. Hasilnya menunjukkan bahawa prestasi murid dalam kumpulan eksperimen adalah lebih tinggi dalam tahap pemikiran geometri van Hiele berbanding murid dalam kumpulan kawalan selepas campur tangan. Oleh itu, lebih banyak penyelidikan harus dilakukan untuk menyediakan pendidik matematik untuk menggunakan pengajaran berasaskan fasa dengan Geogebra dan untuk menyediakan

pelajar dengan pengalaman pengajaran dan pembelajaran matematik yang lebih bermakna.

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## CHAPTER ONE

## INTRODUCTION

### 1.1 Background

This study focuses on middle school geometry. Geometry has always been an indispensable part of the mathematics content of this stage. Clements and Battista (1992) have clearly stated that geometry provides a method for describing and reflecting the external physical environment. This method can be used as a tool for learning mathematics and other sciences. It can strengthen the geometric thinking and contribute to the creative thinking of high-level mathematics. Geometry can help people organize their expressions and describe the world of life. It can also improve their ability to solve problems (Burger & Shaughessy, 1986). Geometry builds an abstract imagination space for us and lets us stay in the ocean of thinking and experience the mysteries brought by mathematics (Hu, 2012). In short, learning geometry helps us deal with various aspects of life.

From the study of ICMI (Mammana & Villani, 1998), a universal phenomenon in today's world geometry courses is that paying attention to the idea of transformation and symmetry have involved various geometric transformations and symmetries since the first grade of primary school in many countries. In contrast, the Chinese curriculum is very weak in this area. Therefore, a change in *The mathematics curriculum standard of compulsory education (2011 Edition)* is to emphasize the role of geometric transformation in China (Bao & Zhou, 2009). Related studies have also shown that geometrical transformations such as translations, rotations, and tools for solving problems in graphic movements open up a new approach for geometrical thinking and reasoning (Bao & Zhou, 2010). The movement of graphics had the function of irreplaceability to cultivate pupils' conception of space and expand the activity of geometry (Kong & Lu, 2005). In short, China has always attached great importance to the teaching of middle school geometry.

Several critical issues are found related to the research on geometry. Firstly, in the six years of primary school mathematics learning, the pupils are mainly in the quantitative relationship (Qi, 2013). Pupils are comfortable with calculations, but lack the ability of geometric thinking and rarely study the property of geometry, even the geometrical thinking, and reasoning. Xing (2016) said that in the transition from primary to junior high school, the geometry curriculum had changed a lot. He said that for pupils, the learning objects and learning methods had changed a lot and they cannot transfer and associate knowledge in the original cognitive structure. At this time, the teacher needs to rethink the geometric thinking level and reasoning of the pupils and rationally teach geometry with geometric thinking. The National Committee of Mathematics Teachers (2000) emphasized that conceptual understanding should be emphasized in mathematics teaching so that pupils can apply what they have learned in various situations. The concept does not only require many pupils to like mathematics, but also the geometric mathematical reasoning and thinking in the process of learning geometric proof must be improved (Qi, 2013).

With the continuous development of information technology with multimedia technology as the core, it is a general trend to use information technology to promote education modernization (Gao, 2011). Gao stated that the ultimate purpose of each element of learning geometric thinking information is the application of technology of teaching geometry (2011). The application, that is, the effective integration of technology and curriculum and the achievement of significant results, can promote education reform and development (Shang, 2008). In the basic concept of *The mathematics curriculum standard of compulsory education* (2011), it is proposed that the extensive application of modern information technology profoundly affects the content of mathematics courses, mathematics teaching, and mathematics learning. *The mathematics curriculum standard of compulsory education* (2011) requires a change in teaching from a traditional way of imparting knowledge to a process of recreating and experiencing the formation of knowledge. *The mathematics curriculum standard of compulsory education* (2011) also requires achieving a new way of inspiring pupils' geometric thinking and cultivating their ability to apply technology. *The mathematics curriculum standard of compulsory education* (2011), promoting the organic integration of IT and plane geometry is emphasized. That is, promoting information technology use to present course content is important. It is difficult to teach how to use scientific calculators and various mathematical education technology tools as much as possible, and strengthen the combination of mathematics teaching such as plane geometry and information technology. *The mathematics curriculum standard of compulsory education* (2011) encourages pupils to use computers, calculators and the tools of information and communications technology to explore and achieve the ability of geometric thinking. So, it is necessary to integrate IT and mathematics, especially plane geometry.

As widely known, in the plane geometry of seventh grade, the pupils had been contacted and studied sporadically in elementary school, but the relationship was relatively fragmentary and had not formed systematic learning (Liu, 2009). Van Hiele's theory was proposed by Pierre Van Hiele and Dina VanHiele-Geldof based on long-term practical research (Gemert, 2015). Van Hiele's theory holds that there exists a very close relationship between pupils' geometric thinking level and the effectiveness of teachers' classroom teaching. Therefore, when the junior middle school mathematics teacher implements the plane geometry teaching, it is necessary to use the Van Hiele theory to teach based on the pupils' existing thinking level. This method is useful to improve quality in teaching junior plane geometry.

Geogebra is a combination of Geometry and Algebra designed by Hohenwarter (2007) of the University of Atlanta, Florida. The software provides multi-window operation and multiple representations of knowledge. Moreover, Geogebra offers a dynamic presentation, which is very convenient by using some elements, such as straight lines, curves, vectors, and functions. Geogebra can display and explore the trajectory generation process. The feature is dynamic. In the dynamic demonstration of Geogebra, the boring mathematics class can become vivid (Andraphanova, 2015).

**1.2 Statement of the Problem**

The first issue is that math teachers do not focus on developing pupils' geometric thinking. Zhao's study (2016) have shown that van Hiele geometric thinking level of pupils is weakness in China, so there is the reason to believe that the study of the geometric thinking level of Chinese pupils on the basis of van Hiele geometric thinking level is important. Wu (2014) concluded that the junior high school students don't reach high level of the geometric thinking level. He concluded that pre-service school education and post-employment education defects are the reasons for forming the low level of geometric thinking (Wu, 2014). *The mathematics curriculum standard of compulsory education* (2011) pointed out that the evaluation aims to fully understand the pupils' geometric thinking. Secondly, the evaluation is to improve pupils' geometric thinking. Third, the evaluation is to establish evaluation systems with diverse evaluation goals and multiple assessment methods. Gao (2013) stated the following conclusion that the level of cognitive geometric thinking in classroom teaching is consistent with the teaching ability, which is higher than the student's cognitive level, classroom teaching helps pupils to improve their geometric thinking level and most middle school pupils can reach the geometric thinking level 3 after classes. Lu (2014) obtained the following conclusions on the geometric thinking level in the seventh, eighth, and ninth grade pupils. The eighth grade pupils have the higher thinking level than pupils in grade seven. Most of the ninth grade pupils reach the level of their thinking. At the level, only a few are at the level of reasoning. There is a significant correlation between the level of pupils' geometric thinking and mathematics achievement. In *The mathematics curriculum standard of compulsory education* (2011), the evaluation of mathematics learning should focus on pupils. Moreover, the results of learning must focus on the process of their learning. Therefore, they must attach importance to the pupils' geometric thinking level. A change in *The mathematics curriculum standard of compulsory education*, it emphasizes the role of geometric thinking (Bao & Zhou, 2009).

Secondly, Chinese mathematics geometry curriculum does not refer to geometric thinking widely. In China, Lu (2014) stated that the use of van Hiele geometric thinking level had issues. The main issue is that van Hiele levels derived from Netherlands, which has a large gap with the Chinese cultural background. Zhang and Huang (2009) stated that the dramatic changes in plane geometry content in the development of *The mathematics curriculum standard of compulsory education* (2011) have also led to widespread debate among scholars. However, China is still one of the countries with the most plane geometry content in middle school and the most emphasis on geometric teaching. At present, the research theories of thinking level that are relatively mature and highly recognized internationally include SOLO classification, Piaget and Inhelder research on the concept of children's space and van Hiele geometric thinking levels. Xing (2016) concluded that the SOLO taxonomy aims to help pupils achieve a higher level of learning by analyzing the pupils' existing outcomes, and is suitable for all subjects and has a wide range of applications. Huang and Li (2010) pointed that Piaget and Ingeld research on the concept of children's space aims to examine the developmental changes of individuals with age and the impact of cognitive activities on pupils' cognitive development, but because of the lack of school factors. Zhao (2016) pointed that van Hiele geometric thinking level was founded in the practice of middle school geometry teaching and emphasized teaching design based on pupils' thinking level and paying attention to the changes in pupils' geometric thinking.

The third issue is lack of using technology on geometric teaching in China. The integration of the technological tools in the instructional process was useful in helping the pupils to acquire higher geometric thinking (Abdul Halim & Effadi, 2013). Traditional method refers to the teaching method that the teacher uses the system to explain in detail so that the pupils can master a large amount of knowledge (Bao &

Zhou, 2009). Bao and Zhou pointed out that teachers are the center of teaching activities and the content of teaching activities and pupils' achievements are the reflection of teachers' teaching level (Bao & Zhou, 2009). The classroom is the main environment of teaching and the stage for teachers to perform (Lu, 2014). It was useful especially when pupils learn collaboratively and constructively through social interaction and self-exploration (Pfannkuch, 2008). Technology such as the computers and learning software are used to replace the traditional methods to stimulate the high-level of geometric thinking process among the pupils (Dou & Qiu, 2003). Li (2013) pointed out that in the traditional classroom teaching with chalk and blackboard, teachers have to spend much time on blackboard writing and drawing, which reduces the time for teachers to explain knowledge and pupils to practice, reduces the teaching effect, and has a single teaching method, which is difficult to cultivate pupils' interest in learning. He also pointed out that multimedia can be easily used to input text, symbols, formula theorems on slides and create their drawings. Moreover, the teachers can design teaching methods, make teaching content on slides, and set up a projection mode, according to the teaching process slides. Because of these, the teachers greatly save blackboard time and can speed up teaching progress, increase teaching capacity and improve teaching efficiency. Hu (2008) put forward the teaching strategy of information technology in the teaching content. *The mathematics curriculum standard of compulsory education* (2011) pointed out that the wide application of modern information technology is having an impact on the content of mathematics courses and mathematics geometric thinking. Gao (2011) pointed out that the tools are Geogebra, the Geometer's Sketchpad and so on. He stated that the Geometer's Sketchpad is suitable for CAI courseware in the fields of geometry and physics. It uses points, lines, and circles as basic elements. However, in Geogebra, users can directly perform non-standard ellipse mapping and tangential operations by clicking the button, which is very simple and convenient. Zuo, Tian, and Yuan (2010) pointed that the Geometer's Sketchpad only focuses a dynamic change in the shape of the plane geometry, which is a disadvantage for revealing the subtle properties of the graphic, especially the analytic geometry and function parts. Geogebra combines algebra and geometry to achieve a dynamic demonstration of the simultaneous changes in graphics and algebraic equations. Gao (2011) pointed out that Geogebra merged the two departments of algebra and geometry to achieve simultaneous changes in graphics and algebraic equations, realizing a dynamic presentation.



### 1.3 Theoretical Framework

In this study, the researcher collected data based on the pupils' pre and post Van Hiele Achievement Test's results which are both observable and measurable. Van Hiele theory is the underlying theory use in the research. Battista (2007) suggested that van Hiele geometric thinking theory as the best description of pupils' thinking about two-dimensional shapes. This theory will help to quantify, collect data and information for this study. Van Hiele put forward that all pupils should go through all the five geometric thinking levels in order to conduct the concepts of geometry (Fuys & Tischler, 1988). Van Hiele concluded some generalities in his model that characterize the use of the model to guide the educators to make instructional decisions (Crowley, 1987).

There are five characteristics of the van Hiele geometric thinking levels are described (Abdullah & Zakaria , 2013).

**Sequential**: The level of pupils' geometric thinking must be taught and gradually increased in order from low to high, and it is impossible to reach a higher level directly across a certain level.

**Advancement**: The geometric thinking level develops from one level to a higher level is not continuous, and there is an essential difference between the two levels.

**Intrinsic and extrinsic:** As usual, the high level of thinking that pupils exhibit is the result of implicit learning at a lower level.

**Linguistic:** Each level of thinking has its linguistic sign system. The linguistic sign system of high level of thinking is a further improvement of the lower level symbol system.

**Mismatch:** Pupils at all levels of thinking have appropriate teaching. If pupils fail to receive appropriate teaching, it is difficult to achieve the desired learning effect. Therefore, teachers should grasp the learning level of pupils in a timely and accurate manner before adjusting the teaching activities and adjust the teaching strategies according to the pupils' learning situation.

In the section of theoretical framework, the researcher made some basic assumptions concerning the subject of this study, research methodology, mathematical, and psychological terms. The assumptions made upon the subject of this study include the Grade Seven pupils would be active and neutral in learning mathematics, and had not learnt about plane geometry before this study was conducted. However, the software of Geogebra was introduced briefly to the pupils before the study was conducted. The researcher assumed that the pupils would try their best to complete the instructional tasks and Van Hiele Achievement Test. The researcher assumed that Grade Seven pupils mastered the second van Hiele geometric thinking level if they would be to analyze and measure types of angles and lines, and then solve simple problems involving angles and lines in the Van Hiele Achievement Tests given. On the other hand, it was assumed the pupils would have mastered the three van Hiele geometric thinking levels if they would be able to analyze and construct the relationships of angles and lines, and then to solve simple problems involving the relationships of angles and lines based on the properties of angles and lines in Van Hiele Achievement Test.

In this study, the researcher also assumed that the pupils in both the control group and the experimental group had similar family background that their parents would not help them to reinforce on what they had learnt in school during the intervention period. The researcher further assumed that pupils in both groups would not communicate about their learning of plane geometry during the intervention process and they learnt merely from the instructions given by the researcher. The pupils from the experimental group obtained higher van Hiele geometric thinking levels about plane geometry by using Geogebra through phase-based instruction, however, pupils from the control group obtained higher van Hiele geometric thinking levels about plane geometry through traditional teaching given without other factors.

The researcher assumed that the usage of phase-based instruction through Geogebra would play certain roles in the teaching and learning process such as collecting, processing, keeping, recalling and using information. Therefore, the instruction required longer duration for the pupils to get used to it compared to the traditional teaching. The researcher assumed that the pupils in the control group would not require a long time to adapt themselves in traditional teaching since they had been taught in that way. Although the intervention provided for both the pupils from the experimental group and the pupils from the control group was six 80 minutes lesson, phase-based instruction using Geogebra on pupils in the experimental group might need to be complemented with an instructional module.

In the section of analyzing the data obtained from this research, the researcher assumed that the pupils must get at least 7 out of 10 items correctly for the first ten items in pre and post Van Hiele Achievement Test to achieve van Hiele Level 2 (analysis) of geometric thinking. Similarly, the pupils must score 7 out of 10 items correctly in the eleventh item to twentieth item in pre and post Van Hiele Achievement Test to achieve van Hiele geometric thinking Level 3 (ordering). The scoring criterion is made after referring to Mayberry's (1981) and Usiskin's (1982) scoring criterion, in which the pupils' passing score for each level should be more than 60 percent. Furthermore, the pupils must possess van Hiele geometric thinking Level 2 before they could possess van Hiele geometric thinking Level 3. A pupil who scored less than 7 out of 10 items in the first ten items would be classified into Level 1 even though the pupil scored more than 7 out of 10 items in the last ten items (Level 3) in Van Hiele Achievement Test. That is to say, the pupils would be categorized as Level 1 in van Hiele geometric thinking levels if they had not mastered the analysis level (Level 2) in van Hiele geometric thinking levels, even though they seemed to have been proficient in the ordering level (Level 3) in van Hiele geometric thinking levels.

### 1.4 Research Purpose and Research Objective

This study aims to examine the effect of phase-based instruction in plane geometry using Geogebra on the geometric thinking of Grade Seven pupils in China. This study wants to find out whether phase-based instruction in plane geometry using Geogebra can be useful for Grade Seven pupils' geometric thinking in China. The study has the following three objectives:

1. To examine the difference in pre van Hiele geometric thinking levels between the pupils in the experimental group and in the control group;

2. To examine the difference in van Hiele geometric thinking levels among the pupils in the experimental group before and after phase-based instruction using Geogebra;

3. To examine the difference in post van Hiele geometric thinking levels between the pupils in the experimental group and the control group.

### 1.5 Research Questions and Research Hypotheses

Especially, this research focuses on solving the following research questions:

1. Is there any significant difference in pupils' geometric thinking between the control group and the experimental group before treatment?

2. Is there any significant difference in pupils' geometric thinking concerning plane geometry before and after phase-based instruction using Geogebra?

3. Is there any significant difference in pupils' geometric thinking between the control group and the experimental group after treatment?

The level of significance used in this study was 5% or 0.05. The research hypotheses were made based on the research questions stated above:

(a) There is no difference about mean rank of van Hiele geometric thinking levels in the Grade Seven pupils in the experimental group and the Grade Seven pupils in the control group before treatment.

(b) There is no difference in the pupils' mean van Hiele geometric thinking levels about plane geometry before and after phase-based instruction with Geogebra in plane geometry in China.

(c) There is no difference in the pupils' mean rank of van Hiele geometric thinking levels between the control group and the experimental group after treatment in teaching plane geometry.

## 1.6 Definition of Terms

### Van Hiele geometric thinking levels

The van Hiele geometric thinking levels were developed by Van Schleich (Gemert, 2015) based on long-term practical research. Lee and Kim (2012) introduced the van Hiele geometric thinking levels and applied the theory to design corresponding teaching courseware for the improvement of pupils' visual level.

The van Hiele geometric thinking levels have a very close relationship between the pupils' geometric thinking level and the effectiveness of teachers' classroom teaching. In this research, the Van Hiele levels were measured using the Van Hiele Achievement Test that consists of geometric thinking.

### Phase-based instruction

Phase-based instruction according to Van Hiele involves five learning phases. The first one is information phase, the second one is guided orientation phase, the third one is explication phase, the fourth one is free orientation phase and the last one is integration phase (Abdullah & Zakaria , 2013).

**Traditional method**

The traditional method refers to the teaching method whereby the teacher uses the system to explain in detail, so that the pupils can master a large amount of knowledge (Bao & Zhou, 2009). The form is relatively simple. Generally, the teacher is standing on the podium, and the pupils passively accept the teaching. Therefore, traditional teaching methods are often dubbed as cramming. Classroom is the main environment of teaching and the stage for teachers to perform (Bao & Zhou, 2009). Li (1997) pointed out that mathematical tradition teaching method can be considered as a way through various measures teachers can highlight the discovery, exploration, research and other cognitive activities in pupils' mathematics learning process, so that the mathematics learning process becomes more and more related to pupils finding problems, asking questions and solving problems. This is a learning method of traditional teaching method (Li, 1997).

**Geogebra**

Gabriela-Simona Antohe described the use of GeoGebra in geometry trajectory modeling (Antohe, 2009). Ljubica (2009) studies the importance of open source in international teaching software and how it is implemented using GeoGebra. The concept of dynamic geometry in the presentation of calculus was presented (Ljubica, 2009).

**Plane geometry**

Plane geometry is a branch of geometry that studies the properties of plane graphics, such as shape, size, position, etc (Wu & Han, 2012). Bao (2003) believes that geometry helps foster good thinking habits and helps develop deductive reasoning and logical reasoning capabilities. Plane geometry refers to geometry constructed according to Euclidean geometry (Liang, 2010). Plane geometry studies the geometric structure and metric properties (area, length, angle, position relationship) of straight lines and conic curves (i.e., conic, ellipse, hyperbola and parabola) on the plane (Wu & Han, 2012).

**Geometric Thinking**

Geometric thinking is one of the mathematical ways to solve mathematical problem. Through creative thinking activities, geometric thinking is used to examine mathematical objects, finally having a good understanding of the nature and regularity of the mathematical objects. The creative mathematics geometric thinking mainly contains guessing, intuition and imagination (Wei, 2015). Wei argued that, as a kind of exploratory thinking way, mathematical guessing is usually used to look for math laws as well as find the final solutions. The way of guessing analogized, empirically summarized, weakened, or strengthened the theorem conditions, mathematical imagination is the image thinking's specific application in mathematical consciousness activities (Wei, 2015).

**Operational definition**

In the quantitative evaluation of Van Hiele thinking level, most of the studies use the number of questions answered by the subject in the test performance as the standard for quantitative evaluation. For example, Usiskin (1982) quantitatively evaluates the level of Van Hiele thinking level at a certain level. The number of questions is three-fifths or four-fifths of all the questions to meet the standard of thinking. Ding and Liu (2013) used four-fifths of the standard to reach the level. In the research, Level 1 was given to the pupils who know the knowledge of plane geometry. Level 2 was given to the pupils who were able to recognize the figure of plane geometric. Level 3 was given to the pupils who are able to analyze the figures based on their specific geometric properties.

In this study, the researcher chose the Van Hiele Achievement Test that had been subject to reliability and validity test (Zhao, 2016). Zhao (2016) had investigated the integrity, systemic, normative, feasibility, and effectiveness of the test volume through a pilot test to ensure the validity and higher reliability of the test. Therefore, this instrument can measure the pupils' van Hiele geometric thinking level.

**1.7 Limitations and Delimitation of the Study**

This study has some limitations and delimitations. There are two limitations related to the research design and data collection method. On the other hand, there are three delimitations related to the psychological construct, setting and research sample.

Firstly, the limitation is associated with research design. This study used quasi-experimental research design. The pupils were not assigned randomly into the experimental group and the control group although they were from two intact mixed-ability classes. Lack of randomization may cause the pupils to be different at the baseline and may subject to concern of internal validity.

Secondly, the limitation is associated with the method of data collection. The researcher taught the pupils in the both control group and experimental group. This might cause a problem of validity of the data, but it could reduce extraneous variable that different instructors might have different teaching experiences and understanding about this study.

There are three delimitations related to the psychological construct, setting and research sample. First delimitation is associated with the psychological construct, which is teaching. The experiments in this study are conducted in real classroom teaching. Therefore, there are uncontrollable factors such as the state of a teacher's class, pupils' psychological changes, classroom environment, and so forth. These will have an impact on the effectiveness of teaching. The teaching of plane geometry is evaluated from the teacher's perspective to compare the effectiveness of phase-based instruction using Geogebra on pupils' van Hiele geometric thinking levels and the effectiveness of traditional teaching on pupils' van Hiele geometric thinking levels. This study does not take into account on how pupils learn plane geometry and how pupils think about plane geometry. The phase-based instruction is assumed to be more useful than the traditional teaching if the pupils who learn through phase-based instruction using Geogebra scored better than the pupils who learn through traditional method in post van Hiele Achievement Test and vice versa.

The second delimitation is associated with the mathematical learning content. The learning areas taught were only the elementary content of plane geometry. The complex of plane geometry is not included in the study. On the other hand, pupils were only taught to recognize, compare and solve simple problems involving plane geometry.

There are other related cognitive processes that were not examined in this study. Therefore, results of this research could not be generalized to all mathematics topics in the primary school curriculum. This research had been carried out for six weeks, which might be insufficient for some of the pupils to get used to Geogebra. Teachers need to have a process from contacting Geogebra to applying to skilled use. Due to the limitations of time and teaching content, many functions of Geogebra are not available.

The third delimitation is associated with the sample chose. The sample of the study was taken in the one school. The number was not large enough. The relevant investigation was only conducted in the first grade of junior high school. The generality of the survey results was insufficient, which would have an impact on the results of this study. This study was restricted to 112 Grade Seven pupils in one of the public schools in YanCheng in China. The pupils from public schools were not included in this study although they may have been studying similar mathematics syllabus. The number of participants in the sample in this study was small that the percentage of the pupils might be too small for the results to be statistically generalized to the population. It can only be generalized to all the Grade Seven pupils in YanCheng.

## 1.8 Significance of the Study

The study will contribute to math teachers. From the point of view of mathematics teachers, Van Hiele geometric thinking level provides evaluation criteria for the study of middle school mathematics geometry courses. This research provides teachers with a balanced teaching process and student learning outcomes, and helps pupils solve problems in geometric learning. Although the foreign research on Van Hiele theory has been quite mature, and the practical application of the theory is quite extensive, the Chinese research on this theory is still in a relatively weak position. The research of this thesis not only has important guiding significance for enriching and perfecting the teaching theory of mathematics geometry in middle school, but also has certain reference value in the field of middle school new curriculum reform.

Hence this study will contribute to curriculum development. Jin (2012) states that the evaluation aims to fully understand the pupils' mathematics learning process, to stimulate pupils' learning and to improve teachers' teaching; to establish an evaluation system with multiple evaluation objectives and various evaluation methods. Pupils should be regarded as the core of mathematics learning evaluation. Pupils' learning process, learning level, emotions and attitudes toward the learning activities should be considered. Moreover, it is very important to help pupils have a better understanding of themselves as well as help them to build up self-confidence. The evaluation of middle school pupils should be diverse, and the Van Hiele theory can be used to judge the pupils' geometric thinking level and provide additional evaluation criteria for the middle school mathematics geometry curriculum.

Additionally, the study will contribute to mathematics educational researchers; the study will offer the source of literature review especially in China. Although the international research based on Van Hiele theory has been quite mature, and the practical application of the theory is quite extensive, the research and application of this theory in China is still lacking. The researchers may extract some different views and perspectives from the statistical data in this study. They could contrast the results in this study to the others in the similar research area. The researchers may amend Geogebra usage in order to conduct future study in various mathematical content to enhance pupils' geometric thinking level.

This study believes that Geogebra has unique advantages and potential as a learning strategy. Since the beginning of development and promotion, the software has been widely used in Europe for free. In addition, the comprehensive functions of algebra, geometry, and statistics are very powerful. Geogebra has become an effective tool to change teachers' mathematics teaching methods and improve pupils' mathematics learning efficiency in Europe and North America.

### 1.9 Summary

Chapter one gives an overview for this research report. It gives the background of the study and identifies some critical issues associated with mathematics education and the area related to the study. This chapter covers the research purpose, research questions and research hypotheses. Next the definition of terms, possible limitations, and the research significance have been explained. In the following part, Chapter Two is related to the detailed literature review. In third chapter, the research methodology is discussed. In fourth chapter, findings of the research are interpreted and elaborated. For the fifth chapter, the research discussion, conclusions and implications are explained. In the last part, all related references have been attached in the Appendices.

# CHAPTER TWO

# LITERATURE REVIEW

## 2.1 Introduction

In this section, the researcher discusses the reasons for using van Hiele geometric thinking theory as the basis of the research, the conceptual framework of the research. And then past researches which examine the geometrical concepts teaching with dynamic geometry software are explained. Some previous studies related to the instruction of plane geometry are shown. Previous researches conducted through phase-based instruction using Geogebra are also included.

The study is conducted on the basis of van Hiele theory (Usiskin, 1982). The theory is related to geometry teaching and learning process, it was regarded as the joint theoretical framework. In this framework, learners' plane geometry learning levels were analyzed in this study. In the section of literature review, the research findings on ICT integration in the field of geometric thinking were discussed. Furthermore, this section points out the present knowledge gaps in the aspect of integrating mathematics education and technology.

## 2.2 Van Hiele Geometric thinking Theory

Both Dina Van Hiele Geldof and Pierre Van Hiele were mathematics teachers in a secondary school in the Netherlands. The theory stems from the doctoral dissertation written by Van Hiele and his wife in 1957. Both of them had been working diligently on the frontline of education. During their work as middle school mathematics teachers in the Netherlands, they found out in their teaching that the geometrical cognitive level of teachers and pupils is not consistent. The mathematics symbolic language and professional knowledge used by teachers in the teaching process often exceed the pupils' level of geometric thinking. Hence pupils cannot fully understand the teaching content of teachers.

In Li's study, the research theory of thinking level with high recognition in the world includes: SOLO classification; Piaget and Inhelder's research on the concept of children's space; Van Hiele geometric thinking theory (Li, 2016). The SOLO taxonomy aims to help pupils achieve a higher level of learning by analyzing the pupils' existing outcomes, and is suitable for all subjects and has a wide range of applications (Li, 2016). Piaget and Inhelder's research on the concept of children's space aims to examine the evolution of individuals' cognitive development with age and the impact of cognitive activities on pupils' cognitive development, but because of the lack of school factors and the inaccurate use of parts (Zhang, 2005). Van Hiele geometric thinking theory was found by Van Hiele in the practice of middle school geometry teaching. Emphasis is placed on teaching design based on pupils' thinking level and paying attention to the changes in pupils' geometric thinking (Li & Zhu, 2005). It is a research theory specifically aimed at middle school geometry teaching. The research on middle school geometry teaching has a strong application value. It can be seen from the above analysis that it is more applicable and feasible to choose Van Hiele theory for the plane geometry teaching.

More experts and scholars use this theory to evaluate the geometric thinking level of pupils at different stages, measure the geometric ability of pupils and apply them to guide geometric design. Among them, Usiskin (1982), Hoffer (1983), Burger (1986), and Shaughnessy (1986). From the perspective of teaching, the theoretical peers also provide evaluation criteria for the study of geometry mathematics in middle school. The theory draws the difference between the teaching of teachers and the understanding of students, and explains why pupils have difficulties in geometric thinking. Although the foreign research on Van Hiele theory has been quite mature, and the practical application of the theory is quite extensive, Chinese research on this theory and its application is still lacking. The research of this thesis not only has important guiding significance for enriching and perfecting the teaching theory of mathematics geometry in middle school, but also has certain reference value in the field of junior high school new curriculum reform in China.

### 2.3 Assessment of Van Hiele Theory

Zhang (2005) expounded the origin, changes, development and current status of geometry, and emphasized that geometry teaching has become a core issue in contemporary education reform. Zhang analyzed in detail the changing times of the content of geometry teaching, and clearly points out that the social environment is not conducive to geometry teaching (2005). It concluded that many pupils have been puzzled by why they must learn geometry. The article also answers the purpose of implementing geometric education in order to be able to appreciate and understand the infinite value of rational thinking and improve everyone's level of thinking. This fully demonstrates that geometric learning contributes to the development of thinking. At the end of the article, the author concludes that different people learn geometry in different ways.

Li and Zhu (2005) argued that in the process of geometry teaching, it is necessary to be good at using dynamic geometry software, which can help teachers to teach more effectively. The article compares the geometry teaching content in the 1959 and 2001 editions of the mathematics syllabus for middle and primary schools in Singapore to demonstrate that the mathematics syllabus will continue to increase the intensity of thinking skills teaching and the wider use of modern technologies. Teachers should make corresponding adjustments to these phenomena. Although the article did not conclude what the primary and secondary school geometry curriculum should teach, it reveals that geometry is nothing but an existence. When teachers teach mathematics, they should infuse geometry at any time and place. The article further emphasizes the importance of cultivating logical thinking.

Wei (2013) emphasized that the study of the Kasner polygon problem is not just a category of higher mathematics, but also has a certain degree of relevance to the learning content of middle school mathematics. Based on Van Hiele geometric thinking levels, this article experimentally studied 12 pupils from grade one to grade three in primary schools to verify that pupils of different grades have different levels of understanding and mastery of Kasner Square. Through the test results, the article analyzed in detail what level of thinking ability each grade student has and the changing trend of thinking ability of the same level pupils and pupils of different grades. Although the statistical criteria used in the analysis of the results of this article come from the Usiskin study, and actually the Usiskin research process contains more test questions than the one used in this article, the feasibility of this test is still of significance.

Li (2007) in her Ph.D. dissertation based on the pupils' knowledge of the inference ability and development of the geometry problem in the seventh to ninth grades, concluded that pupils in each grade have differences in development of their geometric reasoning ability. In order to promote improved student reasoning ability, the article further developed a hierarchical teaching strategy that meets pupils' different skills in different reasoning methods. The paper introduces five van Hiele geometric thinking levels when it comes to the structure of geometric reasoning ability. The author also compares the theory with Piaget's theory.

According to Usiskin (1982) school geometry has at least the following four characteristics: (1) Geometry is the construction of intuitive, graphical and spatial; (2) Geometry is the study of reality and the physical world; (3) Geometry is a mathematical representation or other medium that cannot be visually or physically represented; (4) Geometry is an example of a mathematical system.

Chen (2014) believed that the mathematics curriculum in middle and primary schools should focus on geometry. The reasons for this include: (1) Geometry can provide pupils with more opportunities to appreciate the world. Geometry exists in everything in nature. In fact, almost everything created by human beings is composed of geometric elements; (2) Geometry exploration can develop problem-solving skills. Space exploration is an important form of problem solving, and learning mathematics aims exactly to solve problems; (3) Geometry plays a key role in learning in other fields of mathematics and has close links with other fields of mathematics; (4) Geometry is essential in life and it is used almost every day of life.

There are two core contents of Van Hiele theory: the one is the geometric thinking five levels, and the other is the corresponding five teaching stages.

For example, Mayberry (1988) studied the essence of five levels and the student's organization at each level. Usiskin (1982) measured the student's geometric ability based on the Van Hiele theory. Fuys (1988) investigated effect of the theory of teaching; Burger and Shaughnessy (1986) investigated the effectiveness of these levels in describing the level of pupils' geometric thinking, and the effect of pupils' external behavior on each level.

Lee and Kim (2012) introduced the theory to design corresponding teaching courseware for pupil's visual level improvement for developing clear and rich content.

Olive (1991) designed a set of geometry courses using the LOGO programming language based on Van Hiele's theory and used this course to study how pupils came to learn geometric concepts.

Zhang (2005) expounded the origin, changes, development and current status of geometry, and emphasized that geometry teaching has become a core issue in contemporary education reform. In this article, Zhang (2005) analyzes in detail the changing times of the content of geometry teaching, and clearly points out that the social environment is not conducive to geometry teaching. It concludes that many pupils have been puzzled by why they must learn geometry.

Ding and Liu (2013) introduced the five geometric thinking levels of the Van Hiele couple, namely the 1-5 level, and then pointed out that although these five thinking levels are based on geometric learning. Ding and Liu (2013) still revealed people's understanding of the objective and the general law of things. The authors believed that the Van Hiele and his wife's geometry learning theory has a powerful guiding role in mastering the pupils' learning situation and designing a reasonable teaching process. The function is a combination of geometry and algebra. This article is new and different. Under the guidance of Van Hiele's theory, the teaching of function concept is taken as an example to analyze its teaching method. It provides a powerful teaching method for middle school mathematics teachers to make use of the van Hiele's theory to design the teaching process as well as teaching guidance.

## 2.4 Geometric Thinking Levels

Level 1: Recognition

Pupils can recognize the figure through the overall outline, and can manipulate its geometric composition elements (such as edges, corners); can draw or imitate the figure, use standard or non-standard name to describe the geometric figure; can solve geometric problems according to the operation of the shape. It is not possible to analyze the graph using the feature of the graph or the name of the feature, nor can the summary of the graph be discussed. For example, a child may say that a graphic is a circle because it looks like a wheel. At the level of visualization, the object of student reasoning is to confirm the classification of the figure by intuitively recognizing the same shape. If he is stating that the figure is circular, the student's meaning is this figure has what I have learned as a circular shape. The result of this reasoning is that the conceptualization of the graphs is based on a clear confirmation of their nature. After this conceptual construction, the pupils are at level 1 (Ding & Liu, 2013).

Level 2: Analysis

At this stage, the image becomes the background. The pupils begin to establish the graphical nature through measurement, observation, drawing, and so forth, and use these characteristics to solve the geometric problem, but they cannot explain the relationship between the properties, nor can they understand the definition of the graphic; comparing two shapes, using a certain property to do the graphic classification, but they cannot explain the relationship between certain properties of the graph, and cannot derive the formula and use the formal definition. For example, pupils will know circular inscribed quadrilaterals, but cannot understand the complementary nature of the inscribed quadrilaterals of circles (Ding & Liu, 2013).

Level 3: Ordering

Pupils can establish relationships between graphs and graphs. They can propose informal inferences and understand the elements of constructing graphs. They can further explore the intrinsic properties of graphs and their inclusion relationships, and use the formulas and definitions as well as the nature of discoveries to make deductions. However, it is impossible to establish the results of the proof from unfamiliar premises, nor to establish the intrinsic relationship between theorems. For example, after the student solves the fact that the central angle of the same arc doubles that of the circle, they will push out the right angle of the circle at which the diameter is opposite. Therefore, pupils can make some informal explanations but they cannot be systematically proved (Luo, 2016).

Level 4: Deduction

At this level, pupils build theorems in the axiomatic system. They can understand the importance of the proofs and understand the meanings of undefined terms, theorems, and axioms, and make sure that geometric theorems are needed. Formal logic derivation can only be established, understanding and solving geometric problems must have sufficient or necessary conditions; they can guess and try to confirm their guess with deduction, can use logical reasoning to explain geometric axioms, definitions, theorems, and so forth, and can also infer new theorem, the establishment of the network of theorems, can compare the different proofs of a theorem; can understand the causal relations in the proof, for example, the five elements of the vertical trail theorem: 1) line segment or straight line passing through the center; 2) perpendicular to the string; 3) equally splitting strings; 4) flattened strings for superior arcs; 5) flattened strings for inferior arcs; as long as two of them are satisfied, the remaining three can be rolled out. At this level, pupils can perform formal reasoning by logically resolving geometric statements, including definitions, axioms, and theorems. The object of reasoning is the relationship of the nature of the graphical classification. The product of reasoning is to establish the relationship between relations and a geometric system with logical chains (Luo, 2016).

Level 5: Rigor

At the fifth level, the pupils reason in the mathematical system. Even if they do not have a reference model, they can study geometry, and they can also formalize relationships. The product of their reasoning is the establishment of a geometric axiom system, and its detailed elaboration and comparison. Such as Euclidean geometry and non-Euclidean geometry comparison (Luo, 2016).

## 2.5 Phases of Learning

Van Hiele and his wife emphasized that "the progress of pupils from one level to the next level is seldom dependent on biological development or maturity; on the contrary, its development is affected by teaching process as well as learning process. Teachers are promoting this. This kind of progress plays a special role, especially in providing the desired guidance." (Tao, 2016) Teachers play a crucial role in developing pupils' level of geometric thinking. Corresponding to geometry learning, they have proposed the following division of the teaching stage, namely that pupils need to pass the following five stages under the guidance of a teacher to continuously exceed the existing level and reach a new height.

Phase 1: Information/Inquiry

Pupils begin to become familiar with the relevant content; teachers make necessary explanations about the content, and expose pupils to relevant content. During this stage, teachers should discuss or inquire about how pupils understand these words, and provide information to guide pupils to engage in purposeful actions or obtain relevant knowledge (Tao, 2016).

Phase 2: Directed Orientation

The goal of this stage of teaching is to allow pupils to actively explore (such as origami, measurement, etc.) so as to be able to reach out to the main links of the network of relationships that they wish to form; the role of the teacher is to guide pupils through the careful arrangement of activities. For appropriate exploration, at this time pupils are engaged in actual operations, and teachers should choose those objects, concepts and methods as well as more obvious materials and tasks (Tao, 2016).

Phase 3: Explication/Explanation

At this stage, pupils begin to clearly understand the relationship they want to learn and describe it in their own language. Teachers should guide pupils to use their own language to study objects (geometric objects and concepts, relationships, patterns, etc.) in their own language. The discussion will enable pupils to achieve a clear level of understanding. In addition, once pupils have demonstrated a clear understanding of the learning object and have discussed it in their own language, teachers should introduce relevant mathematical terms (Luo, 2016).

Phase 4: Free Orientation

At this stage, pupils encounter multi-step assignments or assignments which they can accomplish with different methods. Pupils obtain experience to solve problems. By determining the direction of the learning field on their own, they are increasingly clear about the relationship between learning objects and can apply relationships to solve problems. Teachers are mainly responsible for choosing the right material and geometry problems (preferably there are multiple solutions) and encouraging pupils to think and explain the questions, and give an introduction to the related terms, some concepts, and processes of solving problems according to the need. According to Van Hiele (1984): This stage is free for pupils to explore. The scope of the survey is that most pupils know, but pupils still need to find their direction quickly (Luo, 2016).

Phase 5: Integration

Pupils integrate their knowledge into relevant networks that can be easily described and applied, review their own methods, and form a point of view to build a summary of all the content they have learned. Objects and relationships are unified and internalized into one. A new thinking area is formed. Teachers are mainly responsible for encouraging pupils to consolidate the geometric knowledge they have learned, make a comprehensive review of what pupils understand, and further emphasize consolidation of the application of the mathematical structure as a framework. After completion of the fifth stage, pupils' thinking on the content of learning has reached a new level (Tao, 2016).

## 2.6 Conceptual Framework

The conceptual framework (Figure 2.1) illustrates the connections of variables that are helpful in achieving the objective of the study.

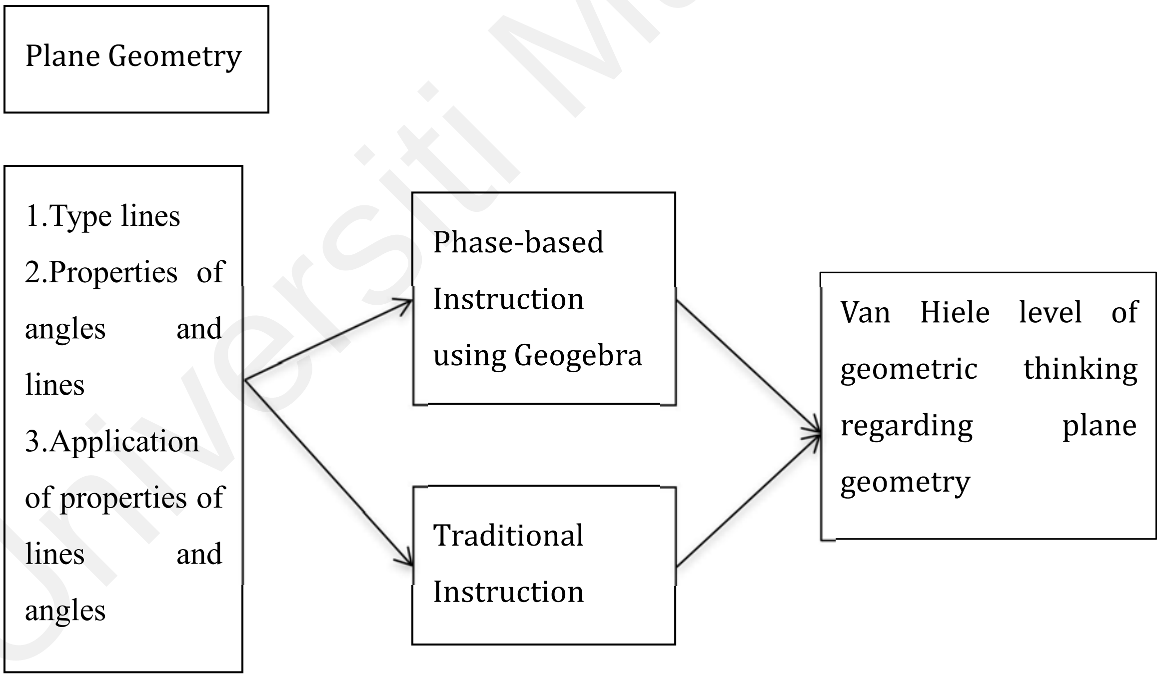


Figure 2.1. *Conceptual framework*

The first column of the conceptual framework shows the mathematics content of plane geometry for secondary school pupils. This study taught the types, properties and application of properties of lines and angels. The study used the phase-based instruction

(Table 1) for the experimental group meanwhile used the traditional instruction for the control group. The second column of the conceptual framework is the instruction using Geogebra in this study and another way is traditional method. Phase-based instruction using Geogebra is also known as the independent variable in the research. The contrast independent variable is phase-based and traditional instruction. The third column of the conceptual framework shows the dependent variable in the research, which is a subject that is measured, which is the Van Hiele geometric thinking levels of the Grade Seven pupils in China.

Usiskin (1982) stressed on the role of instructions in helping the pupils to progress from the lower geometric thinking levels to the higher geometric thinking level. Fuys and Tischler (1988) explained the phase-based instructions, which were put forward by van Hiele and suggested the learning tasks as illustrated in Table 2.1.

Table 2.1

*Phase-Based Instructions and Suggested Learning Tasks*

| Phase-Based Instructions | Suggested Learning Tasks |
|---|---|
| Information | Pupils work with examples and non-examples. |
| Guided Orientation | Pupils do tasks involving different relations such as folding and measuring. |
| Explanation | Pupils know the concepts. |
| Free Orientation | Pupils can transfer information by knowing properties of one kind of shape and investigating these properties for another shape |
| Integration | Pupils summarizes and reflects on his or her learning and actions |

The phase-based instruction is on the basis of the five phases Van Hiele (1986). The five phases are appropriate and sometimes even indispensable for research and teaching purposes. For pupils, the four phases seem to be more appropriate. The first phase is information. It is a basic step for pupils. In this phase, pupils work with examples and non-examples.

In the second phase, which is known as guided orientation, pupils do tasks involving different relations such as folding and measuring. In the third phase, the pupils know the concepts. The fourth phase is free orientation. In this phase, pupils can transfer information by learning properties of one kind of shape and investigating these properties for another shape. The fifth phase is known as Integration. Pupils summarize and reflect on their learning and actions.

The controlled group will be taught by using the traditional instruction. In the research, traditional instruction is defined as an instruction that is not involved in using Geogebra with phase-based instructions. Pupils in control group will use their mathematics textbook and exercise books to do questions based on Chinese Middle School Standard Curriculum.

## 2.7 Traditional Method in Geometric Thinking

*The mathematics curriculum standard of compulsory education* (2011) described the mathematical teaching method request. According to the standard, students should make efforts to explore and learn mathematics knowledge to solve mathematical problems. At first, students should raise specific mathematical questions according to some mathematical facts and come up with assumptions. Moreover, students should actively look for proper mathematical laws, which could be used to answer the questions they raise. Finally, students should show how to use these mathematical laws to solve the mathematical problems.

Li (1997) pointed that mathematical tradition teaching method can be considered as a way that teachers can highlight the discovery, exploration, research and other cognitive activities in the process of pupils' mathematics learning through various measures, so that the mathematics learning process makes more and more pupils find problems, ask questions and solve problems. This is a learning method of traditional teaching method.

Xu (2016) pointed that the traditional teaching method takes the problem as the carrier. The teacher creates the situation of exploring the problem based on the pupils' prior knowledge. He make the hypothesis through observation and analysis, and further guides the teacher to the established mathematical concepts and conclusions. The pupils can gradually explain the argument hypothesis, and finally get a new knowledge of mathematics. During this period, pupils realized that the mathematical concept and the tortuous process of conclusions, the use of hypothesis recognition and the use of logical dialectical thinking, which helps pupils to understand the intuitive and rigorous relationship of mathematics, and to develop the spirit of scientific practice and mathematical thinking ability.

## 2.8 Geometric Thinking in Plane Geometry

Guo (2008) pointed out in the cognitive experiment of plane geometry, although junior high school pupils have differences in knowledge level, the difference in the degree of knowledge acquisition and application is not obvious. The level of geometric thinking of junior high school pupils has matured and the pupils have abstract logical thinking, but the specific images will affect accepting new knowledge. Educators should try to coordinate the contradiction between the pupils' geometric thinking level and the logic of the knowledge they have learned.

Tian (2006) showed that many junior high school pupils only understand the concept and theorem of geometric knowledge on the surface without forming an overall understanding, and there is fear of geometric proof. In deductive reasoning, junior high school pupils can imitate ready-made topics, but they often feel at a loss when they encounter similar unsolved problems with multiple steps of thinking or problems.

Wu (2014) reached the following conclusions by conducting a questionnaire survey among mathematics teachers from middle schools. On the whole, the geometric knowledge level of those mathematics teachers is not high, and the knowledge of geometric thinking disciplines is not balanced. The shortcomings of junior high school education are the reasons why the mathematics knowledge of junior high school mathematics teachers is not high.

*The mathematics curriculum standard of compulsory education* (2011) is to emphasize the role of geometric thinking ((Bao & Zhou, 2009). It can be seen that China has always paid great attention to the teaching of middle school plane geometry.

**2.9 Teaching Geometric Thinking using Technology**

Some previous researches have investigated the integration of dynamic geometry software into the teaching of plane geometry. *Junior high school mathematics* *curriculum standard* (2011) points out that the wide application of modern information technology is having a profound impact on the content of mathematics courses, mathematics teaching, and mathematics learning, especially for the plane geometry. *Junior high school mathematics curriculum standard* (2011) pays high attention to promote the organic integration of information technology and plane geometry. The use of information technology to present course content is important. It is difficult to present in previous teaching, use scientific calculators and various mathematical education technology tools as much as possible, and strengthen the combination of mathematics teaching such as plane geometry and information technology. In the *Junior high school mathematics curriculum standard* (2011), the *standard* encourages pupils to use computers, calculators and the tools of information communication and technology to explore and achieve the ability of geometric thinking. So, it is necessary to integrate information technology and mathematics, especially plane geometry.

In the history of teaching technology in Chinese teaching, from the earlier computer-aided teaching to the current information technology as well as curriculum integration, the researchers believe that the application of domestic information technology in teaching has experienced the following three stages of development (He & Wu, 2008). In the research, it is concluded that there are three stages of integration technological tools into the plane geometry in China. There are CAI (computer-assisted instruction) stage, CAL (computer-assisted learning) stage and IITC (integrating information technology into the curriculum) stage (He & Wu, 2008). CAI (computer-assisted instruction) stage is computer-aided teaching stage. From the early 1960s to the mid-1980s, this stage mainly used the fast calculation, graphic animation, and simulation of computer to help teachers solve some important problems and difficulties during the process of teaching. CAL (computer-assisted learning) stage is computer-aided learning stage. From the mid-1980s to the mid-1990s, this stage was gradually transferred from auxiliary teaching to auxiliary learning, emphasizing how to use computers as a tool to assist pupils in learning. For example, using computers to help collect information, counseling and answering questions, self-testing, and helping to arrange learning plans, that is, not only using computer-assisted teacher teaching, but also emphasizing the use of computers to assist pupils in autonomous learning. IITC (integrating information technology into the curriculum) stage, that is, information technology and curriculum integration stage. Since the mid-1990s, integrating information technology with various subject courses has become a research topic that everyone pays attention to.

In 1995, the National Computer Education Research Center of Primary and Secondary Schools introduced the mathematical software Geometric Sketchpad from the United States. The usage of Geometric Sketchpad in mathematics teaching has achieved certain results (Xu, 2009).

In 2002, the Ministry of Education's Basic Curriculum Development Center launched the Z+Z Intelligent Education Platform for Experimental Research on National Mathematics Curriculum Reform. These projects have promoted the research of applying information technology to mathematics teaching in China (Li, 2004).

In recent years, it has presented a dynamic mathematical software with rapid development momentum， such as Geogebra. At present, this software may not be well known to the first-line mathematics teachers in mainland of China, but it has been widely used in mathematics teaching in Taiwan and Hong Kong, and it has achieved fruitful research results (Xu, 2009). Although the mainland-related research has just started, the development momentum is rapid. In 2011, Geogebra College was established in Beijing Normal University, Tianjin Geogebra College and Nanjing Geogebra College were also established. Throughout the development of recent decades, China has made a lot of efforts and attempts to apply information technology to mathematics teaching. China has achieved certain developments and achievements, but there is still a long way to continue to explore and practice (Xu, 2009). It is worth noting that applying information technology to inquiry teaching has been paid little attention and reference. The professional research in this field is lacking in China.

## 2.10 Usage of Geogebra

Gao (2011) made full use of the characteristics of Geogebra to promote pupils' understanding of mathematics concepts in the process of participating in the classroom, emphasizing that pupils should actively participate in the teaching process and give full play to their enthusiasm in the classroom. The article also provided some suggestions for designing electronic textbooks.

Li (2014) studied the role of Geogebra as a tool in assisting pupils to understand conceptual learning in mathematics. This paper conducted the teaching design and teaching experiments with the understanding of the probability of the learning definition. It is found that the Geogebra can help pupils to improve the level of understanding of the concept of learning, so that pupils can be from the empirical understanding to the structural understanding to cultural understanding. At the same time, pupils can form a perfect cognitive structure including forming a new conceptual structure in the mind, and internalizing it into its own knowledge structure, establishing a complete representation, and constructing a schema about this concept. Some strategies for the understanding of Geogebra-assisted mathematics learning were devised and illustrated that these strategies were effective.

Li (2013) proposed the use of advantages of Geogebra to change the drawbacks of traditional teaching and stimulate the pupils' learning motivation and interest in learning dynamic geometry. Geogebra improved the traditional junior high school dynamic geometry teaching. And Geogebra guided pupils to dynamic geometry academic performance.

Koyuncu, Akyuz and Cakiroglu (2015) investigated how mathematics teachers in the future solve plane geometry problems with paper-and-pencil as well as technology. This research investigated how the prospective mathematics teacher, after understanding the related operations of Geogebra, chose to solve the problem of plane geometry in dynamic geometry software and environment of both paper and pen. The results show that even though most participants choose to use algebraic methods in the paper-and-pen environment, they are more inclined to use geometric solutions in the Geogebra environment even if the algebraic approach can be solved. This shows that changing the environment may encourage pupils to find other solutions, which will also help pupils understand issues better. So, solving problems by making use of more than one environment more beneficial.

## 2.11 Teaching Geometric Thinking using Phase-based Instruction with Geogebra

Wu and Zhou (2013) used Geogebra in the four aspects of derivative concept teaching based on the APOS theory, found that activities, processes, objects and schemas, and explored the application of Geogebra in concept teaching. They believed that under the dynamic geometric environment created by Geogebra, using the advantages of software technology, it was possible to break through the limitations of pupils' cognitive level and achieve good teaching results.

Rumanova (2015) concluded that the creativity and power of geometric task design in teaching. This article focused on the geometric task design, visualization of geometric problems, and the creativity needed in the use of information and communication technology (ICT). The article presents various issues. A total of 21 teacher-training pupils participated in the survey. Research attempted to familiarize normal pupils with different teaching methods, because researchers believed it is crucial for normal pupils to be aware of problems in the teaching process. Normal pupils were becoming more familiar with inlays in using the teaching software Geogebra to solve problems. Teaching embedded in mathematics is considered a suitable way to motivate pupils.

Kutluca (2013), in his research, he studied the effect of Geogebra, a dynamic geometry software based on stage teaching, on the Van Hiele geometric thinking comprehension of 11th-grade students. The study used a pre-test and control group quasi-experimental method. The sample was 42 pupils in the 11th grade from the 2011 to the 2012 spring semester. The Van Hiele Achievement Test has a total of 25 items. This result can be interpreted as a phase-based instruction for the experimental group as well as a learning environment, which is provided by Geogebra. In fact, unlike the control group, pupils from the experimental group had a chance to move a given graph, create their own geometry, experiment with various shapes, test and construct their own knowledge. In addition, the experimental group can actively participate in the teaching process, sharing ideas, discussing existing results with peers, and ultimately constructing their own knowledge system.

Wei (2013) used phase-based instruction with Geogebra software to assist junior high school mathematics teaching, and compared it to the traditional method on the topic of the effects of learning graphics rotation and quadratic function in junior high school. And then pupils did the questionnaires of satisfaction with Geogebra. After the analysis of the questionnaires, it was found that Geogebra stimulated pupils' desire to learn mathematics and innovation, and cultivate pupils' ability of innovation, analysis and problem solving. The research method was a quasi-experimental research method, which adopted the unequal group before and after measurement design. The experimental sample was two classes in the grade three of Yunxi School in Honggutan District, Nanchang City, Jiangxi Province. The experiment lasted for five weeks and had 25 class hours for the experimental class.

## 2.12 Teaching Plane Geometry in Junior High School

"Graphics and Geometry" is one of the four major sections of mathematics in junior high schools (Zhang, 2012). The others are "number and algebra", "statistics and probability", and "comprehensive and practical" (Zhang, 2012). This is a focus of educational reform. In China, the education sector has repeatedly discussed the reform of the middle school geometry curriculum. For example, the middle school mathematics teaching professional committee pointed out in the 1995 annual meeting report that Plane geometry is a controversial content of current mathematics teaching content in junior high schools. Most people with the view of retaining the plane geometry believed that the planar geometry knowledge is conducive to the pupils' logical thinking training, but it is recommended reducing the difficulty of the proof. A few people do not need to set up separate subjects, but replace them with other methods (Zhang, 2012). The dramatic changes in geometric content during the development of the *The mathematics curriculum standard of compulsory education* have also led to widespread debate among scholars (Tang & Zhang, 2005). However, China is still one of the countries with the most geometric content in middle school and the most emphasis on geometric teaching.

Guo (2008) pointed out that although there are differences in the level of knowledge among junior high school pupils in the cognitive experiments of similar triangles, the differences in the extent and application of knowledge acquisition are not obvious. In addition, the level of geometric cognition of junior high school pupils have matured and have abstract logical thinking, but the specific images will still affect accepting new knowledge. Therefore, educators should try to coordinate the contradiction between the pupils' cognitive level and the logic of the knowledge they have learned.

Tian (2006) showed that many junior high school pupils only understand the concept and theorem of geometric knowledge on the surface without forming an overall understanding, and there is fear of geometric proof. In deductive reasoning, junior high school pupils can imitate ready-made topics, but they often feel at a loss when they encounter similar unsolved problems with multiple steps of thinking or problems.

The curriculum standards from primary school to junior high school to senior high school have gradually increased the requirements for pupils to learn about plane geometry knowledge. Plane geometry knowledge is especially important in mathematics learning.

### 2. 13 Research gap

Lu (2014) obtained the conclusions on the geometric thinking level in the grade seventh, eighth, and ninth pupils. Kutluca (2013) researched the impact on dynamic geometry software Geogebra in the level of understanding of the grade 11th student Van Hiele gromrtry thinking level with phase-based instruction. Gao (2013) drew the following conclusion that the level of cognitive geometric thinking in classroom teaching is consistent with the teaching ability, which is higher than the student's cognitive level. Classroom teaching helps pupils to improve their geometric thinking level and most middle school pupils can reach the geometric thinking level 3 after class. There are not many researches done on teaching plane geometry to Chinese secondary school pupils, specifically the phase-based instruction with using Geogebra on the basis of Van Hiele geometric thinking level (Xu, 2016). Therefore, this is the research gap.

### 2.14 Summary

According to the previous studies, the researchers found that Geogebra could contribute to enhance the understanding of mathematical concept, mathematical formula and property (Rincon, 2009). There have not been any researches done to teach plane geometry to Chinese secondary school pupils, specifically based the geometric thinking with using Geogebra on Van Hiele level. This is the research gap. The Chinese researcher suggested that Geogebra assisted mathematics teaching to improve the pupils' space imagination ability (Wei, 2013). Therefore, this research investigates using Geogebra in the process of improving pupils' plane geometry of Van Hiele geometric thinking levels.

In the following the chapters, the researcher illustrated the design in details based on the previous researches. To elaborate and interpret the research findings, discussion, conclusions and its implications referring to the previous researches.

# CHAPTER THREE

# METHODOLODY

## 3.1 Introduction

This research aimed to investigate the difference in middle school pupils' Van Hiele geometric thinking levels using phase-based instruction with Geogebra. Specifically, the following three questions are answered in the research: (a) Before teachers from China use Geogebra in the plane geometry teaching, whether pupils in grade seven from the control group and experimental groups differ from each other in Van Hiele geometric thinking levels or not, (b) Before and after teachers from China use Geogebra in the plane geometry teaching, whether pupils in grade seven make a difference in Van Hiele geometric thinking levels about plane geometry or not, (c) Before and after pupils in grade seven accept traditional teaching, whether their Van Hiele geometric thinking levels about plane geometry change or not, (d) After teachers from China use Geogebra in the plane geometry teaching, whether pupils in grade seven from the control group and experimental groups differ from each other in Van Hiele geometric thinking levels or not

This chapter explained the methodology of the research such as research design, population and sample as well as sampling technique. The researcher explained the procedures of data collection in detail, and then illustrated a diagram to the flow of the study. This paper designed six activities to guide the Grade Seven pupils from the experimental group to gradually improve their own Van Hiele geometric thinking levels. Van Hiele Achievement Test was the instrument in this study, which was from Zhao's study. The researcher represented the validity and reliability of instruments, explained the data analysis method used in this chapter. This chapter included a final section of summary.

**3.2 Research Design**

In this study, a pre-test and post-test design was used, which was quasi-experimental and non-equivalent. In the opinion of Wallen and Fraenkel (2000), conducting a quasi-experimental research is conducive to establish causality. Gribbons and Herman (1997) hold that quasi-experimental studies are similar to randomized controlled trials or traditional experimental designs. However, in their point of view, the former lack some elements, which could be used to assign to control or treatment randomly. This study was a quasi-experimental designed for non-equivalent control group, which was because it was impossible to divide students into groups randomly because of the different class hours (Figure 3.1).

| | | | |
|---|---|---|---|
| Experimental group | O1 | X1 | O2 |
| Control group | O1 | X2 | O2 |

O1 stands for the pre-Van Hiele Achievement Test

O2 stands for the post Van Hiele Achievement Test

X1 stands for the plane geometry using phase-based instruction with Geogebra

X2 stands for the plane geometry using traditional approach

Figure 3.1. *Research design*

In this study, the researcher compared between the groups on the Grade Seven pupils. The researcher can examine the improvement of pupils according to Van Hiele geometric thinking levels by using non-equivalent pretest-posttest design. According to Shadish, Cook and Campbell (2002), researchers using quasi-experiments may still have a modest control over the selection and scheduling of measures, the implementation of non-randomized allocations, the comparative comparison groups and some aspects of how to arrange treatment. Trochim (2001) puts forward in his study that he made the threat of external effectiveness minimize by using quasi-experimental design, because natural environments don't have the same human problems as well-controlled laboratory environments. As a quasi-experiment, this study is a natural experiment. The results of this study could be applied to other settings and subjects, allowing some generalization of the population.

In the research, the participants were Grade Seven pupils from two intact mixed-ability classrooms. Both classes were not streamed. All of them were thirteen years old. The researcher used coin-tossing to assign one of the classes as the experimental group and another class of pupils as the control group. Pupils from the control group learnt the plane geometry in a traditional way while pupils from the experimental group learnt the plane geometry using phase-based instruction with Geogebra. The control group consisted of 54 pupils. They were taught by using the traditional method of learning the plane geometric for six weeks. On the other hand, the experimental group consisted of 58 pupils. They were exposed to Geogebra for 80 minutes lesson for six weeks. This study focused on finding out the differences in Van Hiele geometric thinking levels between pupils who learnt plane geometry using Geogebra and others who learnt plane geometric though the traditional instruction.

### 3.3 Population and Sample

The population in this study included two classes of Grade Seven pupils in the public schools in China. The population was Grade Seven pupils from a public school in YanCheng in China. In this study, the participants were Grade Seven pupils from two intact mixed-ability classrooms. Both classes were not streamed. All of them were thirteen years old. The researcher used coin-tossing to assign one of the classes as the experimental group and the other class of pupils as the control group. This study chose the convenient samples. In order to do the purposes of research, some researchers chose more simple samples to conduct the survey (Ratnayake, Joyce & Webb, 2012). These simple samples are convenient samples.

The school approximately has 2655 pupils from Grade Seven pupils to Grade Nine pupils. Each of the Grade Seven pupils' classes comprised 54 to 60 pupils. In this school, most of the pupils are Chinese and several are South Korean. There are fifteen classes in each standard. This study is conducted with the two classes in Grade Seven. In the two classes, students vary in ability. All of them were around thirteen years old. They were selected to provide a rich data, in which all pupils have not learnt the plane geometry and have not been exposed to Geogebra before.

### 3.4 Data Collection Procedure

The research procedures were illustrated as show in Table 3.1. At the first, the researcher chose the pre-Van Hiele Achievement Test (on Appendix A) to exam pupils initial Van Hiele geometric thinking levels regarding plane geometry in the experimental and the control group at the same time.

Table 3.1

*Research procedures*

| Groups | | Research Procedures |
|---|---|---|
| Experimental Group | 1 | Pre Van Hiele-Achievement Test |
| | 2 | Lesson of introducing Geogebra |
| | | Lesson1 Knowing about lines and angles |
| | | Lesson2 Knowing about complementary angles and supplementary angle |
| | | Lesson3 Knowing about parallel lines |
| | | Lesson4 Knowing about alternate interior angles |
| | | Lesson5 Analyze parallel lines |
| | | Lesson6 Analyze the properties of parallel lines |
| | 3 | Post Van Hiele Achievement Test |
| Control Group | 1 | Pre Van Hiele-Achievement Test |
| | 2 | Lesson of traditional method |
| | | Lesson1 Knowing about lines and angles |
| | | Lesson2 Knowing about complementary angles and supplementary angle |
| | | Lesson3 Knowing about parallel lines |
| | | Lesson4 Knowing about alternate interior angles |
| | | Lesson5 Explore the conditions of parallel lines |
| | | Lesson6 Explore the properties of parallel lines |
| | 3 | Post Van Hiele Achievement Test |

To give Five days after test of the pre-Van Hiele Achievement, the researcher introduced the software of Geogebra to the pupils in the experimental group using smart board. The researcher demonstrated on the usage of each tool in the toolbox, which included the Selection Arrow tool, point tool, Compass tool, Straightedge tools, and Text tools. The terms of describing mouse activities, such as Point, Click, Double-Click, and Drag were also explained. Then, the researcher guided the pupils to rotate the shape drawn as well. At last, they explored Geogebra in pairs and completed the tasks provided whining 30 minutes in the computer lab. The researcher walked around to provide guidance if necessary.

The second week, the pupil from experimental group experienced intervention of plane geometry using Geogebra whereas the pupils in the control group underwent instruction of plane geometry using the traditional instruction. The instruction for each Geogebra activities for the pupils in the experimental group utilized van hiele theory to guide the experimental group's pupils to carry on higher van Hiele geometric thinking levels. Geogebra was used as a main tool of instruction because of its significant advantage in allowing the pupils to learn through exploration.

The instruction received by the pupils in the control group was totally different from pupils in the experimental group. The other class of pupils in the control group learnt using the textbook and worksheets by the Ministry of Education in China. The samples of lesson plan and worksheets provided for the pupils in the control group are attached in Appendix C.

After six weeks of the intervention period, the post Van Hiele Achievement Test (Appendix A) was given to pupils in both groups to evaluate on their van Hiele geometric thinking levels towards the concept of plane geometry. During the evaluation process using the pretest and posttest, both group of pupils were allowed to use Geogebra so that they possessed the same condition and environment during the Grade Seven pupils' assessment. The flow of the research procedures is illustrated in Figure 3.2.

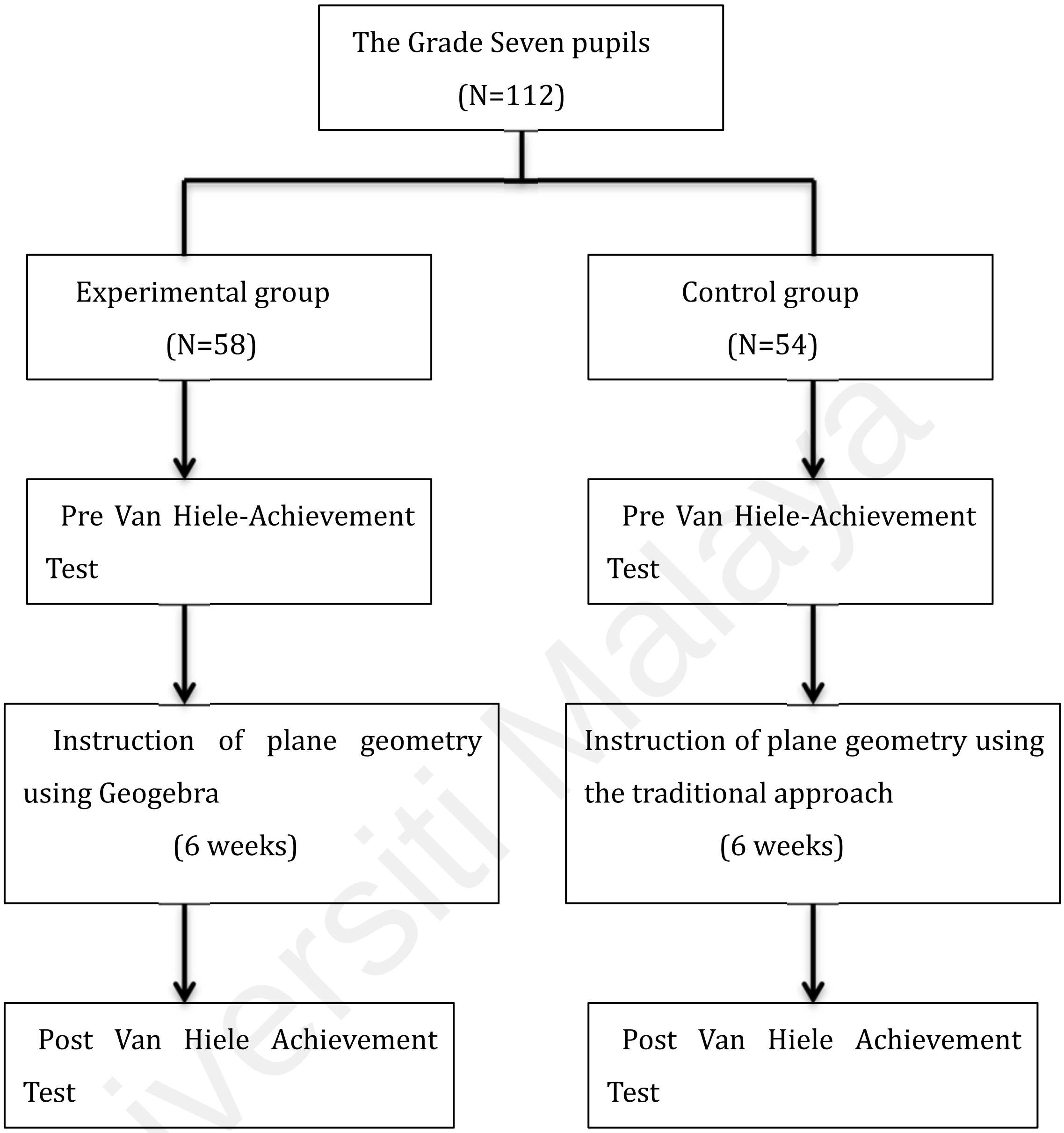


Figure 3.2. *Flow of the stages in the quasi-experimental research procedures*



### 3.5 Instructional Activities

There were total of six activities for the pupils to do. The activities included Lesson 1 (Knowing about lines and angles), Lesson 2 (Knowing about complementary angles and supplementary angle), Lesson 3 (Knowing about parallel lines) and Lesson 4 (Knowing about alternate interior angles), Lesson 5 (Analyzing parallel lines) and

Lesson 6 (Analyzing the properties of parallel lines). These activities were designed basically based on Van Hiele geometric thinking levels. Van Hiele determined some generalities in his model that characterized the use of the model to guide the educators to make instructional decisions (Crowley, 1987). This study described the Van Hiele's geometric thinking phases in each experimental group activities.

The six activities with phase-based instruction with Geogebra were attached in Appendix B. As an example: in the Lesson 1, the topic is knowing about lines and angles. The time of the whole class was 80 minutes. The learning objectives are knowing about the meaning of lines and angles and applying knowledge of today's lesson. The whole lesson arranged with phase-based instruction using Geogebra divided the time into 4 parts, and every part had 20 minutes. There is information phase (20mins), orientation phase (20mins), explication phase (20mins) and free orientation phase (20mins). In the information phase, teacher asked the pupils that if a person wanted to walk to point B from point A, which way was the shortest (Figure 3.3)? Then teacher told the student a fact: The line segment between two points is the shortest. Teacher showed the half line FG and the half line FH in the lesson1.ggb. Then teacher told the student: When two straight lines meet, an angle is formed at the vertex.

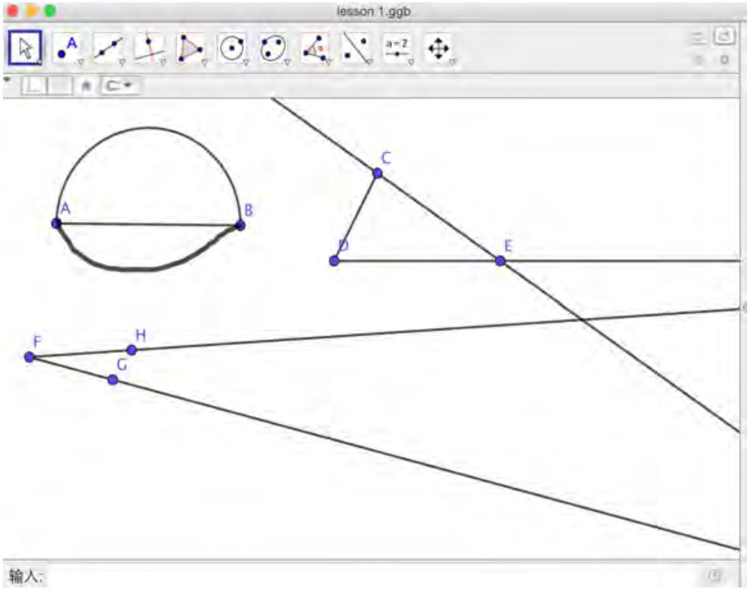


Figure 3.3. *Example of lesson 1*

In orientation phase (20mins), CD is line segment. DE is half line (*Figure 3.3*) and CE is straight line. Then teacher guided the pupils to explore the meaning of lines by moving the point in Geogebra. Teacher gave Problem 1 (Appendix B) and then the pupils answered Problem 1. In explication phase (20mins), teacher gave the Problem 2 (Appendix B) and then the pupils used the knowledge to draw the Problem 2 on Geogebra by themselves. In free Orientation Phase (20mins), the pupils observed the figure shown in Problem 3 and Problem 4 (Appendix B) and solved the problems by themselves. After the pupils solved the problems, a discussion was facilitated by teacher.

The following 5 lessons are attached in Appendix B. The researcher explained more in details in tables.

## 3.6 Instrumentation

**Van Hiele Achievement Test**

In the quantitative evaluation of Van Hiele thinking level, most of the studies used the number of questions answered by the subject in the test performance as the standard for quantitative evaluation. For example, Usiskin (1982) quantitatively evaluated the level of Van Hiele thinking level at a certain level. The number of questions was three-fifths or four-fifths of all the questions to meet the standard of thinking. Senk (1989) used the answer to the number of questions as four-fifths of the test to pass the standard. Zhu, Guo, Xinsen, & Cao (1996) used more than two-thirds of the correct answers in each concept question while Ding and Liu (2013) used four-fifths of the standard to reach the level.

In order to answer all the research questions stated in chapter one, the researcher adopted Van Hiele Achievement Test from Zhao's study to assess the Grade seven pupils' Van Hiele geometric thinking levels before and after the intervention. The pre and post Van Hiele Achievement Test (Appendix A) assessed the Grade seven pupils on the plane geometry and properties of plane geometry. Zhao (2016) had done the reliability and validity test of the items. He had done the pilot test. A structural reliability analysis was performed on the pilot test using Cronbach Alpha method using SPSS17.0 to obtain Table 3.2. In the table, the Cronbach's Alpha of the test items is 0.792, which is close to 0.8. So the Van Hiele Achievement Test is highly reliable and can be used as the test.

Table 3.2

*Reliability and Validity Test*

| Cronbach's Alpha | Cronbach's Alpha based on standardized items | Conclusion |
|---|---|---|
| 0.792 | 0.792 | Higher Reliability |

The pupils needed to understand the plane geometry well including the occurrence of plane geometry in daily life problems. The distribution of items in the pre and post Van Hiele Achievement Test was illustrated in Table 3.3. The test comprised of twenty multiple-choice items. As Mayberrry's (1981) and Usiskin's (1982) scoring criteria suggested the passing criterion for each level should be 60 percent or above. In this research the scoring criteria for the pupils achieve Level 2 in Van Hiele test if and only if they can answer at least 7 out of 10 items correctly for the first ten items and Level 3 in van Hiele test if and only if they score 7 out of 10 items correctly for the last ten items.

Table 3.3

*Distribution of items in the Pre and Post Van Hiele Achievement Test*

| Question Number | Description | Level | Question Type | Criterion for Level |
|---|---|---|---|---|
| 1 | Perpendicular lines | Level2 | Analysis | |
| 2 | Area of rectangle | Level2 | Measurement | |
| 3 | Similar figures | Level2 | Analysis | |
| 4 | Obtuse angle | Level2 | Analysis | |
| 5 | Linear pair | Level2 | Measurement | 7 of 10 |
| 6 | Parallel line | Level2 | Analysis | |
| 7 | Circle terminology | Level2 | Analysis | |
| 8 | Parallel line | Level2 | Analysis | |
| 9 | Right angle | Level2 | Measurement | |
| 10 | Perpendicular lines | Level2 | Measurement | |
| 11 | Equilateral triangle | Level3 | Deduction | |
| 12 | Properties of parallelogram | Level3 | Deduction | |
| 13 | Perpendicular line | Level3 | Deduction | |
| 14 | Properties of parallelogram | Level3 | Deduction | |
| 15 | Perimeter of parallelogram | Level3 | Deduction | 7 of 10 |
| 16 | Similar triangles | Level3 | Deduction | |
| 17 | Definition of circle | Level3 | Deduction | |
| 18 | Perpendicular line | Level3 | Deduction | |
| 19 | Supplementary angles | Level3 | Deduction | |
| 20 | Perpendicular line | Level3 | Deduction | |

**Reliability and Validity of Instrument**

There are several aspects of validity and reliability to be considered in conducting this study. Golafshani (2003) stated as "validity determines whether a study actually measures what it is trying to measure or the veracity of the outcome" (p. 599). The type of validity is relevant to this study including content validity, translation validity, internal validity and external validity and others.

In order to ensure the content validity, this paper constructed the items in the instrument of Van Hiele Achievement Test by adapting and modifying the items in the Van Hiele Geometry Test developed by Usiskin (1982). After constructing the items in Van Hiele Achievement Test, the instruments were tested for the reliability and validity. Zhao (2016) fulfilled the instruments of content validity in China.

The threat of mortality did not affect the internal validity of this research as all pupils fully participated in the study, which meant there was no missing values in both the before Van Hiele Achievement Test and after Van Hiele Achievement Test. All lessons were conducted within two months, in which each lesson was fixed as 80 minutes lesson.

Subject characteristics threat when the pupils were not randomly divided into the control group and the experimental group that can be minimized by administering pre-Van Hiele Achievement Test before the study was conducted. The researcher would know whether Van Hiele's geometric thinking levels were significantly different in the initial stage between the experimental group and the control group, which would answer the first research question. The results of the inferential analysis showed that there was no significant difference in Van Hiele geometric thinking level between the two groups.

Hence, the researcher assumed that this threat to the internal validity was controlled. Furthermore, the pupils' pre-Van Hiele Achievement Test score was determined as the covariate when comparing the mean of the post Van Hiele Achievement Test score of the pupils in both groups.

Fraenkel, Wallen and Hyun (as stated in Akgül, 2014) stated that "testing threat refers to the fact that a pretest can make pupils more aware, sensitive, and responsive towards the subsequent treatment" (p.81). This study administered the post Van Hiele Achievement Test for six weeks after giving the pre-Van Hiele Achievement Test. The pupils might not be able to recall the questions posed in pre-Van Hiele Achievement Test easily. Therefore, the testing threat to the internal validity of this study can be minimized.

External validity means to the degree of which research results could be extended from sample to population (Akgül, 2014). As convenience sampling was administered to choose the school and the pupils, the results of the research might not be generalizable to the population of the study. The generalization might be limited to the national public schools that possessed similar ICT facilities and the pupils with similar socioeconomic background in Yancheng in China. This study might subject to the threat of ecological validity as the pupils in the experimental group underwent instruction in the computer lab whereas the pupils in the control group underwent instruction in the regular classroom setting. However, the threat can be minimized so that the number of pupils in both groups, the seating arrangement and the lighting in both locations were almost similar.

This study analyzed the reliability of the Van Hiele Achievement Test by using Cronbach's Alpha test in SPSS version 17.00 and obtained a Cronbach's Alpha value of 0.792. Considering an instrument with Cronbach's Alpha value above 0.70 as reliable, this meant that the instruments could be properly used to get data from the subjects with the same features as the subjects in the group. That meant the instruments could be properly used to get data from the subjects with same features as the subjects in the group. External validity means the degree of which research results could be extended from sample to population (Calder, Phillips, & Tybout, 1982). After experiments in a more natural environment and random sampling, external validity could be improved. This study used natural setting to overcome the external validity administered convenience sampling to select the school and the pupils, therefore the results of the research cannot be generalized to the population in the research. The generalization might be limited to the national public school with similar socioeconomic background in Yancheng in China.

### 3.7 Data Analysis Method

After gathering all the data, the researcher used descriptive statistics and inferential statistics to answer all of the research questions and used the significant value of 0.05 for this study to compare the differences in pupils' van Hiele geometric thinking levels. As the data obtained from this study is ordinal data, the researcher used a non-parametric test to conduct the inferential analysis. Ordinal data is a state of measurement, which distinguished the variable of middle order in the same category of cases (McCullagh, 1980). Ordinal data can be ordered, but cannot be added or subtracted, such as grade level variables (Agresti, 2013). Ordinal data can decide the order, and the value of variables can rank the subjects. For example, we can divide educational level into university, senior high school, junior high school, elementary school and illiterate; we can divide factory scale into large, medium and small; we can divide age into young, middle and old. The values of these variables cannot only distinguish differences and similarities, but also distinguish the size or height of the research object.

This study used quantitative data analysis to analyze the data obtained through Van Hiele Achievement Test. At the beginning, the pupils' pre and post Van Hiele Achievement Test were marked. The items that were done correctly by the pupils were labeled as "1" while items that were answered wrongly were labeled as "0" according to the descriptions under the title of instrumentation. Based on the criterion as shown in Table 3.3, van Hiele geometric thinking levels of the pupils was determined.

In answering the first research question and the third question whether there is any significant difference in the pupils' van Hiele geometric thinking levels between the control group and the experimental group before and after with treatment, the study used Mann-Whitney Test to compare the van Hiele geometric thinking levels of the Grade Seventh pupils in the control and experimental groups before and after the intervention period. The study used Mann-Whitney Test instead of independent samples t-test because of the data measured at the ordinal level. Secondly, the independent variables were composed of two independent and categorical groups such as the experimental group with phase-based instruction using Geometry and the control group with traditional method. Thirdly, there was no relationship between the observations in each group. That meant that there were different participants in the experimental group with phase-based instruction using Geometry and the control group with traditional method with no participant being in two groups. Fourthly, the two variables were not normally distributed (show in Table 3.4 & 3.5) in the two groups before and after Van Hiele Achievement Test.

Table 3.4

*The test showed whether the groups are normally distributed*

| | Groups | K-S | P |
|---|---|---|---|
| Pretest | Experimental | 0.447 | 0.000 |
| | Control | 0.480 | 0.000 |

Table 3.5

*The test showed whether the groups are normally distributed*

| | Groups | K-S | P |
|---|---|---|---|
| Posttest | Experimental | 0.507 | 0.000 |
| | Control | 0.428 | 0.000 |

In order to answer the second research question raised above, the study conducted Wilcoxon signed-rank test by using SPSS software to compare the difference in the Grade Seventh pupils' van Hiele geometric thinking levels before and after using Geogebra in teaching plane geometry. This case measured the data at the ordinal level. Secondly, the independent variables were composed of two related and categorical groups, such as the experimental group before the phase-based instruction with Geogebra and the experimental group after the phase-based instruction using Geogebra. That meant that the Van Hiele Achievement Test has been measured on two occasions on the experimental group. Thirdly, the distribution of the differences between the experimental group before the phase-based instruction with Geogebra and the experimental group after the phase-based instruction with Geogebra was symmetrical in shape. Therefore, the study used Wilcoxon signed-rank test to look at the change in van Hiele geometric thinking levels regarding plane geometric among pupils from the experimental group.

### 3.8 Summary

This was a quasi-experimental study towards 112 the Grade Seventh pupils in public schools in YanCheng in China. Pupils were guided with phase-based instruction using Geogebra to do the pre-test and post-test, when they are completed before and after instructional activities. Then data was collected on the van Hiele geometric thinking levels regarding the plane geometry. This study analyzed the data obtained by making use of inferential statistics and reported it in Chapter Four.

# CHAPTER FOUR

# FINDINGS

## 4.1 Introduction

This chapter presents the analysis of Van Hiele Achievement Test that correspond to the three hypotheses of this study. The first part discusses the descriptive analysis of Van Hiele Achievement Test of the two groups. The second part shows the results of the inferential test between pupils' pre and post Van Hiele Achievement Test and further compares the effect of using phase-based instruction with Geogebra on plane geometry between the two groups.

## 4.2 Descriptive Analysis of Van Hiele Achievement Test

After conducting the test, the number and percentage of the pupils in the control group and the experimental group who reached Level 1, Level 2, and Level 3 in the both pretest and posttest were calculated and shown in the following Table 4.1.

Table 4.1

*Number and Percentage of Pupils in the Experimental and Control Groups Acquiring Level 1, Level 2 and Level 3 in the Pre and Post Van Hiele Achievement Test*

| Van Hiele Achievement Test | Groups | Van Hiele geometric thinking levels | | | | | |
|---|---|---|---|---|---|---|---|
| | | Level 1 | | Level 2 | | Level 3 | |
| | | *N* | *%* | *N* | *%* | *N* | *%* |
| Pretest | Control (n = 54) | 42 | 77.8 | 12 | 22.2 | 0 | 0.0 |
| | Experimental (n = 58) | 42 | 72.4 | 15 | 25.9 | 1 | 1.7 |
| Posttest | Control (n = 54) | 14 | 25.9 | 3 | 5.6 | 37 | 68.5 |
| | Experimental (n = 58) | 2 | 3.5 | 6 | 10.3 | 50 | 86.2 |

The study gave Pre Van Hiele-Achievement Test to examine the initial Van Hiele geometric thinking levels among the pupils in the two groups before the intervention. In pre-Van Hiele Achievement Test, out of the 54 pupils in the control group, 42 (77.8%), 12 (22.2%), and none (0.0%) of the pupils had reached van Hiele Level 1, Level 2, and Level 3 of geometric thinking respectively. On the other hand, in the experimental group 42 (72.4%), 15 (25.9%), and 1 (1.7 %) of the pupils acquired van Hiele Level 1, Level 2, and Level 3 of geometric thinking respectively. It seemed that the pupils in the experimental group were slightly better than the control group pupils.

The study gave post Van Hiele Achievement Test to the experimental group, whereby 37 (68.5%) of the pupils in the control group had achieved Van Hiele geometric thinking Level 3 after the instruction of plane geometry using traditional approach. But, 14 (25.9%) of them were still at Level 1. Moreover, 50 (86.2%) of the pupils in the experimental group had acquired Van Hiele geometric thinking Level 3 but

2 (3.5%) of the pupils were still at Level 1. The experimental group had better performance than the control group in the post Van Hiele Achievement Test. Therefore, it could be interpreted that using phased-based instruction with Geogebra had helped the experimental group pupils to gain higher Van Hiele geometric thinking levels in contrast to the pupils from the control group who were taught using the traditional approach. Therefore, instruction using the traditional approach might not be used for all of the pupils. By contrast, using phased-based instruction with Geogebra might be a more effective instructional method to help pupils acquire higher van Hiele geometric thinking levels. After the intervention, the pupils' geometric thinking reached a higher van Hiele level.

**4.3 Inferential Analysis of Van Hiele Achievement Test**

To answer the three research questions and to test the three research hypotheses, the researcher used SPSS to record and analyze van Hiele geometric thinking levels among pupils in both the control and experimental groups. The researcher used Mann-Whitney Test instead of independent samples *t*-test because the data were measured at the ordinal level, and involved two categorical independent variables. There is no relationship between the observations in each group and the two variables are not normally distributed (Table 3.4 & 3.5). Because the data did not conform to normality, the researcher used non-parametric tests. Wilcoxon signed-rank test was used instead of paired samples *t*-test because the data were measured at the ordinal level and was composed of two categorical independent variables. The researcher used Wilcoxon signed-rank test to show the difference before and after the Van Hiele Level

Achievement test in the experimental group. Mann-Whitney *U* test was used to assess the difference in value of pre-test and post-test results between the experimental group and the control group. Chapter One presented the research questions and research hypotheses. There were no missing values for pre-Van Hiele Achievement Test and post Van Hiele Achievement Test so analysis could proceed.

*Question 1: Is there any significant difference in the pupils' geometric thinking between the experimental group and the control group before treatment?*

$H_0$ = There is no difference in the Grade Seven pupils between the experimental groups and the Grade Seven pupils in the control group possessed similar mean rank of geometric thinking before this study was conducted.

$H_1$ = There is significant difference between the Grade Seven pupils in the experimental groups and the Grade Seven pupils in the control group possessed similar mean rank of geometric thinking before the study.

The researcher gave Pre Van Hiele-Achievement Test to assess the initial van Hiele geometric thinking levels between two groups before the intervention. The researcher used Mann-Whitney *U* test to analyze the first research question. The results of Mann-Whitney *U* test (see Table 4.2) for pre Van Hiele Achievement Test showed that there is no difference in van Hiele geometric thinking levels between the control group and the experimental group (Mdn=1), U ($n_1$=54, $n_2$=58)= 1476.00, z=0.70, $p$=0.49 at the significant level of 0.05.

Table 4.2

*Result of Mann-Whitney U test for the Experimental Group and the Control Group in Pre Van Hiele-Achievement Test*

| Pre-Test | n | Mdn | Mean Rank | Sum of Ranks | *U* | *z* | *p* | *r* |
|---|---|---|---|---|---|---|---|---|
| Control Group | 54 | 1 | 54.83 | 2961.00 | 1476.00 | 0.70 | 0.49 | 0.08 |
| Experimental Group | 58 | 1 | 58.05 | 3367.00 | | | | |

The result of the Z value is 0.70 with a p-value of 0.49 ($p > 0.05$). Therefore, there was no statistically significant difference, that is, there was no significant difference between the experimental group and the control group before the experiment. Therefore, H0 was accepted. It can be considered that the pre-test of the actual control group and the pre-test of the experimental group came from the same whole, that is to say, there is no difference between the control group and the experimental group before the test, and the test can be compared.

The pupils in the experimental group performed a little bit higher than the pupils in the control group in pre-Van Hiele Achievement Test. The difference, however, was not significant. As a result, the null hypothesis of the research was not rejected. Result of the Mann-Whitney *U* test was not statistically significant at the 0.05 level. As a result, the researcher concluded that pupils in both the control group and the experimental group possessed similar van Hiele geometric thinking levels on the topic of plane geometry before the study.

*Question 2: Is there any significant difference in the pupils' geometry thinking concerning plane geometry before and after using Geogebra?*

$H_0$ = There is no difference in the pupils' van Hiele geometric thinking levels concerning plane geometry before and after using Geogebra in plane geometry in China.

$H_1$ = There is a difference in the pupils' van Hiele geometric thinking levels concerning plane geometry before and after using Geogebra in plane geometry teaching in China.

In order to answer the second research question, the researcher conducted pre-Van Hiele Achievement Test before using Geogebra. The researcher gave the same post Van Hiele Achievement Test after using Geogebra and carried out Wilcoxon signed-rank test to find out whether van Hiele geometric thinking levels on the topic of plane geometry using Geogebra has significant effect on pupils.

Table 4.3

*Result of Wilcoxon signed-rank test for Difference in Van Hiele Levels of geometric Thinking for the Experimental Group*

| Experimental Group | N | Mdn | $\bar{x} \pm s$ | $z$ | $p$ | $r$ |
|---|---|---|---|---|---|---|
| Pre | 58 | 1 | 1.29±0.50 | 6.69 | 0.000 | 0.85 |
| Post | 58 | 3 | 2.83±0.46 | | | |

The table indicated that the pupils who were using phase-based instruction with Geogebra (Mdn = 3) achieved significantly higher scores than before using phase-based instruction with Geogebra (Mdn = 1) on plane geometry. The value of z is 6.69 and *p*-value is 0.000 less than 0.05. The difference in van Hiele geometric thinking levels on the topic of plane geometry before and after using phase-based instruction with Geogebra was significant at a level of 0.05 as $p < 0.05$. The value of *r* was 0.85. This value showed that the effect size of using phase-based instruction with Geogebra on the topic of plane geometry was large based on Rosenthal (1991).

Therefore, there was a statistically significant difference. As a result, the experimental group performance in the post-test was higher than for the pre experiment group. The difference is statistically significant. This result suggested to reject the null hypothesis. It can be considered that the experimental group pre-test has a different score from the post-test. After the instruction, the pupils performed better.

*Question 3: Is there any significant difference in the pupils' geometric thinking between the control group and the experimental group after treatment?*

$H_0$ = There is no difference in the pupils' mean rank of geometric thinking between the control group and the experimental group after using Geogebra in plane geometry teaching in China.

$H_1$ = There is a difference in the pupils' mean rank of geometric thinking between the control group and the experimental group after using Geogebra in plane geometry teaching in China.

The result of research question 2 had shown that the pupils in both the control and the experimental groups have significantly improved their van Hiele geometric thinking levels. The researcher further compared the van Hiele geometric thinking levels among the pupils in two groups in the post Van Hiele Achievement Test. The data obtained were analyzed by using SPSS. The result was shown in Table 4.4.

Table 4.4

*Result of Mann-Whitney U test for the Experimental Group and the Control Group in Post Van Hiele Achievement Test*

| Post-Test | n | Mdn | Mean Rank | Sum of Ranks | *U* | *z* | *p* | *r* |
|---|---|---|---|---|---|---|---|---|
| Control Group | 54 | 3 | 50.65 | 2735.00 | 1250.00 | 2.53 | 0.01 | 0.27 |
| Experimental Group | 58 | 3 | 61.95 | 3593.00 | | | | |

The result of the Mann-Whitney *U* test indicated that the difference in van Hiele geometric thinking levels between the pupils in the control group (Mdn = 3) and the pupils in the experimental group (Mdn = 3) was significant, $U$ ($n_1$=54, $n_2$=58) = 1250.00, $z$ = 2.53, $p$ = 0.01 at the 0.05 significance level. There was a statistically significant difference in geometric thinking levels between the experimental and control group after the intervention. Therefore, the result of post experimental group was significantly different than for the pre-experimental group. Thus the null hypothesis was rejected, indicating a difference in the pupils' mean rank of geometric thinking between the control group and the experimental group after using Geogebra in plane geometry teaching in China. The experimental group performed better than the control group.

The experimental group mean rank (61.95) was higher than of the control group (50.65). The result showed that the performance of pupils in the experimental group was higher in mean rank of van Hiele geometric thinking levels than the pupils in the control group after treatment. The value of $r$ was 0.27, indicating a small effect size on the pupils' van Hiele geometric thinking levels in post Van Hiele Achievement Test according to Rosenthal (1991).

## 4.4 Summary

In this chapter, the descriptive analysis of Van Hiele Achievement Test showed that most of the pupils in two groups were at van Hiele geometric thinking Level 1 before the study was conducted. The data using Mann Whitney $U$ test indicated that the difference in van Hiele geometric thinking levels between the two groups was not significant before the study was conducted. After the intervention, 86.2% of the pupils in the experimental group acquired van Hiele geometric thinking Level 3. And then

68.5% of the pupils in the control group acquired van Hiele geometric thinking Level 3. In research question 2, the results of Wilcoxon signed-rank test indicated that pupils whether in the experimental groups or in the control groups have improved significantly in the aspect of their van Hiele geometric thinking level for the topic plane geometry. However, using Mann Whitney *U* test proved that experimental group pupils' post van Hiele geometric thinking levels were significantly higher than van Hiele geometric thinking levels of pupils from the control group after the intervention.

Further discussion, conclusions and implications on the basis of the research findings are explained in Chapter Five.

## CHAPTER FIVE

## DISCUSSION, CONCLUSIONS, AND IMPLICATIONS

### 5.1 Introduction

In this chapter, the researcher summarized the important points of this study. This research focused on phase-based instruction with Geogebra software to assist the Grade Seven pupils, and recorded the comparative experiments of Geogebra software in classroom teaching. It compared Geogebra software assisted teaching and traditional teaching of plane geometry on the Grade Seven pupils.

The researcher presented Summary of the findings based on each research question and made further discussion on the findings of this study.

### 5.2 Summary of the study

This study aimed at determining the effect of using phase-based instruction with Geogebra on the Grade Seventh pupils' van Hiele geometric thinking levels. At the same time, this study has described the implementation of using Geogebra in plane geometry teaching. Before conducting the research, the researcher had given the three research hypotheses:

1.There is no difference in the Grade Seven pupils between the experimental groups and the control group possessed similar mean rank of van Hiele geometric thinking levels before the research.

2.There is no difference in the pupils' mean van Hiele geometric thinking levels concerning plane geometry before and after phase-based instruction with Geogebra in teaching of plane geometry in China.

3.There is no difference in the pupils' mean rank of van Hiele geometric thinking levels between the control groups and the experimental groups after treatment in teaching of plane geometry.

The research design was a non-equivalent pretest-posttest design. It was carried out on 112 Grade Seventh pupils from two classes in a public school in YanCheng; in the two classes, students vary in their abilities. All of the pupils were 13 years old. They came from different economic background in a national type Chinese public school in YanCheng. The instrument used in this study was the Van Hiele Achievement Test. The study used a quasi-experimental research methodology, which was a non-equivalent group design. Hill, Bloom, Black, and Lipsey (2008) stated that as an empirical study, the quasi-experimental study is mainly applied to estimate how the intervention affects the targeted population.

At the same time, the researcher used Grade Seven pupils' mathematics textbook and the worksheets conducted by the researcher to teach plane geometry to the control group pupils. The researcher carried out all the instruction in the classroom. The control group pupils were not allowed to use any computers during the instructional process.

The researcher used Mann Whitney *U* test and Wilcoxon signed-rank test to analyze the significance of the difference in van Hiele geometric thinking levels among the pupils in the control group and the experimental group. Mann Whitney *U* test was used to evaluate the difference in value of pre-test and post-test score between the experimental group and the control group. The Wilcoxon signed-rank test was used to determine the difference before and after the Van Hiele Level Achievement test in both the control group and the experimental group.

### 5.3 Summary of Research Findings

This study used the phase-based instruction with Geogebra to teach plane geometry to Grade Seven pupils. This study presented summary of the research findings according to the corresponding research questions followed by the discussion based on findings on each research question.

*Question 1: Is there any significant difference in the pupils' geometric thinking between the experimental group and the control group before treatment?*

The Mann Whitney *U* test (Table 4.2) shows that for pre-Van Hiele Achievement Test there is no difference in van Hiele geometric thinking levels between the control group and the experimental group (Mdn = 1), U ($n_1$=54, $n_2$=58) = 1476.00, z = 0.70, $p$ = 0.49 at the significant level of 0.05.

*Question 2: Is there any significant difference in the pupils' geometric thinking concerning plane geometry before and after phase-based instruction with Geogebra?*

The Wilcoxon signed-rank test result in Table 4.3 revealed that the pupils who were using phase-based instruction with Geogebra (Mdn = 3) performed significantly higher than before using phase-based instruction with Geogebra (Mdn = 1) on plane geometry. The experimental group pupils have improved significantly in terms of their van Hiele geometric thinking levels after the phase-based instruction of plane geometry with Geogebra.

*Question 3:* Is there any significant difference in the pupils' geometric thinking between the experimental group and the control group after with treatment?

The Mann Whitney *U* test (Table 4.4) showed that the difference in van Hiele geometric thinking levels between the pupils in the control group (Mdn = 3) and the pupils in the experimental group (Mdn=3) was significant, $U$ ($n_1$ = 54, $n_2$ = 58) = 1250.00, $z$ = 2.53, $p$ = 0.01 at the significance level of 0.05.

## 5.4 Discussion

In this section, the findings of the research are discussed. The Mann Whitney $U$ test result on the pupils' pre van Hiele Achievement Test indicated that the van Hiele geometric thinking levels among pupils in both the control and the experimental groups did not differ significantly before this study was conducted. There was no significant difference between the experimental group and the control group before the experiment. The possible reason of finding could be that the two groups of pupils from two parallel classes in the same school who have similar background performed slightly similarly. We can use the result to carry on the following study. The pupils in the experimental group (58.05) seemed to possess slightly higher van Hiele geometric thinking levels than the pupils in the control group (54.83) before the intervention but the difference was not significant.

The first research question's finding is similar with the study conducted by Dimakos and Zaranis (2010). In their studies, the pupils' geometrical achievement before the study was conducted did not differ significantly between the control group and the experimental group although the experimental group pupils seemed to perform a little bit better than the control group pupils. Similarly, Zhao (2016) concluded that van Hiele levels of pupils in the control and experimental groups did not differ significantly before the instruction. Johnson (2002) found that no matter using dynamic software teaching or using traditional teaching, pupils' achievements did not change obviously.

According to Johnson, that was because teachers did not make good use of the dynamic software or they did not know how to make good use of it.

The result was consistent with Qi (2013). The research selected two parallel classes of the grade as the sample of the experiment and assigned 2 classes to the control group and the experimental group, including 52 pupils from the experimental group and 50 pupils from the control group. The pre-test scores of the two classes were 0.888 ($p > 0.05$), indicating that the pupils from the two groups did not have obviously different pre-test scores (Qi, 2013).

The Wilcoxon signed-rank test result revealed that the van Hiele geometric thinking levels of pupils from the experimental group improved greatly after they had undergone the phase-based instruction of plane geometry with Geogebra. The pupils achieved higher geometric thinking level after phase-based instruction with Geogebra.

The possible reasons for this finding could be that the phase-based instruction with Geogebra improved performance of the pupils in the experimental group. After 6 weeks phase-based instruction with activities, the pupils have obtained their knowledge and improved their van Hiele geometric thinking level. The result possibly showed that using phase-based instruction with Geogebra had improved the van Hiele geometric thinking level for plane geometry.

The result was in conformity with previous findings that the pupils in the experimental group scored significantly in the posttest compared to their pretest after conducting Geogebra activities (Zhao, 2016). This result was expected, that is, using phase-based instruction with Geogebra to teach pupils could help the pupils to master van Hiele geometric thinking levels. Additionally, the Geogebra software allowed the pupils to analyze plane geometry easily besides enabling them to explore the property of plane geometry conveniently. In the study, for the experimental group after two weeks of teaching, the test of the rotating unit was carried out. The researcher analyzed statistically the test scores of the experimental group $p = 0.002 < 0.05$, indicating that the variance was homogeneous. Therefore, the experimental group's pre-test and post-test results had great differences (Qi, 2013).

In this study, 50 out of 58 pupils in the experimental group obtained higher van Hiele geometric thinking levels after phase-based instruction of plane geometry using Geogebra whereas the remaining two pupils possessed similar van Hiele geometric thinking levels before and after intervention. Two pupils remained in van Hiele geometric thinking Level 1 before and after phase-based teaching with Geogebra and another pupil obtained van Hiele geometric thinking Level 2 in pre-Van Hiele Achievement Test and remained in the same level in the post Van Hiele Achievement Test. This result is similar with the study done previously on phase-based instruction using Geogebra (Zhao, 2016), which showed that not all of the pupils could achieve similar van Hiele geometric thinking levels after phased teaching with Geogebra.

Different from the control group, the rank mean of pupils from the other group was higher, because they accepted phased teaching with Geogebra which might help the pupils reflect and examine whether their learning methods were effective as well as how they should accomplish their learning tasks more effectively. The control group pupils did not have the same opportunity as the experimental group pupils. The teaching method they accepted was restricted by limited time and space. After all, it took much time for teachers to draw diagrams on the blackboard, which took up a lot of space.

Li and Zhu (2005), Li (2013) and Koyuncu, Akyuz, and Cakiroglu (2015) reported that phase-based instruction using Geogebra effectively improved the van Hiele geometric thinking level of pupils. Their findings were similar with those in this study. This result agreed with the study conducted by Li and Zhu (2005) that reported the pupils from the experimental group performed significantly better than those from the control group in terms of their geometrical achievement in the posttest compared to their pretest after carrying out the inductive Geogebra activities. In the research, the statistical comparison between the experimental group's post-test and post-test results showed that $p = 0.046 < 0.05$. As a result, the experimental group's post-test result was greatly different from that of the control group. Qi (2013) concluded that the experimental group's mathematics learning scores was different from that of the other group in the unit test for the rotation chapter. The Geogebra software-assisted rotation chapter can improve the pupils' academic performance more than the traditional story-telling teaching mode. The experimental group had a greater improvement than the control group.

The researcher could provide guidance easily to pupils when they faced difficulties in using Geogebra and therefore using Geogebra did not distract the pupils in the experimental group during learning of plane geometry. Furthermore, the researcher provided short notes for them to refer easily on the main usage of Geogebra so that they feel confident in using the software.

### 5.6 Conclusions

In this study, the results supported the hypothesis that pupils' geometric thinking van Hiele levels about the topic of plane geometry would differ significantly after phase-based instruction on plane geometry using Geogebra. Additionally, the van Hiele geometric thinking levels among the experimental group pupils differed significantly after phase-based instruction of plane geometry using Geogebra.

The third finding is that teachers do not widely use integration of technological tools in the instructional process in helping pupils to acquire higher geometric thinking in China. In the experimental group, the pupils' van Hiele geometric thinking levels improved significantly after phased teaching of plane geometry with Geogebra. It seemed that the phase-based activities designed using Geogebra are useful to guide pupils in learning plane geometry. This result was similar with the research done by Li and Zhu (2005), who concluded that in the geometry teaching process it is necessary to be good at using dynamic geometry software for more effective teaching. Geogebra played a very important role in encouraging the pupils' participation in investigating geometric concepts and increasing their desire to learn, especially in a collaborative learning environment. Li (2013) proposed the use of advantages of Geogebra itself to change the drawbacks of traditional teaching and stimulate the pupils' learning motivation and interest pupils in learning dynamic geometry, and to improve the time-consuming, less efficient, inflexible class and many other shortcomings of the traditional junior high school dynamic geometry teaching. Wei (2013) analyzed in detail what level of thinking ability each grade student has and the changing trend of thinking ability of the same level pupils and pupils of different grades.

Additionally, the first issue is that teaching practices among mathematics teachers do not focus on developing pupils' geometric thinking. And the second issue is that teachers do not widely use van Hiele geometric thinking level in the Chinese mathematics geometry curriculum. The result of this study encouraged the teachers, the educators and the curriculum developers of the Ministry of Education in the department of mathematics geometry curriculum to pay more attention to improving the pupils' geometric thinking. This result was aligned with the previous study by Gao (2013) who concluded that the level of cognitive geometric thinking in classroom teaching was consistent with the teaching ability, which was higher than the student's cognitive level. The classroom teaching helped pupils to improve their geometric thinking level. And then the most middle school pupils can reach the geometric thinking level 3 after class. Zhao's study (2016) has shown that teachers must attach great importance to pupils' geometric thinking level, attach great importance to pupils' attitudes and emotions they demonstrated in their mathematical activities, and finally help pupils to have a better understanding of themselves and build up self-confidence.

## 5.7 Implications for Instruction

The phase-based instruction using Geogebra was a way to teach plane geometry among the Grade Seven pupils. From the results obtained as shown in Chapter Four, phase-based instruction using Geogebra could help the Grade Seven pupils to acquire higher van Hiele geometric thinking levels about plane geometry. As a result, a number of implications could be deduced for mathematics educators.

First, the difference in van Hiele geometric thinking levels between the two groups was significant during post Van Hiele Achievement Test, which suggested that phased teaching with Geogebra could improve more effectively middle school pupils' van Hiele geometric thinking levels compared to the traditional approach. This result might encourage more middle school teachers to use Geogebra while teaching plane geometry in providing more meaningful teaching and learning experiences for the middle school pupils. This result was consistent with the study conducted by Li and Zhu (2005), which argued that in geometry teaching it was necessary to be good at using dynamic geometry software to facilitate effective teaching. Li (2014) studied the role of Geogebra as a tool in assisting pupils to understand conceptual learning in mathematics. This study conducted the teaching design and teaching experiments with the understanding of the probability of the definition of learning. Geogebra was found to help pupils improve the level of understanding of the learning concept, so that pupils could shift from empirical understanding to structural understanding and cultural understanding. The researcher devised some strategies for the understanding of Geogebra-assisted mathematics learning and illustrated that these strategies were effective. They found that phased teaching with Geogebra had successfully improved from grade one to grade three pupils' geometric thinking about the Kasner polygon significantly. This research provided teachers with a balanced teaching process and student learning outcomes, and helped pupils solve problems in geometric learning. Although the international research on Van Hiele theory has matured, and the practical application of the theory is quite extensive, the Chinese research on this theory is still relatively weak. The research of this thesis has important guiding significance for enriching and perfecting the teaching theory of mathematics geometry in middle school. In order to strengthen the learning of information technology, the teachers suggested to produce some complicated Geogebra courseware. Teachers need to master the Geogebra script. Therefore, teachers must be skilled in showing pupils the development of knowledge information technology capabilities in the classroom. Even in the classroom, teachers can give more opportunities for pupils to operate. From the questionnaires and interviews of pupils in this study, pupils were very eager to implement the inquiry classroom, and they are very eager to participate in the inquiry with Geogebra. Therefore, when designing teaching procedures, teachers should try to let pupils operate and have an inquiry experience.

Second, the curriculum developers were encouraged to use dynamic geometry tools in middle school mathematics instruction, specifically the software of Geogebra. Because the use of dynamic geometry environments was not only for pupils but also for everyone to explore geometric ideas in more effective ways compared to the exploration using paper-and-pencil (Battista, 2007). It is also important to provide continuing professional development for mathematics teachers from middle schools to understand the stage-based teaching by using van Hiele levels to stimulate pupils' geometric thinking in the process of teaching geometric content with help from dynamic geometry teaching software. *The mathematics curriculum standard of compulsory education* (2011) stated that the main purpose of evaluation was to comprehensively understand the pupils' mathematics learning process, to stimulate learning, to improve teachers' teaching and to establish an evaluation system with multiple evaluation objectives and various evaluation methods. Mathematical learning evaluation should focus on pupils, including their learning process, learning level, and learning attitudes and emotions during the whole learning process. The evaluation of middle school pupils should be diverse, and the Van Hiele theory can be used to judge the geometric thinking level of pupils and provide additional evaluation criteria for the middle school mathematics geometry curriculum. As can be seen from this study, different mathematical software offer different advantages. Mathematics geometry curriculum might wish to write as many software as possible in the textbook for the secondary school teachers to choose according to their specific teaching conditions.

Besides the pupils' van Hiele geometric thinking levels after phase-based instruction with Geogebra suggested that it had significant improvement. The pre-designed Geogebra activities, which utilized van Hiele five learning phases were useful in helping pupils to progress sequentially to the higher level Qi (2013). He suggested that using Geogebra encouraged the pupils to learn through discovery by visualizing and analyzing the problem and further making conjectures before attempting a proof.

Additionally, the study will contribute to mathematics educational researchers; the study will offer the source of literature review especially in China. Although the foreign research on the basis of Van Hiele theory has quite matured, and the practical application of the theory is quite extensive, the research and application of this theory in China is still relatively weak. The results in this research were similar with Liu (2014). She found that the field of studies is very lacking. Her study focused on instruction with Geogebra to provide for further investigation. This research might provide some different views and perspectives from the statistical data. In China, the researcher could contrast the results in this study to the others in the similar research field. The researchers might apply usage of Geogebra in conducting their future studies in various mathematical content to enhance pupils' level of geometric thinking.

### 5.8 Recommendation for Further Research

The results in this research demonstrated that phase-based instruction with Geogebra can be a useful tool in teaching planar geometry in secondary schools. First, in this study, the researcher focused only on two intact-mixed ability classes of pupils in the national type middle Chinese school. For further research on phase-based instruction using Geogebra in the both geometry learning and teaching in high schools, researchers may consider different samples of pupils such as international school pupils. Pupils with different background might give different results and further elaborate the findings of this study.

Second, simple random sampling techniques were more suitable for use to obtain normal distributed data with equal variances between groups of pupils in future research. If the data conformed to normality, parametric tests such dependent samples *t*-test, paired samples *t*-test could be used; the researcher could use Analysis of Variances (ANOVA) and others in analyzing the data. The results obtained would be more robust to be generalized to the population.

In this study, the researcher only chose the instruction of the Grade Seven plane geometry. For future studies, researchers can also use different geometrical content or pupils of different ages. It would be interesting to find out whether phase-based instruction using Geogebra in different geometrical content on pupils with different cognitive level would give encouraging results. It is possible to compare the similarities and differences between the pupils' van Hiele geometric thinking level and their geometric achievements.

Moreover, there were many findings of developing pupils' geometrical thinking with Geogebra in previous researches in China. And so many findings of improving pupils of van Hiele level of geometrical thinking have been done in previous researches in China. Future studies are encouraged to modify the pre-designed instructional materials for phase-based instruction of plane geometry using Geogebra to accommodate other mathematical concepts such as algebra. For further research, Geogebra can be used to conduct instruction on concept of algebra. For instance, in the conical curve in the middle school textbook, the parabola is the deepening of the quadratic function. We can study whether the pupils who have achieved good results after the test using Geogebra achieved good results in the study of the high school conic curve. Instructing pupils to acquire higher mathematical thinking might be useful, such as algebraic thinking. More research is needed to provide mathematical educators with knowledge on phase-based instruction with Geogebra and to provide pupils with more meaningful mathematics teaching and learning experiences.

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