EFFECTIVENESS OF MAYER'S PROBLEM SOLVING MODEL WITH VISUAL REPRESENTATION TEACHING STRATEGY IN ENHANCING YEAR FOUR PUPILS' MATHEMATICAL PROBLEM SOLVING ABILITY

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**DISSERTATION SUBMITTED IN FULFILMENT
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ABSTRACT

This study aims to determine the effectiveness of Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy in enhancing Year 4 students' mathematical problem solving ability in Klang Valley, and its interaction with gender. A quasi-experimental research design was employed in this study. A sample of 203 Year 4 students consisting of 101 males and 102 females from a private school in Klang Valley were drawn using a convenient sampling technique with two classes assigned as the experimental groups namely Mayer's problem solving Model (MM) and MMVR groups, and the other one as the control group. The MM group was given Mayer's problem solving Model (MM) teaching strategy treatment, while the MMVR group was given Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy treatment respectively. The control group on the other hand did not receive any treatment. Students' mathematical problem solving ability before and after treatments was measured using an instrument called Mathematical Problem Solving Ability Test (MPSAT). The study found that MMVR teaching strategy was effective in improving students' mathematical problem solving ability because the paired samples statistics for pretest and posttest scores of problem solving ability for MMVR group shows the mean scores of MPSAT in MMVR group for pretest (M = 75.41, SD = 3.70) and posttest (M = 115.86, SD = 6.47) is different, with statistically significant mean increase for mathematical problem solving ability in MMVR group, t (57) = 41.05, $p < .0005$. Also, the results of one-way ANCOVA analysis reveals that there is a statistically significant difference in the mean of the posttest score in MPSAT between the three groups, F (2, 171) = 291.44, $p < .0005$ ($\eta_p^2 = .77$, observed power = 1), with the adjusted means of posttest scores of MPSAT for MM, MMVR and Control groups were 105.84 (SE = .94), 116.14 (SE = .93), and 85.94 (SE = .89) respectively. However, the study were not able to show that there is significant interaction between problem solving teaching strategy and gender on students' mathematical problem solving ability because the result of two-way ANCOVA analysis indicates that the interaction effect between gender and the groups on mathematical problem solving ability was not statistically significant, $F(2, 169) = .018$, $p = .98$ ($p > .05$), with a minimal effect size ($\eta_p^2 = .00022$). The findings suggest that MMVR teaching strategy can be an effective approach in improving students' mathematical problem solving ability. The study suggests that teachers should incorporate Mayer's problem solving Model with Visual Representation teaching strategy into their lesson to help students improve their mathematical problem solving ability.

KEBERKESANAN STRATEGI PENGAJARAN BERASASKAN MODEL PENYELESAIAN MASALAH MAYER DENGAN PERWAKILAN VISUAL DALAM MENINGKATKAN KEBOLEHAN MURID TAHUN 4 MENYELESAIKAN MASALAH MATEMATIK

ABSTRAK

Kajian ini bertujuan untuk menentukan keberkesanan strategi pengajaran berasaskan Model penyelesaian masalah Mayer dengan Perwakilan Visual (MMVR) dalam meningkatkan kebolehan murid Tahun 4 menyelesaikan masalah matematik di Lembah Klang, dan interaksinya dengan jantina. Reka bentuk penyelidikan kuasi eksperimen telah digunakan dalam kajian ini. Sampel sebanyak 203 murid tahun 4 yang terdiri daripada 101 murid lelaki dan 102 murid perempuan dari sebuah sekolah swasta di Lembah Klang telah dipilih melalui teknik persampelan konvinien dengan dua kelas dipilih sebagai kumpulan eksperimen iaitu kumpulan Model penyelesaian masalah Mayer (MM) dan kumpulan MMVR, dan kelas ketiga sebagai kumpulan kawalan. Kumpulan MM telah diberi rawatan strategi pengajaran berasaskan Model penyelesaian masalah Mayer (MM), manakala kumpulan MMVR diberikan rawatan strategi pengajaran berasaskan Model penyelesaian masalah Mayer dengan Perwakilan Visual (MMVR). Kumpulan kawalan tidak menerima sebarang rawatan.

Kebolehan murid menyelesaikan masalah matematik sebelum dan selepas rawatan diukur dengan menggunakan instrumen yang dikenali sebagai Ujian Penyelesaian Masalah Matematik (MPSAT). Kajian mendapati bahawa strategi pengajaran MMVR berkesan dalam meningkatkan kebolehan murid menyelesaikan masalah matematik kerana statistik sampel berpasangan bagi ujian pra dan pos bagi kumpulan MMVR menunjukkan min skor MPSAT bagi kumpulan MMVR untuk ujian pra ($M = 75.41$, $SD = 3.70$) dan ujian pos ($M = 115.86$, $SD = 6.47$) adalah berbeza, dengan purata peningkatan statistik yang signifikan bagi kebolehan menyelesaikan masalah matematik bagi kumpulan MMVR, t (57) = 41.05, p <.0005. Selain itu, keputusan analisis ANCOVA satu hala mendedahkan bahawa terdapat perbezaan statistik yang signifikan bagi skor ujian pos MPSAT antara tiga kumpulan, F (2, 171) = 291.44, p <.0005 (η_p^2 =. 77, kuasa yang diperhatikan = 1), dengan nilai MPSAT untuk kumpulan MM, kumpulan MMVR dan kumpulan Kawalan adalah 105.84 (SE = .94), 116.14 (SE = .93), dan 85.94 (SE = .89). Walau bagaimanapun, kajian ini tidak dapat menunjukkan interaksi yang signifikan antara strategi penyelesaian masalah dan jantina ke atas kebolehan murid menyelesaikan masalah matematik kerana hasil analisis ANCOVA dua hala menunjukkan bahawa kesan interaksi antara jantina dan kumpulan ke atas kebolehan murid menyelesaikan masalah matematik tidak signifikan secara statistik, F (2, 169) = .018, p = .98 (p> .05), dengan saiz kesan minimum (η_p^2 = .00022). Hasil penemuan menunjukkan bahawa strategi pengajaran MMVR boleh menjadi pendekatan yang berkesan dalam meningkatkan kebolehan murid menyelesaikan masalah matematik. Kajian menunjukkan bahawa guru harus menggabungkan strategi pengajaran Model penyelesaian masalah Mayer dengan Perwakilan Visual ke dalam pengajaran mereka untuk membantu meningkatkan kebolehan murid dalam menyelesaikan masalah matematik.

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CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Mathematics is the mother of all science. As a matter of fact, the world cannot move without mathematics. Mathematics fulfil most of the human needs related to different aspects of everyday life. Moreover, every person requires a knowledge of mathematics in day to day life for various purposes. In fact, from the earliest period when civilization begins man used mathematics for different purpose mainly for getting the answer of 'how many', 'how big', 'how far', 'how much', and so on. The need of using mathematics in daily life started from that period and it continues until now. Therefore, it is worth studying the area of mathematics due to its imperative position in each individual's life to face the challenges of his day-to-day existence as properly for the newly fashioned technological world of these days and day after today.

It is a great reality that in nearly every country, mathematics occupies a central place in the school curriculum. From the primary through the junior secondary to the senior secondary school levels of the educational system, mathematics has becomes a core subject. Principles and Standards for School Mathematics to set forth goals and recommendations for mathematics education in the prekindergarten through grade twelve, was released by the National Council of Teachers of Mathematics (NCTM, 2000). NCTM (2000) described that there are ten mathematical standards in the document that students should know and be able to do across the prekindergarten through grade twelve years.

To emphasize, five of these are considered content standards that address mathematical topic areas such as algebra and geometry, whereas another five are about mathematical processes, such as problem solving, reasoning and proof, connection, communication and representation. A greater emphasis on mathematical processes was placed by NCTM (2003), especially on problem solving as they mentioned that problem solving is the heart of any solid mathematics curriculum. Hence, this study focused on problem solving due to its important role in contemporary mathematics education.

1.2 Mathematical Problem Solving

Problem solving plays an outstanding position in present day mathematics education. The need for learners to end up successful problem solvers has turn out to be a dominant theme in many national standards (AAAS, 1993; MoNE, 2013; NCSS, 1997; NCTE, 1996; NCTM, 1989, 1991, 2000, & 2003). For example, NCTM (1989, p.23) states: *"Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity. Problem solving is not a distinct topic, but a process that should permeate the entire program and provide the context in which concepts and skills can be learned".*

At the same time, NCTM (1989) also stated that: *"Problem solving as more than a vehicle for teaching and reinforcing the mathematical knowledge and helping to meet everyday challenges. It is a skill which can enhance logical reasoning"*. To put in another way, men and women can function optimally in society only when they are capable to determine what algorithm a question requires, and from time to time need to be in position to improve their own rules in a scenario where an algorithm cannot be applied directly. Instruction in mathematics now should not be constrained to simple mastery of algorithms or the improvement of certain mathematical skills, but it should engage learners in problem solving. It ought to involve novices in the investigation via exploring, conjecturing, analyzing and testing.

Besides, problem solving in mathematics is the basic goal to be achieved in mathematics learning. The Malaysian curriculum reported that problem solving is the major center of attention in mathematics teaching and learning (Bahagian Pembangunan Kurikulum (BPK), 2016). Therefore, the teaching and learning process need to involve problem solving skills that are comprehensive and also masking the whole curriculum. At the present time, the mathematics curricula in schools are designed to improve mathematical thinking among students which stress on mathematical processes such as problem solving, reasoning, communicating mathematically and making relationships and representations. Solving each routine and non-routine problems is stressed to produce learners who are thoughtful, creative and innovative, competitive in the globalization era, and capable of going through 21st century challenges (MOE, 2015).

Problem solving is well recognized as forming the essence of mathematics. Aydogdu and Ayaz (2015) reported that problem solving contributes to mathematics itself and it is the centre of the mathematics curriculum. Allowing problem solving into mathematical teaching at all levels, such as the application of mathematics to everyday conditions is advocated in order to lead to new mathematical knowledge. New mathematical knowledge in this context is allowing students to accumulate a deeper conceptual understanding of the subject via their lively participation in constructing their own expertise and understanding.

Mann and Enderson (2017) reported that an understanding develops in the course of the procedure of solving problems in which important math concepts and skills are embedded. Introducing principles and competencies in problem-solving contexts evokes thinking and reasoning about mathematical ideas. Students who think and reason about mathematical ideas, learn to join these new thoughts to thoughts in the past that discovered by them, and it increases their understanding. These values have been echoed many times over in the Department for Education and Employment (2000; 1999).

Problem solving is a complicated human endeavour. It is appreciably greater than the implementation of well-learned approaches or the easy recall of facts. Problem solving includes the development of sequential procedures that build techniques in addition to the application of the structure (Hesse, Care, Buder, Sassenberg, & Griffi, 2015). Problem solving also entails arranging quite a few cognitive and metacognitive processes, finding out and performing suitable methods, and regulating behaviour for the varying demands of problems (Abdullah, Ahmed, Abd Rahman, Mun, & Mokhtar, 2017).

International studies, like PISA defined problem solving as relates to individuals working alone on resolving problem situations where a method of solution is not immediately obvious (OECD, 2012). NCTM (2000) on the other hand referred to that problem solving as an integral part of mathematics, through which students are involved in formulating, and solving multipath problems that encompass a significant amount of effort. By doing so, students' ways of thinking, habits of persistence and curiosity will be improved (Mairing, 2016).

1.3 Mathematical Problem Solving Ability

Problem solving ability is a capability of the person to use the cognitive skill for understanding of the problematic situations and its decision in case when no apparent way of answer is introduced (OECD, 2008). Its parts are also the individual's willingness to deal with these conditions so that he/she can grow his/her personal plausible as a constructive and thoughtful citizen.

Moreover, problem solving ability is more than a copy of the maintained knowledge where it carries mobilization of the cognitive and practical skills, creativity and different psychological sources such as attitudes, motivation and values (OECD, 2003). As mentioned, it is needed to have the expertise for the solving and successful resolution, it is also viable to communicate about the cognitive basis. However, that cannot be understood as something closed, but as a dynamically growing process of the problem solving due to the fact the phase of the ability to solve the problems is an ability to actively acquire and use the new knowledge in a direct contact with a barrier or a difficulty and action performed on it, or gaining new information from other sources that are additionally needed for the successful solution of the problem.

The ability of problem solving is one of the important factors to make the students literate in mathematics. This ability is quintessential to the community, and it is very essential in mathematics, no longer solely for those who will explore or learn mathematics, but also for those who will apply in other studies and in everyday life. The ability to solve mathematical problems has long been recognized as a vital aspect of math competency, however only recently it has been mirrored in school curricula (Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016).

As some distance returned as 1980, the NCTM proposed an overhaul of present math instruction in schools across the United States. An agenda for action was a recommendation for school mathematics of the 1980s outlined eight fundamental changes with an instructional focus on problem solving. The report described the current curriculum as embodying a back-to-basics philosophy where maths competence is erroneously tied to foundational computational capabilities and it known as for a shift in centre of attention to problem evaluation and interpretation (NCTM, 1980).

The Council referred to problem solving ability as the measure of each personal and national mathematical competence (NCTM, 1980). These recommendations for increased mathematical problem-solving ability were echoed by NCTM in 1989 (NCTM, 1989) and again in 2000 (NCTM, 2000). Twenty years after their initial report NCTM still found itself, advocating for the advancement of the curriculum beyond the rote acquisition of procedural and declarative knowledge.

1.4 Importance of Mathematical Problem Solving

This study focussed on problem solving due to its several significance to an individual, society and the nation. The Malaysia Education Blueprint 2013-2025 emphasise that every student needs to possess a spirit of inquiry and learn how to continue acquiring knowledge for the duration of their lives, to be in a position to connect different portions of knowledge, and, most importantly to create new knowledge. Therefore, every student needs to master a range of important cognitive skills. One of them is problem solving and reasoning which promotes the capability to anticipate and approach issues critically, logically, inductively, and deductively to make decisions (Malaysia Education Blueprint, 2013).

Besides, the Standard Curriculum for Primary Schools (KSSR), emphasizes students to be exposed to problem solving as early as in Year 1 (BPK, 2016). Emphasis on the improvement of mathematical thinking is constructed and developed through the teaching and learning in the classroom primarily based on the following principles, which are problem solving, communication, reasoning, making connections, making representations and the application of technology in mathematics (BPK, 2016). The KSSR archives similarly referred to the significance of problem solving as the main focus in the teaching and learning of mathematics. Teaching and learning need to involve problem solving skills comprehensively and throughout the whole curriculum.

The development of problem solving capabilities needs to be emphasized so that students are in a position to solve many issues effectively. The various uses of regular problem solving strategies consisting of multiple steps needed to be expanded to enhance problem solving ability. In carrying out learning activities to build problem solving abilities, problems based on human activities must be introduced (BPK, 2016). Through these activities, pupils can use mathematics when going through new conditions and reinforce themselves in dealing with various challenges each and every day.

Yuan (2016) stated that teaching through problem solving contributes to the realistic use of mathematics by supporting an individual to advance the facility to be extra adaptable. For instance, when technology breaks down, the ones who are good at solving problems, can easily adapt to the adjustments and unexpected problems in their careers and other aspects of their lives. This is because, problem solving is a natural process which stimulates and develops higher order thinking skills such as analysis, synthesis, evaluation and judgment which are needed to solve complex real-life issues.

Teaching higher order thinking via problem solving offers students with applicable life skills and affords them an added benefit of supporting them improve their content knowledge, lower order thinking skills, and self-esteem (Surya & Syahputra, 2017; Swartz, & McGuinness, 2014). It can thus, helps people to switch into new work environments at this time when most are probably to be confronted with various profession changes throughout a working lifetime (NCTM, 1989). Along with the NCTM report, previous researches stated that, *"School should focus its efforts on preparing people to be good adaptive learners, so that they can perform effectively when situations are unpredictable and task demands change"* (Carberry, Cheese, Husbands, Keep, Lauder, Pollard, Schleicher & Unwin, 2015, p.7).

Shah (2010), Thompson (2011), and Henderson Hurley and Hurley (2013), on the other hand stated that problem solving is a vehicle for enhancing critical thinking. Solving a problem requires students to think critically in order to determine and improve their personal approach based on what they learnt and developed in previous problems in a new situation where algorithm cannot be directly applied. National Education Association (NEA, 2002), reported that today's citizens have to be lively imperative critical thinkers if they are to examine evidences, evaluate competing claims, and make sensible decisions. Today's 21st century families ought to sift through a significant array of information involving financial, health, civic, even leisure activities to formulate potential plans of action.

The solutions to international problems, such as global warming, requires critical thinking ability. In everyday work, employees must employ critical thinking to better serve customers, develop better products, and continuously improve themselves within an ever-changing global economy. In fact, economists Frank Levy and Richard Mundane have described the new world of work in which the most appropriate jobs, the ones least possibly to be automated or outsourced, are those that require professional thinking and complex communication (NEA, 2002). According to the AMA, 2010 Critical Skills Survey, 73.3 percent of enterprise executives polled identified critical thinking as a precedence for employee development, talent management, and succession planning.

Apart from that, effective mathematical problem solvers are additionally flexible and fluent thinkers. They are inclined to take on a challenge and persevere in their quest to make sense of a situation and solve a problem. Effective problem solvers will be curious, searching for patterns and connections, and are reflective in their thinking. They be aware of more than one way to solve any problems, and will actively recognize which strategies are more efficient than others beneath specific circumstances. These characteristics are desired for all individuals in each of their professional and personal lives (NCTM, 1989 & 2000). In that case, these traits assist people not only in gaining knowledge of new things more easily, but additionally help them be able to make experience of their existing knowledge.

1.5 Teaching, Learning and Assessing Mathematical Problem Solving

Mathematical problem solving is an important process to learn and also a complex subject to be taught (Bishara, 2016). Teachers included problem solving in the mathematics classroom by means of using distinct procedures (Van de Walle, Karp & Bay-Williams, 2010). Teachers teach problem solving as a content material or as one chapter added into the textbook (Ministry of Education Singapore, 2012), or teach problem solving as a process (Polya, 1945). Another approach is that teachers teach problem solving strategies focusing on teaching certain strategies such as guess-and-check, working backwards, drawing a diagram, and others. In a lesson about problem solving, students work on a problem and then share with the class how using one of these techniques helped them resolve the problem (Evans & Swan 2014).

There is also an approach where teachers teach through problem solving where the lesson would start with the teacher setting up the context and introducing the problem (Van de Walle, 2007). Students then work on the problem whilst the teacher monitors their progress. Then the teacher starts a whole-class discussion. Similar to different approaches of teaching problem solving, the teacher might also call the students to share their ideas, but, instead of ending the lesson there, the teacher will ask students to think about and compare the different ideas, to discuss which ideas are incorrect and why, which ideas are correct, which ones are similar to each other, which ones are extra efficient or more elegant. Through this discussion, the lesson allows students to learn new mathematical ideas or procedures.

Problem solving goes beyond the typical thinking and reasoning students appoint while solving exercises (Cabanilla, Acob, & Josue, 2013; Polya, 1945; Robbins, 2011; Wismath, Orr & Zhong, 2014). Means, problem solving demands thinking deeply about concepts, their associated representations, practicable solution procedures, associated context or cultural knowledge, and creating problem models (Edwards-Lies, 2010). A range of models are proposed that describe the processes that problem solvers use from the beginning till they end their tasks (Mayer, 2002; Montague & Applegate, 1993; Polya, 1945).

One of the problem solving model that can enhance the problem solving ability is Mayer's problem solving model. Richard Mayer (1985) proposed a problem solving model that explains the problem-solving process which occurs in four stages specifically problem translation and problem integration (student' representation of the problem); and solution planning, and solution execution (specific strategies used in the problem). The indispensable problem-solving process requires students to first acquire the meaning of the problem and implications of the text. Next, the student develops an appropriate representation of the problem. Finally, the student links this representation to the best strategy for solving the problem (Mayer, 1985).

The ground-breaking moment in problem solving came when George Polya (1957) wrote his book *How To Solve It*. Polya not only outlines the various steps involved in mathematical problem solving, but gives various examples and solutions to problems and various strategies for solving mathematical problems. In Polya's (1981) formulation, the teacher is the key. *"The teacher emphasizes the value of problem solving as a means to learning mathematics"* (Cabanilla, Acob, & Josue, 2013, p. 38). It means teacher set the right kind of problems for a given class and provide the appropriate amount of guidance to make the class meaningful. Polya used to be quick to point out that students need help to develop problem solving ability (understand the problem, make a plan, carry out the plan, and look backwards) and it needs to be taught correctly by teachers.

Schoenfeld (1982) on the other hand, created a higher perception of how students solve problems, as well as a better understanding of how problems should be solved and how problem solving should be taught. Schoenfeld's model for mathematical problem solving is based totally on Polya's model. The model consists of five episodes, namely, reading, analysis, exploration, planning/implementation and verification. For Schoenfeld, the problem solving process is subsequently a dialogue between the problem solver's prior knowledge, his attempts, and his ideas along the way. As such, the solution path of a problem is an emerging and contextually dependent process.

During the past 20 years, there has been an increasing emphasis on assessing problem solving by analysing the cognitive processes that students use while engaged in problem solving. The multi-dimensionality of the problem-solving process is certainly made evident as attempts are made to look at all thinking done to solve a problem. One of the most frequent techniques used for this purpose is the think-aloud technique (Geography, 2012; Ozcan, Imamoglu, & Bayrakli, 2017). The think-aloud technique is used to acquire verbal protocols of ideas and thoughts that occur to a problem solver. Usually, an audio tape recording is made and some coding procedure is used to document key cognitive behaviours of the participant.

1.6 Teaching and Learning Mathematical Problem Solving through Visual Representation Strategy

Successes in problem solving and achievement measures are influenced by the degree to which students are supported to attain facility with representations. Problem representation is critical to successful problem solving (Krawec, 2014). Analysis of the NCTM standards stated representing a problem as a critical prerequisite to deep understanding. Both paraphrasing and visually representing the problem provide concrete evidence of how students conceptualize what they read. Lavy (2007) stated that the effective tool in learning mathematics is through visual representation which provide an alternative mass resource almost at some stage in the media as the representation of the simplified version of mathematical language in particular in delivering the process of solving problems.

The use of visual representation techniques in solving mathematical problems has been broadly used among Singaporean and Japanese School curricular focusing on the elementary school as the groundwork of exposure to the mind of creativity and criticism (Murata, 2008). As a result, in order to produce a better communication between mathematical ideas, visual method such as tape diagram and simple picture been used in assisting the students in connecting ideas across the problems given (Ho & Lowrie, 2014).

Effective problem solvers consider a range of representations that are suitable for completing a task (Lesh & Zawojewski, 2007). Instruction that permits students to consider a range of representations to complete a task and share them has been proven to have high quality outcomes on students' achievement (Jitendra, Nelson, & Pulles, Kiss & Houseworth, 2016). Hott, Isbel and Montani (2014) reported that the use of pictorial representations in teaching to solve mathematical problems to be effective. Creating an instructional context that stimulates mathematical discussions among problem solvers enhances their ability to solve problems and use a variety of representations. The teacher is the critical factor in making such learning environment (Cabanilla, Acob, & Josue, 2013). This goes along with Polya's (1981) formulation that mentioned teacher as the key to produce successful problem solvers.

The manufacture and exchange between representations is essential to apprehend mathematics word problems. Word problem solving as well as comprehension and text interpretation are present among the skills in the mathematical competence model where problem solving is one of the acquisition skills. Representation and presentation are viewed as a components of mathematical competence which belongs to acquisition and communication skills respectively (Debrenti, 2015). Four representations, namely verbal representation, image informational, decorative images, and provide a number line yields a significant effect on the ability of students solving math problems.

In the beginning of the 1980s, Richard Mayer has made enormous contributions to word problem solving using representation. Further expanding theory on a schema, Mayer confirmed that students do compare problems at hand to the schema for previously solved problems (Mayer, 1985). Furthermore, when students lack a schema for a problem they are facing, the students' representation of the problem is far more likely to be incorrect (Mayer, 1983). Incorrect representation of a problem is likely to produce an incorrect solution. In contrast, Mayer points out the fact that typical problem-solving instruction tends to focus on facts and algorithms rather than on correct representation (Mayer, 1989). Mayer's emphasis on the importance of the representation phase is echoed in this study, as model drawing is exceptionally an action taken in the representation phase of problem solving.

1.7 Equality in Teaching and Learning Mathematical Problem Solving

Teaching and learning of mathematical problem solving should be cultivated to every student regardless their background. Education is universally acknowledged as a vital human right because it incredibly affects the socio-economic and cultural elements of a country. Equality in education will amplify the workforce of the nation, therefore growing countrywide income, economic productivity and Gross Domestic Product (GDP) (OECD, 2012). It reduces fertility and infant mortality, improves infant health, increases life expectancy and increases standards of living. These are elements that allow economic steadiness and growth in the future (OECD, 2012).

The 2013 United Nations Educational, Scientific and Cultural Organization (UNESCO) Annual Report states: *"Gender equality and education are fundamental* *and inalienable human rights"* (p. 13). The report promotes gender equality *to*, *in* and *through* education so as to ensure women's and men's, girls' and boys' equal access to learning opportunities, fair treatment in the learning process, equitable outcomes as well as access to opportunities in all spheres of life.

Gender equality is achieved when female and male learners are treated and benefited from education equally, so that they can fulfil their potential and turn out to be empowered to make a contribution to and benefit from social, cultural, political and economic development equally. This is in accordance to the Universal Declaration of Human Rights in 2016 Report which states: "*Everyone has the right to education. Education shall be free, at least in the elementary and fundamental stages. Elementary education shall be compulsory. Technical and professional education shall be made generally available and higher education shall be equally accessible to all on the basis of merit" (p.114)* (UNESCO, 2013). Education permits girls and boys, women and men to take part in social, economic and political life and is a base for improvement of a democratic society.

1.8 Statement of Problem

Mathematics was assessed as a major domain in PISA 2012. In addition to the "content" subscales (with the "uncertainty" scale re-named as "uncertainty and data" for improved clarity), three new subscales were developed to assess the three processes in which students, as active problem solvers, engage (OECD, 2012). The PISA report shows that Malaysian students can only solve 29.1% of the problems correctly (OECD, 2012). This rate of achievement is below the OECD average, which is 45.5%. Malaysian students' poor performance did not end with that, but it continues until the 2015 PISA assessment (PISA, 2016). The weighted response rate among the initially sampled Malaysian schools fell short of the standard PISA response rate of 85% which the results were not been comparable to those of other countries or to results for Malaysia from previous years. Therefore, in PISA 2015, Malaysian students' scores fell short of full recognition. The result from PISA 2018 on the other hand revealed that only 2% of Malaysian students able to score at Level 5 or higher in mathematics (PISA, 2018). This rate of achievement is also below the OECD average, which is 11%.

The status of Malaysia's ranking in TIMSS 2011 reported a plummeting trend in the position of Malaysia in the Mathematics subject where the rank fell from 16th (1999) to 10th (2003), 20th (2007) and 26th (2011) (Stephen, Lydia, Maria, Katherine, Westat & Judy, 2016). TIMSS assesses students' mathematical thinking in three cognitive domains which are knowing, applying, and reasoning. TIMSS 2015 International Results in Mathematics showed that Malaysian students' average score is 465 which is significantly lower than the centre point of the TIMSS 8th grade scale, which is 500 (Stephen, Lydia, Maria, Katherine, Westat & Judy, 2016). It has been observed in TIMSS 2015 result report that Malaysian students are not performing well in solving Grade 4 problems which involves arithmetic (Mullis, Martin, Foy & Hooper, 2016).

According to Wulandari (2015), both PISA and TIMSS assessment test on student's problem solving ability which provide useful information that helps a country in monitoring and evaluating the curriculum and instruction in schools. The TIMSS and PISA overall results were for 14 and 15 years old pupils. The results from PISA and TIMSS indicate that the ability of solving mathematical non-routine problems among Malaysian students remains at a low level.

Students' poor performance in problem solving can be due to the teaching and learning process in the school system. Umugiraneza, Bansilal, and North (2017) reported that the use of a variety of effective teaching procedures and styles is encouraged to motivate adapt-ability and lifelong learning in the teaching and learning process of problem solving. Therefore, students' ability to solve mathematical problems will largely depend on whether they have been taught using effective strategies or not.

Many findings from the Teacher Education and Development Study in Mathematics (TEDS-M) stated that pre-service teachers have concern solving abstract problems and problems requiring multiple steps (Tatto, Peck, Schwille, Bankoy, Senk, Rodrigues, Ingyarson & Reckase, 2011). They had been probably to have challenge in solving multi-step problems with complex linguistic or mathematical relations, and relating equivalent representations of concepts. In particular, they found recognizing faulty arguments and justifying or proving conclusions/ final answer is challenging (Tatto et al., 2011). In fact, the secondary analysis of the TEDS-M results has also indicated that Malaysian pre-service mathematics teachers at each of the primary and secondary level have low mathematics content knowledge (Leong, Chew & Abdul Rahim, 2015).

Related to mathematics learning in the classroom, Siswono (2008) reported that mathematics learning process is still going on conventionally and tends to be mechanistic. It means that students listen, imitate or copy exactly the same way what the teacher gives without initiative. The above finding shows that there is no effective problem solving strategy that could assist students in solving mathematical problem effectively. Teachers use different kind of strategies which are not helping their students to improve their problem solving skill.

The use of visual representations has been proven to be a positive approach to enhance students' problem solving ability. However, it has been cited that students appear reluctant to use visual representations to solve mathematics problems unless specifically directed to (Teahen, 2015). Reasons for this encompass the understanding that diagrams are a teacher approach for teaching (Uesaka, Manalo, & Ichikawa, 2007) and that visual reasoning is considered of low value (Arcavi, 2003).

There are three different kinds of difficulties on visualization faced by students especially those with learning difficulties (Garderen & Montague, 2003; Hamidreza Kashefi, Nor Athira Alias & Mohamad Fahmi Kahhar, 2015). First, the frequency of using any visual representation strategies in solving problem is very low, second, the quality of diagram and the interrelationship with the problem statement is very poor, and thirdly, they use very limited visual strategies in problem solving such as organize, plan, monitor, compute and justify (Garderen & Montague, 2003; Hamidreza Kashefi, Nor Athira Alias & Mohamad Fahmi Kahhar, 2015). However, all those weaknesses showed by the students with learning difficulties is not a major problem as these can be fixed through visualization.

Strategies on how to teach problem solving had been prevalent since time immemorial. Mayer's (1980s) problem solving model had successfully introduced and tested by-time. Mayer's problem solving model has visual representation component which past studies proved that it is an effective strategy to improve students' problem solving ability. However, Mayer's problem solving model not fully employing visual representation strategy throughout the model as only Mayer's first stage of problem representation converts a problem from words into an internal representation to an external representation (Mayer, 1985). This is insufficient to solve a problem accurately because, the more the visual representations include appropriate relational and numerical components, the closer they would fall on the accurate solution of the problem (Krawec, 2014).

Based on the past studies, it has been hypothesized that the use of visual representation and Mayer's problem solving model respectively could enhance students' problem solving ability. Teachers, as a facilitator of students in the dynamic classroom situation, must learn how to tailor such approaches, methodologies and strategies that would suit best the need of their students. Therefore, researcher incorporated visual representation into Mayer's problem solving model in order to assist teachers in enhancing students' problem solving ability.

However, there is no sufficient evidence to prove that incorporating visual representation into Mayer's problem solving model could enhance students' problem solving ability. This research therefore intended to determine the effectiveness of Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy in enhancing students' mathematical problem solving ability. This study also applied another teaching strategy called as the Mayer's problem solving Model (MM) in order to compare the effectiveness of Mayer's problem solving Model (MM) when incorporated with Visual Representation strategy (VR).

Gender equality is believed to achieve when female and male learners have equal access to learning opportunities, and are treated and benefit from education equally. However, PISA report published that there is an inequality in students' performance in problem solving in regards of gender (OECD, 2012). The report says that boys outperform girls in problem solving with 23 out of 65 participated countries. Besides, an overview of gender patterns in educational attainment based on national examination results of primary education (UPSR), lower secondary education (PMR) and upper secondary education (SPM), reveal that girls perform better in all the national examinations and across all types of schools (Jelas, Salleh, Mahmud, Azman, Hamzah, Hamid, Jani, & Hamzah, 2014). Therefore, this research aimed to study the problem solving ability based on the gender.

Besides, Ramful and Lowrie (2015) reported that there is a relationship between spatial visualization and gender. Koscik, O'Leary, Moser, Andreasen, and Nopoulos (2009) attributed the observed gender variations to purposeful and morphological differences in the brains of males and females. Researchers suggested that biological sex differences lead to gender differences in cognitive processing and adopting a certain problem solving strategy for mental rotation tasks (Hirnstein, Hausmann & Gunturkun, 2008). For example, males and females adopt different mental strategies to evaluate response options in each item (Hirnstein, Hausmann & Gunturkun, 2008).

Above studies show that gender disparity still exist in mathematical problem solving. The use of Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy could enhance students' problem solving ability regardless of gender. However, there is no sufficient evidence to prove the effectiveness of the teaching strategy in regards of the gender. Therefore, this study is intended to examine the interaction of MMVR teaching strategy with the gender factor in enhancing students' mathematical problem solving ability.

1.9 Objective of the Research

The primary purpose of this study is to determine the effectiveness of Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy in enhancing Year 4 students' mathematical problem solving ability, and also its interaction with gender. This study aimed to achieve the following objectives:

1. To determine the effectiveness of MMVR teaching strategy in enhancing Year 4 students' mathematical problem solving ability.
2. To determine the effectiveness of MMVR teaching strategy in enhancing Year 4 students' sub-components of mathematical problem solving ability which are understanding of the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability.
3. To determine the interaction between MMVR teaching strategy and gender in enhancing Year 4 students' mathematical problem solving ability.

1.10 Research Questions

Based on the objectives of the study, the research questions are as follow:

1. Does the mathematical problem solving ability of Year 4 students in MMVR group improve significantly after the MMVR treatment?
2. Does the sub-components of mathematical problem solving ability which are understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability of Year 4 students in MMVR group improve significantly after the MMVR treatment?
3. Is there any significant difference in the mathematical problem solving ability of Year 4 students in MM group, MMVR group and control group after the treatments, after controlling the pretest score?

4. Is there any significant difference in the sub-components of mathematical problem solving ability which are understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability of Year 4 students among MM group, MMVR group and control group after the treatments, after controlling the pretest scores?
5. Is there any significant interaction between problem solving teaching strategy and gender on Year 4 students' mathematical problem solving ability after controlling the pretest score?

1.11 Research Hypothesis

To answer the research questions, the following hypotheses were developed:

RQ1 H_0: The mean of mathematical problem solving ability scores of Year 4 students in MMVR group is not different after MMVR treatment.

H_1: The mean of mathematical problem solving ability scores of Year 4 students in MMVR group is higher after MMVR treatment.

RQ2 H_0: The mean of understanding the problem ability score, the mean of devising a plan ability score, the mean of carrying out the plan ability score, and the mean of looking back ability score of Year 4 students in MMVR group are not different after MMVR treatment.

H_1: The mean of understanding the problem ability score, the mean of devising a plan ability score, the mean of carrying out the plan ability score, and the mean of looking back ability score of Year 4 students in MMVR group are higher after MMVR treatment.

RQ3 H_0: The mean of mathematical problem solving ability scores of Year 4 students is not different in MM group, MMVR group and control group after the treatment.

H_1: The mean of mathematical problem solving ability scores of Year 4 students is different in MM group, MMVR group and control group after the treatment.

RQ4 H_0: The mean of understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability scores of Year 4 students are not different among MM group, MMVR group and control group after the treatments.

H_1: The mean of understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability scores of Year 4 students are different among MM group, MMVR group and control group after the treatments.

RQ5 H_0: There is no interaction between problem solving teaching strategy and gender on Year 4 students' mathematical problem solving ability.

H_1: There is an interaction between problem solving teaching strategy and gender on Year 4 students' mathematical problem solving ability.

1.12 Research Paradigm

This research is primarily based on the positivist paradigm as the researcher holds an epistemological belief of a positivist within a dualist and objectivist view. Being objectivist is an integral aspect of any competent inquiry (Creswell, 2009). The knower and the object to be recognized are different entities. Neither of them exerts influence on the other. Positivists are interested in facts and hold that research should be value free. Threats to validity are controlled through preventive procedures. Causal relationships can be established and therefore generalization and replicability become possible.

Secondly, the researcher aims at explaining relationships "of what?." Cause and effect relationship is one of the tenets of the positivist paradigm (Creswell, 2014). Experimental designs appear to grant an umbrella to provide an explanation for this causal relationship (Creswell, 2014). Questions and hypotheses are examined and proven through experiments. The researcher is seeking for a cause-effect relationship between the independent variable, which is the intervention and cause of any improvement, and the dependent variable, the outcome of the intervention.

The attribution of the impact to the independent variable can be warranted via the manipulation of different variables that may additionally threaten research validity. Moreover, a deductive approach is followed. Accordingly, terms such as intervention and treatment come to be key words in the scientific paradigm. Based on this reality, the independent variable, or intervention and treatment, is the cause of any change in the performance or behavior of subjects.

Thirdly, the researcher used data collection methods to collect quantitative, numerical data that can be tabulated and analyzed statistically. According to Creswell (2013), four main kinds of data are gathered in quantitative research. Individual performance is the first type. It consists of norm referenced tests, criterion-referenced tests, intelligence and aptitude tests. The second type of data measures individual attitude and uses an affective scale. Observation of character behavior is the third type of gathered data. Researchers can use behavioral checklist to record observation about individual behavior. The last type of data is factual. The researcher relied on public documents or school records to collect data about a sample. Creswell (2013) consents with Dornyei (2007) on the great significance of choosing the sample in quantitative studies. Both of them started out their chapters about collecting quantitative data by addressing the troublesome of random sampling. Dornyei (2007) contends likewise that sampling is important as it can guarantee generalizable findings.

1.13 Theoretical Framework

This study is grounded by way of several theories and the theoretical framework comprises Problem Solving Model by Polya (1945) and Mayer's Problem Solving Model (1985), which viewed problem solving as a complex, multiple-step cognitive process.

This study aimed to enhance students' problem solving ability. The definition of problem solving ability for this study is derived from Problem Solving Model by Polya (1945) which explains the ability that every problem solver should pose when solving non-routine problems. George Polya, the founder of modern theory in mathematical problem solving, developed a detailed treatise on general heuristics for solving mathematical problems in his 1945 book titled *How To Solve It*. To become a successful problem solver, students should be able to understand the problem given, devise an accurate plan for the problem, carrying out the plan, and checking back the answer (Polya, 1945).

Next, Mayer's Problem Solving Model presents the conceptual framework for this study by walking the students through the four steps of solving mathematical problems. Mayer (1985) viewed problem solving as a complex, multiple-step cognitive process which requires one to associate previous experiences to the problem at hand and further act upon the solution. Through employing this teaching model, students learn how to translate and integrate mathematical problem, planning for the solution, and finally execute the planning in order to help them become successful problem solvers. Also, Mayer has emphasis on the importance of representation when solving problems. In this study, Mayer's Problem Solving Model was used as a teaching approach along with Visual Representation strategy to enhance students' problem solving ability.

1.14 Significance of the Research

This study will benefit three main stakeholders namely mathematics educators, mathematics policy makers and mathematics education researchers. Also, this study will add into existing literature in regards of mathematics education.

For mathematics educators, this study will grant them with guidelines to teach problem solving in mathematics effectively. This study addresses the mathematical needs of students who are going through difficulties in solving mathematical problems by means of offering them with a heuristic process of mathematical problem solving strategy called 'Visual Representation' incorporated in Mayer's Problem Solving Model, that used to be empirically examined to determine its effectiveness at increasing problem solving ability. Educators may find Mayer's Problem Solving Model implemented in this study, is a valuable teaching strategy.

The usage of Mayer's Problem Solving Model with Visual Representation (MMVR) teaching strategy will introduce a technique of problem solving instruction, by providing a meaningful way for students to improve their problem solving abilities. In addition, the results of this study may be useful for many educators who are willing to make their instruction by incorporating representations into their mathematics classroom. The outcomes of this study will draw teachers' attention to the necessity of the usage of visual representation along with problem solving model, in solving mathematical problems.

Secondly, this study will grant beneficial information for the policy makers to determine on new policy strategies to enhance the usage of visual representation in teaching and learning of mathematics. The information from this study will provide the policy makers with a better understanding on how students use representations in problem solving, as well as information as to where and how problem solvers have difficulty using visual representations. In doing so, the study will provide improved and informed direction for both teaching and curriculum design of instruction involving visual representation. Educators, thus, will include visual representation in their lesson plan for mathematics, and textbooks or activity books particularly when teaching problem solving.

Thirdly, this study will be useful to those researchers who are interested in doing research in teaching and learning mathematical problem solving. It will provide comprehensive information that can be helpful in conducting more research pertaining to visual representations in teaching mathematical problem solving particularly among primary school students. This study will provide researchers with an avenue to address related classroom problems such as teachers' lack of visual representation skills, how teachers control the mental pictures of students, difficulties in learning appropriate skills, and inadequate time for planning or fitting visual representation into existing curricula.

Finally, this study adds to the literature on the use of MMVR teaching strategy in solving mathematical problems. It is one of the few studies focusing on the impact of MMVR teaching strategy on the teaching and learning of mathematical problem solving of Year 4 students. Hence, the results of this study will stimulate further research studies that integrate visual representation strategy in learning mathematical topics, along with specific instructional model.

1.15 Limitation and Delimitation of the Research

There are several limitations in this study. First is the feasibility in implementing an experimental design for this study. Some of the most essential questions in educational policy cannot feasibly be evaluated via experiments, even though one could in theory design an experiment to test a particular question. It is difficult to imagine schools or parents agreeing to the random assignment of students for this study design. In situations like these, a quasi-experimental methods can frequently be adopted to tease out the parameter of interest.

Secondly is the time duration of this study. A related issue is that experiments are frequently conducted on newly implemented teaching strategy. This is challenging if the short-term measurable influences do no longer proxy the long-run affects very well. In general, when a researchers intervene with students, researchers are eventually fascinated in enhancing long-run outcomes such as lifetime wages or other measures of well-being. Since researchers typically do no longer favour to wait twenty or extra years for wage data to be available, they look for impacts on short-run measures such as test scores that are thought to be good proxies for the longer-term outcomes. Sometimes, though, the long-term outcomes are not properly envisioned with the aid of short-run measures.

The third limitation is on the external validity of this study. A limitation of both experiments and well-identified quasi-experiments is whether the estimated impact would be similar if the teaching strategy have been replicated in any other area or focused on a unique team of students. Researchers regularly do little or nothing to tackle this point and have to possibly do more. One straightforward method is to report a comparison of the experiment's control group to the population of interest and reweigh the sample to be representative of that population. This again suggests the need for extra experimentation across a wider variety of settings.

Lastly, due to the characteristics of quasi-experimental method, researcher ensured that both the experimental and control groups experience the same external events. To avoid any physical or mental changes that may occur within students over time, researcher selected students who mature or change at the same rate (e.g. same age) in the course of the experiment. Also, to keep away from both control and experimental groups communicate with each other, researcher kept the two groups as separate as feasible all through the experiment.

This study was set to certain boundaries. First, this study delimited to the process of problem solving and it is not focused on other mathematical processes such as reasoning and proof, communication, connections, and representation. Past researches clearly noted that students are dealing with difficulties in problem solving. Their overall mathematical achievement is affected by their low performance in problem solving. Therefore, this study solely focused in mathematical problem solving.

Secondly, this study involved basic arithmetical topics only. Following the Standard Based Curriculum for Primary Schools (KSSR), students start to learn problem solving involving the basic arithmetical topics. Therefore, students can master on how to solve the problem with the most basic arithmetical topics such as numbers, length and time before applying to other advanced contents such as algebra and geometry.

Thirdly, this study was delimited to Year 4 private school students who were studying the Standard Based Curriculum for Primary Schools (KSSR). The reason of choosing private school students is that, the process of entering the private school to conduct the research is easier and faster compared to a public school. By doing this, the researcher saved more time. School with a KSSR curriculum has been chosen for this study due to the Malaysian public school students' low performance in PISA 2012 to 2018 and TIMSS 2015 assessments. The study delimited to Year 4 students because they are the youngest group of students who can be monitored easily in their visual representation ability, and also who will be available for participating in this study based on authority rules of the school.

Lastly, this study was set to certain boundaries based on the sample selection, the uniqueness of the setting, and the timing of the experiment. Researcher restrained claims about groups to which the outcomes cannot be generalized. Next, researcher recommended other/ future researchers to replicate the study at later times to determine if equal outcomes appear as in the earlier time.

1.16 Definition of Terms

This section presents the definition of key terms used in this study.

Ability to carry out the plan according to Polya (1945) is solve the equation students came up in their "devise a plan" step, and check each step to ensure the step in correct. In this study, ability to carry out the plan is operationally defined as the score in the Mathematical Problem Solving Ability Test (MPSAT).

Ability to devise a plan according to Polya (1945) is to come up with a way to solve the problem by making a representations, considering past problems, and choosing a suitable operation to solve the problem. Ability to devise a plan in the present study is operationally defined as the score in the Mathematical Problem Solving Ability Test (MPSAT).

Ability to look back according to Polya (1945) is examine the solution obtained by checking the result, derive the solution differently, and using the result for some other problem. Ability to carry out the plan in this study is operationally defined as the score in the Mathematical Problem Solving Ability Test (MPSAT).

Ability to understand the problem according to Polya (1945) is reading, interpreting and comprehending the problem in order to find the data, unknown, and hidden condition of the problem. Ability to understand the problem in present study is operationally defined as the score in the Mathematical Problem Solving Ability Test (MPSAT).

Mathematical problem solving according to Lesh and Zawojewski (2007), is a task, or goal-directed activity, turns into problem or problematic when the problem-solver, which might also be a collaborating group of specialists, needs to develop a more productive way of thinking about the given situation. In this study, mathematical problem solving is operationally defined as a cognitive process in which students are engaged in solving mathematical problems.

Mathematical problem solving ability according to Polya (1945) involves the ability to understand the problem (identify the data, conditions, and unknown), devise a plan (choose applicable and appropriate strategy (ies) with reason), carry out the plan (show and prove the accuracy of each step), and look back (explain the accuracy of the solution). Mathematical problem solving ability in this study is operationally defined as the mean score in the Mathematical Problem Solving Ability Test (MPSAT).

Mayer's Problem Solving Model (MM) *a*ccording to Mayer (1985) consisting of two major phases in mathematical word problem solving: problem representation and problem solution. Problem representation is composed of two sub-stages: problem translation, which relies on linguistic skills needed to comprehend what the problem is saying, and problem integration, which depends on the ability to mathematically interpret the relationships among the problem parts to form a structural representation. The second general phase, problem solution, is composed of the sub-stages solution planning, determining which operations to use and the order in which to use them, and solution execution, carrying out the planned computations in order to solve the problem. Mayer's Problem Solving Model (MM) in this study is operationally defined as using the Mayer's problem solving model to teach students to solve mathematical problems successfully.

Mayer's Problem Solving Model with Visual Representation (MMVR) is operationally defined as the teaching approach that combines the Mayer's Problem Solving Model (MM) with the use of Visual Representation (VR) strategy along the model. The MMVR teaching approach enables students to use visual representation to give solution to each step in the mathematical problem solving worksheets (See Appendix B). Mayer's Problem Solving Model with Visual Representation (MMVR) in this study is operationally defined as using the Mayer's problem solving model integrated with visual representation strategy into each step of Mayer's problem solving model to teach students to solve mathematical problems successfully.

Teaching strategy according to Akdeniz (2016), is the ways and approaches followed by the teachers, to achieve the fundamental aims of teaching. Teaching strategy in this study is defined as the Mayer's Problem Solving Model (MM) and Mayer's Problem Solving Model with Visual Representation (MMVR) instruction.

Visual Representation is defined as the construction and formation of internal images (e.g., mental images) and/ or external images (Van Garderen & Montague, 2003). In this study, visual representation in operationally defined as the constructions of external images such as drawings of objects, pictures or diagrams.

1.17 Summary

This chapter described the purpose of the study, the research questions, some background and rationale for conducting the study, and included basic conclusions, and assumptions that can be drawn regarding problem solving ability, visual representation and teaching strategies of this study. Terms used throughout the research study were defined.

Chapter two will discuss the literature about learning theories, problem solving theories and their respective models that involved in this study, and visual representations and their role within teaching. The factor affecting the problem solving ability will be discussed thoroughly in the next chapter. Finally, the relationships between these variables are then outlined as the conceptual framework of this study in Chapter 2.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The objective of this chapter is to answer the research questions stated for this study in Chapter 1. In regard to that, this literature review has four main focus: (a) Polya and Mayer's problem solving Models, (b) key concepts of mathematical problems solving ability, teaching, gender and visualization (c) past research on mathematical problem solving ability, teaching and gender, (d) conceptual framework of the study.

2.2 Theories

A theory is a contemplative and rational kind of abstract or generalizing thinking, or the results of such thinking. Theories guide the enterprise of finding facts instead of attaining goals, and are neutral concerning other choices amongst values (Udo-akang, 2012). A theory can be a body of knowledge, which might or might not be related with particular explanatory models. The use of a specific theory presents the thinking process, the underlying principles, and the operational roadmap, for research design decisions.

In excellent research designs, the language of the theorist is without a doubt evident throughout the research steps and methodology. The theory helps the researcher to frame the research queries and directs the search of the literature. The theory's perspective ought to be evident throughout the proposal and last product and to be meaningful, especially in the discussion of the outcomes and implications for practice. Therefore, this study is grounded by using two problem solving models, namely Polya's problem solving model and Mayer's problem solving model. Polya's problem solving model is primarily based on Polya’s theory of problem solving and Mayer’s problem solving model is based on Mayer’s theory of problem solving.

2.2.1 Polya’s Problem Solving Model

Since Descartes, mathematicians and educators have considered methods to efficiently understand and educate the problem solving process (Banerjee, 2010). George Polya, the founder of modern-day theory in mathematical problem solving, developed an exact treatise on familiar heuristics for solving mathematical problems in his 1945 book titled *How To Solve It* (Polya, 1945). Widely cited as a seminal work in problem solving, Polya's work breaks down mathematical problem solving into four easy steps which are to understand the problem, devise a plan, carry out the plan, and look back.

Step	Primary Activity
Understand the problem	Reading, interpreting, comprehension
Devise a plan	Making a representation, considering past problems, choosing an operation
Carry out the plan	Operations on numbers
Look back	Checking on computation and reasonableness of solution

Figure 2.1. Polya's four steps problem solving model (1945)

Polya's first step, understanding the problem, asks the solver to perceive the data, hidden conditions and unknown of the problem with the aid of reading, interpreting, and comprehending the problem story (Polya, 1945). Polya suggests isolating the condition of the problem into smaller parts if necessary. Polya also includes recommendation to draw a diagram of the problem in order to facilitate understanding (Polya, 1945), early advice germane to this study. Schematic diagrams include germane information from the problem that supports an accurate problem solution (Edens & Potter, 2008). These diagrams, as opposed to pictorial representations, are positively correlated with improved problem-solving performance (Stylianou & Silver, 2004).

In Polya's second step, making a plan, the solver is recommended to consider previous knowledge before actually solving the problem (Polya, 1945). Polya encourages the solver to relate the problem at hand to similar preceding problems. The identical search for preceding knowledge is undertaken for the unknown in the problem. Polya stated that the ability at selecting a fine strategy is best learned by solving many problems. Also, this step has clear connections to schema theory, as the solver accesses schema on problem solving and problem types as a method for choosing the correct mathematical operation (Schoenfeld, 1992).

Polya's third step, carrying out the plan, relates directly to mathematical computation and heuristics for computation. Polya encourages checking computation carefully, referring it to the understanding of the problem generated in the first two steps (Polya, 1945). Polya spends rather little effort in creating this step, as he viewed that most difficulties in problem solving emanated from misunderstandings springing up from understanding the problem or making a plan (Schoenfeld, 1992).

Once the solution is reached, Polya advises the solver to look back. This step encompasses checking calculations, but additionally includes deriving a solution via an alternative method, and checking the context of the problem to be certain that the unknown has been made known (Polya, 1945). In addition, Polya has emphasized to take a look if the strategy used to derive the answer could be used for other similar problems. Polya's four steps provide an early model for the problem solving process. Many later researchers and theorists in problem solving started out their investigations with references to this foundational work (Department of Education, 2008; Leong, Tay, Toh, Quek, & Dindyal, 2011; Mayer, 1983; NCTM, 2000; Porter, McMacken, Hwang, & Yang, 2011).

Lasak (2017) in his study mentioned that students taught in traditional mathematics education environments are preoccupied by using exercises rules, and equations that needed to be learned, however they are limited when it comes to the use in unfamiliar conditions such as solving real life mathematics projects. In distinction to conventional mathematics classroom environments, a Polya's problem solving process offers students with opportunities to enhance their capabilities to adapt and change methods to fit new situations. Furthermore, students taking part in learning mathematical procedures related with communication, representation, modelling and reasoning.

Mathematics instructors agree in that routine problems are as necessary as non-routine problems in teaching problem solving. Polya's observation based on the reviewed literature is that typically non-routine problems develop the problem solving skill and this skill develops the skill of using them in real life situations (Polya, 1957). In Polya's further statements, he stated that in order to develop problem solving skill, it is important to teach on how to solve routine problems. However, in order to develop critical thinking and creative skills he also added that non-routine problems should also be included in teaching. Furthermore, since non-routine problems require that one or two of problem solving strategies are used, this is additionally really helpful in this aspect. It helps to develop critical and creative thinking. (Mabilangan, Limjap, & Belecina, 2011).

Polya's descriptions of his problem solving strategies which were too broad, and it is only applicable theoretically, not empirically has caused to not to choose

Polya's problem solving model as a teaching approach for this study (Schoenfeld, 2013). When Polya's books were read by mathematicians, they felt his descriptions of problem solving strategies are right, however, had not yet been possible to teach students to use those strategies efficiently (Schoenfeld, 2013). For instance, even if it sounds like a sensible strategy, when try to solve an easier related problem, but it turns out that, there are at least a dozen different ways to create easier related problems when depending on the original problem.

Polya's name for any particular strategy was in fact a label that identified a *family* of strategies as each of these is a strategy in itself. So that, the students could "take part" a family by discovering the primary strategies that fell under its umbrella once a student understood it. Teacher could teach each of those certain strategies (e.g., solving problems that had integer parameters by looking at what happened for n = 1, 2, 3 . . . ; looking at lower-dimensional versions of complex problems; etc.), and they are said that they have mastery of family strategies that Polya had named when the students had learned each of those example.

It is meant that, understanding and teaching Polya's strategies was an empirical one and no longer a theoretical challenge (Schoenfeld, 2013). In order to promote an efficient problem solving process, a teaching strategy of this Mayer's problem solving model been chosen. On the other hand, to evaluate students' problem solving ability for this study, Polya's problem solving model was chosen. The motive is, standard manifestations of mental ability can be focused on in problem solving via mathematical means and can be related to the heurism recognized from the analyses of approaches via Polya.

2.2.2 Mayer's Problem Solving Model

Beginning in the 1980s, Richard Mayer has made enormous contributions to problem solving. Mayer's emphasis on the significance of the representation section is echoed in this study, as model drawing is notably an action taken in the representation phase of problem solving. Mayer outlines a number of special kinds of knowledge required in the act of problem solving (Mayer, 1982). Linguistic and factual knowledges concern how to encode sentences, such as grammar, along with history knowledge connected to the problem context. Schematic knowledge is handy to problem types, such as the aforementioned combine, alternate and contrast problem types in elementary mathematics (Amnueypornsakul & Bhat, 2014). Algorithmic understandings concern on how to perform frequent repetitive techniques in computation, such as the steps accompanied in column addition, long division or multiplying fractions (Mayer, 1982).

Mayer (1983) viewed problem solving as a complex, multiple-step cognitive system which requires one to associate preceding experiences to the problem at hand and further act upon the solution. He argued that a problem has to be paraphrased, comprehensively understood, and then visually integrated into a theoretically correct and complete schematic structure in order to reach the solution. He identified problem representation and problem solution as two major processes involved in solving mathematical problems (Mayer, 1985). This model is consolidated in the figure below.

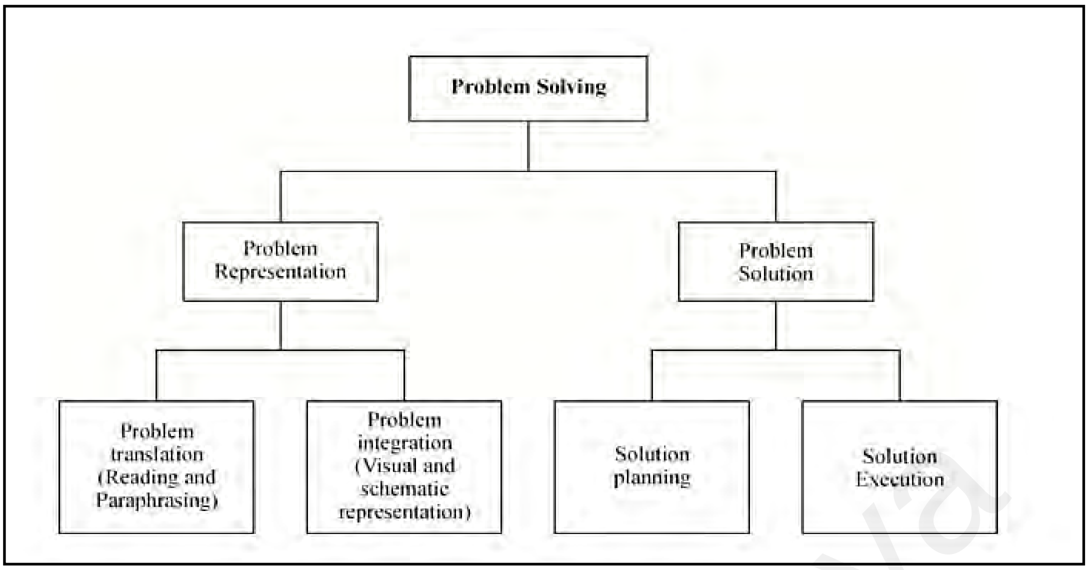


Figure 2.2. Mayer's problem solving model (1985)

Problem representation is composed of two sub-stages together with problem translation and problem integration whereas problem solution consists of sub-stages including solution planning and solution execution. Strongly linked to successful problem solving are the two sub-stages of problem representation which are essential precursors to a conceptually correct and comprehensive interpretation of the problem (Krawec, 2014). As a conceptual framework to situate this study, we used Mayer's approach to problem solving.

Actions taken during this stage, are to move the internal representation to an external representation. For instance, blocks might be manipulated to show the actions of a problem, or a solver might draw a picture or a diagram of the elements of the problem during the representational phase of problem solving (Mayer, 1983). Polya's steps of "understanding the problem" is encapsulated by Mayer's problem representation stage. For developing a correct and comprehensive understanding of the problem, problem translation is a necessary prerequisite. For it to be transformed into an understandable form for the problem solver, it involves reading and paraphrasing the problem (often called re-telling). Reading and paraphrasing are the most fundamental procedures for eventual success in problem-solving (Krawec, 2014; Montague, 2003). It will strongly influence the subsequent processes and solution accuracy if an error is made at this point. In mathematics research, paraphrasing does not only require students to re-word the textual content into a familiar form, but it also requires a critical analysis of the textual content for relevance. For instance, in order to understand the problem, the students need to differentiate relevant information from irrelevant information, identify relationships, manipulate and process information based on relevance.

Problem integration, on the other hand, involves using a visual illustration to understand the problem and its structure, and similarly interpret the relationships (Mayer, 1985). Schematic knowledge is most needed at the problem integration step. A solver need access prior knowledge about comparable problem types, discerning how the problem at hand is like problems solved in the past and illuminating a path towards a solution. To bring the internal representation to the external realm, efforts are made here (Mayer, 1989). At this step, students can vary broadly in the level of experience they have with exceptional kinds of word problems (Mayer, 1985).

The solver acts upon the representations generated in the first stage is where the second stage of Mayer's model problem solution stage is. For numerical quantities, mathematics is applied and equations are solved. In order to arrive at the desired numerical solution, algorithms or other strategies are chosen, applied and worked. Finally, by the problem solver within the story context, a numerical solution is interpreted. Polya's “devise a plan” and "carry out the plan" steps map to Mayer's problem-solution stage (Mayer, 1982).

Solution planning followed by solution execution are the two steps broken down from problem solution stage. For the solver to be strategically knowledgeable, solution planning is required. An operation is chosen and how to carry out that operation is considered by the solver (Mayer, 1985). Depending on the computation abilities of the solver coupled with the solver's level of strategic knowledge, mental arithmetic, counting strategies, and pencil-paper algorithms could be applied. Students can differ from each other in terms of their familiarity with general problem-solving strategies in this step (Mayer, 1985).

Solution execution demands algorithmic knowledge, the understanding of the steps required to carry out the computation involved in the word problem is the second step (Mayer, 1985). Mayer suggests that vast variation can take place among students in the sophistication, accuracy and automaticity of their algorithms for fundamental operations (Mayer, 1985).

The complex and interconnected nature of the task of problem solving is pointed out as Mayer's two-stage model of problem solving points. A problem solver can become lost when there are many points along the road. Mayer suggests that second problem solution stage of his model is where much of the researches is done (Mayer, 1983). Since it is easier to examine final solutions when compared to the complexities of how students actually conceive of the problem itself, perhaps the above mentioned statement is true. The work of Mayer informs and guides the inquiry of this study, providing the backbone for the dissertation since Mayer provides a model for understanding the entire problem solving process.

2.3 Key Research Concepts

The key research concept is usually the main idea of the research question. Following are the explanation of main ideas for this study. The main ideas involved for this study are: (a) mathematical problem solving, (b) Asian's perspective on problem solving, (c) problem solving models, (d) types of problems, (e) problem solving ability, and (f) the factors affecting problem solving ability.

2.3.1 Mathematical Problem Solving

The study of mathematical problem solving, usually been a vital area of investigation through researchers. Much emphasis used to be placed on the importance of problem solving in mathematics (Aydogdu & Ayaz, 2015). For example, in the NCTM Standards it is stated: "*Solving problems is not only a goal of learning mathematics but also a major means of doing so. In everyday life and in the workplace, being a good problem solver can lead to great advantages. Problem solving is an integral part of all mathematics learning*" (NCTM, 2000, p. 52). Learners will become better problem solver since they will discover for themselves by learning through problem solving. Brown (2008) referred to problem solving must allow learners to enhance mathematization skills that can come to be generative assets in life beyond the classroom.

Schoenfeld (2008) said that, there must be a goal, a blocking of that goal for the individual, and acceptance of that goal by the individual to be solving a problem. Schoenfeld stated the reason either that there is no blocking or no acceptance of the goal as what is a problem for one student may not be a problem for another. Schoenfeld situated a problem as having been given the description however do not yet have something that satisfies that description. A problem solver is described as a person perceiving and accepting a goal without an immediate means of reaching the goal by Schoenfeld.

Mathematical problem solving is a complicated cognitive exercise. Doing word problems, creating patterns, interpreting figures, developing geometric constructions and proving theorems are separate activities of mathematics problem solving as mentioned by Nfon (2013). While according to Polya (1945), understanding the problem, making a plan, carrying out the plan and looking back are several dynamic activities that defines the process of mathematical problem solving. The latter definition is utilized to the discussion in this review. Walden (2015) mentioned, the exploration of the unknown land where the journey and not the destination is the goal is linked to mathematical problem solving process. It entails active and greater order learning.

Besides, Kuzle (2013) defined mathematical problem solving as the process of deciphering a scenario mathematically, which usually includes some iterative cycles of expressing, testing, and revising mathematical interpretation and of sorting out, integrating, modifying, revising or refining clusters of mathematical ideas from quite a number of topics inside and beyond mathematics. Learners are required to use their prior knowledge to develop and generate mathematical solutions on their own from particular situation. As mentioned by Naveri, Pehkonen, Hannula, Laine and Heinila (2010), a situation is said to be a problem when a person must combine new data in a new way to solve the problem.

In mathematical problem solving it is believed that two types of thoughts, spatial inductive thought and verbal-logical deductive thought are important. (Lohman & Lakin, 2009). In order to derive answers on mathematical problem solving tests, students might apply a number of general strategies such as a solution rubric, a logical mathematical reasoning, a trial-and-error approach and an outright guess during the process. Mathematical problem solving was divided into four cognitive phases which are translating, integrating, planning and execution by Mayer (2003). According to Montague (2006), two stages such as problem representation and problem execution are the two stages involved in mathematical problem solving process. Both of them regarded representing the problem correctly as the basis for understanding the problem and making a plan to solve the problem.

A mathematical problem solver, had to be able to identify and manage a set of appropriate strategies such as heuristics, techniques, shortcuts and so on to solve the problem and not only required cognitive abilities to understand and represent a problem situation, to create algorithms to the problem, to process different types of information, and to execute the computation (Lohman & Lakin, 2009). Someone who received information and a goal without an immediate means to achieve the goal is a problem solver according to Luckin, Baines, Holmes & Holmes (2017). The mathematical problem solver must develop a base of mathematics knowledge and organize it, create an algorithm and generalize it to a specific set of applications, and use heuristics in order to achieve the goal.

Today, a number of strategies and skills are applied in solving a problem, as problem solving strategies are not specific to a man or woman problem. The concept of horizontal mathematization through which the learners come up with mathematical tools which can assist them to organize and solve a problem located in a real-life situation and vertical mathematization, which lets learners in finding shortcuts and discovering connections between principles and techniques and then applying these discoveries (Ekowati and Nenohai, 2016). These two processes of mathematization in problem solving can become a medium for developing pupils' mathematics and simultaneously help enhance independent decision-making and independence of thought and action if it is used efficiently.

2.3.2 Asians' Perspective on Problem Solving

From a focus on conceptual learning and problem solving in the late 1980s, Singapore Mathematics Curriculum (SMC) has evolved to include inquiry-based activities and a focus on creative and critical thinking in solving mathematical problems (Kheong, 2009). Problem solving is central to the SMC framework (Soh, 2008). Problem solving in the center and five interdependent, necessary elements (attitudes, metacognition, processes, concepts and skills) are surrounded in the Pentagon model which is still the central design on which the school mathematics curriculum in Singapore is based on. Problem solving is defined as acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real-world problems by SMC (Ministry of Education Singapore, 2012).

One aspect of the essential processes involved in acquiring and applying mathematical knowledge is heuristics as outlined in the Singapore Curriculum Framework. One of these heuristics, 'using a diagram or model', grew to be synonymous with Singapore mathematics, and researchers from the West coined it 'bar modelling'. This method helped learners to make sense of a problem by visualizing it, then organizing information that leads to the solution is what that was believed by the curriculum planners in Singapore. Its reliance on visual or concrete representations, which is based on the work of Bruner (2009) is what may have resulted from the model method. Successful implementation of the SMC is therefore contingent upon the participating teachers' beliefs about mathematics education and learning, and whether or not these beliefs are aligned with the philosophy underpinning the SMC, which is premised on a problem solving methods as espoused in the curriculum statements.

The course of study of mathematics for lower secondary school in Japan emphasized the "social need" after the Second World War. (Isoda, 2010; Nagasaki, 2007b). For instance, from 1950, mathematics textbook has contents consists of everyday-life related chapters such as "our school", "our food" and "our dwelling" for lower secondary school is "Mathematics for everyday (Nichijo no Suugaku, in Japanese)" (Souma, 1997). A project from "our food", for instance, encourages students to have a look at the components of the food Japanese people consume and its nutrients; how much rice does one individual eat? How much of the energy absorption is from one portion of rice? To resolve this type of problems, the textbook suggests the use of percentages and diagrams to display the factors and it shows how to calculate energy absorption using the four basic arithmetic operations.

Here in Japan, a tool to solve students' everyday-life related problem is mathematics. Solving text problems is transformed from solving these everyday life related problems as a goal of mathematics education (Nagasaki, 2011). Problem solving is described as an effective method to foster student's ability of logical thinking and explains a process of solving text problems which influenced by George Polya's four steps for solving mathematical problems in "How to solve it" in a plan for the curriculum in mathematics for elementary school (Polya, 1957).

Key Learning Areas, Generic Skills and Values, and Attitudes are the three interconnected components of Hong Kong Curriculum Framework. Collaboration, communication, creativity, critical thinking, information technology, numeracy, problem solving, self-management and study skills are included as Generic Skills and mathematics is one of the Key Learning Areas. Interestingly, the priority for 2001-2006 was communication, critical thinking and creativity is what that was indicated by the Basic Education Curriculum Guide (Education Department HKSAR, 2002). While Hong Kong has a coherent curriculum with high expectations, which values learning and training in basic skills and fundamental concepts, students are acknowledged to have low self-efficacy and poor attitudes, particularly in mathematics by teachers who have good pedagogical content knowledge (Wardlaw, 2008).

Furthermore, there is an examination orientation, the mathematics curriculum is dense and compact, and the teaching and learning is rushed. Problem-solving approaches in teaching mathematics for which teachers in Hong Kong are more aware of, but there remains limited evidence of implementation. Students are continued to lead by those teachers who try to engage students in discussion, mathematical reasoning and problem solving, on a predetermined solution pathway rather than allowing more open investigation and exploration of mathematical ideas (Mok, Cai & Fung, 2005). "*Whole-class teacher-pupils interaction and highly structured group/pair work*" is what that observations in Year 1 classrooms were characterized by (Mok & Morris, 2001). Little use of group work or open-ended questions suitable for exploratory problem solving in the lessons of Hong Kong secondary schools was noted more recently by Mok and Lopez-Real (2006).

The Philippine Basic Education observes the Kindergarten plus 12 years to complete its Basic Education Program is meant so, because the Philippine Basic Education observes the Kindergarten plus 12 years to complete its Basic Education Program. (Magayon & Tan, 2016). Low achievement scores of Filipino students in the National Achievement Test and the international test known as the Third

International Mathematics and Science Study (TIMSS) has reflected and caused the Philippine Basic Education to take this move (Magayon & Tan, 2016). A negative results by the use of Filipino as the first language of mathematics students in the Philippines has created discussions and some studies regarding it. In the study of Bernardo (2002), the effect of solving worded problems in Mathematics with the usage of the first language (Filipino) is identical as when the second language (English) is used. Problem solving and learning strategies are correlated where problem tests written in the first language can facilitate learning (Ong, Liao, and Alimon, 2009). This occurs when students are given problem-solving tests written in their native language. Filipino as students use more learning strategies, it is meant that more cognitive resources for comprehension of the problem test are able to be allocated than understanding the language in mathematical problems.

2.3.3 Problem Solving Models/ Theories

The enactive stage where concrete objects are directly manipulated (e.g., physically joining sets of blocks together), the iconic stage where mental representations and visualization become key (e.g., generating a picture of the blocks and joining them together), and thirdly, the symbolic stage where children can use symbols rather than images of the objects (e.g., using the symbols 3 + 4 to join sets together) are the three levels of representation proposed by Bruner (1965) as a developmental model which saw children progress. The development of problem solving models which highlight the importance of creating internal representations is the basis provided by Bruner's developmental theory.

A two-step model for solving mathematics word problems were discussed by Kintsch and Greeno (1985). This model was primarily based on the written words, the textual content base, and from this developing a summary problem model, which may want to be used to solve the problem. Reusser (1990) added an intermediary step to this model, which is the situation model. Reusser felt the initial model involved leaping from the text base to having a useful mathematical equation besides any thought for the situation of the model. The initial model was not effective for modelling to younger students or low achieving students, but may be effective for competent problem solvers or students who had a strong understanding of different types of word problems.

The situation model (Reusser, 1990) entails the problem solver creating an intellectual model of the situation of the problem which can then be used to create the mathematics information needed to solve the problem, referred to as the mathematical problem model by Reusser. Reusser's model placed higher emphasis on the context of the problem than preceding models and highlighted the importance of understanding the problem in its setting.

Linguistic elements needed to be given more emphasis than just the mathematical elements when solving mathematical word problems (English and Halford's, 1995). This view is consistent with Reusser's thoughts. A three-step approach for solving computational problems were proposed by them. As part of the process of comprehension, these three steps are the problem-text model where the reader constructs a superficial representation of the text. Next is the problem-situation model where the reader forms an intellectual representation of the problem. This includes the reader mapping the facts in the text onto a familiar situation that they have skilled or can relate to. Lastly, a mathematical model from which they can solve the problem is translated from the problem-situation model into a mathematical model by the reader.

They stated that, they will have difficulty in solving the problem if one of the three stages is incorrect. The quality of the problem-situation model is the key to success (English & Halford, 1995) with it being at this step where an intellectual model is created which brings the data from the text together and creates a relationship between the data in the problem and the lacking data (the answer) (Lucangeli, Tressoldi, & Cendron, 1998). The more accurate they can be in producing the mathematical model and then in solving the problem only if the better mental mode students can generate. This procedure of including the era of a mental model is frequently the step that is left out in the teaching of problem solving (English & Halford, 1995; Gervasoni, 1999) and it is left up to students to do this stage on their own, but many students are not in the position to do this efficiently (Arnoux & Finkel, 2010).

Besides, Montague has developed seven problem solving processes into a program for solving maths word problems called Solve It! Through research on cognitive strategy instruction. (Montague, 2003). Both the phases of the process and the functions required to carry them out can be illustrated by the integration of Montague's model with Mayer's model. Students reading the problem for understanding and then paraphrasing the problem in their own words is what problem translation consists of specifically. Student visualizes the problem by making a schematic representation is what problem integration is. The student then *hypothesizes* or makes a sketch to solve the problem and *estimates* a realistic reply all through the solution planning stage. Students compute or do the arithmetic, and then check to make sure everything is right is where the final stage, solution execution is.

Students are instructed to paraphrase the math word problem in order to monitor their understanding of the text in Solve It! (Montague, 2003). Unfortunately, students' ability to carry out this method was not explicitly measured in the *Solve It!* research. Impact of students' comprehension of word problems on their ability to solve them, is therefore impossible to be determined. On the other hand, a better understanding of how students solve problems, as well as a better understanding of how problems should be solved and how problem solving should be taught has been created by Schoenfeld (1982).

A list of characteristics of good problem has been created by Schoenfeld (1982). If the problem is easily understood, and does not require specific knowledge to get into, it can be approached from a number of different ways, should serve as an introduction to important mathematical ideas, and should serve as a starting point for rich mathematical exploration and lead to more good problems (Schoenfeld, 1982). Schoenfeld stated that the problem solving process is ultimately a dialogue between the problem solver's prior knowledge, his attempts, and his ideas along the way (Schoenfeld 1982). As such, the solution path of a problem is a rising and contextually based process. This can be viewed in Schoenfeld's (1982) description of a fine problem solver.

There are however, two consequences of Schoenfeld's work. The existence of problems for which the solver does not have "access to a solution schema" is the first of these. Schoenfeld acknowledges that problem solving heuristics are, in fact, personal entities that are dependent on the solver's prior knowledge as well as their understanding of the problem at hand. Hence, the problems that a person can solve through his or her personal heuristic are finite and limited. The second consequence is that if a person lacks the solution schema to solve a given problem she or he may still solve the problem with the help of luck.

According to Krulik and Rudnick (1987), problem solving is not an algorithm. "*The existence of a problem implies that the individual is confronted by something he or she does not recognize, and to which he or she cannot merely apply a model. A problem will no longer be considered a problem once it can easily be solved by algorithms that have been previously learned" (p. 3)*, (Krulik & Rudnick, 1987). Additionally, advocates of problem solving imply that algorithms are inferior models of thinking because they do not require thought on a high level, nor do they require deep understanding of the concept or problem. Algorithms only require memory and routine application. Further, they are not useful for solving new problems (Krulik & Rudnick, 1987).

Krulik and Rudnick argue that educators need to teach a method of thought that does not pertain to specific or pre-solved problems or to any specific content or knowledge. A heuristic is a process or a set of guidelines that a person applies to various situations. Heuristics do not guarantee success as an algorithm does (Krulik & Rudnick, 1987), but what is lost in effectiveness is gained in utility. They distinguish between algorithms and heuristics. Unlike employing an algorithm, using a heuristic requires the problem solver to think on the highest level and fully understand the problems. Krulik and Rudnick (1987) also prefer heuristics to algorithms because the latter only applies to specific situations, whereas a heuristic applies to many as yet undiscovered problems.

Krulik and Rudnick's problem solving heuristic consisting few stages namely read, is when one identifies the problem; explore, is when one looks for patterns or attempts to determine the concept or principle at play within the problem; select a strategy, is where one draws a conclusion or makes a hypothesis about how to solve the problem based on what he or she found in steps one and two; solve the problem where the selected method been applied to the problem; and review and extend where students verify his or her answer and looks for variations in the method of solving the problem (Krulik & Rudnick, 1987).

A part from that, Liljedahl (2008) stated that problems, are tasks that cannot be solved by direct effort and will require some creative insight to solve. Some problems take longer, especially at the beginning. But if we build “thinking classrooms”, then students learn content in a fraction of the time and retain the material better (Liljedahl & Sriraman, 2015). This framework is predicated on a desire to design a “classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together and constructing knowledge and understanding through activity and discussion. It is a space wherein the teacher not only fosters thinking but also expects it, both implicitly and explicitly” (Liljedahl & Sriraman, 2015, p.2).

To enhance the "thinking classroom”, Liljedahl employs AHA! experience into the classroom (Liljedahl, 2005). Simply put, the AHA! experience is the experience of having an idea come to mind with ‘characteristics of brevity, suddenness, and immediate certainty’ (Liljedahl, 2005, p.8). The AHA! experience encompasses all that leads up to illumination in the process of invention, discovery, and creativity with no consideration for the validity of the ensuing insight (Liljedahl, 2005). It begins with the initiation phase during which the solver attacks the problem intentionally and directly, relying on past experiences, intuition, and imagination in the selection and evaluation of directions of attack. This willful effort then wanes as the process gives itself over to the incubation phase during which time the conscious mind of the solver is distracted away from the problem. This is followed by illumination where an idea as to the solution or method towards a solution suddenly appears, filling the solver with a sense of certainty, relief, and joy (Liljedahl, 2005).

2.3.4 Types of Problems

Very often the terms 'problem' and 'non-routine problem' are used interchangeably in opposition to what is commonly called a routine problem. Routine problems usually one- or two-step problems which require the reproduction and application of a fixed solution procedure, whereas non-routine problems require productive thinking and can be approached in more or less sophisticated ways (Kolovou, Ven den Heuvel Panhuizen, & Bakker, 2011).

In some literatures, problems are named non-routine problems in order to highlight that when solving a problem, it requires a novel idea from the student (Milgram, 2007). In TIMMS 2011 framework, non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned (Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009). So if the student knows what method, algorithm, technique or formula to use for solving a task, then that task is not a problem, it is a routine exercise. Thus, it is possible that the same task is a problem for one student and it is an exercise for another one. Also, a problem is no longer considered a problem for that student, who already solved it (Ozturk & Guven, 2016).

While learning mathematics, pupils solve exercises and problems in order to deeper the acquired knowledge and develop their mathematical skills. Surya, Fauzi, and Shahputra (2017) stated the differences between exercise and problem, where an individual is faced with a problem when he encounters a question he cannot answer or a situation he is unable to resolve using the knowledge immediately available to him. A problem differs from an exercise in that the problem solver does not have an algorithm that, when applied, will certainly lead to a solution.

In order to be able to solve non-routine problems, students' ability in solving mathematical problems has to be developed. According to PISA evaluators, problem solving ability is an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious (PISA, 2012). Besides, problem solving ability involves the ability to use the acquired knowledge in a new way, the ability to learn new things which are useful for the problem and to discover new methods for the solution. So the transfer of knowledge and skills to new situation is essential. Creative thinking and critical thinking are important components of problem solving ability (Mayer, 1992).

In Romania, most of the problems given on national Mathematics tests require to apply formulas or algorithms. These problems have a mathematical formulation, they don't have any connection with real life (Marchis, 2013). Thus, teachers are tempted to solve many routine problems that their pupils obtain good results on these tests. But most of the pupils who pass these tests and even they get good marks don't have a good problem solving ability, they have just learned some techniques, methods or formulas and they know which one to use for a specific problem. Another reason, that teachers do not solve non-routine problems in the classroom is that they are not confident in their problem solving ability and they are not comfortable with handling pedagogical demands required for this type of problem solving activity (Marchis, 2013). A study on how primary school teachers in Romania develop their pupils' problem solving skills shows that three quarters of the teacher's guide pupils in order to understand the problem and encourage them for self-control during problem solving; only one third of the respondents encourage their students to solve the problems with more methods. Almost three quarters of the primary school teachers state that they give interesting, real-life problems in class.

2.3.5 Problem Solving Ability

To obtain the ability in problem solving, one must have a lot of experience in solving various problems. A question or a math problem is said to be a problem if the solution requires some creativity, understanding and thinking/ imagination of everyone facing the problem. The mathematical problem is usually a matter of the story, proving, create or find a mathematical pattern. According to the NCTM (2000), the problem solving ability is not just a goal of learning mathematics, but also a major tool to perform or work mathematically.

There are many interpretations about problem solving ability in mathematics. Among these, Polya's opinion is the most referred by many maths observers (Apulina & Surya, 2017). Polya defines that problem solving interpret as an attempt to find a way out of a difficulty to achieve a goal that is not so immediately achievable. Maths problem is a challenge when need a solution that requires creativity, understanding and original thought or imagination. Therefore, it can be concluded that problem solving is the ability in each person that varies depending on what is seen, observed, in mind and in their minds according to the incident in real life.

By Polya (1945), there are four steps in solving the problem, namely: (1) understand the problem: in this activity is to formulate: what is known, what is asked whether the information sufficient, condition (condition of) what should meet, restate the original problem in a more operational (solvable) way. (2) planning the solution: the activities carried out in this step is trying to find or recall issues you have solved that has similarities with the properties that will be solved, look for patterns or rules, draw up resolution procedures. (3) carry out the plan: the activities in this step are performed the procedures that have been created in the previous step to the settlement. (4) to look back the procedures and results of the settlement: activities in this step is analysing and evaluating whether the procedures applied and the results obtained are correct, whether there are other procedures that are more effective, whether procedures have created can be used to solve similar problems, or whether the procedures generalizations can be made.

The development of problem solving ability among school children has been a persistent goal of mathematics education community for over a century. There has been a fundamental shift in mathematics education from an emphasis on knowledge and procedural skills to a focus on the active process of extending and applying known concepts in new contexts and problem solving (Schoenfeld, 2008). A recent trend in Swedish elementary schools is an increasing interest to teach mathematics in an outdoor setting (Milrad, 2010). Teachers believe that this particular approach motivates the children more than solving problems in textbooks, thus offering new ways to introduce and work with mathematical problem solving. Teaching mathematics in an outdoor setting usually refers to school children solving practical problems using whichever forms of mathematics they find appropriate (Lovgren, 2007). It is concluded that in mathematics education, doing mathematics is not about reproducing ready-made mathematics, but about developing problem solving abilities.

Presenting a problem and developing the skills needed to solve that problem is more motivational than teaching the skills without a context. Such motivation gives problem solving special value as a vehicle for learning new concepts and skills or the reinforcement of skills already acquired Aydogdu and Ayaz (2015). Mathematical problem solving creates a context which simulates real life and therefore justifies the mathematics rather than treating it as an end in itself.

The National Council of Teachers of Mathematics recommended that problem solving be the focus of mathematics teaching because, they say, it encompasses skills and functions which are an important part of everyday life (NCTM, 1980). Furthermore, it can help people to adapt to changes and unexpected problems in their careers and other aspects of their lives. More recently the Council endorsed this recommendation with the statement that problem solving should underlie all aspects of mathematics teaching in order to give students experience of the power of mathematics in the world around them. They see problem solving as a vehicle for students to construct, evaluate and refine their own theories about mathematics and the theories of others.

Problem solving is, however, more than a vehicle for teaching and reinforcing the mathematical knowledge and helping to meet everyday challenges. It is a skill which can enhance logical reasoning. Individuals can no longer function optimally in society by just knowing the rules to follow to obtain a correct answer. They also need to be able to decide through a process of logical deduction what algorithm, if any, a situation requires, and sometimes need to be able to develop their own rules in a situation where an algorithm cannot be directly applied. For these reasons problem solving can be developed as a valuable skill in itself, a way of thinking rather than just as the means to an end of finding the correct answer (NCTM, 1980).

Many writers have emphasized the importance of problem solving as a means of developing the logical thinking aspect of mathematics. If education fails to contribute to the development of the intelligence, it is obviously incomplete. Yet intelligence is essentially the ability to solve problems: everyday problems, personal problems (Polya, 1980). Modern definitions of intelligence talk about practical intelligence which enables the individual to resolve genuine problems or difficulties that he or she encounters and also encourages the individual to find or create problems thereby laying the groundwork for the acquisition of new knowledge (Ferrando, Ferrandiz, Llor, & Sainz, 2016).

Those who are skillful in problem solving can experience a range of emotions associated with various stages in the solution process. Mathematicians who successfully solve problems say that the experience of having done so contributes to an appreciation for the power and beauty of mathematics, and the joy of banging your head against a mathematical wall, and then discovering that there might be ways of either going around or over that wall (Aydogdu & Ayaz, 2015). Whole school year observe changes in the pupils' ability to be able to transfer knowledge to unfamiliar situations.

2.4 Factors Affecting Mathematical Problem Solving Ability

The following sections will highlight the factors affecting mathematical problem solving ability. These factors include gender differences, teaching of mathematical problem solving, and use of visualization in the teaching and learning mathematical problem solving.

2.4.1 Gender Factor on Mathematical Problem Solving Ability

The last decades, psychologists have grappled with the nature and the origin of sex differences in behavior and cognition (Asante, 2010). Research on sex differences, its causes and consequences is not only of educational interest, but concerns general academic policy. Sex variations in mathematics performance and ability remain a concern as scientists searching for to address the underrepresentation of female at the highest levels of mathematics (Asante, 2010). Gender differences in mathematics problem solving learning are not clear during the elementary school years, but female begin to fall behind males during the intermediate school years, and they fall further behind during the high school years (Asante, 2010).

Despite the fact that female students work harder, and they are more eager to learn mathematics compared to male students (Brandell & Staberg, 2008), researchers, who have conducted studies about the students with different grades, point out that mathematics is a male domain (Brandell, Leder, & Nystrom, 2007; Brandell & Staberg, 2008; Sumpter, 2012). It is viewed that male students like mathematics and regard mathematics as an important part of their future and therefore are more successful in mathematics. On the other hand, the thought female students should study harder in mathematics leads to the thinking that the male students are more prone to mathematics than female students (Brandell & Staberg, 2008).

There are many factors contributes to gender difference in mathematical problem solving. In this area, spatial abilities were of major concern. Another line of research paid attentions to speed of problem solving, in which a Math-Retrieval speculation is still in hot argument among some scholars (Zhu, 2007). Arends and Richard (2008) defined that there are differences of cognitive ability between male and female. The males are more rational, has enthusiasm directed to the things that intellect, abstract, such that they are better in logical thinking and more critical. Meanwhile, the females are more accurate and detail in making decisions, her memory is better, more emotional, and interested more in verbal skills. Based on these findings, it can be assumed that females and males have different patterns of mathematical problem solving.

Mathematical success of male students depend on spatial (three dimensional) abilities, while success of female students depend on oral abilities (Pnina, Klein, Esther Adi-Japha, & Simcha Hakak-Benizri, 2010). Imitation reasoning is effective on success of female students in mathematics and female students tend to use standard methods in mathematical reasoning (Sumpter, 2016). Similarly, while female students learn standard algorithms to be successful in mathematics, male students can think creatively with their ability that comes from birth (Leslie, Cimpian, Meyer, & Freeland, 2015).

Since many mathematical problems on standardized tests are multi-step and require some systematic approach, students could arrive at a correct solution by choosing and combining a set of appropriate strategies. Strategy flexibility is important for successful performance on standardized tests (Elia, van den Heuvel-Panhuizen & Kolovou, 2009). Only focusing on test sores might not reveal gender differences in problem solving patterns, investigating gender differences in strategy use might shed some light on researching gender patterns of mathematical problem solving.

Different types of problems make different sets of demands on people's mathematical reasoning (Nunes, Bryant, Evans, Gottardis, & Terlektsi, 2015). Wolbers and Hegarty (2010) declare that gender factor creates a difference in terms of learning strategies used by students in mathematical reasoning. For example, male students tend to use metric calculation and focus on the main aspects of the subjects, while female students prefer to use classical and familiar strategies (Ruggiero, Sergi, & Iachini, 2008; Wolbers & Hegarty, 2010). Female students tend to use the strategies they have learned from their teachers while male students develop different strategies and think more abstract. In the calculations that requires addition and subtraction, female students calculate by using their fingers while male students are doing mental computation. Male students include many possibilities in their thinking and therefore, they try to use different strategies (Sumpter, 2016). This assertion is expressed by a participant teacher in Sumpter's (2016) research as male students push all the buttons on the calculator and think that this will help them.

Studies where the interaction between gender and problem solving beliefs are addressed point out that the variable of gender can have an effect on mathematical problem solving beliefs (Duatepe Paksu, 2008; Giovanni & Sangcap, 2010; Piskin Tunc & Haser, 2012; Saglam & Dost, 2014; Soyturk, 2011; Ugurluoglu, 2008). Giovanni and Sangcap (2010), for example, found a significant difference in favor of male students as a result of their study on mathematical problem solving beliefs of university students. Piskin Tunc and Haser (2012) examined the beliefs of primary school teacher candidates regarding mathematics education and concluded that the beliefs of teacher candidates regarding mathematics education differ according to gender. In the study of Soyturk (2011), he concluded that the beliefs of primary school teacher candidates regarding mathematical problem solving significantly differ in favor of female students.

2.4.2 Teaching Problem Solving

There are so many devices for effective teaching and an effective technique can ensure effective learning. It is being felt that there should be new methods of teaching and learning (Unal, 2017). Teachers have many roles, from planning classroom activities, to instructing, disciplining, motivating and guiding students. Teachers are also expected to both use teaching techniques effectively and to have modern management skills in classroom environments in order to establish learning that can be defined as permanent changes in behavior (Kahyaoglu & Yangin, 2007). Those elements which most affect students' learning and performance are not only teachers' attitudes, choice of methodology, and the content of curriculum, but also students' socioeconomic background, behavior, and personal characteristics (Santos-Trigo, 2007; Tatar & Dikici, 2008). Effective teaching, therefore, must place equal emphasis on teacher, student, environment, curriculum and different factors.

The following principles may provide guidance for effective classroom practices in supporting the teaching of mathematical problem solving. First, it is recommended that teachers build on children's natural interest in mathematics, and on their intuitive and informal mathematical knowledge. They should encourage inquiry and exploration to foster problem solving and mathematical reasoning (Guclu & van Gerven, 2014). Second, teachers are expected to use both formal academic lessons and daily activities as natural vehicles for developing children's problem solving ability. Providing a mathematically rich environment and incorporating the language of mathematics throughout the school day could be effective. Third, it is encouraged that teachers establish partnerships with parents and other caregivers in order to support children's mathematical development (Soylu, 2009).

Besides, to develop into expert problem solvers, students need to first encounter problems that engage them and give them opportunities to develop the skills they need to learn. The types of problems that benefit students the most are the ones that perplex them. For a problem to have the greatest benefit for students, it must be challenging enough to require the regulation of cognitive and metacognitive strategies. One way in which teachers can improve students' problem solving skills is by having them focused on processes rather than outcomes. A psychology professor, Langer, points out that thinking about outcomes often inhibits students in problem solving. A process orientation questioning "How do I do it?" instead of "Can I do it?" helps students actively think of different ways in which a problem might be solved instead of focusing on the many possibilities for failure (Stice, 2011).

Teaching problem solving is not only about providing a model and real problems to students, but also about the guidance of the teacher (Jose, 2017). Teachers need to exemplify and discuss their actions and thoughts as they solve a problem and focus not only on what is being done but additionally on why the choice was made. Besides modelling teacher's own problem solving process and asking questions, orchestrating whole class discussions can advance mathematical learning in cognitively demanding tasks (Stein, Engle, Smith, & Hughes, 2008). A model for discussion facilitation consists of the following 5 practices: (1) anticipating likely student responses to cognitively demanding mathematical tasks, (2) monitoring students' responses to the tasks throughout the explore phase, (3) selecting particular students to present their mathematical responses during the discuss-and-summarize phase, (4) purposefully sequencing the student responses that will be displayed, and (5) helping the class make mathematical connections between different students' responses and between students' responses and the key ideas. Bor-de Vries and

Drijvers (2015) underline these suggestions and add for example the importance of developing a safe learning surroundings and giving sufficient time to think. These suggestions led to the third design criterion where the teacher should make the problem solving method explicit, ask questions and fade the use of questions, and orchestrate whole class discussions in a safe learning environment.

Even though problem solving is described (Drijvers, 2015; Van Streun, 2014), it is not yet known how problem solving skills of students can be promoted in daily practice and how teachers can implement teaching problem solving in their lessons. This problem is very relevant nowadays. According to Doorman, Drijvers, Dekkar, van den Heuvel-Panhuizen, Lange, and Wijers (2007) *"problem solving in secondary mathematics education has only a marginal position*" (p. 411) and work needs to be done. In addition, society, focused on knowledge, is increasingly shaped by the rapid emergence of ICT. This development suggests that other skills and capabilities are necessary to function in society, the so-called '21st century skills'. Problem solving is one of these necessary skills (Van den Oetelaar & Lamers, 2012).

The issue of intellectual authority is central to the comparison between how mathematics is known in school and how it is known in the discipline. In the classroom, the teacher and the textbook are the authorities, and mathematics is not a subject to be created or explored. In school, the truth is given in the teacher's explanations and the answer book; there is no zig-zag between conjectures and arguments for their validity, and one could hardly imagine hearing the words maybe or perhaps in a lesson. Knowing mathematics in school therefore comes to mean having a set of unexamined beliefs, whereas Polya (1945) suggest that the knower of mathematics needs to be able to stand back from his or her own knowledge, evaluate its antecedent assumptions, argue about the foundations of its legitimacy, and be willing to have others do the same.

Teachers tell students, whether their answers are right or wrong, however few teachers engage students in a public analysis of the assumptions that they make to get their answers. Even when teachers give an explanation rather than simply stating a rule to be followed, they do not invite students to have a look at the mathematical assumptions behind the explanation, and it is unlikely that they do so themselves (Coe, Aloisi, Higgins, & Major, 2014). In conventional mathematics lessons, students believe that the teacher knows which answers are right, and teachers believe that the paths to these answers can be found in rules in books. That teachers and students think this way about mathematical knowledge and how it is acquired is both a cause and a logical consequence of the ways in which knowledge is regarded in school mathematics lessons.

Bor-de Vries and Drijvers (2015) investigated, by working together with several teachers, what a teacher can do to enhance students' problem solving skill. This resulted in practical tips for and characteristics of suitable learning activities and teacher guidance. Bor-de Vries and Drijvers pointed out that in choosing or designing activities it is important to connect to prior knowledge and experience of students and to differentiate if necessary. They named the following characteristics of activities that are problems as the activity has a surprising element, the method to solve the problem is unknown and asks for creativity, the activity is not too much structured, multiple steps are necessary to obtain a solution and every student has to be able to solve the problem to some extent. Concrete and practical tips for designing such activities are adapting tasks from school textbooks by leaving out sub questions, forwarding more challenging tasks, looking critical at the context of an activity or task, variating in different activities, and designing a task with knowledge and skills from previous chapters. These suggestions led to the second design criterion which is learning activities in the lessons need to be non-routine and experienced as problems by the students.

2.4.3 Visual Representation and Mathematical Problem Solving

Representation is one of the process standard should enable students to know and do from kindergarten to K-12 (Istadi, Kusmayadi & Sujadi, 2017). The representation standard should enable students to create and use representations to organize, record, and communicate mathematical ideas. Representations can be expressed in the form of visual, verbal, and symbolic. Visual representations consist of illustrate, show, or work with mathematical ideas using diagrams, pictures, number lines, graphs, and other maths drawings. Verbal representations include using language (words and phrases) to interpret, discuss, define or describe mathematical ideas, bridging informal and formal mathematical language. Symbolic representations include recording or working with mathematical ideas using numerals, variables, tables, and other symbols (Huinker, 2015; NCTM, 2000). Other representation standard should enable students to select, apply, and translate among mathematical representations to solve problems. For instance, translations between mathematical problems and mathematical representations as well as translations among mathematical representations. Thus, students must be able to move flexibly in between form of representations (NCTM, 2000; Huinker, 2015). It suggests using representations could help to facilitate when solving problem more efficient. For instance, visual representation could facilitate in solving algebraic problems than using symbolic representation, despite previous students were more likely to prefer symbolic representations over visual representations (Mielicki, Marta, & Wiley, 2016).

Some benefits of representations could motivate students' mathematical ideas, especially in problem solving ability (NCTM, 2000; Sajadi, Parvaneh, & Rostamy-Malkhalifeh, 2013; Yee & Bostic, 2014). Besides in solving the problem, representations useful in understanding the abstract concepts of mathematics. For instance, in transition between arithmetic and algebra by geometric representations as well as in teaching factoring second-degree polynomials (Cabahug, 2012; Panasuk & Beyrnevand, 2011). Representations are not only to goal curriculum standard, however an important aspect to improving students' educational value as follows: (1) to help students consolidate their understanding and improve skills, (2) to help teachers and students enrich their concept of mathematics and mathematical teaching, (3) to help students overcome the psychological barriers, (4) to help teachers assess students' learning result, and (5) to help teachers improve their own literacy (Zhe, 2012). Research related to translation among mathematical representations showed students were successful than the pre-service teachers in understanding of functions, likewise in representing the fractions on number lines through other representations (Bannister; Biber, 2014). From the research, representations were used in analyzing the understanding of prime numbers, making generalizations on algebra material, as well as in representing the law of cosines without using the Pythagorean theorem and trigonometry (Zeljic & Dabic, 2014).

NCTM (1989) advised that students needed to understand and improve mathematical concepts and operations. In other words, using different representations of information in classroom strengthens learning methods and improves their successes by the referral to more than a few sources of information (NCTM, 2000).

Representing information visually is considered an efficient representation process in mathematics education, especially in problem solving (Guler & Ciltas, 2011). The importance of using visual representations in mathematics education can be explained with the contribution it makes to the development of understanding and intuitional perspectives. The use of visual representations in the problem solving process may not always be effective and in some situations it may even lead to incorrect solutions (Guler & Ciltas, 2011), however creating visual representations which emphasize spatial relationships in the process of solving mathematical problems may contribute to problem solving success. Therefore, a teaching method which is directed to create this kind of visual representations in the process of problem solving is important for students. It is known that mathematical problems have greater than one solution. The solutions offered by teachers significantly affect the solutions which their students are going to use in solving similar problems. Similarly, the preferences of teachers for problem solving affects the choice of assistive instruments used in these solutions and the creation of the figures representing the situation expressed in the problem.

The level of learning process is divided into three which include enactive, iconic and symbolic. Enactive is the crucial level of visualization which performs the connection between the practices and formal level of understanding or in other word the mediator of the communication (Deliyianni, Monoyiou, Elia, Georgiou, & Zannettou, 2009). Diagram or picture that the student use or construct to enhance their understanding will automatically generate a big picture in their mind to dig up the solution of the problem (Deliyianni et al., 2009). It does not only help them to establish a relationship of mathematical images but also an effective way in solving any problem syntactically, semantically and pragmatically perspective (Hamidreza

Kashefi, Nor Athira Alias, & Mohamad Fahmi Kahhar, 2015). The well-form use of pictorial signs will make a good perspective of syntactic, meaningful used of pictorial signs will show semantic perspective while a pictorial signs that being used to think, communicate and learn will give a pragmatic perspective.

Word problem solving is one of the important components of mathematics problem solving which incorporate real-life problems and applications (Ahmad, Tarmizi, & Nawawi, 2010). In order to master mathematical word problem solving, they need the support of thinking strategies that will govern the interpretation and manipulation of information through language skills and visual capabilities in working memory (Abdullah, Zakaria, & Halim, 2012). This is because mathematical word problems include worded items and their structure makes them difficult to solve. The problems need to be analyzed and interpreted as the basis for selection and decision making. To achieve this goal, students need to be guided and exposed to strategic thinking and representation skills so that mathematical problem solving skills can be achieved effectively.

Solving problem using word seems to be very difficult if the students cannot do relation between the known and unknown mainly when the student faced troublesome to understand the problem text given (Boonen, van der Schoot, van Wesel, de Vries, & Jolles, 2013). The comprehension of the student can also refine by use of visualization to simulate the student thinking varies rather than focusing on symbolism and formalism approach (Lavy, 2007). The effective tools in learning mathematics is through visual which provide an alternative mass resource almost throughout the media as the representation of the simplified version of mathematical language especially in delivering the process of solving problem (Lavy, 2007). This technique has been widely used in Singaporean and Japanese School curricular focusing on the elementary school as the basis of exposure to the mind of creativity and criticist (Murata, 2008). As the result, communication of mathematical ideas using visual such as tape diagram and simple picture aiding the student in connecting ideas across the problem given (Ho & Lowrie, 2014). Moreover, the improvement in tackling techniques of the problem in mathematics improved the skill of thinking among students.

2.4.4. Use of Visual Representation in Teaching and Learning Problem Solving

Theoretical perspectives on teaching students mathematics increasingly emphasize that teachers should possess adequate mathematical knowledge for teaching (MKT), namely the collective mathematical knowledge, skills and attitudes needed to support student learning (Ball, Thames, & Phelps, 2008). This consists of both pedagogical content knowledge, knowing a variety of effective ways to present and represent mathematical content, taking account of learner characteristics and common misconceptions and difficulties in learning the subject matter, as well as specialized mathematical subject matter knowledge as a teacher needs to teach particular content (Ball, Thames & Phelps; Hill, Ball & Schilling, 2008).

With respect to visual representations, teachers need to recognize what is involved in using particular representations and when they are appropriate to use (Ball, Thames & Phelps, 2008). In the context of the present study, teachers need to be aware that visual representations should be used to support the first phase of the word problem solving process which is problem comprehension and that arithmetical representations are only appropriate in the problem solution phase. Moreover, teachers should be able to use more than one representations and link different representations to each other and to underlying ideas (Ball, Thames & Phelps, 2008; Dreher & Kuntze, 2015).

Research shows that the effectiveness of teachers' mathematical instructional practices depends largely on the quality of teachers' MKT (Ball, Thames & Phelps; Hill, Ball & Schilling, 2008). Unfortunately, there is relatively little literature about teachers' understanding of and ability with visual representations, and we have found no literature that specifically addresses how teachers teach students to construct visual representations to support mathematical problem solving process. It cannot be assumed that teachers are able to do this, indeed, visual representations are reported to be problematic for teachers. For example, Orrill, Sexton, Lee and Gerde (2008) reported that middle grade mathematics teachers in the US are uncomfortable with visual representations, and that this relates to their incomplete knowledge about using and interpreting such representations.

Besides, Turner (2008) discovered that beginning elementary school teachers in the UK frequently have challenge in choosing and using visual representations such as number lines and hundred squares, and that their choices are based on superficial attractiveness rather than the suitability of the representations for the mathematics they prefer children to learn. In Germany, Dreher and Kuntze (2015) discovered that even secondary school mathematics teachers do not fully understand the role and use of different forms of visual representations for learning about and teaching fractions.

In the present case, in which visual representations should be used to support non-routine mathematical problems, teachers may not know what kind of representations should be made or in which phase of the problem solving process to use them. Teachers may also have difficulty in developing visual representations accurately which is correctly and completely. Incorrect and/or incomplete visual representations are referred to as inaccurate visual. Furthermore, research shows that it is more effective to teach students to construct their own visual representations than to provide them ready-made, as this contributes to skill adaptivity (Boonen, Van Wesel, Jolles, & Van der Schoot, 2014).

Representations help students assign meanings to the mathematics principles they are learning, before they learn to use formal notation and work with abstract ideas (Murata 2008; Murata, Aki & Sailaja Kattubadi 2011). Thus, careful selection and use of representations help facilitate students' learning processes by supporting a variety of mathematical practices. By representing ideas, teachers and students create a common space to carefully analyze and critique their thinking more concretely, constructing and revising their problem-solving processes together

A representation-rich classroom also invites more students to take part in mathematizing, as a result creating the culture of equality. It will grant access to students who may be otherwise marginalized for one reason or another such as the second-language speakers, students with less financial resources, and so on, and help them engage in mathematics in ways that make sense to them (Fuson, Karen & Aki Murata, 2007). By visually representing their ideas and seeing others' ideas, students have improved opportunities to explore mathematics. Also, by participating in discussions, students will come to understand their own thinking better and learn how to articulate it.

Representation of problems requires careful thinking and planning on the teacher's part to make sure each representation used is meaningful to students (Murata & Stewrat, 2017). Also, although multiple representations are usually helpful, make sure they relate with one another in ways that highlight core mathematics concepts, instead of unintentionally distracting students' attention. In planning lessons, teachers may anticipate possible student responses and, on the basis of the set of responses, design a number of possible representations to guide student learning in the lessons. Thinking about how to emphasize core mathematics principles in the relationships is also important. Stewart's lessons exemplify how to support the students' sense-making method of place value, one of the most challenging math concepts in elementary school classrooms, through the effective use of multiple representations.

Thus, teaching needs to focus on the construction process like how to make the representation, rather than offering a representation as a given entity. Finally, teachers have to encourage students to use visual representations in a diverse, adaptive/flexible and functional way. This refers to being able to use different kinds of visual representations and to switch between them such that the representation fits the structural characteristics of the problem and is useful for helping to solve it. Indeed, the ability to deal flexibly with multiple representations and move adaptively between them is seen as being essential for successful mathematical problem solving (Acevedo Nistal, van Dooren, Clarebout, Elen, & Verschaffel, 2009; Dreher & Kuntze, 2015). However, as referred to above, these aspects are frequently problematic for teachers (Dreher & Kuntze, 2015; Orrill et al., 2008; Turner, 2008). In short, is important to establish visual representation in mathematical classroom as it is critical to the viability of this method for supporting mathematical problem solving in schools, as well as providing important indications for teacher professionalization programs.

2.4.5 Gender as Factor that Contributes to Visual Representation

Spatial ability has a vital role in our daily interaction with environment, such as navigation, recognizing and manipulating objects, academic tasks, and recalling locations. Spatial ability is one of the several relatively autonomous human intellectual competencies and is considered essential in representing information in problem solving. A positive relationship between success in mathematics and spatial ability is often emphasized (Gunderson, Ramirez, Beilock & Levine, 2012). NCTM (2000) emphasized the importance of spatial abilities in mathematics education and noted that spatial ability was important and included 2D and 3D objects' mental representation and manipulation with the perception of different perspectives of the objects.

Gender differences in spatial ability are well documented in the scientific literature (Halpern, 2007). Early researchers in this area have traditionally reported a male advantage over female on standard tests of spatial ability, at least after adolescence. Females are much less likely to get high scores in Mental Cutting Test (Nemeth, Soros, & Hoffmann 2007). Turgut and Yilmaz (2012) on the other hand stated that boys have a higher spatial ability than girls which may be caused by biological and/or environmental factors. And the related literature shows that there is a significant male advantage on mental rotation tasks at every age (Pietsch & Jansen, 2012). Turgut and Nagy-Kondor (2013) found that there is not a significant difference between male and female groups' scores in spatial visualization of prospective elementary mathematics teachers. Although the existence of gender differences in cognitive ability is still debated among the researchers, the psychological and the social sciences studies widely acknowledge that males and females differ in spatial ability (Khine, 2016). Indeed, it is one of the most robust and consistently found phenomenon of all cognitive gender differences (Halpern, 2011). While there is individual variability within each gender, on average males score higher than females on tests that measure visual-spatial ability.

Evolutionary psychology seeks to make sense of gender differences in human cognition by considering the role of evolutionary selection arising from the division of labor between men and women in traditional hunter-gatherer societies (Khine, 2016). Men would be required to travel long distances in order to track and hunt animals, a task requiring strong spatial perception and navigation skills (Buss, 2015). In contrast, women fulfilled the role of the gatherer of more local food and assumed childrearing duties. This role had less need for spatial proficiency but emphasized other adaptive traits such as nurturing and fine-motor skills. Over successive generations, evolutionary forces may have developed sex-specific proficiencies in spatial ability, giving males a strong advantage over females with such tasks (Buss, 2015).

Support for the position of evolutionary psychology comes from cross-cultural studies of cognitive gender differences. A large body of research has shown that spatial differences are consistently found in all countries (Janssen & Geiser, 2012). Furthermore intelligence - including spatial ability is a highly heritable trait (Sternberg, 2012), meaning that it can be passed down from one generation to the next. Nevertheless, some researchers question the validity of evolutionary and genetic factors, arguing that at the genetic level men and women are identical with the exception of the sex chromosome (Hyde, 2014). Such arguments do not take into account other biological differences. For instance, the expression of sex hormones might be an important factor linked to genetic and evolutionary gender differences (Hines, 2015a).

Males outperformance of females on measures of visuospatial abilities has been implicated as contributing to gender differences on standardized exams in mathematics and science (Halpern, 2007). An evolutionary account of gender differences in mathematics and science supports the conclusion that although gender differences in math and science performance have not directly evolved, they could be indirectly related to differences in interests and specific brain and cognitive systems.

Sex hormones such as androgens and oestrogens could be a biological explanation for gender differences in spatial ability. Berenbaum and Beltz (2011) mentioned that production of sex hormones greatly increases with the onset of puberty, and is associated with a range of psychological and behavioural changes as well as differences in brain development. While both males and females produce these sex hormones to some degree, greater androgen production is typically found in males while greater estrogen and progesterone production is present in females. Such a difference starts early, with differences in testosterone concentration of foetuses found as early as eight weeks gestation (Hines, 2010).

2.5 Related Past Researches

Previous researches are needed to consider limitations, inconsistencies or addressing conclusions made by others in relates of this study. The previous researches involved in this study are on: (a) mathematical problem solving ability, (b) teaching of mathematical problem solving ability, and (c) gender factor on mathematical problem solving ability.

2.5.1 Previous Research on Mathematical Problem Solving Ability

The development of problem solving ability among school children has been a persistent goal of mathematics education community for over a century, however, the issue of how develop problem solving skills among learners continues to be a major dilemma. Apulina and Surya (2017) analyzed the student's mathematics problem solving ability of class 6 SMP Negeri 4 Pancurbatu on Quadrilateral. The type of the study is qualitative descriptive. The subject of the study was 31 students of class 6-1 SMP Negeri 4 Pancurbatu 2016/2017 Academic Year. The instruments of the study were Mathematics Problem Solving Ability Test. The result showed that the percentage of students' problem solving ability in first indicator of problem solving was 75.08%, the second indicator was 66.12 %, the third indicator was 29.03%, and the fourth indicator was 24.19%. The results showed that most students of 6 grade of SMP Negeri 4 Pancurbatu have not been able to solve the problem given to fulfil all phases of the indicators of the problem solving abilities. They are unable to solve the problem well.

Besides, Effat Alvi and Haleema Mursaleen (2016) examines the beliefs, processes and difficulties associated with mathematical problem solving of Grade 9 students in his study. Consistent with the constructivist notions, he framed the study within Mayer's work who approached problem solving as a process that is largely influenced by problem representation and problem solution. He conducted semi-structured interviews with 12 Grade 9 students and further engaged them in solving five different problems. The findings revealed that students struggle with solving mathematical problems due to five major reasons. These include making sense of the problem statement, conceptual understanding, contextualization and visualization of the problem, and critical thinking and reasoning.

In the study conducted by Angateeah in 2017, the cognitive processes undergone by Mauritian students who have difficulties in solving word problems were explored. A questionnaire of three non-routine word problems was administered to 190 grade 8 students, of different abilities. 15 students were interviewed to gauge the cognitive processes used while solving the problems. Montague's (2003) framework for problem solving was used to analyse the data. All students could read the problems. High Achievers (HA) are wrong due to careless errors. While HA demonstrates good problem solving skills, some exhibit overconfidence. Average Achievers (AA) suffer from procedural errors while Low Achievers (LA) face difficulties mainly in visualizing and representing the problem.

Tambychik, and Meerah (2010) conducted a study in Malaysia that discuss the major mathematics skills and cognitive abilities in learning that caused the difficulties in mathematics problems solving among students from students' point of view. The study was carried out on three focused group samples that were selected through purposeful sampling. A mixed qualitative and quantitative approach is used in order to have a clearer understanding. Apart from the questionnaire given, focused group interviews were carried out. Interviews were recorded and transcribed. Data finding was analysed descriptively. Data findings showed that respondents lacked in many mathematics skills such as number-fact, visual-spatial and information skills. Information skill was the most critical. The deficiency of these mathematics skills and also of cognitive abilities in learning inhibits the mathematical problem solving. This understanding on how the deficits influenced the problem solving is expected to give effective guidelines in preparing diagnostic instruments and learning modules in order to develop the mathematics skills.

Bahar (2013) conducted a study to investigate the influence of cognitive abilities on mathematical problem solving performance of students. The author investigated this relationship by separating performance in open-ended and closed situations. Multiple regression analyses were performed to predict students' problem solving performance. Intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability constituted independent variables whereas mathematical problem solving performance scores in closed and open-ended problems were the dependent variables. The author found that mathematical problem solving performance (MPSP) in closed problems was correlated significantly with cognitive variables, including mathematical knowledge, quantitative ability, verbal ability, general intelligence, general creativity, and spatial ability. Similarly, MPSP in open-ended problems was correlated significantly with several cognitive abilities, including verbal ability, general creativity, spatial ability, mathematical knowledge and quantitative ability. The author concluded that closed and open-ended problem requires different cognitive abilities for reaching correct solutions. In addition, when combining all of these findings, the author proposed that the relationship between cognitive abilities and problem solving performance may vary depending on the structure (type) and content of a problem.

Metacognitive skills play an important role in solving mathematical problems. However, there is a lack of empirical studies on the role of metacognitive skills in solving mathematical problems, particularly non-routine ones. Therefore, Abdullah, Rahman and Hamzah (2017) conducted a study to identify students' metacognitive skills and the impact of such skills on non-routine mathematical problem solving. By using a quantitative method, a total of 304 students in Johor Bahru district were involved in the study. A Self-Monitoring Questionnaire (SMQ) and a mathematical test were used in data collection. Data were analyzed using descriptive and inferential statistics such as frequency, percentage, mean, the Mann-Whitney U test, and the Kruskal-Wallis H test. Results showed that the level of the students' performance in solving non-routine mathematical problems was very low. There was also a significant difference in the metacognitive skills among students with different performance levels in solving non-routine mathematical problems, and the study concluded that these metacognitive skills should be emphasized in the problem solving process.

Moreover, Delima (2017) investigated if there is influence of problem solving ability of students' mathematical thinking, and to know how strong problem solving ability affect students' mathematical thinking. This research used descriptive quantitative method, which a population is all students that taking discrete mathematics courses both in the department of Information Systems and department of mathematics education. Based on the results of data analysis showed that there is an influence of problem solving ability to students mathematical thinking either at the department of mathematics education or at department of information systems. In this study, it was found that the influence of problem solving ability of students' mathematical thinking which take place at the mathematics education department is stronger than at information system department. This is because, at mathematics education department, problem solving activities more often performed in courses than at department of information system.

Jose (2017) conducted a study which aimed to learn how problem solving skills of students in mathematics education can be promoted. He focused on explicitly providing a problem solving model to students, problems which have to be experienced as real problems, and the role of the teacher in guiding the problem-solving process of students. A design-based research approach is adopted to develop a series of 36 lessons during nine weeks. The designed lessons were taught in five grade 8 classes by three teachers, including the researcher herself. A pre- and post-test was conducted with 121 students. Also, mini-interviews with groups of four students about the awareness of the students' own problem solving process were carried out each week. Results show that the problem solving skills of students significantly improved and the awareness of students' own problem solving process and skills increased. Therefore, it can be concluded that a well-implemented multidimensional approach, focused on explicitly providing a problem-solving model, activities that are real problems, and the guidance of the teacher, promote problem-solving skills of students in secondary mathematics education.

Sala and Gobet (2016) conducted a study to see if playing chess enables children to improve their mathematical problem solving ability. They ran two experiments that used a three-group design including both an active and a passive control group, with a focus on mathematical ability. In the first experiment involving 233 students, a group of third and fourth graders was taught chess for 25 hours and tested on mathematical problem solving tasks. Participants also filled in a questionnaire assessing their meta-cognitive ability for mathematics problems. The group playing chess was compared to an active control group (playing checkers) and a passive control group. The three groups showed no statistically significant difference in mathematical problem solving or metacognitive abilities in the post-test. The second experiment involving 52 students broadly used the same design, but the Oriental game of Go replaced checkers in the active control group. While the chess-treated group and the passive control group slightly outperformed the active control group with mathematical problem solving, the differences were not statistically significant. No differences were found with respect to metacognitive ability. These results suggest that the effects (if any) of chess instruction, when rigorously tested, are modest and that such interventions should not replace the traditional curriculum in mathematics.

The above findings support the problem statement of this study saying that students having difficulties in solving mathematical problems. It is due to the cognitive ability of the students. Students' performance level in solving mathematical problems are different based on their cognitive ability level. Therefore, it is worth studying on problem solving due to its importance in current education system.

2.5.2 Previous Research on Teaching Mathematical Problem Solving

In the last three decades, there has been a great deal of educational research on mathematical problem solving, and this research has deepened our understanding of problem solving and related pedagogical issues immensely. A study conducted by Prabawanto (2017) aims to investigate the enhancement of students' mathematical problem solving through teaching with metacognitive scaffolding approach. This research used a quasi-experimental design with pretest-posttest control. The subjects were pre-service elementary school teachers in a state university in Bandung. In this study, there were two groups: experimental and control groups. The experimental group consists of 60 students who acquire teaching mathematics under metacognitive scaffolding approach, while the control group consists of 58 students who acquire teaching mathematics under direct approach. Students were classified into three categories based on the mathematical prior ability, namely high, middle, and low. Data collection instruments consist of mathematical problem solving test instruments. By using mean difference test, two conclusions of the research were obtained. First, there is a significant difference in the enhancement of mathematical problem solving between the students who attended the course under metacognitive scaffolding approach and students who attended the course under direct approach, and second, there is no significant interaction effect of teaching approaches and ability level based on the mathematical prior ability toward enhancement of students' mathematical problem solving.

Ali, Hukamdad, Akhter and Khan (2010) conducted a study to investigate the effects of using problem solving method on students' achievement in teaching mathematics at elementary level. Pre-test post-test design was used in the study. Results were analyzed using mean, standard deviation and t-test. From the findings it was observed that the use of problem solving method enhanced the achievement of the students in mathematics. The result showed that there was significant difference between the effectiveness of traditional teaching method and problem solving method in teaching of mathematics at elementary level. The study recommended that the teachers should be encouraged to employ problem solving method in teaching mathematical concepts like set, information handling and geometry. Regular training, workshops and seminars should be arranged for teachers to give them knowledge and understanding of problem based learning.

Similar study was conducted by Perveen (2010) to determine the effect of the problem solving approach on academic achievement of students in mathematics at the secondary level. The secondary school students studying mathematics constituted the population of this study. The students of 10th class of Government Pakistan Girls High School Rawalpindi were selected as a sample for the study. Sample size consisted of 48 students who were equally divided into an experimental group and a control group on the basis of pre-test. Treatment of the planned problem solving approach is the guideline of Polya's (1945) heuristic steps of the problem solving approach. After the treatment, post-test was used to see the effects of the treatment. A two-tailed t-test was used to analyze the data, which revealed that both the experimental and control groups were almost equal in mathematics base at the beginning of the experiment. The experimental group outscored the control group significantly on the post-test.

Rudd (2010) conducted a study to examine the effects of teaching heuristic, problem solving, reasoning and strategies on seventh grade students' perceptions and level of achievement in mathematics. The researcher examined students' ability to solve non-routine problems in novel contexts. Two seventh-grade math classes participated in the study. One of the classes acted as a control group and received their standard problem-solving instruction. The other class acted as the intervention group which received explicit instruction on heuristic problem-solving reasoning and strategies. The results of this study showed that the students were taught the heuristic reasoning and problem-solving strategies significantly improved in their level achievement compared to those that were not. The results also showed that for the group of students that received the intervention, there was a significant improvement in their positive perception of their problem solving abilities.

An action research study was conducted by Lopez (2008) to examine the influence mathematical strategies had on middle school students" mathematical ability. The purpose of this action research study was to observe students mathematical abilities and to investigate whether teaching students' problem solving strategies in mathematics will enhance students' mathematical thinking and their ability to comprehend and solve word problems. The study took place in an urban school in Orlando, Florida in the fall of 2004. The subjects were 12 eighth grade students assigned to the intensive math class. Quantitative data were collected. Students' took a pre and post-test designed to measure and give students practice on mathematical skills. Students worked individually on practice problems, answered questions daily in their problem solving notebook and mathematics journals. Results showed the effectiveness of the use of direct instruction and problem solving strategies on at-risk students.

Besides, Surya and Syahputra (2017), investigated on the relevant strategies to improve students' mathematical problem solving ability. Their study aimed to determine the increase in mathematical problem solving ability of students taught with problem-based learning model is better than the increase in mathematical problem solving ability of students taught by conventional teaching. Research conducted in the form of quantitative research with experimental approach to true experimental design randomized control group pretest-posttest. The research sample was determined by random technique that became the experimental class (VII7) and the control class (VII8). The test results of quantitative data shows that the normal distribution of data, which can be analysed by statistical one sample t-test at $\alpha = 0.05$ significances, based on data analysis of N-Gain score of mathematical problem solving ability obtained = 3. 7 and = 1. 67 or > is 3. 7 > 1. 67, it means that $H_0$is rejected and consequently H_1 is accepted. From the results of this study concluded that the increase in mathematical problem solving ability of students who received the application of problem-based learning model is better than students who received conventional learning the material opportunities.

Marchis (2012) investigated on Hungarian 3th grade primary school textbooks from Romania. These textbooks are analysed using two classifications. The first classification is based on how much creativity and problem solving skills pupils need to solve a given task. In this classification, problems are grouped in three categories: routine problems, grayarea problems and puzzle-like (non-routine) problems. The results show that most of the problems from textbooks are routine-problems. Only about 15% of the problems are more difficult, which can be solved in few steps, but even these problems are not challenging. The second classification divide problems based on how the operation chain they have to solve is given: by the numbers, by text or in a word problem. The results show that there are big differences in the percentage of problems from these three categories in different textbooks. In one of the studied textbook halves of the problems are word problems, in the other one only one quarter. Thus, Marchis concluded that teachers should choose the most appropriate textbook when teaching problem solving for their students.

The above findings support the statement of this study saying that an effective teaching strategy is needed to improve students' problem solving ability. Classroom teachers should be encouraged to employ different types of effective problem solving method which suits well their students in teaching mathematical problem solving to students with varied cognitive abilities.

2.5.3 Previous Research on the Gender Factor on Mathematical Problem Solving Ability

In the past few decades, research has repeatedly reported gender differences in mathematical problem solving performance. Rasiman (2015) conducted a qualitative research which aimed to formulize the levelling of students' critical ability in solving mathematics problem based on gender. The data collection was gathered by interview based task. The research subjects were eleventh grader students of SMA Islam Sultan Agung Semarang. This research revealed that for more critical students, the female student was able to solve a problem correctly and carefully. Besides, she was looking back her works, while the male student was able to solve a problem correctly, but he did not careful and did not look back his works. For critical students, while planning the problem they tend to be trial and error, and the female student looking back her works, but the male student did not. For the less critical students, both male and female were able to create the plan by writing the formula several times, but they were not able to solve the problem correctly.

Dannawi (2013), in his study, investigate about the importance of teaching problem solving and critical thinking in schools and including it in the curriculum, to prepare students achieve the best quality of thinking in the society and be more involved in the society. The study focuses on the gender difference and the use of cooperative learning in teaching problem solving, how it effects on their performance. The study was on 10th graders in U.A.E., Dubai private school, under the implementation of cooperative learning. A comparative data collected by pre-test and a post-test to study the student's achievement as well as a questionnaire to study the cultural background of the students. The study showed there is significance in performance in aspects, gender and student's achievement, but no significance in culture.

In the study conducted by Hornburg, Rieber, and McNeil (2017) in examining the gender as a potential source of variation in children's formal understanding of mathematical equivalence, the hypothesis stated was that girls would perform more poorly than boys. An integrative data analysis was conducted with 960 second and third graders across 14 previously conducted studies of children's understanding of mathematical equivalence. Measures included problem solving, problem encoding, and equal sign definition. Overall, children performed poorly on all measures. As predicted, girls were less likely than boys to solve mathematical equivalence problems correctly, even though there were no gender differences in calculation accuracy. In addition, girls were more likely than boys to use the ‘‘add-all” strategy, an incorrect strategy that has been shown to be more resistant to change than other incorrect strategies. There were not statistically significant differences for encoding or defining the equal sign, suggesting that deficits may reflect girls’ tendency to follow taught algorithms.

Bassey (2007) conducted a study of gender difference and mathematics achievement of rural senior secondary students. The sample of the study included 2000 students out of which 1000 were male and 1000 were female. Multiple choice mathematics achievement test (MAT) used to assess the mathematical ability of children. The result revealed that there was gender difference in mathematical problem solving ability of children it was due to difference in nurturing practices and environment provision provided by parents to their children.

Kolawole (2007) explored the study to examine the gender issues and academic performance of senior secondary school students in mathematics computation task in Nigeria. The sample of the study included 500 students. Self-structured questionnaire developed by researcher, which included fifty multiple choice questions with five options was used to assess the problem solving ability of children’s. The result indicated that boys performed significantly better than girls in mathematics computation task. Girls in school performed better than their counterparts in mixed schools in mathematics computation task.

Tella (2007) designed a study to know the impact of motivation on student’s academic achievement and learning outcomes in mathematics among secondary school students in Nigeria. The study included 450 high school children out of which 260 were male and 190 were female. Questionnaire developed by researcher used to assess the mathematical ability of children. The result of the study showed that boys performed significantly better than girls in mathematical task.

Babakhani (2011) in his research initialled the effect of teaching the cognitive and metacognitive strategies in verbal math problem solving performance of primary school students with verbal problem solving difficulties. The sample included 60 students out of which 30 were boys and 30 were girls. Verbal math problem solving test used to assess the problem solving ability of children. The study included two groups, one experimental group which included (15 girls and 15 boys) and other group included (15 girls and 15 boys). The result of the study showed that teaching of cognitive and met cognitive strategies significantly improved performance of experimental group in both gender and also no significant differences between boys and girls in either applying strategies or effectiveness of teaching in problem solving.

Mathematical reasoning is a common activity, which involves induction, deduction, association, and inference methods, as well as how learners interact with each other to solve the problems (Erdem, 2011). Erdem and Soylu (2017) conducted a study to determine the relationship between mathematical reasoning ability with gender. The study uses cross-sectional design, which was conducted with 409 of 8th, 9th and 10th grade students attending to middle school and high school in different provinces of Turkey from different socio-economic environments. Mathematical Reasoning Test (MRT) was used for the data collection. Independent group's t-test was applied in order to analyze the relationship between mathematical reasoning and gender. The analysis shows that male students perform significantly better than female students in mathematical reasoning. The study concluded that it is very important to take encouraging steps to ensure that women are interested in mathematics instead of discouraging attitudes in society.

The above findings support the statement of this study saying that both male and female students perform differently in mathematical problem solving tasks. It could be due to the different cognitive ability in both gender. Therefore, it is worth studying on the effective problem solving strategy that equip both gender to solve mathematical problem solving tasks effectively.

2.6 Conceptual Framework

The conceptual framework was constructed based on the review of the literature on the theories, key variables, the models, and past researches related to the study.

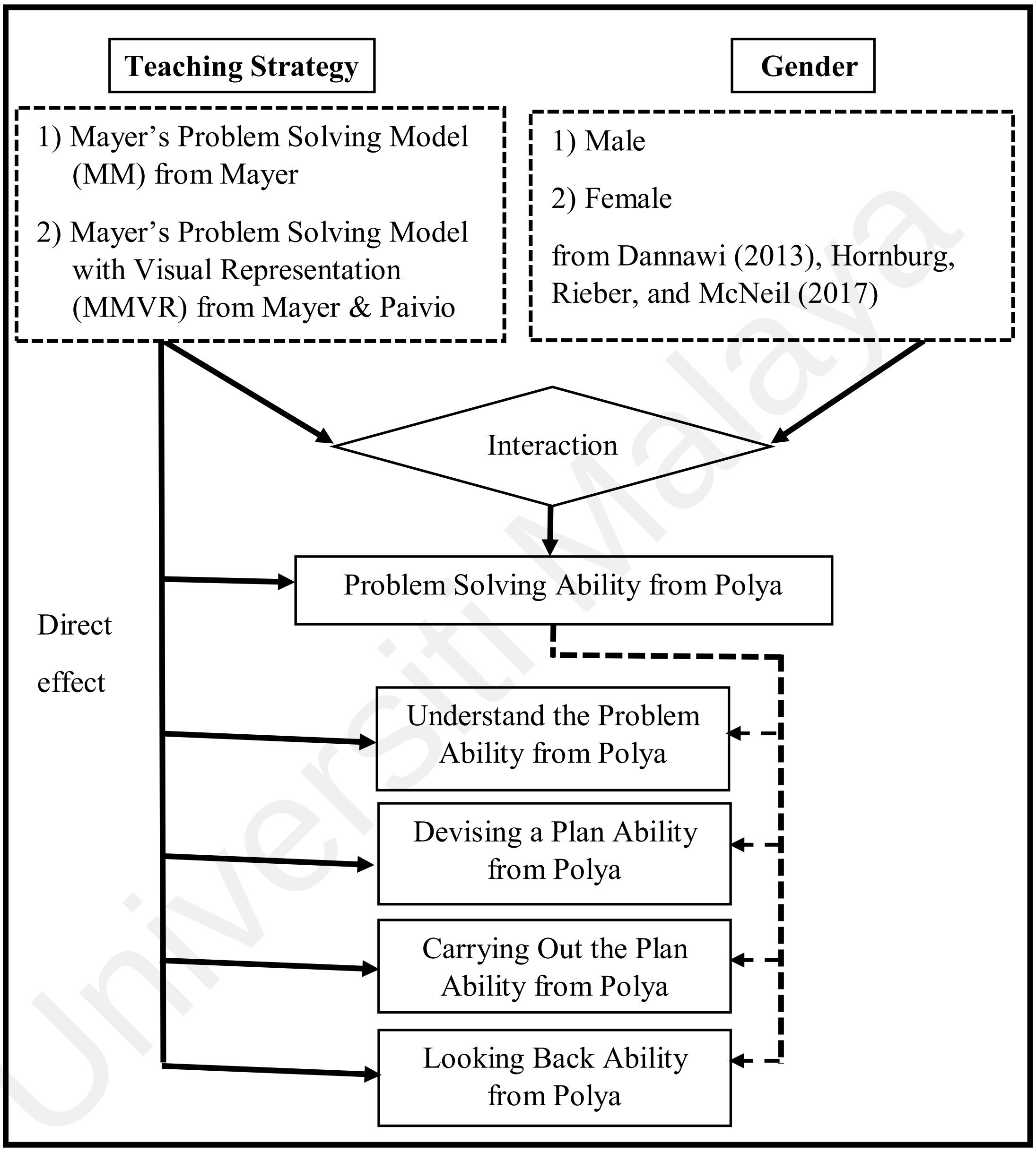


Figure 2.3. Conceptual framework

This research aimed to determine the effectiveness of Mayer's problem solving Model with Visual Representation (MMVR) on students' mathematical problem solving ability. Therefore, two variables namely independent and dependent variables involved in this study. There are two independent variables used in this study which are the teaching strategy and gender. The dependent variable of this study is students' problem solving ability.

There are two teaching strategies involved in this study namely Mayer's Problem Solving Model (MM) teaching strategy and Mayer's Problem Solving Model with Visual Representation (MMVR). Both teaching strategies were developed based on Mayer's problem solving model (1985). Through employing this teaching strategies, students practice on how to translate and integrate mathematical problem, planning for the solution, and finally execute the planning in order to help them become successful problem solvers. Visual representation strategy was adopted into MMVR teaching strategy based on its importance in problem solving. According to Paivio (1971), visual representation become meaningful in the context of problem solving, and enables students to strengthen their mental schemes during solving problems by which users not only to solve the problem but also enable them to reflect on their solution. Visual representation became a tool to help students to build up mental schemes, either assimilating already familiar schemas or producing new schemas that allow the students to achieve a cognitive goal.

Gender is considered as one of the independent variable in this study due to the past studies that showed there is a significant interaction between gender and teaching strategies of problem solving. Mathematics is generally viewed as a difficult subject to all and more to girls. While gender choices are made more freely today, inequality remains in mathematics occupation. Researches have repeatedly reported on the gender differences in mathematical problem solving performance (Dannawi, 2013; Hornburg, Rieber, & Mcneil, 2017).

The dependent variable of this study which is students' problem solving ability was developed based on Polya's problem solving model (1957). According to Poya (1957), to become a successful problem solver, students should be able to understand the given problem, devise an accurate plan for the problem, carrying out the plan, and checking back the answer. In this study, students' problem solving ability has been evaluated based on Polya's problem solving model.

The hypothesis in this study were tested based on the direct and interaction effects of the variables. The direct effect involved Mayer's Problem Solving Model with Visual Representation (MMVR) teaching strategy, with students' problem solving ability including the problem solving sub-constructs abilities namely Understand the Problem ability, Devise a Plan ability, Carry out the Plan ability, and Looking Back ability. The interaction effect in this study involved gender with students' problem solving ability including the problem solving sub-constructs abilities namely Understand the Problem ability, Devise a Plan ability, Carry out the Plan ability, and Looking Back ability. The direct and interaction effects in this study related in a way that this research tests the interaction between Mayer's Problem Solving Model with Visual Representation (MMVR) teaching strategy and gender in improving students' mathematical problem solving ability including the problem solving sub-constructs abilities namely Understand the Problem ability, Devise a Plan ability, Carry out the Plan ability, and Looking Back ability after students undergo the treatment.

2.7 Summary

This literature review was segmented into few sections initiated with describing about theories involved in this study, key concepts on mathematical problem solving, teaching of mathematical problem solving, factors affecting mathematical problem solving ability, the literature concerning visual representation and its effects on students' mathematics problem solving ability. This chapter closed with the constructed conceptual framework based on the review of the literature on theoretical framework and research framework of this study.

The following chapter presents research methodology of the study, the subjects, sampling technique, research instruments, procedure of data gathering, and statistical treatment that was used for accurate data analysis and interpretation.

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter focuses on the research design employed to answer the research questions. The chapter begins with the rationale for adopting the quantitative methodology in general and the research design to carry out the study. The ensuing sections contain information about the details on the selection of population and sample, treatments, instruments, data collection procedures and data analysis procedures. Furthermore, issues pertaining to the validity, reliability and ethics were addressed before concluding with a summary of the chapter.

3.2 Research Methodology

An approach for undertaking a research study is vital in order to provide the best answers for all the research questions and to make sure the data to be collected is sufficient, valid, and reliable. Since this study follows the positivism research paradigm, the methodology that had been used to carry out this study was quantitative research methodology. Quantitative research is built on a positivist paradigm of research. Here, the researcher is external to the research site and is the controller of the research process (Bernard, 2011). The data gathered from quantitative research can be used to look for cause and effect relationships and therefore, can be used to make predictions.

3.3 Research Design

There are two main types of quantitative research designs namely experimental and non-experimental research design. The differences between them are, experimental research is when a researcher is in a position to manipulate the predictor variable and subjects to identify a cause-and-effect relationship. On the other hand, non-experimental research is the label given to a study when a researcher cannot control, manipulate or alter the predictor variable or subjects, but instead, relies on interpretation, observation or interactions to come to a conclusion (Creswell, 2014).

Since this study examine the effect of Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy on Year 4 students' mathematical problem solving ability, the research design used for this study was experimental research design. According to Ross and Morrison (2014), a primary approach used to investigate causal (cause/effect) relationships and to study the relationship between one variable and another are known as an experimental research design. This is a traditional type of research that is quantitative in nature. In short, experimental research is used by researchers to compare two or more groups on one or more measures. They identify the effects of causes by implementing interventions in a controlled environment (Ross & Morrison, 2014). Finally, they help the researcher to be able to offer explanations for outcomes. To answer hypotheses, experimental designs are used in this way. To address a specific question, researcher formulate a testable statement known as hypothesis. The researcher next designed an experimental study which support or disprove the hypothesis.

There are three types of experimental research design namely true experimental, quasi- experimental, and pre-experimental. Participants are randomly assigned to either the treatment or the control group in a true experiment, whereas they are not assigned randomly in a quasi-experiment. Participants are randomly assigned to treatment group only for pre-experimental design because they fail to include control group in the design (White & Sabarwal, 2014). Since researcher controlled the assignment to the treatment condition in the present study due to some limitation, therefore, quasi-experimental research design was chosen for this study.

Quasi-experiment design or also known as non-equivalent control group design is a strong experimental research design (Johnson & Christensen, 2008) as it allows variables which may influence the results. In order to collect baseline data of the students' ability to solve problems, a pre-test was used before a week of the intervention. These data then was compared to the post-tests conducted after the intervention for each group. To examine whether any gains made by the participants, the final test was used after a week of the final intervention session.

Simply stated, quasi-experiments work well in natural settings (Newby, 2010). Quasi-experimental research may be more feasible because it often does not have the time and logistical constraints associated with many true experimental designs. Intervention sessions were designed to replicate a classroom mathematics teaching session with a small group for this study. This keeps the situation as natural as possible for the students. Students were still in their own school, and in a classroom setting even though they were removed from their classes for the sessions.

Quasi-experiments use convenient sampling technique instead of randomly selected sampling (Newby, 2010). In convenient sampling technique, students are selected because of their convenient accessibility and proximity to the researcher, and it is known as a non-probability sampling technique. According to Creswell (2012), quasi-experiments are experiments that lack random assignment of units to conditions but that otherwise have similar purposes and structural attributes to randomized experiments. The generalizability of the results to a larger population might be limited due to the lack of random assignment into test groups which leads to non-equivalent test groups. Conclusions about causality are less definitive in quasi-experimental designs apart from the lack of randomization and the reduced internal validity (Creswell, 2013).

Figure below describe the quasi-experimental design which also known as the non-equivalent control group design using Campbell and Stanley (1963) notation.

$Experimental_1$	O_1	X_1	O_4
$Experimental_2$	O_2	X_2	O_5
Control	O_3		O_6

Figure 3.1. The Pre-test—post-test non-equivalent group design of the study based on Campbell and Stanley (1963) notation

According to Campbell and Stanley (1963) notation, *X* represents the exposure of a group to an experimental variable or event, the effects of which are to be measured. *O* refers to the process of observation or measurement. *X*s and *O*s in a given row are applied to the same person/ groups whereas *X*s and *O*s vertical to one another are simultaneous. Parallel rows separated by a dashed line represent groups not equated by non-random assignment.

For this study, O_1, O_2, and O_3 represents pretest scores for MPSAT while O_4, O_5, and O_6 represents posttest scores for MPSAT. An additional interventions from researcher applied to the experimental group was Mayer's Problem Solving Model (MM), which is represented by X_1 and Mayer's Problem Solving Model with Visual Representation (MMVR) which is represented by X_2. MM intervention involved the students in MM Group being instructed in the use of Mayer's problem solving Model only. MMVR intervention involved MMVR group of students being instructed in the use of Mayer's problem solving Model with Visual Representation. This was to allow comparisons between the two teaching strategies. Control group on the other hand did not undergo any additional intervention from researcher.

Each group had ten treatment sessions. Ten sessions are intended as the researcher wanted to see if any changes in problem solving ability could be achieved in a short period of time, with time restrictions of busy teachers being an important consideration if such interventions are to be easily implemented in school settings. The intervention sessions was designed to replicate a classroom mathematics teaching session with a small group lasting forty-five minutes, keeping the situation as natural for the students as possible (Newby, 2010).

Even though the students had been moved out from their classes for the sessions, they still been in their own school, with the teachers they familiar with. In order to hold the students' mathematics program as usual as possible, the intervention sessions happened outside the students' usual mathematics lessons. The intervention sessions involved the researcher modelling the favoured strategy and then the students having opportunities to use the strategy.

3.4 Threat to Internal Validity

Internal validity refers to the validity of the findings within the research study. It is principally concerned with controlling the extraneous variables and outside influences that may additionally influence the consequences (Mohajan, 2017). This is particularly necessary in experimental research to make sure that the experimental treatment is, in fact, responsible for a change in the dependent variable. This is necessary if the study is going to be able to determine a causal relationship. Therefore, the researcher have to plan to control or eliminate the influence of different variables in order to be assured when making conclusions about the relationship between experimental treatment and dependent variable (Mohajan, 2017).

In this study, researcher desired to determine if there was a causal relationship between Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy in improving mathematical problem solving ability of Year 4 students. Therefore, researcher would need to consider other factors that may improve students' mathematical problem solving ability and attempt to eliminate those influences in the experimental group.

Quasi-experimental research have a tendency to have numerous threats to their internal validity, only those that regarded to create problems in interpretation for either of the preceding studies were discussed. Pre-existing factors and other influences are not taken into account due to the fact variables are less controlled in quasi-experimental research. If other variables are not controlled, the researcher cannot be assured that the treatment was the sole factor causing the outcome. Statistical analyses may not be meaningful due to the lack of randomization and the threats to internal validity (Creswell, 2013).

The lack of random assignment into test groups leads to non-equivalent test groups which can limit the generalizability of the results to a larger population. It must be possible to assume that the sample used in the research is representative of the general population to which the research results would apply in order for a study to be generalizable to a wider context (Polit & Beck, 2010). In this research context, the researcher ensured that the several factors were in place to guard against threats to internal validity in the form of subject variability because it was not possible to randomly assign subjects to different groups. Therefore, this experimental design controlled situations that may threaten internal validity of the experiment such as testing, history, maturation, mortality, regression, and diffusion of treatment (Johnson & Christensen, 2008).

Firstly, testing refers to any changes that may manifest in students' scores during a second sitting of a test due to having already sat the test and being more familiar with the style and content of the tests. Researcher consequently took a longer time interval between administrations of the outcome of the test (Creswell, 2013).

Secondly, history refers to any different events that might take place between a pre-test and a post-test which could have an effect on scores such as the classroom teaching or a homework task (Creswell, 2013). For example, students' post-test math score improvement may have been caused by their preparation for a math exam at their school, rather than the remedial math program (Furtak, Seidel, Iverson, & Briggs, 2012). Researcher therefore had both the experimental and control groups experience the same external events (Creswell, 2013).

Thirdly, maturation includes any physical or mental change that may additionally take place within students over time which include learning, ageing, and fatigue (Creswell, 2013). Maturation threat refers to the possibility that observed effects are caused by natural maturation of subjects such as a general improvement in their intellectual ability to understand complex concepts, rather than the experimental treatment (Furtak et al., 2012). Researcher consequently choses students who mature or change at the same rate (e.g. same age) during the experiment (Creswell, 2013).

Then, mortality refers to students drop out during an experiment due to many possible reasons. The outcomes are thus unknown for these students (Creswell, 2013). Mortality threat also refers to the possibility that subjects may be dropping out of the study at differential rates between the treatment and control groups due to a systematic reason, such that the dropouts were mostly students who scored low on the pretest. If the low-performing students drop out, the results of the posttest will be artificially inflated by the preponderance of high-performing students (Furtak et al., 2012). Researcher recruited a large sample to account for dropouts or compare those who drop out with those who continue in terms of the outcome in order to overcome this (Creswell, 2013).

Next, regression, refers to students with intense scores are chosen for the experiment. Naturally, their scores will in all likelihood change in the course of the experiment. Scores, over time, regress towards the mean (Creswell, 2013). Regression threat also called a regression to the mean, refers to the statistical tendency of a group's overall performance on a measure during a posttest to regress toward the mean of that measure rather than in the anticipated direction (Furtak et al., 2012). For instance, if students scored high on a pretest, they will have a tendency to score lower on the posttest (closer to the mean) because their high scores (away from the mean) during the pretest was possibly a statistical aberration. This problem tends to be more prevalent in non-random samples and when the two measures are imperfectly correlated (Furtak et al., 2012). Therefore, researcher selected students who did not have intense scores as entering requirements for the experiment (Creswell, 2013).

Lastly, diffusion of treatment refers to event where students in the control and experimental groups communicate with each other. This communication can impact how each group score on the outcomes (Creswell, 2013). This could happen because individuals in the control groups and treatment groups talk to each other about the treatment. As such, this is usually an issue in research involving training or informational programs (Gundersen & Svartdal, 2008). To resolve this issue, researcher kept the two groups as separate as possible during the experiment (Creswell, 2013).

To further ensure that stipulations for the groups were even, students had been randomly assigned to one of the two groups. The groups then had been checked to make sure they are similar in the number of girls and boys, English language learners, Year 4s, and scores on the pre-test. This form of matching is another way of providing control in an experiment (Creswell, 2012).

3.5 Threat to External Validity

External validity refers to the extent to which the results of study can be generalized or applied to other members of the larger population being studied. The characteristics of individuals selected for the sample, the uniqueness of the setting, and the timing of the experiment influenced the threat to external validity (Creswell, 2009). The random selection of participants and random assignment of the study participants into groups is critical for this reason, so that the members of the study are truly representative of the larger population. This experimental design therefore controlled situations that may threaten external validity of the experiment such as interaction of selection and treatment, interaction of setting and treatment, and interaction of history and treatment.

First, interaction of selection and treatment refers to an event where researcher cannot generalize the outcome of this study due to students who do not have the characteristics of the sample of this study due to the fact of the narrow characteristics of sample in the experiment (Creswell, 2013). If subjects are drawn from a too restrictive sample or an unrepresentative sample, then obviously more replication will be required to generalize the results with confidence. When it comes to the representativeness of a sample, some variables may be disregarded, while we must pay attention to some other variables such as age, level of education and gender (Druckman, Green, Kuklinski & Lupia, 2011c). Therefore, researcher restrained claims about groups to which the results cannot be generalized (Creswell, 2013).

Next, interaction of setting and treatment refers to an event where researcher cannot generalize the outcome of this study due to students in different settings due to the fact of the characteristics of the setting of sample in an experiment (Creswell, 2013). Increasing the diversity of circumstances or situations in which a particular phenomenon is investigated can heighten external validity. Exploring a particular process in a variety of settings can prove particularly helpful for discovering contextual boundaries on particular processes and illustrating the particular dimensions of its operation (McDermott, 2011). Researcher therefore restrained claims about groups to which the results cannot be generalized.

Lastly, interaction of history and treatment refers to an event where researcher cannot generalize the results to past or future situations due to the results of an experiment are time-bound (Creswell, 2013). When many studies of one problem are conducted, the results can vary. Several studies might find an effect of the number of bystanders on helping behavior, whereas a few do not. To make sense out of this, there is a statistical technique called meta-analysis that averages the results of two or more studies to see if the effect of an independent variable is reliable (McDermott, 2011). A meta-analysis essentially tells us the probability that the findings across the results of many studies are attributable to chance or to the independent variable. If an independent variable is found to have an effect in only one of 20 studies, the meta-analysis will tell you that that one study was an exception and that, on average, the independent variable is not influencing the dependent variable. If an independent variable is having an effect in most of the studies, the meta-analysis is likely to tell us that, on average, it does influence the dependent variable (McDermott, 2011). Researcher therefore suggested other/ future researchers to replicate the study at later times to determine if the same results occur as in the earlier time.

3.6 Population and Sample

Population is the broader group of people to whom the researcher intend to generalize the results of the study, while sample will always be a subset of the population. Exact population depends on the scope of the study. This study intended to determine the effectiveness of Mayer's problem solving Model with Visual Representation teaching strategy in improving mathematical problem solving ability of Year 4 students. There are about 7877 primary schools in Malaysia with 2719044 students (MOE, 2015). The population is very huge and it would consume more time and costing if researcher intended to do research on the population. It is impossible to assess every single student of a population, so a group of people (smaller in number than the population) which known as sample is selected for this study. On the basis of information obtained from the sample, the inferences are drawn for the population. The process through which a sample is extracted from a population is called as sampling.

Convenient sampling technique was chosen instead of probabilistic sampling due to some reasons. First, since the design of this study is a quasi-experimental and due to the nature of the design, students were not assigned randomly. Students were selected based on their convenient accessibility and proximity to the researcher. Secondly, only students who poor with mathematical problem solving ability were chosen for this study. Besides, the students are those who are able to read and comprehend English words in order to understand the mathematical problems linguistically. Those students were chosen based on school teachers' advice. This is because, school teachers know better of their students' performances. The third reason for choosing this sampling method is because the permission that obtained from the school principal to conduct the research on the particular school. Fourthly, students must be willingly involved in the pilot study, not by forcing, as well as parents should give their consent to allow their children to join the pilot study of this research. Moreover, this sampling technique enables the researcher to achieve the sample size in a relatively fast and inexpensive way.

A total of 203 Year 4 students from a private primary school who were being taught using KSSR syllabus were chosen to participate in this study. The student population in the school reflected the multiracial citizens in Malaysia. The selection of the students begun with the researcher seeking permission from the principal to conduct the study. A consent letter explaining the general nature of the project was sent to parents in a month prior to the experiment and request for permission was included. After getting the consent from the parents of 203 Year 4 students to participate in the study, the researcher equally divided the students into three groups to form three intact classes. Two classes were selected as the sample for the experimental groups while the other class as a control group. Hundred and thirty-six (136) students were selected as the sample for the experimental groups to undergo the Mayer's problem solving Model (MM) teaching strategy and Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy respectively.

3.7 Treatments

Two treatments were involved in order to answer the research questions for this study. The intervention of the treatment was administered after school hours with selected hundred and thirty-six students (N = 136) from Year 4. Another sixty-seven students (N = 67) from Year 4, from another class became the control group where they did not receive extra instruction on how to solve mathematical problems after the school hours. The experimental groups students (N = 136) underwent mathematics lesson during school hours as normal, together with extra instructions which were Mayer's problem solving Model (MM) and Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy after the school hours. Whereas, the control group students (N = 67), underwent mathematics lesson during school hours as normal, without any additional instruction after school hours.

The first treatment, which is called as Mayer's problem solving Model (MM) teaching strategy, was intended to provide the instructional assistance based on

Mayer's (1985) four step problem solving model. The MM teaching strategy can help students decide and what to do when solving mathematical problems. Students learn how to translate the mathematical problems, integrated the information presented, developed logical plans to solve problems, and carried out the plans in an appropriate manner. The second treatment, Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy, required students to use both MM and Visual Representation (VR) at the same time in order to examine whether the use of MM and VR can enhance students' problem solving ability or not.

Before the treatment, two-hundred and three students (N = 203) from Year 4 were given a pre-test to measure their initial problem solving ability. Regardless their problem solving ability from the pre-test result, these students were equally divided with regard to gender into three groups. Means, MM group consisted of thirty-four (34) male and female students respectively, MMVR group also consisted of thirty-four (34) male and female students respectively, and control group consisted of thirty-three (33) male and thirty-four (34) female students. Once the groups formed, they were assigned into two treatment groups (MM group and MMVR group) and one control group. The two treatment groups have been taught on solving mathematical problems where the MM group was undergo the treatment using problem solving worksheet (PS Worksheet (MM)) (See Appendix B), whereas the MMVR group was using (PS Worksheet (MMVR)) (See Appendix B).

The sub-dimension problem solving abilities derived from Polya's problem solving model, namely: understand the problem ability, devise a plan ability, and carry out the plan ability was examined in every ten consecutive sessions during the treatment. Polya's another sub-dimension problem solving ability namely: looking back ability was not taken place in the ten sessions of the treatment. This is because, Mayer's problem solving Model did not emphasize on the looking back criteria. Thus, the focus of the treatment was on the other three abilities stated by Polya which were understand the problem ability, devise a plan ability, and carry out the plan ability. In each session, students underwent all the treatments in Mayer's (1985) problem solving model, namely: problem translation, problem integration, solution planning, and solution execution. Table below shows how the treatment of this study aligns with students' problem solving ability.

Table 3.1

Treatment and Problem Solving Ability Alignment

Treatment		Problem Solving Ability
Problem Representation ➢ Problem Translation ➢ Problem Integration	→	Understand the Problem
Problem Solution ➢ Solution Planning ➢ Solution Execution	→	Devise a Plan Carry out the Plan

At "understand the problem" stage, students were asked to identify the data, condition and unknowns from the problems. It is important to understand the given problem, as well as to know what the problem is asking them to find. The "devise a plan" stage basically asked students to find appropriate strategy that could be used to solve the problems and to explain why the strategy would be appropriate. Students were asked to choose the strategy that they will use to solve the problem. At "carry out the plan" stage, students were asked to solve the problem using the chosen strategy. They were asked to show their steps and check their results, explaining their reasoning as they go. The venue was outside the students' regular classroom. A special session was planned with the respective school teachers. The following table gives a session-by-session description of how the treatments were administered for each treatment group. In addition, it provides outline of this study component.

Table 3.2

Outline of the Study

Sessions	Treatment	Assessment
Before Treatment		Pre-Test
Session 1 Session 2 Session 3 Session 4 Session 5 Session 6 Session 7 Session 8	Problem Translation Problem Integration Solution Planning Solution Execution	
Session 9 Session 10		
After Treatment		Post-Test

3.7.1 MM Group Treatments

MM Group: Sessions 1 and 2, Understand the Problem Stage:

During the first session of treatment, understand the problem, the focus was on how students in the MM group translate and integrate the problem by working in group. Students were placed in 2 groups, 34 students in each group. To initiate these sessions, the teacher first discussed the importance of problem solving strategy and introduced MM teaching strategy for this session to the students. Then, the teacher discussed the importance of reading a problem more than once for better understanding.

After the explanation, the teacher demonstrated a question on how to understand a problem using the MM teaching strategy (See Appendix A). Then, students were given PS Worksheet (MM) (See Appendix B) which consist of directions for understanding the problem. According to the PS Worksheet (MM), the first question requires students to re-write the given problem in their own words. Students did this question after they worked in pairs to practice re-tell the problems in their own words (both students took turn to re-tell the problem in their own words to make sure students understand the problem story). Those students who became a listener had the responsibility to correct the understanding of their partner if their partner's understanding diverted from the actual problem story. The second question was drawing a picture/ schematic diagram. Based on Mayer's second stage "problem integration", efforts to bring the internal representation to the external representation are made here. Therefore, students were required to visually represent the problem in order to facilitate solution planning. The third question required students to list down the data, condition and unknown of the given problem.

Once students completed all the questions, they were needed to compare their answers. If their answers did not tally, or did not meet the common understanding, then they discuss and explain to their partner on why they did not get the similar answers. Each student in every group took turns to explain their answer to their partner. Students had to come out with their final answers upon agreement of both partners. Then, at the end of the session, the teacher discussed the answer.

MM Group: Sessions 3, 4 and 5, Devise a Plan Stage:

In the third, fourth and fifth sessions of treatment, devise a plan, the focus was on the notion that there is more than one way to solve a mathematical problem. For the third session, teacher guided students to think in terms of numerical and graphical methods, and also explained the strategies that been used to solve problems in Strategy Sheet in PS Worksheet (MM) (See Appendix B). Referring to the Strategy Sheet in the PS Worksheet (MM), teacher explained the students on how to devise a plan for the same example of the problem taken from session 1 (See Appendix A).

For session four and five, with referring to Strategy Sheet, students were required to work in the same group to answer the questions stated in the PS Worksheet (MM). According to the PS Worksheet (MM), the first question requires the students to write down the strategy (ies) that they have selected. Then, they had to explain on the reasons for their selection. In these sessions, students can select more than one strategy, and explain their reasons for each chosen strategy if possible. This is to ensure students familiar with every strategy they have chosen. Then, at the end of the session, the teacher discussed the answer.

MM Group: Sessions 6, 7 and 8, Carry out the Plan Stage:

In the sixth, seventh and eight sessions of treatment, carry out the plan, students were involved in solving mathematical problems and checking the results. The methods of good problem solvers highlighted by Whimbey and Lochhead (1982) such as having a positive attitude, being concerned about accuracy, breaking the problem into parts, avoiding guessing, and being active when solving problems were discussed during this treatment, using PS Worksheet (MM) (See Appendix B). Teacher carried out the plan using a selected strategy for a question that been used from session 1 (See Appendix A).

Next, students carried out the selected strategy to solve the problem in the PS Worksheet (MM) in the same group. According to the PS Worksheet (MM), the question requires students to solve the problems using the selected strategy. Based on the strategies students selected from sessions four and five, students were required to choose only one strategy that they think the best to solve the problem. Together with that, they were required to write an explanation/reason for each of their working steps when solving the problem. In addition of that, students were required to make sure that they have understood the underlying logic before applying any formulas, understood what the actual situation is, and only after fulfilling the two prior steps can they start making the mathematical computations.

While working in the group to solve the problems, student discussed on how to explain the steps. For those students in the group who became the listener while his/her partner is explaining, they had the responsibility in helping the partner who fails to give a full explanation of the strategy. For instance, if the listener's partner who performed computations or applied formulas that are inappropriate and lead to wrong answers or has not spelled out situations with full understanding, the listener must insist and ask the solver to show a table or diagram which illustrates step-by-step, the relationships between the facts in the problem. This method will help both students in each group to become good mathematical problem solvers. Then, at the end of the session, the teacher discussed the answer.

MM Group: Sessions 9 and 10:

For these sessions, students were regarded to work in groups of two to solve two different types of problems. This time, students were paired with different partner, not with the same partner from sessions 1 to 8. By referring to the example provided, and by referring to their own work from sessions 1 to 8 in PS Worksheet (MM) (See Appendix B), students were attempted to solve the given problems by themselves without or with less guidance from the teacher. Students were instructed to follow the problem solving steps taught to them in previous lessons. Then, at the end of the session, the teacher discussed the answer.

3.7.2 MMVR Group Treatments

The MM and MMVR are two different treatments in this study, however, both of them were designed based on the four structured steps of Mayer's mathematical problem solving model. Students in MMVR group follow the instructions and solve the given problems in the problem solving worksheet same as the students in MM group. However, MMVR group engage with different problem solving worksheet namely PS Worksheet (MMVR) (See Appendix B). The lesson plan for both of the groups are mostly the same. What made these two treatments different from one another was the adoption of visual representation strategy throughout the Mayer's mathematical problem solving model during the process of solving the problems in the PS Worksheet (MMVR).

MMVR Group: Sessions 1 and 2, Understand the Problem Stage:

During the first session of treatment of understand the problem, the focus was on how students in the MMVR group translate and integrate the problem by working in groups. Students were placed in 2 groups, 34 students in each group. To initiate these sessions, the researcher first discussed the importance of problem solving strategy and introduced MMVR teaching strategy for this session to the students. Then, the teacher discussed the importance of reading a problem more than once for better understanding.

After the explanation, the teacher demonstrated a question on how to understand a problem using MMVR teaching strategy (See Appendix A). Then, students were given PS Worksheet (MMVR) (See Appendix B) which consisted of directions for understanding the problem. According to the PS Worksheet (MMVR), the first question requires the students to re-write the given problem in their own words. Students did this question after they worked in pairs to practice re-tell the problems in their own words (both students took turns to re-tell the problem in their own words to make sure students understand the problem story). Those students who became a listener had the responsibility to correct the understanding of their partner if their partner's understanding diverted from the actual problem story. The second question is drawing a picture/ schematic diagram. Students were required to visually represent the problem in order to facilitate solution planning. The third question requires students to list down the data, condition and unknown of the given problem.

Once students completed all the questions, they were needed to compare their answers. If their answers did not tally, or did not meet the common understanding, then they must discuss and explain to their partner on why they did not get the similar answers. Each student in every group took turns to explain their answer to their partner. Students came out with their final answers upon agreement of both partners. Then, at the end of the session, the teacher discussed the answer.

MMVR Group: Sessions 3, 4 and 5, Devise a Plan Stage:

In the third, fourth and fifth sessions of treatment which was devise a plan, the focus was on the notion that there is more than one way to solve a mathematical problem. For the third session, teacher guided students to think in terms of graphical methods, where students were asked to use visual representation as a strategy to solve the problems. Referring to the Strategy Sheet in PS Worksheet (MMVR) (See Appendix B), teacher explained the students on how to devise a plan for the same example of the problem taken from session 1 (See Appendix A).

For session four and five, with referring to Strategy Sheet, students were required to work in the same group to answer the questions stated in the PS Worksheet (MMVR). According to the PS Worksheet (MMVR), the first question requires the students to write down which strategy is sufficient to plan a solution for the problem, whether drawing a tree diagram, a diagram or a picture. Then, they had to explain on the reasons for their selection. In these sessions, students can select more than one strategy, and explain their reasons for each chosen strategy if possible. Then, at the end of the session, the teacher discussed the answer.

MMVR Group: Sessions 6, 7 and 8, Carry out the Plan Stage:

In the sixth, seventh and eight sessions of treatment which was carry out the plan, students were involved in solving mathematical problems and checking the results. During this treatment, the methods of good problem solvers highlighted by Whimbey and Lochhead (1982) such as having a positive attitude, being concerned about accuracy, breaking the problem into parts, avoiding guessing, and being active when solving problems were discussed using PS Worksheet (MMVR) (See Appendix B). Teacher carried out the plan using a selected strategy for a question that been used from session 1 (See Appendix A).

Next, students carried out the selected strategy to solve the problem in the PS Worksheet (MMVR) in the same group. According to the PS Worksheet (MMVR), the question requires students to solve the problems using the selected strategy. Based on the strategies students selected from sessions four and five, students were required to choose only one strategy that they think the best to solve the problem. Basically, for this group of treatment, students were required to solve the given problem using illustrations whether in the form of a tree diagram, a diagram or a picture. Together with that, they were required to write an explanation/reason for each of their working steps when solving the problem. In addition of that, students were required to make sure that they have understood the underlying logic before applying any formulas, understood what the actual situation is, and only after fulfilling the two prior steps can they start making the mathematical computations.

While working in the group to solve the problems, students discussed on how to explain the steps. For those students in the group who became the listener while his/her partner is explaining, they had a responsibility in helping the partner who fails to give a full explanation of the strategy. For instance, if the listener’s partner who performed computations or applied formulas that are inappropriate and lead to wrong answers or has not spelled out situations with full understanding, the listener must insist and ask the solver to show a table or diagram which illustrates step-by-step, the relationships between the facts in the problem. This method will help both students in each group to become good mathematical problem solvers. Then, at the end of the session, the teacher discussed the answer.

MMVR Group: Sessions 9 and 10:

For these sessions, students were regarded to work in groups of two to solve two different types of problems. This time, students were paired with different partner, not with the same partner from sessions 1 to 8. By referring to the example provided, and by referring to their own work from sessions 1 to 8 in PS Worksheet (MMVR) (See Appendix B), students attempted to solve the given problems by themselves without or with less guidance from the teacher. Students were instructed to follow the problem solving steps taught to them in previous lessons. Teacher discussed the answer in the end of the session.

Students realized that the role of Visual Representation was significant in making the process easier, understandable and enjoyable for students. The following table summarizes the role of visual representation in MMVR group of treatment.

Table 3.3

The Role of Visual Representation in MMVR Treatment

Problem Solving Ability	Summary of Visual Representation Role in Each Step on Solving "Problem 1"
Understand the Problem	The visual representation role is started when students find data, conditions and unknown from the problem. For example, in "Problem 1" (See Appendix B), through visual representation, students drew the picture of the story problem. The illustration, then gives an overall idea of the problem, thus eased the students to find the data, conditions and unknown of the problem. To illustrate a problem, students have to take into account of every single sentence of the problem. Due to this, students will not miss any of the important information or sentences that they will need to use in the upcoming stages of problem solving. Visual representation encourages students to solve the problem by illustration rather than just reading the problem story.
Devise a Plan	The visual representation strategy used in the problem gives students the idea on how things are related to each other. From there, students can further integrate the data and unknown of the problem with the given conditions. For example, the illustration of the problem provides students an idea on how the working method is going to be in order to find the answer.
Carry out the Plan	Students drew many types of illustrations to carry out the problem solving process in an organized manner. Also, during this process, students maintained the relationship between the data and unknown of the problem. There will be no any important information will be omitted through this process.
Looking Back	Visual representation helps students to come out with another strategy to check the answer quickly and easily.

The following table summarizes how both MM and MMVR treatments were carried out for each intervention session:

Table 3.4

Summary of MM and MMVR Treatments for 10 Intervention Sessions

Sessions	MM Treatment	MMVR Treatment
	Pre-test	
1 & 2	Understand the Problem: ✓ Teacher explained using an example. ✓ Students worked with their partner using **PS Worksheet (MM).** ✓ Students wrote again the problem in their own words. ✓ Students drew picture/ schematic diagram to further explain the problem. ✓ Students found data, condition, and unknown of the problem using the illustration. ✓ Teacher discussed the answer in the end of the session.	Understand the Problem: ✓ Teacher explained using an example. ✓ Students worked with their partner using **PS Worksheet (MMVR).** ✓ Students wrote again the problem in their own words. ✓ Students drew picture/ schematic diagram to further explain the problem. ✓ Students found data, condition, and unknown of the problem using the illustration. ✓ Teacher discussed the answer in the end of the session.
3	Devise a Plan: ✓ Students went through the different types of strategies (**arithmetically and graphically**) in the strategy sheet provided by the teacher in PS Worksheet (MM). ✓ Teacher explained using an example.	Devise a Plan: ✓ Students went through the different types of strategies (**graphically**) in the strategy sheet provided by the teacher in PS Worksheet (MMVR). ✓ Teacher explained using an example.
4 & 5	Devise a Plan: ✓ Students worked with their partner using **PS Worksheet (MM).** ✓ Students wrote down the strategy(ies) that they had selected. ✓ Students explained the reasons for their selection. ✓ Teacher discussed the answer in the end of the session.	Carry out the Plan: ✓ Students worked with their partner using **PS Worksheet (MMVR).** ✓ Students wrote down the strategy(ies) that they had selected, between a **tree diagram, a diagram** or a **picture**. ✓ Students explained the reasons for their selection. ✓ Teacher discussed the answer in the end of the session.

Table 3.4 (continued)

Sessions	MM Treatment	MMVR Treatment
6	Carry out the Plan: ✓ Students went through the characteristics of a good problem solver in the **PS Worksheet (MM).** ✓ Teacher explained on how to carry out the plan with the chosen strategy using an example.	Carry out the Plan: ✓ Students went through the characteristics of a good problem solver in the **PS Worksheet (MMVR).** ✓ Teacher explained on how to carry out the plan with the chosen strategy using an example.
7 & 8	Carry out the Plan: ✓ Students worked with their partner using **PS Worksheet (MM).** ✓ Students solved the problem using the chosen strategy. Students showed the steps one-by-one and with explanation. ✓ Teacher discussed the answer in the end of the session.	Carry out the Plan: ✓ Students worked with their partner using **PS Worksheet (MMVR).** ✓ Students solved the problem using the chosen strategy. Students showed the steps one-by-one and with explanation. ✓ Teacher discussed the answer in the end of the session.
9 & 10	✓ Students worked with their partner to solve mathematical problems in **PS Worksheet (MM)** using the method that will be taught from Sessions 1 to 8 ✓ Teacher discussed the answer in the end of the session.	✓ Students worked with their partner to solve mathematical problems in **PS Worksheet (MMVR)** using the method that will be taught from Sessions 1 to 8 ✓ Teacher discussed the answer in the end of the session.
	Post-test	

3.8 Instrument

Instrument refers to a tool which researcher used to obtain data from respondents for the purpose of research work. The research instrument in this study is called Mathematical Problem Solving Ability Test (MPSAT). The purpose of the instrument used for this study is to collect data on how far Year 4 students could answer problem solving questions in order to determine their problem solving ability which includes understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability.

MPSAT was adapted from Mathematical Processing Instrument (MPI) by Hegarty and Kozhevnikov (1999). In the pilot study done by Hegarty and Kozhnevnikov, the MPI gave internally consistent measures of problem solving success (Cronbach's a = .78) and solution strategy which is the tendency to use visual-spatial representations (Cronbach's a = .72). The items been revised by the researcher to improvise the grammar and to lengthier the questions/ added on some sentences which prompt students to draw when solving the questions. Also, the mathematical values (numbers) been changed in some items. Table 3.5 shows examples of the original and revised items of MPSAT instrument. The original items are derived from Mathematical Processing Instrument (MPI) by Hegarty and Kozhevnikov (1999).

Table 3.5

Examples of Original and Revised Items of MPSAT

Original Item	Revised Item
Four young trees were set out in a row of 10 meters apart. A well was situated beside the last tree. A bucket of water is needed to water two trees. How far would a gardener have to walk altogether if he had to water the four trees with one bucket?	Five young trees were set out in a row of 10 meters apart. A well was situated beside the last tree. A bucket of water is needed to water three trees. How far would a gardener have to walk altogether if he had to water five trees with one bucket?
A square (A) has an area of 1 square meter. Another square (B) has sides twice as long. What is the area of B?	A square (A) has an area of 4 square meter. Another square (B) has sides twice as long as square (A). Square (C) has sides twice as long as square (B). What is the area of square C?

Table 3.5 (continued)

Original Item	Revised Item
A hitchhiker set out on a journey of 60 miles. He walked the first 5 miles and then got a lift from a lorry driver. When the driver dropped him he still had half of his journey to travel. How far had he traveled in the lorry?	A hitchhiker set out on a journey of 70 miles. He walked the first 5 miles and then got a lift from a lorry driver. When the driver dropped him he still had half of his journey to travel. He then got a lift from a motorcyclist and travelled for another 10 miles. How far had he traveled so far?
On one side of a scale there are three pots of jam and a l00g weight. On the other side there are a 200 g and a 500 g weight. The scale is balanced. What is the weight of a pot of jam?	On one side of a scale there are three pots of jam and l00g weight. On the other side there are a 200 g and a 500 g weight. The scale is balanced. What is the weight of a pot of jam?
A balloon first rose 200 meters from the ground, then moved 100 meters to the east, then dropped 100 meters. It then traveled 50 meters to the east, and finally dropped straight to the ground. How far was the balloon from its original starting point?	Remain the same.

All items were carefully designed to be solved by devising more than two strategies based on the system of coding in the MPSAT rubric, which also inclusive of visual representation strategy as one of the chosen strategy of solving the MPSAT questions. In addition, the MPSAT items were intentionally designed to align closely with the four steps of mathematical problem solving model as suggested by Mayer (1985). Each item of MPSAT was developed to collect students' responses so that the researcher will be able to determine students' overall problem solving ability and students' abilities to understand the problem, devise a plan, carry out the plan, and look back on the obtained solution prior and after the treatments. The following table 3.6 presents each items of MPSAT in terms of their mathematics content.

Table 3.6

Mathematics Content of MPSAT Items

Item	Mathematics Content
1	Application of Arithmetic (Length)
2	Application of Arithmetic (Area)
3	Application of Arithmetic (Length)
4	Application of Arithmetic (Mass)
5	Application of Arithmetic (Length)

3.9 Discriminant and Difficulty Index

In the development of an instrument, an item analysis is considered as an important phase. Item analysis will reveal if an item is too easy, too difficult, failing to show a difference between skilled and unskilled examinees, or even scored incorrectly. Item discrimination and item difficulty are the two most common statistics reported in an item analysis (Taib & Yusoff, 2014). The measure of the proportion of examinees who responded to an item correctly known as item difficulty, while the measure of how well the item discriminates between examinees who are knowledgeable in the content area and those who are not known as item discrimination (Mahjabeen, Alam, Hassan, Zafar, Butt, Konain, & Rizvi, 2018). Difficulty index is measured by dividing the number of students who got correct answers by the total number of students, while discrimination index is measured by subtracting the number of students in the lower group who got the item correct from the number of students in the upper group who got the item correct (Taib & Yusoff, 2014). In this study, the discriminant and the difficulty index values were measured from the pilot test with 30 students.

Table below shows the discriminant index measured from the pilot test. Students considered getting "Correct Answer" when they are able to score 2, 3, and 4 points (according to MPSAT Rubric) in every parts under understand the problem ability, devise a plan ability, carry out the plan ability, and looking back ability. The discriminant index is then measured using partial score item analysis.

Table 3.7

Discriminant Index of MPSAT Items

Question	Sub-construct	Item Difficulty for Top 27%	Item Difficulty for Bottom 27%	Discriminant Index
1	Understand the Problem	0.27	0	0.27
	Devise a Plan	0.21	0	0.21
	Carry Out the Plan	0.33	0	0.33
	Looking Back	0	0	0
2	Understand the Problem	0.25	0	0.25
	Devise a Plan	0.24	0	0.24
	Carry Out the Plan	0.28	0	0.28
	Looking Back	0	0	0
3	Understand the Problem	0.25	0	0.25
	Devise a Plan	0.25	0	0.25
	Carry Out the Plan	0.25	0	0.25

Table 3.7 (continued)

Question	Sub-construct	Item Difficulty for Top 27%	Item Difficulty for Bottom 27%	Discriminant Index
3	Looking Back	0	0	0
4	Understand the Problem	0.25	0	0.25
	Devise a Plan	0.38	0	0.38
	Carry Out the Plan	0.23	0	0.23
	Looking Back	0	0	0
5	Understand the Problem	0.33	0	0.33
	Devise a Plan	0.21	0	0.21
	Carry Out the Plan	0.25	0	0.25
	Looking Back	0	0	0

According to Mahjabeen et al. (2018), the discriminant index below 0.2 is considered poor, between 0.21 to 0.24 is acceptable, between 0.25 and 0.35 is good, and more than 0.36 is considered excellent. Table 3.7 above indicates that the discriminant index for most of the sub-constructs except for Looking Back sub-construct, mostly fall under acceptable and good categories. The discriminant index for Looking Back sub-construct is below 0, and this could be due to many students who unable to perform looking back steps when solving problems.

Table below shows the difficulty index measured from pilot test. Students considered getting "Correct Answer" when they are able to score 2, 3, and 4 points (according to MPSAT Rubric) in every parts under understand the problem ability, devise a plan ability, carry out the plan ability, and looking back ability. The difficulty index is then measured using partial score item analysis.

Table 3.8

Difficulty Index of MPSAT Items

Question	Sub-construct	Correct Answers	Total Responses	Difficulty Index
1	Understand the Problem	9	30	52%
	Devise a Plan	9	30	52%
	Carry Out the Plan	6	30	56%
	Looking Back	0	30	0%
2	Understand the Problem	8	30	51%
	Devise a Plan	6	30	40%
	Carry Out the Plan	7	30	50%
	Looking Back	0	30	0%
3	Understand the Problem	6	30	56%
	Devise a Plan	8	30	53%
	Carry Out the Plan	7	30	53%
	Looking Back	0	30	0%
4	Understand the Problem	7	30	55%
	Devise a Plan	6	30	59%
	Carry Out the Plan	7	30	51%
	Looking Back	0	30	0%
5	Understand the Problem	7	30	58%
	Devise a Plan	5	30	53%

Table 3.8 (continued)

Question	Sub-construct	Correct Answers	Total Responses	Difficulty Index
5	Carry Out the Plan	6	30	52%
	Looking Back	0	30	0%

According to Mahajabeen et al. (2018), the difficulty index below 30% indicates that the items are too difficult, 30% to 40% are average, 50% to 60% are good, and more than 70% are considered to be too easy. Table 3.8 above indicates that the difficulty index for most of the sub-constructs except for Looking Back sub-construct are fall under 'good' category. The difficulty index for Looking Back sub-construct is 0%, and it could be due to the student's poor looking back ability when solving problems.

3.10 Scoring Rubric

MPSAT Rubric:

Scoring rubrics are descriptive scoring schemes which used to assess students' performance. The scoring rubric in this study is called Mathematical Problem Solving Ability Test (MPSAT) Rubric. The purpose of scoring rubric used for this study is to effectively measure Year 4 students' problem solving ability which includes understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability. MPSAT rubric was adapted from Starkey (2010). The rubric was designed accordingly to score students' responses to pre and post-tests depending on how accurate and to what extent they manage to complete and respond to each question. Students may receive one, two, three, or four points for every section according to their problem solving performances.

The first component of the rubric, "Understand the Problem", consisted of three parts, which are identifying the data, conditions and unknown. According to the rubric, students who can identify all, most and just a few of the known and unknown data, will be scored 4, 3, and 2 respectively. In addition, students who fail to identify any of the data, condition and unknown will be scored 1. Table below provides details of the "Understand the Problem" component of the rubric.

Table 3.9

Rubric (Understand the Problem Component)

Understand the Problem				
	(1)	(2)	(3)	(4)
Identify Data – student write down the key words/clue from the problem	No data is identified	Yes, just a few of the important data are identified	Yes, most of the important data are identified	Yes, all the important data are identified
Identify Condition – students identifies conditions or assumptions that are important to understanding this problem	No conditions listed and no mention that there are none	Yes, identifies one condition, but not really important to this problem or is not hidden	Yes, identifies the conditions for the problem, but does not explain why important for this problem	Yes, identifies the conditions for the problem and explains why important for this problem
Identify Unknown – student correctly identifies what the problem is asking them to find	No unknown is identified	Completely wrong unknown is identified	Almost identifies the unknown (maybe no units stated)	Yes, correctly identifies the unknown (with units)

The second component of the rubric called "Devise a Plan", consisted of two parts, namely, selecting strategy and explanation of appropriateness. For instance, if students identify the best strategy(ies), they will be able to score 4. Students who able to find appropriate strategy(ies), but not the best one, they will be able to score 3.

Students will score 2 if they find inappropriate strategy(ies), and score 1 if they find none. Best strategy(ies) is considered when students are able to fully solve the problems accurately using the chosen strategy. Appropriate strategy(ies) is considered when students able to fully solve the problem, but not all the steps with accuracy, using the chosen strategy. Table below provides details of the "Devise a Plan" component of the rubric.

Table 3.10

Rubric (Devise a Plan Component)

Devise a Plan				
	(1)	(2)	(3)	(4)
Strategy Selection – student decides on an appropriate strategy(ies)	No strategy identified	Chooses a totally inappropriate strategy(ies)	Chooses an appropriate strategy(ies), but, maybe not the best one	Chooses best strategy(ies)
Explanation of Appropriateness – student explains why he/she thinks these strategies are appropriate/ reasonable	No explanation	Attempts to explain why choose the strategy(ies) but not substantive	Partly gives a substantive explanation	Yes, gives a complete and reasonable explanation

The third component of the rubric called "Carry out the Plan", consisted of two parts, namely, solves and check Steps. For instance, students who show some steps, but it lacks some important steps will get 2 scores. Students who show working steps but do not explain reasoning will get 3 scores and students who show all key steps and explains reasoning will get full marks, or 4. In addition, students who fail to show any attempts to solve the problem and explain reasoning will get 1 score. Table below provides details of the “Carry out the Plan” component of the rubric.

Table 3.11

Rubric (Carry out the Plan Component)

Carry out the Plan				
	(1)	(2)	(3)	(4)
Solves – student solves the problem using the chosen strategy	No attempt made to solve the problem	Attempts to solve the problem, but makes a major mistake	Answer is correct, but unit is missing, or has made a minor error	Yes, correctly solves the problem, includes units
Check Steps – student shows steps and explain reasoning	No steps shown and no reason given	Student shows some steps, but some important steps are missing	Student shows work but does not explain reasoning	Students shows all key steps and explain reasoning

The fourth component of the rubric called “Looking Back”, consisted of four parts, namely, achievement of goals, errors, different strategy, and what they learned. In achievement of goal part, students will be able to score 4 if they include a substantive reflection, score 3 marks if they include reflection but fail to give substantive reasons, score 2 marks if they partly reflect without explaining reasons for their answers, and score 1 mark if students failed to reflect on their solution. Moreover, students who reflect on a specific thing they learned that will help them with similar type of problems in future will be eligible to gain full marks. However, students will get 3 marks for reflecting on general learning, 2 marks for partly reflects, and will be scored 1 if they fail to reflect on their answer. Table below provides details of the “Looking Back” component of the rubric.

Table 3.12

Rubric (Looking Back Component)

Looking Back				
	(1)	(2)	(3)	(4)
Achievement of Goal – student reflects on whether they think they reached the goal of the problem and how they know	No reflection	Partly reflects, gives just yes or no, but no reasons for their answer	Yes, includes a reflection, but not substantive reason	Yes, includes a substantive reflection; includes how they know
Errors – student identifies their mistakes or identifies confusing steps	No reflection	Partly reflects, lists an error or two, but gives no explanation	Reflects on errors but does not give good reasons for why made an error or why could be confusing	Reflects on errors appropriately and/or identifies what steps may confuse others and why
Different Strategy – student reflects on whether there may be different strategy for this problem	No reflection	Partly reflects	Students says there probably a different strategy, but they cannot identify it or recognizes that they have a good strategy but cannot say why	Student identifies a different strategy after seeing the solution or explains why there is none
What Learned – student reflects on what they learned from solving/attempting to solve this problem	No reflection	Partly reflects	Yes, reflects on general learning	Yes, reflects on the specific thing they learned that will help them with similar type problems in future

In the end, all the scores are totalled to calculate the overall score and each group of scores is averaged to give an average "Understand the Problem" score, average "Devise a Plan" score, average "Carry out the Plan" score, and average "Looking Back" score.

3.11 Instruments Validity and Reliability

Researcher should employ qualified instruments in order to draw a conclusion based on the information they obtained. According to Gay, Mills, and Airasian (2009), the validity and reliability of the instruments are two essential elements that must be carefully established in an instrument used in the research.

3.11.1 Validity of the Instruments

Validity can be viewed as the core of any form of assessment that is trustworthy and accurate. According to Gay et al. (2009), validity is the extent to which an instrument measures, what it is supposed to measure and performs as it is designed to perform. Therefore, content validity were established for this study.

Content validity in accordance to Zohrabi (2013) is associated to a type of validity in which different elements, capabilities and behaviors are sufficient and effectively measured. It measures the comprehensiveness and representativeness of the content of an instrument. Experts in that field of research will review the research instruments and the data. Based on the reviewers' comments, the unclear and obscure questions can be revised and the complicated items reworded.

Content validity is chosen for this study because it will be the representative of the Mathematical Problem Solving Ability Test (MPSAT). Content validity of MPSAT depends on the adequacy of a specified domain of content that is sampled. It refers to the degree that the MPSAT covers the content that it is supposed to measure for example the mathematical problem solving ability among Year 4 students.

To examine the content validity in judgment stage, two very experienced mathematics teachers were chosen, one is an expert mathematics teacher with more than 15 years of experience, and another one is head of mathematics department with more than 25 years of experience. Both the teachers were asked to evaluate each item of the MPSAT based on the content suitability and assess the general domain of mathematical problem solving abilities. In addition, to make sure the items are within the ability of students to answer, the difficulty level and suitability of the terms used were studied by the panel.

Basically, they satisfied with the contents of MPSAT items which is in accordance with the Year 4 KSSR curriculum. They also found that the questions are suitable for the students' academic level and both reported that the MPSAT questions clearly assess the four steps of problem solving as stated by Mayer (1985).

3.11.2 Reliability of the Instruments

A test is considered as being reliable when it can be used by using a range of different researchers under stable conditions, with constant results and the results do not vary. Reliability reflects consistency and replicability over time. Furthermore, the more measurement errors occur, the less reliable the test are, and this considered as the degree to which a test is free from measurement errors (Mohajan, 2017). Therefore, two reliabilities namely the test-retest and inter-rater reliability were established for this study.

The first type of reliability test is the test-retest reliability, also called as stability, which answers the question, "Will the scores be stable over time." The extent to which similar results are obtained on two separate occasions or a test-retest procedure, is known as the stability of the instrument (Polit & Beck, 2010). A test or measure is administered. Sometime later the same measure or test is re-administered to the same or highly similar group.

Since it is difficult for us to trust that the data provided by the measure is an accurate representation of the participant's performance, a test-retest reliability was chosen for this study. Besides, this study aimed to examine the impact of an intervention on students' problem solving ability. Without the confidence that the measure we have chosen is reliable, it is difficult to ascertain whether differences in performance pre and post-intervention are genuine due to the intervention provided and not an artefact of the tool. A tool with low reliability can therefore mask the true effects of an intervention, which could have serious ramifications on the conclusions drawn, and therefore the future progression of that intervention.

Test-retest calculates the correlation coefficient (r) which measures the strength of relationship. A measurement tool providing the same data output at every time point would therefore produce a perfect linear correlation of Pearson's $r = 1$.

In this study, Mathematical Problem Solving Ability Test (MPSAT) was administered using the test-retest reliability to check on the stability of the instrument. Stability (test-retest) reliability was established using the data which was collected from the pilot test. 30 Year 4 students participated in two pilot tests which were administered separately during two weeks' time in order to measure the stability of the score over time. The reliability of the MPSAT was calculated by Pearson correlation. Table 3.13 presents the results of test-retest reliability.

Table 3.13

Pearson Correlation Matrix

		Judge 1	Judge 2
Judge 1	Pearson Correlation	1	.926
	Sig. (2-tailed)		.000
	N	30	30
Judge 2	Pearson Correlation	.926	1
	Sig. (2-tailed)	.000	
	N	30	30

From Table 3.13, the test-retest results indicated that the MPSAT scores are significantly stable over time (Pearson's r = .926, $p < .00005$). Therefore, the MPSAT has a strong stability.

Inter-rater reliability on the other hand is the level of agreement between raters or judges. If everyone agrees, IRR is 1 (or 100%) and if everyone disagrees, IRR is 0 (0%) (McHugh, 2012). Different trained raters, use a standard rating form, which measure the object of interest consistently. Inter-rater agreement answers the question, "Are the raters consistent in their ratings?" The reliability coefficient will be high, if the observers rated similarly. This type of reliability is important in the fact that it represents the extent to which the data collected in the study are correct representations of the variables measured.

For the purpose of this study, researcher as judge 1 and the researcher's colleague as judge 2 independently scored the students' responses to the five questions. Students' responses was obtained from the pilot test, and was photocopied in order for two judges to score them. In order to measure the reliability of measurements or ratings, the Intraclass Correlation Coefficient (ICC) was run. For the purpose of assessing inter-rater reliability and the ICC, the raters rated students' responses using the MPSAT rubric. The following table (Table 3.14) shows the result related to inter-rater reliability of the rating for MPSAT using the rubric of this study.

Table 3.14

Intraclass Correlation Coefficient

		95% Confidence Interval		F Test with True Value 0			
	Intraclass Correlation	Lower Bound	Upper Bound	Value	df1	df2	Sig
Single Measures	.926	.901	.933	56.508	29	29	.000
Average Measures	.968	.934	.963	56.508	29	29	.000

From Table 3.14, a very high degree reliability was found between two raters (judges) scoring of the MPSAT using the MPSAT rubric. The average measure ICC was .968 with a 95% confidence interval from .934 to .963, $p < .00005$. Therefore, 97% of the variance in the mean of these raters is real. It can be concluded that the inter-rater reliability results showed that MPSAT rubric scores correlated 97% of the time.

3.12 Pilot Test

The pilot study participants were 30 students of Year 4 who is taking KSSR curriculum. These are the students who have already learned the application of arithmetic involving length, area, and mass from Year 2 in their school. These students did not participate in the actual study. After consulted with their mathematics teacher, students who are eligible to take part into the pilot test because of their sufficient and relative pre- requisite knowledge were chosen for this pilot test. Furthermore, these students are not from the same school that the actual study conducted. Pilot test was conducted about 2 weeks before the data collection process. The duration between first and second test (using the same MPSAT instrument) is 2 weeks.

Researcher conducted the test under similar conditions as planned for actual data collection. Time was recorded to see how long each test is been completed by the students. Next, researcher paid attention to instances when students shy or not ready to answer or ask for clarification, as this may be an indication that questions or answers are too vague, difficult to understand or have more than one meaning. Researcher made a note of where this occurs.

The time, space and questions difficulties were considered to revise after receiving feedbacks from experts and after administering the pilot test. And also, all items were carefully designed to be solved by devising more than two plans (strategies) based on the system of coding in the MPSAT rubric.

3.13 Data Collection Procedures

This study was conducted with Year 4 private school students in Kuala Lumpur at the ending of the second term of 2018. The data was collected using Mathematical Problem Solving Ability Test (MPSAT) before and after the treatment sessions.

Permission was obtained by giving the consent letter from the selected school first for the data collection purposes. All 203 students were given a pretest (O1, O2, and O3) consisted of five problem solving questions. The pre-test took about 30 minutes. Students were required to write their answers on the MPSAT paper provided to them. Students' responses to each question were collected and scored using MPSAT rubric. Their test result was kept by the researcher.

After a week of the pre-test session, 136 students who were conveniently chosen for this study were divided to undergo treatments using two types of teaching strategies namely Mayer's problem solving Model (MM), and Mayer's problem solving Model with Visual Representation (MMVR) instruction. Ten sessions of treatments were conducted for 3 weeks. Another 67 students were remain as a control group for this study.

After a week of the last/ tenth session on the treatment, an equivalent final posttest (O4, O5, and O6) was administered with the same number of students who undergo the ten sessions of the treatment. The post-test took about 30 minutes. Students were required to write their answers on the MPSAT paper provided to them. Students' responses to each question were collected and scored using MPSAT rubric. Both pre-posttests score was compared for data analysis purposes. The outline of this study has been presented in the Table 3.2 earlier.

3.14 Data Analysis Procedures

The purpose of research is the discovery of prevalent ideas based upon the observed relationship between variables (Best & Kahn, 2009). To attain this purpose, statistical analysis was done. The descriptive analysis data are described with the assistance of statistical measurements. Description of data through mere descriptive analysis does not provide conclusive results. It only helps to describe the properties of a specific sample under study. Hence, in order to achieve conclusive results, hypotheses formulated was tested in the research.

Prior to the testing of any hypotheses associated to the research questions of this study, the assumptions required to run hypothesis tests were examined. This is because, different hypothesis tests make distinctive assumptions about the distribution of the random variable being sampled in the data. These assumptions must be considered when selecting a test and when interpreting the results. For example, the *z*-test (ztest) and the *t*-test (ttest) both assume that the data are independently sampled from a normal distribution.

If the normality assumption is violated, an alternative technique will be used, which is a non-parametric test will be conducted. If the data are independently sampled from a normal distribution, then the hypotheses are tested statistically with the assistance of statistical techniques. Inferential statistical techniques are used to test the hypotheses and on that basis, it is determined whether the hypotheses are accepted or not. This system of analysis that follows description of data to provide conclusive results is called inferential analysis. Thus, in this study, the data obtained was analyzed using the inferential analysis procedure. The data which obtained from the Mathematical Problem Solving Ability Test (MPSAT) is a score which evaluated using the MPSAT Rubric (See Appendix D).

3.15 Scoring

Data which has been collected from the students' responses using Mathematical Problem Solving Ability Test (MPSAT) were scored. The researcher went through the students' responses one by one and gave scores that match with the MPSAT rubric accordingly. That means, if the answering characteristics shown in the responses match any of the rubric components, then the appropriate score will be assigned.

According to the MPSAT rubric, "Understand the Problem" consists of three parts, whereas "Devise a Plan" and Carry out the Plan" consist of two parts respectively. "Looking Back" consists of four parts. Therefore, students are more likely to obtain more scores in the "Looking Back" section in comparison to other sections. In order to eliminate the threat of this inequality, the mean score of "Understand the Problem, "Devise a Plan", "Carry out the Plan", and "Looking Back" sections, and also the total of the overall score was calculated.

3.16 Analysis of the Test Scores

In this study, the scores of the pre- and posttests were analyzed inferentially to answer five research questions. The statistical analysis was done by means of the Statistical Package for the Social Sciences (SPSS) version 19. In order to conduct ANCOVA tests to explore the research questions (Q3 and Q5) of this study, the General Linear Model (GLM) was used. In addition, the post hoc test was run when the overall significant difference in group means was obtained. Therefore, the post hoc test was conducted to confirm whether the differences occurred between groups. The post hoc test was run with Bonferroni adjustment so as to control for Type I error across the three pairwise comparisons, thus, the adjustment is for the number of possible pairwise post hoc comparison (Pallant, 2010).

The analysis was done using several different hypotheses tests and the following presents the types of test conducted for each research question.

Table 3.15

Data Analysis Method of Each Research Questions

Research Questions	Assumptions Tested	Statistical Test
RQ1: Does the mathematical problem solving ability of Year 4 students in MMVR group improve significantly after the MMVR treatment?	• Dependent variable must be continuous (interval/ratio). • The observations are independent of one another. • The dependent variable should be approximately normally distributed. • The dependent variable should not contain any outliers.	<u>If Assumption Pass</u> Paired-samples *t*-test One-tailed <u>If Assumption Fail</u> Wilcoxon test
RQ2: Does the understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability of Year 4 students in MMVR group improve significantly after the MMVR treatment?	• Dependent variables must be continuous (interval/ratio). • The observations are independent of one another. • The dependent variables should be approximately normally distributed. • The dependent variables should not contain any outliers.	<u>If Assumption Pass</u> Paired-samples *t*-test One-tailed <u>If Assumption Fail</u> Wilcoxon test
RQ3: Is there any significant difference in the mathematical problem solving ability of Year 4 students in MM group, MMVR group and control group after the treatments, after controlling the pretest score?	• Dependent variable and covariate variable(s) should be measured on a continuous scale. • Independent variable should consist of two or more categorical, independent groups. • Independence of observations, which means that there is no relationship between the observations in each group or between the groups themselves. • No significant outliers. • Residuals should be approximately normally distributed for each category of the independent variable. • Needs to be homogeneity of variances.	<u>If Assumption Pass</u> One-way ANCOVA <u>If Assumption Fail</u> Kruskal-Wallis test

Table 3.15 (continued)

Research Questions	Assumptions Tested	Statistical Test
RQ3: Is there any significant difference in the mathematical problem solving ability of Year 4 students in MM group, MMVR group and control group after the treatments, after controlling the pretest score?	• Needs to be homogeneity of regression slopes, which means that there is no interaction between the covariate and the independent variable. • Covariate should be linearly related to the dependent variable at each level of the independent variable. • Needs to be homoscedasticity. • Covariates are different between independent variable.	<u>If Assumption Pass</u> One-way ANCOVA <u>If Assumption Fail</u> Kruskal-Wallis test
RQ4: Is there any significant difference in the understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability of Year 4 students among MM instruction group, MMVR instruction group and control group after the treatments, after controlling the pretest scores?	• Two or more dependent variables should be measured at the interval or ratio level. • One independent variable should consist of two or more categorical, independent groups. • One or more covariates are all continuous variables. • No relationship between the observations in each group of the independent variable or between the groups themselves. • Homogeneity of regression slopes. • Homogeneity of variances and covariance. • No significant multivariate outliers in the groups of your independent variable in terms of each dependent variable. • No significant univariate outliers in the groups of your independent variable in terms of each dependent variable. • Here should be multivariate normality.	<u>If Assumption Pass</u> One-way MANCOVA <u>If Assumption Fail</u> Kruskal-Wallis test

Table 3.15 (continued)

Research Questions	Assumptions Tested	Statistical Test
RQ4: Is there any significant difference in the understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability of Year 4 students among MM instruction group, MMVR instruction group and control group after the treatments, after controlling the pretest scores?	• Covariates are different between independent variable. Homogeneity of covariance matrices.	If Assumption Pass One-way MANCOVA If Assumption Fail Kruskal-Wallis test
RQ5: Is there any significant interaction between problem solving teaching strategy and gender on Year 4 students' mathematical problem solving ability after controlling the pretest score?	• Dependent variables should be measured at the interval or ratio level. • One independent variable should consist of two or more categorical, independent groups. • All samples are drawn independently of each other. • No significant outliers. • Residuals should be approximately normally distributed for each combination of groups of the two independent variables. • All populations have a common variance. • Covariate should be linearly related to the dependent variable for each combination of groups of the independent variables. • Needs to be homoscedasticity. • Homogeneity of regression slopes.	If Assumption Pass Two-way ANCOVA If Assumption Fail Friedman test

The first research question was analyzed using a one-tailed paired-samples *t*-test. The dependent *t*-test called the paired-samples t-test in SPSS statistics compares the means between two related groups on the same continuum, dependent variable. Therefore, to test whether the means of overall scores of problem solving ability on the students' posttests will be improved after using MMVR teaching strategy among students in MMVR group, a paired *t*-test was run. In addition, based on the research hypothesis, there is a predicted direction indicated that the mean of post-test score will be higher than the mean of pre-test scores in MMVR group. Hence, the type of the test was a one- tailed test.

For second question, a one-tailed paired-samples *t*-test was conducted using SPSS to test whether the means of overall scores of understanding of the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability on students' post-tests will be improved after using MMVR teaching strategy among students in MMVR group. Furthermore, there is a predicted direction based on the research hypothesis indicated that the mean of post-test score will be higher than the mean of pre-test scores in MMVR group. Hence, the type of the test was a one- tailed test.

In order to answer the third research question, a One-way ANCOVA was conducted to determine whether the mean of the posttest scores of Mathematical Problem Solving Ability Test (MPSAT) of Year 4 students is different between the MM group, MMVR group, and control group after controlling for the pre-test scores. The ANCOVA has the additional benefit than One-way ANOVA. It allows the researcher to statistically control for a third variable, which is sometimes known as a confounding variable, which may be negatively affecting the results (Pallant, 2010). This third variable that could be confounding the results is the covariate that the researcher includes in an ANCOVA. ANCOVA is in purpose to increase the precision of comparison between groups by reducing within-group error variance, and to adjust comparisons between groups for imbalances by eliminating confounding variables. Therefore, by making the result of the initial pretest of the three groups a covariate, the ANCOVA allows the researcher to control for the effect of these initial differences statistically so that the researcher can get a more accurate picture of the effect of the two treatments.

For research question four, a One-way multivariate analysis of covariance (MANCOVA) was conducted to determine whether the mean of total average of the posttest scores of understand the problem ability, devise a plan ability, carry out the plan ability, and looking back ability of Year 4 students in MPSAT is different between the MM group, MMVR group, and control group after controlling for pre-test scores. Unlike the One-way ANCOVA, which tests for differences in the mean values of the dependent variable between the groups of the independent variable, the One-way MANCOVA tests for the linear composite or vector of the means between the groups of the independent variable (Pallant, 2010). Essentially, two or more dependent variables are combined to form a new dependent variable in order to maximize the differences between the groups of the independent variable (Pallant, 2010).

For research question five, a 2x3 factorial analysis of covariance (ANCOVA) was conducted to compare the mean differences between groups that have been split on two independent variables called factors while controlling for the effects of the covariate. Therefore, by making the result of the initial pretest scores of the three groups a covariate, the two-way ANCOVA allows the researcher to determine whether the effect of one of the independent variables of this study, which is gender or teaching treatment, on the dependent variable which is the problem solving ability is the same for all values of another independent variable and vice versa.

Finally, a Post hoc tests with Bonferroni adjustment was conducted for research question 3 and 4. Post-hoc is meant to analyze the results of the experimental data. They are often based on a familywise error rate (FEW). FWE is also known as alpha inflation or cumulative Type I error (Kutner, Nachtsheim, Li & William, 2005). FWE error represents the probability that any one of a set of comparisons or significance tests is a Type I error.

Multiple comparisons arise when a statistical analysis involves multiple simultaneous statistical tests, each of which has a potential to produce a "discovery" (Kutner et al., 2005). A stated confidence level generally applies only to each test considered individually, but often it is desirable to have a confidence level for the whole family of simultaneous tests. Failure to compensate for multiple comparisons can have important real-world consequences. As more tests are conducted, the likelihood that one or more are significant just due to chance (Type I error) increases. Hence, the Bonferroni simply calculates a new pairwise alpha to keep the familywise alpha value at 0.05 (or another specified value).

The Bonferroni is the most commonly used post hoc test, because it is highly flexible, very simple to compute, and can be used with any type of statistical test. For research question 3 and 4, the teaching strategies (MM and MMVR) are a new way to teach students on problem solving, and the control group is the traditional or standard way of teaching problem solving in schools. Students' problem solving ability are compared in terms of MM, MMVR and traditional way of teaching strategies. As more strategies are compared, it becomes increasingly likely that the problem solving ability will appear to differ on at least one strategy due to sampling error alone.

Therefore, post hoc test was conducted to confirm where the differences occurred between groups. The post hoc test was run with Bonferroni adjustment so as to control for Type I error across the three pairwise comparisons; thus, the adjustment is for the number of possible pairwise post hoc comparison (Pallant, 2010).

3.17 Ethics

Both students and their parents should give active consent (Johnson & Christensen, 2008). Active consent means that the parents and students had to sign and return the forms agreeing to be part of the study, as opposed to passive consent (Johnson & Christensen, 2008) where participants only return forms if they wish not to be part of the research. A consent letter outlining what the purpose of the study was, and what the students will be requested to do were given to both the students and their parents, and were asked for permission (See Appendix F).

The students will also had the study explained verbally to ensure they understood what is being asked from them. Part of the conditions that all participants, including the principal, teachers, students, and parents agreed to, was the right to withdraw consent at any time up to a given date when all data had been collected. Pseudonyms will be used in the document to ensure confidentiality of the participants.

The most important issue confronting researchers is the ethical treatment of research participants and the right to protection from physical and mental harm (Johnson & Christensen, 2008). In order to have the minimal effect on the participants, the researcher consulted with teachers when working out session times to ensure they are not missing out on what they or the teacher deemed to be essential or fun as well as ensuring the least interruption to the students. To ensure the participants did not miss out on their normal mathematics instruction which their peers are receiving, all sessions were performed outside the normal mathematics class sessions.

The researcher was the only person to have access to the coding system. The coding system consisted of a table containing each student's name along with a randomly assigned student's number, which was the three-digit number between 001 and 200 randomly chosen using an online random number generator. The table was stored separately from the data in a hard copy format and was stored in a locked filing cabinet in the researcher's locked office. No one other than the researcher will see this table and the student names were not used in data analyses or in the reporting of results.

Finally, researcher avoided any discrimination against students on the basis of sex, race, ethnicity, or other factors that are not related to their scientific competence and integrity. Researcher aimed in educating, mentoring, and advising students, as well as promoting their welfare and allowing them to make their personal decisions during the intervention session.

3.18 Summary

This chapter has outlined and justified the research paradigm and methods used in this study. In addition, this chapter has also discussed the participants, the method, and the data collecting tools used as well as how the data will be analyzed. Ethical considerations as well as validity and trustworthiness of the design and data were also discussed.

The next chapter will introduce the study results drawn from analysis of the data collected using the statistical analysis tools mentioned in this chapter. It presents the findings resulting from an intervention of three teaching strategies in enhancing Year 4 students' problem solving ability.

CHAPTER 4

RESULTS

4.1 Introduction

This chapter presents the demographic profile of samples and the results of the data analysis of the study. It presents the results of the tests used for each of the hypotheses related to the research questions. Prior to the testing of each of these hypotheses related to the research questions of this study, the assumptions required to run hypotheses testing were examined. All hypotheses were evaluated at the 5% level of significance.

4.2 Demographic Profile of Samples

Descriptive analysis was conducted on the demographic data of the independent variables, included treatments and gender. This section presents the results of this analysis.

Analysis by Gender

Table 4.1 below shows the distribution of samples according to gender.

Table 4.1

Samples' Distribution by Gender

	Before Data Screening	After Data Screening
Gender	*f*	*f*
Male	101 (49.8%)	87 (49.7%)
Female	102 (50.2%)	88 (50.3%)
Total	203 (100%)	175 (100%)

As shown in Table 4.1 above, out of 203 samples selected to participate in this study, 101 (49.8%) are male students, while the remaining 102 (50.2%) are female students. After the data screening due to outliers, the total number of male and female students reduced to 175. Out of 175 students, 87 (49.7%) are male students, while the remaining 88 (50.3%) are female students. Outliers are unusual values in a dataset, and they can distort statistical analyses and violate the assumptions if it is not been removed from the dataset (Lalitha & Kumar,2012).

<u>Analysis by Treatment</u>

Table 4.2 below shows the distribution of samples according to treatment.

Table 4.2

Samples' Distribution by Treatment

Treatment	Before Data Screening *f*	After Data Screening *f*
MM	68 (33.5%)	57 (32.6%)
MMVR	68 (33.5%)	58 (33.1%)
Control	67 (33.0%)	60 (34.3%)
Total	203 (100%)	175 (100%)

As shown in Table 4.2 above, out of 203 samples, 68 (33.5%) students were assigned to be in MM and MMVR treatment groups respectively, while the remaining 67 (33.0%) students were assigned to be in a control group. After the data screening due to outliers, the total number of students reduced to 175. Out of 175 students, 57 (32.6%) students were from MM treatment group, 58 (33.1%) students were from MMVR treatment group, while the remaining 60 (34.3%) students were from control group.

The analysis of research questions for this study is based on the sample size of 175, which is after the data screening. According to Krokhmal (2011), if the population follows the normal distribution, then the sample size, N, can be either small ($N < 30$) or large ($N > 30$). Therefore, if we assume that our populations are normal, then we are always safe when making the parametric assumptions about the sampling distribution, regardless of sample size (Krokhmal, 2011).

4.3 Results of Analysis for Research Question 1

This section presents the results of the paired-samples *t*-test used to answer research question one which stated 'Does the mathematical problem solving ability of Year 4 students in Mayer's problem solving Model with visual representation (MMVR) group improve significantly after the Mayer's Problem Solving Model with Visual Representation teaching strategy (MMVR) treatment?'. There are four assumptions that need to be tested prior to the analysis using paired-sample *t*-test. The following presents the results of the tests of assumption.

<u>Assumption 1: Dependent variable must be continuous (interval/ratio)</u>

The scores obtained from MPSAT in MMVR group are from an interval scale. Therefore, the first assumption is met.

<u>Assumption 2: The observations are independent of one another</u>

This assumption was tested using a scatterplot for three groups namely MM, MMVR, and Control group. The following figure shows the result of the analysis.

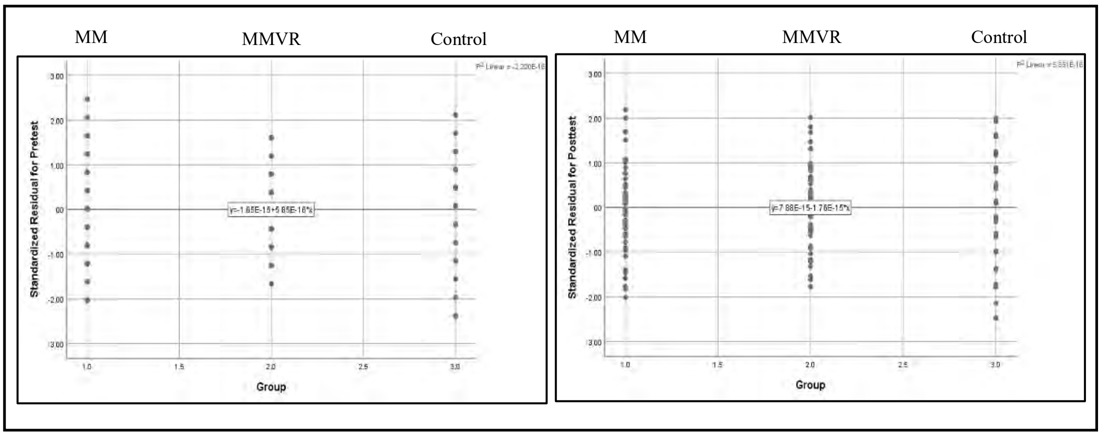


Figure 4.1. Assumption of independent observations for pretest and posttest scores of problem solving ability for MM, MMVR, and Control groups

Figure 4.1 shows that the points fell relatively randomly above and below the horizontal reference line at 0 for pre- and post-test of all three groups. Therefore, the assumption of independence has been met.

<u>Assumption 3: The dependent variable should be approximately normally distributed</u>

This assumption was tested using the Kolmogorov-Smirnov test. The Kolmogorov-Smirnov test is used to decide if a sample comes from a population with a specific distribution (Chakravart, Laha, and Roy, 1967). The Kolmogorov-Smirnov test has the advantage of making no assumption about the distribution of data. Also, it is an exact test (the chi-square goodness-of-fit test depends on an adequate sample size for the approximations to be valid) (Chakravart, Laha, and Roy, 1967). The following table shows the result of the analysis.

Table 4.3

Assumption of Normality for Pretest and Posttest Scores of Problem Solving Ability for MMVR Group

	Group	*df*	*p*
Pretest	MMVR	58	.20
Posttest	MMVR	58	.061

From Table 4.3, the MMVR group resulting a *p*-value for pretest which is .20 ($p > .05$), and a *p*-value for posttest which is .061 ($p > .05$). Therefore, the null hypotheses stating that the sample data comes from normal population is retained at the 5% of significance level. Hence, the assumption of normality is met.

<u>Assumption 4: The dependent variable should not contain any outliers</u>

This assumption was tested using a boxplot. The following figure shows the result of the analysis.

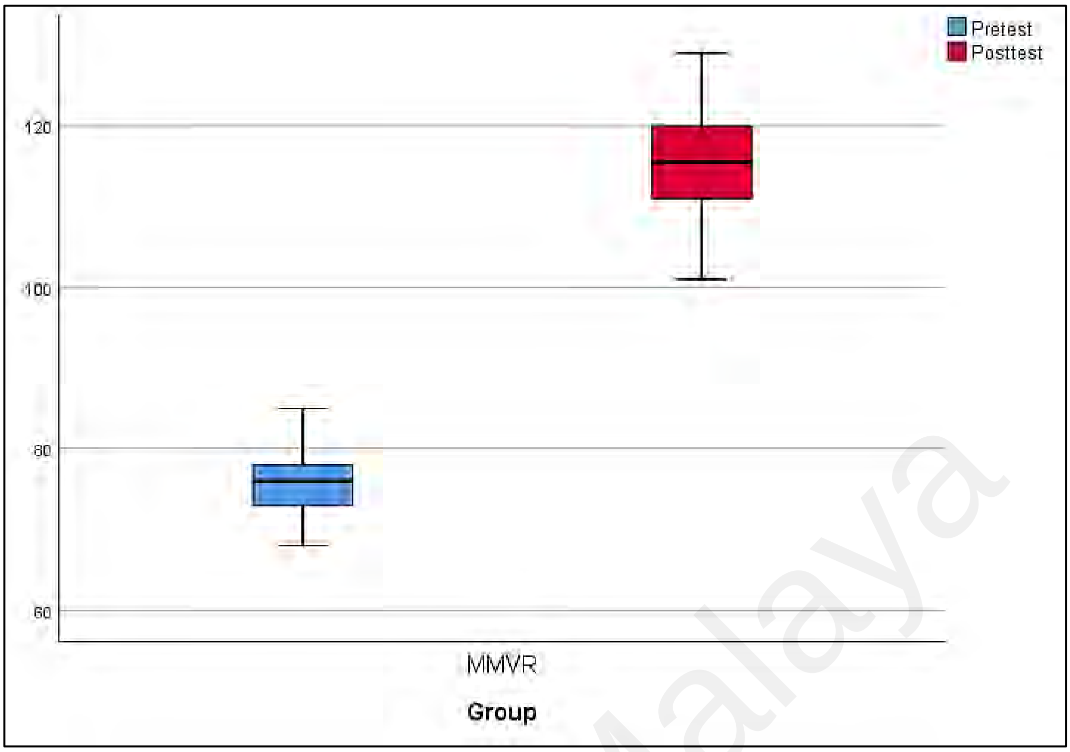


Figure 4.2. Assumption of no outlier for pretest and posttest scores of problem solving ability for MMVR group

From Figure 4.2, the result of testing the assumption revealed that there were no outliers in MMVR group for pretest and posttest scores. Therefore, the assumption of normality is met.

Since the required assumptions were met, the Paired Samples t-Test was conducted, and the following section will present the results of this test.

Results of Paired Sample t-Test analysis

Table below presents the descriptive analysis for MMVR group.

Table 4.4

Paired Samples Statistics for Pretest and Posttest Scores of Problem Solving Ability for MMVR Group

	M	*N*	*SD*
Pretest	75.41	58	3.70
Posttest	115.86	58	6.47

Table 4.4 shows the mean scores of MPSAT in MMVR group for pretest (*M* = 75.41, *SD* = 3.70) and posttest (*M* = 115.86, *SD* = 6.47) is different.

Table below presents the results of Paired Samples *t*-Test for MMVR group.

Table 4.5

Paired Samples t-Test analysis for Pretest and Posttest Scores of Problem Solving Ability for MMVR Group

				95% Confidence Interval of the Difference				
	M	*SD*	*SEM*	Lower	Upper	*t*	*df*	*p*
Posttest - Pretest	40.45	7.51	.99	38.47	42.42	41.05	57	<.0005

Table 4.5 indicates the statistically significant mean increase for mathematical problem solving ability in MMVR group, 40.45, 95% CI [38.47, 42.42], t (57) = 41.05, $p < .0005$, with an effect size $d = 5.39$. Based on the 5% level of significance, the null hypothesis that the mean of the pretest scores and posttest scores of Year 4 students in MPSAT are not different in MMVR group is rejected. Thus, the data provide sufficient evidence to conclude that the Year 4 students' mathematical problem solving abilities in MMVR group improved significantly after MMVR treatment.

4.4 Results of Analysis for Research Question 2

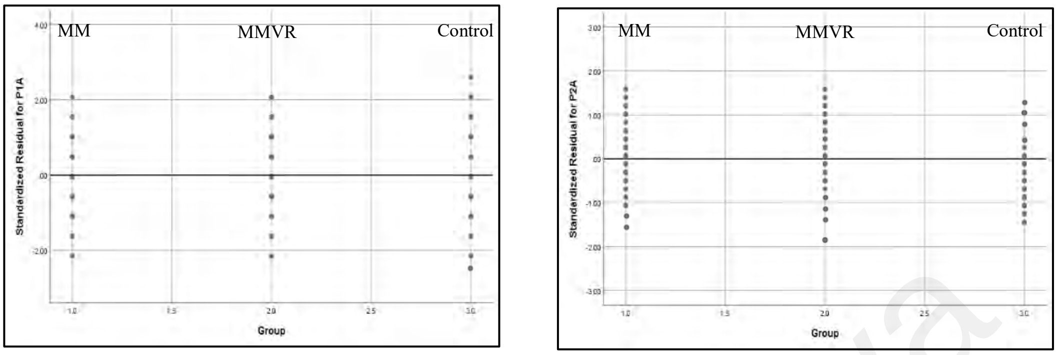
This section presents the results of the paired-samples t-test used to answer research question two which stated 'Does the understanding of the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability of Year 4 students in MMVR group improve significantly after the MMVR treatment?'. There are four assumptions that need to be tested prior to the analysis using paired-sample t-test. The following presents the results of the tests of assumption.

Assumption 1: Dependent variable must be continuous (interval/ratio)

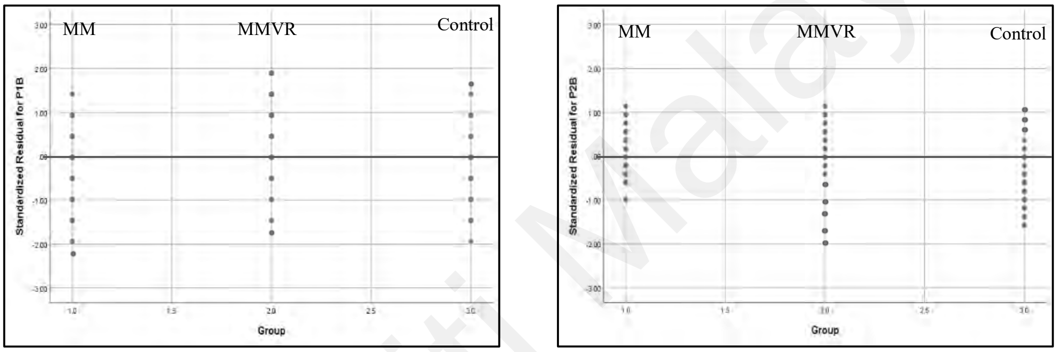
The scores obtained from MPSAT in MMVR group are from an interval scale. Therefore, the first assumption is met.

Assumption 2: The observations are independent of one another in each group of the independent variable

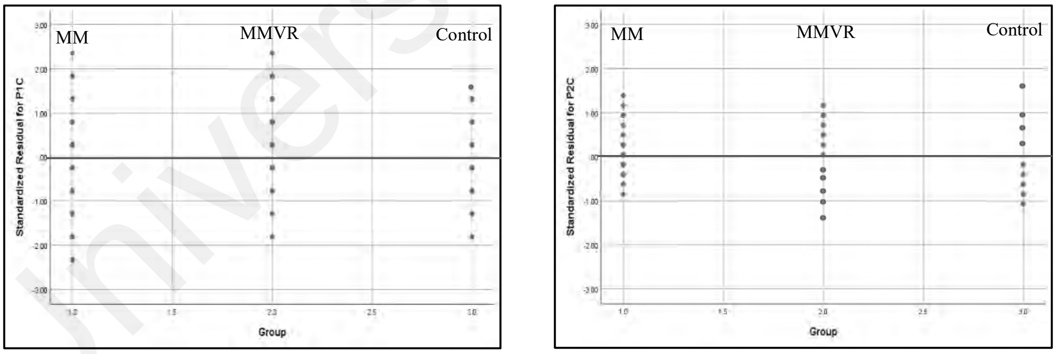
This assumption was tested using a scatterplot for understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability for three groups namely MM group, MMVR group, and Control group. The following figure shows the result of the analysis.



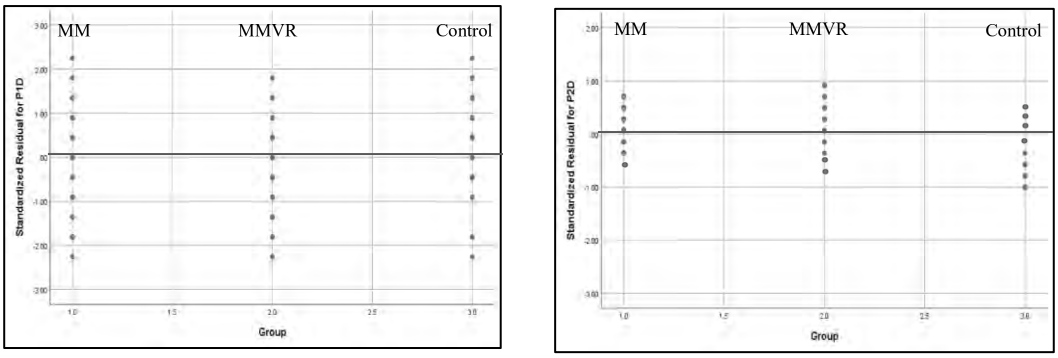
Understand the Problem



Devise a Plan



Carry Out the Plan



Looking Back

Figure 4.3. Assumption of independent observations for pretest and posttest scores of understanding of the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability for MM, MMVR, and Control groups

Figure 4.3 shows that the points fell relatively randomly above and below the horizontal reference line at 0 for pre- and post-test for MM group, MMVR group, and Control group. Therefore, the assumption of independence has been met.

<u>Assumption 3: The dependent variable should be approximately normally distributed</u>

This assumption was tested using the Kolmogorov-Smirnov test. The following table shows the result of the analysis.

Table 4.6

Assumption of Normality for Pretest and Posttest Scores of Understanding of the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MMVR Group

		Group	*df*	*p*
Pretest Total Average	Understand the Problem	MMVR	58	.090
	Devise a Plan	MMVR	58	.101
	Carry out the Plan	MMVR	58	.313
	Looking Back	MMVR	58	.094
Posttest Total Average	Understand the Problem	MMVR	58	.053
	Devise a Plan	MMVR	58	.174
	Carry out the Plan	MMVR	58	.200
	Looking Back	MMVR	58	.061

From Table 4.6, the MMVR group resulting a p-value for pretest total average understand the problem ability which is .090 (is greater than the significance level of .05); a *p*-value for pretest total average devise a plan ability which is .101 ($p > .05$); a *p*-value for pretest total average carry out the plan ability which is .313 ($p > .05$); and a *p*-value for pretest total average looking back ability which is .094 ($p > .05$). The MMVR group resulting a *p*-value for posttest total average understand the problem ability which is .053 ($p > .05$); a p-value for posttest total average devise a plan ability which is .17 ($p > .05$); a p-value for posttest total average carry out the plan ability which is .200 ($p > .05$); and a p-value for posttest total average looking back ability which is .61 ($p > .05$). Therefore, the null hypotheses stating that the sample data comes from normal population is retained at the 5% of significance level. Hence, the assumption of normality is met.

<u>Assumption 4: The dependent variable should not contain any outliers</u>

This assumption was tested using a boxplot. The following figures show the results of the analysis.

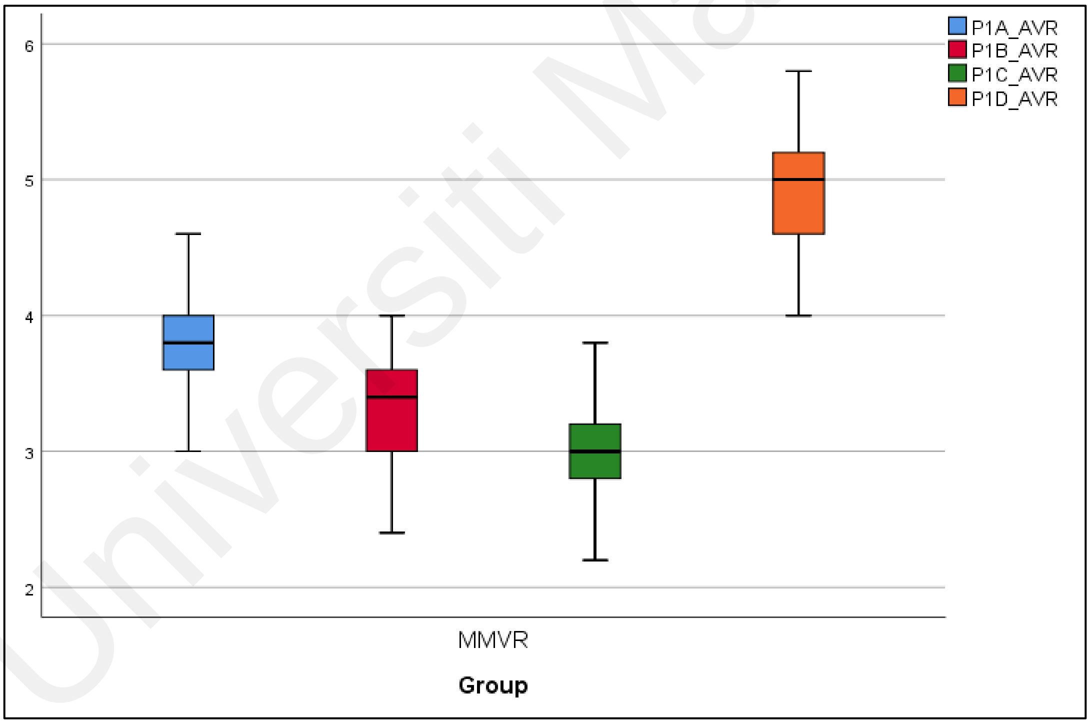


Figure 4.4. Assumption of no outlier for pretest total average scores of problem solving ability for MMVR group

From Figure 4.4, the result of testing the assumption revealed that there were no outliers in MMVR group for pretest total average scores for understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability. Therefore, the assumption of normality is met for pretest total average scores in MMVR group.

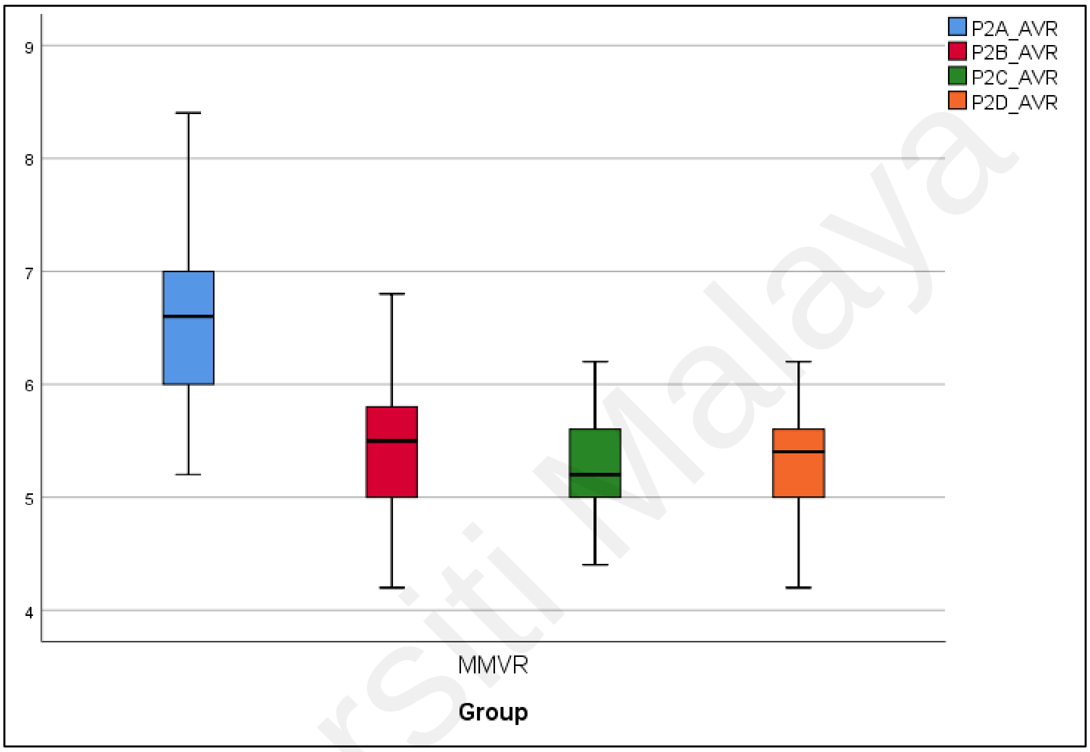


Figure 4.5. Assumption of no outlier for posttest total average scores of problem solving ability for MMVR group

From Figure 4.5, the result of testing the assumption revealed that there were no outliers in MMVR group for posttest total average scores for understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability. Therefore, the assumption of normality is met for posttest total average scores in MMVR group.

Since the required assumptions were met, the Paired Samples t-Test was conducted, and the following section will present the results of this test.

Results of Paired Sample t-Test analysis

Table below presents the descriptive analysis for MMVR group.

Table 4.7

Paired Samples Statistics for Pretest and Posttest Scores of Understanding of the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MMVR Group

		M	*N*	*SD*	*SEM*
Understand the Problem	Pretest Total Average	3.83	58	.34	.04
	Posttest Total Average	6.58	58	.79	.10
Devise a Plan	Pretest Total Average	3.32	58	.36	.047
	Posttest Total Average	5.50	58	.62	.082
Carry out the Plan	Pretest Total Average	3.06	58	.36	.047
	Posttest Total Average	5.25	58	.43	.056
Looking Back	Pretest Total Average	4.88	58	.42	.055
	Posttest Total Average	5.27	58	.44	.058

Table 4.7 shows the mean scores of MPSAT in MMVR group for pretest total average of understand the problem (*M* = 3.83, *SD* = .34) and posttest total average of understand the problem (*M* = 6.58, *SD* = .79) is different; for pretest total average of devise a plan (*M* = 3.32, *SD* = .36) and posttest total average of devise a plan (*M* = 5.50, *SD* = .62) is different; for pretest total average of carry out the problem (*M* = 3.06, *SD* = .36) and posttest total average of carry out the plan (*M* = 5.25, *SD* = .43) is different; and for pretest total average of looing back (*M* = 4.88, *SD* = .42) and posttest total average of looking back (*M* = 5.27, *SD* = .44) not greatly differed.

Table below presents the results of Paired Samples *t*-Test for MMVR group.

Table 4.8

Paired Samples t-Test analysis for Pretest and Posttest Scores of Understanding of the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MMVR Group

					95% Confidence Interval of the Difference				
		M	*SD*	*SEM*	Lower	Upper	*t*	*df*	*p*
Understand the Problem Ability	Posttest Total Average – Pretest Total Average	2.76	.47	.061	2.63	2.88	44.98	57	.000
Devise a Plan Ability	Posttest Total Average – Pretest Total Average	2.19	.29	.037	2.11	2.26	58.43	57	.000

Table 4.8 (continued)

					95% Confidence Interval of the Difference				
		M	*SD*	*SEM*	Lower	Upper	*t*	*df*	*p*
Carry out the Plan Ability	Posttest Total Average – Pretest Total Average	2.19	.12	.016	2.16	2.22	141.23	57	.000
Looking Back Ability	Posttest Total Average – Pretest Total Average	.39	.079	.010	.37	.41	37.67	57	.000

Table 4.8 shows the MMVR group resulting a statistically significant mean increase for understand the problem ability, 2.76, 95% CI [2.63, 2.88], t (57) = 44.98, $p < .0005$, with an effect size $d = 5.87$; for devise a plan ability, 2.19, 95% CI [2.11, 2.26], t (57) = 58.43, $p < .0005$, with large effect size $d = 7.55$; and for carry out the plan ability, 2.19, 95% CI [2.16, 2.22], t (57) = 141.23, $p < .0005$, with large effect size $d = 18.25$. Meanwhile, there is no significant mean increase for looking back ability, 0.39, 95%

CI [0.37, 0.41], *t* (57) = 37.67, *p* = .11 (*p* > .05) with an effect size of 4.94. Based on the 5% level of significance, the null hypothesis that the mean of the pretest total average scores and posttest total average scores of Year 4 students in MPSAT are not different in MMVR group is rejected for understand the problem ability, devise a plan ability, and carrying out the plan ability. The data provide evidence to conclude that the understand the problem ability, devise a plan ability, and carrying out the plan ability of Year 4 students in MMVR group improved significantly after MMVR treatment.

4.5 Results of Analysis for Research Question 3

This section presents the results of the test of assumption and the result of one-way ANCOVA used to answer research question three which stated 'Is there any significant difference in the mathematical problem solving ability of Year 4 students in Mayer's Problem Solving Model (MM) instruction group, Mayer's Problem Solving Model with Visual Representation (MMVR) instruction group and control group after the treatments, when controlled for the pretest?'. There are ten assumptions that need to be tested prior to the analysis using one-way ANCOVA. The following presents the results of the tests of assumption.

Assumption 1: Dependent variable and covariate variable(s) should be measured on a continuous scale.

The scores obtained from MPSAT in MM, MMVR, and Control groups are from a continuous interval scale. Therefore, the first assumption is met.

Assumption 2: Independent variable should consist of two or more categorical, independent groups

The scores for this question derived from three categorical independent groups which are MM, MMVR, and Control groups. Therefore, the second assumption is met.

Assumption 3: Independence of observations, which means that there is no relationship between the observations in each group or between the groups themselves.

This assumption was tested using a scatterplot for three groups namely MM, MMVR, and Control group. The following figure shows the result of the analysis.

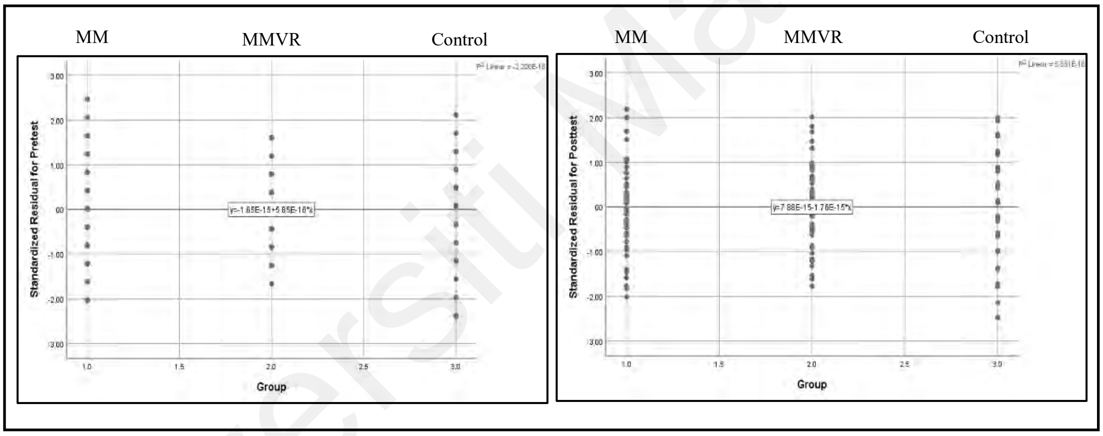


Figure 4.6. Assumption of independent observations for pretest and posttest scores of problem solving ability for MM, MMVR, and Control groups

From figure 4.6, it shows that the points fell relatively randomly above and below the horizontal reference line at 0 for all the three groups. Therefore, the assumption of independence has been met.

Assumption 4: No significant outliers

This assumption was tested using a boxplot for posttest scores of three groups namely MM, MMVR, and Control group. The following figure shows the result of the analysis.

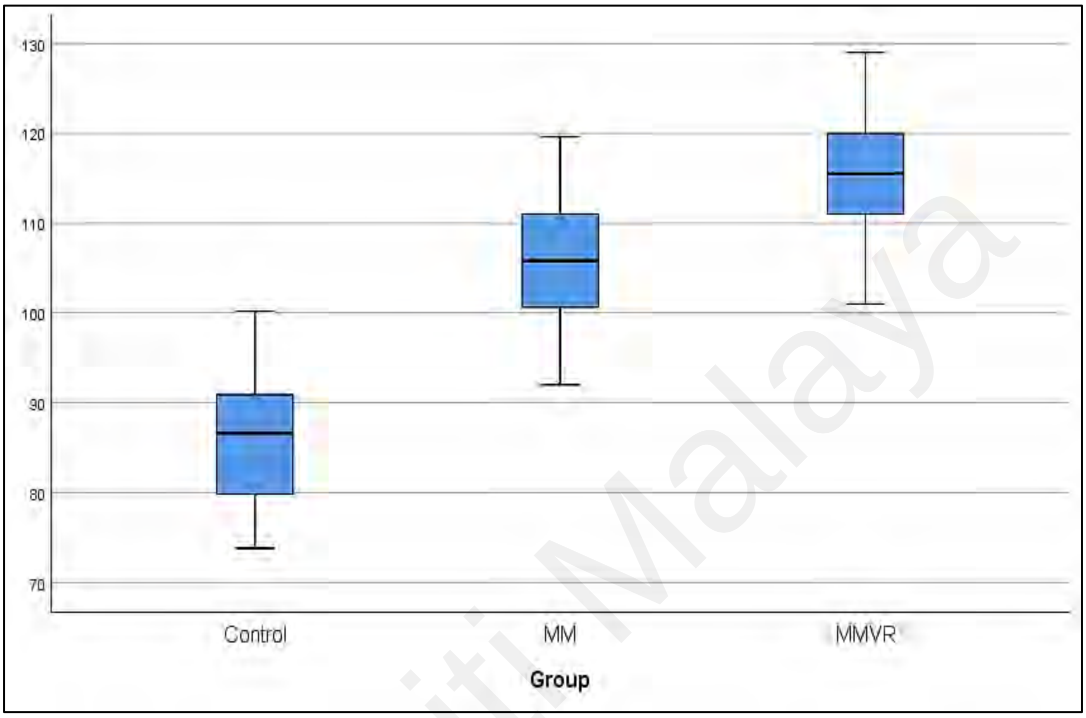


Figure 4.7. Assumption of no significant outliers for posttest scores of problem solving ability for MM, MMVR, and Control groups

The result of testing the assumption revealed that there were no outliers in MM, MMVR, and Control group for posttest scores. Therefore, the assumption of normality is met.

Assumption 5: Residuals should be approximately normally distributed for each category of the independent variable

This assumption was tested using Kolmogorov-Smirnov test. The following table shows the result of the analysis.

Table 4.9

Assumption of Normality for Posttest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

	Group	*df*	*p*
Standardized Residual for Posttest	MM	57	.065
	MMVR	58	.71
	Control	60	.096

From Table 4.9, the MM group resulting a *p*-value for standardized residual for posttest which is .065 ($p > .05$), a *p*-value for standardized residual for posttest for MMVR group which is .071 ($p > .05$), and a *p*-value for standardized residual for posttest for Control group which is .096 ($p > .05$). Therefore, the null hypotheses stating that the sample data comes from normal population is retained at the 5% of significance level for the three groups. Hence, the assumption of normality is met.

Assumption 6: Needs to be homogeneity of variances

This assumption was tested using Levene's test of homogeneity of variance - covariance. The following table shows the result of the analysis.

Table 4.10

Assumption of Homogeneity of Variance for Posttest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

F	*df1*	*df2*	*p*
.66	2	172	.52

From Table 4.10, there is a homogeneity of variance-covariance, as assessed by Levene's test of homogeneity of variance, F (2, 172) = .66, p = .52 (p > .05). This means that the variance of the dependent variable is equal across groups. Hence, the assumption of homogeneity of variance is met.

Assumption 7: Needs to be homogeneity of regression slopes, which means that there is no interaction between the covariate and the independent variable

This assumption was tested using a General Linear Model (GLM) procedure. The following table shows the result of the analysis.

Table 4.11

Assumption of Homogeneity of Regression Slopes for Posttest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

Source	*df*	*F*	*p*
Group * Pretest	2	.18	.84
Error	169		

From Table 4.11, there is no interaction between the covariate (pretest) and the independent variable (group) since the interaction term was not statistically significant, F (2, 169) = .18, p = .84 (p > .05).

<u>Assumption 8: Covariate should be linearly related to the dependent variable at each level of the independent variable</u>

This assumption was tested using a scatterplot for three groups namely MM, MMVR, and Control group. The following figure shows the result of the analysis.

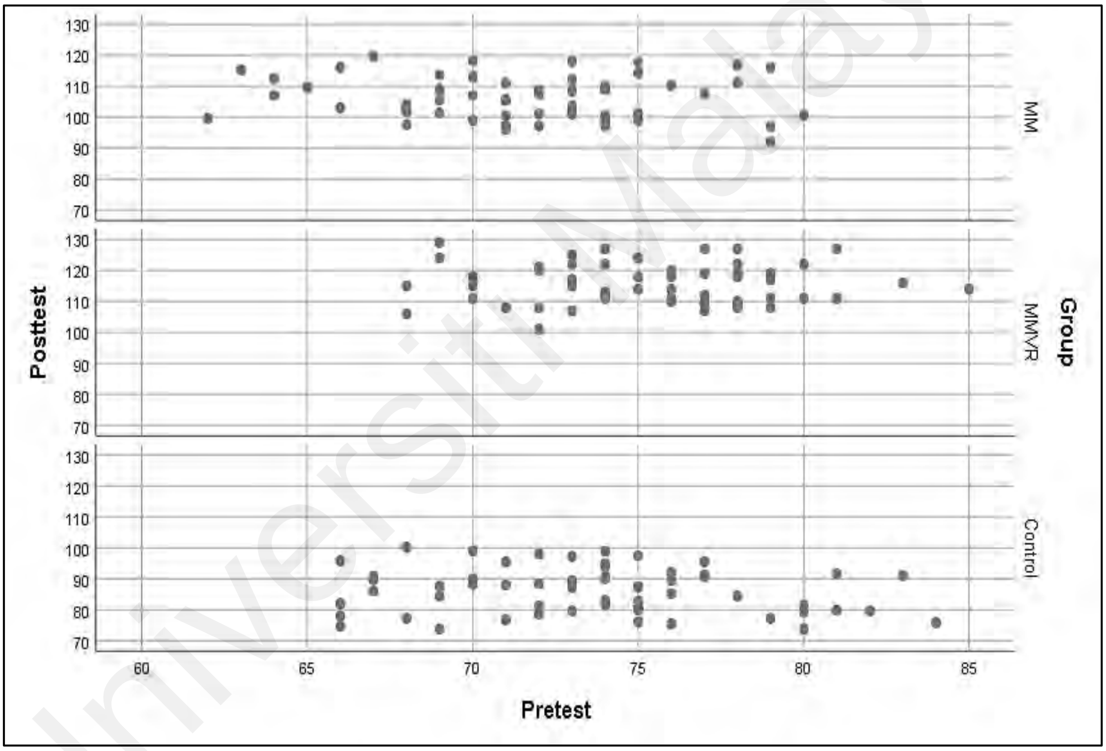


Figure 4.8. Assumption of linearity for posttest scores of problem solving ability for MM, MMVR, and Control groups

From Figure 4.8, it shows that the assumption of linearity was fulfilled because there was a linear relationship between the post-test and pre-test scores as a covariate for MM, MMVR, and control groups. Therefore, the assumption of linearity is met.

Assumption 9: Needs to be homoscedasticity

This assumption was tested using a scatterplot for three groups namely MM, MMVR, and Control group. The following figure shows the result of the analysis.

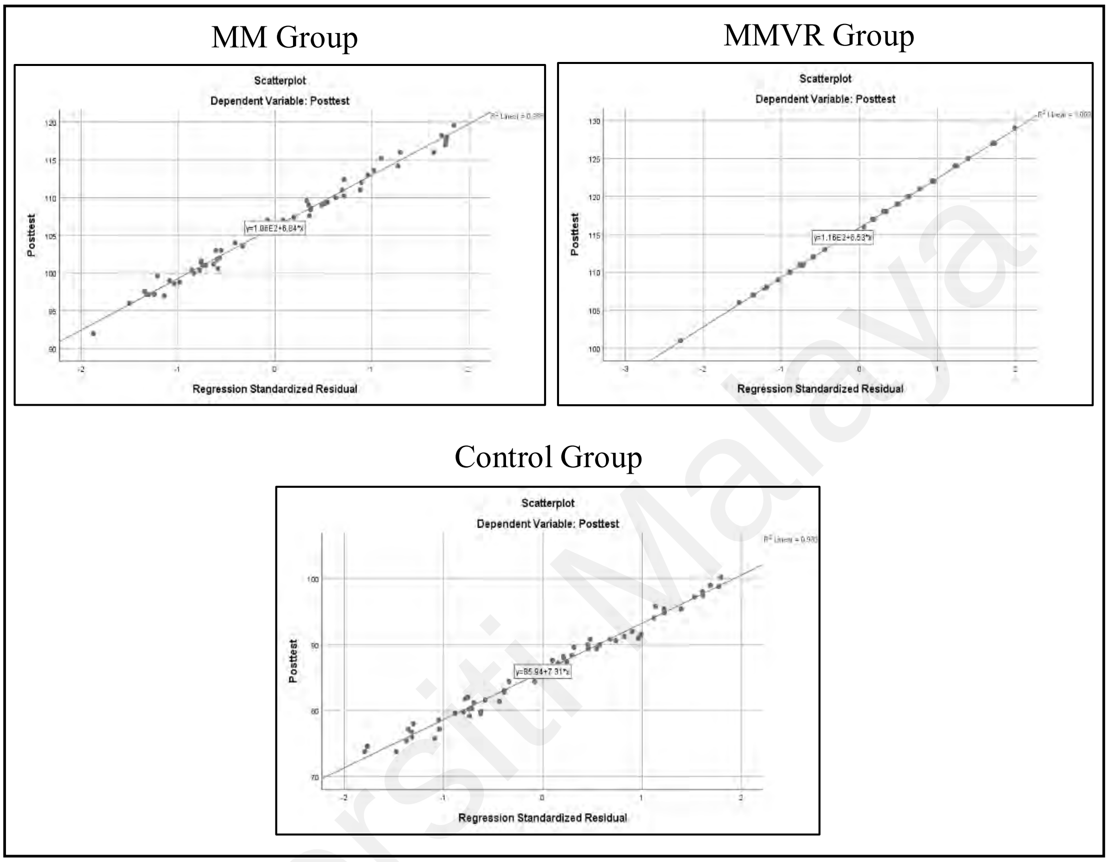


Figure 4.9. Assumption of homoscedasticity for posttest scores of problem solving ability for MM, MMVR, and Control groups

From Figure 4.9, it shows that the assumption of homoscedasticity was fulfilled because the distance of dots is constant (not increasing/ decreasing) along the straight line for MM, MMVR, and control groups. Therefore, the assumption of homoscedasticity is met.

Assumption 10: Covariates are different between independent variable

This assumption was tested using a General Linear Model (GLM) procedure. The following figure shows the result of the analysis.

Table 4.12

Tests of Between-Subjects Effects for Pretest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

Source	*df*	*F*	*p*
Group	2	11.75	$< .0005$
Error	172		

The Tests of Between-Subjects Effects analysis presented in Table 4.12 above shows that there is a statistically significant difference in the mean of the pretest score in MPSAT between the three groups, F (2, 172) = 11.75, $p < .0005$.

Since the required assumptions were met, the inferential analyses on post-test scores were conducted, and the following section will present the results of this test.

Results of One-way ANCOVA analysis

Table below presents the tests of between-subject effects.

Table 4.13

Tests of Between-Subjects Effects for Posttest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

Source	Type III Sum of Squares	*df*	Mean Square	*F*	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power
Corrected Model	27604.624[a]	3	9201.541	194.533	.000	.773	583.598	1.000
Intercept	6768.232	1	6768.232	143.089	.000	.456	143.089	1.000
Pretest	52.851	1	52.851	1.117	.292	.006	1.117	.183
Group	27354.876	2	13677.438	291.44	.000	.772	578.318	1.000
Error	8088.423	171	47.301					
Total	1871889.080	175						
Corrected Total	35693.048	174						

The Tests of Between-Subjects Effects analysis presented in Table 4.13 above has adjustment for pretest scores and found that there is a statistically significant difference in the mean of the posttest score in MPSAT between the three groups, F (2, 171) = 291.44, $p < .0005$, with large effect size and strong power ($\eta_p^2 = .77$, observed power = 1). The effect size suggests that about 77% of the variation in posttest scores can be accounted for by instructional methods. Therefore, the null hypothesis that the mean of the post-test scores of Year 4 students in MPSAT are not different between MM, MMVR, and control groups after controlling for the pretest is rejected at 5% of significance level.

Table below presents the breakdown of the estimated marginal means of MPSAT for Control, MM, and MMVR groups.

Table 4.14

Estimated Marginal Means for Posttest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

Group	*M*	*SE*
MM	105.84	.94
MMVR	116.14	.93
Control	85.94	.89

From Table 4.14, the adjusted means of posttest scores of MPSAT for MM, MMVR and Control groups were 105.84 (*SE* = .94), 116.14 (*SE* = .93), and 85.94 (*SE* = .89) respectively. These differences shown in the figure below that can be easily visualized by the generated plots of estimated marginal means of post-test scores of MPSAT for three groups.

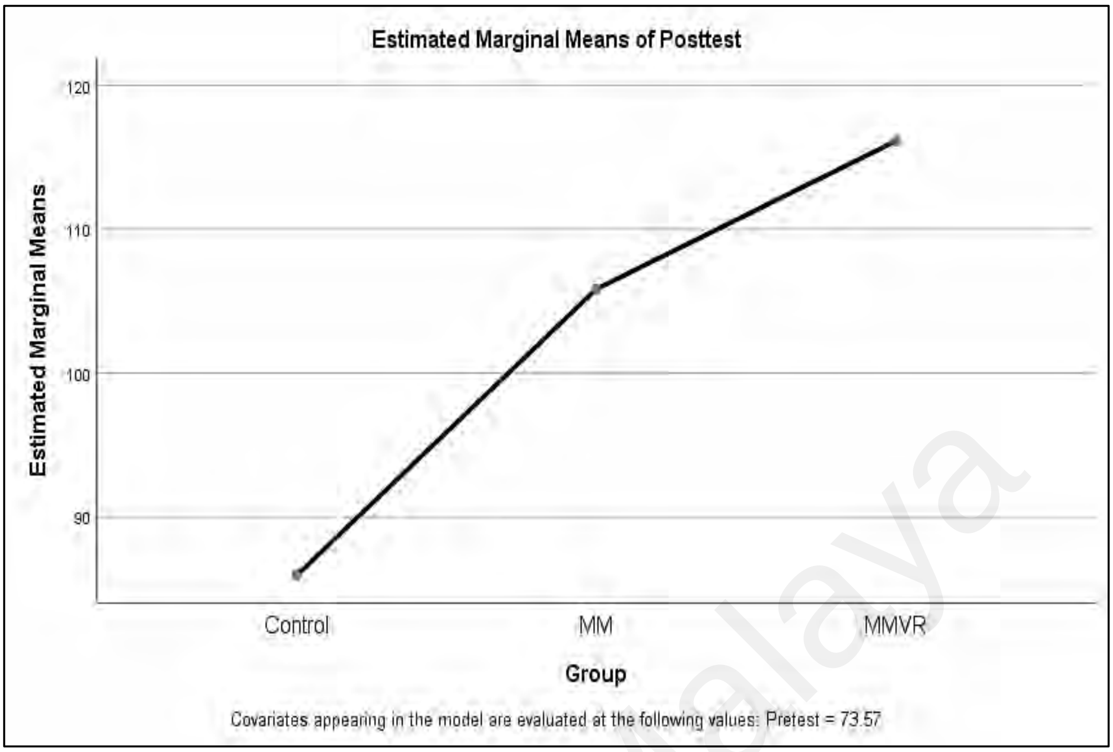


Figure 4.10. Estimated marginal means for posttest scores of MPSAT for MM, MMVR, and Control groups

Figure 4.10 illustrates the estimated marginal means of posttest scores where the adjusted mean of posttest scores for MMVR group was higher than the adjusted mean of posttest score of MM and control group respectively after the treatments.

Table below presents the results of the Bonferroni post hoc test, which allows the researcher to discover which specific means differed.

Table 4.15

Post Hoc Analysis for Posttest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

(I) Group	(J) Group	*MD* (I-J)	*p*
MM	Control	20.18	.000
	MMVR	-9.74	.000
MMVR	Control	29.92	.000
	MM	9.74	.000
Control	MM	-20.18	.000
	MMVR	-29.92	.000

Post hoc analysis was performed with a Bonferroni adjustment as shown in the Table 4.15 above. The table gives a significant level for mean differences between MM, MMVR, and Control groups. There was a significant difference in posttest scores between MMVR and MM treatments, between MMVR treatment and Control group, and between MM treatment and Control group. The mean of posttest scores for the MMVR teaching strategy was different from the MM teaching strategy with 9.74, and was different than for the control group with 29.92. Meanwhile, the mean of posttest scores for MM teaching strategy was different than for the control group with 20.18. Therefore, the null hypothesis that the mean of the posttest scores of Year 4 students in MPSAT are not different between MM, MMVR, and control groups after controlling for the pre-test is rejected at 5% of significance level.

4.6 Results of Analysis for Research Question 4

This section presents the results of the test of assumption and the result of one-way MANCOVA used to answer research question four which stated 'Is there any significant difference in the understanding of the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability of Year 4 students between Mayer's Problem Solving Model (MM) instruction, Mayer's Problem Solving Model with Visual Representation (MMVR) instruction, and control groups after the treatments when controlled for the pretest?'. There are 11 assumptions that need to be tested prior to the analysis using one-way MANCOVA. The following presents the results of the tests of assumption.

<u>Assumption 1: Two or more dependent variables should be measured at the interval or ratio level</u>

The scores of Understanding the Problem, Devising a Plan, Carrying out the Plan, and Looking back which are obtained from MPSAT in MM, MMVR, and Control groups, are from an interval scale. Therefore, the first assumption is met.

<u>Assumption 2: One independent variable should consist of two or more categorical, independent groups</u>

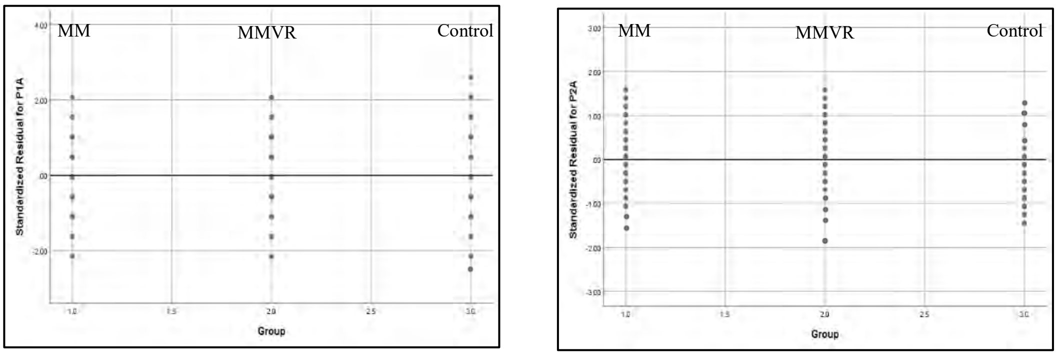
The scores of Understanding the Problem, Devising a Plan, Carrying out the Plan, and Looking back are derived from three categorical independent groups which are MM, MMVR, and Control groups. Therefore, the second assumption is met.

Assumption 3: One or more covariates are all continuous variables

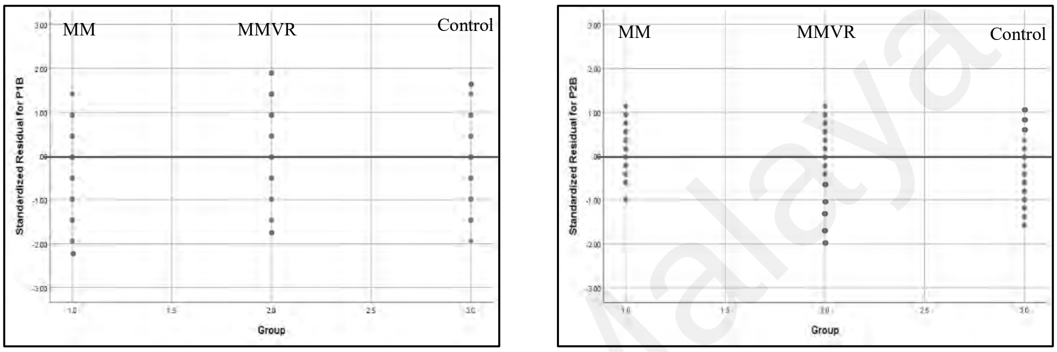
The scores of Understanding the Problem, Devising a Plan, Carrying out the Plan, and Looking back which are obtained from MPSAT in MM, MMVR, and Control groups, are all continuous interval variable. Therefore, this assumption is met.

Assumption 4: No relationship between the observations in each group of the independent variable or between the groups themselves

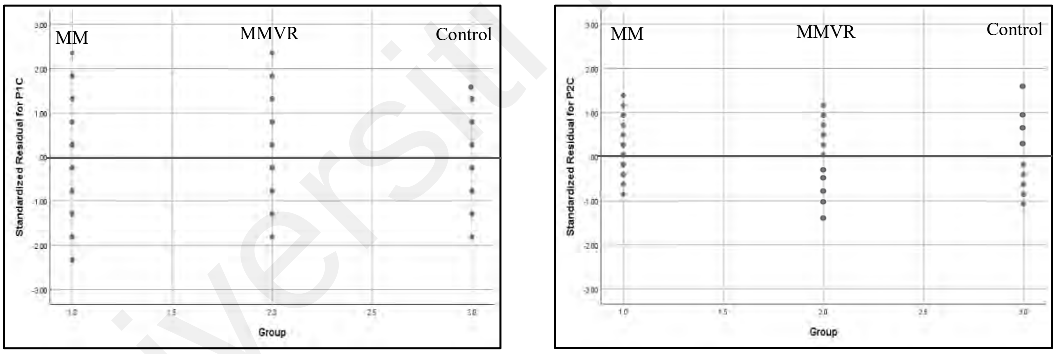
This assumption was tested using a scatterplot for four sub-dimension of problem solving abilities for three groups namely MM, MMVR, and Control group. The following figure shows the result of the analysis.



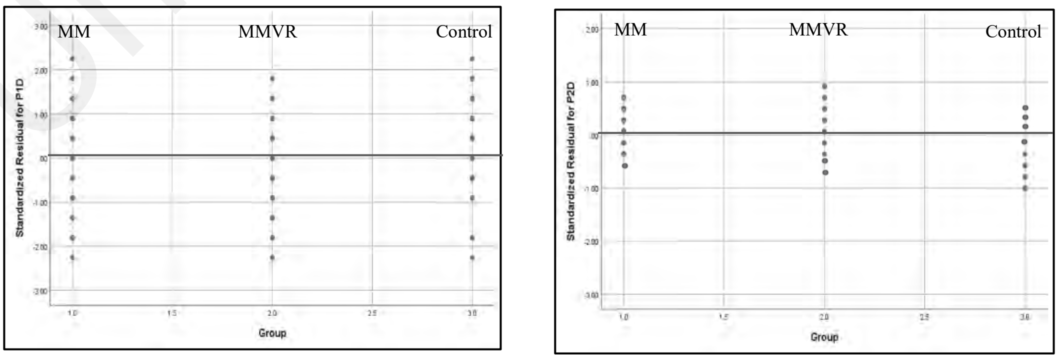
Understand the Problem



Devise a Plan



Carry Out the Plan



Looking Back

Figure 4.11. Assumption of independent observations for pretest and posttest scores of understanding of the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability for MM, MMVR, and Control groups

From Figure 4.11, it shows that the points fell relatively randomly above and below the horizontal reference line at 0 for each sub-dimension of problem solving ability for all three groups. Therefore, the assumption of independence has been met.

Assumption 5: Homogeneity of regression slopes

This assumption was tested using a General Linear Model (GLM) procedure. The following table shows the result of the analysis.

Table 4.16

Assumption of Homogeneity of Regression Slopes for Posttest Scores of Understanding the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MM Group, MMVR Group and Control Group

Source	Dependent Variable	*df*	*F*	*p*
Group * ToAvUnderstandTheProblem *ToAvDeviseaPlan*ToAvCarryOutthePlan* ToAvLookingBack	Posttest Total Average Understand the Problem	3	.52	.46
	Posttest Total Average Devise a Plan	3	.33	.63
	Posttest Total Average Carry Out the Plan	3	.21	.75
	Posttest Total Average Looking Back	3	.18	.81

Table 4.16 (continued)

Source	Dependent Variable	*df*	*F*	*p*
Error	Posttest Total Average Understand the Problem	171		
	Posttest Total Average Devise a Plan	171		
	Posttest Total Average Carry Out the Plan	171		
	Posttest Total Average Looking Back	171		

From Table 4.16 above, there is no interaction between the average posttest scores of four sub-dimension problem solving abilities and the average pretest scores of four sub-dimension problem solving abilities since the interaction term was not statically significant for Understand the Problem, F (3, 171) = .52, p = .46 (p > .05), was not statistically significant for Devise a Plan, F (3, 171) = .33, p = .63 (p > .05), was not statistically significant for Carry Out the Plan, F (3, 171) = .21, p = .75 (p > .05), and was not statistically significant for Looking Back F (3,171) = .18, p = .81 (p > .05). Therefore, the assumptions of homogeneity of regression slopes is met.

Assumption 6: Homogeneity of variances - covariance

This assumption was tested using Levene's test of homogeneity of variance - covariance. The following table shows the result of the analysis.

Table 4.17

Assumption of Homogeneity of Variance-Covariance for Posttest Scores of Understanding the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MM Group, MMVR Group and Control Group

	F	*df1*	*df2*	*p*
Posttest Total Average Understand the Problem	.32	2	172	.73
Posttest Total Average Devise a Plan	1.80	2	172	.17
Posttest Total Average Carry Out the Plan	.74	2	172	.48
Posttest Total Average Looking Back	1.09	2	172	.34

From Table 4.17, there is a homogeneity of variance-covariance, as assessed by Levene's test of homogeneity of variance for Understand the Problem ability, $F = .32$, $p = .73$ ($p > .05$); for Devise a Plan ability, $F = 1.80$, $p = .17$ ($p > .05$); for Carry Out the Plan ability, $F = 0.74$, $p = .48$ ($p > .05$); and for Looking Back ability, $F = 1.09$, $p = .34$ ($p > .05$). This means that the variance of the dependent variable of four sub-dimension problem solving abilities are equal across groups. Hence, the assumption of homogeneity of variance is met.

Assumption 7: No significant multivariate outliers in the groups of your independent variable in terms of each dependent variable

This assumption of no multivariate outliers was tested using a boxplot of Mahalanobis Distance for three groups namely MM, MMVR, and Control group. The following figure shows the result of the analysis.

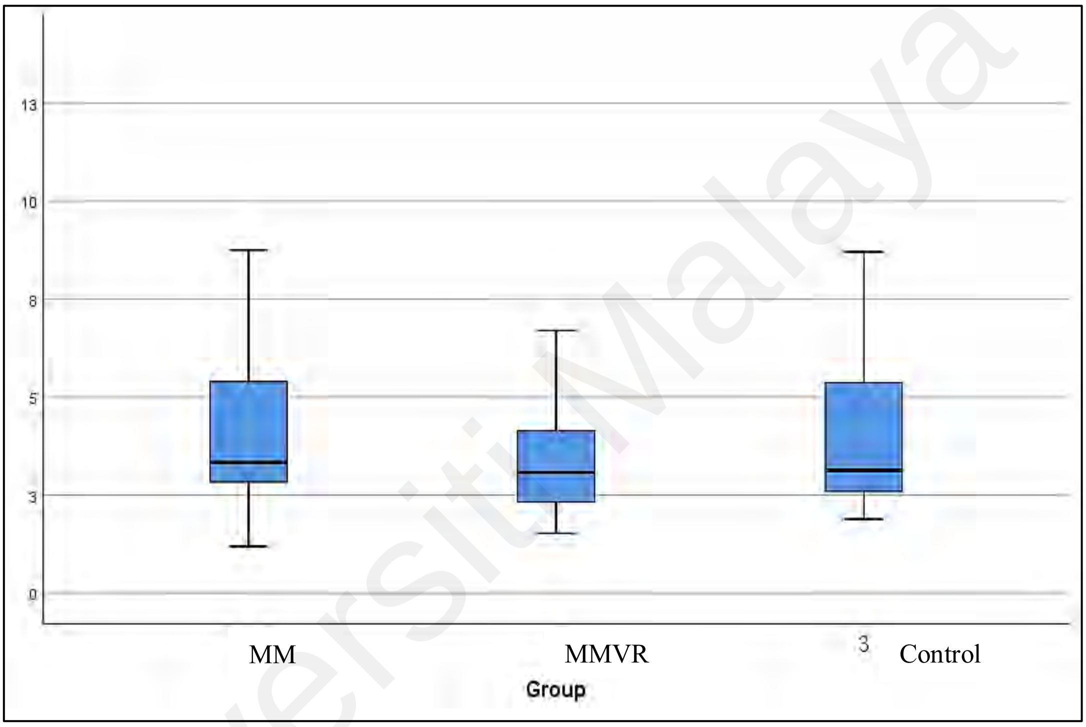


Figure 4.12. Assumption of no significant multivariate outliers for posttest scores of problem solving ability for MM, MMVR, and Control groups

The result of testing assumption showed in Figure 4.12 above indicated that there were no multivariate outliers, as assessed using Mahalanobis Distance. Therefore, the assumption is met.

Assumption 8: No significant univariate outliers in the groups of your independent variable in terms of each dependent variable

This assumption of no univariate outliers was tested using a boxplot of total average posttest scores for Understanding the Problem, Devising a Plan, Carrying out the Plan, and Looking back, which are obtained from MPSAT in MM, MMVR, and Control groups. The following figure shows the result of the analysis.

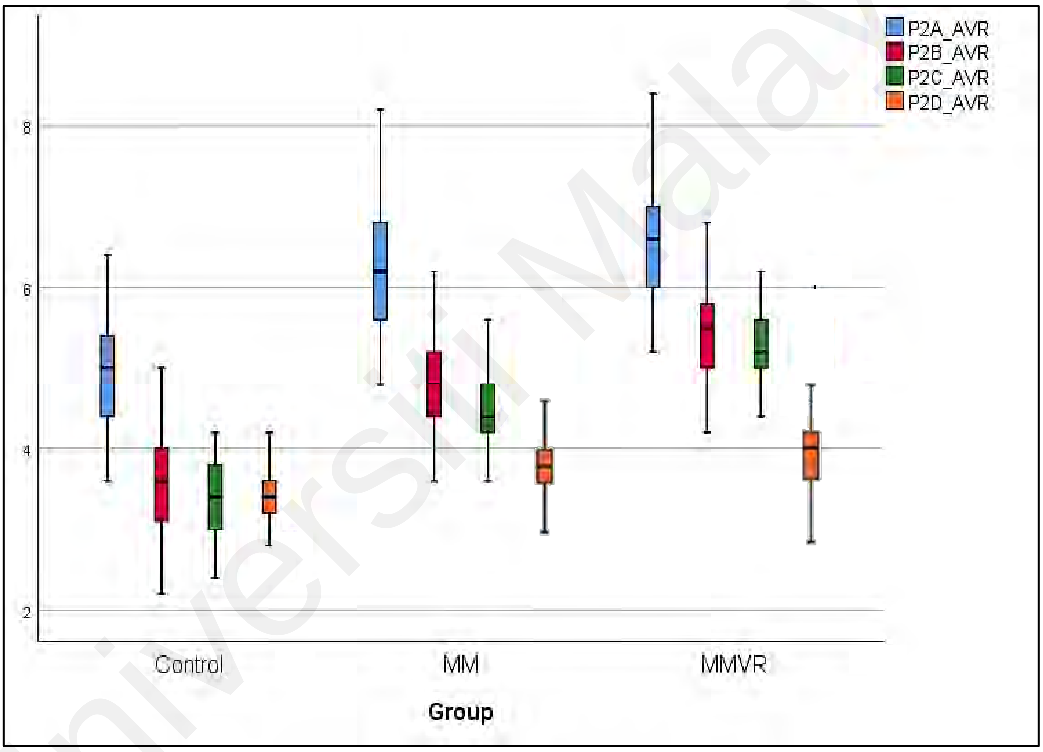


Figure 4.13. Assumption of no significant univariate outliers for posttest average scores of understanding of the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability for MM, MMVR, and Control groups

The result of testing assumption showed in Figure 4.13 above indicates that there were no univariate outliers of total average posttest scores of understand the problem, devise a plan, carry out the plan, and looking back, as assessed by boxplot. Therefore, the assumption is met.

Assumption 9: Should be multivariate normality

This assumption was tested using Kolmogorov-Smirnov test. The following table shows the result of the analysis.

Table 4.18

Assumption of Multivariate Normality for Posttest Scores of Understanding the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MM Group, MMVR Group and Control Group

	Group	Statistic	*df*	*p*
Posttest Total Average Understand the Problem	Control	.97	60	.20
	MM	.96	57	.093
	MMVR	.96	58	.053
Posttest Total Average Devise a Plan	Control	.97	60	.23
	MM	.97	57	.89
	MMVR	.97	58	.214
Posttest Total Average Carry Out the Plan	Control	.96	60	.051
	MM	.96	57	.88
	MMVR	.97	58	.20

Table 4.18 (continued)

	Group	Statistic	*df*	*p*
Posttest Total	Control	.97	60	.811
Average Looking	MM	.97	57	.60
Back	MMVR	.98	58	.616

From Table 4.18 above, the Control group resulting a *p*-value for the total average of posttest scores for Understand the Problem ability which is .20 ($p > .05$), a *p*-value for the total average of posttest scores for Devise a Plan ability which is .23 $p > .05$), a *p*-value for the total average of posttest scores for Carry Out the Plan ability which is .051 ($p > .05$), and a *p*-value for the total average of posttest scores for Looking Back ability which is .111 ($p > .05$). Meanwhile, the MM group resulting a *p*-value for the total average of posttest scores for Understand the Problem ability which is .093 ($p > .05$), a *p*-value for the total average of posttest scores for Devise a Plan ability which is .89 ($p > .05$), a *p*-value for the total average of posttest scores for Carry Out the Plan ability which is .88 ($p > .05$), and a *p*-value for the total average of posttest scores for Looking Back ability which is .60 ($p > .05$). Finally, the MMVR group resulting a *p*-value for the total average of posttest scores for Understand the Problem ability which is .053 ($p > .05$), a *p*-value for the total average of posttest scores for Devise a Plan ability which is .214 ($p > .05$), a *p*-value for the total average of posttest scores for Carry Out the Plan ability which is .20 ($p > .05$), and a *p*-value for the total average of posttest scores for Looking Back ability which is .116 ($p > .05$). Therefore, the null hypotheses stating that the sample data comes from normal population is retained at the 5% of significance level for the three groups. Hence, the assumption of normality is met.

Assumption 10: Covariates are different between independent variable

This assumption was tested using a General Linear Model (GLM) procedure. The following table shows the result of the analysis.

Table 4.19

Tests of Between-Subjects Effects for Pretest Scores of Understanding the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MM Group, MMVR Group and Control Group

Source	Dependent Variable	*df*	*F*	*p*
Group	Pretest Total Average Understand the Problem	2	5.670	.003
	Pretest Total Average Devise a Plan	2	32.151	< .0005
	Pretest Total Average Carry Out the Plan	2	9.840	< .0005
	Pretest Total Average Looking Back	2	6.802	.001
Error	Pretest Total Average Understand the Problem	172		
	Pretest Total Average Devise a Plan	172		
	Pretest Total Average Carry Out the Plan	172		

Table 4.19 (continued)

Source	Dependent Variable	*df*	*F*	*p*
Error	Pretest Total Average Looking Back	172		

The Tests of Between-Subjects Effects analysis presented in Table 4.19 above showed that there is a statistically significant difference in the mean of the total average of posttest scores in MPSAT for Understand the Problem ability, F (2, 172) = 5.670, p = .003 (p < .05); Devise a Plan ability, F (2, 172) = 32.151, p < .0005; Carry Out the Plan ability, F (2, 172) = 9.840, p < .0005; and Looking Back ability F (2, 172) = 6.802, p = .001 (p < .05), between MM, MMVR, and Control groups.

Assumption 11: Homogeneity of covariance matrices

This assumption was tested using Box's M test of equality of covariance matrices. The following table shows the result of the analysis.

Table 4.20

Assumption of Homogeneity of Covariance Matrices for Posttest Scores of Understanding the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MM Group, MMVR Group and Control Group

Box's M	*F*	*df1*	*df2*	*p*
46.173	4.933	20	105810.475	.162

From Table 4.20 above, there is a homogeneity of covariance matrices, Box's M = 46.173, p > 0.001. Therefore, the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups. Hence, the assumption of covariance matrices is met.

Since the required assumptions were met, the one-way MANCOVA test was conducted, and the following section will present the results of this test.

<u>Results of One-way MANCOVA analysis</u>

Table below presents the multivariate tests for posttest scores of problem solving sub-constructs for MM, MMVR, and Control groups.

Table 4.21

Multivariate Tests for Posttest Scores of Understanding the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MM Group, MMVR Group and Control Group

	Value	*F*	Hypothesis *df*	Error *df*	*p*	Partial Eta Squared	Observed Power
Wilks' lambda	.009	398.65	8.00	330.00	< .0005	.81	1.00

A one-way MANCOVA from Table 4.21 above revealed a significant multivariate main effect, Wilks' $\Lambda = .009$, F (8, 330) = 398.65, $p < .0005$; with large effect size and strong power (η_p^2 =. 81, observed power = 1). Therefore, the null hypothesis that the mean of total average of the posttest score of understand the problem ability, devise a plan ability, carry out the plan ability, and looking back ability of Year 4 students in

MPSAT was not different among MM group, MMVR group and control group when controlling for pretest scores is rejected at 5% of significance level.

Given the significance of the overall test, the univariate main effects were examined. Table below presents the univariate ANOVA tests of understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability.

Table 4.22

Tests of Between-Subjects Effects for Posttest Scores of Understanding the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MM Group, MMVR Group and Control Group

Source	Dependent Variable	Type III Sum of Squares	*df*	Mean Square	*F*	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power
Corrected Model	P2A_AVR	189.613[a]	6	31.602	1176.970	.000	.977	7061.818	1.000
	P2B_AVR	182.761[b]	6	30.460	1932.043	.000	.986	11592.260	1.000
	P2C_AVR	137.451[c]	6	22.909	2607.454	.000	.989	15644.722	1.000
	P2D_AVR	151.834[d]	6	25.306	3508.206	.000	.992	21049.236	1.000
Intercept	P2A_AVR	.207	1	.207	7.717	.006	.044	7.717	.789
	P2B_AVR	.658	1	.658	41.711	.000	.199	41.711	1.000
	P2C_AVR	.028	1	.028	3.144	.078	.018	3.144	.422
	P2D_AVR	.233	1	.233	32.290	.000	.161	32.290	1.000
Group	P2A_AVR	45.398	2	22.699	845.389	.000	.910	1690.778	1.000
	P2B_AVR	35.582	2	17.791	1128.455	.000	.931	2256.910	1.000

Table 4.22 (continued)

Source	Dependent Variable	Type III Sum of Squares	*df*	Mean Square	*F*	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power
Group	P2C_AVR	22.977	2	11.489	1307.636	.000	.940	2615.272	1.000
	P2D_AVR	56.504	2	28.252	3916.673	.13	.979	7833.346	0.118
Error	P2A_AVR	4.511	168	.027					
	P2B_AVR	2.649	168	.016					
	P2C_AVR	1.476	168	.009					
	P2D_AVR	1.212	168	.007					

*P2A_AVR = Posttest Total Average Understand the Problem
P2B_AVR = Posttest Total Average Devise a Plan
P2C_AVR = Posttest Total Average Carry Out the Plan
P2D_AVR = Posttest Total Average Looking Back

From Table 4.22, the Tests of Between-Subjects Effects as well as the follow-up Univariate ANOVAs showed that there is a statistically significant difference in the mean of the total average of posttest scores in MPSAT for Understand the Problem ability, F (2, 168) = 845.39, $p < .0005$, with large effect size and strong power (η_p^2 = .91, observed power = 1); Devise a Plan ability, F (2, 168) = 1128.46, $p < .0005$, with large effect size and strong power (η_p^2 = .93, observed power = 1); and Carry Out the Plan ability, F (2, 168) = 1307.64, $p < .0005$, with large effect size and strong power (η_p^2 = .94, observed power = 1) between MM, MMVR, and Control groups. The effect size shows that 91% of the total variation in Understand the Problem total average posttest scores, 93% of the total variation in Devise a Plan total average posttest scores, and 94% of the total variation in Carry Out the Plan total average posttest scores, is accounted by the treatments effect in the MM and MMVR groups. Whereas, the Looking Back ability, F (2, 168) = 316.67, p = .13 (p > .05), with small effect size and weak power (η_p^2 = .15, observed power = 0.118) was not statistically significantly different between the MM, MMVR, and Control groups, after controlling for the pretest scores. The effect size shows that only 15% of the total variation in Looking Back total average posttest scores is accounted for by the treatments effect in the MM and MMVR groups. The null hypothesis that the mean of the posttest scores of Understanding the problem, Devising a Plan, Carrying out the Plan, and Looking Back abilities are not different between MM, MMVR, and control groups after the treatments is partially rejected at 5% of significance level.

Table below presents the results of the Bonferroni post hoc test, which allows the researcher to discover which specific means differed.

Table 4.23

Post Hoc Analysis for Posttest Scores of Understanding the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MM Group, MMVR Group and Control Group

Dependent Variable	(I) Group	(J) Group	*MD* (I-J)
Posttest Total Average	Control	MM	-1.32
Understand the Problem		MMVR	-1.63
	MM	Control	1.32
		MMVR	-.30
	MMVR	Control	1.63
		MM	.30

Table 4.23 (continued)

Dependent Variable	(I) Group	(J) Group	*MD* (I-J)
Posttest Total Average	Control	MM	-1.30
Devise a Plan		MMVR	-1.95
	MM	Control	1.30
		MMVR	-.65
	MMVR	Control	1.95
		MM	.65
Posttest Total Average	Control	MM	-1.09
Carry Out the Plan		MMVR	-1.87
	MM	Control	1.09
		MMVR	-.77
	MMVR	Control	1.87
		MM	.77
Posttest Total Average	Control	MM	-.63
Looking Back		MMVR	-.89
	MM	Control	.63
		MMVR	-.26
	MMVR	Control	.89
		MM	.26

Post hoc analysis was performed with a Bonferroni adjustment as shown in the Table 4.23 above. The table gives a significant level for mean differences between MM, MMVR, and Control groups for four sub-dimension problem solving abilities. There was a significant difference in total average posttest scores between MMVR and MM treatments, between MMVR treatment and Control group, and between MM treatment and Control group for Understand the Probem, Devise a Plan, and Carry Out the Plan abilities. Meanwhile there was no significance difference been observed for Looking Back ability. The results revealed that the mean of total average posttest scores for Understand the Problem ability in the MMVR group was different than for the MM group with .30 and was different than the control group with 1.63. The mean of total average posttest scores for Understand the Problem ability in MM group was different than that of the control group with 1.32. The mean of total average posttest scores for Devise a Plan ability in MMVR group was different than the MM group with .65 and was different than control group with 1.95. The mean of total average posttest scores for Devise a Plan ability in MM group was different than control group with 1.30. The mean of total average posttest scores for Carry Out the Plan ability was different in MMVR group than MM group with .77 and was different than control group with 1.87. The mean of total average posttest scores for Carry Out the Plan ability was different in MM group than control group with 1.09. Meanwhile, for Looking Back ability, although the mean of total average posttest scores in MMVR group was different than MM group with .23, but it was not statistically significant, and although the mean of total average posttest scores in MMVR group was different than control group with .89, but it was not statistically significant. The mean of total average posttest scores for Looking Back ability in MM was different than control group with .63, but was also not statistically significant. Therefore, the null hypothesis that the mean of total average of the posttest score of Understand the Problem, Devise a Plan, Carry Out the Plan, and Looking Back abilities of Year 4 students in MPSAT was not different between the MM, MMVR, and control groups when controlling for pretest scores is rejected at 5% of significance level.

Table below presents the breakdown of the estimated marginal means of MPSAT for understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability for Control, MM, and MMVR groups.

Table 4.24

Estimated Marginal Means for Posttest Scores of Understanding the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability for MM Group, MMVR Group and Control Group

Dependent Variable	Group	*M*	*SE*
Posttest Total Average Understand the Problem	MM	6.54	.33
	MMVR	6.21	.36
	Control	5.07	.35
Posttest Total Average Devise a Plan	MM	5.03	.32
	MMVR	5.28	.25
	Control	3.61	.24
Posttest Total Average Carry Out the Plan	MM	4.60	.28
	MMVR	5.09	.24
	Control	3.42	.23
Posttest Total Average Looking Back	MM	3.62	.22
	MMVR	4.11	.21
	Control	3.38	.23

Table 4.24 shows that the adjusted mean of total average posttest score of Understand the Problem ability when controlling for pretest score for control group (*M* = 5.07, *SE* = .35) was different than for the MM group (*M* = 6.54, *SE* = .33), and MMVR (*M* = 6.21, *SE* = .36) group; Devise a Plan ability for control group (*M* = 3.61, *SE* = .24) was

different than for the MM group (*M* = 5.03, *SE* = .32), and MMVR (*M* = 5.28, *SE* = .25) group. The adjusted mean of total average posttest score of Carry Out the Plan ability was also different between the control (*M* = 3.42, *SE* = .23), MM (*M* = 4.60, *SE* = .28), and MMVR (*M* = 5.09, *SE* = .24) group; and also, the total average score of posttest of Looking Back ability was different between the control group (*M* = 3.38, *SE* = .23), MM group (*M* = 3.62, *SE* = .22), and MMVR group (*M* = 4.11, *SE* = .21) . These differences can be easily visualized by the generated plots of estimated marginal means of post-test scores in terms of Understand the Problem, Devise a Plan, Carry Out the Plan, and Looking Back, as shown in figure below.

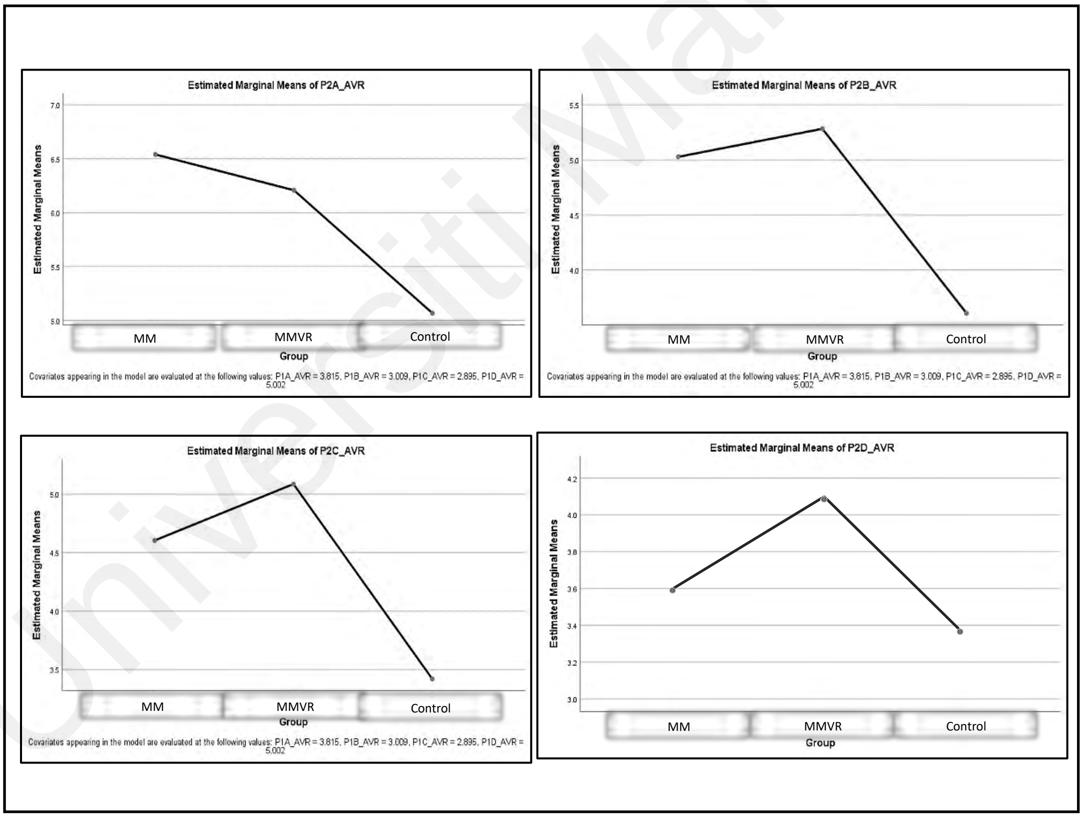


Figure 4.14. Estimated marginal means of total average posttest scores for understanding of the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability for MM, MMVR, and Control groups

Figure 4.14 illustrates the estimated marginal means of total average posttest scores for each sub-dimension of mathematical problem solving ability where the adjusted mean of posttest score for MMVR group was higher than the adjusted mean of posttest score of MM and control group respectively after the treatments.

4.7 Results of Analysis for Research Question 5

This section presents the results of the test of assumption and the result of the 2X3 factorial analysis of covariance (ANCOVA) used to answer research question five which stated 'Is there any significant interaction between problem solving teaching strategy and gender on mathematical problem solving ability of Year 4 students when controlled for the pretest?'. There are eight assumptions that need to be tested prior to the analysis using ANCOVA. The following presents the results of the tests of assumption.

Assumption 1: Dependent variables should be measured at the interval or ratio level

The scores obtained from MPSAT in MM, MMVR, and Control groups for male and female groups are from an interval scale. Therefore, the first assumption is met.

Assumption 2: One independent variable should consist of two or more categorical, independent groups

The independent variable of this question for each MM, MMVR, and Control group consisting of two categorical independent groups, which are male and female groups. Therefore, the second assumption is met.

Assumption 3: All samples are drawn independently of each other

This assumption was tested using a scatterplot for male and female for three groups namely MM, MMVR, and Control group. The following figure shows the result of the analysis.

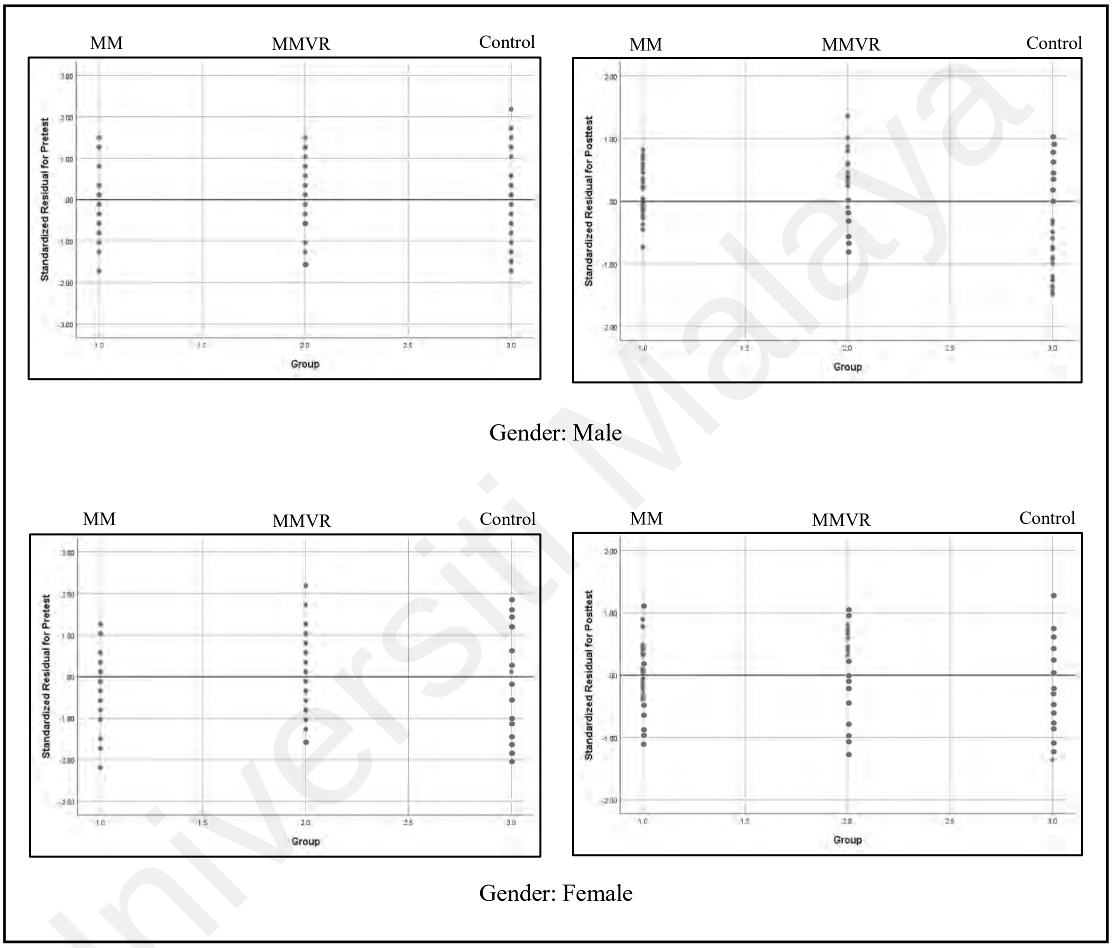
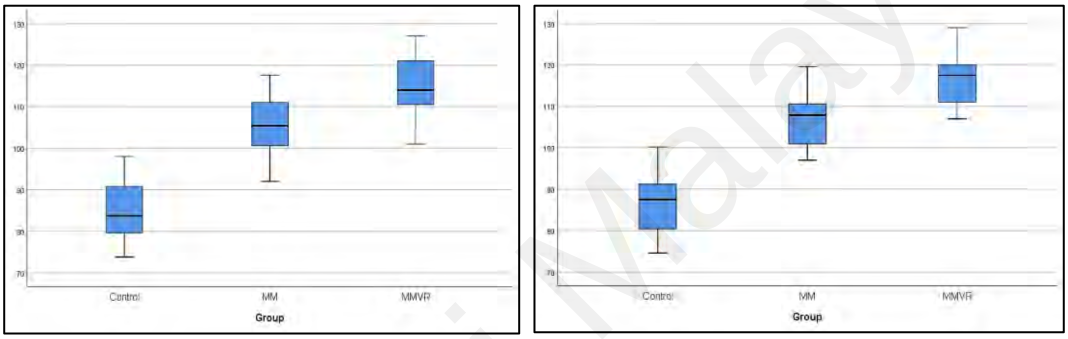


Figure 4.15. Assumption of independent observations for posttest scores of problem solving ability of male and female students for MM, MMVR, and Control groups

The assumption of independence has been met as shown by the scatterplots from Figure 4.15 above. It suggests the evidence of independence with relative randomness of points above and below the horizontal reference line at 0. Therefore, the assumption of independence has been met.

Assumption 4: No significant outliers

This assumption was tested using a boxplot for posttest scores of three groups namely MM, MMVR, and Control groups for male and female. The following figure shows the result of the analysis.



Gender: Male   Gender: Female

Figure 4.16. Assumption of no significant outliers for posttest scores of problem solving ability of male and female students for MM, MMVR, and Control groups

From Figure 4.16, the result of testing the assumption revealed that there were no outliers in MM, MMVR, and Control group for posttest scores by gender. Therefore, the assumption is met.

Assumption 5: Residuals should be approximately normally distributed for each combination of groups of the two independent variables

This assumption was tested using Kolmogorov-Smirnov test. The following table shows the result of the analysis.

Table 4.25

Assumption of Normality for Posttest Scores of Problem Solving Ability for Male and Female in MM Group, MMVR Group and Control Group

Gender	Group		Statistic	*df*	*p*
Male	Control	Standardized Residual for Posttest	.96	30	.14
	MM	Standardized Residual for Posttest	.97	29	.088
	MMVR	Standardized Residual for Posttest	.96	28	.20
Female	Control	Standardized Residual for Posttest	.95	30	.20
	MM	Standardized Residual for Posttest	.94	28	.20
	MMVR	Standardized Residual for Posttest	.95	30	.20

From Table 4.25, the Control group resulting a *p*-value for standardized residual for posttest for male which is .20 ($p > .05$) and a *p*-value for standardized residual for posttest for female which is .20 ($p > .05$).

Meanwhile, the MM group resulting a *p*-value for standardized residual for posttest for male which is .14 ($p > .05$) and a *p*-value for standardized residual for posttest for female which is .20 ($p > .05$). Lastly, the MMVR group resulting a *p*-value for standardized residual for posttest for male which is .088 ($p > .05$) and a *p*-value for standardized residual for posttest for female which is .20 ($p > .05$). Therefore, the null hypotheses stating that the sample data comes from normal population is retained at the 5% of significance level for the three groups, for male and female. Hence, the assumption of normality is met.

<u>Assumption 6: All populations have a common variance</u>

This assumption was tested using Levene's test of homogeneity of variance. The following table shows the result of the analysis.

Table 4.26

Assumption of Homogeneity of Variance for Posttest Scores of Problem Solving Ability for Male and Female in MM Group, MMVR Group and Control Group

F	*df1*	*df2*	*p*
.36	5	169	.87

From Tabl4 4.26, there is a homogeneity of variance, as assessed by Levene's test of homogeneity of variance, (F (5, 169) = .36, p = .87 ($p > .05$). This means that the variance of the dependent variable is equal for male and female. Hence, the assumption of homogeneity of variance is met.

Assumption 7: Covariate should be linearly related to the dependent variable for each combination of groups of the independent variables

This assumption was tested using a scatterplot for male and female, under three groups namely MM, MMVR, and Control group. The following figure shows the result of the analysis.

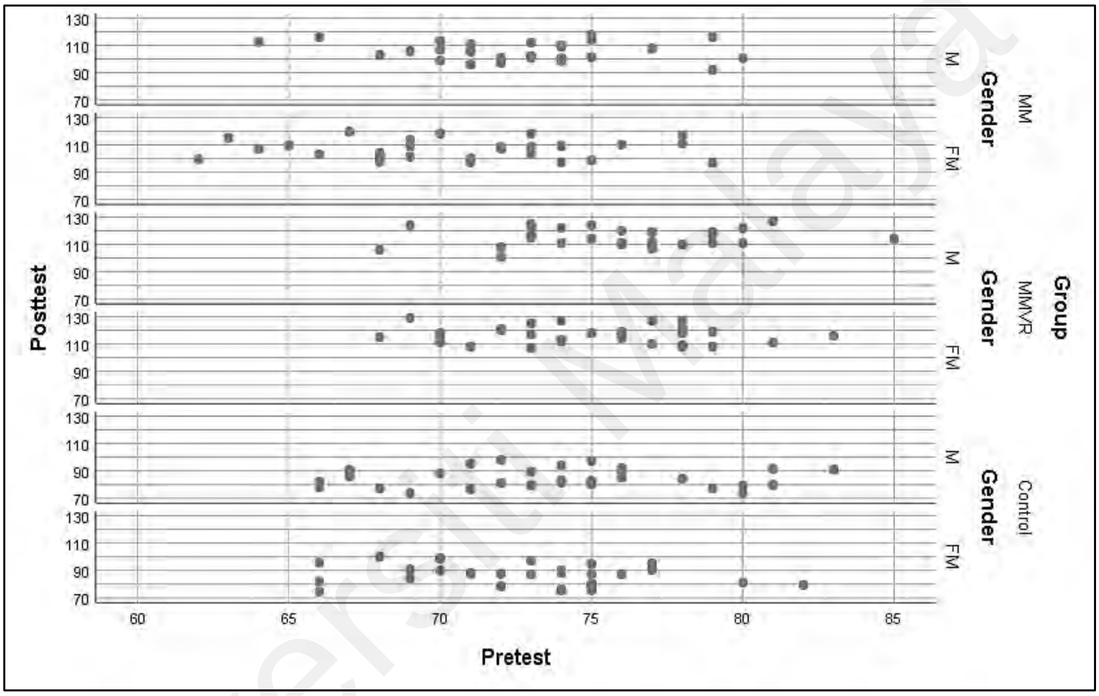


Figure 4.17. Assumption of linearity for posttest scores of problem solving ability of male and female students for MM, MMVR, and Control groups

From the Figure 4.17, it indicates that there was a linear relationship between the posttest scores and pretest score as a covariate for MM, MMVR, and control groups as well as gender. Therefore, the assumption of linearity was met.

Assumption 8: Needs to be homoscedasticity

This assumption was tested using a scatterplot for male and female, under three groups namely MM, MMVR, and Control group. The following figure shows the result of the analysis.

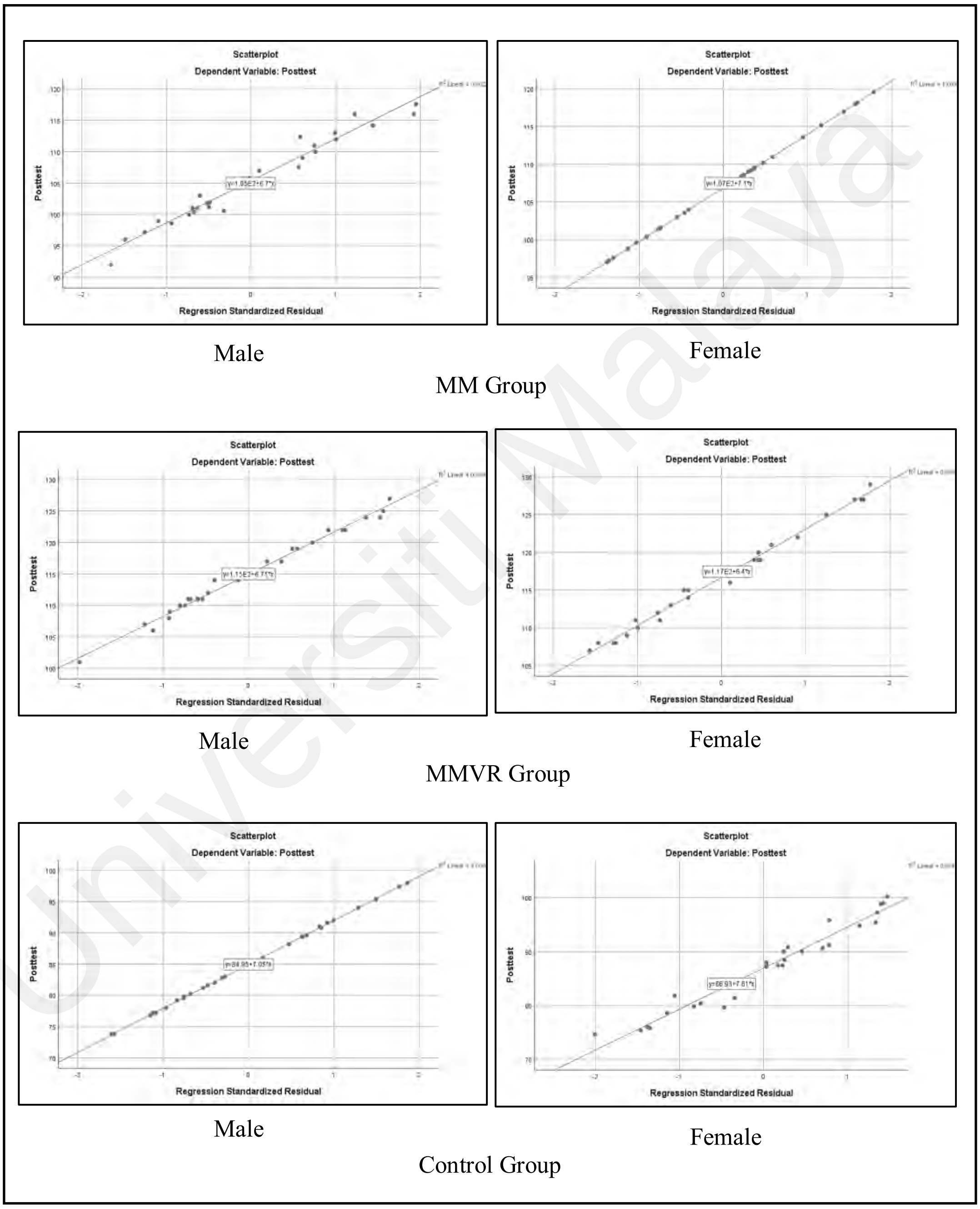


Figure 4.18. Assumption of homoscedasticity for posttest scores of problem solving ability of male and female students for MM, MMVR, and Control groups

From Figure 4.18, it shows that the assumption of homoscedasticity was fulfilled because the distance of dots is constant (not increasing/ decreasing) along the straight line for MM, MMVR, and control groups for male and female. Therefore, the assumption of homoscedasticity is met.

Assumption 9: Needs to be homogeneity of regression slopes, which means that there is no interaction between the covariate and the independent variable

This assumption was tested using a General Linear Model (GLM) procedure. The following table shows the result of the analysis.

Table 4.27

Test of Homogeneity of Regression Slopes for Posttest Scores of Problem Solving Ability for Male and Female in MM Group, MMVR Group and Control Group

Source	*df*	*F*	*p*
Gender * Group * Pretest	3	.60	.61
Error	82		

From Table 4.27, there is no interaction between the covariate (pretest) and the independent variables (gender and group) since the interaction term was not statistically significant, F (3, 82) = .60, p = .61 (p > 05). Hence, the assumption of homogeneity of regression slopes is met.

Since the required assumptions were met, the two-way ANCOVA test was conducted, and the following section will present the results of this test.

Results of Two-way ANCOVA analysis

Table below presents the tests of between-subject effects.

Table 4.28

Tests of Between-Subjects Effects for Posttest Scores of Problem Solving Ability for Male and Female in MM Group, MMVR Group and Control Group

Source	*df*	*F*	*p*	Partial Eta Squared
	1	2.78	0.97	.016
Gender * Group	2	.018	.98	.00022
Error	169			

Table above shows the Tests of Between-Subjects Effects and indicates that the interaction effect between gender and the groups on mathematical problem solving ability was not statistically significant, *F* (2, 169) = .018, *p* = .98 ($p > .05$), with a minimal effect size (η_p^2= .00022). The analysis above also indicated that the main effect of gender on mathematical problem solving ability was not statistically significant, *F* (1, 169) = 2.78, *p* = .097 ($p > .05$), with a small effect size η_p^2 =. 016. Therefore, the null hypothesis stating that there is no interaction between MM and MMVR instruction and gender on Year 4 students' mathematical problem solving ability is retained at 5% of significance level.

Table below presents the breakdown of the estimated marginal means of MPSAT between Control, MM, and MMVR groups, and gender.

Table 4.29

Estimated Marginal Means for Posttest Scores of Problem Solving Ability for Male and Female in MM Group, MMVR Group and Control Group

Gender	Group	*M*	*SE*
M	MM	105.39	1.28
	MMVR	114.96	1.30
	Control	84.95	1.26
FM	MM	106.89	1.30
	MMVR	116.70	1.26
	Control	86.93	1.26

From Table 4.29, the analysis of interaction between teaching strategies and gender showed that the adjusted means of mathematical problem solving ability scores in the MM group for male is 105.39 (*SE*=1.28), and female is 106.89 (*SE*=1.30). The adjusted means of mathematical problem solving ability scores in the MMVR group for male is 114.96 (SE=1.30) and female is 116.70 (*SE*=1.26). The adjusted means of mathematical problem solving ability scores in the Control group for male is 84.95 (*SE*=1.26) and female is 86.93 (*SE*=1.26). These differences can be seen clearly by the estimated marginal means plot of posttest scores of male and female students in three groups as the pretest scores adjusted as covariate, as in the figure below.

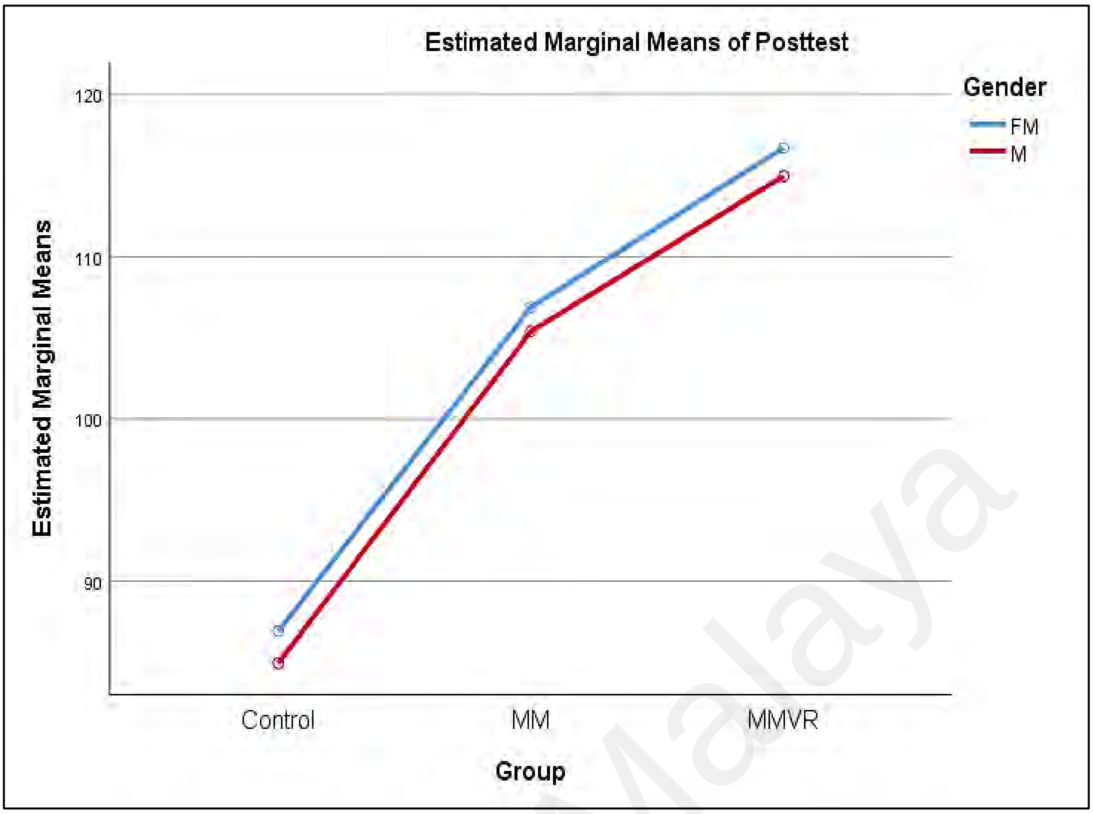


Figure 4.19. Adjusted means for posttest scores of problem solving ability of male and female students for MM, MMVR, and Control groups

Figure 4.19 illustrates the estimated marginal means of posttest scores where the adjusted mean of posttest scores for MMVR group was higher than the adjusted mean of posttest score of MM and control group respectively after the treatments for both male and female.

Table below presents the results of the Bonferroni post hoc test, which allows the researcher to discover which specific means differed.

Table 4.30

Post Hoc Analysis for Posttest Scores of Problem Solving Ability for Male and Female in MM Group, MMVR Group and Control Group

(I) Group	(J) Group	*MD* (I-J)	*p*
MM	MMVR	-9.74	.000
	Control	20.18	.000
MMVR	MM	9.74	.000
	Control	29.92	.000
Control	MM	-20.18	.000
	MMVR	-29.92	.000

Post hoc analysis was performed with a Bonferroni adjustment as shown in the Table 4.30 above. The table gives a significant level for mean differences between MM, MMVR, and Control groups for both male and female. The MM group was associated with the mean of mathematical problem solving ability score of 20.18, higher than the control group. The MMVR group was associated with the mean of mathematical problem solving ability score of 9.74 higher than the MM group, and 29.92 higher than the control group.

4.8 Summary

This chapter has presented and discussed the results of the tests used for each of the hypotheses related to the research questions. Also presented was the assumptions tested before running each of the hypotheses testing analysis.

The next chapter will present the summary of the findings that attempt to answer the research questions. It will present a comprehensive discussion on the major findings of the research based on the objectives of the study. Also, next chapter will discuss on the implications of the study, and the recommendations for further research.

CHAPTER 5

DISCUSSIONS

5.1 Introduction

This study was conducted to determine the effectiveness of Mayer's problem solving Model with visual representation teaching strategy in enhancing Year 4 students' mathematical problem solving ability. This chapter summarizes the major findings of the research based on the objectives of the study. The chapter also presents a comprehensive discussion of the major findings of the research and provides the conclusions. This chapter elaborates on the implications of the study and the recommendations for further research. This chapter ends with the conclusion of the study.

5.2 Summary of the Findings

This section presents the summary of the findings based on the research questions of this study. The first finding of this section is on the effect of Mayer's problem solving Model with visual representation teaching strategy on mathematical problem solving ability of Year 4 students, whereas, the second finding is on the effect of Mayer's problem solving Model with visual representation teaching strategy on the understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability of Year 4 students. The third finding is on the differences in mathematical problem solving ability of Year 4 students in MM group, MMVR group, and Control group after the Mayer's problem solving Model with visual representation teaching strategy treatment. The fourth finding is on the differences in the understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability of Year 4 students among MM group, MMVR group, and Control group after the Mayer's problem solving Model with visual representation teaching strategy treatment. The last finding of this study is on the interaction between problem solving teaching strategy and gender on mathematical problem solving ability of Year 4 students.

5.2.1 The First Finding

This study found that there is an improvement in mathematical problem solving ability of Year 4 students after students undergoing Mayer's problem solving Model with visual representation teaching strategy treatment for three weeks. This finding proves the research hypothesis that the mathematical problem solving ability of Year 4 students will improve after students undergoing Mayer's problem solving Model with visual representation teaching strategy treatment.

This is based on the posttest scores (M = 115.86, SD = 6.47) that is higher compared to the pretest scores (M = 75.41, SD = 3.70). The results of the one-tailed paired-samples t-test indicates a statistically significant increase in mathematical problem solving ability from pretest to posttest scores among students in Mayer's problem solving Model with visual representation group with (M = 40.45, SD = 7.51), t (57) = 41.05, $p < .005$.

5.2.2 The Second Finding

This study also found that there is an improvement in the understanding the problem ability, devising a plan ability, and carrying out the plan ability of Year 4 students after students undergoing Mayer's problem solving Model with visual representation teaching strategy treatment for three weeks. This finding partially proved the research hypothesis that the understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability of Year 4 students will improve after students undergoing Mayer's problem solving Model with visual representation teaching strategy treatment.

This is based on the total average posttest scores of understanding the problem ability (M = 6.58, SD = .79) that is higher compared to the total average pretest scores (M = 3.83, SD = .34). For devising a plan ability, the total average posttest scores (M = 5.50, SD = .62) is higher compared to the total average pretest scores (M = 3.32, SD = .36). For carrying out the plan ability, the total average posttest scores (M = 5.25, SD = .43) is higher compared to the total average pretest scores (M = 3.06, SD = .36). Meanwhile, for looking back ability, the total average posttest scores before the treatment (M = 5.27, SD = .44) is slightly higher compared to the total average pretest scores after the treatment (M = 4.88, SD = .42). This results of the one-tailed paired-samples t-test indicates a statistically significant increase in understanding the problem ability from total average pretest to total average posttest scores among students in Mayer's problem solving Model with visual representation group with (M = 2.76, SD = .47), t (57) = 44.98, $p < .005$; a statistically significant increase in devising a plan ability from total average pretest to total average posttest scores among students in Mayer's problem solving Model with visual representation group with (M = 2.19, SD = .29), t (57) = 58.43, $p < .005$; and a statistically significant increase in carrying out the plan ability from total average pretest to total average posttest scores among students in Mayer's problem solving Model with visual representation group with (M = 2.19, SD = .12), t (57) = 141.23, $p < .005$. Meanwhile, there is no significant mean increase for looking back ability from total average pretest to total average posttest scores among students in Mayer's problem solving Model with visual representation group with (M = 0.39, SD = .079), t (57) = 37.67, $p < .05$.

5.2.3 The Third Finding

This study also found that there is difference in mathematical problem solving ability of Year 4 students between MM group, MMVR group and Control group after students undergoing Mayer's problem solving model teaching strategy and Mayer's problem solving model with visual representation teaching strategy treatments for three weeks. This finding proves the research hypothesis that the mathematical problem solving ability of Year 4 students will be different between MM group, MMVR group, and Control group after students undergoing the Mayer's problem solving model teaching strategy and Mayer's problem solving model with visual representation teaching strategy treatments.

This is based on the result of post hoc analysis that indicates a significant difference in posttest scores between MMVR and MM treatments, between MMVR treatment and Control group, and between MM treatment and Control group. The mean of posttest scores for the MMVR teaching strategy was different from the MM teaching strategy with 9.74, and was different than for the control group with 29.92. Meanwhile, the mean of posttest scores for MM teaching strategy was different than for the control group with 20.18. The result obtained for the adjusted means of mathematical problem solving ability for MM group, MMVR group, and Control group were 105.84 (SE = .94), 116.14 (SE = .93) and 85.94 (SE = .89) respectively.

The results of the one-way ANCOVA indicates that there is a significant univariate main effect F (2, 171) = 291.44, $p < .005$, with large effect size and strong power (partial $\eta 2$ = .77, observed power = 1) at the significance level of .05) which conclude that Mayer's problem solving model teaching strategy treatment and Mayer's problem solving model with visual representation teaching strategy treatment have greatly improved Year 4 students' mathematical problem solving ability.

5.2.4 The Fourth Finding

The fourth finding of this study is that there are differences in the understanding the problem ability, devising a plan ability, and carrying out the plan ability among MM group, MMVR group, and Control group after students undergoing Mayer's problem solving model teaching strategy and Mayer's problem solving model with visual representation teaching strategy treatments for three weeks. This finding proves the research hypothesis that the understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability of Year 4 students will be different between MM group, MMVR group, and Control group after students undergoing the Mayer's problem solving model teaching strategy and Mayer's problem solving model with visual representation teaching strategy treatments.

This is based on the result of post hoc analysis that indicates a significant difference in total average posttest scores between MMVR and MM treatments, between MMVR treatment and Control group, and between MM treatment and Control group for Understand the Probem, Devise a Plan, and Carry Out the Plan abilities. Meanwhile there was no significance difference been observed for Looking Back ability. The results revealed that the mean of total average posttest scores for Understand the Problem ability in the MMVR group was different than for the MM group with .30 and was different than the control group with 1.63. The mean of total average posttest scores for Understand the Problem ability in MM group was different than that of the control group with 1.32. The mean of total average posttest scores for

Devise a Plan ability in MMVR group was different than the MM group with .65 and was different than control group with 1.95. The mean of total average posttest scores for Devise a Plan ability in MM group was different than control group with 1.30. The mean of total average posttest scores for Carry Out the Plan ability was different in MMVR group than MM group with .77 and was different than control group with 1.87. The mean of total average posttest scores for Carry Out the Plan ability was different in MM group than control group with 1.09. Meanwhile, for Looking Back ability, although the mean of total average posttest scores in MMVR group was different than MM group with .23, but it was not statistically significant, and although the mean of total average posttest scores in MMVR group was different than control group with .89, but it was not statistically significant. The mean of total average posttest scores for Looking Back ability in MM was different than control group with .63, but was also not statistically significant.

The results based on the analysis of univariate ANOVA showed that there was a statistically significant differences in the posttest scores of Mathematical Problem Solving Ability Test between the three groups for understand the problem ability, F (2, 168) = 845.39, $p < .005$, with large effect size and strong power (partial $\eta 2$ = .91, observed power = 1); Devise a Plan ability, F (2, 168) = 1128.46, $p < .005$, with large effect size and strong power (partial $\eta 2$ = .93, observed power = 1); and Carry Out the Plan ability, F (2, 168) = 1307.64, $p < .005$, with large effect size and strong power (partial $\eta 2$ = .94, observed power = 1). Whereas, there is no statistically significant difference in the post-test scores of Mathematical Problem Solving Ability Test between the three groups for Looking Back ability, F (2, 168) = 316.67, $p > .05$, with small effect size and weak power (partial $\eta 2$ = .15, observed power = 0.118).

The results of the one-way MANCOVA indicates that there is a significant multivariate main effect Wilks' $\Lambda = .009$, $F(8, 330) = 398.65$, $p < .005$; with large effect size and strong power (partial $\eta 2$ =. 81, observed power = 1) at the significance level of .05), which conclude that Mayer's problem solving model teaching strategy treatment and Mayer's problem solving model with visual representation teaching strategy treatment have greatly improved Year 4 students' devising a plan ability, carrying out the plan ability, and looking back ability.

5.2.5 The Fifth Finding

This study found that there is no significant interaction between Mayer's problem solving Model and Mayer's problem solving Model with visual representation teaching strategies with gender on Year 4 students' mathematical problem solving ability. This finding were not able to prove the research hypothesis that there is significant interaction between Mayer's problem solving model teaching strategy and Mayer's problem solving model with visual representation teaching strategy with gender on Year 4 students' mathematical problem solving ability.

This finding is based on the result of analysis of two-way ANCOVA that indicates that the interaction effect between gender and teaching strategies on mathematical problem solving ability was not statistically significant, $F(2, 169) = 0.18$, $p = .98$, with a minimal effect size and power (partial $\eta 2 = .00022$, observed power= .035). The results of analysis of the main effect indicates that the main effect of gender on mathematical problem solving ability was not statistically significant, (1, 169) = 2.78, $p = .097$, with a small effect size partial $\eta 2$ =. 016.

The result of post hoc analysis indicates a significant level for mean differences between MM, MMVR, and Control groups for both male and female. The MM group was associated with the mean of mathematical problem solving ability score of 20.18, higher than the control group. The MMVR group was associated with the mean of mathematical problem solving ability score of 9.74 higher than the MM group, and 29.92 higher than the control group.

5.3 Discussion

In this section, the discussion of the findings is presented. The discussion is divided into three sections based on the three objectives of this study. The first section discusses the effectiveness of Mayer's problem solving model with visual representation teaching strategy on Year 4 students' mathematical problem solving ability, the second section discusses the effectiveness of Mayer's problem solving model with visual representation teaching strategy on Year 4 students' understanding the problem ability, devising a plan ability, carrying out the plan ability, and looking back ability. The third section discusses the interaction between Mayer's problem solving model with visual representation teaching strategies and gender on Year 4 students' mathematical problem solving ability.

5.3.1 Effectiveness of Mayer's Problem Solving Model with Visual Representation Teaching Strategy on Year 4 Students' Mathematical Problem Solving Ability

The findings from this study showed that mathematical problem solving ability of Year 4 students in MMVR group has improved after students has undergone Mayer's problem solving model with visual representation teaching strategy treatment, and this improvement is caused by Mayer's problem solving model with visual representation treatment. The evidence of the result from this study suggests that the use of visual representation along with Mayer's problem solving model is effective in improving students' ability in solving non-routine mathematical problems.

This finding proves the research hypothesis of this study that saying Mayer's problem solving model with visual representation teaching strategy treatment could improve students' ability in solving mathematical problems. This hypothesis was based from past studies that says Mayer's problem solving model and visual representation teaching strategy could improve students' mathematical problem solving ability. Such past studies are, Mayer (1985), found that for a problem solver to reach problem solution accurately, problem representation and problem solution play a major processes in solving mathematical problems. Ho and Lowrie (2014) found that communication of mathematical ideas using visual such as simple picture aiding students in connecting ideas across the problem given, hence, improve the tackling techniques of mathematical problems among students. Besides, to be a successful problem solver, one should be able to deal flexibly with multiple representations and move adaptively between them (Acevedo Nistal et al., 2009; Dreher & Kuntze, 2015). Also, representing information visually is considered an efficient representation techniques in mathematics education, particularly in problem solving (Guler & Citas, 2011). The findings from this study hence confirmed the hypothesis suggested by past studies that saying Mayer's problem solving model and visual representation teaching strategy improve students' mathematical problem solving ability.

There are several explanations on why the Mayer's problem solving model with visual representation teaching strategy treatment is effective in improving mathematical problem solving ability of Year 4 students. First explanation is that, students in MMVR group use drawing/ visual representation strategy from the first to fourth phase of Mayer's problem solving Model which are problem translation, problem integration, solution planning, and solution execution phases. Students in MMVR group were taught by the teacher in each class session to draw the story problem in order to understand the problem story, choosing drawing strategy as a method to solve the problem, and draw pictures in every steps when solving the problem. Students in MMVR group were encouraged to follow the same method when they solve post MPSAT questions. Meanwhile, students in MM group only use drawing/ visual representation strategy in the first and second phases of Mayer's problem solving model which are problem translation and problem integration phases. This means, students in MM group were only taught by the teacher to use drawing strategy in understanding the problem story stage.

During the post MPSAT session, many students in MMVR group used visual representation method in problem translation, problem integration, solution planning, and solution execution phases as taught to them during the previous ten treatment sessions. From the posttest result of MPSAT, it can be seen that students in MMVR group who solved the MPSAT questions using visual method tend to achieve more accurate solutions for the given problems compared to students in MM group. Therefore, it can be stated that visual representation was a factor, where it is a possibility that would explain the varying results of the MM and MMVR groups on MPSAT. This adds to the work completed by Ho and Lowrie (2014) which reported that students much more preferred to use visual method when solve difficult problems.

Figure below shows how Student A from MMVR group and Student B from MM group solved Question 1 from post MPSAT.

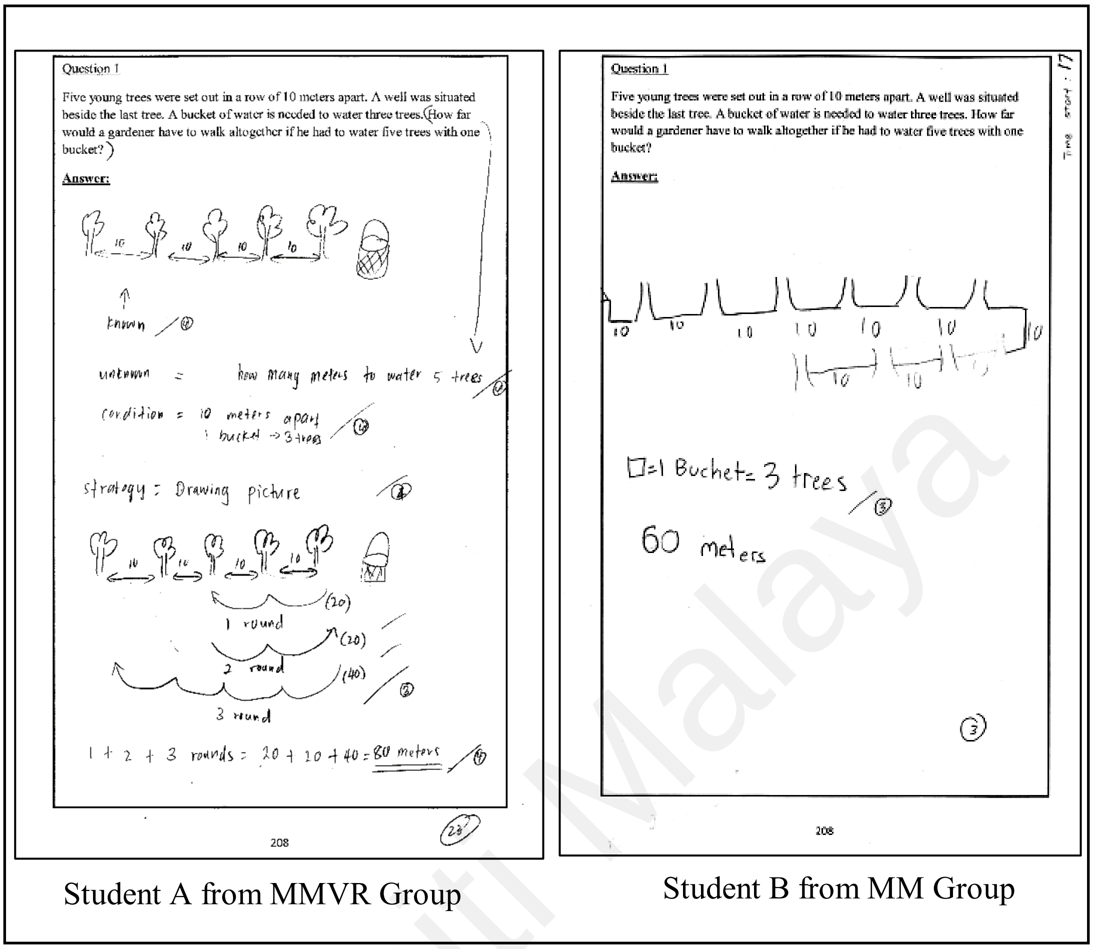


Figure 5.1. Comparison of student A and student B solved question 1 of post MPSAT

From the figure above, student A from MMVR group able to achieve an accurate solution for the question while student B from MM group unable to solve the question accurately. Student A has used the drawing technique in problem representation and problem solution phases, therefore the student manage to solve the questions accurately. Meanwhile, student B unable to draw out the problem story completely, hence, the student unable to carry out the problem solution phase accurately where the student unable to decide which strategy to be used to complete the solution.

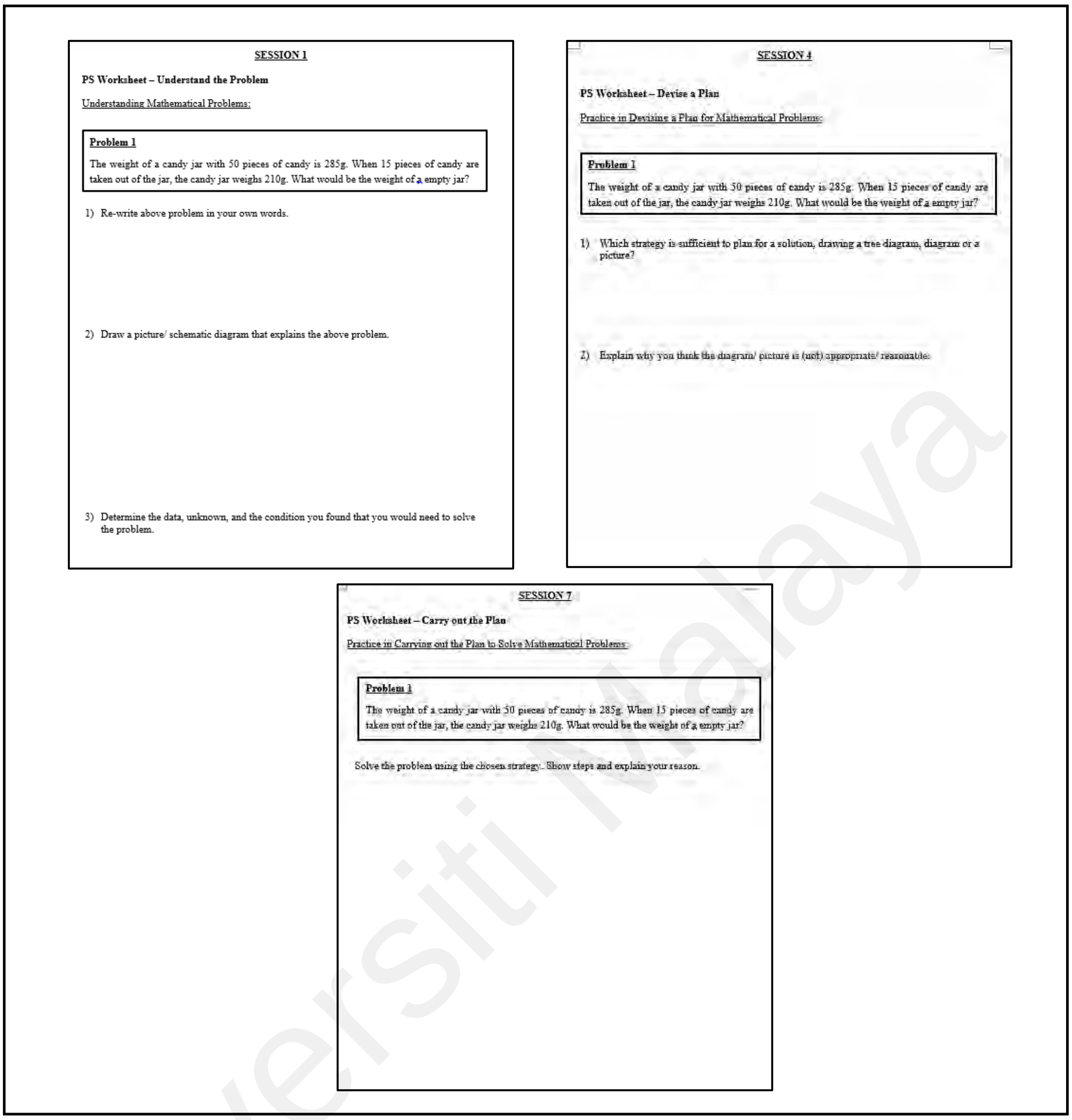
Mayer's problem solving model with visual representation teaching strategy worked across all types of word problems in MPSAT. This is because, students' accuracy in solving mathematical problems improved for all types of wordings and operations. This is an effective result in relation to Mayer's problem solving model teaching strategy, including schema-based strategy, that involve knowing the appropriate format to use for the different types of question, as it at times involves students knowing few strategies and being able to apply it to all types of word problems. This add to the previous study that found having simply one strategy lessens the demands on students' thinking as they know they have one strategy which can assist with word problems without spending any thinking or time on determining which strategy or structure is needed for a particular problem (Funke, Fischer & Holt, 2017).

Besides, one of the issue identified by many students at different stages was that only visualising created too much 'clutter' in their brain and that drawing helped to 'free up space'(Teahen, 2015). This ties in with theories that working memory has a capacity and can only hold so much information (Cowan, 2014). Trying to create a mental picture, work out the equation and then calculate the equation seemed too much to keep in the working memory for some students. Students' feedback suggested that the use of drawings to get some of the information down or to help with the calculation of the problem enables the students to be less stressed and be more accurate in their working. This result ties in with Raghubar, Barnes, and Hecht's (2010) study who discovered that the use of pen and paper likely lessens the load on working memory. It may be that drawings help more with lessening the load on the working memory and with calculation accuracy than in helping with the comprehension of the question.

It has been argued in previous research that students only use visual representations to solve mathematics problems when directed (Teahen, 2015), and the reasons for this include the perception that diagrams are a teacher strategy for teaching (Uesaka, Manalo, & Ichikawa, 2007) and that visual reasoning is seems of low value (Arcavi, 2003). Students' fluency with different kinds of representations and in translating information among them has evoked due to the past studies which concerned with visual representations. In this study, as discussed earlier, students were encouraged to use visual representation strategy from the first to fourth phase of Mayer's problem solving Model which are problem translation, problem integration, solution planning, and solution execution phases. This teaching method shows the possibility that students' ability to engage in the process of relating and translating information when dealing with representations is governed by the type of strategy taught by teachers in solving mathematical problems.

Another explanation for the improvement of the problem solving ability of Year 4 students after the Mayer's problem solving Model with visual representation treatment is could be due to the Mayer's problem solving Model with visual representation Worksheet (Figure 5.2 below) that was developed for the purpose of this study. This add to the study by Alfieri, Brooks, Aldrich, and Tenenbaum (2011) that found guidance in the form of worksheets, worked examples, scaffolding, and elicited explanations will be very beneficial to students especially in students' cognitive development. Mayer's problem solving Model with visual representation worksheets might accommodates these guidance.

Figure below shows Mayer's problem solving model with visual representation strategy scaffoldings in the Mayer's problem solving Model with visual representation worksheet.



SESSION 1

PS Worksheet – Understand the Problem

Understanding Mathematical Problems:

Problem 1

The weight of a candy jar with 50 pieces of candy is 285g. When 15 pieces of candy are taken out of the jar, the candy jar weighs 210g. What would be the weight of a empty jar?

1) Re-write above problem in your own words.

2) Draw a picture/ schematic diagram that explains the above problem.

3) Determine the data, unknown, and the condition you found that you would need to solve the problem.

SESSION 4

PS Worksheet – Devise a Plan

Practice in Devising a Plan for Mathematical Problems:

Problem 1

The weight of a candy jar with 50 pieces of candy is 285g. When 15 pieces of candy are taken out of the jar, the candy jar weighs 210g. What would be the weight of a empty jar?

1) Which strategy is sufficient to plan for a solution, drawing a tree diagram, diagram or a picture?

2) Explain why you think the diagram/ picture is (not) appropriate/ reasonable.

SESSION 7

PS Worksheet – Carry out the Plan

Practice in Carrying out the Plan to Solve Mathematical Problems:

Problem 1

The weight of a candy jar with 50 pieces of candy is 285g. When 15 pieces of candy are taken out of the jar, the candy jar weighs 210g. What would be the weight of a empty jar?

Solve the problem using the chosen strategy. Show steps and explain your reason.

Figure 5.2. Sample of Mayer's problem solving model with visual representation worksheet

During each treatment session, every student was given a set of worksheet comprises of problem solving questions and instructions on how to solve the problems using Mayer's problem solving Model with visual representation strategy. The worksheet instructs students to only use visual representation strategy when solving the mathematical problems in the worksheet. The worksheet designed in a way that students are required to use visual representation strategy in every step of Mayer's problem solving phase which are understanding the problem step, devising a plan step, and carrying out the plan step.

This explanation is tie with Stein et al. (2008) which reported that modelling teacher's own problem solving process and asking questions, orchestrating whole class discussions can advance mathematical learning in cognitively demanding tasks. In this study, teacher facilitate discussion using Mayer's problem solving Model with visual representation Worksheet which specifically designed for MMVR group, and administer students on how they solve the problems in every treatment session. This makes the teacher-students participation in the classroom learning is active, hence, makes the teaching objective to be achievable among students.

It is also been argued that the mathematics learning process is still going on conventionally/ traditionally and tends to be mechanistic. It means that students listen, imitate or copy exactly the same way what the teacher gives without initiative (Siswono, 2008). Students' difficulty to make connections within and across questions is the major factor influencing the effectiveness of learning problem solving. Also reported, pre-service teachers facing challenges in solving abstract problems and problems demanding multiple steps (Tatto et al., 2011). In the current study, as discussed above, students are provided with Mayer's problem solving Model with visual representation worksheet which requires both teacher and students to engage with the worksheet in every class when teaching the problem solving topic. Students are given prescribed guidance on how to solve the problems step by step using visual representation method. This study revealed that, when there is a special worksheet guiding students on solving certain task, the learning becomes effective. Also, for teachers, the lesson plan used in this study helped teachers to give an effective and intensive instructions for students on how to solve non-routine problems. Therefore, step-by-step instructional manual could help even pre-service teachers to effectively teach problem solving in the classroom.

Students' overall mathematical problem solving ability in the MMVR group and MM group were not only different, but also the overall mathematical problem solving ability in MMVR group was higher than that of the MM group. This improvement of mathematical problem solving ability among MMVR group students is due to the employment of visual representation strategy in problem representation and problem solution stages when solving mathematical problems.

5.3.2 Effectiveness of Mayer's Problem Solving Model with Visual Representation Teaching Strategy on Year 4 Students' Understanding the Problem Ability, Devising a Plan Ability, Carrying Out the Plan Ability, and Looking Back Ability

This study also found that the understanding the problem ability, devise a plan ability, and carrying out the plan ability of Year 4 students in MMVR group has improved after these students has undergone Mayer's problem solving model with visual representation teaching strategy treatment. It was found that the use of visual representation along with Mayer's problem solving model is effective in improving students' ability in understanding non-routine mathematical problems, devising a plan for them, and carrying out the plan, and this improvement is caused by Mayer's problem solving model with visual representation treatment.

This finding proves the research hypothesis that saying Mayer's problem solving model with visual representation teaching strategy treatment could improve students' ability in understanding mathematical problems, devising a plan, and carrying out the plan. This hypothesis was based from past studies that says Mayer's problem solving model and visual representation teaching strategy could improve students' mathematical problem solving ability in terms of understanding the problem ability, devising a plan ability, and carrying out the plan ability. Such past studies are, Polya (1945) in his 1945 book titled *How To Solve It*, stated that to become a good problem solver, one should possess an ability to understand the problem, devise a plan, carry out the plan, and look back. Polya's steps of "understanding the problem" is encapsulated by Mayer's problem representation phase, and Polya's "devise a plan" and "carry out the plan" steps map to Mayer's problem solution phase (Mayer, 1982). Therefore, students who undergo Mayer's problem representation and problem solution phases could able to understand the problem given, devise a plan, and carry out the plan, as stated by Polya (1945). Mielicki, Marta, and Wiley (2016) found that visual representations could help to facilitate students solving problems more efficiently including understanding the problem, devising a plan, and carrying out the plan. The use of visual representations in the problem solving process may not always be effective, and in some situations, it may even lead to incorrect solutions. Therefore, creating visual representations which emphasize spatial relationships in the process of solving mathematical problems which includes understanding the problem, devising a plan, carrying out the plan, and looking back, may contribute to problem solving success (Guler & Citas, 2011). The findings from this study hence confirmed the hypothesis suggested by past studies that saying Mayer's problem solving model and visual representation teaching strategy improve students' ability in understanding mathematical problems, devising a plan, and carrying out the plan.

There are several explanations on why Mayer's problem solving model with visual representation teaching strategy treatment is effective in improving the understanding the problem ability, devise a plan ability, and carrying out the plan ability of Year 4 students. First explanation is that, students in MMVR group use drawing/ visual representation strategy from the first to fourth phase of Mayer's problem solving Model which are problem translation, problem integration, solution planning, and solution execution phases. Students in MMVR group were taught by the teacher in each class session to draw the story problem in order to understand the problem story, choosing drawing strategy as a method to solve the problem, and draw pictures in every steps when solving the problem. Students in MMVR group were encouraged to follow the same method when they solve post MPSAT questions. Meanwhile, students in MM group only use drawing/ visual representation strategy in the first and second phases of Mayer's problem solving model which are problem translation and problem integration phases. This means, students in MM group were only taught by the teacher to use drawing strategy in understanding the problem story stage.

From the posttest result of MPSAT, it can be seen that students in MMVR group who solved the MPSAT questions using visual method tend to achieve more accurate solutions for the given problems compared to students in MM group. Therefore, it can be stated that visual representation was a factor, where it is a possibility that would explain the varying results of the MMVR and MM groups on understanding the problem ability, devising a plan ability, and carrying out the plan ability.

Students in MMVR group made a great improvement in the area of understanding the problem stage, which is the first stage in solving mathematical problems. Understanding the problem stage is in light of the problem translation and problem integration phases in Mayer's problem solving Model. With Mayer's problem solving model with visual representation teaching strategy, students in MMVR group use visual representation strategy to draw picture of the story problem in order to find more relevant information from the problem story. Since students in MMVR group were only exposed to drawing pictures or tree diagram throughout the treatment sessions, this might have resulted in the increased capability of visual representation skill of each student that allowed them to visualize more representations in their brain during the understanding the problems phase. The representations is mediated by the joint activity of the individual's verbal and imagery systems, therefore, students in MMVR group were able to transform numeric data into visual representation in the shortest time with the highest accuracy level. This result added to Krawec's (2014) work which found that the more visual representations include appropriate relational and numerical components, the closer students would fall on the accurate solution of the problem.

Beside, students in MMVR group also made progress in the area of devising a plan and carrying out the plan abilities. The reason could be, students in MMVR group were only given an option of choosing problem solving strategy which is through graphically, where only drawings of picture or tree diagram are allowed when solving the mathematical problems. This result supports the research by Macnab, Phillips, and Norris (2012) which found that visual representation give an enormous contribution solution spontaneously and functionally to the world of mathematics whether non-routine or routine problems.

The results of the study showed that students' ability in understanding mathematical problems, devising a plan for the problems, and carrying out the chosen plan were not only different, but higher than students' abilities in MM group. This improvement among MMVR group students is due to the employment of visual representation strategy in understanding the problem stage, devising a plan stage, and carrying out the plan stage when solving mathematical problems.

However, it was found that the use of visual representation along with Mayer's problem solving model is not that effective in improving students' ability in looking back the solution. The reason is that, Mayer's problem solving model consisting of two major phases in mathematical word problem solving which are problem representation (problem translation and problem integration) and problem solution (solution planning and solution execution). By employing this model, students learn how to translate and integrate mathematical problem, planning for the solution, and finally execute the planning in order to help them become successful problem solvers. There is no looking back stage which requires students to check back their solution. Therefore, during the treatment session, students were not taught to look back their solution because it is not in the problem solving model as proposed by Mayer. Hence, students could not able to improve their looking back ability in the post MPSAT questions.

In this study, results were inconsistent with the suggestion made earlier that incorporating visual representation into Mayer's problem solving model could improve students' problem solving ability because the more the visual representations include appropriate relational and numerical components, the closer students would fall on the accurate solution of the problem (Krawec, 2014). As discussed above, it appeared that students made improvement in understanding the problem, devising a plan, and carrying out the plan ability, but not in the looking back ability due to the nature of Mayer's problem solving Model. The inconsistency of this result makes it difficult to compare this work to previous work done by Mayer (1985) who had found a model that describe the processes that problem solvers use from the beginning until they finish their tasks successfully.

5.3.3 Interaction between Mayer's Problem Solving Model and Mayer's Problem Solving Model with Visual Representation Teaching Strategy and Gender on Year 4 Students' Mathematical Problem Solving Ability

This study do not find sufficient evidence to conclude that there is interaction between gender and teaching strategies in improving Year 4 students' mathematical problem solving ability.

There is no sufficient evidence to support that there is an interaction between problem solving teaching strategy and gender on students' mathematical problem solving ability. Based from past studies, there is an interaction between problem solving teaching strategy and gender on students' mathematical problem solving ability. Such past studies are, Hirnstein, Hausmann, and Gunturkun (2008) reported that gender differences in cognitive processing and adopting a certain problem solving strategy for mental rotation tasks is influenced by biological sex differences. Ramful and Lowrie (2015) on the other hand reported that there is a connection between spatial visualization and gender. Mayer's problem solving Model with visual representation teaching strategy in this study requires the use of spatial visualization when solving mathematical problems. Therefore, Mayer's problem solving Model with visual representation teaching strategy is assumed to be interacted with gender based on the study conducted by Ramful and Lowrie (2015). Also, Wolbers and Hegarty (2010) reported that male and female tend to adopt different teaching strategies when comes to solving mathematical problems. For example, male students develop different strategies from the strategies taught by their teachers, and think more abstract to solve mathematical problems while female students tend to use the strategies they have learned from their teachers. This study supported by Sumpter (2016), who reported male students try to use different strategies when solving mathematical problems because they include many probabilities in their thinking.

However, there are few studies that failed to show that there is an interaction between gender and teaching strategies in improving students' mathematical problem solving ability. Such studies are, Adeleke (2008) reported that the problem solving performance of male and female students using Conceptual Learning Strategy (CLS) and Procedural Learning Strategy (PLS). A sample of 124 science students assigned into CLS, PLS and Conventional Method (CM) groups were involved in the study making use of pretest, posttest control group design. Findings of the study showed that the two learning strategies had not interacted with the performance of boys and girls in problem solving. The study therefore concluded that when training on how to solve mathematical problem solving questions is carried out in a strategies manner, both boys and girls will perform equally well without significant difference.

Besides, the results reported by Ejodamen (2018) who studied on the effect of Mastery Learning Strategy (MLS) on male and female students shows that there is no interaction between gender and teaching strategies in improving students' mathematical problem solving ability. A sample size of 78 (43 male, 35 female) students from two selected schools participated in this study. A 50-item multiple choice Basic Technology Achievement Test (BTEAT) was used. Both the experimental and control groups received pre-test and post-test. It was found out that there was a not significant interaction effect between gender and instructional strategy (MLS), on students' academic achievement in BTE. Based on the findings, it was concluded that MLS is an effective instructional strategy that significantly enhances gender academic achievement in BTE.

There are several explanations on why the study was not able to prove that there is an interaction between the problem solving teaching strategy and gender on students' mathematical problem solving ability. The first explanation is that, students in MM group, MMVR group, and Control group were given the same amount of work, drilling practice, and similar type of worksheets by the teacher, regardless their gender. Therefore, this might give an equal opportunity for both male and female students in this study to excel in problem solving. This assumption adds to Carrington, Tymms, and Merrell's (2008) work that reported, teachers' consistency and equal support in students learning regardless gender play an important part in students' school performance. In 2006, a panel convened by the National Academies released a report that blamed bias for the gender gap in mathematics and science (Fogg, 2006). The panel concluded that women are underrepresented in positions of maths and science due to biases, discrimination, and outdated institutional structures (Fogg, 2006). The panel's findings concluded that they are unable to find any significant biological differences between men and women in performing science and mathematics that can account for the lower representation of women in these fields.

Besides, teacher in MM group, MMVR group, and Control group also give an equal attention for all students regardless their gender. During the treatment session, teacher makes sure both male and female students participate in the group activities, class discussion, and were administrated equally to check if they follow the instructions in the worksheets correctly or not. This brings to a possible reason on why there is no interaction between teaching strategies and gender in this study. This belief aligned with the study conducted by Samuelsson (2016) which states teachers make a difference in students' performance since they are responsible for the interactional conditions in classroom. From this study, it can be said that some teaching strategy may not be interacted with gender.

It has been argued that gender differences in cognitive processing and adopting a certain problem solving strategy for mental rotation tasks is due to the biological sex differences (Hirnstein, Hausmann & Gunturkun, 2008). However, this study shows that when equal attention, amount of work and type of class activities given to students regardless gender disparity, both male and female students could perform well in mental rotation task. The result of this study inconsistence with the study by Ramful and Lowrie (2015) which reported that there is a relationship between spatial visualization and gender.

Also, the sample size used in this study included a small number of participants (averagely 30 male and 30 female students) for each treatment group which gives insufficient of data to prove the interaction between gender and teaching strategies of this study. Smaller sample size raises the issue of generalizability to the whole population of the research in terms of the research method (Harry & Lipsky, 2014; Thompson, 2011). The decision on the size should reflect the quality of the sample in this wide interval although the sample size between 30 and 500 at 5% confidence level is generally sufficient for many researchers (Thomson, 2004).

Finally, gender itself might be a factor that does not have a direct effect to the teaching strategies used in problem solving. This is because, Al Shabibi (2017) reported that gender does not influence the differences on metacognitive skills and mathematical problem solving ability among 6th grade students of varying levels of achievement (learning disabled, average achievers and high achievers). The study sample included 90 students in grade six with 30 students enrolled in a learning disability program, 30 average-achieving students and 30 high-achieving students. The results showed that there is no gender differences in both mathematical problem solving and metacognitive skills.

The result of this study also consistent with the study by Noureen and Sheikh (2016) which reported that both male and female students have shown an equivalent problem solving proficiency in mathematics. This study involved a large sample of 1500 grade 6 students (public sector schools) from four districts of the Punjab province. The study sought to find out the gender differential mathematical problem solving performance of the students. It was found that boys and girls have shown equivalent problem solving proficiency in algebra, area and perimeter, whole number and volume and surface area.

Nor'ain and Mohan (2016), on the other hand, conducted a study to explore the link between scientific reasoning skills and mathematics performance as measured by students' responses to a series of novel problems. A total of 351 Year 11 students from 14 Malaysian secondary schools participated in the present study. Results indicated that there is no differences between both scientific reasoning skills and problem solving ability with gender.

5.4 Implications

In this section, the implications of the study is presented. There are four implications of the study. Implications for classroom practice are discussed in the first section. The second section discusses the implications for in-service teachers' professional development followed by the third section which discusses the implications for pre-service teachers' education. This section follows with a discussion of the implications for policy makers and curriculum developers, and end with contribution to theories.

5.4.1 Implications for Classroom Practice

The implication of Mayer's problem solving Model with visual representation teaching strategy for classroom practice is that teachers can modify their current pedagogy or lesson plan to increase students' performance in problem solving. Mathematics teachers need to know about effective pedagogies in order to include them in their repertoire. Teachers tend to adapt their instructional practices to the overall characteristics of their students. Enhanced activities should be used in classes with a high proportion of students with a different problem solving ability background. As it helps provide students with appropriate levels of cognitive challenges, such adaptation should be encouraged (Topping, 2005). If teachers do not change their teaching practices, there is no increase in student achievement in problem solving. As the literature discussed in this paper informs that it is the teaching practice or pedagogy that affect students' performance in problem solving.

A positive learning environment is important for both students and teachers (Lounkaew, 2013). In order to make a positive teaching and learning environment, teachers need extra support, through interventions that consider teachers' individual characteristics and competences and the features of individual classes. Therefore, the teaching strategy, lesson plan, and worksheet developed in this study could help existing math teachers to make their teaching more effective with positive outcomes. Teachers could use the Mayer's problem solving Model with visual representation lesson plans utilizing the components of explicit instruction that was prepared for this study (See Appendix B). It is important to note that strategy instruction within the explicit teaching cycle follows a pattern that is evidenced in each of the prepared lesson plans (Hudson & Miller, 2006).

The use of worksheets (See Appendix B) containing of leading questions in this study, encouraged students to achieve all three (except Looking Back ability) sub-dimension abilities using only visual representation method. Teaching and learning based on Mayer's model will help students to gain access to strategic knowledge, to guide them as they apply strategies, and regulate their use of strategies and their overall performances as they solve problems. It may also teach them the habit of mind to follow the leading questions even without the assistance of the worksheets. Therefore, students will appreciate the leading questions that will guide them understand the problem, devise a plan, and carry out the plan when solving the problems.

Questions need to be in higher order and exploratory in order to allow students to construct their own knowledge and understanding (Moursund, 2003). Moreover, to allow students to develop their personal understanding though answering the questions, questions need to be open-ended, rather than simple closed questions, where the answers are already pre-determined. Students need to be given the opportunity to gradually learn processes and construct their own answers. Teachers can promote the questions from Mayer's problem solving Model with visual representation worksheets to encourage students to gradually construct their understanding in solving mathematical problems.

The low-cost, easy, and efficient use of the Mayer's problem solving Model with visual representation teaching strategy is an important implication for classroom teachers. The cost of implementing this strategy was low in that two reams of paper were used to photo copy all materials including the lesson plans and the worksheets, file folders for each student, management chart to record students' score, and scoring sheet to score students answer/ solution on the given mathematical problems. In rural and urban schools, teachers will benefit from using strategies that are low cost as additional resources may not be available to them to purchase more expensive learning strategy curriculums.

Also, a positive teacher-student relations is being reported for teachers who exchange ideas and information and co-ordinate their practices with other teachers at their school (OECD, 2014). Co-operation in conjunction with improving teacher-student relations, leads to a positive school culture. Therefore, it may be reasonable to encourage existing math teachers to share and learn together on how to implement Mayer's problem solving Model with visual representation teaching strategy to improve students' mathematical problem solving ability in their schools. Positive teacher-student relations are closely related to teachers' job satisfaction, at least at the individual teacher level (OECD, 2014)

Teachers need to provide activities which are engaging and challenging the learners. Mayer's problem solving Model with visual representation teaching strategy and its materials are engaging and challenging for students. This makes students to participate more in the class activities and discussion. This demands a board array of work which is differentiated to the students' intellect. Teachers need to offer scope of activities which makes accustomed effort and activity falls on the learners' responsibility. In order to assure that all students apply mental effort and take an active role in their own learning, a differentiation is a critical implication in the classroom. Such opportunities would afford learner engagement and optimise the possibility of effective lasting learning taking place (Pritchard, 2009). Personalisation is also crucial to ensure all learners, despite genetic and innate differences which may affect their learning are accounted for. If a student is set tasks which do not require thought or challenge, learning constructively will fail.

5.4.2 Implications for In-Service Teachers' Professional Development

This study found that Mayer's problem solving Model with visual representation teaching strategy is effective in enhancing students' mathematical problem solving ability. Therefore, the use of Mayer's problem solving Model with visual representation teaching strategy when solving mathematics word problems leads to many implications and considerations for in-service teachers' professional development program.

Professional development that has shown an impact on student achievement is focused on the content that teachers teach. Content-focused professional development generally treats discipline-specific curricula such as mathematics, science, or literacy (Doppelt, Mehalik, Schunn, Silk & Krysinski, 2008). It is most often job embedded, meaning the professional development is situated in teachers' classrooms with their students, as opposed to generic professional development delivered externally or divorced from teachers' school or district contexts. This type of professional development can provide teachers the opportunity to study their students' work, their existing teaching methodology, hence test out new curriculum with their students (Doppelt et al., 2008). Therefore, schools can organize programs that in-service math teachers could participate in analysing their teaching where the program focusses in specific math content which is problem solving, and its new pedagogy which is using Mayer's problem solving Model with visual representation teaching strategy. The analysis of this program could be scaffolded by any relevant professional development facilitators.

Teacher collaboration is an important feature of well-designed professional development as schools have increasingly structured teaching as a collaborative community endeavour (Allen, Pianta, Gregory, Mikami, & Lun, 2011). Collaboration can span a host of configurations from one-on-one or small-group interactions to schoolwide collaboration to exchanges with different professionals beyond the school (Allen at al., 2011). Any school who successfully created an interactive classroom using Mayer's problem solving Model with visual representation teaching strategy's materials such as its worksheets and lesson plans, could host a program that designed to improve teacher-student interactions. In-service teachers who intended to develop their profession could participate in an initial training workshop in those schools followed by follow-up coaching from a remote mentor. For teachers who are in remote or rural schools and who did not have access to professional learning opportunities more readily available in suburban or urban areas, this model of professional development could be promising.

Also, professional development that utilizes models of effective practice has proven successful in supporting student's achievement and promoting teacher learning. Instructional and curricular models help teachers to have a vision of practice on which to anchor their own learning and growth (Heller, Daehler, Wong, Shinohara, & Miratrix, 2012). Video or written cases of teaching, demonstration lessons, unit or lesson plans, observations of peers, and curriculum materials including sample assessments and student work samples can be included as the various kinds of modelling (Heller et al., 2012). Therefore, during professional development program, facilitators could focus on pedagogical mathematical problem solving content knowledge, utilizing two different interventions which are Mayer's problem solving Model and Mayer's problem solving Model with visual representation teaching strategies. This will show the comparison of the effectiveness between those two strategies. Hence, in-service teachers can know which effective problem solving strategy and its materials to be employed to use in their classroom.

To offer more support to in-service teachers, professional development would include discussions of problems that can and have emerged when employing Mayer's problem solving Model with visual representation teaching strategy. While it is acknowledged that students have to access a variety of problem solving model in order to solve mathematical problems successfully, this research provides evidence that explicitly teaching students the benefits of Mayer's problem solving Model with visual representation teaching strategy and explaining it in different ways that students can utilise these materials for learning will ensure better teaching delivery and outcomes. This study also indicated that in reality both the teaching and learning involved in utilising new problem solving model is not static and instead may shift between processes and stages in a dynamic way depending on the experience, motivation and disposition of the teacher and the learner. What makes this model useful is its ability to associate the most appropriate instructional delivery with the cognitive stage of the learner. Promoting awareness and understanding of the relationship between the problem solving model and the needs and goals of students is essential to increasing engagement and satisfaction.

5.4.3 Implications for Pre-Service Teachers' Education

Emphasis needs to be given to the role that visualisation can play in mathematics education, during the initial teacher education. The focus during teacher education can be on the transfer of effective teaching strategies between curriculum areas. The use of visual representations strategy along with Mayer's problem solving Model can be used in mathematics to increase students' achievement. Therefore, this approach needs to be made explicit within pre-teacher trainings.

Teacher education as vocational training would inhibit constructive responses to problems of identifying formation in becoming a teacher. A training approach emphases the development of discrete and technical teaching skills and often assumes a single definition of a 'good teacher' which centres on the demonstration of skills. In this view, pre-service teachers can be trained on how to use teaching materials especially Mayer’s problem solving Model with visual representation lesson plan and worksheets effectively and also on how to create or adapt these materials according on student’s ability in order to attract their interest and make them eager to learn.

Teaching method should move towards students-centered, as well as should become more reflective in preparing and delivering the lesson. Pre-service teachers therefore can be trained on how to effectively manage a following the lesson plan and worksheets (See Appendix A and B) developed for this study. Also, they can be trained on how to be flexible and to utilise a variety of strategies and teaching approaches to ensure the best possible mastery of different aspects of the subject content especially in the problem solving. These will show them so many aspects in teaching that are not defined in the syllabus and curriculum of the school, yet they are important elements of teaching.

Sawyer (2004) assert that effective teaching must be adaptive. This quality has been called “adaptive metacognition” (Lin, Schwartz, & Hatano, 2005), “thoughtfully adaptive” teaching (Duffy, 2005), or “adaptive expertise” (Bransford, Darling-Hammond, & LePage, 2005). Teachers who possess this quality, independent of the name, are able to use curricular tools and apply their professional knowledge flexibly to fit the particular students interacting with particular ideas in particular circumstances. In order for teachers to innovate, or adapt, they must learn to access the appropriate knowledge and implement this knowledge in a way that fits specific students in particular circumstances. This process has been referred to as 'Pedagogical Reasoning' by Shulman (2006). Therefore, pre-service teachers should be trained on how to use Mayer's problem solving Model with visual representation lesson plan and its materials so then they can apply this method under any circumstances when they teach problem solving.

Studies on teacher development suggest that the pre-service teacher develops from an initial preoccupation with self to a focus on tasks and teaching situations and finally to a consideration of pupil learning ((Burn, Hagger & Mutton, 2003). This highlights the implication and challenges for teacher educators in terms of course structure and curricula and the need to be responsive to individual learners. Mayer's problem solving Model with visual representation teaching strategy thus will have implications for course structures and pre-service teacher development throughout their program of study and consider whether the pre-service teacher curriculum takes into account the complexity and diversity of pre-service to graduate teacher development and identity.

5.4.4 Implications for Curriculum Developers

The results in this study show that visualising can help students increase their achievement in a short time and the importance of this needs to be emphasised in official documents. This could be achieved with a greater focus on the teaching model strategy and an increased emphasis on the importance of the using imagery step in curriculum.

Mayer's problem solving Model with visual representation teaching strategy intervention was implemented to improve elementary school students' problem solving ability in this study. Using the Mayer's problem solving Model with visual representation worksheets, students were able to receive individual corrective feedback on their strategy use and were provided with ample opportunities to practice using the strategy. Therefore, for students failing the secondary school mathematics curriculum, there is a need for a resource room, supplemental material, or supported inclusion model to provide intensive-explicit instruction for students from any level who struggling in solving mathematical word problems to improve their foundation in problem solving.

While results demonstrated that students' ability to discriminate between relevant and irrelevant information was a contributing factor to problem solving accuracy which is one of the problem solving ability defined by Polya (1945), the practical relevancy of students' discrimination abilities should be debated. Most classroom math textbooks focus each lesson on a specific skill and provide practice questions based on that skill (Bell, Bretzlauf, Dillard, Hartfield, Isaacs & McBride, 2007). Instruction thus amounts to procedural repetitions of the same process where problems rarely include math skills extraneous to the lesson, and even less often include information irrelevant to solving the problem (Massey, Montague, & Fults, 2009; van Garderen, 2008). Consequently, when students are confronted with irrelevant information, they may assume its relevancy based on past experience. Therefore, curriculum developers should take into consideration to develop a study materials includes combination of Mayer's problem solving Model with visual representation instruction focusing on traditional textbook-based math skills with project-based problem solving activities in order to improve both specific and generalized problem solving ability.

Mathematics teachers have encountered problem in implementing of problem solving skill. The problem solving questions can only be answered and resolved by clever students. Whereas, many students who are weak and moderate cannot answer questions that require certain problem solving strategies. The reason students are less capable of solving mathematical problems is because they were not challenged to think at a higher level. It is again about designing a curriculum using challenging and engaging questions and activities to yield successful outcome in Mathematics subject. The changes in the curriculum, pedagogy and assessment should be introduced to develop students' thinking skill in the classroom so that students can make decisions, solve problems, innovate and create ideas. Therefore, it is important to include Mayer's problem solving Model with visual representation instructions to address the problems faced by teachers in applying thinking skill through the teaching and learning of problem solving for year four students.

Time allocation for a subject plays an important role in ensuring the curriculum is implemented effectively. If a subject has minimum time allocation, the teacher has less time to engage pupils to think, reflect, explore and solve problems. The lack of time allocation leads to insufficiency to carry out the process of teaching and learning of problem solvng effectively. The lack of time management skill by teachers impacted on the utilization of time. Various negative impacts such as teacher-centered and content based teaching and learning were influenced the utilization of time. It also resulted in pupils becoming less interested in learning Mathematics. The gap between lessons also meant that most pupils tend to forget the previous lesson and also fail to complete the homework given. This has led to insufficient mastery of the knowledge, skills and special values of the subjects. According to Stabback (2016), a good quality of curriculum must have a balance between time allocation and content. Therefore, employing Mayer's problem solving Model with visual representation instruction into the Mathematics lesson plan eases the teachers to do effective time management when teaching problem solving.

5.4.5 Contribution to Study

From the theoretical aspect, the findings of this study are in congruence with Mayer's (1985) mathematical problem solving models by which students learn mathematical problem solving by translating problem, integrate problem, plan for solution, and execute planned solution. Students made at least 70% of improvement in their problem solving ability after they undergo Mayer's problem solving Model and Mayer's problem solving Model with visual representation teaching strategies treatments. A very minimal improvement can be observed for students in the control group which does not use any kind of problem solving model or worksheets to equip them in the learning process. This result strengthen Mayer's theory that viewed problem solving as a complex, multiple-step cognitive process (Mayer, 1983), and it does requires problem representation skill in which a student builds a mental representation of the problem, and problem solution skill in which a student devises and carries out a plan for solving the problem (Mayer, 1992).

The key to success in problem solving according to Polya (1945) is based on student's ability of understanding the problem, devising a plan, carrying out the plan and looking back on the solutions obtained. 70% of the abilities were achieved by most of the students in this study. The analysis of student's problem solving ability using Mathematical Problem Solving Ability Test rubric indicated that the stages and key functions in problem solving identified by Polya were in standardized, detailed, and analytical response format where it ease the teachers to give scores between four scoring levels. This enhances the theory of Polya that saying to become a successful problem solver, students should be able to understand the problem given, devise an accurate plan for the problem, carrying out the plan, and checking back the answer (Polya, 1945).

5.5 Recommendation for Future Research

While the results of this study are promising, there are several limitations and delimitations for future research that should be considered. The following recommendations have been made for further research that would add to the general knowledge on the teaching and learning of mathematical problem solving. Many further studies could be conducted as a follow-up to further investigate the effectiveness of Mayer's problem solving Model with visual representation teaching strategy on Year 4 students' mathematical problem solving ability.

There were several limitations in this study. Firstly, this study was limited in terms of data collection method. This study employed a quasi-experimental research design due to convenient sampling technique. Some of the most important questions in educational policy cannot feasibly be evaluated via quasi-experiments method only. Therefore, for future studies, it is recommended to employ a mixed methods research design so as to yield deeper outcomes concerning students' mathematical problem solving ability especially in relates to visual representation ability. For example, the research question like "Does Mayer's problem solving Model with visual representation teaching strategy affect students' problem solving ability?" could be addressed quantitatively, while the research question like "Which aspects of the Mayer's problem solving Model with visual representation Worksheet were particularly helpful for students to learn problem solving?" could be addressed qualitatively. This is because, the researcher might be interested to know more in what other effects the students who used the Mayer's problem solving Model with visual representation Worksheet had, especially effects that might not have been anticipated.

Secondly, this study was limited in terms of the time duration of the study where researcher looked for impacts on short-run measures. The treatment in this study was administered over 10 consecutive sessions. Therefore, it is recommended for future studies to consider longer treatment periods than current study for more accurate results using visual representation. Carrying out the study over more than 10 sessions would show whether greater progress could be made, and especially on the type of questions where the students in this study still struggled. Helping students become successful problem solvers may also increase their overall confidence in their ability to perform certain mathematical tasks (Koch, 2018).

Lastly, this study was limited in terms of the external validity where this study only implemented in one of the private school in Klang Valley area. The students selected for this study might be exposed to many other enrichment and tuition centers to help in their mathematics performances. While these limitations limit generalizability of findings, therefore, future research should continue to investigate Mayer's problem solving Model with visual representation teaching strategy for teaching problem solving skills to students outside Klang Valley (Kuala Lumpur) area, means in other geographic locations like Terengganu, Kelantan, and Pahang, preferably those students in the rural area who face challenges to get additional support in their school mathematics performance.

There were several delimitation in this study. Firstly, this study was delimited to the process of problem solving only and it is not focused on other mathematical strands such as reasoning and proof, communication, and connections. Therefore, it would be worth exploring other areas of mathematics such as reasoning and proof, communication, connections to see if the teaching strategy used in this study works across all the strands taught in mathematics.

Secondly, this study was delimited to arithmetical topics only in problem solving. Therefore, future research should focus on the effectiveness of Mayer's problem solving Model with visual representation teaching strategy in improving students' problem solving ability on other mathematical topics such as equations involving algebras to see if this intervention is workable to other mathematical topics or not.

Thirdly, current study was delimited in teaching problem solving using Mayer's problem solving Model with visual representation teaching strategy to private school students who studying KSSR curriculum. This limits the generalizability of these results to other students who studying international curriculum, or to those students who studying in local schools. Therefore, future research should be conducted with different type schools which are national schools in Malaysia with KSSR curriculum, and international schools with international curriculum like 'International General Certificate of Secondary Education' (IGCSE) curriculum in order to determine if the Mayer's problem solving Model with visual representation teaching strategy is an effective intervention for teaching problem solving skills for students from different type of schools and curriculum.

Also, this study was delimited to only Year 4 students with a small number of participants (averagely 30 male and 30 female students) for each treatment group. This limited the generalizability of the results of this study to a different grades students and to a larger population of students. Therefore, future research should replicate the study at later times to students from other grades like Year 5 or 6, with a larger sample size which is more than 100 for each treatment group to determine if the same results occur as in the earlier time, especially when the study is on the interaction effect of gender and teaching strategy. Having a larger sample size is important to conduct a study on the gender issue in order to yield an in-depth results. Having a smaller sample size in this study did not provide much information if gender issue did pose a significant impact on the students' problem solving ability in regards of gender.

5.6 Conclusion of the Study

The results from PISA and TIMSS indicated that the ability of solving mathematical problems among Malaysian students remains at a low level. Students' poor performance in problem solving is due to the teaching and learning process in the school system. This research therefore aimed to determine the effectiveness of problem solving strategy namely Mayer's problem solving Model with visual representation teaching strategy in improving students' mathematical problem solving ability. Based on a quantitative analysis of Mayer's problem solving Model with visual representation teaching strategy in response to Year 4 students' problem solving ability, it has been showed that the use of visual representation along with Mayer's problem solving model is effective in improving students' ability in solving mathematical problems for both male and female students.

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