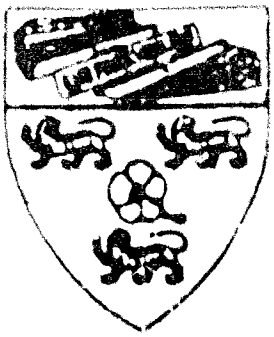

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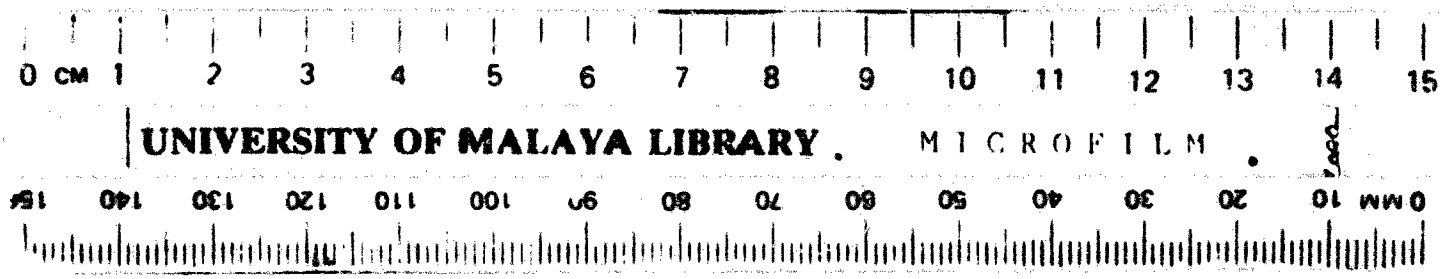
STRUCTURES OF DIFFERENTIABLE MANIFOLDS

by

TU YONG ENG



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INTRODUCTION

This dissertation consists of four chapters. Chapter 1 introduces the basic concept which will be used in the later part of the dissertation. Chapter 2 is devoted to the study of G-structures. Chapter 3 is on the f-structure and its related structures. Almost tangent structures and almost cotangent structures are discussed in Chapter 4.

In this dissertation, the manifolds are assumed to be real, paracompact and differentiable. It is well known that such a manifold admits a metric. Hence, it has an underlying Riemannian structure.

A result on the integrability of a G-structure is as follows: A G-structure is integrable if and only if there exists a permissible n-frame $\{v^1,\ldots,v^n\}$ on each open set U_α of a coordinate covering $\{(U_\alpha,x_1,\ldots,x_n)\}$ such that if c_{ij}, determined by $\frac{\partial}{\partial x_i} = \sum_{j=1}^{n} c_{ij}v^j$, then we have $\frac{\partial c_{ij}}{\partial x_k} = \frac{\partial c_{kj}}{\partial x_i}$ for all i, j and k.

We studied also an integrable almost complex structure on S^2. Furthermore, based on a set of transition functions on $\mathbb{R}P^{2n-1}$, we conclude that $\mathbb{R}P^{2n-1}$ is orientable.

In Chapter 3, we discussed the concept of f-structures, globally framed f-structures, (f,k) structures and bi-f structures. These structures are defined by certain (1,1) tensor fields. In this chapter, the relations among these structures and G-structures are discussed. It is shown that $S^{2p+1} \times S^{2q+1}$ admits a bi-f structure.

Almost tangent structure and almost cotangent structures are discussed in Chapter 4. We studied the basic properties of these structures. Based on the G-structure, we showed that the almost tangent manifold admits an almost cotangent structure and an almost complex structure. Similarly, the almost cotangent structure gives rise to an almost tangent structure and an almost complex structure.