TECHNICAL EFFICIENCY PERFORMANCE OF MALAYSIAN PUBLIC RESEARCH UNIVERSITIES: FUZZY DATA ENVELOPMENT ANALYSIS

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THESIS SUBMITTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

FACULTY OF BUSINESS AND ECONOMICS
UNIVERSITI MALAYA
KUALA LUMPUR

2023

UNIVERSITI MALAYA

ORIGINAL LITERARY WORK DECLARATION

Name of Candidate: SABER ABDELALL MOHAMED AHMED

Matric No: EVA180007 & Student ID: 17029156

Name of Degree: DOCTOR OF PHILOSOPHY

Title of Degree: TECHNICAL EFFICIENCY PERFORMANCE OF MALAYSIAN PUBLIC RESEARCH UNIVERSITIES: FUZZY DATA ENVELOPMENT ANALYSIS

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TECHNICAL EFFICIENCY PERFORMANCE OF MALAYSIAN PUBLIC RESEARCH UNIVERSITIES: FUZZY DATA ENVELOPMENT ANALYSIS

ABSTRACT

Of late, university technical efficiency measurement via Data Envelopment Analysis (DEA) is widely applied when the technique enables universities as the decision-making units (DMUs) to identify the top performers among them being evaluated and discover the alternative ways to spur their operations to become one of the best performers. Despite decreasing trends of government funds to finance universities' operational and research expenditures, the public research universities in Malaysia (PRUMs) continue to shoulder the responsibility to be a leader in innovation, produce world class research outputs, that includes high impact research publications, and to increase their world ranking and international reputations. Good ranking universities would attract high-quality local students and from abroad. Technical efficiency is an alternative approach to best measure the PRUMs performance at the global level while providing valid data and information for future framework of long-term plans. This is not only crucial for the decision makers of PRUMs but also to National Higher Education.

This research employs Data Envelopment Analysis (DEA) to measure the technical efficiency of five PRUMs for four successive academic years 2018/2019 to 2021/2020. Based on past research supported by scholars in higher education studies, this study three input variables selected are the number of full-time-equivalent (FTE) staff, number of full-time-equivalent students and ratio of FTE international students to FTE students. With the growing importance for universities to achieve higher international

ranking, that this study proposes the international QS world ranking indicators as the output variables, namely: Teaching Reputation, Research Reputation, and the citations percentage ratio. For the international ranking indicators are beyond the control of research universities, this study aims to measure the technical efficiency of public research universities in Malaysia (PRUMs) by employing the non-parametric Fuzzy Data Envelopment Analysis (FDEA) methodology. By utilizing Fuzzy DEA (FDEA) model with algorithm, this study observes the PRUMs efficiency status during the study period and the expected average efficiency for the next 2021/2022 academic year. To further benchmark the efficiency of PRUMs at the international arena, this study assesses PRUMs' technical efficiency by grouping them together with selected public research universities in Asia (APRU).

The contribution of this study is two-fold. First, while many studies on performance measurements of high education institutions (HEI) studies apply DEA methods, this study is among the very few that employ Fuzzy DEA approaches. This study integrates the concept of fuzzy set theory with the traditional DEA by introducing an algorithm to measure the technical efficiency of PRUMs in the form of fuzzy linear programming (FLP) models. Secondly, while DEA model requires precise input and output data of HEIs, this study utilizes the QS World University Ranking research output indicators as the output variables of the Fuzzy DEA model. Clearly the QS metric is beyond the control of any DMUs, thus the application of FDEA suits the criteria to measure the technical efficiency performance of PRUMs at the international level.

Findings from this study reveal, while all PRUMs are expected to be fully efficient in the next 2021/2022 academic years, this is not the case when benchmarked against the selected Asian public universities. Areas of improvement can be observed by determining the respective variables that need further analysis to achieve full efficiency.

Keywords: DEA, fuzzy DEA, fuzzy linear programming (FLP), technical efficiency, university ranking, public research universities.

PRESTASI KECEKAPAN TEKNIKAL UNIVERSITI PENYELIDIKAN AWAM MALAYSIA: ANALISIS PENYELUBUNGAN DATA KABUR

ABSTRAK

Sejak kebelakangan ini, pengukuran kecekapan teknikal universiti melalui analisis penyelubungan data kabur digunakan secara meluas apabila teknik ini membolehkan universiti sebagai unit membuat keputusan (DMUs) untuk mengenalpasti universiti berprestasi terbaik dalam kalangan universiti yang dinilai. DMU juga boleh menemui cara merangsang operasi untuk menjadi salah satu universiti terbaik. Walaupun terdapat trend penurunan dalam peruntukan dana kerajaan membiayai perbelanjaan operasi dan penyelidikan, universiti penyelidikan awam di Malaysia (PRUM) terus memikul tanggungjawab untuk menjadi peneraju dalam inovasi, menghasilkan penyelidikan bertaraf dunia, termasuk penerbitan penyelidikan berimpak tinggi, dan meningkatkan kedudukan di persada dunia dengan meningkatkan reputasi antarabangsa. Kedudukan universiti yang baik akan menarik lebih ramai pelajar berkualiti tinggi dari dalam dan luar negara. Kecekapan teknikal adalah pendekatan alternatif untuk mengukur prestasi PRUM di peringkat global sambil menyediakan data dan maklumat yang sah untuk rangka kerja masa depan termasuk rancangan jangka panjang. Ini bukan sahaja penting untuk PRUM sendiri tetapi juga kepada pihak Pengajian Tinggi Negara.

Penyelidikan ini menggunakan analisis penyelubungan data untuk mengukur kecekapan teknikal lima PRUM untuk empat tahun akademik berturut 2018/2019 hingga 2021/2020. Berdasarkan penyelidikan masa lalu yang disokong oleh para sarjana dalam kajian kecekapan pendidikan tinggi, tiga pemboleh ubah input dipilih adalah bilangan

staff, bilangan pelajar dan nisbah pelajar antarabangsa. Dengan meningkatnya kepentingan universiti untuk memperbaiki kedudukannya di peringkat antarabangsa , kajian ini mencadangkan tiga petunjuk kedudukan dunia QS antarabangsa sebagai pembolehubah output, iaitu: Reputasi Pengajaran, Reputasi Penyelidikan, dan nisbah peratusan petikan. Oleh kerana indikator kedudukan antarabangsa adalah di luar kawalan PRUM, kajian ini bertujuan mengukur kecekapan teknikal PRUM dengan menggunakan metodologi bukan parametrik analisis penyelubungan data kabur (FDEA).

Dengan menggunakan model analisis penyelubungan data kabur ber-algoritma, kajian ini memerhatikan status kecekapan PRUM dalam tempoh kajian dan kecekapan purata dijangkakan untuk tahun akademik seterusnya 2021/2022. Untuk menanda aras kecekapan PRUM di persada antarabangsa, kajian ini menilai kecekapan teknikal PRUM dengan mengelompokkannya bersama universiti penyelidikan awam Asia yang terpilih (APRU).

Terdapat dua sumbangan utama kajian ini. Pertama, sementara banyak kajian pengukuran prestasi kajian institusi pendidikan tinggi (HEI) yang dijalankan menggunakan kaedah DEA, sangat sedikit yang menggunakan pendekatan FDEA. Kajian ini mengintegrasikan konsep teori analisis penyelubungan data tradisional dan analisis penyelubungan data kabur dengan algoritma untuk mengukur kecekapan teknikal PRUM dalam bentuk model pengaturcaraan linear data kabur (FLP). Kedua, sementara model DEA memerlukan data input dan output yang tepat, kajian ini menggunakan indikator keluaran penyelidikan QS World University Ranking sebagai pemboleh ubah output model FDEA. Jelas sekali metrik QS adalah di luar kawalan mana-mana DMU, oleh itu penggunaan FDEA adalah bersesuaian dengan kriteria untuk mengukur prestasi kecekapan teknikal PRUM di peringkat antarabangsa.

Dapatan daripada kajian ini mendedahkan, walaupun semua PRUM dijangka mencapai kecekapan sepenuhnya pada tahun akademik 2021/2022 akan datang, ini nampaknya tidak berlaku apabila ditanda aras terhadap universiti awam Asia (APRU) yang terpilih. Walaubagaimanapun, ruang penambahbaikan boleh dikenalpasti dengan menentukan pembolehubah mana yang memerlukan analisis lanjut bagi mencapai kecekapan penuh.

Katakunci: Analisis penyelubungan data (DEA), analisis penyelubungan data kabur (FDEA), pengaturcaraan linear data kabur (FLP), kecekapan teknikal, kedudukan universiti, universiti penyelidikan awam.

ACKNOWLEDGEMENTS

First and foremost, I would like to dedicate this work to the cherished souls of my parents. Without their unwavering support and guidance, I would not have been able to accomplish this endeavor. Thank you, my beloved mother and father.

The journey of writing this research has been a fascinating and extensive experience. However, I could not have completed this thesis without the care and assistance of my beloved wife, Resa. She constantly reminded me that this research was a gift for both of us, encouraging me to write it with precision and dedication. I sincerely hope that I have done justice to her expectations.

I extend my heartfelt gratitude to my supervisors, Dr. Muzalwana, Associate Professor Dr. Rohana and Associate Professor Dr. Noor Fadiya.

To Dr. Muzalwana, my esteemed master teacher, I extend my deepest gratitude. Your presence throughout my academic journey, from my master's degree to the completion of my Ph.D., has been instrumental. Your unwavering support and belief in my abilities have been a source of calm and self-assurance. Dr. Wana, you are an extraordinary, exceptional, and highly knowledgeable teacher.

Dr. Rohana, you ignited the spark that led me to explore the efficiency of the faculties at the University of Malaya. Your understanding and provision of supportive data for the study have been invaluable. Our interactions have consistently generated new ideas, inspiring me to delve deeper into this research and future studies. Thank you, Dr. Rohana.

Dr. Fadiya, I am at a loss for words to express my appreciation. Without your guidance, I would not have been able to publish any articles. You have provided me with a wealth of experience in the research domain, and I am immensely grateful for your mentorship.

I am also indebted to all the individuals and institutions who have provided me with unconditional supports and encouragements during my pursuit of a Doctor of Philosophy degree.

Each person mentioned here has made a significant contribution to this study, whether through their time, knowledge, or simply granting me the freedom to conduct my research. The merits of this research are attributed to their contributions, while any shortcomings remain under my responsibility. I express my sincere gratitude to all of them.

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LIST OF SYMBOLS AND ABBREVIATIONS

QS	: World Ranking Top Universities
PRUM	: Public Research Universities in Malaysia
UM	: Universiti Malaya
USM	: Universiti Science Malaysia
UKM	: Universiti Kebangsaan Malaysia
UPM	: Universiti Putra Malaysia
UTM	: Universiti Teknologi Malaysia
HUK	: University of Hong Kong
HKUST	: Hong Kong University of Science & Technology
KU	: Kyoto University
SNU	: Seoul National University
FDU	: Fudan University China
RU	: Research Universities
EECA	: Top 200 universities in Emerging Europe and Central Asia
BRICS	: Strongest universities in the emerging markets of Brazil, Russia, India, China, and South Africa
WR	: The QS World Ranking
AR	: The QS Asian Ranking
HEI	: Higher Educational Institution
DM	: Decision Makers
DEA	: Data Envelopment Analysis
DMU	: Decision-Making Unit

CCR	: Charnes, Cooper and Rhodes model
LP	: Linear Programming
FDEA	: Fuzzy Data Envelopment Analysis
IFDEA	: Intuitionistic Fuzzy DEA
FFDEA	: Full Fuzzy DEA
TFN	: Triangular Fuzzy Number
sAPRU	: selected Public Research Universities in Asia
FLP	: Fuzzy Linear Programming
CRS	: Constant Returns to Scale
VRS	: Variable Returns to Scale
T.E	: Technical Efficiency
RHS	: Right Hand Side
MCDM	: Multi-Criteria decision-making method
AHP	: Analytical Hierarchical Process
BCC	: Banker, Charnes and Cooper model
FDH	: Fuzzy decision hierarchical model
ERP	: Enterprise Resource Planning
SBM	: Slack-based measure
FCCR	: Fuzzy Charnes, Cooper and Rhodes model
CGM	: The center of gravity method
CCP	: Chance-Constrained Programming
IPR	: Intellectual property rights
DF	: Degree of freedom
CSP	: China Statistical Press

KAM	: Kourosh and Arash model
R&D	: Research and Development
ECM	: Efficiency Contribution Measure
PCA	: Principal Component Analysis
COLS	: Corrected ordinary least squares
FTE	: Full Time Equivalent
MCO	: Movement Control Order
PCC	: Person Correlation coefficient
CWUR	: The Center for World University Rankings
ε	: very small positive number
θ	: Radial Measure of Technical Efficiency

CHAPTER 1: INTRODUCTION

1.1 Background of the study

For many universities around the world, it is a very important issue for these universities to improve their position in the world ranking list for example to be the top 100 universities. It is also important for any university to be on the leading list in its own local or national arena. As an example, the QS World University Rankings of Universiti Malaya (UM) for the academic year of 2020/2021, is 65th (top 100 universities) and its local Asian ranking is 8th (top 10 university in Asia). More importantly, UM need to keep on improving its world ranking, at least, to enter the 50 top university list and keep on improving its regional rankings in the best of 10 Asian universities or to be best of these ten.

For the last 3 decades the Malaysian universities and the Asian university systems have shown much improvement in terms of rankings, especially the public research universities in Malaysia (PRUM) and also for all Asian public research universities. These universities play very important roles in the Malaysian society as the PRUM are now set to be the leader in innovation, to establish and enhance centers of excellence in selected areas of the nation (MOHE, 2021). These research universities are expected to produce world class research outputs, to generate high impact research effective publications. These PRUMs are also expected to attract graduate students of high standards locally and abroad for these institutions should have the facilities to provide conducive environment for research work to be undertaken (MOHE, 2021).

1.2 Public Research Universities in Malaysia (PRUM)

As the name indicate, a research university is one where the academic faculty is consistently doing research. Through the Ministry of Higher Education Malaysian government envisaged research activities would contribute to economic development by developing knowledge-linking activities that enhance science and technology transfer, commercialization, workers, and professional competencies (Komoo, et al., 2008). This led to the transformation of 4 Malaysia public universities to research universities in 2005. The transformation of public universities churns out 5 research universities in total, 4 comprehensive universities and 11 focused universities. The research universities focus on research but also academically orientation and, as such, the ratio of undergraduates to postgraduates is around 50:50. Comprehensive universities offer various courses and different fields of study and the ratio of undergraduates to postgraduates is 70 to 30. The focused universities are further subdivided into technical universities, education universities, management universities and defense universities. Almost all universities offer a full range of academic programs in undergraduate, master's, and doctoral programs.

The emphasis to reinforce the Research & Development (R&D) culture within the Malaysian public universities was clearly underlined in the National Higher Education Strategic Plan 2007 – 2020 (Sheriff, N.M. & Abdullah, N. 2017). Within the plan are for the PRUMs to meet the nation's development needs to develop human capital and raise, as well as teaching and learning at every level of society with the hope of attracting students from around the globe to pursue higher education in Malaysia. In a nutshell, this plan aims to produce quality human capital characterized by knowledge, skills, creativity, innovativeness and increase Malaysia's stature and competitiveness in the international arena.

The five public research university in Malaysia (PRUM) are Universiti Malaya (UM), Universiti Sains Malaysia (USM), Universiti Kebangsaan Malaysia (UKM), Universiti Putra Malaysia (UPM) and Universiti Teknologi Malaysia (UTM). These are the universities that were established as research universities (RU) in 2015. These PRUMs shoulder the responsibility to meet the national goals: (1) to be a leader in innovation, (2) to set up and enhance centers of excellence in prioritized areas, (3) to produce world class research outputs, (4) to generate high impact research publications, (6) to attract graduate students of high standards, and (7) to provide a conducive environment for research (MOHE, 2022).

The important role of universities in a country is to create high quality human resources so that the human resources can easily adapt to rapidly changing working environment and industry demand overtimes. Good quality human resources or qualified people are generally ‘produced’ from high-quality educational institutions. As such, students or parents who are looking for good quality education, or even, companies need to ensure their prospective employees are those who graduated from the universities that are deemed of good reputation too. Thus, these university rankings are the best indicator for these people to refer to as good or high-quality universities.

The existence of research universities in the pursuit of delivering knowledge should be protected by the government (Krull, 2005) because only the governments would have the capacity. Like the other public universities, PRUM are mainly funded by the government. For more conducive research working environment and equipped with good facilities, PRUMS could stand better chance to get additional funding (MOHE, 2022). At the same time, the government expect the PRUMs to increase the output and quality of research to improve the international ranking and reputation of Malaysian universities (Ibrahim et al., 2015), which will attract more high-quality local students and from overseas.

However, there are many universities to choose from, and this makes it hard for one to come to a firm decision. University rankings are the standards performance measurement for the universities made based on several pre-determined factors, such as research excellence, teaching quality and graduate employability. The best education institutions are then ranked based on their performance.

There are many university rankings but not all university rankings use the same factors. The most popular world university rankings are the Quacquarelli Symonds, - QS World University Rankings, Times Higher Education (THE) and Academic Ranking of World Universities (ARWU) on which different assessment criteria when ranking universities. QS rankings put emphasis on how international a university is by measuring the international students and faculty members ratio). This ranking is also well-accepted by employers and other academics, too. Thus, QS world university ranking will be discussed more thoroughly with regards to this study.

1.3 The QS World and Asian University Ranking

They can compare the ranking of universities for a specific region, by subject area, or based on factors such as reputation or research citations, or just by looking at the world ranking (Top Universities, 2022) for example the QS World University Rankings.

From the marketing viewpoint, ranking of universities can be used as promotional material to attract new students (Nazarko et al., 2008; Nazarko et al., 2009). University rankings also have a fair share in influencing the government, external stakeholders, and other corporate institutions as it is useful to assist their decision-making in providing research funding (Hasnan, 2019). The QS World University Rankings® discover the world's top universities with the most widely read university comparison of their kind. It acts as a catalyst to attract the best brains and talent looking for the best place to teach and provides important information to potential international partners for academic and research collaboration. Students, parents, or potential employees aiming to enhance certain labor skills or choosing new employees who graduated from the best universities, can compare the ranking of universities in a specific region, by subject area, or based on factors such as reputation or research citations (TopUniversities, 2022). Generally, higher ranked universities would be among the choices listed by the parents, prospective students, and employees.

The QS Top Universities Ranking works in different and parallel ways to determine top ranking universities in many categories. Some of the prominent ones are the QS World University Rankings (world's top 800 universities overall); QS University Rankings Asia (top universities in Asia); QS World University Rankings by Faculty (leaders in five broad faculty areas) and QS World University Rankings by Subject (strongest in 42 individual subject area. The QS University Rankings has a specific set of indicators with different ratio for each indicator as presented in Table 1.1 below (TopUniversities, 2022):

Table 1.1: Six Indicators for the QS World University Rankings

	The indicator	The Ratio
1	Academic Reputation from Global Survey	40 %
2	Employer Reputation from Global Survey	10 %
3	Faculty Student Ratio	20 %
4	Citations per faculty from Scopus	20 %
5	International faculty ratio	5%
6	International student ratio	5%

Based on these six performance indicators, the rankings are designed to assess universities in four areas: research, teaching, employability, and internationalization. Each of the six indicators carries a different weighting when calculating the overall efficiency scores. Four of the indicators, namely Faculty Student Ratio, Citations per faculty from Scopus, International faculty ratio and international student ratio are based on 'hard' data, and the remaining two Academic Reputation and Employer Reputation are based on major global surveys (TopUniversities, 2021).

The Asian QS University Rankings indicators differ somewhat from the QS World University Rankings®. The Asian QS Rankings is based on feedback collected from the region, the expert assessment of important factors in the region and based on the availability of data, too. The research excellence aspect is based on data from Scopus, the world's largest abstract and citation database of peer-reviewed academic literature (TopUniversities, 2022). The indicators of Asian region ranking are shown below.

Table 1.2: Ten indicators are drawn together to form this regional ranking.

	The indicator	**The Ratio**
1	Academic Reputation from Global Survey	30 %
2	Employer Reputation from Global Survey	20 %
3	Faculty Student Ratio	15 %
4	Citations per Paper from Scopus	10%
5	Papers per Faculty from Scopus	10%
6	Proportion of staff with PhD 5%	5%
7	Proportion of International Students	2.5 %
8	Proportion of International Faculty	2.5 %
9	Proportion of Inbound Exchange Students	2.5 %
10	Proportion of Outbound Exchange Students	2.5 %

The 2021 QS World University Rankings is the most outstanding achievement for all five research universities because it was for the first time ever, all five Malaysian research universities have been ranked among the world's top 200 (Chan, 2020).

The following table shows the QS World (WR) and Asian ranking (AR) for PRUM for the past five years.

Therefore, in general, ranking of universities has some encouraging benefits, as not only does it boost their reputation, but it can also be used as an advertising material to entice more new students from local and abroad (Nazarko et al., 2008). It is also the requirement from the working industry for qualified and well-trained human resources to have received quality education in highly ranked universities (TopUniversities, 2022). From the indicators employed by the QS origination to produce rankings of the universities, the university ranking can be used as the reflection of quality of a university or higher education institutions (Nazarko et al., 2009).

Times Higher Education (THE), Academic Ranking of World Universities (ARWU) and Center for World University Rankings (CWUR).

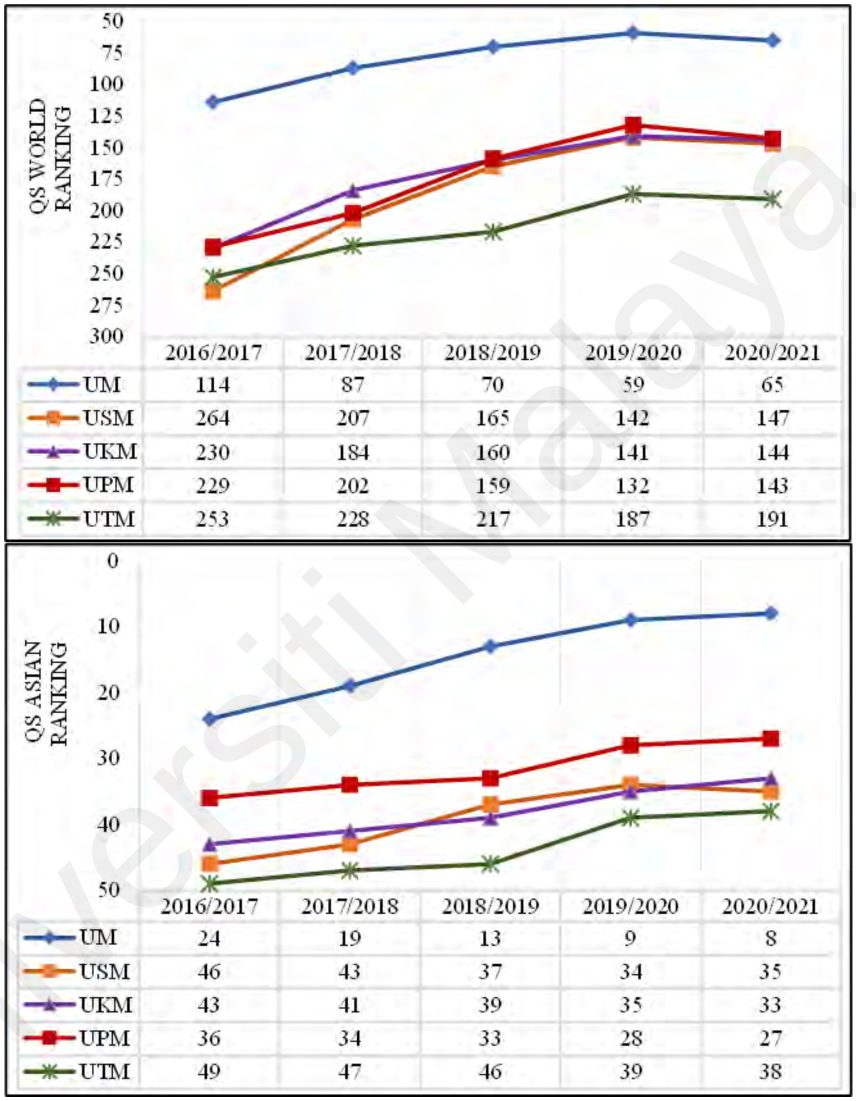


Figure 1.1: World Ranking & Asian Ranking for PRUMs, QS 2017- 2021 (TopUniversities, 2022)

1.4 PRUM Challenges and Efficiency-based ranking.

For the past decade, there has been a growing pressure for the higher education sector in many countries across the globe to increase their efficiency and to improve the quality of its service and activities for the students (Zafiropoulos & Vrana, 2008). From the previous studies, it was emphasized that many public universities, not only in Malaysia but in different countries had been facing with decreasing trends of government funds to finance their operational and research expenditures. This is important especially when involving public funds, the governments would only channel funds the public HEIs at levels deemed necessary (Cooper et al., 2011) and that the HEIs need to be transparent in using the state funding (Zafiropoulos & Vrana, 2008). There are also issues asserting the public resources not being allocated in a way that promotes efficiency, nor it meet the established goals set for the higher education sector. Moreover, the returns on investment in the higher education sector is often characterized by time-lags of decades (Zafiropoulos & Vrana, 2008). In other cases, there were also demands from the society, media, and other stakeholders for the budget execution, planning and management of the universities to be accomplished at an increased level of transparency (Gajda, 2009).

In the case of Malaysia, a huge amount totaling to more than RM14 billion that was allocated to the Higher Education Ministry as announced in the Malaysian 2021 Budget Report. This includes RM50 million for infrastructure and equipment replacement in the public universities alone (MOHE, 2022). An earlier study has shown that 70% to 90% of the total funds for the research and innovations activities within PRUMs are sourced from the government (Amran, F.H. et. Al., 2014). With the national goals set for the PRUMs, these universities obviously stand to gain additional government funding for research activities, research management, PRUM incentive grants and specialized research services like patenting, intellectual property rights (IPR) and repository (MOHE, 2022)., this may indicate that the Malaysian government wants to see that there will be no disruptions in the research and innovations activities of the Research Universities. But again, with the issues and challenges faced by the HEIs abroad, as discussed earlier, the same pressures are put on the PRUMs alike. These PRUMs need to reorganize activities and priorities, increasing research output and quality and to increase their international ranking and reputations (MOHE, 2022).

For research output is the main evaluation criteria in determining a university's world ranking, measuring the performance of PRUMs based on efficiency-based ranking is an alternative tool to enhance accountability and transparency of these institutions (Nooraini & Noordini, 2017). Important to note, when the focus of transforming the Malaysian Higher Education is to be made on outcomes and performance as underlined in the Malaysia Education Blueprint (2015-2025), efficiency measurement of the public HEIs and PRUMs alike is no longer an option (Nooraini & Noordini, 2017).

Efficiency measurement can provide detailed information on universities' performances which enable the government to make the future framework of long-term plans based on valid data (Mahmudah & Lola, 2016). Therefore, it is very important for the decision makers (DM) of PRUM to know the efficiency levels of the current year and the expected efficiency scores for the succeeding year. Based on the expected efficiency scores changes can be made in the capacity of certain input and output variables to increase the efficiency scores and hence further improve the university ranking. In turn, this could increase the efficiency for each university or for the whole higher educational system in the country (Cooper et al., 2011).

1.5 University Efficiency Measurement

So different QS origination indicators can guide the researchers in measuring the university efficiency scores based on a different set of input/output variables (TopUniversities, 2022). More importantly, it is how to create an efficiency measurement model based on these criteria for each set of indicators, according to which QS section of indicators are suitable for the university under study. The most important sets of indicators could be based on the world and/or Asia, as listed in Table 1.3, or on another set of indicators that suits the topic of study (TopUniversities, 2022). With a selection of variables for certain indicators, it is easier to measure and predict the efficiency scores for any university or HEI (TopUniversities, 2022) in the not-too-distant future.

1.5.1. Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a nonparametric method in operations research and economics (Charnes et al., 1978) that can be used to measure the efficiency of universities' performances. Under the concept of DEA, a decision-making unit (DMU) is an individual or a unit group of individuals that hold responsible to measure the efficiency of the whole department, institution, or organization. As such a decision-maker (DM) is a higher-level employee, usually in leadership, who makes challenging decisions and has very critical roles impacting how the company operates. These are the employees who hold strong decision-making roles, acquire the knowledge and critical thinking skills on how to effectively solve problems or help to find solutions to problems. They can effectively weigh possible options and decide on the outcomes that best benefit the company and its employees.

In DEA approach, the primary DEA model designed by Charnes, Cooper, and Rhodes in 1978 assessed the comparative efficiencies of any organization, thus, coined as the CCR-model (Charnes et al., 1978). The DEA method utilizes Linear Programming (LP) techniques to perform the relative connection between the outputs and inputs set for the Decision-

Making Units (DMUs) hence formulate an effective production boundary as an inference in a vision frame for the finest practices or experimental methods or DMUs. DEA methods calculates the efficiency score of each DMU by comparing along with all the other DMUs in the case study (institution, association, firm, or any) involving itself. These relative or termed as technical efficiency scores which evaluate the relative amount of weighted outcome after adding together all outputs and the summation of weighted outcome after combining all inputs to achieve the maximum optimality.

The other type is allocative efficiency. Technical efficiency refers to producing as much output as technology and input usage is needed or by using as little input as required by the technology and output production (Fried et al., 2008). This type of efficiency can either be an output orientation or an input orientation. On the other hand, allocative efficiency refers to the combination of inputs to produce a given quantity of outputs given the prevailing input prices (Coelli et al., 2002). Thus, the fundamental DEA model works from the combination of outputs and inputs variables, that are mostly fixed, selected for the organizations under study or the DMUs in DEA context.

1.5.2. DEA input and output variables

Generally, the Data Envelopment Analysis (DEA) method measures the efficiency scores of universities based on a set of input and output data.

DEA method is a popular approach because it can handle efficiency estimation and assessment not only for the previous and current, but also for the following year based on expected efficiency scores. The efficiency scores can, then, help the HEIs to improve their ranking based on the available resources and flexible variables (Cooper et. al., 2011).

Avkiran (2001) and Ozden (2008) reviewed previous studies measuring the efficiency of universities and listed the inputs and outputs applied in those studies. Some of those studies measure technical efficiency of universities while the others measure the allocative efficiency with various input and output variables used in each study. Among the earliest studies are by Bessent et al. (1983) and Tomkins and Green (1988).

Although both studies measured allocative efficiency, each study utilized different sets of input and output variables. Bessent et al. (1983) applied three input variables namely, revenue from state government, number of students completing a program and employer satisfaction with student training.

Tomkins and Green (1988) utilized the number of full-time employees, personnel costs, operating costs and other costs. Later. Beasley (1990) and Johnes (1993) were among the earliest to measure technical efficiency of higher education where both studies used research income as the input variable and research outputs as the output variable. The other

DEA studies measuring the technical efficiency of HEIs employ number of academics and number of students as the input and number of students as the output variables (Abbott & Doucouliagos, 2003; Kutlar & Babacan, 2008; Gökşen et al., 2015; Ahmed et al., 2021). The latest by Mahmudah and Lola (2016) utilized ranked data in their study.

In establishing DEA model, there are no specific guidelines to deal with the selection of variables (Niranjan and Andrew, 2011), but it is to the users' own perspectives, judgment, and expertise. In the case of choosing the input and output variables to determine the performance of education sector, variables involving price educational outputs like profits are hardly used. But with efficiency analysis methods such as DEA, as DMUs, the universities can choose other inputs and outputs based on their own objectives, for example, applying the key drivers critical to success to be the input or outputs for DEA model (Gökşen et al., 2015; Avkiran, 2001). In many cases, the input and output variables used are those contributing to performance and efficiency in higher education like number of academic staff and non-academic staff, number of undergraduate and graduate enrolments. Small different in selection of input and output variables is observed include number of accredited programs, ranking data, number of local and international student, number of student employment and amount of state funding (Gökşen et al., 2015; Mahmudah & Lola, 2016; Olariu & Brad, 2017; Ahmed et al., 2021).

It is also known that DEA approach produce good relation between productivity of the university and its efficiency make it easier to put DMUs in its suitable ranks (Coelli et al., 1998). Basically, productivity also examines the relationship between output and input in a given production process (Coelli et al., 1998). Productivity is expressed in terms of output versus input ratio (Coelli et al., 1998) and this concept of productivity is closely related to that of efficiency. The terms productivity and efficiency are often used interchangeably, although efficiency does not have the same precise meaning as production does. While efficiency is also defined in terms of a comparison of two components (inputs and outputs), the highest productivity level from each input level is recognized as the efficient situation. Coelli et al. (1998) further suggest that efficiency reflects the ability of an organization to obtain maximum output from a given set of inputs. If a university or HEI is obtaining maximum output from a set of inputs, it is said to be an efficient university or HEI (Rogers & Wright, 1998) because output is only a measure of the joint power of inputs to achieve results (Zhu, 2003).

According to Zhu (2003) where the product is the human, and the services are introducing knowledge of human being and develop the minds, it is very critical issue to measure the efficiency of the public research universities, where important research services are rendered. Therefore, DEA benchmarking approach has proven to help the HEIs, particularly the research universities, to be more efficient in their operations hence improving their standards and ranking (Rey & Racionero, 2010, Mahmudah & Lola. 2016).

DEA approach has enabled efficiency measurement to be made from multiple viewpoints through different combinations of input and output variables. As discussed earlier, in most cases, the outputs would be the variables that are more critical to success (Gökşen et al., 2015; Avkiran, 2001). It is also known that HEIs had more influence on their achieved results than the amount of their resources (Gökşen et al., 2015; Avkiran, 2001). Therefore, the focus for HEIs is more on technical efficiency where it correlates how much total output can be achieved from a set of total input.

Technical efficiency is commonly used as reported in the literature by Färe and Lovell (1978) and other HEIs DEA literatures particularly the output-increasing approach (Johnes, 1993; Abbott & Doucouliagos, 2003; Flegg et al., 2004; Kutlar & Babacan 2008). If a university is obtaining maximum output from a set of inputs, it is said to be efficient (Rogers, 1998) because output is a measure of the joint power of inputs to achieve results (Zhu, 2003).

DEA approach establishes a good relationship between productivity and efficiency of a university, hence make it easier to put DMUs in its suitable ranks (Coelli et al., 1998). In measuring research productivity, total academic staff research is the commonly applied input variables (Beasley, 1995) while research results produced ratings of DMUs (Beasley, 1990; Johnes, 1993) and impact of research are the commonly employed output variables (Johnes & Yu, 2008). As DMU, universities can identify the areas that require improvement within the university thus further look into the possibilities of developing those areas (Aoki, 2010). This can help to contribute decisions on fund allocation among the organizational units (Leitner et al., 2007; Taylor & Harris, 2004; McMillan & Datta, 1998; Bradley et al., 2006; Nazarko et al., 2008)

1.6 Problem Statement

The performance of public universities has been in the interest of many people locally and abroad. Public universities' performance also will reflect the performance of the respective higher education systems. In the latest Universitas 21 Ranking 2020, the Malaysian higher education system is ranked 27th overall (Universitas 21, 2021). Universitas 21 Ranking is the ranking of 50 National Higher Education Systems from all continents, are evaluated and basically ranked overall and on each of four modules: Resources, Environment, Connectivity and Output. The performance of Malaysian higher education in Universitas 21 Ranking, nevertheless, is not that impressive after it remains exactly at the same rank (27th) after more than 5 years (Chang Da Wan, 2018). On the major input and output measures, Malaysia secured ranks of 15th for Resources, 9th for

Environment, 31st for Connectivity and 45th for Output. While Malaysia Higher Education did well on its share in the National R&D expenditure as its rank improves from 8th (2015) to 6th (2020), in terms output measures, the latest ranking is only 45 out of 50 scoring only at 27.6%. This brought to the same observation made in 2015 ranking where although many resources were invested into the higher education, this had not yielded the anticipated output level. Therefore, it is timely to measure the performance of PRUMs considering huge amount of money has been allocated to these universities and at the same time more areas of improvement can be identified based on the output of these universities.

Data Envelopment Analysis (DEA) is a popular method applied by many researchers to measure the universities' performances in terms of its efficiency and productivity. DEA implementation, however, has its own limitations. DEA benchmarking technique adopts a frontier approach and therefore it is sensitive to outliers (Guo et al., 2000). In real life situation, however, the observed values of the input and output data are sometimes imprecise or vague (Hatami-Marbini et al., 2010) and this situation can lead to measurement inaccuracies, unquantifiable and incomplete (Cheng et al., 2017) whereas for a meaningful and reliable DEA results, the input and output data must be accurately measured (Wen & Li, 2009).

Definition of terms used as input and output variables are sometimes not the same. For example, the number of students could be the number of registered students which are not the same as the number of student enrollment. Even there are slight differences in these two terms as explained by different universities. Some variables are just ambiguous by the very nature of the matter, and some are only available in the form of linguistic data or qualitative data (Mahmudah & Lola, 2016). As another example to measure university efficiency where an output variable that has been chosen is "Number of Graduate Students". The question likely to arise is that "is there any difference between graduate students from university A and university B entering the labor market? Here it is related to the quality level of students graduated from different universities. This data is not available or cannot be determined and hardly put in numerical form or cannot be estimated in regular method. It could only be measured in terms of qualitative or ordinal form. These are termed as crisp input and output data where in most cases are indispensable in the conventional DEA applications.

To overcome information uncertainty and complex decision-making problems where DMUs find it difficult to express their preferences by using exact numbers (Zhang et al., 2014) many researchers have proposed fuzzy DEA approach (Hatami-Marbini *et al.*, 2010). In a fuzzy DEA approach, fuzzy set theory has been proposed to extend the traditional DEA models into a fuzzy framework known as Fuzzy Data Envelopment Analysis (FDEA) (Wen & Li, 2008). The technique of Triangular Fuzzy Number (TFN) expresses the vagueness and the uncertainty of information in fuzzy terms for information processing in performance evaluation (Zhang et al., 2014).

Therefore, Data Envelopment Analysis (DEA) method is used to measure the technical efficiency scores of the PRUMs while Fuzzy Data Envelopment Analysis (FDEA) can deal with the undetermined data. With this technique, the inherently imprecise problems definition can be solved with a construction of mathematical solutions or algorithm (Wen & Li, 2008) to gauge the PRUMs' efficiency levels of the current year and the expected efficiency scores for next specific years. Based on the expected efficiency scores changes can be made in the capacity of certain input and output variables to increase the efficiency scores for the institution of the coming year, and hence further improve the HEIs in terms of ranking.

Because among the main concern put forth by the MOHE on PRUMs is to improve their international ranking, this study also attempts to benchmark the PRUMs against a selected of Asian public research universities. The research universities from abroad are assumed as homogeneity among the PRUMs in terms of the nature of the operations and the conditions under which they operate as research universities.

In measuring of DMU efficiency, the CCR (1978) DEA as explained earlier has enabled each DMU to determine the most favorable weights for its own efficiency by accepting the resulting efficiency assessment. The resulting model also effectively facilitates the use of available resources and input variables of the HEIs. This could increase the efficiency for each university (as individual DMU) in terms of its total average efficiency score (Cooper et al., 2014). More importantly, the topic on research university efficiency measurement, in particular, dealing with imprecision and vagueness of input/output data of research university as DMUs is obviously under researched.

1.7 Research Questions

At the end of this research, the answers to the following research questions on university efficiencies can be attained.

1. What are the technical efficiency scores of PRUMs for the latest academic years (2017/2018, 2018/2019, 2019/2020, and 2020/2021)?
2. What is the algorithm to convert crisp data of input and output variables to fuzzy data in the form of Triangular Fuzzy Numbers (TFNs)?
3. What is the most suitable Fuzzy Data Envelopment Analysis (FDEA) model to measure the technical efficiency under uncertainty in input and output variables?
4. What are the expected efficiency scores of PRUMs for the next academic year 2021/2022?

5. What are the technical efficiency scores of PRUMs when benchmarked against a selected group of research universities in Asia?

1.8 Research Objectives

The main aim of this research is to propose a fuzzy DEA model with algorithm to measure the technical efficiency scores and to observe the status (Efficient/Inefficient) of the Public Research University in Malaysia (PRUMs), based on a specific set of variables supported by experts. This followed by the aim to estimate the technical efficiency scores and the status (Efficient/Inefficient) for the PRUMs when taking into consideration a selected public research universities in Asia (sAPRU), and also their respective efficiency scores by using the Fuzzy DEA model (FDEA).

Therefore, the research objectives of this study are as follows:

1. To measure the technical efficiency scores of the PRUMs for the latest academic years (2017/2018, 2018/2019, 2019/2020, and 2020/2021).

2. To identify the algorithm which can be used to convert crisp data of input and output variables to fuzzy data in the form of TFNs for an FDEA model of this study.

3. To propose a suitable FDEA model to measure the technical efficiency under uncertainty in input and output variables.

4. To estimate the expected efficiency score of PRUMs for the next academic year 2021/2022.

5. To estimate the technical efficiency scores of PRUMs when benchmarked against a selected group of research universities in Asia.

1.9 Significance and Contribution of the Research

The methodological significance of this study divides into three parts. First, many studies concerned the performance of the high education institutions (HEI), by using DEA methods, but very few numbers of research employed Fuzzy DEA approaches in efficiency estimation of the HEIs. This study integrates the concept of fuzzy set theory with the traditional DEA by introducing an algorithm to measure the efficiency of public research universities in Malaysia in the form of fuzzy linear programming (FLP) models.

Secondly, while DEA applications on HEIs are generally on available fixed data, for example, the number of enrolled students where DEA model requires precise inputs and output. This is unlikely for real-life applications where one or more variables may be qualitative, linguistic data or crisp. So, this study deals with the usually imprecise, unclear, or uncontrollable input or output data.

Thirdly, this study estimates the efficiency of the research universities based on vague, imprecise, or uncontrollable variables. By proposing a fuzzy DEA model, the input variables represent the university system, and the output indicators are consistent with the World University Ranking section's criteria. The world ranking data are the uncontrollable variables which also justify the application of FDEA in this present study. This is the strong point of the research that not only the study establishes a model to estimate the efficiency of PRUMs based on World University ranking indicators, but an attempt has been made to benchmark the PRUMs against a selected of Asian Public Research universities. Estimating the efficiency of a pool of DMUs especially HEIs from different countries was conducted by Solmaz et al., (2020).

1.10 Definition of Terms

The following terms are used in this study:

Data envelopment analysis (DEA) is a nonparametric method in operations research and economics that is used to empirically measure the productive efficiency of decision-making units (DMUs).

Linguistic data is the data that has the methodological function for being considered in research, for example, a graduate student from the university can be described based on level of quality or number of years employed after graduation.

Crisp data is the undetermined data (characteristics are blurred) also called vague data, for example, the number graduated students in the next academic year.

Fuzzy data is a set of Linguistic or Crisp data that is within the number of Fuzzy theories set.

FDEA is the Fuzzy DEA where some input or output data are Fuzzy data.

Dis-fuzzifying is converting fuzzy findings into real *n*.

PRUMs is the public research universities in Malaysia: Universiti Malaya, Universiti Sains Malaysia, Universiti Kebangasaan Malaysia, Universiti Putra Malaysia, Universiti Teknologi Malaysia

Selected APRUs is the selected public research universities in Asia which include PRUMs, University of Hong Kong, Hong Kong University of Science & Technology, Kyoto University, Seoul National University and Fudan University China.

1.11 Organization of chapters

The study is organized in the following structure. The following chapter 2 contains the conceptual review of DEA, studies and literature review related to DEA with all concepts and definitions in DEA. Chapter 2 also presents the basic DEA models and some empirical studies by applying DEA to the higher education sector.

Chapter 3 presents the background to a popular technique for efficiency measure analysis of organizations under uncertainty circumstances with crisp variables called fuzzy variables and using DEA models, and then converted to be Fuzzy DEA models by researchers. A specific model was chosen to be applied in this study and a brief discussion on fuzzy theorems justifies the fuzzy theorem will be utilized in this study.

Chapter 4, this chapter delineates all the variables (3 inputs and 3 outputs) that are employed for the PRUMs and sAPRU case. All empirical results are tabulated for every academic year under the study followed by the respective explanation based on each table.

The succeeding chapter 5 introduces an algorithm to be applied in converting crisp data to fuzzy data by utilizing the Triangular Fuzzy Numbers Theorem (TFNs). R-soft coding is applied if the number of variables and DMUs increased to be big number, as the expanded fuzzy DEA Model (CCR-FDEA model), launched to measure the fuzzy efficiency score for each DMU under the study (PRUM and sAPRU case). The CCR-FDEA model will estimate the expected efficiency scores for the next academic year 2021/2022. Also, this chapter compares the results of the study and the respective QS-World Ranking of each DMU under the study.

The last chapter, chapter 6, concludes the empirical results presented in chapters 4 and 5. It also introduces several recommendations and suggests the policy implications together with future research directions and some potential ideas for further study in the same area.

CHAPTER 2: THEORETICAL REVIEW OF DEA AND FUZZY DEA METHODS

2.1 Introduction

This chapter presents the theoretical empirical review on DEA and Fuzzy DEA methods. This begins with the conceptual review of DEA approach where production technology and changes in output and input levels bring about returns to scale conception. This chapter also explain the foundation of technical efficiency measurement and discuss various types of DEA models and different efficiency-based ranking methods before a comprehensive discussion on fuzzy DEA methods including the theory of fuzzy numbers.

2.2 Conceptual Review of DEA

The original idea of Data Envelopment Analysis (DEA) was coined by Farrell in 1957 when he introduced DEA to study the labor productivity. Later in 1978, Charnes, Cooper and Rhodes further extended Farrell's (1957) work by transforming the fractional linear measure of efficiency into a mathematical programming approach to measure relative efficiency from a single-input ratio to a multiple-input multiple-output ratio. Later the basic model further developed to estimate inefficiencies by Banker et al., (1984). Their published research work has been considered as the major references and resources for the researchers that came after that in both theoretical and practical aspect of DEA. They became the pioneers in measuring the efficiency and productivity of Decision-Making Units (DMU), in education (Bessent, 1982), banking (Thanassoulis, 1999; Ebrahimnejad et al., 2014), manufacturing (Wahab et al., 2008), logistics (Xu et al., 2009), telecommunication (Cooper et al., 2001) and many others.

In DEA, the organizational unit under study is called a DMU (Decision Making Unit). The definition of DMU allows flexibility in its use over a wide range of possible applications. Generically a DMU is regarded as the entity responsible for converting inputs into outputs and whose productivity performances are to be evaluated. As such, in management science literature, productivity and performance measurement have traditionally been concerned with some factors (inputs and outputs), processes, or machines rather than the organizational whole. Measuring the ratio of total output to a particular input, for example, indicates the partial factor productivity. The research of productivity drove the development of other measures that incorporates and in complement with other important factors in aggregated form. These measures offer insight into the technical and financial performance of an organization.

2.2.1 Production Technology and Function

A production technology is defined as the set (X, Y) such that inputs. $X = (x_1, x_2, \ldots x_i) \in R_+^i$ are transformed into outputs $Y = (y_1, y_2, \ldots y_j) \in R_+^j$.

In production theory, the change in output levels due to changes in input levels is termed as returns to scale. Returns to scale can be constant or variable. Constant Returns to Scale (CRS) implies a certain proportional increase in input levels results in an increase in output levels by the same proportion. Figure 2.2 shows the linear relationship between the inputs and outputs. On the other hand, Variable Returns to Scale (VRS) implies that an increase in the input levels need not necessarily result in a proportional increase in output levels. This means, the output levels can increase (increasing returns to scale) or the output levels can decrease (decreasing returns to scale) by a different proportion than the input increment.

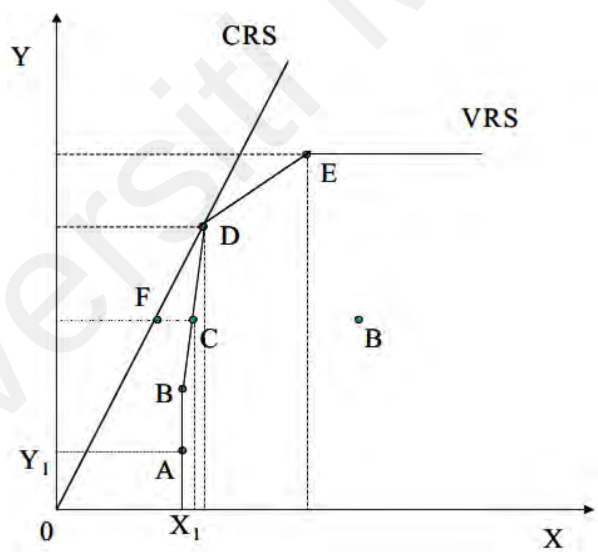


Figure 2.1: Constant and Variable Returns to Scale.

The linear relationship between inputs and outputs for CRS is replaced by a curve with a changing slope. Figure 2.1 also shows the piecewise linear curve with varying slopes where, as the VRS curve increases the production technology displays increasing returns to scale (from B to D), but the slope decreases as the production technology displays decreasing returns to scale (From D to E). Where the curve has a zero slope (from point E to ∞) the production technology experiences no increase in output for any further increase in input. On the curve from X_1 to A, the output jumps from 0 to Y_1 for an input usage of X_1.

The source of the "Constant and Variable Returns to Scale" Figure 2.1, theory is the field of microeconomics, specifically in the study of production functions and their relationship with inputs and outputs. The concept of returns to scale refers to the relationship between the increase in the scale of production and the resulting change in the level of output. The theory was first developed by the economist Alfred Marshall in his book "Principles of Economics" in 1890 and has since been expanded upon and refined by other economists, including Paul Samuelson and Kenneth Arrow. The concept is widely used in modern economic analysis, particularly in the fields of industrial organization, international trade, and development economics.

2.2.2 Measures of Efficiency

The concept of an *isoquant* can be utilized to explain the foundation of efficiency measurement. Isoquant serves as the standard of comparison for the firms, and this is the essence of the relative efficiency concept.

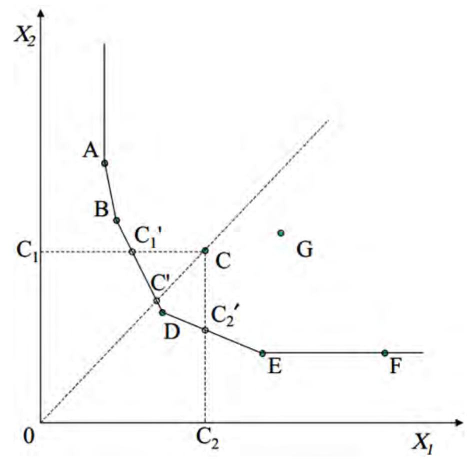


Figure 2.2: Isoquant for Output Level Y^0

As in Figure 2.2, each point on the isoquant represents a unique production technology where the farther the isoquant is from the origin in the positive quadrant, the greater is the output level. With several firms, lying on the isoquant and each producing the same output Y^0 would consume the least different combinations of inputs X_1 and X_2. Any firms consume more inputs to generate the same output enveloped by the isoquant, these firms are inefficient.

The concept of isoquant illustrated in Figure 2.2 bring about two ways to measure the efficiency for a given firm, (i) radially and (ii) non-radially. From Figure 2.2, firm C which is considered inefficient, and that C` be a virtual firm in the convex combination to firms B and D. As both C and C` lie on the same ray through the origin, the radial measure of technical efficiency for C is:

$$TE_{\text{Radial}}(C) = \frac{OC`}{OC} \tag{2.1}$$

For C to become efficient it must operate at C` 's input levels, that C must radially or equi-proportionately reduce its inputs to C` 's levels. But because an equi-proportional reduction in inputs may not always be feasible, the non-radial measure of efficiency is more appropriate. The non-radial measure of technical efficiency for C is:

$$TE_{Non\text{ - }Radial}\,(C) = \frac{C_1 C_1^{`}}{C_1 C} \qquad (2.2)$$

$$TE_{Non\text{ - }Radial}\,(C) = \frac{C_2 C_2^{`}}{C_2 C} \qquad (2.3)$$

While maintaining the same level of output and not altering the input levels, the inputs are reduced individually and non-radially by different proportions to reach the efficient subset ABDE as shown in Figure 2.2 which will produce different efficiency scores for each input.

2.3 Technical Efficiency

The concepts of radial and non-radial presented above are associated with two definitions of technical efficiency (Färe & Lovell, 1978). The radial measure of technical efficiency (Farrell, 1957) defines by reducing a unit, the efficiency is the difference between unity (100% efficiency) and the maximum equi-proportional reduction in inputs, while maintaining the production of originally specified output levels. If this difference is zero, then the unit is efficient, else it is inefficient. The second definition based on non-radial is Koopmans (1951) definition of technical efficiency.

Here, the firm is technically efficient if and only if an increase in one output results in a decrease in another output so as not to compromise the input level or else results in the increase of at least one input. Stated otherwise, the definition implies that a decrease in one input must result in an increase in another input so as not to compromise the output, or else, it must result in the decrease of at least one output. The difference between the two definitions is explained through Figure 2.2. The radial definition states that all firms on the isoquant with the same output level as efficient. However, the non-radial definition deems firm F as inefficient.

Technical Efficiency is the most important term in DEA as such by merging all inputs in a scalar form by converting the multi-elements vector to digital scalar form (equation 2.8) as aggregated scalars. This the first mathematical form of the Technical Efficiency T.E. and the formulation for multiple input-output as follow:

$$T.E. = \frac{\text{Aggregate Output Measure}}{\text{Aggregate Input Measure}} \qquad (2.4)$$

Here, all available resources are consumed to generate the aggregate outputs (Farrell, 1957) as seems in equation (2.1). Also, technical efficiency for any firm is related to the ability of that firm in:

(i) producing the maximum outputs for a constant input usage known as output-increasing efficiency; or

(ii) using the minimum inputs to generate a constant output production known as input-reducing efficiency.

Technical efficiency measurement generally involves comparing a decision-making unit's (DMU's) production plan to a production plan that lies on the efficient production frontier or isoquant (Fried et al., 1993; Färe et al., 1994; Charnes et al., 1994). Comparing the production plans leads to the need for deriving a "standard of excellence" which is to serve as a benchmark. This standard must represent the level of technical efficiency that is achieved through: (1) the least number of inputs and constant outputs (for input-reducing efficiency) and (2) The maximum production of outputs with constant inputs (for output-increasing efficiency).

2.3.1 Input-Reducing Orientation of Technical Efficiency

Consider a decision-making unit (DMU) uses i = 1,2, …, I number of inputs to produce j =1, 2…j, number of outputs. Let the problem of interest be DMU productivity over a period of time (say, one year or twelve months) that means, each month should be represented as one DMU. Denote the input vector for the nth month as $X_n = [x_{in}]$ and the output vector for the nth month as $Y_n = [y_{jn}]$. See Figure 2.3 below.

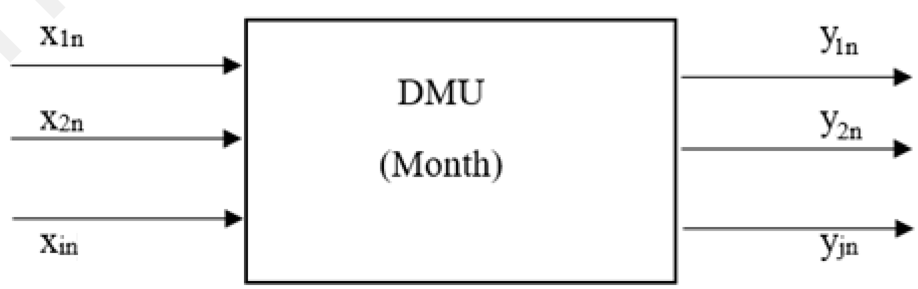


Figure 2.3: Input and Output Vectors for the nth Month

The objective of the DMU per month would be to minimize usage of each input resource and maximize the production of each output i.e., the efficient transformation of inputs into outputs is an efficient production frontier or similar to the isoquant concept (Farrell, 1957). An input-reducing isoquant is defined by “the observations that are efficient relative to the other observations in the data set”. Therefore, the isoquant represents the minimum input usage that is required to produce a constant set of outputs. Such an isoquant is assumed to be convex to the origin and to have a negative slope which assumes virtual production plans that are obtained as a weighted combination of actual production plans.

In other words, given the negative slope assumption, different input mixes can be obtained without compromising the output level. While an increase in any input without change in other inputs must result in the observation moving to a higher isoquant. If the output level remains constant, then it would imply weak disposability of inputs.

A production technology which exhibits constant returns to scale will demonstrate an increase in inputs results in an equi-proportional increase in the output. The resultant input-reducing isoquant then represents the output level where for every output level there exists an input-reducing isoquant. Therefore, the input-reducing isoquant is a function of the output level.

2.3.2 Output-Increasing Orientations of Technical Efficiency

By considering observations in the data set are relatively efficient, an output-reducing isoquant where the production possibility frontier represents the maximum output production possible with consumption of constant inputs. In the output-increasing orientations, the production technology shows negative slope and constant returns to scale as it is concave with respect to the origin.

The concavity property permits the weighted combinations of actual observations while the negative slope permits strong disposability of outputs which means a decrease in any one output keeping all other outputs constant must result in the decrease in the level of inputs. The constant return to scale assumption implies that a decrease in outputs results in an equi-proportional decrease in inputs. The strong disposability of outputs ensures that a decrease in any output without change in other outputs must result in the observation moving to a lower frontier.

In Figure 2.4, all DMUs use the two inputs X_1 and X_2 to produce a constant output set as Y^{θ} to achieve the input-reducing technical efficiency where the output vector is Y^{θ}.

The input-(in) efficiency score of DMU B is given as:

$$\text{TE}_{\text{Input}}(\text{B}) = \frac{\text{OB`}}{\text{OB}} \tag{2.5}$$

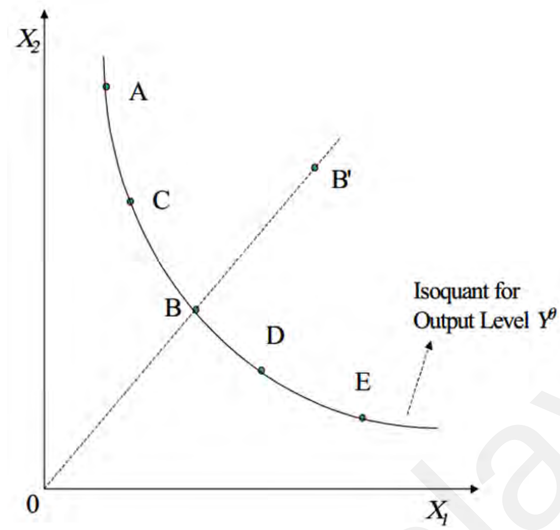


Figure 2.4: Input-Reducing Technical Efficiency

In the same way, the output-increasing case can be illustrated by the figure 2.5 as following:

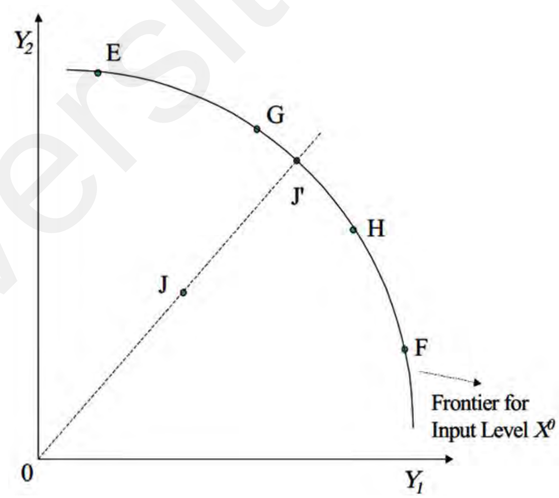


Figure 2.5: Output-Increasing Technical Efficiency

Then, from Figure 2.5 the output-(in) efficiency score of DMU J is given as:

$$TE_{\text{Output}}(B) = \frac{OJ`}{OJ} \tag{2.6}$$

So, DEA approach applies linear programming techniques to observe inputs consumed and outputs produced by Decision-Making Units (DMU) and constructs the "Efficient Production Frontier" as a conclusion based on best practices and empirical techniques. Each DMU's efficiency (in the system) is measured relative to this frontier. In other words, DEA clearly assesses the efficiency of each DMU relative to all the DMUs in the sample (organization, institution, or any) including itself. This relative efficiency is calculated by obtaining the ratio of the weighted sum of all outputs and the weighted sum of all inputs. The weights are selected to achieve maximum optimality for each DMU or known as its technical efficiency.

The other concept of efficiency is allocative efficiency which is also known as price efficiency (Farrell, 1957) measure the ability of a technically efficient DMU to use inputs in proportion that minimize production costs given input prices. Therefore, it is measured as the ratio of the minimum costs required by DMU to produce a given level of outputs and the actual costs of the DMU adjusted for technical efficiency. As such it is defined as "a state in which every good or service is produced up to the point where the last unit provides a marginal benefit to consumers equal to the marginal cost of producing. In the single-price model, at the point of allocative efficiency where price is equal to marginal cost." Due to the mixture of optimal input which gives to a definition of input price thus allocative efficiency is known as price efficiency (Lewin & Morey, 1981) as stated earlier.

2.4 DEA Models

DEA is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogeneous DMUs by evaluating their relative efficiency (Hatami-Marbini et al., 2011). In DEA approach, the input and output variables need not to be in the same units of measurement indicating that DEA is invariant to scaling of the variables (converting, inputs and outputs vectors to scales). The DEA methodology not only enables inefficient DMUs to be identified, but also it can determine the sources and amounts of inefficiency of the respective inputs and/or outputs.

DEA model development has gone through a lot of enhancement for the last six decades where many models have been introduced in various fields including education and many other industries. These models were introduced to help DMUs to work on the optimal efficiency which will support the maximum output or profit of the system in which the DMU is operating. one is the CCR Model.

2.4.1 The CCR Model

Charnes, Cooper, and Rhodes (1978) introduced the original and among the earliest and most famous DEA model with the linear programming algorithm. The CCR DEA model was named as the initial letters of their names. The CCR model requirements are the full information on inputs and outputs for a set of homogenous DMUs. The model is a fractional linear program that compares the efficiency of each DMU with all possible linear combinations of the other DMUs (including the one under consideration). The CCR model focuses on reducing the inputs to reduce the cost and increasing the output as whole to get the optimal scale assuming this scale is constant. This is the best solutions possible (input-reducing and output-increasing orientations). In mathematical terms, consider a set of *n* DMUs, where DMU *j* has a production plan (X_j, Y_j) *with* $X_j = (x_1, x_2, \ldots, x_m)$ inputs and $Y_j = (y_1, y_2, \ldots, y_s)$ outputs.

Let $U = (u_1, u_2, \ldots, u_m)$ and $V = (v_1, v_2, \ldots, v_s)$ be weight vectors. Let the variables be defined as:

c = DMU whose technical efficiency is being measured

x_{ik} = quantity of input *i* consumed by DMU k

y_{jk} = quantity of output *j* produced by DMU k

ui = weight assigned to input i

v_j = weight assigned to output *j*

ε = very small positive number

Then, the original CCR model is then written as:

$$Max \quad \frac{\sum_{j=1}^{s} v_j y_{jc}}{\sum_{i=1}^{m} u_i x_{ic}} \tag{2.7}$$

$$subject\ to \quad \frac{\sum_{j=1}^{s} v_j y_{jk}}{\sum_{i=1}^{m} u_i x_{ik}} \leq 1, \qquad k = \{1,2,\ldots,n\} \tag{2.8}$$

$$u_i \geq \varepsilon\ , \qquad i = \{1,2,\ldots,m\} \tag{2.9}$$

$$v_j \geq \varepsilon \,, \qquad\qquad j = \{1,2,\ldots,s\} \qquad (2.10)$$

Ratio Form of the CCR Model (M1)

For this model, the efficiency of DMU c is measured as an aggregate weighted sum of outputs, then divided by an entirety weighted sum of inputs. This efficiency should maximize the objective function subject to the efficiencies of all units and bounded above by 1 (less than 1). However, the main property of the DEA model is that the values of u and v are not fixed exogenously but are chosen (by the determined model). This is to maximize the efficiency of the DMU under consideration in comparison to the other DMUs which must also carry the same weights. In other words, the weights (values) are chosen so that each DMU is shown in the best possible light. Noteworthy, the weights will not necessarily be the same for each DMU.

2.4.2 The CCR Linear (Primal) Model

The CCR model is later enhanced by Boussofiane, Dyson, and Thannasoulis (1991) from a fractional linear program and can be re-written as a linear program with (s + m) variables and (n+ s + m+1) as several constraints. This model is known as the CCR Linear (Primal) model and written as:

$$Max \quad \sum_{j=1}^{s} v_j y_{jc} \tag{2.11}$$

$$subject\ to \quad \sum_{i=1}^{m} u_i x_{ic} = 1 \tag{2.12}$$

$$\sum_{j=1}^{s} v_j y_{jk} - \sum_{i=1}^{m} u_i x_{ik} \leq 0 \qquad k = \{1,2,\ldots,n\} \tag{2.13}$$

$$- u_i \leq - \varepsilon\ , \qquad i = \{1,2,\ldots,m\} \tag{2.14}$$

$$- v_j \leq - \varepsilon\ , \qquad j = \{1,2,\ldots,s\} \tag{2.15}$$

CCR Linear (Primal) Model

The CCR Primal Model has many more constraints than the other model that the linear programs is able to solve more difficult problems.

2.4.3 The CCR Linear Dual Model

In the interest of problem related to shadow price in the linear model, Boussofiane, Dyson, and Thannasoulis (1991) further re-write the CCR Linear Primal Model into the CCR Dual Model. In linear programming, the shadow price of a constraint is the difference between optimized value of objective function and the value of the objective function when the right-hand side (RHS) of the constraint is increased by one unit. The dual model has as many constraints as there are inputs and outputs similar like the primal model but in most of cases the number of DMUs is much greater than the number of inputs and outputs.

The dual of the CCR model is written as:

$$Min\ \theta_c \tag{2.16}$$

$$subject\ to: \quad \theta_c\, x_{ic} - \sum_{k=1}^{n} z_k x_{ik} \geq 0 \quad i=\{1,2,\ldots,m\} \tag{2.17}$$

$$\sum_{k=1}^{n} z_k y_{jk} \geq y_{jk}, \qquad j = \{1,2,\ldots,s\} \tag{2.18}$$

$$\theta_c,\ z_c \geq 0 \qquad k = \{1,2,\ldots,n\} \tag{2.19}$$

CCR Dual Model

where, θ_c = radial measure of technical efficiency, z_c = activity levels associated with inputs and outputs of DMU k. The optimal solution denoted as θ_c^*, is the degree of input efficiency of DMU_c. A new weight vector z = (z_1, z_2, … z_k) is unique for each DMU. The z_k's are the activity levels and characterize the level of performance of an efficient virtual DMU_c` against which the performance of $DMU_c$ is compared. The dual seeks to find values of $z_k$ so as to construct a composite (virtual) unit $DMU_c$` with outputs Σ z_k y_k , and inputs Σ z_k x_k that outperforms DMU_c. If both DMU_c and DMU_c` are found to perform at the same level, then DMU_c is considered to be efficient and designated an input-efficiency score of one.

If DMU_c utilizes more inputs than $DMU_{c`}$, then $DMU_c$ is considered inefficient and given an input- efficiency score less than one. This is so because it is possible for $DMU_{c`}$ to produce the same output using lesser input than DMU_c. In this case, the optimal values of z_k will construct a virtual unit that outperforms DMU_c.

In this model where a constraint is an obligation or binding, a shadow price will be positive and where the constraint is non-binding the shadow price will be zero. In the solution to the primal model therefore a binding constraint implies that the corresponding unit has an efficiency of

1 and there will be a positive shadow price or dual variable. Then positive shadow prices in the primal, or positive values for its variables in the dual model, correspond to and identify the peer group for any inefficient unit.

Figure 2.6 shows the CCR production function for the simple case, one-input (X) and one output (Y). This is because that the constant returns to scale assumption this model gives identical results for both input-reducing and output-increasing orientations.

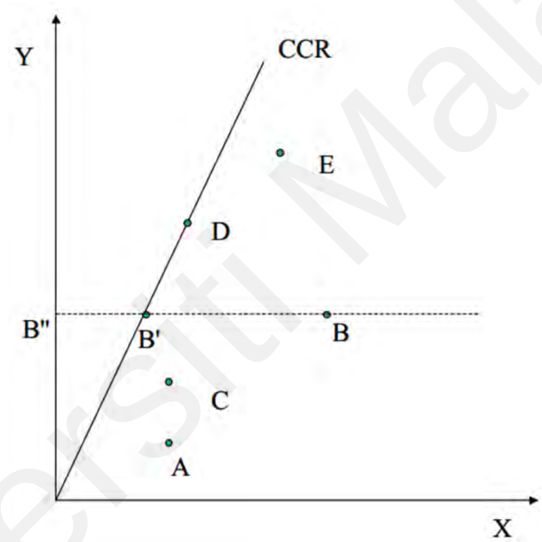


Figure 2.6: CCR Production Function

From the graph, the scores are a distance measure between each DMU and its horizontal projection (input orientation) or vertical projection (output orientation) onto the CCR production function. For example, the input-reducing efficiency score for DMU B is:

$$TE_{Iutput}(B) = \frac{B''B`}{B''B} \qquad (2.20)$$

2.4.4 The CCR Model with Slacks (M4)

With more proper modifications to the CCR model, Boussofiane, Dyson, and Thannasoulis (1991) provide the decision-maker with input and output target (scales) values that would transform inefficient units as efficient. Here, a new virtual unit, which represent targets for *DMU*c, which will satisfy which part will make the unit efficient, the target values the CCR dual model should be rewritten as the following to be a CCR model with slacks:

$$Min\ \theta_c - \varepsilon \left(\sum_{i=1}^{m} e_i + \sum_{j=1}^{s} r_j\right) \tag{2.21}$$

$$Subject\ to{:}\ \sum_{k=1}^{n} z_k x_{ik} + e_i = \theta_c\, x_{ic}, \quad i = \{1,2,\ldots,m\} \tag{2.22}$$

$$\sum_{k=1}^{n} z_k y_{jk} - r_j = y_{jc}, \qquad j=\{1,2,\ldots s\} \tag{2.23}$$

$$\theta_c,\ z_k,\ e_i,\ r_j \geq 0 \qquad \forall\ i, j, k \tag{2.24}$$

The CCR Model with Slacks

In the CCR Model with slacks, e_i and r_j are the slack variables introduced to convert the constraints from inequalities to equalities. So DMU_c is efficient when the slacks are equal to zero. When DMU_c is inefficient, then the input-efficiency score $\theta_c^* \leq 1$ and/or $(e_i, r_j) > 0$.

2.4.5 The BCC Model

In the DEA model developed by Banker, Charnes, and Cooper (1984), they used a new experimental or empirical production function to compute efficiency under the assumption of variable returns. Here, the notion is a proportional increase in inputs need not necessarily produce a proportional increase in outputs. This model can generally show how the scale of operation of a DMU impacts its efficiency or inefficiency. When DMUs fail to achieve the best possible output levels and/or usage of excessive amounts of inputs, they are deemed technically inefficient. This is commonly known as the BCC model that addresses efficiency as made up of technical physical efficiency and scale efficiency whilst the CCR model addresses aggregate (technical and scale) efficiency. Technical and scale efficiency can be explained from Figure 2.7.

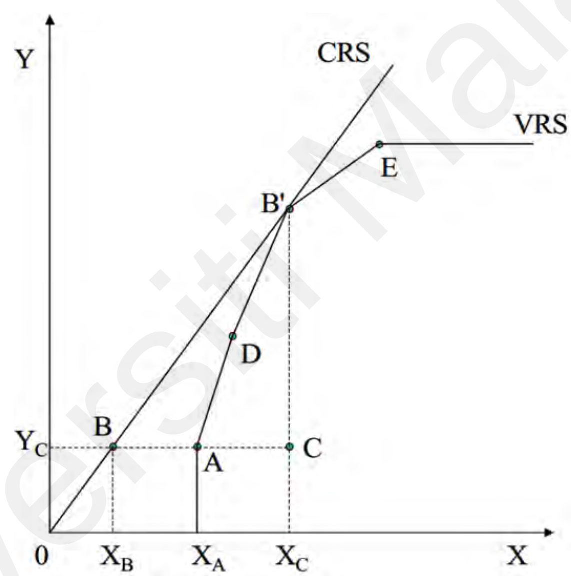


Figure 2.7: Technical Efficiency and Scale Efficiency

In Figure 2.7, DMU C is considered inefficient as it is enveloped by the efficient frontiers. The input-reducing technical efficiency for C at its scale of operation is given as X_A/X_C where C must reduce its inputs to A's level to become efficient as both produce the same output. B is the most aggregate efficient unit in the production possibility set because it achieves both technical and scale efficient or aggregate efficient. B and B` lie on the same line, hence, same slope and therefore the same numerical productivity. C’s aggregate efficiency can be calculated by comparing it with B` or B, and therefore the aggregate efficiency of C is given as:

$$\frac{OX_B}{OX_c} = \frac{OX_B}{OX_A} * \frac{OX_A}{OX_c} \qquad (2.25)$$

(Read as: Aggregate Efficiency = Scale Efficiency * Technical Efficiency)

The BCC model introduces an additional convexity constraint where the constraint restricts the sum of the activity levels of the input and output factors to one and restricts the virtual DMU to be of the same scale size as the DMU under consideration. This model concentrates on maximal movement toward the frontier line by a proportional reduction of inputs (input-reducing) or by proportional increment of outputs (output increasing) (Charnes et. al., 1994). The mathematical form of the two-stage BCC model can be represented as the following:

$$Min\ z_0 = \theta - \varepsilon \textstyle\sum_{j=1}^{s} s_j^{+} - \varepsilon \sum_{i=1}^{m} s_i^{-} \qquad (2.26)$$

$$Subject\ to:\ \theta_c\, x_{ic}, - s_i^{-} = \textstyle\sum_{k=1}^{n} z_k x_{ik}\,, \qquad i = \{1,2,\dots,m\} \qquad (2.27)$$

$$\textstyle\sum_{k=1}^{n} z_k y_{jk} - s_j^{+} = y_{jc}\,, \qquad j=\{1,2,\dots s\} \qquad (2.28)$$

$$\textstyle\sum_{k=1}^{n} z_k = 1 \qquad (2.29)$$

$$z_k,\ s_j^{+},\ s_i^{-} \geq 0 \qquad \forall\ i, j, k \qquad (2.30)$$

The BCC Model: Input-Orientation

In this formula, the objective function (*Min* z_0) contains both the variable θ and the non-Archimedean (infinitesimally small) constant ε. Equation (2.29) represents the additional convexity constraint.

The dual of this formulation would show that ε acts as a lower bound for the dual multipliers. The scalar variable θ is the proportional reduction of all inputs for the DMU under consideration which would then improve its efficiency. The simultaneous reduction of all inputs should cause a radial movement toward the envelopment surface. Therefore, a DMU is efficient under two conditions, if and only if (i) $\theta^*= 1$, and (ii) all slacks are zero.

The radial efficiency measure (input orientation), thus computed by the BCC model) can be arrived at by means of a two-stage process, that means first, should the maximal reduction in inputs given by θ^* is calculated. This, however, does not guarantee that any of the DMUs will move onto the efficient subset through the fixed, proportional (equi-proportional) reduction in inputs. Therefore, the second stage helps to determine the input surplus e+ and the output slack r-. So easily decision-makers can thus identify causes and quantities of inefficiencies through nonzero-slacks and a θ^* value less than 1.

Figure 2.8 shows the differences of the *Production Functions* between CCR model and BCC. There are several other modifications to CCR DEA model, extensions of Dual Model like the slacks-based models. Some of these can be referred to Cooper, Seinford and Tone (2007).

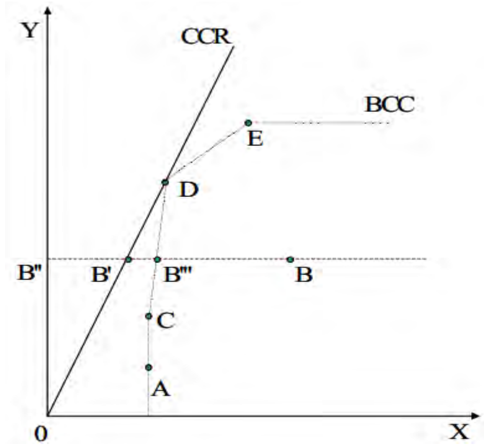


Figure 2.8: CCR and BCC Production Functions

Much later are the super efficiency DEA measurement models and the integration of the two; Slacks-Based Models of efficiency and Super Efficiency, and Super Slacks-Based of Efficiency Model (SupSBM) and linearized integration of the SBM model but are not in the scope of this study and therefore are not included in this review. So that, all the above could be applied in FDEA model that will be developed in this study.

2.5 Fuzzy logic approach

The application of fuzzy logic in ranking the DMUs utilizes the fuzzy information in DEA in order to achieve the full ranking of DMUs (Wen & Li, 2009). The core of this method is the fuzzy DEA model where a hybrid intelligent algorithm and fuzzy simulations are used together with a genetic algorithm. Another way is to combine fuzzy logic and DEA (Karsak, (1998) & Hougaard, (1999)) but expert knowledge is also needed for the evaluation (Adler, 2002).

A more recent study reviews the applications of DEA efficiency ranking methods in various fields of study (Aldamak & Zolfaghari, 2017). The authors argue that most of these approaches can be considered to offer post-analysis to normal DEA models to achieve a satisfactory final ranking. Although the applications of fuzzy approach in generating a ranking is said not been able to provide an accurate assessment, Hatami-Marbini et al., (2011) have provided an excellent fuzzy DEA review by classifying the present methods in the literature and proposed the tolerance approach as they claimed have been widely applied in practice.

2.6 Fuzzy Data Envelopment Analysis (FDEA) Model

While the natural of DEA models require precise input and output data, in the real-world problems data which are available is usually imprecise or unclear and could be in the form of qualitative. For example, in the HEI cases, the changing academic year will have changes in the number of graduated students, unexpected expenses or number of papers published, etc. (Tavana et al., 2021a). Fuzzy DEA (FDEA) has integrated the concept of fuzzy set theory with the traditional DEA by representing imprecise and vague data with fuzzy sets. Similar to other DEA models, FDEA models also take the form of fuzzy linear programming models (Peykani et al., 2018).

Fuzzy set theory, first established by Zadeh (1965), is a well-known tool to represent this type of data. The word “fuzzy” refers to objects which are not clear or are ambiguous. Such activities in any process, function, or event that is changing continuously and cannot always be defined as either true or false, are defined to behave in a fuzzy behavior. Therefore, in a fuzzy set theory, data are based on the concept of ‘‘degree of membership”, that ranges from 0 to 1, compared to the traditional binary sets (‘true” or ‘‘false”, 0 or 1). Sengupta (1992) introduced his first work using fuzzy theory in DEA to measure the relative efficiencies of a set of decision-making units (DMUs) with common crisp inputs and outputs. His study stated these fluctuating data can be represented as linguistic variable characterized by fuzzy numbers.

There are many fuzzy sets-based methods which have been proposed in DEA in the last three decades. Hatami, et al., (2017) reviewed the concept of fuzzy from the previous FDEA studies and introduce further explanations on fuzzy set theory in DEA. From his work, FDEA models are generally represented as FLP models with fuzzy coefficients (i.e., fuzzy input-output data) and crisp decision variables. For FDEA Model, it is so important to know that some of these variables can be in terms of categorical variables where in some studies these variables are termed to be as membership functions. But again, since FDEA models take the form of FLP problems, different FDEA approaches have been developed in different ways corresponding to the FLP models (Hatami, et al., 2017).

In general, the linear programming (LP) DEA models are converted into fuzzy LP (FLP) models when the input and/or output data are characterized by fuzzy numbers. Based on this concept, Hatami, et al.,

(2011) classifies the applications of fuzzy set theory in DEA into four sub-areas groups (1) tolerance approach, (2) α-level based approach, (3) fuzzy ranking approach, (4) Possibility Approach. Later, the researchers added another two methods, fuzzy arithmetic, and the fuzzy random type-2. Refer to Figure 2.9. To our recent best knowledge, this is the latest classification of FDEA approaches, as most recently stated by Peykani, et al., (2019) in their study on Fuzzy Data Envelopment Analysis Approach for Ranking of Stocks in the Tehran Stock Exchange.

Figure. 2.9: The Classification of Fuzzy DEA field of study (Source, Peykani et al., (2019), based on Hatami, et al., (2017))

According to Hatami et al., (2017). Out of the six approaches, the tolerance approach is the most powerful and commonly used method.

In general, the FLP problems can be classified into the following six categories to handle fuzzy data:

(1) Both decision variables and the right-hand-side of the constraints are characterized by fuzzy numbers.

(2) The coefficients of the decision variables in the objective function are characterized by fuzzy numbers.

(3) The coefficients of the decision variables in the constraints and the right-hand-side of the constraints are characterized by fuzzy numbers.

(4) The decision variables, the coefficients of the decision variables in the objective function and the right-hand-side of the constraints are characterized by fuzzy numbers.

(5) The coefficients of the decision variables in the objective function, the coefficients of the decision variables in the constraints and the right-hand-side of the constraints are characterized by fuzzy numbers.

(6) FLP models when all the parameters and variables are characterized by fuzzy numbers.

The following sub-sections briefly explain the six Fuzzy DEA methods reviewed by Hatami et al., (2017).

2.6.1 The tolerance-approach.

Sengupta, (1992) was the first FDEA model that used the concept of fuzziness in DEA modeling by defining tolerance levels on constraint violations. The limitation behind the tolerance approach is related to the design of a DEA model with a fuzzy objective function and fuzzy constraints which may or may not be satisfied (Triantis & Girod, 1998) and further improved on the tolerance model was made by Kahraman and Tolga (1998).

Let us assume that n DMUs consume varying amounts of m different inputs to produce s different outputs. Assume that $\tilde{x}_{ij}$ (i =1, 2, ….m) and $\tilde{y}_{rs}$ (r = 1, 2, ...s) represent, respectively, the fuzzy input and fuzzy output of the jth DMU$_j$ (j = 1,2,…n). The primal and its dual fuzzy CCR models in input-oriented version can be formulated as:

<u>Primal CCR model (input-oriented)</u>

$$
\begin{aligned}
\min \quad & \theta_p \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le \theta_p \tilde{x}_{ip}, \quad \forall i, \\
& \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \tilde{y}_{rp}, \quad \forall r, \\
& \lambda_j \ge 0, \quad \forall j.
\end{aligned}
\qquad (2.31)
$$

<u>Dual CCR model (input-oriented)</u>

$$
\begin{aligned}
\max \quad & \theta_p = \sum_{r=1}^{s} u_r \tilde{y}_{rp} \\
s.t. \quad & \sum_{i=1}^{m} v_i \tilde{x}_{ip} = 1, \\
& \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \le 0, \quad \forall j, \\
& u_r, v_i \ge 0, \quad \forall r, i.
\end{aligned}
\qquad (2.32)
$$

Where v_i and u_r in model (2.31) are the input and output weights assigned to the ith input and rth output. If the constraint $\sum_{j=1}^{n} \lambda_j = 1$, is adjoined to (2.32), a fuzzy BCC model is obtained and this added constraint introduces an additional variable, $\tilde{u}_0$, into the dual model which these models are respectively shown as follows:

$$
\begin{aligned}
\min \quad & \theta_p \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le \theta_p \tilde{x}_{ip}, \quad \forall i, \\
& \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \tilde{y}_{rp}, \quad \forall r, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \ge 0, \quad \forall j.
\end{aligned}
\qquad (2.33)
$$

$$
\begin{aligned}
\max \quad & w_p = \sum_{r=1}^{s} u_r \tilde{y}_{rp} + u_0 \\
s.t. \quad & \sum_{i=1}^{m} v_i \tilde{x}_{ip} = 1, \\
& \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} + u_0 \le 0, \quad \forall j, \\
& u_r, v_i \ge 0, \quad \forall r, i.
\end{aligned}
\qquad (2.34)
$$

Tolerance (1992) FDEA model (1st FDEA Model)

The tolerance approach fuzzifies the inequality or equality signs, but it does not treat fuzzy coefficients directly. Again, this disadvantage of this model is related to the design of a DEA model with a fuzzy objective function and fuzzy constraints where it may or may not be satisfied (Triantis & Girod, 1998). Although in most production processes fuzziness is present both in terms of not meeting specific objectives and in terms of the imprecision of the data, the tolerance approach provides flexibility by relaxing the DEA relationships while the input and output coefficients are treated as crisp (Hatami-Marbini et al., 2011).

2.6.2 The α-level based approach

The α-level approach, in general, transforms the FDEA model into a pair of parametric programs for each α-level (Hatami-Marbini, et al., 2017 & Kao and Liu, (2000). One of the most cited α-level approach's studies is by Chen and Klein (1997) who used method for ranking fuzzy numbers to convert the FDEA model to a pair of parametric mathematical programs for a given level of α. The α-level based approach provides fuzzy efficiency but requires the ranking of the fuzzy efficiency sets as proposed by Meada et al., (1998). Triantis and Girod (1998) followed up by introducing the fuzzy LP approach to measure technical efficiency based on Carlsson and Korhonen (1986) framework. Their approach involved three stages: First, the imprecise inputs and outputs were determined by the decision maker in terms of their risk-free and impossible bounds. Second, three fuzzy CCR, BCC and FDH models were formulated in terms of their risk-free and impossible bounds as well as their membership function for different values of α. Third, they illustrated the implementation of their fuzzy BCC model in the context of a preprint and packaging line which inserts commercial pamphlets in newspapers. Furthermore, their paper was clarified in detail the implementation road map by Girod and Triantis (1999).

Kao and Liu (2000) followed up on the basic idea of transforming a fuzzy DEA model to a family of conventional crisp DEA models and developed a solution procedure to measure the efficiencies of the DMUs with fuzzy observations in the BCC model. Their method found approximately the membership functions of the fuzzy efficiency measures by applying the α-level approach and Zadeh's extension principle (Zadeh 1978, Zimmermann 1996). Saati, Memariani, and Jahanshahloo (2002) developed a fuzzy CCR model as a possibilistic programming problem and changed it into an interval programming problem by means of the α -level based approach. Afterward, some fuzzy DEA-based extension has been done using Saati et al., (2002). Then Triantis (2003) extended his earlier work on fuzzy DEA (Triantis & Girod, 1998) to fuzzy non-radial DEA measures of technical efficiency in support of an integrated performance measurement system. Triantis (2003) also compared his method to the radial technical efficiency of the same manufacturing production line, which was described in detail by Girod, (1996), Girod and Triantis, (1999).

In 2010 Hatami-Marbini et al., developed their method such as a four-phase fuzzy DEA framework based on the theory of displaced ideal (Hatami-Marbini et al., 2010), and the same teamwork later in 2013, develop and use a positive-normative use of fuzzy logic in a NATO enlargement application by using the α-level based approach (Hatami-Marbini et al., 2013). All above researchers tried to transform the fuzzy DEA model to a pair of parametric mathematical programs and used the ranking fuzzy numbers method proposed by Chen and Klein (1997) to obtain the performance measure of the DMU. Solving this model at the given level of α-level produced the interval efficiency for the DMU under consideration.

Several such intervals could be used to construct the corresponding fuzzy efficiency. Assume that there are *n* DMUs under consideration. Each DMU consumes varying amounts of *m* different fuzzy inputs to produce *s* different fuzzy outputs. Specifically, DMU_j consumes amounts $\tilde{x}_{ij}$ of inputs to produce amounts $\tilde{y}_{rj}$ of outputs. In the model formulation, $\tilde{x}_{ip}$ and $\tilde{y}_{rp}$ denote, respectively, the input and output values for the DMUp. To solve the fuzzy BCC model (2.33), Kao and Liu (2000) proposed a pair of two-level mathematical models to calculate the lower bound $(w_p)_\alpha^L$ and upper bound $(w_p)_\alpha^u$ of the fuzzy efficiency score for a specific α-level as follows:

$$
(w_p)_\alpha^L = \min_{\substack{(X_{ij})_\alpha^L \le x_{ij} \le (X_{ij})_\alpha^U \\ (Y_{rj})_\alpha^L \le y_{rj} \le (Y_{rj})_\alpha^U \\ \forall r,i,j}} \left\{ \begin{aligned} \tilde{w}_p = \max\ & \sum_{r=1}^{s} u_r y_{rp} + u_0 \\ s.t.\ & \sum_{i=1}^{m} v_i x_{ip} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0, \quad \forall j, \\ & u_r, v_i \ge 0, \qquad \forall r,i. \end{aligned} \right. \tag{2.35}
$$

$$
(w_p)_\alpha^U = \max_{\substack{(X_{ij})_\alpha^L \le x_{ij} \le (X_{ij})_\alpha^U \\ (Y_{rj})_\alpha^L \le y_{rj} \le (Y_{rj})_\alpha^U \\ \forall r,i,j}} \left\{ \begin{aligned} \tilde{w}_p = \max\ & \sum_{r=1}^{s} u_r y_{rp} + u_0 \\ s.t.\ & \sum_{i=1}^{m} v_i x_{ip} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0, \quad \forall j, \\ & u_r, v_i \ge 0, \qquad \forall r,i. \end{aligned} \right. \tag{2.36}
$$

Where and $[(x_{ij})_\alpha^L, (x_{ij})_\alpha^U]$ and $[(y_{rj})_\alpha^L, (y_{rj})_\alpha^U]$ are α-level form of the fuzzy inputs and the fuzzy outputs, respectively. This two-level mathematical model can be simplified to the conventional one-level model as follows:

$$
\begin{aligned}
(w_p)_\alpha^L = \max\ & \sum_{r=1}^{s} u_r (Y_{rp})_\alpha^L + u_0 \\
s.t.\ & \sum_{r=1}^{s} u_r (Y_{rp})_\alpha^L - \sum_{i=1}^{m} v_i (X_{ip})_\alpha^U + u_0 \le 0, \\
& \sum_{r=1}^{s} u_r (Y_{rj})_\alpha^U - \sum_{i=1}^{m} v_i (X_{ij})_\alpha^L + u_0 \le 0, \ \forall j, j \ne p, \\
& \sum_{i=1}^{m} v_i (X_{ip})_\alpha^U = 1, \quad u_r, v_i \ge 0, \quad \forall r,i.
\end{aligned} \tag{2.37}
$$

$$
\begin{aligned}
(w_p)_\alpha^U = \max\ & \sum_{r=1}^{s} u_r (Y_{rp})_\alpha^U + u_0 \\
s.t.\ & \sum_{r=1}^{s} u_r (Y_{rp})_\alpha^U - \sum_{i=1}^{m} v_i (X_{ip})_\alpha^L + u_0 \le 0, \\
& \sum_{r=1}^{s} u_r (Y_{rj})_\alpha^L - \sum_{i=1}^{m} v_i (X_{ij})_\alpha^U + u_0 \le 0, \ \forall j, j \ne p, \\
& \sum_{i=1}^{m} v_i (X_{ip})_\alpha^L = 1, \quad u_r, v_i \ge 0, \quad \forall r,i.
\end{aligned} \tag{2.38}
$$

Next, a membership function is built by solving the lower and upper bounds $[(w_p)_\alpha^L, (w_p)_\alpha^U]$ of the α-levels for each DMU using models (2.37) and (2.38). Kao and Liu (2000) have used the ranking fuzzy numbers method of Chen and Klein (1997) to rank the obtained fuzzy efficiencies. Also, Kao and Liu (2000) used the same method they proposed to calculate the efficiency scores by considering the missing values in the fuzzy DEA based on the concept of the membership function in the fuzzy set theory.

In their approach, the smallest possible, most possible, and largest possible values of the missing data are derived from the observed data to construct a triangular membership function. They demonstrated the applicability of their approach by considering the efficiency scores of 24 university libraries in Taiwan with 3 missing values out of 144 observations. Kao, et al., (2003) further introduced a method for ranking the fuzzy efficiency scores without knowing the exact form of their membership function. In this method, the efficiency rankings were determined by solving a pair of nonlinear programs for each DMU.

Kao and Liu (2003) used the maximum set–minimum set method of Chen (2000) into the fuzzy DEA model proposed by Kao and Liu (2000) and built pairs of nonlinear programs and ranked the DMUs with fuzzy data. They used the earlier fuzzy DEA solution to transform fuzzy DEA models to bi-conventional crisp DEA models by a set of α-level values.

Kuo and Liang (2011) applied a fuzzy DEA method to evaluate the performance of multinational corporations in the face of volatile exposure to exchange rate risk. They employed the fuzzy DEA model suggested by Kao and Liu (2000) to the information technology industry in Taiwan. Li and Yang (2008) proposed a fuzzy DEA-discriminant analysis methodology for classifying fuzzy observations into two groups based on the work of Sueyoshi (2001). They used the Kao and Liu (2000) method and replaced the fuzzy linear programming models by a pair of parametric models to determine the lower and upper bounds of the efficiency scores. By applying the Kao and Liu (2000) method and the fuzzy analytical hierarchy procedure, Chiang and Che (2010) proposed a new weight-restricted fuzzy DEA methodology for ranking new product development projects at an electronic company in Taiwan.

Saati et al., (2002) suggested a fuzzy CCR model as a possibility programming problem and transformed it into an interval programming problem using α-level based approach. The resulting interval programming problem could be solved as a crisp LP model for a given α with some variable substitutions. Model (2.39) proposed by Saati et al., (2002) is derived in a particular case where the inputs and outputs are triangular fuzzy numbers:

$$\begin{aligned}
\max \quad & w_p = \sum_{r=1}^{s} y'_{rp} \\
s.t. \quad & \sum_{r=1}^{s} y'_{rj} - \sum_{i=1}^{m} x'_{ij} \le 0, \qquad \forall j, \\
& v_i(\alpha x^m_{ij} + (1-\alpha) x^l_{ij}) \le x'_{ij} \le v_i(\alpha x^m_{ij} + (1-\alpha) x^u_{ij}), \qquad \forall i, j, \\
& u_r(\alpha y^m_{rj} + (1-\alpha) y^l_{rj}) \le y'_{rj} \le u_r(\alpha y^m_{rj} + (1-\alpha) y^u_{rj}), \qquad \forall r, j, \\
& \sum_{i=1}^{m} x'_{ip} = 1, \quad u_r, v_i \ge 0, \qquad \forall r, j.
\end{aligned} \tag{2.39}$$

Where $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^m, x_{ij}^u)$ and $\tilde{y}_{rj} = (y_{rj}^l, y_{rj}^m, y_{rj}^u)$ are the triangular fuzzy inputs and the triangular fuzzy outputs, and x'_{ij} and y'_{rj} are the decision variables obtained from variable substitutions used to transform the original fuzzy model proposed into a parametric LP model with $\alpha \in [0, 1]$. Then, Saati and Memariani (2005) suggested a procedure for determining a common set of weights in fuzzy DEA based on the α-level method proposed by Saati et al., (2002), with triangular fuzzy data. In this method, the upper bounds of the input and output weights were determined by solving some fuzzy LP models and then a common set of weights was obtained by solving another fuzzy LP model.

Azadeh, et al., (2007) proposed an integrated model of fuzzy DEA and simulation to select the optimal solution between some scenarios which obtained from a simulation model and determined optimum operators' allocation in cellular manufacturing systems. They used a fuzzy DEA model to rank a set of DMUs based on the Saati et al., (2002)'s method. In addition, they clustered fuzzy DEA ranking of DMUs by fuzzy C-Means' method to show a degree of desirability for operator allocation. Ghapanchi, et al., (2008) employed fuzzy DEA to evaluate the Enterprise Resource Planning (ERP) package performance. In their approach, inputs and outputs indices were first determined by experts' opinions which were evaluated using linguistic variables characterized by triangular fuzzy numbers and then a set of potential ERP systems was considered as DMUs. They applied a possibility-programming approach proposed by Saati et al., (2002) and obtained the efficiency scores of the ERP systems at different α values.

Hatami-Marbini and Saati (2009) developed a fuzzy BCC model which considered fuzziness in the input and output data as well as the u_0 variable. Consequently, they obtained the stability of the fuzzy u_0 as an interval by means of the method proposed by Saati et al., (2002). Then, Hatami-Marbini et al., (2010a) used the method of Saati et al., (2002) and proposed a four-phase fuzzy DEA framework based on the theory of displaced ideal. Saati and Memariani (2009) developed a fuzzy slack-based measure (SBM) based on the α-level approach. They transformed their fuzzy SBM model into the LP problem by using the approach proposed by Saati et al., (2002).

Hatami-Marbini et al., (2010b) proposed a fuzzy additive DEA model for evaluating the efficiency of peer DMUs with fuzzy data by utilizing the Saati et al., (2002)'s α-level approach. Moreover, they compared their model to the method of Jahanshahloo et al., (2004a) and demonstrated the advantages of their proposed model. Liu (2008) developed a fuzzy DEA method to find the efficiency measures embedded with assurance region (AR) concept when some observations were fuzzy numbers. He applied an α-level approach and Zadeh's extension principle (Zadeh, 1978 & Zimmermann, 1996) to transform the fuzzy DEA/AR model into a pair of parametric mathematical programs and worked out the lower and upper bounds of the efficiency scores of the DMUs. The membership function of the efficiency was approximated by using different possibility levels. Thereby, he used the Chen and Klein (1997) method for ranking the fuzzy numbers and calculating the crisp values. By considering that the relative importance of the inputs and outputs as $\frac{L_{I\delta}}{U_{Iq}} \leq \frac{v_\delta}{v_q} \leq \frac{U_{I\delta}}{L_{Iq}}$, $\delta < q=2,\ldots,m;$ and $\frac{L_{O\delta}}{U_{Oq}} \leq \frac{u_\delta}{u_q} \leq \frac{U_{O\delta}}{L_{Oq}}$, $\delta <$ q=2,…,s ; respectively. The two parametric mathematical programs proposed by Liu (2008) are as follows:

$$
\begin{aligned}
(W_p)_\alpha^L = \max \ & \sum_{r=1}^{s} u_r (y_{rp})_\alpha^L \\
s.t. \ & \sum_{r=1}^{s} u_r (y_{rj})_\alpha^U - \sum_{i=1}^{m} v_i (x_{ij})_\alpha^L \leq 0, \quad \forall j, j \neq p, \\
& -v_\delta + I_{\delta q}^L v_q \leq 0, \ v_\delta - I_{\delta q}^U v_q \leq 0, \ \forall \delta < q, \\
& -u_\delta + O_{\delta q}^L u_q \leq 0, \ u_\delta - O_{\delta q}^U u_q \leq 0, \ \forall \delta < q, \\
& \sum_{i=1}^{m} v_i (x_{ip})_\alpha^U = 1, \quad u_r, v_i \geq 0, \quad \forall r, j.
\end{aligned} \tag{2.40}
$$

$$
\begin{aligned}
(W_p)_\alpha^U = \max \ & \sum_{r=1}^{s} u_r (y_{rp})_\alpha^U \\
s.t. \ & \sum_{r=1}^{s} u_r (y_{rj})_\alpha^L - \sum_{i=1}^{m} v_i (x_{ij})_\alpha^U \leq 0, \quad \forall j, j \neq p, \\
& -v_\delta + I_{\delta q}^L v_q \leq 0, \ v_\delta - I_{\delta q}^U v_q \leq 0, \ \forall \delta < q, \\
& -u_\delta + O_{\delta q}^L u_q \leq 0, \ u_\delta - O_{\delta q}^U u_q \leq 0, \ \forall \delta < q, \\
& \sum_{i=1}^{m} v_i (x_{ip})_\alpha^L = 1, \quad u_r, v_i \geq 0, \quad \forall r, j.
\end{aligned} \tag{2.41}
$$

where, $I_{\delta q}^L = \frac{L_{I\delta}}{U_{Iq}}$, $I_{\delta q}^U = \frac{U_{I\delta}}{L_{Iq}}$, $O_{\delta q}^L = \frac{L_{O\delta}}{U_{Oq}}$ and $O_{\delta q}^U = \frac{U_{O\delta}}{L_{Oq}}$,

Jahanshahloo et al., (2009a) proposed some corrections to the Liu's (2008) model. Liu and Chuang (2009) applied the fuzzy DEA/AR model suggested by Liu (2008) and evaluated the performance of 24 university libraries in Taiwan based on the method proposed by Kao and Liu (2000b). Guh (2001) used a fuzzy DEA model like Kao and Liu (2000a) to approximate the fuzzy efficiency measures. However, Kao and Liu (2000a) developed their model under the Variable Returns to Scale (VRS) assumption and Guh (2001)'s model was developed under the CRS assumption.

Entani et al., (2002) proposed a DEA model with an interval efficiency consisting of the efficiencies obtained from the pessimistic and the optimistic viewpoints. They also developed this approach for fuzzy input and output data by using α-level sets. Hsu (2005) applied a simple fuzzy DEA model to balanced scorecard with an application to multinational research and development projects. The fuzzy DEA method included both crisp and linguistic variables processed by a four-step framework. Liu et al., (2007) developed a modified fuzzy DEA model to handle fuzzy and incomplete information on weight indices in product design evaluation. They transformed fuzzy information into trapezoidal fuzzy numbers and considered incomplete information on indices weights as constraints.

They used an α-level approach to convert their fuzzy DEA model into a family of conventional crisp DEA models. Saneifard et al., (2007) developed a model to evaluate the relative performance of DMUs with crisp data based on $l_2 - norm$. They used the ranking fuzzy numbers method of Jiménez (1996) to determine a crisp α-parametric model and solve the fuzzy $l_2 - norm$ model. Jahanshahloo et al., (2007b) developed a fuzzy $l_1 - norm$, model with trapezoidal fuzzy inputs/outputs that were initially suggested by Jahanshahloo et al., (2004c) for solving the crisp data in DEA. They applied the ranking fuzzy numbers method of Jiménez (1996) to the fuzzy $l_1 - norm$, model and obtained a crisp α-parametric model. Allahviranloo et al., (2007) introduced the notion of fuzziness to deal with imprecise data in DEA. They proposed fuzzy production possibility set with constant returns to scale to calculate the upper and lower relative efficiency scores of the DMUs by using the α-level approach.

Lotfi et al., (2007c) applied the method of DEA-discriminant analysis proposed by Sueyoshi (1999) to the imprecise environment. They first modified Sueyoshi's model with crisp data and then developed it to be fuzzy inputs and outputs based on the concept of α-level approach.

Karsak (2008) proposed an extension of Cook et al., (1996)'s model to evaluate crisp, ordinal and fuzzy inputs and outputs in flexible manufacturing systems by determining the optimistic (the upper bound) and pessimistic (the lower bound) of the α-level of the membership function of the efficiency scores. Azadeh et al., (2008) used a triangular form of fuzzy inputs and outputs instead of the crisp data and proposed a fuzzy DEA model for calculating the efficiency scores of the DMUs under uncertainty with application to the power generation sector. They transformed the fuzzy CCR model into a pair of parametric programs using the α-level approach and found the lower and upper bounds of the efficiency for different α-values. Their contribution to the fuzzy DEA literature is in the development of the membership functions and not the crisp measure of the efficiencies. They used the α-level to transform the fuzzy DEA model into a series of conventional crisp DEA models.

Azadeh and Alem (2010) later used this fuzzy DEA method (Azadeh et al., 2008) for vendor selection problems which was taken from Wuy and

Olson (2008). Noura and Saljooghi (2009) proposed an extension of a definite class of weight function in fuzzy DEA based on the principle of maximum entropy to provide circumstances for the compatibility and stability in the ranking of interval efficiency scores of DMUs at various α values. Wang et al., (2009b) proposed a fuzzy DEA–Neural approach with a self-organizing map for classification in their neural network. They used the upper and lower bounds of efficiency score at different possibilistic levels in their model. Lotfi et al., (2009a) developed two methods for solving fuzzy CCR model with respect to fuzzy, ordinal, and exact data. They used an analogue function to transform the fuzzy data into exact values in the first method.

In the second approach, they applied an α-level approach based on Kao (2006)'s method to obtain the interval efficiency scores for DMUs. Tlig and Rebai (2009) proposed an approach based on the ordering relations between LR-fuzzy numbers to solve the primal and the dual of FCCR. They suggested a procedure based on the resolution of a goal programming problem to transform the fuzzy normalization equality in the most primal of FCCR.

Zerafat Angiz et al., (2010a) show the advantages and shortcomings of the fuzzy ranking approach, the defuzzification approach, the tolerance approach, and the α-level based approach. They proposed an α-level approach to retain fuzziness of the model by maximizing the membership functions of inputs and outputs. They also compared their results with the results from Saati et al., (2002).

2.6.3 The Fuzzy Random Type-2 Fuzzy Set

Qin et al., (2009) developed a DEA model with type-2 fuzzy inputs and outputs to deal with linguistic uncertainties as well as numerical uncertainties with respect to fuzzy membership functions. Based on the expected value of fuzzy variable, they used a reduction method for type-2 fuzzy variables and built a fuzzy DEA model by means of the generalized credibility measure. Qin and Liu (2009) proposed a class of fuzzy random DEA (FRDEA) models with fuzzy random inputs and outputs when randomness and fuzziness coexisted in an evaluation system and the fuzzy random data were characterized by known possibility and probability distributions. They also proposed a hybrid genetic algorithm and stochastic simulation approach to assess the objective function of the proposed DEA.

Qin and Liu (2010) also proposed another approach like the method proposed in (Qin & Liu 2009). They included the chance functions in the objective and constraint functions which were subsequently converted to the equivalent stochastic programming forms and solved with a hybrid genetic algorithm and Monte Carlo simulation method. Zerafat et al., (2010b) proposed an alternative ranking approach based on DEA in the fuzzy environment to aggregate preference rankings of a group of decision makers. They applied their method to a preferential voting system with four stages. Although they considered the data as ordinal relations, stage 1 defined a fuzzy membership function for ranking a set of alternatives to find the ideal alternative.

In the 2nd stage they used the fuzzy DEA model proposed in Zerafat et al., (2006) to obtain the ideal solution. In the last two stages, they proposed a method to aggregate the results to a single score using subjective weights obtained from comparative judgments for ranking the alternatives Several fuzzy DEA models that do not fall into the fuzzy ranking approach, the tolerance approach, the α-level based approach, or the possibility approach categories. Hougaard (1999) extended scores of technical-efficiency that used in DEA for fuzzy intervals and showed how the fuzzy scores allow the decision maker to use scores of technical-efficiency in combination with other sources of available performance information such as expert opinions, key figures, etc.

Guo et al., (2000) proposed a self-organizing fuzzy aggregation model and ranked a group of entities with multiple attributes based on the concept of DEA. Sheth and Triantis (2003) introduced a fuzzy goal DEA framework to measure and evaluate the goals of efficiency and effectiveness in a fuzzy environment. They defined a membership function for each fuzzy constraint associated with the efficiency and effectiveness goals and represented the degree of achievement of that constraint. Hougaard (2005) introduced a simple approximation for the assessment of efficiency scores with regards to fuzzy production plans. This approach did not require the use of fuzzy LP techniques and had a clear economic interpretation where all the necessary calculations could be performed in a spreadsheet making it highly operational.

Wang et al., (2005) proposed a pair of interval DEA models for dealing with imprecise data, such as interval data, ordinal preference information, fuzzy data, and their mixture. Within their method, the efficiency scores were obtained as interval numbers and a minimax regret approach was used to rank the interval numbers. Uemura (2006) introduced a fuzzy goal based on the evaluation ratings of individual outputs obtained from the fuzzy loglinear analysis and then proposed a fuzzy goal into the DEA.

Luban (2009) proposed a method inspired by Sheth and Triantis's (2003) work and used the fuzzy dimension of the DEA models to select the membership function, the bound on the inputs and outputs, the global targets, and the bound of the global targets. Wang et al., (2009a) proposed two fuzzy DEA models with fuzzy inputs and outputs by means of fuzzy arithmetic. They converted each proposed fuzzy CCR model into three LP models in 25 order to calculate the efficiencies of DMUs as fuzzy numbers. In addition, they developed a fuzzy ranking approach to rank the fuzzy efficiencies of the DMUs.

Mozaffari et al. (2022), discusses the use of multi-stage fuzzy networks in data envelopment analysis (DEA) and DEA with undesirable outputs (DEA-R) to evaluate the efficiency of firms. Specifically, the authors focus on using liquidity ratios as inputs in their analysis. The study finds that the use of multi-stage fuzzy networks improves the accuracy of DEA and DEA-R models, especially when there are undesirable outputs, and suggests that this approach can be useful for assessing the efficiency of firms in the finance industry. The article provides detailed mathematical models and analyses to support their findings.

2.7 Triangular Fuzzy Number

The word "fuzzy" itself refers to objects which are not clear or are ambiguous, that any process, function, or event that is changing continuously cannot always be defined as either true or false. This is known as Fuzzy behavior (Zadeh, 1965). In fuzzy behavior activities, the changing nature is indicated by a number in the range from 0 to 1. (0 % to 100%) where 1.0 (100%) represents absolute truth and 0.0 represents absolute falseness. In the context of this present study, a possible approach to the vague data for the earlier example on "Graduate Student" variable can be obtained by conducting a survey to check the 'quality level' of graduated students. There are two possible approaches to state the quality of graduate students from their employers' viewpoint. First approach, by discrete outcome approach whether Good or Not Good, like 0 or 1, Yes or No, Suitable or Not Suitable or Accepted or Not Accepted. Here, the results can be determined based two outcomes only. The second possible approach is the outcomes are in a range of different quality levels, for example, (from Excellent – Very good – Good – Acceptable – Poor – Very poor). For this, the outcomes are in terms of percentage (from 0% to 100%) which is 'fuzzy' because data is determined by expectations and related to %. This is 'fuzzy logic' that allows one to express knowledge in a rule format close to natural language expression.

Therefore, the set of natural numbers, which formed the basis for theories and calculations, has bridged fuzzy subsets with most of the area in Mathematics, thus, introduced the fuzzy numbers (Anand & Bharatraj, 2017). Fuzzy numbers have been widely used to obtain better results in problems where decision making, and analysis are involved. To bridge the gap between number theory and fuzzy numbers, the theory of triangular fuzzy numbers is introduced and can be explained by Triangular Fuzzy Theory.

Singh and Alu (2023) describes the development of a bi-objective Fuzzy Data Envelopment Analysis (BOFDEA) model to evaluate the performance of decision-making units (DMUs). The authors propose an algorithm to solve the developed model and validate it with two numerical examples. They compare the results obtained from the proposed model with another study and conclude that the proposed methodology is more powerful and effective in ranking DMUs. They also present an education sector application to validate the proposed methodology. However, one major limitation of the proposed BOFDEA models is that they only account for trapezoidal fuzzy numbers (TFNs) and do not consider other types of fuzzy numbers. Additionally, the rankings of DMUs change with the change in alpha values. The article aims to provide new perspectives on the BOFDEA model's solution and its applicability in constant returns to scale (CRS) setting.

2.7.1 Triangular Fuzzy Theory

Fuzzy set theory is a generalization of classical set theory in that the domain of the characteristic function is extended from the discrete set {0, 1} to the closed real interval [0, 1]. Zadeh (1965) defined a fuzzy set as a class of objects with continuum grades of membership. Later, many researchers reform these sets of fuzzy theory and are introduced many theorems based on Zadeh definition, one of these scientists is Zimmermann, who introduced many of these preliminaries or theorems (Zimmermann, 2001), one of these theories is “Triangular Fuzzy Theory”. In 2017, Anand and Bharatraj introduced full definitions and concepts in Triangular Fuzzy Number Theory (TFN).

2.7.2 Triangular Fuzzy Number Definitions

Triangular Fuzzy Number (TFNs) can be defined in the following forms:

Definition 1: a fuzzy number $\tilde{A} = (a, b, c)$, where $c \geq b \geq a$ is called triangular fuzzy number if its linear membership function $\mu_{\tilde{A}}(x)$ is given by (Edalatpanah & Shahabi, 2012).

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{x-c}{b-a} & b \leq x \leq c \\ 0 & \text{otherwise} \\ 1 & \end{cases} \tag{2.42}$$

Definition 2: Let a fuzzy number Ã = (*a, b, c*) be a triangular fuzzy number. Then Ã is called a non-negative fuzzy number if and only if $a \geq 0$. (Rodríguez et al., 2016).

Definition 3: Let a fuzzy number Ã = (*a, b, c*) be a triangular fuzzy number. Then Ã is called an unrestricted fuzzy number if $a, b, c \in R$. (Rodríguez et al., 2016).

Definition 4: Let Ã = (*a, b, c*) and $\tilde{B}$ = (*d, e, f*) as two triangular fuzz⊖y numbers, then should be satisfied:

(i) $\tilde{A} \oplus \tilde{B} = (a, b, c) \oplus (d, e, f) = (a + d, b + e, c + f)$,

(ii) $\tilde{A} \ominus \tilde{B}$ = (a, b, c) ⊖ (d, e, f) = (a − f, b − e, c − d),

(iii) $\tilde{A} \otimes \tilde{B}$ = (min (γ), be, max (γ)) where, γ = {ad, af, cd, cf}

(Rodríguez, R.M, et al., 2016).

Definition 5: If Ã = (*a, b, c*) and $\tilde{B}$ = (*d, e, f*) as two triangular fuzzy numbers. Then these numbers are equal if and only if $a = d$, $b = e$ *and* $c = f$. (Rodríguez et al., 2016).

Definition 6: If Ã = (*a, b, c*) is a triangular fuzzy number. Then the ranking function of Ã is defined as follows: $R(\tilde{A}) = \frac{1}{4}(a + 2b + c)$,

(Rodríguez et al., 2016).

Definition 7: consider $\tilde{A} = (a, b, c)$ and $\tilde{B} = (d, e, f)$ as two triangular fuzzy numbers, then: (i) $\tilde{A} \leq \tilde{B}$ if and only if Re $(\tilde{A}) \leq$ R $(\tilde{B})$.

(ii) $\tilde{A} < \tilde{B}$ if and only if Re $(\tilde{A}) <$ R $(\tilde{B})$.

(Rodríguez et al., 2016).

Definition 8: A triangular fuzzy number can also be defined as $\tilde{A} = (a, b, c)$ which is referred to as a left right (L-R) fuzzy number. *a* is the central value, *b* is the left width (spread), and *c* is the right width (spread).

Then the linear membership function also has the following form (Sotoudeh et al., 2016):

$$\mu\tilde{A}(x) = \begin{cases} \frac{x-a+b}{b} & \text{a-b} \leq \text{x} \leq \text{a} \\ \frac{a-x+b}{b} & \text{a} \leq \text{x} \leq \text{a} + \text{c} \\ 0 & \text{otherwise} \end{cases} \qquad (2.43)$$

Definition 9: From definitions 1 to 8, let redefine fuzzy number $\tilde{A} = (a, b, c)$, where $c \geq b \geq a$ *as following:*

$\tilde{A}$ (Lower, Medium, Upper) be denoted as a triplet $\tilde{A}$ (y_L, y_M, y_U) is a fuzzy number for each output in PRUM case, that is used in the Fuzzy DEA Linear Programming model (LP.) later in this chapter and finding the fuzzy efficiency scores.

Figure 2.10 illustrates an example of a triangular fuzzy number where, $\tilde{A}$ is a fuzzy number and $\tilde{A} = (a, b, c)$, that is exactly represented the fuzzy number belongs to PRUM case. The algorithm will be introduced to show how this fuzzy number form will be chosen for this study in the methodological chapter.

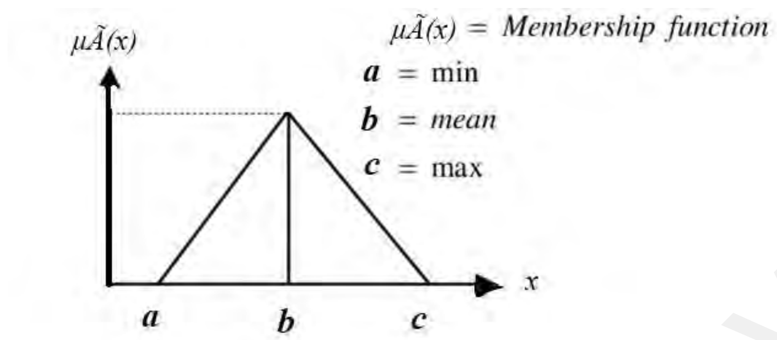


Figure 2.10: A triangular Fuzzy Number

2.8 Conclusion

This chapter describes the different efficiency ranking methods including the fuzzy logic in ranking the DMUs which is the core of this method and hybrid intelligent algorithm and fuzzy simulations are used together with a genetic algorithm. The applications of DEA efficiency ranking methods will be further discussed in the next chapter on empirical review of DEA methods in various industries.

Notably, the review on fuzzy DEA models by Hatami-Marbini et al., (2013) they have provided an excellent account by classifying the present FDEA methods in the literature which is selected for this study. A review on the empirical work of DEA in HEI sector will be presented in the following chapter.

CHAPTER 3: EMPIRICAL REVIEW OF DEA EFFICIENCY MEASUREMENT IN HIGHER EDUCATION SECTOR

3.1 Introduction

This chapter includes discussions on previous studies of performance and efficiency measurement of the public HEIs. First by looking at the importance of measuring the performance of publics HEIs and followed by the empirical review on DEA methods applied in public HEIs and academic research institutions. DEA applications for HEIs in Malaysia, Asia and from different areas and Fuzzy DEA applications in HEI setting are discussed next. This was followed by a short discussion on the choice and selection of input and output variables of DEA models for HEIs settings. This chapter finally discusses the research gaps in the field of DEA applications especially in the context of PRUMs and sAPRU, before concluding with the research direction of this study.

3.2 Performance Measurement of Public Higher Education Sector

The activities of public sector institutions are not subject to highly competitive pressure as with the profit-oriented and private counterparts (Munteanu & Andrei-Coman 2011). But the pressure arises and become issues when it comes to the efficiency of money allocated from state financial resources to these HEIs. It was claimed that lack of objective criteria for the assessment of the sector lead to state money distribution are not related with efficiency of its management by these public HEIs.

Decisions on providing the financial resources to the HEIs are influenced by the expectation on the achievement of several goals, that include economic and social mission entrusted to the HEIs (Munteanu & Andrei-Coman 2011).

The public sector is often characterized by the complexity of the sector's environmental instability and frequently related to political and legal changes. These were also influenced by the decision on the multitude and the ambiguity goals of various stakeholders and their contradicting expectations (Nazarko et al., 2009).

Measuring HEI efficiency has been recommended in response to issues of the increased awareness of accountability, value for money and cost control within HEIs (Athanassopoulos, 1997). Lately, the importance to create stimuli for the rational management of public funds by HEI was emphasized and for the quality improvement of HEI services too (Nazarko & Šaparauskas, 2014) with increased in fund allocations. Therefore, with the limited financial resources, current regulations and supervisions of HEI spending, there needs to be some form of indicators on HEI performance as the guidelines on division of public money to the HEIs (Nazarko & Šaparauskas, 2014).

In Malaysia, the public universities are mainly funded by the government. With the establishment of public research universities, through the respective ministry, the government are putting many pressures on these universities, to reorganize their activities and priorities to, among others, increase the research output and quality, to achieve critical mass in critical areas like science and technology, as well as to improve the international ranking and reputation of Malaysian universities (Ibrahim et al., 2015). The research universities in Malaysia also stand better chance to get additional funding for research activities, research management and quality assurance, RU incentive grants and specialized services like IPR, patenting and repository (MOHE, 2022).

The proposed research performance standards and measures of the public research universities set are to be benchmarked against the global standards. These standard measures would allow for comparison and ranking based on the selected quality indicators. More importantly, as clearly stated, the performance will determine the research funding allocation because performance indicator is focusing on research excellence. Therefore, the research universities should aim to maximize their status and research output based on their research resources (Henkel, 1999). With much consideration put on the inputs and outputs of research universities, the efficiency measurement could be suggested as the performance measurement approach.

3.3 DEA efficiency measurement of higher education institutions (HEIs)

3.3.1 The earlier DEA studies on HEIs

The whole framework of the DEA has been adapted for multi-input; multi-output production functions as applied in many industries. The major developments of DEA in the 1970s and 1980s were documented by Seiford and Thrall in 1990. After nearly two decades, Emrouznejad et al., (2008) produced an extensive evaluation of efficiency and productivity research encompassing the first 30 years theoretical developments in DEA work.

There have been many books and journal articles written on DEA, too, or applying DEA on various sets of problems. Various applications of DEA to public organizations such as schools, banks, hospitals, armed services, shops, airports, and others also have been reported (Seiford, 1996) where there are more than 800 references on this subject alone. In managerial applications, DMUs may include banks, department stores and supermarkets, and extend to car makers, hospitals, schools, public libraries and so forth.

Emrouznejad, et al., (2008) have produced an extensive evaluation of efficiency and productivity research comprising analysis of the first 30 years of scholarly literature in DEA that encompass the theoretical developments as well as "real-world" applications from its inception to the year 2007. More recently, Ahn and Vazquez, (2016) delved into the developments in DEA applications in the public sector including health care systems, educational institutions, and governmental bodies to private organizations like banks and service providers.

In the United Kingdom, the higher education management and HEI issues has received a lot of attention for the past few decades (Glass, et al., 2006), thus studies in the UK provides many instances of DEA to assess the effectiveness or productivity of higher education (Glass, et al., 2006). The UK is the pioneer in evaluating the university effectiveness and HEI for both public and private sectors. One of the examples is the comparative efficiency analysis undertaken as a response to the increased awareness of the issues of accountability, value for money and cost control.

The other example of DEA applications in HEI is the investigation on the efficiency level and productivity of nearly 200 education providers in England over the period 1999–2003 (Bradley et al., 2010). This study found that student-related variables (such as gender, ethnicity, and age) were generally more important in determining efficiency levels than staff-related variables. It was also established that the local unemployment rate influences provider efficiency. Another British example of DEA application is the examination of the technical efficiency of 45 universities in the period 1980/81–1992/93.

The rise of technical efficiency scores was attributed largely to the gains in pure technical efficiency and congestion efficiency, with scale efficiency playing a minor role (Flegg et al., 2004). A study conducted on a sample of 54,564 graduates from UK universities to assess whether the choice of technique affects the measurement of universities' performance (Johnes, 2003). A methodology developed by Thanassoulis and Portella (2002) allows each individual's DEA efficiency score to be decomposed into two components: first one, attributable to the university, at which a student studied, and the second attributable to the individual student. The results showed that the ranking of universities derived from the DEA efficiencies, which measuring university performance, were not strongly correlated with the university rankings derived from the university effects of the multilevel models. The university efficiency scores derived were largely unrelated to the scores from the individual level, further confirming the results are from a smaller data set (Johnes, 2006a).

However, the university level DEAs provide efficiency scores, which are generally strongly related to the university effects of the multilevel models.

DEA application in British higher education sector is on efficiency and productivity studies of more than 500 English in service training institutions during the period of 5 years (Bradley et al., 2006). Variables describing the number and the quality of students and teachers were used as input variables for a DEA model. Student achievements, measured as the number of students continuing their education and the number of attaining qualifications, were treated as output variables. A study by Casu, Thanassoulis, et al., (2011) evaluate cost efficiency in UK university central administration with a DEA framework.

The problems in defining the unit of assessment and the relationship between the inputs and the outputs are clearly demonstrated. In another research, Glass et al., (2006) computed DEA-based efficiency scores for policy evaluations and possible funding guidance in UK higher education. The aforementioned studies are mostly from the UK on which it can be claimed, UK as the pioneer in DEA efficiency studies on HEIs. Similar review but on the American background, looking at the effectiveness of education institutions has been conducted by Hirao (2012). He compared the efficiency of the top 50 public and private business schools in the United States for the year 2006 with DEA. It was found that although technical efficiencies of private and public schools were both high, scale and overall efficiencies of public schools were lower than those of private schools.

In Canada, the efficiency of 45 HEIs was studied by McMillan and Datta (1998). Three types of Canadian HEI were specified: comprehensive with a medical school, comprehensive without a medical school and primarily undergraduate. Nine different models were used in the analysis. Output variables included among others: number of students sorted by the field of studies, number of sponsored research grants. Input variables consisted of the number of academic staff with the division between the exact science and the humanities, the number of employees obtaining research grants. The authors stress the utility of the DEA method as a benchmarking tool applied by HEI. They recommend that DEA is used to study more homogenous administrative units such as departments. Another similar efficiency assessment was applied for the Canadian universities by using DEA and stochastic frontier methods (McMillan & Chan 2006). The analysis of the rankings revealed that the relative positions of individual universities across sets of several efficiency rankings demonstrated an underlying consistency. High-efficiency and low-efficiency groups were evidenced but the rank for most universities was not significantly different from the others. The results emphasized the need for caution when employing efficiency scores for management and policy purposes, and they recommended looking for confirmation across viable alternatives.

A study on Australian higher education (Madden et al., 1997), compared the established universities to the former colleges of advanced education, specific comparison of the initial and subsequent performance of economics departments was conducted. The findings revealed that while overall performance has improved substantially, further productivity improvements were required for new universities to achieve best practice.

Also, Avkiran (2001) used DEA to examine the relative efficiency of Australian universities. Three performance models were developed: overall performance, performance on delivery of educational services, and performance of fee-paying enrolments. The findings showed that the universities were performing well on technical and scale efficiency, but there was space for improving performance on fee-paying enrolments.

In South Africa, 10 out of 21 public HEI were studied from the perspective of their efficiency during a period of 4 years (Taylor & Harris, 2004). In each of the seven models tested, the output variables consist of number of graduates and indicators characterizing HEI engagement in research. Input variables varied with each model and included: total costs, financial resources, number of students and employees. The differences in efficiency between HEIs indicate four main factors that determine HEI efficiency: increase in the number of students, quality of recruited students, quality of academic staff and the level of fixed costs.

In Taiwan, 18 classes of freshmen English students in the academic year 2004–2006 were examined using DEA (Montoneri et al*.,* 2012). The teaching performance improvement mechanism was designed to identify key performance indicators to help the teachers in their teaching efforts. The sensitivity study highlighted the priority of richness of course contents over the other evaluated indicators. The performance improvement mechanism was designed to help decision-makers to develop their educational policies. Chen and Chen (2011) adopted Inno-Qual performance system (IQPS) by using DEA to evaluate the Inno-Qual efficiency of 99 Taiwanese universities divided into five types (research-intensive, teaching-intensive, profession-intensive, research & teaching-intensive, and education-in-practice-intensive). From the empirical results, researchers found that more than half (73%) of the universities were highly inefficient in improving the Inno-Qual performance. Thus, it was concluded that improving the Inno-Qual efficiency based on results would be helpful to reducing most of the cost expenditures.

3.3.2 More recent DEA studies on HEIs

More recently in 2014, Zhang et al., apply Data Envelopment analysis (DEA) for evaluating the relative efficiency of top 20 universities in China. Nine factors inclusive 4 inputs and 5 outputs are selected to find the ranking of universities in 2013. Kourosh and Arash Model (KAM) is applied while a very small negligible thickness of the efficient frontier is introduced (Khezrimotlagh, 2014). KAM represents that only one university can be efficient with 10-6 degree of freedom (DF) and other DMUs are inefficient with 10-6 -DF while three universities were completely inefficient (Khezrimotlagh, 2014). The suggested KAM rankings are compared with the measured rankings by the China Statistical Press (CSP). According to this study, a significant difference can be seen between the two sets of ranking, which suggests CSP to resurvey its methodology to rank universities of China. That paper concludes some universities had high values of outputs, but they simultaneously used the high values of resources too (Khezrimotlagh, 2014).

The technique of DEA by using KAM appropriately represents which university with less inputs values has higher values of outputs. Selecting more universities with a greater number of factors can be a future challenge to rank universities of China (Khezrimotlagh, 2014).

Leitner et al., (2007) studies with the use of DEA to assess the efficiency of natural sciences and engineering departments in HEIs in Austria. Models developed there consisted of two input variables (number of academic teachers and floor area of the department) and 12 output variables (extramural grants, ratio of completed projects to the total academic staff, number of projects completed by the department, number of exams, diploma students, monographs, reports, presentations and other publications, number of patents obtained, and PhD graduates). It was demonstrated that DEA method surpassed traditional approaches based on a simple calculation of indicators. Based on that study, application of the DEA method does not only allow determining a department's efficiency, but also helps specifying improvement possibilities of department.

Kempkes and Pohl (2010) examined the efficiency of 72 public universities in Germany for the period 1998-2003, with DEA and stochastic frontier analysis. The work referred to the faculty composition of universities as an essential element in the efficiency of higher education. The main finding was that East German universities have performed better in the total factor productivity change compared to those of West German universities. But on mean efficiency scores over the sample period, West German universities still appeared at the top end of relative efficiency outcomes.

A multi-output production function to analyze economies of scope between patents and R&D (Research and Development) was applied in research universities in the US (Chavas et al., 2012). The tradeoffs and/or synergies that arise between traditional university research outputs (articles and doctorates) and academic patents were analyzed. The study also investigates the sources of economies of scope and relative roles of complementarity, scale, and convexity. DEA estimates of scope economies using R&D input and output data from 92 research universities showed significant economies of scope between articles and patents but only modest complementarities except for a few cases. The findings showed how scale effects (for small universities) and convexity effects can contribute to economies of scope (Chavas et al., 2012).

In 2017 Delimiro et.al., study the efficiency of Colombian public universities, by employing the CCR, BCC and SBM models under output orientation. With the main objective is to determine technical, purely technical, scale and mix efficiencies using data acquired from the Ministry of National Education, the results show the extent to which outputs of inefficient Higher Education Institutions (HEIs) could be improved and the possible cause of this inefficiency. The universities were ranked by using a Pareto efficient cross-efficiency model and the overall productivity Malmquist index between 2011 and 2012 was also examined. The results showed Tolima, Caldas and UNAD to be the best-performing universities, with Universidad del Pacífico as the worst performer.

Asl and Ebru (2018) measure the research efficiency and productivity of public universities founded before 1981 in Turkey over the period 2013-2016. Using DEA to assess the relative research efficiency of these universities and Malmquist Total Factor Productivity Index to measure the total factor productivity change with respect to research inputs of universities, the results indicate several universities are relatively efficient as their research inputs declines continuously over the years. Also, the productivity of research activities decreases except in the period 2013-2014. As observed, the 2.3% fall in research productivity of the universities is due to deterioration in both technological and technical efficiency over the years.

Another interesting DEA study that was observed is the efficiency comparison among different countries. Johnes (2006) has explored the advantages and drawbacks of the various methods applied for measuring efficiency in the higher education context. In their study, the effectiveness of education systems in Turkey and European Union countries were analyzed and the relative total measuring activity is the analysis of technical and scale. The research concludes that both state and private universities have their own contribution in realizing these objectives owing to the number of personnel and financial resources as the constraints. They conclude that for effective universities, the most efficient use of limited resources is extremely important. Among the countries of the European Union, countries with value of activity below 1 are Denmark, Italy, Lithuania, Malta, Austria, Portugal, Finland and Izland.

To conclude, data envelopment analysis (DEA) evaluates the relative efficiency of a set of comparable decision-making units (DMUs) with multiple performance measures (inputs and outputs). Classical DEA models rely on the assumption that each DMU can improve its performance by increasing its current output level and decreasing its current input levels (Mehdi & Jana, 2019).

Evidently, DEA has become a popular performance measurement tool for non-profit institutions like universities due to its capability of handling multiple inputs and outputs without a priori assumptions on the monetary values of the inputs and outputs (Johnes & Lu, 2008). The following points outline the advantages of DEA as the tool to measure the efficiency of HEIs.

- Enable comparative efficiency assessment of HEI activities to be made from multiple viewpoints.
- Provide valuable information in supporting the management of HEI.
- Enables the identification of areas requiring improvement and outcomes of DEA also describe the development possibilities in those areas.
- Allows identification of HEI strengths and weaknesses and possible mode of fund allocation among HEI organisational units, or the optimal size of these units. (Nazarko & Šaparauskas, 2014; Leitner et al., 2007; Taylor & Harris, 2004; McMillan & Datta 1998; Bradley et al., 2006 and Nazarko et al., 2008).
- Enlighten the concepts of cost and outcome efficiency to gain further insights into the university operations (Athanassopoulos & Shale 1997).

3.4 Efficiency measurement of academic research institutions

This section discusses the importance of efficiency and performance measurement of academic research institutions or research efficiency in HEIs alike. Generally, an efficient research production is not guaranteed by the usual market correction mechanisms, and because of this, there is a need for tools to quantify research efficiency. Some researchers had argued that an effective performance measurement system is necessary for R&D productivity (Cherchye & Abeele, 2005; Cordero, et al., 2008).

With DEA evaluation tools, the efficiency measurement task of multiple (input and output) dimensions are made much simpler. Khoshnevis and Teirlinck (2018), evaluate the performance of R&D active firms in Belgium using DEA models with ratio data. The input-oriented constant and variable returns to scale DEA models (CRS- and VRS-based models) are applied. Scale efficiency and the respective types of returns to scale have been examined. The firms have also been evaluated based on global, size and sector frontiers. The results of this paper highlight that on average R&D active firms suffer from both technical inefficiency and scale size problems while the average scale efficiency is modest.

According to the size, small-sized firms suffer from scale and technical inefficiency. Medium-sized firms endure scale inefficiency rather than technical inefficiency. Large firms, however, present a higher average scale efficiency and technical efficiency. With regards to the sector of activity, firms in specialized supplier industries tend to outperform other firms in terms of average scale efficiency and average technical efficiency. Firms in science-based industries are also found to underperform on average in terms of VRS and scale efficiency.

Korhonen, Tainio and Wallenius (2001) evaluate the efficiency and performance of academic research at universities and research institutes using data from the Helsinki school of economics. Cherchye and Abeele (2005) analyze the productive efficiency of research in Economics and Business Management Faculties of Dutch universities. Abramo et al., (2011) measure the technical and allocative efficiency of university research activity based on bibliometric data for the five-year period 2004-2008. The technical and allocative efficiency is measured from university's research staff classified according to academic rank as the input and their field-standardized impact of the research product as the output.

There are numerous classic examples of studies on DEA applications in HEIs focusing on research productivity. Among the earliest are Ahn et al., (1988) applied DEA to doctorate-granting universities in the U.S.A by employing three output data: undergraduate and graduate students, federal research grants and contracts; and the three inputs namely, instructional expenditures, overhead expenditures, and physical investments.

Tomkins and Green (1988) evaluated the cost efficiency of UK departments of accountancy for the 1984-1985 using four outputs (one for research and three for teaching activity) and six inputs (three for labor and three for capital). Johnes (1993) assessed the research productivity of 36 UK departments of economics over the period 1984-88, using 32 input-output combinations. In this case the inputs express four distinct levels of departmental staff in roles that involve research activity, while the eight outputs take account of various codifications of the research results produced (articles and letters in academic journals, articles in professional journals, articles in popular journals, authored and edited books, published official reports and contributions to edited works).

Beasley (1995) focused on UK Chemistry and Physics departments using a multi-output and multi-input DEA model that takes account of both teaching and research. Among the outputs, the author also inserts four dummy variables for department ratings (outstanding, above average, average, or below average) as indicated by the University Grants Committee (UGC, 1986). The inputs are all of financial type. Madden et al., (1997) analyzed the effect of policy changes on the efficiency of 15 Australian economics departments from 1987 to 1991, considering both research output, teaching output and a single input for total academic staff.

Abbott and Doucouliagos (2003) estimate technical and scale efficiency of 38 Australian public universities for the year 1995 by considering multiple outputs, subdivided as teaching outputs and research outcomes. Meng et al., (2008) proposed a DEA model featuring hierarchical structure to 15 research institutes of the Chinese Academy of Sciences. Various outputs measured considered are publications, invited talks, awards, patents, reports, external funding, excellent leaders, and graduates to produce relatively comprehensive performance profiles of the research institutes.

In China, relative efficiency in the production of research of 109 regular universities in 2003 and 2004 was analyzed (Johnes & Yu, 2008). Output variables measured the impact and productivity of research. Input variables reflected staff, students, capital, and resources. The mean efficiency was just over 90% when all input and output variables were included in the model, and this fell to just over 80% when student-related input variables were excluded from the model. The rankings of the universities across models and time periods were highly significantly correlated. Further investigation suggested that the mean research efficiency was higher in comprehensive universities compared to specialist universities, and in universities located in the coastal region compared to those in the western region of China. These results offered support for the merger activity, which actually took place in the Chinese higher education.

More recently, Tun et. al. (2020), explores the application of data envelopment analysis (DEA) to measure the performance of higher education institutions in Myanmar. The authors evaluate the efficiency of 25 universities and colleges in the country using input and output data such as the number of students, number of academic staff, and research outputs. The study finds that the overall efficiency of higher education institutions in Myanmar is relatively low, with only a few institutions performing well.

The authors identify the factors that contribute to the low efficiency, such as inadequate funding and lack of research facilities, and suggest ways to improve the performance of the sector. The article provides a detailed description of the DEA methodology used in the study and presents the results in tables and graphs. The findings of the study can be useful for policymakers and education stakeholders in Myanmar and other countries with similar contexts.

3.5 DEA studies on Malaysian HEIs

One of the earliest DEA applications in measuring the efficiency of public universities in Malaysia is by Talib (2005). This study measures the technical and allocative efficiency of 18 public universities for the period of 2001-2003, where UM and USM were the top universities with 100% score of efficiency.

Kuah and Wong (2011) measured relative efficiency for 16 Malaysian universities where they identified lacking exercises in these universities and suggested fitting activities to be taken for development. The application of DEA enabled academics to identify deficient activities in their universities and take appropriate actions for improvement. Commencing the year 2014, there were more DEA efficiency measurement studies on HEIs in Malaysia.

For example, Irliana et al., 2014, also examines the relative efficiency of 20 public universities of Malaysia in the students' transition process in 2011. Three input and five output values are defined to estimate the relative efficiency of the universities through the marketability of the graduated students; either they manage to get a job, or further their studies or being unemployed. Data were gathered from Ministry of Higher Education in Malaysia and Ministry of Education Graduate Tracer Study websites. The results of Constant Returns to Scale (CRS) and Variable Returns to Scale (VRS) DEA models show that University Technology MARA is the most efficient university in Malaysia followed by University Malaysia Terengganu and University of Malaya.

Kao (2015) used the DEA hierarchical structure to measure the efficiency of the Department of Physics at a university. In this study, Kao's hierarchical system is applied and extended in Malaysian universities where the teaching function was further separated into three activities, namely teaching undergraduate, masters, and PhD students. While service to the community is further divided into consultation activities and cooperation activities.

Lim and Anderson, (2016) examine the efficiency of Malaysian public universities in comparison with the private and foreign universities using Data Envelopment Analysis (DEA). The study involves 22 universities (17 public, 1 private and 4 foreign) and based on data between the year 2008 and 2011. Madden et al., (1997) introduced a new study, which develop a theoretical framework to investigate productivity changes in the higher education sector particularly in the Malaysian community college institutions from 2007 to 2010. This study used the technical efficiency in measuring the productivity changes among community colleges in Malaysia.

Rosmaini et. al., (2017) proposed a method to measure the university's effectiveness by using parallel network DEA model. This study illustrates the application of the proposed effectiveness measurement model on 14 faculties in a public Malaysian university. Also, Madden et. al., (2017) in their DEA study estimates a function of the Malmquist total factor productivity index and its components under a variable-returns-to-scale (VRS) assumption to the higher education sector.

As an extension of Kao (2015) study, Kashim et al. (2018), present a study for measuring efficiency of University Utara in Malaysia using a hierarchical network data envelopment analysis model. The latest DEA study on HEIs is by Ahmed et al., (2021) who measure the efficiency of different faculties in UM. This study which also measures the overall efficiency of UM provides empirical evidence as to whether the results is consistent with the international QS Global World University Rankings.

3.6 Fuzzy DEA Application in Higher Education Institutions

Fuzzy Data Envelopment Analysis (FDEA) has the same definition as DEA as in the previous section: It is a non-parametric technique to measure the relative efficiencies of a set of decision-making units (DMUs) with common crisp inputs and outputs The main difference between DEA and FDEA is only the input/output variables which are considered as crisps, which is the per-step to fuzzy is needed before computing the DEA scores.

Based on the CCR- model, Sengupta (1992) introduced his first work using fuzzy theory in DEA. He developed the first fuzzy DEA model where the input and output data of DMUs often fluctuate, therefore, these fluctuating data are represented as linguistic variables and characterized by fuzzy numbers (Sengupta, 1992).

3.7 DEA Input and Output variables in HEI efficiency measurement studies

From the review on past DEA efficiency studies of HEIs, the following table summarizes the input and output variables used in some selected studies. Niranjan and Andrew (2011) stated there are no specific guidelines to deal with the selection of variables for DEA models. Rather, it was to the users' own perspectives, discretion, judgment, and expertise to select the more variables which are more critical to success to be the outputs for their DEA model (Gökşen et al., 2015 & Avkiran, 2001).

As it was assumed that HEIs had more influence on achieved results compared to the amount of their resources. Particularly, in the case of public higher education, variables involving price educational outputs like profits are hardly used. In many cases, the input and output variables used are those contributing to performance and efficiency in higher education like number of academic staff and non-academic staff, number of undergraduate and graduate enrolments. DMUs, like the universities, could identify the areas requiring improvement and further delve into the possibilities of developing those areas within the university (Aoki, 2010). This, in turn, can help to contribute decisions on fund allocation among the organizational units.

Different combinations of input and output variables through Data Envelopment Analysis (DEA) approach has enabled efficiency measurement to be made from multiple viewpoints. Some examples of studies on DEA application in higher education are outlined in table. More recent studies take into account ranked data, number of accredited programs, number of local and international student, number of student employment and amount of state funding as the input and output variables (Gökşen et al., 2015; Mahmudah & Lola, 2016; Olariu & Brad, 2017; Ahmed et al., 2021).

Table 3.1: Used Inputs/Outputs Variables for Some HEI Studies About Measuring Efficiency Using DEA.

Author	Used Input Variables	Used Output Variables	Efficiency Type
Bessent et al., (1983)	- Revenue from State Government. - Number of Students Completing a Program. - Employer Satisfaction with Training of Students.	- Student connect hours. - Number of Full-Time Equivalent instructors. - Square Feet of Facilities for Each Program. - Direct Instructional Expenditures	Allocative
Tomkins and Green (1988)	-Number of Full-Time Employees. - Personnel Costs. - Operating Costs. - Other Costs.	- Number of University Students. - Number of PhD Students. - Total Income. - Number of Publications.	Allocative.
Beasley (1990)	- Research Income. - Expenditure.	- Number of UG Students. - Number of PG Students. - Research Ratings.	Technical
Johnes (1993)	- Research Income.	- Research Outputs	Technical
Stern et al., (1994)	- Operating Costs. - Salaries.	- Research Grants. - Publications. - Graduate Students. - Contact Hours.	Technical
Beasley (1995)	- Research Income. - Personnel Costs. - Operating Costs.	- Number of students (UG+PG). - Number of Publications.	Allocative.
Abbott and Doucouliagos (2003).	-Number of Academic Staff. -Number of Non-Academic Staff. -Operating Costs. - Fixed Expenses	-Number of students. - Total number of Graduate students (Ass, UG & PG). - Amount of research.	Technical

Flegg et al., (2004)	- Number of Faculty Members. - Number of UG Students. - Number of PG Students. - Total Expenses.	- Research & Consultancy Income. - Number of Graduate from UG students. - Number of Graduate from PG students.	Allocative. & Technical
Warning (2004)	- Personnel Costs. - Other Costs.	- Number of Publications (Take indexes) - Number of Students.	Technical
Kutlar and Kartal (2004)	-Number of Academic Staff. -Number of Administrative Staff. - Personnel, Service Procurement & Consumption Expenses. - Acreage.	- Number of Students. - Student Fees. - Projects. - Number of Postgraduate Students.	Allocative. & Technical
Baysal et al., (2005)	- Personnel Costs. - Other Current Expenditures. - Investment Expenses. - Transfers. - Number of Faculty Members.	- Number of UG Students. - Number of PG Students. - Number of PhD Students. - Number of Publications.	Allocative. & Technical
Muzalwana (2005)	- Operating Expenditure. - Number of Graduate Students.	-Number of Undergraduate Students Enrolments. - Publication Counts. - Research Income.	Allocative. & Technical
Kartal et al., (2007)	- General Budget Expenditures. - Expenditures out of Budget. - Number of Professor (Prof.) - Number of Associate Prof. - Number of Assistant Prof. - Number of Assistant Instructor. - Number of Administrative Staff.	-Number of Publications (Take indexes) - University Income. - Number of UG Students. - Number of Graduate from UG students. - Number of PG Students. - Number of Graduate from PG students.	Allocative. & Technical
Kutlar and Babacan (2008)	- General Budget Expenditures. - Expenditures out of Budget. - Number of Professor (Prof.) - Number of Associate Prof.	-Number of Publications (Take indexes) - University Income. - Number of UG Students. - Number of Graduate from UG students.	Allocative. & Technical

	- Number of Assistant Prof. - Number of Assistant Instructor. - Number of Administrative Staff.	- Number of PG Students. - Number of Graduate from PG students.	
Gökşen et al., (2015).	- Outdoor-indoor Area of University. - Number of Academic Staff. - Number of Administrative Staff.	- Number of Publications. - Number of Graduate students.	Technical
Mahmudah and Lola (2016)	- Number of Lecturers. - Total Number of Students. - Number of Departments. - Ratio of Accredited programs	- World Rank. - Presence Rank. - Impact Rank. - Openness Rank. - Excellence Rank.	Technical
Olariu and Brad (2017)	- Number of academic staff - Number of non-academic staff - Number of accredited programs in universities	- number of undergraduate enrolments - number of graduate enrolments - amount of state money for basic institutional funding	Technical
Ahmed et al., (2021)	- Academic Staff Number	- Local Student Number - International Student Number - Employment Student Number	Technical

3.8 Variable Selection for DEA Models

As discussed earlier, DEA itself does not provide guidance for the specification of the input and output variables; rather, the users have their own discretion, judgment, and expertise. On the issues too, Niranjan and Andrew (2012) stated issues likely to arise when selecting variables are: unavailability of data, high dimensional production processes, and the inclusion of irrelevant inputs or outputs. Niranjan and Andrew (2011) have introduced eight variable selection methods to identify the relevant variables and offer guidelines for choosing the most appropriate method for research work.

All approaches are statistical in nature where four of which have already been discussed by Sirvent et al., (2005) and Adler and Yazhemsky (2010). The four remaining methods to be analyzed are: Efficiency contribution measure by Pastor et al., (2002), PCA application to DEA by Ueda and Hoshiai (1997) and Adler and Golany (2001), a regression-based test by Ruggiero (2005) and bootstrapping for variable selection by Simar and Wilson (2001).

Two approaches are forward selection (addition of variables) and backward elimination (removal of variables) (Niranjan, & Andrew, 2011)

1) Efficiency Contribution Measure (ECM): Pastor et al., (2002) develop a method for analyzing the relevance of a variable based on its contribution to efficiency. The variable being tested is called the candidate. Two DEA formulations are considered, one with the candidate variable and one

without it. A binomial statistical test determines if the effect of this variable on the efficiency measure indicates that the candidate variable is important to the production process.

2) Principal Component Analysis (PCA)-DEA: Ueda and Hoshea (1997) and Adler and Golany (2001) independently develop Principal Component Analysis-DEA (PCA-DEA), a general statistical method used to reduce the dimensionality of the data set by expressing the variance structure of a matrix of data through a weighted linear combination of variables. Each principal component accounts for maximal variance while remaining uncorrelated with the preceding principal components. Adler and Golany (2002) give a separate PCA-DEA mathematical formulation to obtain the efficiency estimates in which the principal components replace the original variables. In this method, a percentage of the information is retained for each of the original variables, thus improving the discriminatory power of DEA (Niranjan, and Andrew, 2011).

3) Regression-based test: Ruggiero (2005) suggests a variable selection approach in which an initial measure of efficiency is obtained from a set of known production variables. Efficiency is then regressed against a set of candidate variables; if the coefficients in the regression are statistically significant and have the proper sign (coefficient values should be positive for inputs and negative to outputs), the variables are relevant to the production process. This analysis is repeated, identifying one variable at a

time. The analysis stops when there are no further variables with significant and properly signed coefficients.

4) Bootstrapping for variable selection: Simar and Wilson (2001) discuss a statistical procedure to test the relevance of removing input and output variables as well as the potential for aggregation. Test statistics are calculated, and a bootstrap estimation procedure is used to obtain the critical values for these tests.

5) Banker (1996) lists three statistical tests to indicate the significance of an input or output variable to the production process. The null hypothesis is that the variable being tested does not influence the production process. Simulation studies are conducted, and the results indicate that these tests perform as well as or better than COLS-based tests (Olson et al., 1980). This is true even when the parametric frontier form used in COLS estimation is identical to the one used to generate the simulated data.

6) Fanchon (2003) suggests a recursive method to determine the variables to be included. This five-step approach determines the variable set that best explains output behavior, followed by using DEA iteratively to analyze the increase in the number of efficient observations. To validate the included variables, two more regressions are performed, one with only efficient observations and the other with both efficient and inefficient observations. In each, a high statistical significance of regression coefficients indicates a valid input variable.

7) Jenkins and Anderson (2003) propose a variable reduction method that omits the variables containing the minimum information using partial correlation as a measure of information content. Information in an input or output variable is measured as the variance over a set of production units; zero variation indicates all observed production units have the same value for that variable. Omitting highly correlated variables can have a major influence on efficiency scores, and thus the multivariate statistical approach using partial correlation measures is useful to determine the relevance of a given variable.

8) Dario and Simar (2007) aggregate highly correlated inputs and outputs to reduce the dimensionality of the production possibility space to a single input and a single output using eigenvalues.

While Data Envelopment Analysis (DEA) is used to obtain the scores of efficiencies, Fuzzy approach is applied to address the possibility of errors in determining the input and output variables correctly. This is where the FDEA method can be used in measuring the universities' performances under imprecise inputs and outputs.

3.9 Research Gaps

Based on the theoretical review on DEA concepts and the empirical review on DEA efficiency measurement studies on HEIs, there are two types of gaps identified for this study and listed in the following sections.

3.9.1 Empirical Gaps

While there are many efficiency studies on Malaysian public universities, efficiency-based ranking studies on public research universities are very limited. There are two important issues related to this that there is a need to fill this empirical gap. First is the issue of accountability. The public research universities in Malaysia have better chance to get additional government funding. According to the Ministry of Education, it is very important for the public research universities to increase their research output and research quality as well as to achieve critical mass in critical areas like science and technology. Therefore, with very limited financial resources, current regulations, and supervisions of HEI spending, this can be identified as a critical empirical gap. There needs to be some form of indicators on these universities' performance that can be used as the guidelines on allocation of public money to the HEIs.

The second issue is the need to continuously improve the international ranking and reputation of Malaysian research universities. Ranking of universities, especially the research universities, not only can be used as a promotional material to attract new students, but potential employees are also aiming to recruit new employees who graduated from the best universities. In addition, the proposed research performance standard, and measures for the public research universities in Malaysia have been set to be benchmarked against the global standards. As further stated, HEI performance will determine the amount of research funding allocation.

Review of previous studies on the efficiency research performance indicate there is still a gap in selecting information from the international or global ranking as the input or output variables of research universities. When international ranking and research excellence are the focus of research universities, these universities should aim to maximize their status and research output based on their resources. Hence, in measuring efficiency, information from the international ranking should either be the inputs or outputs to the DEA efficiency-based ranking of the research universities.

3.9.2 Methodological Gaps

Numerous studies have been conducted on studying different aspects of HEI efficiency and performance by using DEA methods. But very few applied Fuzzy DEA approaches on HEI cases (Mahmudah, & Lola, 2016). Amongst others traceable are by Lopes and Lanzer (2002) and Demir (2014). While the former used DEA and fuzzy sets to assess the performance of academic departments in a university, the latter compares the results of the classic DEA and the FDEA in measuring and evaluating activities in high schools in Turkey. Mahmudah and Lola (2016) use Fuzzy Data Envelopment Analysis to measure the performance and efficiency of public and private universities based on Webometrics ranking as the input and output variables.

According to Cooper (2011), choosing the exact DEA variables (inputs / outputs) for the HEI efficiency studies depends on the critical issues that may differ from one university to another or university among the group of universities under study. In the previous section, it is highlighted that the emphasis of the public research universities is to continuously improve their international ranking and research excellence as emphasized by the MOHE, Malaysia. Therefore, in applying the DEA efficiency-based ranking approach, information from the international ranking should be employed as the inputs or outputs of DEA model.

The empirical gap of this study also brings about the methodological gap where in this case the information from the international ranking is the uncontrollable variables of DEA model. Peykani et al., (2019) point out, there are examples where the only data available for efficiency analysis are in the form of qualitative data, imprecise, unclear and could be in the form of qualitative, linguistic data. For the case of HEIs, Peykani et al., (2019) gave examples like during the school year changing the number of academic staff member, unexpected expenses or number of papers published.

The limitation of DEA is that it is sensitive to data. Because DEA is a methodology focused on frontiers or boundaries, small changes in data can change efficient frontiers significantly (Thanassoulis, 2003). Fuzzy DEA solved this problem through the concept of fuzzy set theory by representing imprecise and vague data with fuzzy sets numbers. For this, fuzzy DEA (FDEA) models take the form of fuzzy linear programming models (Peykani et al., 2018).

Fuzzy sets theory can be effectively used to handle vagueness and ambiguity data by using the DEA approach. So, where several input or output data, (not all the variables) are considered as fuzzy, this is a fuzzy DEA (FDEA) case. On the other hand, where all inputs and outputs data (all the variables) are not clear and not precise, the FDEA model will be the full fuzzy DEA (FFDEA) case. In both cases, if any of the variables is crisp and easy to change its value, it is called fuzzy variable and the DMU where it is coming from, it is considered as Fuzzy DMU (Muren & Cui., 2014). This is the most important part in methodological gap because it is not easy to determine which variable exactly must be fuzzy in each study case, so choosing fuzzy variables as input, output or both is on the researcher's judgement or the experts in the field of study like education and agriculture and banking too.

As one cannot find many studies or research in the HEIs field, to estimate the technical efficiency by DEA by using a fuzzy algorithm also recent in this field of research. Even more important is how to determine which fuzzy definition should be applied for fuzzifying the crisp data of fuzzy variables.

3.10 Research Direction

The general aim of this study is to measure the technical efficiency of the public research universities in Malaysia (PRUMs) with DEA methods. As discussed, PRUMs are under pressure to be accounted for the financial resources allocated to them while increasing their research output and the quality of research. Because PRUMs' focus is on research excellence and, also there is need to continuously improve their international ranking hence their global reputation, the proposed standard performance measures will have to be set against the global standards.

More importantly, it is clearly stated that their performance will determine the allocation of research funding. Therefore, the performance indicator of PRUM and its research excellence and status at international level are the direction of research.

In the DEA context, DMUs of HEIs can choose the inputs and outputs based on their own objectives, for example, based the applying the key drivers critical to success as the input or outputs for DEA model (Gökşen et al., 2015; Avkiran, 2001). In many HEI cases, the input variables used are those contributing to performance and efficiency in higher education like number of academic staff and non-academic staff, number of undergraduate and graduate enrolments. With the consideration to increase the international ranking of PRUMs, benchmarking against selected sAPRU, the output variable will be related its research and teaching reputation at international level. Hence, the suitable set of indicators for the efficiency measurement model will be selected according to the QS set of indicators for the PRUMs and sAPRU. With a selection of indicators for the variables, it enables measurement and prediction of the efficiency scores for the PRUMs and sAPRU.

As in real life situation the observed values of the input and output data are sometimes uncontrollable, imprecise, or vague unlike some other which are controllable of can be fixed by DMUs. For example, the number of student intakes in public universities in Malaysia are generally fixed. Whereas some other variables are just ambiguous by their very nature, and some are only available in the form of linguistic data or qualitative data as discussed at length earlier. In this study the data to be derived from the world ranking are beyond the DMU's control and therefore, this efficiency-based ranking study will propose the tolerance approach of fuzzy logic DEA model as the most powerful and commonly used method in most field of study. The details of methodology of this research will be explained in the next chapter.

CHAPTER 4: METHODOLOGY OF RESEARCH

4.1 Introduction

The chapter presents the research methodology and design in detail, based on similar studies on HEIs efficiency studies. In general, this chapter explains the approach employed to meet the research objectives by proposing a suitable FDEA model to measure the technical efficiency under uncertainty in input and output variables. The following section begins with discussion on DMUs and variable selection for the FDEA technique that is based on tolerance approach. This is followed by the data and variables involved in the proposed FDEA model and description of the proposed FDEA. There are two level efficiency analysis of this study which is to measure (1) the technical efficiency of five PRUMs only and (2) the technical efficiency of five PRUMs and five selected public research universities in Asia (sAPRUs) altogether.

4.2 DMUs and Variable Selection

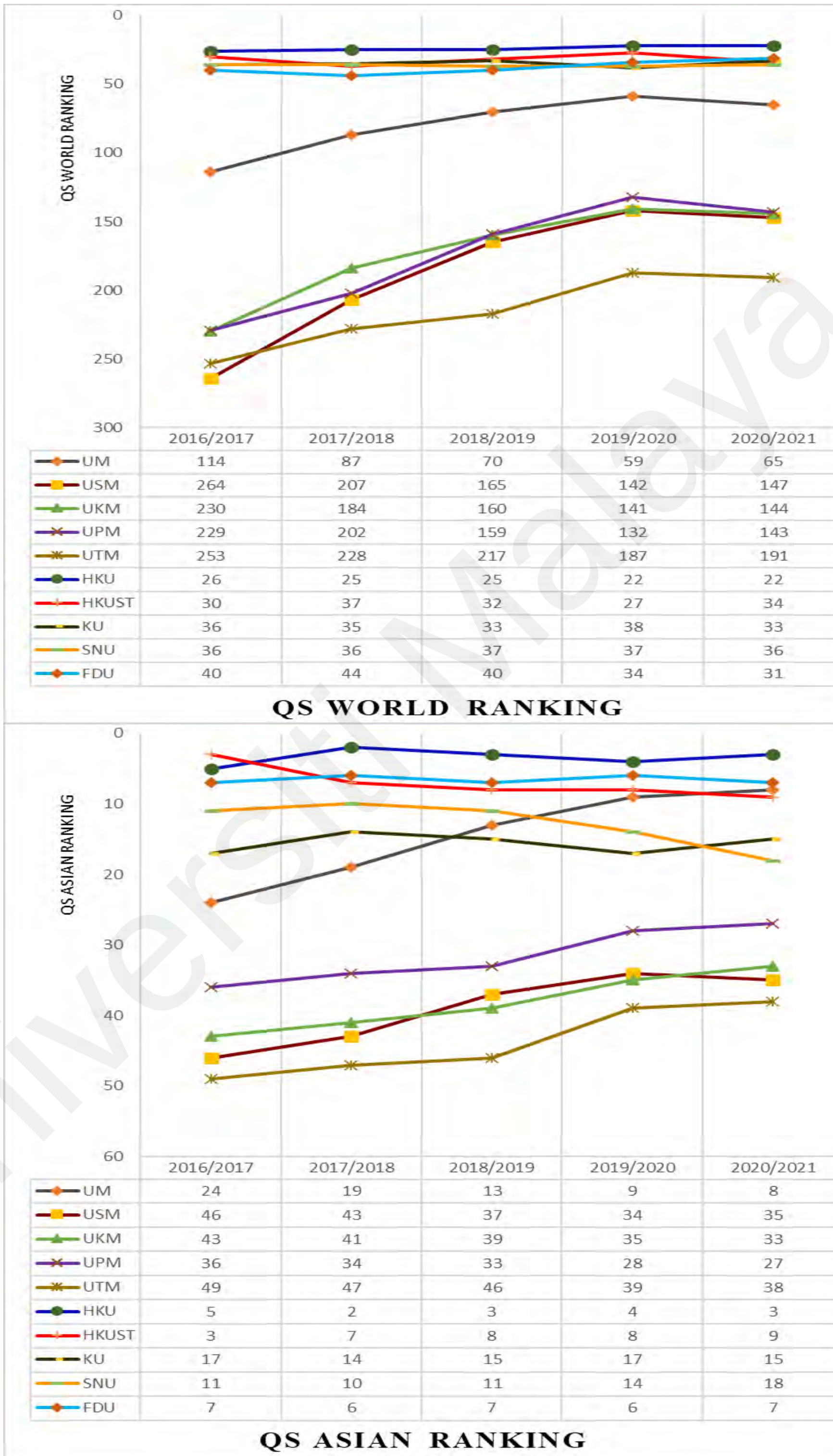
4.2.1 Decision Making Units

In general, the data collected are the panel primary data from the annual reports of the Ministry of Education in Malaysia (Higher Education Department) and the official websites of others 5 Asian universities (DMUs). The DMUs are the 5 public research universities representing all public research universities in Malaysia (PRUM) and for the international benchmarking purpose, five public research universities in Asia (sAPRUs) are selected and put under considerations together with PRUMs. The selection of sAPRUs is based on the highest Asian public university ranking in the World University Research Ranking (WURR) for the public universities in Asia for 2020 (https://worldresearchranking.com/) accessed on 28th January 2022. The WURR index is designed from the existing ones QS, TIMES Higher Education and Academic Ranking of World Universities (ARWU)) to evaluate three key components namely, research multi-disciplinarity, research impact, and research collaborative-ness. Each DMU and its initials are listed in the following table.

Table 4.1: The list of DMUs (5 PRUMs and 5 sAPRUs) of this study

DMU	University Name	Country	Code
1	Universiti Malaya	Malaysia	UM
2	Universiti Sains Malaysia	Malaysia	USM
3	Universiti Kebangsaan Malaysia	Malaysia	UKM
4	Universiti Putra Malaysia	Malaysia	UPM
5	Universiti Teknologi Malaysia	Malaysia	UTM
6	University of Hong Kong	Hong Kong	HUK
7	Hong Kong University of Science & Technology	Hong Kong	HKUST
8	Kyoto University	Japan	KU
9	Seoul National University	Korea	SNU
10	Fudan University China	China	FDU

Based on the QS World (WR) and Asian ranking (AR), the following figure illustrates the position of sAPRU together with PRUMs for the past five years.



	2016/2017	2017/2018	2018/2019	2019/2020	2020/2021
UM	114	87	70	59	65
USM	264	207	165	142	147
UKM	230	184	160	141	144
UPM	229	202	159	132	143
UTM	253	228	217	187	191
HKU	26	25	25	22	22
HKUST	30	37	32	27	34
KU	36	35	33	38	33
SNU	36	36	37	37	36
FDU	40	44	40	34	31

	2016/2017	2017/2018	2018/2019	2019/2020	2020/2021
UM	24	19	13	9	8
USM	46	43	37	34	35
UKM	43	41	39	35	33
UPM	36	34	33	28	27
UTM	49	47	46	39	38
HKU	5	2	3	4	3
HKUST	3	7	8	8	9
KU	17	14	15	17	15
SNU	11	10	11	14	18
FDU	7	6	7	6	7

Figure 4.1: World Ranking & Asian Ranking for PRUM and sAPRU (QS-2017: 2021), Source: TopUniversities, 2021.

Figure above shows for the past five years the selected Asian public research universities are way above most of the PRUMs for both international rankings. However, this is an exceptional case for UM. Unlike the other PRUMs, UM is among the sAPRU in the QS Asian ranking but only appears to approach the sAPRUs series of rankings in the QS World Ranking.

4.3 Variable Selection

To select the variables for FDEA model, a study by Puri and Yadav (2015) on the intuitionistic Fuzzy DEA (IFDEA) approach is referred. Since FDEA models deal with those inputs and output data where in real situations are only available in subjective, linguistic, and vague forms, the approach considers for the optimistic and pessimistic outcome of each variable. This is known as the fuzzy and intuitionistic fuzzy environments. Figure 4.2 illustrates the stages of variable selection and data collection process in the IFDEA study by Puri and Yadav (2015).

Also, Figure 4.2 shows that the selection of input/output variables for FDEA models are made based on review of past literature and can be from expert opinion too. There are 5 selection approaches as shown in Figure 4.2

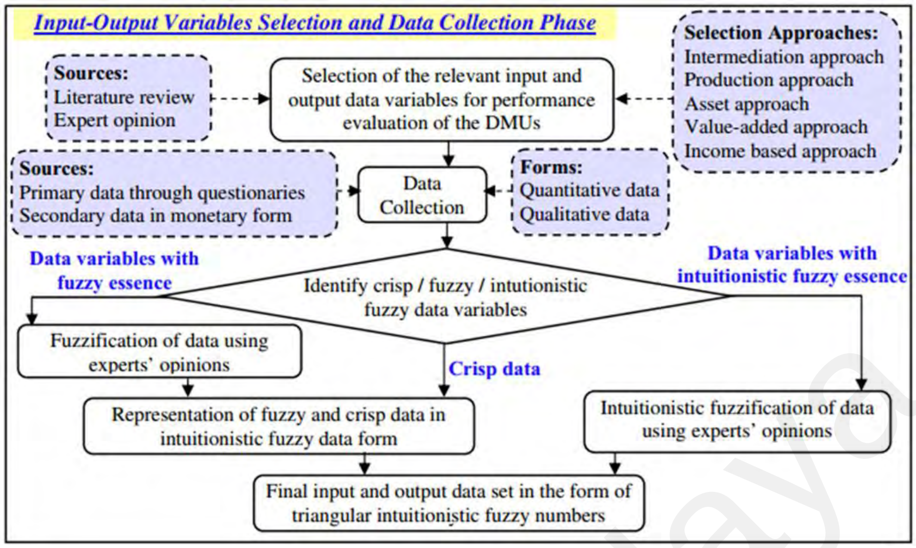


Figure 4.2: Variable Selection and Data collection phase of FDEA model (Source Puri & Yadav, 2015))

Figure 4.2 shows the sources of data (primary and secondary) are based the respective study, but the data can be in form of quantitative and qualitative data. The next step is to identify the crisp, fuzzy or intuitionistic fuzzy nature of data variables. If the data variables are characterized with fuzzy essence, data fuzzification is performed based on experts' opinion. Finally, the input and output data are set to be in the form of triangular fuzzy numbers.

4.4 Data and variable definition

For this study, the input panel data are collected from the annual reports of the universities in Malaysia accessed through the website of the Higher Education Department, Ministry of Education (MOHE, 2022) and from the official websites for other fives universities. Whereas the output data on research, teaching and influence ratio are retrieved from The World

University Rankings website (2022) and QS Top Universities website where all data for all DMUs are consistent in both sources.

4.4.1 Input Variables

The three input variables are used in the FDEA analysis for this study are:

- Input 1: Number of Full Time Equivalent Staff (No. of FTE Staff) inclusive of all: number of academic staff (full-time equivalent FTE), the number of academic staff of international/overseas origin (FTE) and a number of research staff (FTE). All staff numbers are pre-fixed or determined by each DMU (the university).
- Input 2: Number of Full Time Equivalent students (No. of FTE Students): inclusive of all: total number of students (FTE) and number of students of international/overseas origin (FTE) which are all controlled and determined by the university (DMU).
- Input 3: the percentage (%) of International Students: the percentage ratio of FTE international Student to FTE Student.

4.4.2 Output Variables

In this study all the output variables are set as fuzzy numbers because reputations and influences are normally ambiguous and qualitative in nature. Reputations are the beliefs or opinions that are generally held about someone or something or the widespread belief that someone or something has a particular characteristic (Oxford Learner's Dictionaries, 2021). While influence is defined as the capacity to influence the character, development, or behavior of someone or something, or the effect itself (Oxford Learner's Dictionaries, 2021).

The output variables as follows:

- Output 1: Teaching Reputation in % Percentage, the percentage ratio of total number of undergraduate degrees awarded, masters awarded, and doctorates awarded to the total students FTE.
- Output 2: Research Reputation % Percentage. This is based on the volume of research reputation ratio included all, ratio research income, ratio reputation of university survey and research productivity ratio of the university.
- Output 3: Citations % Percentage which depicts the research influence ratio of the university.

Inputs/outputs can be changed up to the type of the study case, in our study case we work on Public Research university in Malaysia and based on QS ranking criteria we believe these are the best variables, also based on peer-view studies on the same area. The decision to include or exclude certain inputs and outputs, such as budget values or local students, will depend on the specific research question being addressed and the characteristics of the DMUs being studied and based on type of efficiency that is measured and, in our case, we investigate the Technical Efficiency (T.E.) and in some cases, including these variables may not be appropriate or relevant to the analysis. In general, the selection of inputs and outputs in DEA should be guided by the specific research question being addressed and the factors that are most relevant to the efficiency of the DMUs being studied. It is important to carefully consider the inclusion or exclusion of each variable and to ensure that the selected inputs and outputs are appropriate and relevant to the analysis.

When selecting the input/output variables for the FDEA model of this study, however, several issues have arisen. Issues like the unavailability of data, high dimensional production processes, and the inclusion of irrelevant inputs or output variables. According to a study, those issues can be addressed based on the expert's discretion, judgment, and experience (Niranjan, & Andrew, 2011). Several variable selection methods have been introduced to identify the relevant number of variables and offer guidelines for choosing the most appropriate number of variables. One of these methods has been utilized by Delimiro et al., (2017) and has been employed in this study for the best and most accurate results of this study.

Under the basic rule, if the number of DMUs, n, is bigger than 10, then n is equivalent to, greater than or equal the maximum between $[(m \times s) + 1$ or $3 \times (m + s)]$, where m is the number of input variables and s the number of output variables. While when n is smaller than or equal 10, then n should be equivalent to smaller than or equal the minimum between $[(m \times s) + 1$ or $3 \times (m + s)]$, in this study trying to follow these guidelines and methods to detect the most relevant variables and its numbers. Based on

Delimiro et al., (2017), since there are 10 public research universities (n = 10) in this study, m and s could be set as, number of input variables $m = 3$ and the number of output variables $s = 3$ then number of DMUs will be n = 10. To check the validation, as $n = 10 \leq min\ [(m \times s) +1 = 10;\ 3 \times (m + s) =18]$.

4.5 The CCR-DEA Conceptual Framework of this study

DEA Model with fuzzy data is the main route to this research methodology, based on CCR-DEA model which is developed by A. Charnes et al., (1978). This DEA model measure the efficiencies of DMUs with crisp or unstable variables (inputs and outputs.). This research proposes a model which is the extension of the CCR model to a fuzzy framework which is suitable for the HEI cases.

The basic structure of a fuzzy inference system called Type-2 was introduced by Karnik and Mendel in 2001 which is adopted in this study. In Fuzzy Logic Systems (FLS), the Type-2 fuzzy logic set is the extension for ordinary fuzzy sets which is characterized in [0,1]. This allows handling of linguistic uncertainties or increased ability to handle inexact information in a logical manner.

The fuzzy framework for the PRUM of this study is developed by researchers as shown by figure 4.2 below.

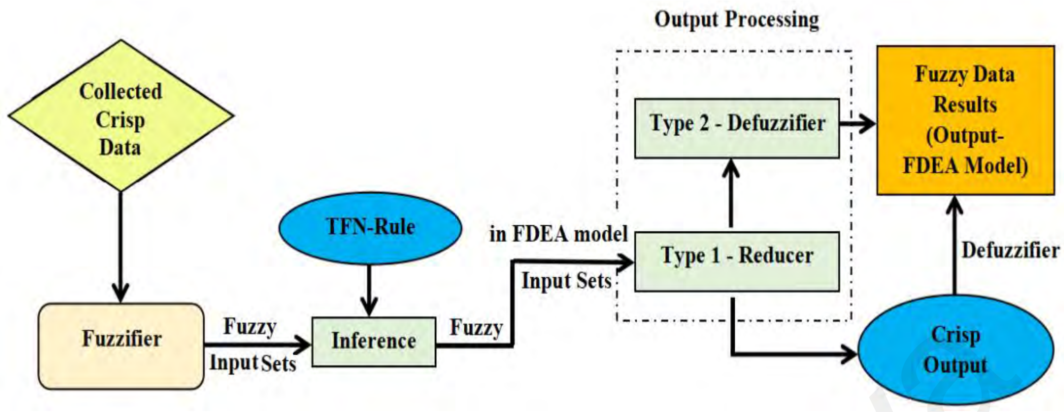


Figure 4.3: The fuzzy framework of the PRUM-sAPRU Case (Source: Author)

In an earlier study (Ahmed et al., 2021), the researcher established a DEA conceptual framework adapted from Thanassoulis in 2003, as shown in the following Figure 4.4

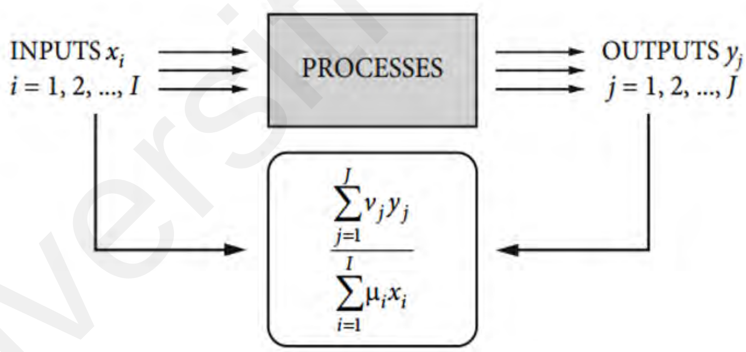


Figure 4.4: DEA conceptual framework

x_i:i-th input, *y_i: j-th output,*

I: number of inputs, *J: number of outputs,*

μ_i: weight of the input x_i, *v_i: weight of the output y_j.*

In a fuzzy system all or some of the variables are crisp or fuzzy numbers where the numbers are hopeful to be expected and credited by the

DM, so in a DEA conceptual framework with fuzzy variables, all the variables in previous framework in Figure 4.4 are re-defined,

$\tilde{x}_i$ *:i-th Fuzzy input,* $\tilde{y}_i$*: j-th Fuzzy output,*

$\tilde{\mu}_i$*: weight of the Fuzzy input* $\tilde{x}_i$*,* $\tilde{v}_i$*: weight of the Fuzzy output* $\tilde{y}_j$*.*

and thus, shown by Figure 4.5 as follows:

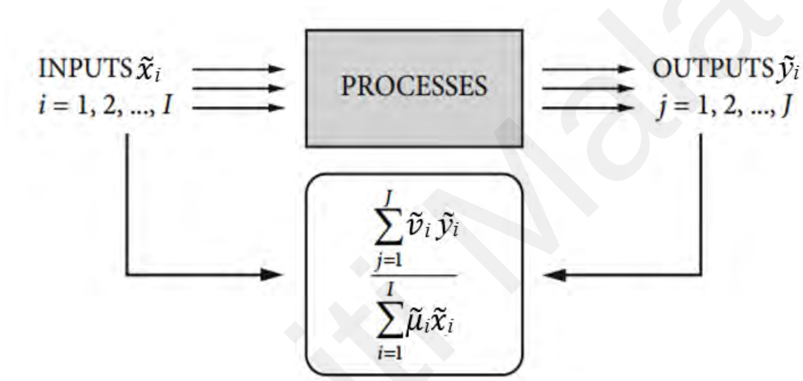


Figure 4.5: DEA conceptual framework by Fuzzy variables.

4.6 The Proposed FDEA approach: Tolerance Approach.

This study proposes the tolerance approach because it is the most powerful and commonly used method in most DEA fields of study for example transportations, banking, education, and others (Hatami-Marbini et al., 2011). Tolerance approach provides flexibility by relaxing the DEA relationships while the input and output coefficients are treated as crisp (Hatami. Marbini et al., 2011b).

The tolerance approach FDEA model is utilized as the concept of fuzziness in DEA modeling of this study by defining the tolerance levels on constraint violations. This approach is applicable in the study because all outputs variables are considered as crisps and could not controlled by decisions makers (DM) for example the research metrics of DMUs in this study, whereas all the inputs can be controlled by DM (Hatami-Marbini et al., 2011b).

Sengupta (1992) introduced the first FDEA model that used the concept of fuzziness by defining the tolerance levels on constraint violations. There is, however, the limitation behind tolerance approach is that the design of a DEA model with a fuzzy objective function and fuzzy constraints which may or may not be satisfied by the model (Triantis & Girod, 1998). So further improvement has been made on the tolerance model by Kahraman and Tolga (1998).

Fuzzy Data Envelopment Analysis (FDEA) is a powerful optimization technique used to evaluate the relative efficiency of decision-making units (DMUs) that have multiple inputs and outputs. FDEA is particularly useful when dealing with data that is imprecise, vague, or uncertain not only linguistic variables (data). In some cases, the outputs of a DMU may be expressed as percentage or ratio data, rather than as linguistic variables. FDEA can still be used in this scenario to evaluate the efficiency of the DMU, provided that the input and output data is appropriately fuzzified. One advantage of using percentage or ratio data is that it is a more precise way of expressing the outputs of a DMU. This allows for more accurate calculations and a more accurate assessment of the DMU's efficiency.

Additionally, using percentage or ratio data may be more appropriate in situations where the outputs are numerical in in nature, such as financial data. However, it's worth noting that using linguistic variables in FDEA, can also be beneficial in some cases. Linguistic variables can help to capture subjective or qualitative information about the DMU's performance, which may not be easily quantifiable using percentage or ratio data. Ultimately, the choice of whether to use percentage or ratio data, or linguistic variables, will depend on the specific context and objectives of the analysis.

Let us assume that n DMUs consume varying amounts of m different inputs to produce s different outputs. Assume that $\tilde{x}_{ij}$ (i =1, 2,….m) and $\tilde{y}_{rs}$ (r = 1,2,….s) represent, respectively, the fuzzy input and fuzzy output of the jth DMU$_j$ (j = 1,2,…n). The primal and its dual fuzzy CCR models in input-oriented version can be formulated as:

<u>Primal CCR model (input-oriented)</u>

$$
\begin{aligned}
\min \quad & \theta_p \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \leq \theta_p \tilde{x}_{ip}, \quad \forall i, \\
& \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{rp}, \quad \forall r, \\
& \lambda_j \geq 0, \quad \forall j.
\end{aligned}
$$

<u>Dual CCR model (input-oriented)</u>

$$
\begin{aligned}
\max \quad & \theta_p = \sum_{r=1}^{s} u_r \tilde{y}_{rp} \\
s.t. \quad & \sum_{i=1}^{m} v_i \tilde{x}_{ip} = 1, \\
& \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \leq 0, \quad \forall j, \\
& u_r, v_i \geq 0, \quad \forall r, i.
\end{aligned}
\tag{4.1}
$$

Where v_i and u_r in model (4.1) are the input and output weights assigned to the i_{th} input and r_{th} output. If the constraint $\sum_{j=1}^{n} \lambda_j = 1$, is adjoined to (4.1), a fuzzy BCC model is obtained and this added constraint introduces an additional variable, $\tilde{u}_0$, into the dual model which these models are respectively shown as follows:

$$\begin{aligned} \min \quad & \theta_p \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \leq \theta_p \tilde{x}_{ip}, \quad \forall i, \\ & \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{rp}, \quad \forall r, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geq 0, \quad \forall j. \end{aligned}$$	$$\begin{aligned} \max \quad & w_p = \sum_{r=1}^{s} u_r \tilde{y}_{rp} + u_0 \\ s.t. \quad & \sum_{i=1}^{m} v_i \tilde{x}_{ip} = 1, \\ & \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} + u_0 \leq 0, \quad \forall j, \\ & u_r, v_i \geq 0, \quad \forall r, i. \end{aligned}$$ (4.2)

Tolerance (1992) FDEA model (1st FDEA Model)

The tolerance approach fuzzifies the inequality or equality signs, but it does not treat fuzzy coefficients directly. This is a disadvantage related to the design of a DEA model with a fuzzy objective function whereby fuzzy constraints may or may not be satisfied (Triantis & Girod, 1998) as stated earlier.

Although in most production processes fuzziness is present both in terms of (1) not meeting specific objectives, and (2) the imprecision of the data; the tolerance approach provides flexibility by relaxing the DEA relationships while the input and output coefficients are treated as crisp (Hatami-Marbini et al., 2011a). By defining the tolerance levels on constraint violations, for the case of this study, all outputs variables are considered as crisps because it cannot be controlled by the DMUs.

For example, the output variables which are the research and teaching reputation as well the influence ratio are all beyond the control of DMUs because they are determined externally. On the other hand, the input variables like the number of international students for the academic year, for example, are controlled by the DMUs. This is particularly true during the time of a pandemic of the Co-vid19 outbreak. There are limits to the number of student intake from abroad, so the ratio of international students is controlled. This is due to the procedures of traveling and entering Malaysia other Asian countries where the DUMs under the study, as the result of the MCOs (Movement Control Orders) during 2020 and 2021.

When the Malaysian Government announced about the Movement Control Order commonly referred to as the MCO, any changes in the inputs are under the discretion of the universities or in other words, controlled by the DMUs. Moreover, the DMUs can reassigned other resources like the number of staff allocated for each faculty for the year. This is what is meant by the input variables are fixed and handled by the DM unit in each university. These are the different fixations and the limitations behind the flexible tolerance approach which are related to the design of a DEA model with a fuzzy objective function and fuzzy constraints.

For all the outputs of the FDEA model are in terms of fuzzy numbers, the Triangular Fuzzy Number (TFN) theorem is to be utilized to generate new data variables (outputs) which are fuzzy in nature. The TFN theorem basically converts the crisp data to fuzzy number at 3 levels: lower bound, medium level, and upper bound where three fuzzy values can be used in a FDEA model (Edalatpanah, Shahabi, 2012).

4.7 Scope and Limitation of FDEA model

The FDEA model of this study is established with three fixed input variables and three fuzzy output variables. Output maximization is used to analyze the model. Presupposing variable return to scale, the FDEA efficiency scores of the selected public research universities in Asia are evaluated. The number of reference set of variables for the DMUs under the study is tested on each DMU for the 4-year data, then calculate the efficient score for each university (DMU) followed by analyzing their efficiency status. Any inefficiency will be investigated for each academic year if exists.

Fuzzy set theory has been used widely to model uncertainty in DEA. As there are many applications of fuzzy set theory in the DEA and most of these approaches are powerful, but they usually have some theoretical and/or computational limitations and sometimes it is applicable to a very specific situation (Soleimani-damaneh et al., 2006).

For example, the tolerance approach uses fuzzy inequalities and equalities instead of fuzzy inputs and fuzzy outputs. Even though models related to this approach are not computationally efficient because this group mostly requires many linear programming models in the possibility approach, the proposed models may not be adapted to other DEA models (Soleimani-damaneh et al., 2006).

Up to researcher knowledge, there is no existing study using the Fuzzy DEA model to estimate technical efficiency. That motivated us to work at this point in the research to forecast the technical efficiency scores and to learn the status of each DMUs. A specific algorithm for HEI cases to convert crisp data to fuzzy data is also derived.

The next section presents the descriptive statistics for the input and output variables. The table reflects the data collected according to each university's annual report and the world ranking report for academic years (2017/2018, 2018/2019, 2019/2020 & 2020/2021) as described in the earlier part of this section.

Due to the possibility of biased efficiency score when DMUs were 5 only so, we decided to add another 5 DMUs. In the next part justification will be provided on the selection of another Asian Public Research University (sAPRU) to the PRUMs. The homogeneity is the base in selection of Asian Public Research Universities (APRU) and Public Research Universities in Malaysia (PRUM) for performance measurement can be justified in several ways as following:

1) sAPRUs and PRUMs are recognized as leading institutions of higher education and research in Asia, and their performance can provide valuable insights into the state of higher education and research in the region. Measuring the performance of APRUs and PRUMs can help identify areas for improvement and best practices that can be shared with other institutions.
2) sAPRUs and PRUMs are known for their research activities, and measuring their research performance can help assess the impact of their

research on society and the economy. The research does that by analyzing many factors such as research output, citation impact, and collaborations with industry partners and other institutions.

3) sAPRUs and PRUMs are significant contributors to human capital development in Asia and Malaysia, respectively. Measuring their performance can help assess their contributions to training the next generation of leaders and professionals and to the overall development of their respective countries.

4) Finally, APRUs and PRUMs are often compared to other similar institutions globally and measuring their performance can help benchmark their performance against international standards. This can help identify areas for improvement and areas where they excel, which can inform strategic planning and decision making.

4.8 Data and Descriptive Analysis

4.8.1 Academic year 2017 / 2018.

Table 4.3 shows data collected from PRUMs and selected APRUs for academic year 2017/2018.

Next in table 4.3; Descriptive statistics the variables of Public Research Universities in Malaysia as DMUs for 2017/2018 academic year, for the inputs shows that the average number of the first input, full-time staff (No of FTE Staff) for each DMUs, as whole, is about 1828 full time staff member for each university, with minimum member of staff 422 for

DMU7 and maximum of number member is 2716 for DMU10, the standard deviation of the input1 No of FTE Staff is 679.54 and variance 461767.

Table 4.2: The PRUMs & sAPRU input and output data: Academic year: 2017/2018

DMU Code	FTE Staff	FTE Student	% Inter. Students	Teach. Rep	Research Rep.	Influence Ratio
1	2,018	21,990	18	31.20	26.60	54.40
2	2,014	25,975	8	32.20	17.70	15.00
3	1,641	19,353	12	30.50	21.40	11.00
4	2,010	25,929	18	26.60	23.70	17.20
5	1,735	18,214	14	31.10	20.40	22.70
6	1,020	18,364	42	69	78	74
7	442	10,214	31	55	68	93
8	2584	22481	8	71.8	78.6	50.9
9	2101	26470	11	69.3	71	61
10	2716	32,859	10	60	58	65

Table 4.3: Descriptive Statistics of DMUs in Academic year 2017/2018

Descriptive Statistics	**FTE Staff**	**FTE Student**	**% Inter. Students**	**Teach. Rep.**	**Research Rep.**	**Influence Ratio**
Mean	1828.1	22184.9	17.2	47.67	46.34	46.42
Std. Deviation	679.54	6168.89	11.07	18.96	26.40	28.34
Variance	461767	38055192	122.62	359.42	697.07	803.35
Minimum	422	10214	8	26.6	17.7	11
Maximum	2716	32859	42	71.8	78.6	93

Also in Table 4.3, second input, full-time staff (No of FTE Students) the average for all DMUs, 22185 students for each university with minimum 10214 students for DMU 7, and maximum of the number of students is

32859 for DMU 10 and the standard deviation of this input (No of FTE Students) is 6168.89 and its variance 38055192.

For the third input the percentage of the international students (International Students %) the average is 17.2 % for all DMUs, with a standard deviation = 11.07 % and its variance 122.6, and the minimum 8% for DMU 2 and the maximum value is 42 % for DMU 6.

Now for the three outputs, (Teaching Reputation %) teaching reputation percentage as the first output, Table 4.3 tells that the mean of output1 is 47.67 % with a standard deviation = 18.96 % and its variance 359.42 and DMU 8 has the maximum percent 71.8 % and on the other hand the minimum value of teaching reputation for DMU 4 with 26.6 %.

The second output, a percentage of the research reputation (Research Reputation %), has average = 46.34 %, with a standard deviation = 26.4 % and its variance 697.07 and DMU 8 has the maximum percent, 78.6 % and on the other side the minimum value of teaching reputation for DMU 2 with 17.7%.

Lastly, the third output, which is the percentage of citations (Citations %), has an average = 46.42% with a standard deviation = 28.34 % and its variance 803.35, DMU 7 has the maximum value of this output which is 93% and the DM U 3 has the minimum value of the Citations % = 11%.

4.8.2 Data for academic year 2018 / 2019

Table 4.4 represents data collected from PRUMs and selected APRUs for academic year 2018/2019.

Table 4.4 PRUM & sAPRU data: Academic year 2018/2019

DMU Code	FTE Staff	FTE Student	% Inter. Students	Teach. Rep	Research Rep.	Influence Ratio
1	1,921	17,095	23	37.00	27.10	59.10
2	1,911	21,394	13	35.60	22.40	18.80
3	1,704	18,904	14	34.30	21.40	18.40
4	2,339	23,853	20	26.90	18.50	18.10
5	1,627	17,726	17	35.90	24.40	23.30
6	1,001	18,122	43	72.60	78.40	73.70
7	440	10,394	31	57	68	94.00
8	2,548	22,420	8	76	78	55.00
9	2,119	26,066	12	75	71	64.00
10	2,819	34,393	11	64	57	69.00

Table 4.5 Descriptive Statistics of DMUs Academic year 2018/2019

Descriptive Statistics	FTE Staff	FTE Student	% Inter. Students	Teach. Rep	Research Rep.	Influence Ratio
Mean	1,842.9	21,036.7	19.2	51.43	46.62	49.34
Std. Deviation	709	6,381	10.73	19.41	25.91	27.61
Variance	502,872	40,718,446	115.07	376.84	671.34	762.29
Minimum	440	10,394	8.00	26.90	18.50	18.10
Maximum	2,819	34,393	43.00	76.00	78.40	94.00

In Table 4.5; Descriptive statistics the variables of The 10 Selected Public Research Universities in Malaysia and Asia as DMUs for 2018/2019 academic year, for the inputs shows that the average number of the first input, full-time staff (No of FTE Staff) for each DMUs, as whole, is almost 1843 full time staff member for each university, with minimum member of staff 440 for DMU 7 and maximum of number member is 2819 and goes for DMU 10, the standard deviation of the input1 No of FTE Staff is 709 with variance 502872.

Also in Table 4.5, second input, full-time students (No of FTE Students) the average for all DMUs, is about 21037 students for each university with minimum 10394 students also for DMU 7, and maximum of the number of students is 34393 for DMU 10 as well. and the standard deviation of this input (No of FTE Students) is 6381 and its variance 40718446. For third input the percentage of the international students (International Students %) the average is 19.2 % for all DMUs, with a standard deviation = 10.73% and its variance 115.07, and the minimum 8% for DMU 8 and the maximum value is 43% for DMU 6, the maximum and minimum are about to be the same as the previous academic year.

Now for the three outputs, (Teaching Reputation %) teaching reputation percentage as the first output, Table 4.5 tells that the mean of output1 is 51.43% with a standard deviation = 19.41% and its variance 376.84 and DMU 8 has the maximum percent 76 % and on the other hand the minimum value of teaching reputation for DMU4 by 26.9 % for DMU 4. The second output, percentage of the research reputation (Research Reputation %), has average = 46.62%, with a standard deviation =25.91% and its variance 671.34 and DMU 6 has the maximum percent, 78.4% and on the other side the minimum value of teaching reputation for DMU 4 with 18.5%.

Then the third output, which is the percentage of citations (Citations %), has an average = 49.34% with a standard deviation = 27.61% and its variance 762.29, DMU 7 has the maximum value of this output, which is 94% and the DMU 4 has the minimum value of the Citations % = 18.1%.

4.8.3 Academic year 2019 / 2020

For academic year 2019/2020 data collected from PRUMs and selected as shown in Table 4.6

Table 4.6: PRUM & sAPRU data: Academic year 2019/2020.

DMU Code	FTE Staff	FTE Student	% Inter. Students	Teach. Rep	Research Rep.	Influence Ratio
1	1,893	15,140	20	41.60	30.50	56.60
2	1,955	20,908	15	35.60	22.70	26.70
3	1,701	17,180	15	34.20	19.60	32.30
4	1,655	20,018	23	32.00	31.40	19.10
5	1,705	19,087	14	36.40	25.20	29.20
6	1,003	18,260	44	69.50	77.20	76.60
7	454	10,125	31	57.40	66.10	89.80
8	2,507	22,566	9	73.70	78.10	59.90
9	2,111	26,182	12	72.30	71.60	66.50
10	2,905	32,537	12	61.90	58.60	68.10

Table 4.7: Descriptive Statistics DMUs Academic year 2019/2020

	FTE Staff	FTE Students	% Inter. Students	Teach. Rep	Research Rep.	Influence Ratio
Mean	1788.9	20200.3	19.5	51.46	48.1	52.48
Std. Deviation	694.09	6,104	10.74	17.16	24.28	24.07
Variance	481757	37,261,137	115.39	294.53	589.31	579.46
Minimum	454	10,125	9	32	19.6	19.1
Maximum	2905	32,537	44	73.7	78.1	89.8

In Table 4.7; Descriptive statistics the variables of the Selected Public Research Universities in Malaysia and Asia as DMUs for 2019/2020 academic year, for the inputs shows that the average number of the first input, full-time staff (No of FTE Staff) for each DMUs, as whole, is almost 1789 full time staff member for each university, with minimum member of staff 454 for DMU 7 and maximum of number member is 2905 and goes for DMU 10, the standard deviation of the input1 No of FTE Staff is 694.09 with variance 481757.

Also in Table 4.7, second input, full-time students (No of FTE Students) the average for all DMUs, is about 20200 students for each university with minimum 10125 students for DMU 7, and maximum of the number of students is 32537 for DMU 10 and the standard deviation of this input (No of FTE Students) is 6104 and its variance 37261137.

For third input the percentage of the international students (International Students %) the average is 19.5 % for all DMUs, with a standard deviation = 10.74% and its variance 115.39, and the minimum 9% for DMU 8 and the maximum value is 44% for DMU 6.

Next about the three outputs, (Teaching Reputation %) teaching reputation percentage as the first output, Table 4.7 tells that the mean of output1 is 51.46% with a standard deviation = 17.16% and its variance 294.53 and DMU 8 has the maximum percent 73.7% and on the other hand the minimum value of teaching reputation for DMU 4 with 32%.

The second output, a percentage of the research reputation (Research Reputation %), has average = 48.1%, with a standard deviation = 24.28% and its variance 589.31 and DMU 8 has the maximum percent, 78.1% and on the other side the minimum value of teaching reputation for DMU 3 by 19.6%.

Then the third output, which is the percentage of citations (Citations %), has an average = 52.48% with a standard deviation = 24.07% and its variance 579.46, DMU 7 has the maximum value of this output, which is 89.8% and the DMU 4 has the minimum value of the Citations % = 19.1%.

4.8.4 Data for academic year 2020 / 2021

Table 4.8 represents data collected for academic year 2020/2021. In Table 4.9; Descriptive statistics the variables of the Selected Public Research Universities in Malaysia and Asia as DMUs for 2020/2021 academic year, for the inputs shows that the mean of the first input, full-time staff (No of FTE Staff) for each DMUs, as whole, is almost 1750 full time staff member for each university, with minimum member of staff 462 for DMU 7 and maximum of number member is 2910 and goes for DMU 10, the standard deviation of the input1 No of FTE Staff is 677.18 with variance 458568.5.

Also in Table 4.9, a second input, full-time student (No of FTE Students) the average for all DMUs, is about 20476 students for each university with minimum 9976 students for DMU 7, and maximum of the number of students is 32597 for DMU 10 and the standard deviation of this input (No of FTE Students) is 6138.94 and its variance 37686549.7.

Table 4.8: PRUM & sAPRU Data: Academic year 2020/2021.

DMU Code	FTE Staff	FTE Student	% Inter. Students	Teach. Rep	Research Rep.	Influence Ratio
1	1,903	15,794	20	39.30	31.50	60.00
2	1,967	21,039	14	34.70	23.30	32.20
3	1,709	17,601	16	35.30	21.40	42.50
4	1,648	19,937	25	33.30	29.70	24.40
5	1,694	19,988	17	30.40	24.30	38.80
6	996	18,135	43	67.50	73.30	80.30
7	462	9,976	30	52.10	63.00	88.90
8	2,434	22,935	11	77.90	79.90	60.80
9	1,772	26,757	11	72.40	73.80	68.80
10	2,910	32,597	13	65.90	65.60	73.30

Table 4.9: Descriptive Statistics DMUs Academic year 2020/2021

	FTE Staff	FTE Student	% Inter. Students	Teach. Rep	Research Rep.	Influence Ratio
Mean	1749.5	20475.9	20.0	50.9	48.6	57.0
Std. Deviation	677.18	6138.94	10.14	18.45	24.36	21.62
Variance	458569	37686550	102.9	340.4	593.5	467.6
Minimum	462	9976	11.00	30.40	21.40	24.40
Maximum	2910	32597	43.00	77.90	79.90	88.90

For third input the percentage of the international students (International Students %) the average is 20% for all DMUs, with a standard deviation = 18.45% and its variance 340.4, and the minimum 11 % for DMU 8 and DMU 9 and the maximum value is 43% for DMU 6. Next about the three outputs, (Teaching Reputation %) teaching reputation percentage as the first output, Table 4.9 tells that the mean of output1 is 50.9% with a standard deviation = 18.45% and its variance 340.4 and DMU 8 has the maximum percent 77.9% and on the other hand the minimum value of teaching reputation for DMU5 with 30.4%.

The second output, percentage of the research reputation (Research Reputation %), has average = 48.6%, with a standard deviation = 24.36% and its variance 593.5 and DMU 8 also, has the maximum percent, 79.9% and on the other side the minimum value of teaching reputation for DMU 3 with 21.4%.

Then the third output, which is the percentage of citations (Citations %), has an average = 57% with a standard deviation = 21.62% and its variance 467.6. DMU 7 has the maximum value of this output, which is 88.9% and the DMU 4 has the minimum value of the Citations % = 24.4%.

4.9 Pearson Correlations Matrix.

It is most important that to stress on the difference between the common and the statistical meaning of the word "significance" when The Person Correlations coefficient is used for variables of a study, with consideration that this study findings are measuring and estimating the efficiency scores for sAPRU, and it is not research studying specific phenomena, behaviors, or relationships between some factors. So that this study has no hypothesis (Freedman, et al., 2007). In Person Correlation coefficient, statistical significance ("p-value") is the probability of a more extreme test statistic than the one calculated from the observed or collected data, under a given model. From this viewpoint Person Correlation tells us something about the data and not about a "truth". A low p-value of PCC (high statistical significance) means that the model is clearly unable to describe all features of the data well.

Given the context of the model and the source/generation and type of the data, this finding may be an indication that the model is unsuited to describe the data. When the model is a restricted version of a larger model that must be able to describe all features of the data, then we can attribute the un-suitedness to this restriction. This restriction is usually called the "null hypothesis", the hypothesis that is "tested".

A low p-value is then interpretable as an indication that the restriction (the "null hypothesis") makes the model unsuited to explain the data, and that therefore this hypothesis should be rejected and that the full (unrestricted) model should better be used to explain the data, because of that high statistical significance or low statistical significance of PCC is not applicable in this study and has no effects on the calculations results or on findings (Benesty, et al., 2009).

Based on above, Table 4.10 shows the relationships between the input variables, between the output variables (all shown in highlighted borders). and all between the six variables, but important to note that the results significance or does not have any effects on the results and findings.

Table 4.10: Pearson Correlations Matrix of the Input/Output Variables of 4 Academic Years

Academic Year 2017/2018

Correlations	Input Data			Output Data		
	FTE Staff	FTE Student	Int Student	Teaching Reputation	Research Reputation	Research Influence
FTE Staff	1	.869**	-.803**	0.023	-0.125	-0.368
FTE Student	.869**	1	-0.603	0.040	-0.107	-0.277
Int Student	-.803**	-0.603	1	0.233	0.377	0.564
Teaching Reputation	0.023	0.040	0.233	1	.975**	.735*
Research Reputation	-0.125	-0.107	0.377	.975**	1	.817**
Research Influence	-0.368	-0.277	0.564	.735*	.817**	1

Academic Year 2018/2019

Input Data			Output Data		
FTE Staff	FTE Student	Int Student	Teaching Reputation	Research Reputation	Research Influence
1	.857**	-.774-**	0.015	-0.161	-0.357
.857**	1	-.573-*	0.242	0.068	-0.118
-.774-**	-.573-*	1	0.090	0.252	0.449
0.015	0.242	0.090	1	.972**	.734**
-0.161	0.068	0.252	.972**	1	.816**
-0.357	-0.118	0.449	.734**	.816**	1

Academic Year 2019/2020

FTE Staff	1	.848**	-.793**	0.130	-0.036	-0.247
FTE Student	.848**	1	-0.529	0.342	0.217	-0.057
Int Student	-.793**	-0.529	1	0.121	0.277	0.404
Teaching Reputation	0.130	0.342	0.121	1	.970**	.805**
Research Reputation	-0.036	0.217	0.277	.970**	1	.825**
Research Influence	-0.247	-0.057	0.404	.805**	.825**	1

Academic Year 2020/2021

1	.823**	-.747*	0.183	0.005	-0.280
.823**	1	-0.563	0.433	0.296	-0.057
-.747*	-0.563	1	-0.009	0.146	0.354
0.183	0.433	-0.009	1	.972**	.715*
0.005	0.296	0.146	.972**	1	.793**
-0.280	-0.057	0.354	.715*	.793**	1

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

From Table 4.10, for the Academic Session 2017/2018 the input variable FTE Staff is highly positive significant (at 0.01 level) and correlated the FTE Students and highly negative significant correlated International Students, but FTE Staff shows no significant correlation at 0.05 or 0.01 levels with the other variables (all output variables). FTE Student is highly positive significant with FTE Staff but there is no significant correlated (at both 0.01 and 0.05 levels) with all other variables. But International Students has highly negative significant correlation with FTE Staff at 0.01 level only and not significant with all other variables at 0.05 nor 0.01 levels. The output variable Teaching Reputation is highly positive significant with Research Reputation and Research Influence. While Research Reputation is highly significantly correlated (at both 0.01 and 0.05 levels) with Teaching Reputation and Research Influence variables. Finally, Research Influence only is highly significantly correlated at 0.01 with Teaching Reputation and highly significant at 0.05 level with Research Reputation.

For the Academic Session 2018/2019, the correlation matrix in Table 4.10, for the input variable FTE Staff is highly positive significant (at 0.01 level) and correlated the FTE Students and highly negative significant correlated International Students, but FTE Staff shows no significant correlation at 0.05 or 0.01 levels with the other variables (all output variables). FTE Student is highly positive significant with FTE Staff and highly negative significant with international students at 0.05 level, but there is no significant correlation (at both 0.01 and 0.05 levels) with all other variables. Then, International Students has highly negative significant correlation with Research Reputation and Research Influence at both 0.01 and 0.05 levels and not significant with all other variables at 0.05 nor 0.01 levels. The output variable Teaching Reputation is highly positive significant with other 2 outputs only Research Reputation and Research Influence. Also, Research Reputation is highly significantly correlated (at both 0.01 and 0.05 levels) with Teaching Reputation and Research Influence variables. Finally, Research Influence is highly significantly correlated at (at both 0.01 and 0.05 levels) with Teaching Reputation and with Research Reputation outputs.

For session 2019/2020, the correlations matrix for the set of data in pairs (one to one), Table 4.10, for the input variable FTE Staff is highly positive significant (at both 0.01 and 0.05 levels) and correlated the FTE Students and highly negative significant correlated International Students, but FTE Staff shows no significant correlation at 0.05 or 0.01 levels with the other variables (all output variables). FTE Student is highly positive significant with FTE Staff at both levels 0.01 and 0.05, but there is no significant correlation (at both 0.01 and 0.05 levels) with all other variables. Also, International Students has highly negative significant correlation with only FTE Staff input at both 0.01 and 0.05 levels and not significant with all other variables at 0.05 nor 0.01 levels. The output variable Teaching Reputation is highly positive significant with other 2 outputs only Research

Reputation and Research Influence. Also, Research Reputation is highly significantly correlated (at both 0.01 and 0.05 levels) with Teaching Reputation and Research Influence variables. Finally, Research Influence is highly significantly correlated at (at both 0.01 and 0.05 levels) with Teaching Reputation and with Research Reputation outputs.

Finally, Table 4.10 for Academic session 2020/2021, shows for the input variable FTE Staff is highly positive significant (at both 0.01 and 0.05 levels) and correlated the FTE Students and highly negative significant correlated International Students, but FTE Staff shows no significant correlation at 0.05 or 0.01 levels with the other variables (all output variables). The table also shows International Students input negatively significantly with FTE Staff at 0.05 α-level, while it is not correlated to all other variables. The output variable Teaching Reputation is highly positive significant with other 2 outputs only Research Reputation and Research Influence. Also, Research Reputation is highly significantly correlated (at both 0.01 and 0.05 levels) with Teaching Reputation and Research Influence variables. Finally, Research Influence is highly significantly correlated at (at both 0.05 and 0.01 levels) with Teaching Reputation and with Research Reputation respectively. In summary, there is no clear correlation pattern between input-input, input-output, and input-output data set for all four academic years. While some significant correlations are identified between two variables for one Academic year, there is no correlation between the same variables for another Academic year.

4.10 The FDEA CCR Model Flow chart

The diagram (flowchart) in Figure 4.6 below shows the whole framework in establishing the Fuzzy DEA CCR model for the DMUs of this study:

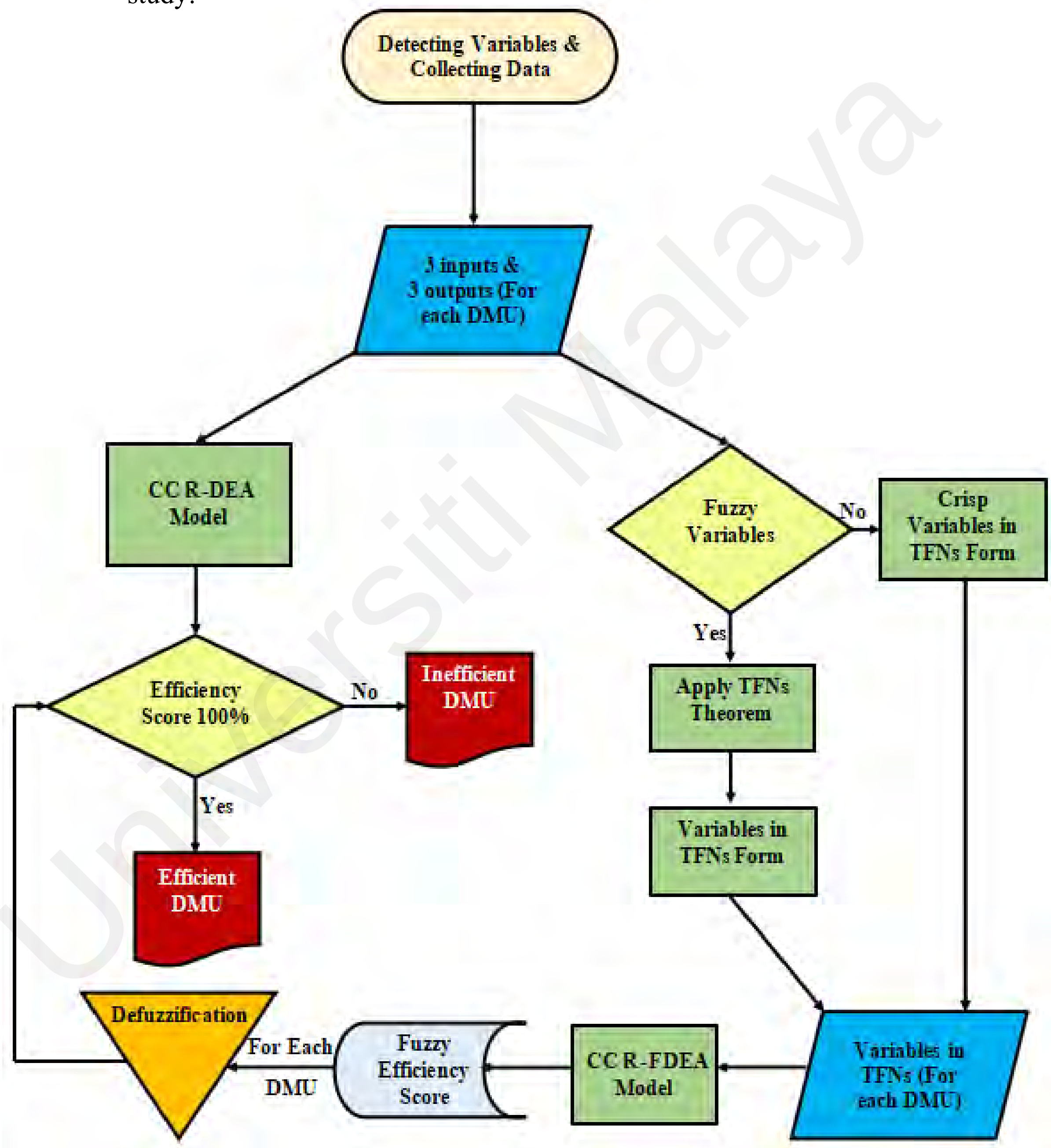


Figure 4.6: FDEA CCR Flowchart for PRUMs & sAPRUs (Source: Author)

4.11 Conclusion

This methodology chapter defines the selected input and output variables of the five PRUMs and the five selected sAPRUs followed by the CCR DEA conceptual framework and the proposed fuzzy DEA design approach that utilize the tolerance approach. The fuzzy logic method approach of DEA will be applied because the output variables obtained from the university global ranking are fuzzy in nature and cannot be controlled. Detailed data of each input and output data for each DMUs and the Pearson Correlation analysis between each variable are also presented. The FDEA CCR model flowchart showing the framework for the DMUs of this study is also included.

The following chapter presents the preliminary findings of CCR-CRS DEA technical efficiency of the PRUMs, and selected APRUs level of efficiency analysis and the stages involved in establishing the Fuzzy DEA model of PRUMs and selected APRUs under period of study.

CHAPTER 5: RESULTS AND DISCUSSIONS ON TECHNICAL EFFICIENCY AND FUZZY DEA EFFICIENCY OF MALAYSIAN PUBLIC RESEARCH UNIVERSITIES

5.1 Introduction

This chapter discusses the application of the DEA models specified in chapter 3 to evaluate the technical efficiency of PRUMs for the last 4 consecutive academic years (2017/2018, 2018/2019,2019/2020 and 2020/2021). This is followed by the Triangular Fuzzy Number (TFN) deduction for each crisp variable for the case of PRUM. By applying the Linear Programming (LP) fuzzy DEA model by Lertworasirikul et al., (2003b), this chapter further demonstrates the estimation of expected technical efficiency scores of the PRUMs for the academic year 2021/2022. In the next part of the chapter, the technical efficiency PRUMs will be benchmarked against the selected group of public research universities in Asia (sAPRU) for the same 4 consecutive academic years (2017/2018, 2018/2019, 2019/2020 and 2020/2021). This is followed by the assessment of the Fuzzy technical efficiency scores of PRUM when benchmarked against the sAPRU for the academic year 2021/2022.

5.2 DEA Empirical Results

This study applies the concept of Constant Returns to Scale (CRS) to evaluate the efficiency of DMUs. This well-known technique has been applied in many DEA studies (Hatami-Marbini et al., 2017), (Aldamak and Zolfaghari, 2017), (Lai and Hwang, 1992), Lee (2004) and Lee et al., (2005), and this DEA constant returns to the scale approach are called CCR-CRS (Charnes et al., 1978). This CCR-CRS DEA model assume input-oriented model where the inputs are minimized with the limitation on the lower amount of the outputs (Guzik, 2009). The equations of constraints for the CRS model are in the form of linear programming models. As large number of conditions and restrictions would have the negative impact on the solution of the problem, it would be more convenient and more practical to construct the dual models of linear programming for the model and the advantage is that this model uses the same data but with less restrictions (Jablonsky & Dlouhy, 2004).

Based on the above, the model 4.2 would be expanded to Model 5.1.

$$
\begin{aligned}
&\theta_p{}^* = \max \sum_{r=1}^{s} u_r y_{rp} \\
&s.t: \\
&\sum_{i=1}^{m} v_i x_{ip} = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad \forall j \\
&u_r, v_i \geq 0 \qquad \forall r, i.
\end{aligned}
\tag{5.1}
$$

In this model (5.1) each DMU has s = 3 outputs and m=3 outputs, i.e., each DMU uses inputs x_{ij} (i = 1, 2, 3), to obtain s=3 outputs y_{rj} (r=1,2,3). Here u_r (r = 1, 2, 3) and v_i (i = 1, 2, 3), are the weights of the i_{th} input and

r_{th} output. This fractional program is calculated for every DMU to find out its best input and output weights. To simplify the computation, the nonlinear program shown as (4.2 in chapter 4) can be converted to a linear program (LP) and the model is called the CCR model (5.1) above.

CCR (Constant Returns to Scale) and BCC (Variable Returns to Scale) are two common types of DEA (Data Envelopment Analysis), Scale assumptions of CCR assumes that all DMUs operate under constant returns to scale, while BCC allows for variable returns to scale. So based on the inputs, outputs, and research objectives the DMUs under the study being evaluated operate under constant returns to scale, then CCR may be more appropriate. Conversely, if there are reasons to believe that firms have variable returns to scale, then BCC may be more suitable. Besides, CCR is an input-oriented model, meaning that it seeks to maximize the outputs produced from a given set of inputs. BCC is both input and output-oriented, meaning that it seeks to minimize inputs needed to produce a given level of outputs or maximize outputs given a fixed set of inputs. In this study the input efficiency is the primary interest therefore CCR may be preferred, whereas if output efficiency is more important, BCC may be preferred. So CCR-model is more appropriate than BCC-model in case of PRUM and sAPRU. Another reason is data availability; the choice between CCR and BCC can also depend on the availability and quality of data. CCR requires fewer data points than BCC, and it is easier to estimate.

5.2.1 PRUMs DEA Efficiency Scores

In this research it is assumed that all five public research universities in Malaysia are all homogeneous. The efficiency scores generated by the Solver-365 software are arranged in 4 sets, according to the four academic years (2017/2018), (2018/2019), (2019/2020) and (2020/2021) and for each set, the efficiency scores for each DMU are based on the data given in Table 4.2, Table 4.4, Table 4.6, and Table 4.8. The results for each academic year will be discussed in the succeeding sections. There are two sets of results generated: (1) the efficiency scores and efficiency status of each DMU for each academic year; and (2) the average of efficiency score and efficiency status.

5.2.2 Efficiency status for PRUM in Academic Year 2017/2018

Table 5.1: Efficiency scores with DEA for academic year 2017/2018

DMU	Efficiency	Efficiency Status
DMU1	100%	Efficient
DMU2	100%	Efficient
DMU3	100%	Efficient
DMU4	77%	Inefficient (refers to DMU1, DMU2, DMU3 &DMU5)
DMU5	100%	Efficient

Table 5.1 shows all other DMUs are efficient for academic year 2017/2018 based on the efficiency scores, except for DMU4 which scores 77%, indicating it is inefficient when referred to the other DMUs (DMU1, DMU2, DMU3 and DMU5).

5.2.3 Efficiency status for PRUM in Academic Year 2018/2019

Table 5.2: Efficiency score using DEA for academic year 2018/2019

DMU	Efficiency	Efficiency Status
DMU1	100%	Efficient
DMU2	100%	Efficient
DMU3	100%	Efficient
DMU4	89%	Inefficient (refers to DMU1, DMU2, DMU3 & DMU5)
DMU5	100%	Efficient

Similar to Table 5.1, Table 5.2 shows DMUs 1, 2 3, and 5 are efficient for academic year 2018/2017 based on the model results and data provided for all variables, except for DMU4 which depicts inefficiency when scoring at 89 %. This could mean that although the scores show some improvement, further improvements are needed for the score to be 100% fully efficient. Therefore, DMU4 is inefficient when referred to other DMUs (DMU1, DMU2, DMU3 and DMU5).

5.2.4 Efficiency status for PRUM in Academic Year 2019/2020

Table 5.3: Efficiency score using DEA for academic year 2019/2020

DMU	Efficiency	Efficiency Status
DMU1	100%	Efficient
DMU2	93%	Inefficient (refers to DMU1, DMU3, DMU4 &DMU5)
DMU3	100%	Efficient
DMU4	100%	Efficient
DMU5	100%	Efficient

For academic year 2019/2020 as shown in Table 5.3, four DMUs are efficient based on the results of model, but for this academic year DMU2 decreased to be 93%, also although it looks efficient, but need more improvements from decision makers in the University to increase this score and back to be 100% efficiency score like before. So DMU2 is inefficient refers to other DMUs (DMU1, DMU3, DMU4 and DMU5) for academic year 2019/2020.

5.2.5 Efficiency status for PRUM in Academic Year 2020/2021

Table 5.4: Efficiency score for the academic year 2020/2021

DMU	Efficiency Score	Efficiency Status
DMU1	100%	Efficient
DMU2	100%	Efficient
DMU3	100%	Efficient
DMU4	100%	Efficient
DMU5	100%	Efficient

In this academic year 2020/2021, Table 5.4 indicates all DMUs are fully efficient with a 100% score for each. This could be because all DMUs have performed better like highest efficiency score, or 100 % efficiency is achieved for DMU2 as for the others' previous scores.

5.2.6 Average efficiency score of each DMU for all Academic years: (2017/2018), (2018/2019), (2019/2020) and (2020/2021)

The next Table 5.5 and Figure 5.1 show the average efficiency scores of all DMUs for all four academic years. DMU1, DMU3 and DMU5 all have achieved full efficiency in the average scores for the four academic years. The same set of data is to be applied in the FDEA model to estimate the efficiency of the next academic year 2021/2022. Any efficiency estimation scores which are less than 100% or even is very close to full efficiency score will indicate inefficiency. Table 5.5 and Figure 5.1 indicate for two DMUs: DMU2 and DMU4 are inefficiency in one or more academic years. DMU2 is not fully efficient by scoring efficiency level of 93% for the academic year (2019/2020) and DMU4 was not efficient in two academic years (2017/2018) and (2018/2019) where it failed to get the full efficiency scores when only scored 89% and 77%, respectively. Henceforth, the average scores of these two DMUs for the period under study were less than 100% as depicted in Table 5.5.

Table 5.5: Efficiency Scores for the 4 academic years and average score for each university.

DMU	**2017/ 2018**	**2018/ 2019**	**2019/ 2020**	**2020/ 2021**	**Average**	**Efficiency Status**
DMU1	100%	100%	100%	100%	100%	Fully Efficient
DMU2	100%	100%	93%	100%	98%	Not Fully Efficient
DMU3	100%	100%	100%	100%	100%	Fully Efficient
DMU4	89%	77%	100%	100%	92%	Not Fully Efficient
DMU5	100%	100%	100%	100%	100%	Fully Efficient

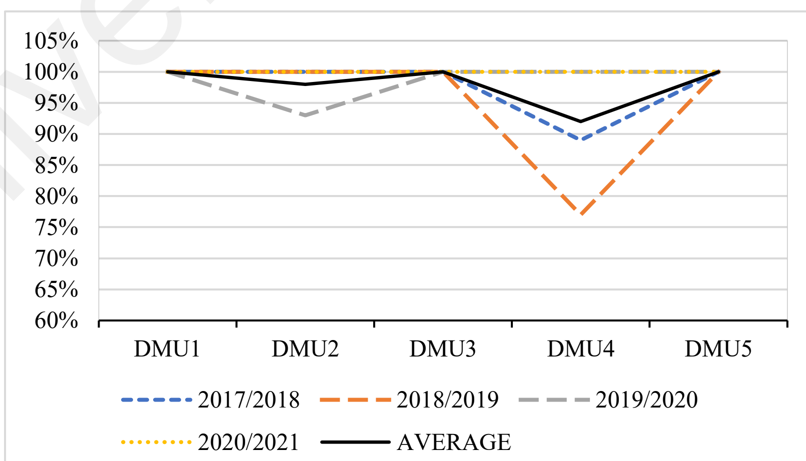


Figure 5.1: Efficiency Scores and Average Efficiency Score of DMUs for 4 academic years.

5.3 Estimating the Technical Efficiency of PRUM with Fuzzy DEA

By using the same PRUM data for the 4 consecutive academic years (2017/2018, 2018/2019,2019/2020 and 2020/2021), the second part of this study applies the Fuzzy Data Envelopment Analysis model. To establish the model for this study, the process starts with deducing the Triangular Fuzzy Numbers (TFNs) for each crisp fuzzy variable (3 inputs). Then the next step attempts to estimate the technical efficiency score of PRUM for the academic year 2021/2022 by applying the proposed Model by Lertworasirikul et al., (2003b), as Linear Programming (LP) fuzzy DEA model for evaluating the technical efficiency scores.

5.3.1 Fuzzifying crisp data of the Fuzzy variables.

By using the three outputs data (of four academic years 2017/2018, 2018/2019, 2019/2020 and 2020/2021) which are determined as crisp data, these outputs are considered as fuzzy variables. Therefore, it must be converted into fuzzy numbers by applying Triangular Fuzzy Numbers Theory. The Triangular Fuzzy Numbers Theory developed by (Zimmermann, 2001) employed to derive the algorithm as follows.

5.3.2 Algorithm of Finding Fuzzy Numbers with TFN.

For explanation, this algorithm in general, assumed that there exists numeric vector $x_i = (x_1, x_2, ..., x_n)$ where x_i, (i = 1, 2, 3, …integer) is a fuzzy variable with collected crisp data, therefore. to covert x_i to be a fuzzy variable y_i with fuzzy data by using TFNs theorem and its definitions, do the following steps:

- Convert Real Value Numbers of x_i to Triangular Fuzzy Numbers by using R-coding.

- Key in R-soft as:

{`fuzzify(x, y = NULL, method = "mean", err = 0, dimnames = list("x", "y"), ...)`}.

- Run R-soft then get in the results in the form (y_L, y_M, y_U), y_L = lower value, y_M = Medium value and y_U = Upper value for each x_i

- R-soft Details: Converts crisp numbers in *x* to a triangular fuzzy number (TFN). Optionally, values in *y* can be used as grouping elements and are coerced to a factor.

1- Method *mean* calculates the central value of a TFN as the mean of *x* given *y*, and the left and right spreads as standard deviations.

2- Method *median* gives the central values as a median and left and right spread are calculated as distance of the first and third quartile from the median.

3- Method *zero* inserts' zeros to both spreads.

4-Method *error* uses a user-defined numeric value or vector for the spreads. The length of the numeric vector in argument *err* must be in *(1, length (x), 2 * length(x)*).

Using any of the above methods is up to the type of collected crisp data and by doing data analysis can check which method will be more fit to get fuzzy numbers in R-soft, and as a numerical example applying this algorithm using R-coding.

```
library(fuzzyreg)

fuzzify(1:5)

fuzzify(1:6, c(1,1,1,2,2,2), method = "err", err = runif(6) * 1e-3)
```

code 5.1

Then Run R-soft, get the following results in R-soft form:

```
  xc       xl       xr y
1  3 1.581139 1.581139 1
  xc           xl           xr y
1  1 0.0008694394 0.0008694394 1
2  2 0.0004822162 0.0004822162 1
3  3 0.0004193971 0.0004193971 1
4  4 0.0008593754 0.0008593754 2
5  5 0.0009066272 0.0009066272 2
6  6 0.0004363237 0.0004363237 2
```

code 5.2

In 5.2 code, R-soft get back the triangular fuzzy numbers (TFNs)

5.3.3 Determining Fuzzy Numbers for PRUM Case

Based on collecting data for PRUM case where it was assumed that the three output variables are fuzzy, then Table 5.6 represent the crisp data, that should be converted to be triangular fuzzy numbers (TFNs). These triangular fuzzy numbers will be applied in the FDEA model to be explained in the next section.

In this approach, where the data values are close to each other, and its range is small, so that Mean Method in R-soft is used to generate the fuzzy numbers as TFNs.

Table 5.6: Data collected from DMUs latest 4 years (Crisp Data)

DMU (University)	Input			Output (Crisp)		
	No of FTE Staff	No of FTE Students	International Students %	Teaching Reputation %	Research Reputation %	Citations %
DMU1(UM)	1903	15794	20	(39.3, 41.6, 37.28, 31.2)	(30.5, 31.5, 27.1, 26.6)	(60, 56.6, 59.1, 54.4)
DMU2(USM)	1967	21039	14	(34.7, 35.6, 35.6, 32.2)	(23.3, 22.7, 22.4, 17.7)	(32.2, 26.7, 18.8, 15)
DMU3(UKM)	1709	17601	16	(35.3, 34.2, 34.3, 30.5)	(21.4, 19.6, 21.4, 21.4)	(42.5, 32.3, 18.4, 11)
DMU4(UPM)	1648	19937	25	(33.3, 32, 26.9, 26.6)	(29.7, 31.4, 18.5, 23.7)	(24.4, 19.1, 18.1, 17.2)
DMU5(UTM)	1694	19988	17	(30.4, 36.4, 35.9, 31.1)	(24.3, 25.2, 24.4, 20.4)	(38.8, 29.2, 23.3, 22.7)

As an example, given from Table 5.7 for the cell of DMU1 and Teaching Reputation %, it has (39.3, 41.6, 37, 31.2) crisp data, by using TFNs definitions in section **2.7.7** and R-soft Mean method in section **5.3.2** this crisp data value will be (31.2, 37.28, 41.6) TFNs such that:

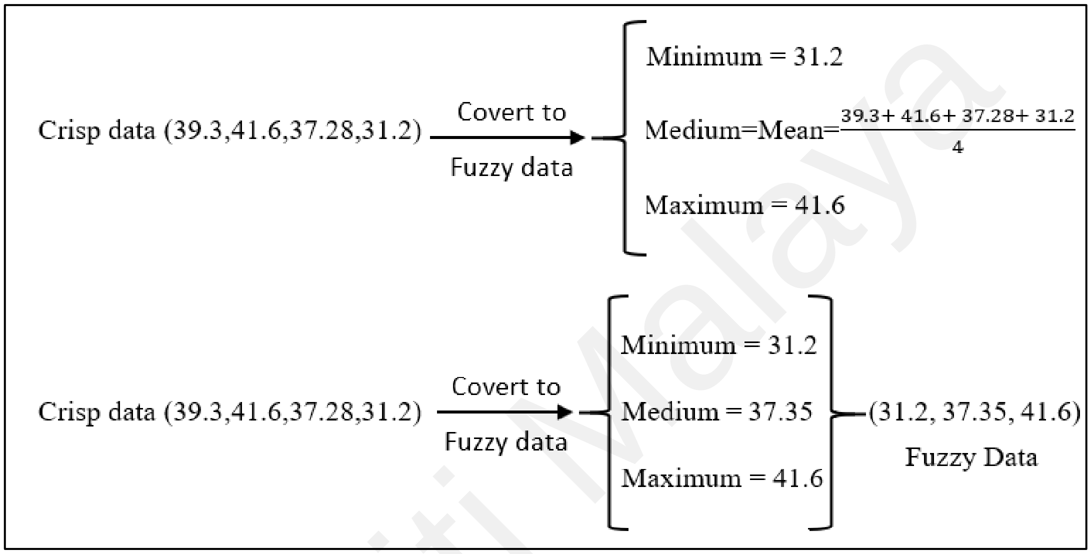


Figure 5.2: Fuzzy data converter

By applying all output data (for the three variables) in the table, we get Table 5.7 fuzzy triangular numbers (TFNs) data as follow:

Table 5.7: Fuzzify: Convert data to TFNs

DMU (University)	Input			Output (Fuzzy)		
	No of FTE Staff	No of FTE Students	International Students %	Teaching Reputation % (y_{1L}, y_{1M}, y_{1U})	Research Reputation % (y_{2L}, y_{2M}, y_{2U})	Citations % (y_{3L}, y_{3M}, y_{3U})
DMU1(UM)	**(1903, 1903, 1903)**	**(15794, 15794, 15794)**	**(20, 20, 20)**	**(31.2, 37.35, 41.6)**	**(26.6, 28.93, 31.5)**	**(54.4, 57.53, 60)**
DMU2(USM)	**(1967, 1967, 1967)**	**(21039, 21039, 21039)**	**(14, 14, 14)**	**(32.2, 34.53, 35.6)**	**(17.7, 21.53, 23.3)**	**(15, 23.18 , 32.2)**
DMU3(UKM)	**(1709, 1709, 1709)**	**(17601, 17601, 17601)**	**(16, 16, 16)**	**(30.5, 33.58, 35.3)**	**(19.6, 20.95, 21.4)**	**(11, 26.05, 42.5)**
DMU4(UPM)	**(1648, 1648, 1648)**	**(19937, 19937, 19937)**	**(25, 25, 25)**	**(26.6, 29.7, 33.3)**	**(18.5, 25.8 , 31.4)**	**(17.2, 19.7, 24.4)**
DMU5(UTM)	**(1694, 1694, 1694)**	**(19988, 19988, 19988)**	**(17, 17, 17)**	**(30.4, 33.45, 36.4)**	**(20.4, 23.58, 25.2)**	**(22.7, 28.5, 38.8)**

Note that, the input data put in the TFNs form (L, M, U) and

all are the same values L= M= U.

5.3.4 Fuzzifying Data of PRUM Case

In this section the fuzzy data the study case PRUM will be listed in the form of TFNs, by splitting Table 5.7 based on three projections - lower, medium, and upper bound, the outcomes as in 3 tables as follow:

Table 5.8: Lower Bound Fuzzy data

DMU	Input			Output		
	No of FTE Staff	No of FTE Students	International Students %	Teaching Reputation % y_{1L}	Research Reputation % y_{2L}	Citations % y_{3L}
DMU1(UM)	**1903**	**15794**	**20**	**31.2**	**26.6**	**54.4**
DMU2(USM)	**1967**	**21039**	**14**	**32.2**	**17.7**	**15.0**
DMU3(UKM)	**1709**	**17601**	**16**	**30.5**	**19.6**	**11.0**
DMU4(UPM)	**1648**	**19937**	**25**	**26.6**	**18.5**	**17.2**
DMU5(UTM)	**1694**	**19988**	**17**	**30.4**	**20.4**	**22.7**

Table 5.8 shows the lower bound of fuzzy data which is the first projection of the three coordinates for outputs in Table 5.7.

Table 5.9: Medium Level Fuzzy data

DMU (University)	Input			Output		
	No of FTE Staff	No of FTE Students	International Students %	Teaching Reputation % y_{1M}	Research Reputation % y_{2M}	Citations % y_{3M}
DMU1(UM)	**1903**	**15794**	**20**	**37.3**	**28.9**	**57.5**
DMU2(USM)	**1967**	**21039**	**14**	**34.5**	**21.5**	**23.2**
DMU3(UKM)	**1709**	**17601**	**16**	**33.6**	**21.0**	**26.1**
DMU4(UPM)	**1648**	**19937**	**25**	**29.7**	**25.8**	**19.7**
DMU5(UTM)	**1694**	**19988**	**17**	**33.5**	**23.6**	**28.5**

Table 5.9 shows the medium level of fuzzy data which is the second projection of the three coordinates for outputs in Table 5.7, and the last table is as follows.

Table 5.10: Upper Bound Fuzzy data

DMU (University)	Input			Output		
	No of FTE Staff	No of FTE Students	International Students %	Teaching Reputation % y_{1U}	Research Reputation % y_{2U}	Citations % y_{3U}
DMU1(UM)	**1903**	**15794**	**20**	**41.6**	**31.5**	**60.0**
DMU2(USM)	**1967**	**21039**	**14**	**35.6**	**23.3**	**32.2**
DMU3(UKM)	**1709**	**17601**	**16**	**35.3**	**21.4**	**42.5**
DMU4(UPM)	**1648**	**19937**	**25**	**33.3**	**31.4**	**24.4**
DMU5(UTM)	**1694**	**19988**	**17**	**36.4**	**25.2**	**38.8**

Table 5.10 shows the upper bound of fuzzy data which is the third projection of the three coordinates for outputs in Table 5.7.

The next sections demonstrate the application of FDEA model based on data in Table 5.8, Table 5.9, and Table 5.10. The fuzzy technical efficiency scores are computed and the last steps of dis-fuzzifying to estimate the technical efficiency score of the academic year (2021/2022) are also included.

5.4 Expanding FDEA Model for PRUM Case

As explained before in Chapter 2 section (2.4.5), the technical efficiency of a DMU is defined as the ratio of sum output weight respect to sum input weight, and this ratio must be between one and zero. Let *p*-th DMU (DMU_p) be under consideration, then the CCR model for the relative efficiency is as follows (Charnes et. al., 1978):

$$\theta_p{}^* = \max \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}}$$

$$s.t.$$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad \forall j$$

$$u_r, v_i \geq 0 \qquad \forall r, i. \tag{5.2}$$

In Model 5.2, each DMU (consider that PRUM case has n = 5 DMUs) uses m =3 (PRUM case) inputs x_{ij} (i = 1, 2, . . . , m), to obtain s =3 (PRUM case) outputs y_{rj} (r = 1, 2, . . . , s). Here u_r (r = 1, 2, . . . , s) and v_i (i = 1, 2, . . . , m) are the weights of the i_{th} input and rth output. This fractional program is calculated for every DMU to find out its best input and output weights. Model 5.2 is nonlinear program and to simplify the calculations, this model 5.2 should convert to be a linear program (LP) and this model is called the L.P- CCR model, where $\theta_p{}^*$, the objective function for model 5.2 and 5.3, will be defined for PRUM case by introducing FDEA in the model (5.4).

$$\theta_p{}^* = \max \sum_{r=1}^{s} u_r y_{rp}$$

$$s.t:$$

$$\sum_{i=1}^{m} v_i x_{ip} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad \forall j$$

$$u_r, v_i \geq 0 \qquad \forall r, i. \tag{5.3}$$

Both models 5.2 and 5.3 are objective function as optimization problem, such that maximize the ratio of weighted output to weighted input. The constraints say that weights of each DMU must not provide an efficiency score larger than 1 to any other DMU which benchmarking DMU method. The most efficient score (the full score or the optimal objective value) is equal to 1.

5.4.1 FDEA Model definition

Before expanding Model 5.3 and define FDEA model, next definition should be considered:

Definition 10: (CCR efficiency definition) according to Mansourirad et al., 2010.

1) DMU_p is CCR-efficient if $\theta_p^*=1$ and there exist at least one optimal u^*, v^* with $u^* > 0$, $v^* > 0$.
2) Otherwise, DMU_p is CCR-inefficient.

In the previous sections Model 5.3 run the 10-times for each academic year by providing data to work out the technical efficiency of 10 DMUs in each year, also if $\theta_{p^*}=1$, then DMU_p is efficient, otherwise it is inefficient, all discussed in chapter 4, and next part with the same definition (definition 1) technical efficiency score will be estimated for academic year 2021/2022, by developing model 5.3 to be fuzzy DEA model for PRUM case as following:

$$\begin{aligned} \theta_p{}^* = \quad & \max \sum_{r=1}^{s} u_r \tilde{y}_{rp} \\ s.t: & \\ & \sum_{i=1}^{m} v_i \tilde{x}_{ip} = 1 \\ & \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \leq 0, \qquad \forall j \\ & u_r, v_i \geq 0 \qquad \forall r, i. \end{aligned} \tag{5.4}$$

Where, $\tilde{x}_{ij}$ (i = 1, 2, 3) are inputs not fuzzy, but put in fuzzy form, as in table 5.2 and $\tilde{y}_{rj}$ (r = 1, 2, 3) are fuzzy outputs for the jth DMU (DMU$_j$), other parameters are the same definition as previous models.

According to Ali, et al. (2018) this fuzzy CCR Model 4.1 (FDEA) is a strong method for evaluating the efficiency of DMUs with imprecise information.

By expanding the Charnes and Cooper (1994) transformation Model 5.4, by using TFNs triangular fuzzy definition, the form $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ could be defined as following, such that $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$ & $\tilde{y}_{ij} = (y_{rj}^L, y_{rj}^M, y_{rj}^U)$ and Model 5.5 can be rewritten to be as the following:

$$\theta_p = \max \textstyle\sum_{r=1}^{s} u_r \otimes (y_{rp}^L, y_{rp}^M, y_{rp}^U)$$

$$s.t.$$

$$\textstyle\sum_{i=1}^{m} v_i \otimes (x_{ip}^L, x_{ip}^M, x_{ip}^U) = (1,1,1)$$

$$\textstyle\sum_{r=1}^{s} u_r \otimes (y_{rj}^L, y_{rj}^M, y_{rj}^U) - \sum_{i=1}^{m} v_i \otimes (x_{ij}^L, x_{ij}^M, x_{ij}^U) \leq (0,0,0) \qquad \forall j$$

$$u_r, v_j \geq 0 \qquad \forall r, i.$$

(5.5)

Where, $\tilde{x}_{ij}$ $(i = 1, 2, \ldots, m)$ and for PRUM case $m = 3$ also, $\tilde{y}_{rj}$ $(r = 1, 2, \ldots, s)$ and for PRUM case $s = 3$ inputs are fuzzy formed only and real fuzzy outputs for the jth DMU (DMU_j). Model 5.5 Fuzzy DEA Model developed to be applied in solver-365 to estimate the technical efficiency for next academic year 2021/2022.

5.5 FDEA Empirical Results for PRUM case

By utilizing the proposed model in section (Model 5.5) and Fuzzy data (TFNs) from tables in section 5.3.4 (Table 5.8, Table 5.9, and Table 5.10), the results from the proposed FDEA model are presented below.

5.5.1 FDEA Efficiency Score of Lower Bound Findings

By applying Model 5.5 for data in Table 4.8 for the lower bound of triangular fuzzy data, the results are as in Table 4.11.

Table 5.11: Lower Bound of (TFNs) Efficiency Score

Lower Bound (TFNs) Efficiency Results		
DMU	**Efficiency score**	**Efficiency Status**
DMU1	100%	Efficient
DMU2	100%	Efficient
DMU3	100%	Efficient
DMU4	100%	Efficient
DMU5	100%	Efficient

Table 5.11 shows that the estimated efficiency score and status for lower bound of TFNs that applied from Table 4.8 where all DMUs are 100% i.e., θ_L^* =1 or from Definition 10 (DMU_L) is CCR-efficient where P = Lower Bound Path.

5.5.2 FDEA Efficiency Score of Medium Level Findings

Again, using Model 5.5 for data in Table 4.9 for medium level of triangular fuzzy data, the next table is generated.

Table 5.12: Medium Level of (TFNs) Efficiency Score

Medium Level (TFNs) Efficiency Results		
DMU	**Efficiency score**	**Efficiency Status**
DMU1	100%	Efficient
DMU2	100%	Efficient
DMU3	100%	Efficient
DMU4	100%	Efficient
DMU5	100%	Efficient

Table 5.12 is exactly, the same as Table 5.11, all estimation score and status of technical efficiency are fully 100 %, this looks good indicator that all DMUs are still fully efficiency at medium level of TFNs data followed by dis-fuzzing after getting all fuzzy TFNs scores. The results of Upper Bound TFNs are shown next.

5.5.3 FDEA Efficiency Score of Upper Bound Findings

Table 5.13: Upper Bound of (TFNs) Efficiency Score

Upper Bound (TFNs) Efficiency		
DMU	**Efficiency score**	**Efficiency Status**
DMU1	100%	Efficient
DMU2	100%	Efficient
DMU3	100%	Efficient
DMU4	100%	Efficient
DMU5	100%	Efficient

Based on Model 5.5, Table 5.10 includes the upper bound of triangular fuzzy data and the next table (Table 5.13) shows that all DMUs are fully efficient, which confirm the results in previous two tables (Table 5.11, Table 5.12), indicating the final computation to estimate the technical efficiency for next academic year 2021/2022 will be accurate. The following section discusses the dis-fuzzy results.

5.6 Dis-Fuzzifying FDEA Empirical Results

From the results in the earlier sections the estimated efficiency scores for year 2021/2022 for 5 DMUs are computed and shown below. Thus, by Dis-fuzzifying the results are in Table 5.14.

Table 5.14: Fuzzy Efficiency Score of (TFNs) Data

(L, M, U) Fuzzy Efficiency Score of (TFNs) Data	
DMU	**Efficiency score (L, M, U)**
DMU1	(100%, 100%, 100%)
DMU2	(100%, 100%, 100%)
DMU3	(100%, 100%, 100%)
DMU4	(100%, 100%, 100%)
DMU5	(100%, 100%, 100%)

The concept of Central Tendency measurements is employed to dis-fuzzify the fuzzy empirical results in this study. A measure of central tendency is one value that tries to describe a set of data by identifying the central position within that set of data. Measures of central tendency or also known as measures of central location consider mean, median and mode as the most popular averages to represent the entire mass of data. The most used one is mean or also known as arithmetic average (Gravetter & Wallnau, 2010).

At this juncture, it is important to highlight that in the case of PRUMs of this study, however, the best method is mode because the interest of describing DMUs is in a discrete categorical manner – either efficient or inefficient, where only the score of 100% fully efficient is considered as efficient. Therefore, mode is preferred in this situation. The final estimation of efficiency scores and efficiency status of 2021/2022 Academic year is presented in Table 5.15 as follows.

Table 5.15: Estimated Efficiency Score for Academic year 2021/2022

Estimated Efficiency Score for Academic year 2021/2022		
DMU	**Efficiency score**	**Efficiency status**
DMU1	100	Efficient
DMU2	100	Efficient
DMU3	100	Efficient
DMU4	100	Efficient
DMU5	100	Efficient

From Table 5.15, the final estimation of efficiency score and status for the academic year of 2021/2022 based on the data collected from the 4 consecutive academic years for each DMU shows that all public research universities in Malaysia are fully efficient.

Compared with the empirical results in Table 5.14, DMU1, DMU3 and DMU5 are fully efficient during all four consecutive academic years, therefore, these DMUs are also expected to be fully efficient in the next academic year (2021/2022) and Table 5.14 confirms the study results. While Table 5.5 proves that DMU2 is only inefficient (93%) in 2019/2020 and is fully efficient in the other academic years, while Table 5.15 depicts the forecasts for next year academic year (2021/2022) for DMU2 to be fully efficient.

Finally, DMU4 demonstrates inefficiency in first two academic years (2017/2018) and (2018/2019) referring to Table 5.5 also tells that DMU4 inefficient, while in the latest 2 academic years (2019/2020) and (2020/2021) both DMU4 has full efficiency score of 100% so it is very reasonable for it to stay fully efficient in next academic year (2021/2022), so that the expectation results for DMU 4 is acceptable.

5.7 Determining Fuzzy Numbers for PRUM and sAPRU Case

In the next table, Table 5.16 outlines the crisp data collected from PRUMs and sAPRUs. Assuming the three output variables are fuzzy, Table 5.16 presents the crisp output data are generated into the triangular fuzzy numbers (TFNs) by using R-soft Mean method.

Table 5.16 Data collected from DMUs latest 4 years (Crisp Data)

DMU	Input			Output (Crisp)		
	No of FTE Staff	No of FTE Students	International Students %	Teaching Reputation %	Research Reputation %	Citations %
DMU1	1903	15794	20	(39.3, 41.6, 37.28, 31.2)	(30.5, 31.5, 27.1,26.6)	(60, 56.6, 59.1, 54.4)
DMU2	1967	21039	14	(34.7, 35.6, 35.6, 32.2)	(23.3, 22.7, 22.4,17.7)	(32.2, 26.7, 18.8, 15)
DMU3	1709	17601	16	(35.3, 34.2, 34.3, 30.5)	(21.4, 19.6, 21.4, 21.4)	(42.5, 32.3,18.4, 11)
DMU4	1648	19937	25	(33.3, 32, 26.9, 26.6)	(29.7, 31.4, 18.5, 23.7)	(24.4, 19.1, 18.1, 17.2)
DMU5	1694	19988	17	(30.4, 36.4, 35.9, 31.1)	(24.3, 25.2, 24.4, 20.4)	(38.8, 29.2, 23.3, 22.7)
DMU6	996	18135	43	(69, 72.6, 69.5, 67.5)	(78, 78.4, 77.2, 73.3)	(74, 73.7, 76.6, 80.3)
DMU7	462	9976	30	(55, 57, 57.4, 52.1)	(68, 68, 66.1, 63)	(93, 94, 89.8, 88.9)
DMU8	2434	22935	11	(71.8, 76, 73.7,77.9)	(78.6, 78, 78.1, 79.9)	(50.9, 55, 59.9, 60.8)
DMU9	1772	26757	11	(69.3, 75, 72.3, 72.4)	(71, 71, 71.6, 73.8)	(61, 64, 66.5, 68.8)
DMU10	2910	32597	13	(60, 64, 61.9, 59.9)	(58, 57, 58.6, 65.6)	(65, 69, 68.1, 73.3)

As an example, from Table 5.16, DMU1's Teaching Reputation %, crisp data is (39.3, 41.6, 37, 31.2) by using TFNs definitions in section **5.2.2** and R-soft Mean method in section **5.2.3** this crisp data value will be (31.2, 37.28, 41.6) TFNs such that:

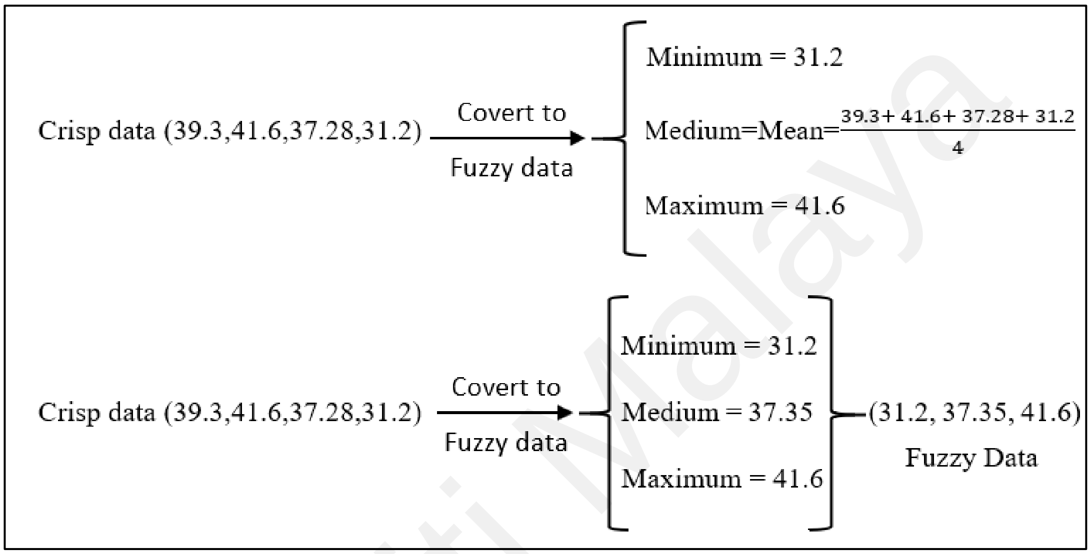


Figure 5.2 Fuzzy data converter

By applying all output data (for the three variables) in the table, we get Table 5.17 fuzzy triangular numbers (TFNs) data as next table. Note that, in this table the input data put in the TFNs form (L, M, U) and all are the same values L= M= U.

Table 5.17: Fuzzifying: Convert data to TFNs

DMU	Input			Output (Fuzzy)		
	No of FTE Staff	No of FTE Students	International Students %	Teaching Reputation % (y_{1L}, y_{1M}, y_{1U})	Research Reputation % (y_{2L}, y_{2M}, y_{2U})	Citations % (y_{3L}, y_{3M}, y_{3U})
DMU1	(1903, 1903, 1903)	(15794, 15794, 15794)	(20, 20, 20)	(31.2, 37.35, 41.6)	(26.6, 28.93, 31.5)	(54.4, 57.53, 60)
DMU2	(1967, 1967, 1967)	(21039, 21039, 21039)	(14, 14, 14)	(32.2, 34.53, 35.6)	(17.7, 21.53, 23.3)	(15, 23.18 , 32.2)
DMU3	(1709, 1709, 1709)	(17601, 17601, 17601)	(16, 16, 16)	(30.5, 33.58, 35.3)	(19.6, 20.95, 21.4)	(11, 26.05, 42.5)
DMU4	(1648, 1648, 1648)	(19937, 19937, 19937)	(25, 25, 25)	(26.6, 29.7, 33.3)	(18.5, 25.8 , 31.4)	(17.2, 19.7, 24.4)
DMU5	(1694, 1694, 1694)	(19988, 19988, 19988)	(17, 17, 17)	(30.4, 33.45, 36.4)	(20.4, 23.58, 25.2)	(22.7, 28.5, 38.8)
DMU6	(996, 996, 996)	(18135, 18135, 18135)	(43, 43, 43)	(67.5, 69.65, 72.6)	(73.3, 76.73, 78.4)	(73.7, 76.15, 80.3)
DMU7	(462, 462, 462)	(9976, 9976, 9976)	(30, 30, 30)	(52.1, 55.38, 57.4)	(63, 66.28, 68)	(88.9, 91.43, 94)
DMU8	(2434, 2434, 2434)	(22935, 22935, 22935)	(11, 11, 11)	(71.8, 74.85, 77.9)	(78, 78.65, 79.9)	(50.9, 56.65, 60.8)
DMU9	(1772, 1772, 1772)	(26757, 26757, 26757)	(11, 11, 11)	(69.3, 72.25, 75)	(71, 71.85, 73.8)	(61, 65.08, 68.8)
DMU10	(2910, 2910, 2910)	(32597, 32597, 32597)	(13, 13, 13)	(59.9, 61.45, 64)	(57, 59.8, 65.6)	(65, 68.85, 73.3)

5.7.1 Fuzzifying Data of PRUM and sAPRU Case

In this section the fuzzy data the study case PRUM and sAPRU will be listed in the form of TFNs, by splitting Table 5.17 the outcomes will be 3 tables as follow:

Table 5.18: Lower Bound Fuzzy data

DMU	Input			Output		
	No of FTE Staff	No of FTE Students	International Students %	Teaching Reputation %	Research Reputation %	Citations % y_{3L}
DMU1	**1903**	**15794**	**20**	**31.2**	**26.6**	**54.4**
DMU2	**1967**	**21039**	**14**	**32.2**	**17.7**	**15.0**
DMU3	**1709**	**17601**	**16**	**30.5**	**19.6**	**11.0**
DMU4	**1648**	**19937**	**25**	**26.6**	**18.5**	**17.2**
DMU5	**1694**	**19988**	**17**	**30.4**	**20.4**	**22.7**
DMU6	**996**	**18135**	**43**	**67.5**	**73.3**	**73.7**
DMU7	**462**	**9976**	**30**	**52.1**	**63.0**	**88.9**
DMU8	**2434**	**22935**	**11**	**71.8**	**78.0**	**50.9**
DMU9	**1772**	**26757**	**11**	**69.3**	**71.0**	**61.0**
DMU10	**2910**	**32597**	**13**	**59.9**	**57.0**	**65.0**

Table 5.18 shows the lower bound of fuzzy data which is the first projection of the three coordinates for outputs in Table 5.17.

Table 5.19: Medium Level Fuzzy data

DMU	Input			Output		
	No of FTE Staff	No of FTE Students	International Students %	Teaching Reputation %	Research Reputation %	Citations % y_{3M}
DMU1	**1903**	**15794**	**20**	**37.3**	**28.9**	**57.5**
DMU2	**1967**	**21039**	**14**	**34.5**	**21.5**	**23.2**
DMU3	**1709**	**17601**	**16**	**33.6**	**21.0**	**26.1**
DMU4	**1648**	**19937**	**25**	**29.7**	**25.8**	**19.7**
DMU5	**1694**	**19988**	**17**	**33.5**	**23.6**	**28.5**
DMU6	**996**	**18135**	**43**	**69.69**	**76.73**	**76.15**
DMU7	**462**	**9976**	**30**	**55.38**	**66.28**	**91.43**
DMU8	**2434**	**22935**	**11**	**74.85**	**78.65**	**56.65**
DMU9	**1772**	**26757**	**11**	**72.25**	**71.85**	**65.08**
DMU10	**2910**	**32597**	**13**	**61.45**	**59.80**	**68.85**

Table 5.19 shows the medium level of fuzzy data which is the second projection of the three coordinates for outputs in Table 5.17 Then the last table.

Table 5.20: Upper Bound Fuzzy data

DMU	Input			Output		
	No of FTE Staff	No of FTE Students	International Students %	Teaching Reputation %	Research Reputation %	Citations % y_{3U}
DMU1	**1903**	**15794**	**20**	**41.6**	**31.5**	**60.0**
DMU2	**1967**	**21039**	**14**	**35.6**	**23.3**	**32.2**
DMU3	**1709**	**17601**	**16**	**35.3**	**21.4**	**42.5**
DMU4	**1648**	**19937**	**25**	**33.3**	**31.4**	**24.4**
DMU5	**1694**	**19988**	**17**	**36.4**	**25.2**	**38.8**
DMU6	**996**	**18135**	**43**	**72.60**	**78.40**	**80.30**
DMU7	**462**	**9976**	**30**	**57.40**	**68.00**	**94.00**
DMU8	**2434**	**22935**	**11**	**77.90**	**79.90**	**60.80**
DMU9	**1772**	**26757**	**11**	**75.00**	**73.80**	**68.80**
DMU10	**2910**	**32597**	**13**	**64.00**	**65.60**	**73.30**

Table 5.20 shows the upper bound of fuzzy data which is the third projection of the three coordinates for outputs in Table 5.17.

The following sections present the application of the FDEA model for these data in tables (Table 5.18, Table 5.19, and Table 5.20) and the results showing the fuzzy technical efficiency scores. The last step is dis-fuzzifying process to estimate the technical efficiency scores for the academic year (2021/2022).

5.7.2 Expanding FDEA Model for PRUM and sAPRU Case

Similarly, as section (5.4.4), the technical efficiency of a DMU is defined as the ratio of sum output weight respect to sum input weight, and this ratio must be between one and zero. Let *p*-th DMU (DMU_p) be under consideration, then the CCR model for the relative efficiency is as follows (Charnes, et. al 1978):

$$\theta_p{}^* = \max \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}}$$

$$s.t.$$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad \forall j$$

$$u_r, v_i \geq 0 \qquad \forall r, i. \tag{5.2}$$

In Model 5.2, each DMU (consider that PRUM and sAPRU) case has n = 10 DMUs) uses m =3 (PRUM and sAPRU case) inputs x_{ij} (i = 1, 2, . . ., m), to obtain s =3 (PRUM case and sAPRU) outputs y_{rj} (r = 1, 2, . . ., s). Here u_r (r = 1, 2, . . ., s) and v_i (i = 1, 2, . . ., m) are the weights of the i_{th} input and *rth* output.

This fractional program is calculated for every DMU to find out its best input and output weights. Model 5.2 is nonlinear program and to simplify the calculations, this Model 5.2 should convert to a linear program as introduced in Model 5.3 which is a linear program (LP) and this model is called the L.P- CCR model, where $\theta_p{}^*$, the objective function for Model 5.2 and Model 5.3, will be defined for PRUM and sAPRU case by introducing the FDEA, next.

$$\theta_p{}^* = \max \sum_{r=1}^{s} u_r y_{rp}$$

$$s.t:$$

$$\sum_{i=1}^{m} v_i x_{ip} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad \forall j$$

$$u_r, v_i \geq 0 \qquad \forall r, i. \tag{5.3}$$

Both models 5.2 and 5.3 present the objective function as optimization problem, such that maximizes the ratio of weighted output to weighted input. The constraints outline that the weights of each DMU must not provide an efficiency score larger than 1 to any other DMU which benchmarking DMU method. The highest efficiency score (the full score or the optimal objective value) is equal to 1.

5.8 FDEA Empirical Results for PRUM-sAPRU case

As introduced in chapter 3, the CRS assumption is applied in evaluating the DMUs in this study where the same assumption of CRS, but DEA/FDEA model requires constant returns to the scale approach are called CCR model. By utilizing the proposed Model 5.6 and plugging in the Fuzzy data (TFNs) from tables in section 5.2.4.1 (Table 5.18, Table 5.19, and Table 5.20), the results from the proposed FDEA model for PRUM-sAPRU case are presented below. The FDEA efficiency scores are presented separately for lower, medium, and upper bounds.

5.8.1 Lower bound of the PRUM-sAPRU FDEA Efficiency Scores

By applying Model 5.6 (by using lower bound notation only) on data in Table 5.18 for a lower bound of triangular fuzzy data, the results are summarized in Table 5.21.

Table 5.21: Lower Bound of (TFNs) Efficiency Score (PRUM-sAPRU)

Lower Bound (TFNs) Efficiency Results		
DMU	**Efficiency score**	**Efficiency Status**
DMU1	100%	Efficient
DMU2	98%	Inefficient (Refer: DMU3, DMU8 & DMU9)
DMU3	100%	Efficient
DMU4	80%	Inefficient (Refer: DMU3, DMU7 & DMU9)
DMU5	95%	Inefficient (Refer: DMU3, DMU7 & DMU9)
DMU6	100%	Efficient
DMU7	100%	Efficient
DMU8	100%	Efficient
DMU9	100%	Efficient
DMU10	100%	Efficient

Table 5.21 shows the estimated efficiency scores and status for a lower bound of TFNs from Table 5.18. The results show that DMUs, DMU 2, DMU 4 and DMU 5 are inefficient but the other DMUs are fully efficient (100%) i.e., $\theta_L^* = 1$ for these all efficient DMUs or from Definition 10 (DMU_L) is CCR-efficient where P = Lower Bound path.

5.8.2 Medium bound of the PRUM-sAPRU FDEA Efficiency Scores

Again, by using Model 5.5 (by using medium notation only) on data in Table 5.19 for a medium level of triangular fuzzy data, the next table is produced (Table 5.22).

Table 5.22: Medium Level of (TFNs) Efficiency Score (PRUM-sAPRU)

Medium (TFNs) Efficiency Results		
DMU	**Efficiency score**	**Efficiency Status**
DMU1	100%	Efficient
DMU2	98%	Inefficient (Refer: DMU3, DMU8 & DMU9)
DMU3	100%	Efficient
DMU4	80%	Inefficient (Refer: DMU3, DMU7 & DMU9)
DMU5	95%	Inefficient (Refer: DMU3, DMU7 & DMU9)
DMU6	100%	Efficient
DMU7	100%	Efficient
DMU8	100%	Efficient
DMU9	100%	Efficient
DMU10	100%	Efficient

Table 5.22 is almost exactly (after approximating to nearest unit %), the same as Table 5.21, all estimation scores and status of technical efficiency are fully 100 %. These show good indicators to all DMUs that all are still fully efficient in the medium bound of TFNs data.

5.8.3 FDEA Efficiency Score of upper Bound Findings (PRUM-sAPRU)

Earlier when using a Model 5.6 (by using upper bound notation only) for data in Table 5.20 for the upper bound of triangular fuzzy data, the efficiency scores on upper bound TFNs are shown in Table 5.23 below.

Table 5.23: Upper Bound of (TFNs) Efficiency Score (PRUM-sAPRU)

Upper Bound (TFNs) Efficiency		
DMU	**Efficiency score**	**Efficiency Status**
DMU1	99%	Inefficient (Refer: DMU3, DMU7 & DMU8)
DMU2	98%	Inefficient (Refer: DMU3, DMU7 & DMU8)
DMU3	100%	Efficient
DMU4	80%	Inefficient (Refer: DMU3, DMU7 & DMU9)
DMU5	95%	Inefficient (Refer: DMU3, DMU7 & DMU9)
DMU6	100%	Efficient
DMU7	100%	Efficient
DMU8	100%	Efficient
DMU9	100%	Efficient
DMU10	100%	Efficient

In the third and the last table (Table 5.23), are the same as Table 5.21 and Table 5.22 where all DMUs achieve the same efficiency scores, but only UM scored 99.4 % (Table 23) which make UM inefficient in this upper bound. On the other hand, the other scores confirm the results in the previous two tables, and this is helpful in the next dis-fuzzifying stage presented as follow.

The final results estimate the technical efficiency for the next academic year 2021/2022 by dis-fuzzying the results shown in Table 5.21, Table 5.22 and Table 5.23 next.

5.9 Dis- Fuzzifying FDEA Empirical Results (PRUM-sAPRU)

From the earlier sections, the estimated efficiency scores for year 2021/2022 for 10 DMUs are Dis Fuzzified and the results are shown below in Table 5.24.

Table 5.24: Fuzzy Efficiency Score of (TFNs) Data (PRUM-sAPRU)

(L, M, U) Fuzzy Efficiency Score of (TFNs) Data		
DMU	**Efficiency score (L, M, U)**	**FDEA Score (Average)**
DMU1	(100%, 100%, 99%)	100%
DMU2	(98%, 98%, 98%)	98%
DMU3	(100%, 100%, 100%)	100%
DMU4	(80%, 80%, 80%)	80%
DMU5	(95%, 95%, 95%)	95%
DMU6	(100%, 100%, 100%)	100%
DMU7	(100%, 100%, 100%)	100%
DMU8	(100%, 100%, 100%)	100%
DMU9	(100%, 100%, 100%)	100%
DMU10	(100%, 100%, 100%)	100%

To dis-fuzzify the fuzzy empirical results in this study or in many other fuzzy DEA studies, the measures of central tendency are utilized. In this case (PRUM-sAPRU), the best method is by employing mode as explained earlier in section 5.6.

Table 5.25: PRUM-sAPRU estimated efficiency scores for Academic year 2021/2022

Estimated Efficiency Score for Academic year 2021/2022		
DMU	**Efficiency score**	**Efficiency status**
DMU1	100%	Efficient
DMU2	98%	***Inefficient***
DMU3	100%	Efficient
DMU4	80%	***Inefficient***
DMU5	95%	***Inefficient***
DMU6	100%	Efficient
DMU7	100%	Efficient
DMU8	100%	Efficient
DMU9	100%	Efficient
DMU10	100%	Efficient

Therefore, when benchmarked against sAPRUs, the final estimation of efficiency scores and efficiency status are shown as in Table 5.25, and Table 5.25 also, lists the final estimation of efficiency scores and the status for the next academic year 2021/2022 based on data collected from the four preceding academic years for each DMU. Based on the results, it could be said that only 2 PRUMs would be fully efficient in the 2021/2022 academic year which are DMU 1 and DMU 3 while the 3 other DMUs are not fully efficiency. DMU 2 is expected to be 98% very close to being fully efficient, DMU 5 with 95 % efficiency score but this is still not fully efficient. Whereas DMU 4 scores the least efficient with 80%. But obviously all sAPRU are likely to be fully efficiency with 100% Efficiency scores for the coming 2021/2022 academic year.

Compared with the empirical results in Table 5.24 and 5.25, DMU1 has Fuzzy efficiency score = (100%, 100%, 99%) so that the scores will be 100 % after de-fuzzifying. Furthermore, the previous efficiency scores of the four academic years 2017/2018, 2018/2019, 2019/2020 and 2021/2022 are all fully efficient, therefore, it is very likely for it to achieve full efficiency, too, in the following academic year 2021/2022 and this supports the results of FDEA for DMU1 Malaya university.

Similarly, DMU3 is fully efficient for all 3 previous academic years, except for the 4th academic year (2020/2021), it decreases to 79%. This large reduction should alert the decisions makers of the DMU3 to procrastinate any potential related problems so that in the next academic year 2021/2022, it could achieve full efficiency score as predicted by this study. The result of fuzzy DEA also indicates the three DMUs: DMU 2, DMU 4, and DMU 5 are expected not to achieve full efficiency in the next academic year 2021/2022. This is because none of the DMUs are fully efficient for the previous 3 academic years.

5.10 Inputs/Outputs Orientation to Increase the Efficiency Score

In CRS model, the technical efficiency scores will be the same for an input or an output orientation (Rajasekar & Deo, 2014). But these values will be different if variable return to scale (VRS) is assumed. Coelli and Perelman (1999) note that, in many situations, the selection of model orientation only has modest effect on the technical efficiency scores calculated based on a VRS model.

In general, the input-oriented model focusses on operation and managerial issues whereas output-oriented model is more associated with planning and strategies (Cullinane, et al., 2005). Specifically, the model's orientation should be chosen in accordance with which variables (outputs or inputs) the decision maker has most control over. For example, the university human resources division (H.R.) may have more control over the teaching staff (input) than over the number of postgraduate students (input) that dependent upon the applications coming in which could have impact on the number of publications may in terms of higher citation percentage and research influence on which both are the output measures. In such a situation, an input orientation will be preferred (Coelli, & Perelman,1999).

Of the same, in the higher educational sector, the required amount of fund allocation may be planned and secured for each higher institution of interest. In this case, at higher level for example ministerial level, the decision maker may want to maximize the output (and therefore choose an output orientation). Otherwise, if the decision maker's mission is to produce a given level of output (e.g., a quota) with the minimum input, hence the decision maker will opt for an input orientation.

In the same context, if the decision maker is not facing any constraints and has full control and management of both input and output, then the model's orientation will depend on the institution (or university concerned).

Therefore, the most important question should be posed - Does the higher institution need to cut costs (input orientation) or does it want to maximize production (output orientation)? - and this question could only be answered based on the results revealed by the proposed model utilized to measure the PRUMs' efficiency scores.

5.10.1 Determining FDEA model orientation

Referring to the estimated efficiency of PRUMs for the academic year 2021/2022 in Table 25.5, the expected efficiency score of DMU 2 is

98%. To increase its efficiency to 100% for the next coming academic year, the decision makes must work in either one of the following approaches:

- Under the input-oriented model, the capacity to change the input of DMU2 (i.e., a reduction) by 2% (100 - 98) is calculated using the original inputs value of 3 variables (inputs). The 2% improvement can be recalculated according to the new values of inputs. From a practical point of view, the capacity to reduce input so as to increase efficiency scores by 2% means that the DMU2 should reduce all inputs by 2% in order for it to become efficient and achieve the 100% efficiency score.
- Under the output-oriented model, the capacity to increase output so as to increase the efficiency scores by 2% (100 - 98) is calculated using the value of efficiency scores (Table 6.2). From a practical point of view, the capacity to increase output by 2% means that the Register Office should increase all outputs by 2% in order to become efficient (Azad, et al., 2018).

Similarly, for DMU4, Table 5.25 shows that the efficiency score is estimated to be 80% and by using the same approach, the decision makers should decrease the inputs by 20% or increase the outputs by 20% in order to become efficient. As for DMU 5, the FDEA model results in Table 5.25 show that the estimated efficiency score of 95%, so the decision makers should decrease the inputs by 5% or increase the outputs by 5% in order to become fully efficient.

For future studies, the decision could be determined based on the quantities of each input variable (or output) for all inefficient DMUs, as to whether the inputs (or outputs) should be decreased or increased to make the DMUs (for example DMU 2, DMU 4 and DMU 5) to achieve full efficiency.

5.11 Comparing QS ranking with FDEA Efficiency Scores

Table 5.26 shows the QS World Ranking and its respective efficiency scores of 5 PRUMs and 5 APRUs gathered from QS World Ranking organization website for the years under the study.

Overall, the DEA efficiency scores of all DMUs support the QS World Rankings for all academic years 2018/2019 to 2020/2021. Obviously, DMU1 is in the top list of PRUMs, and its QS World Ranking is ranked top 100 throughout the consecutive 4 academic years. While DMU3 ranked second in PRUM list for 2017/2018, DMU4 takes over for the next three academic years (2018/2019 to 2020/2021). While all 5 APRU DMUs are ranked in the top 50 universities in QS World Rankings, during the 4 academic years of study. DMU6 maintains the first position for all periods under study and the other 4 sAPRUs change positions with one another. Again, the DEA results of this study where all 5 sAPRU scored fully efficiency (100%) supports the QS World Ranking for all academic years under study.

Table 5.26: Summary of QS-World Ranking and Overall Efficiency Scores of 10 DMUs for past four academic years.

2017/2018			2018/2019			2019/2020			2020/2021		
DMU	QS WR	OES (%)	DMU	QS WR	OES (%)	DMU	QS WR	OES (%)	DMU	QS WR	OES (%)
DMU6	25	84.3	DMU6	25	83.8	DMU6	22	83.7	DMU6	22	86.3
DMU8	35	81.2	DMU7	32	80.6	DMU7	27	82.1	DMU10	31	82.6
DMU9	36	80.6	DMU8	33	80.5	DMU10	34	79.9	DMU8	33	82.3
DMU7	37	80.5	DMU9	37	79.6	DMU9	37	79	DMU7	34	82.2
DMU10	44	77.6	DMU10	40	78.6	DMU8	38	78.9	DMU9	36	81.7
DMU1	87	62.6	DMU1	70	67.1	DMU1	59	70.1	DMU1	65	69.8
DMU3	184	45.5	DMU4	159	48.4	DMU4	132	52.7	DMU4	143	52.2
DMU4	202	43.8	DMU3	160	48.3	DMU3	141	52	DMU3	144	52
DMU2	207	43	DMU2	165	47.9	DMU2	142	51.9	DMU2	147	51
DMU5	228	40.5	DMU5	217	41.4	DMU5	187	44.2	DMU5	191	45.1

QR WS: QS World Ranking; OES (%): Overall Efficiency Scores (%)

The QS ranking score methodology appraises based on the weighted criteria on academic and employer reputations; faculty/student ratio; Scopus citations; international faculty ratio and international student ratio. Both methods apply the ‘hard data’ like FTE students and the percentage ratio of FTE international students to FTE students. However, the DEA approach of this study resembles the QS ranking by employing three QS metric research indicators namely Academic/Teaching reputations (%), research reputations (%) and research citations as the output indicator. This is deemed suitable because as research universities, the PRUM DMUs are heavily involved in research-related activities which are not only research work, but the output would also in terms of research publications, like journal articles which give rise to citation numbers, hence research reputation.

Table 5.27 below presents the correlation coefficient of QS World Ranking, input, and output variables of the DEA model in this study. The correlation analysis for PRUM and APRU research universities are made separately. A quick analysis on the input variables for both PRUM and

APRU universities and QS World Ranking indicate positive correlations between FTE students and world ranking. The research universities from Malaysia and Asian region also present significant inverse relationship between world ranking and international student ratio. Additionally, all output variables, are negatively correlated to world ranking for both PRUM and APRU. However, only APRUs show significant positive correlation between FTE Staff input variable to the world ranking.

Table 5.27: Correlation of coefficient of the input/output variables and the QS World Ranking

PRUMs	World Ranking	FTE Staff	FTE Student	Internl Stud. ratio	Teaching Reputation	Research Reputation
I/P Variables						
FTE Staff	-0.2260	1				
FTE Student	**0.456***	**0.5454***	1			
Internl Stud. Ratio	**-0.559****	0.0064	-0.2877	1		
O/P Variables						
Teaching Reputation	**-0.559****	-0.2442	**-0.7368****	0.0923	1	
Research Reputation	**-0.633****	-0.2224	-0.4416	**0.738****	**0.4664***	1
Research Influence	**-0.801****	0.1077	**-0.5632****	**0.4262^**	**0.5849****	**0.5483***

APRUs	World Ranking	FTE Staff	FTE Student	Internl Stud. ratio	Teaching Reputation	Research Reputation
I/P Variables						
FTE Staff	**0.5426***	1				
FTE Student	**0.4763***	**0.9096****	1			
Internl Stud. Ratio	**-0.804****	**-0.837****	**-0.6793****	1		
O/P Variables						
Teaching Reputation	-0.0825	**0.4862***	0.3905	-0.3134	1	
Research Reputation	**-0.4495***	-0.1283	-0.2951	0.1868	**0.7244****	1
Research Influence	-0.3577	**-0.833****	**-0.6832****	**0.6984****	**-0.7774****	-0.3333

Among the input and output variables, only APRU indicates significant inverse relationship between international student ratio and FTE students. While FTE Student are negatively correlated to Teaching Reputation and Research Influence within PRUMs, APRU only shows negative relationship between FTE Student and Research Influence.

5.12 Summary of the DEA Model Empirical Results

This chapter evaluates the technical efficiency of PRUMs for the last 4 consecutive academic years (2017/2018, 2018/2019,2019/2020 and 2020/2021). The results reveal two PRUMs, DMU2 and DMU4 do not achieve full efficiency in the average efficiency scores. Only DMU1 and DMU5 show full efficiency throughout all four academic years. Next, by applying the Triangular Fuzzy Number (TFN), this chapter further demonstrates the estimation of expected efficiency scores of the PRUMs for the next academic year 2021/2022 where all PRUMs seems to be able to achieve full efficiency based on the Upper, Medium, and Lower Bound scores. Next when the technical efficiency PRUMs are benchmarked against the selected group of public research universities in Asia (sAPRU) for the same 4 consecutive academic years (2017/2018, 2018/2019, 2019/2020 and 2020/2021), findings from the FDEA results indicates two PRUMs are not fully efficient which, again, are DMU2 and DMU4. Similarly, the estimated Fuzzy technical efficiency scores of PRUM shows the same results when benchmarked against the sAPRU for the academic year 2021/2022.

In general, the DEA efficiency scores of all DMUs support the QS World Rankings for all academic years 2018/2019 to 2020/2021 on which DMU1 is the top list of PRUMs and its QS World Ranking is ranked top 100 throughout the consecutive 4 academic years. The correlation analysis show two input variables of DEA model in this study, FTE Student and International Student Ratio are significantly correlated to the QS World Ranking. The next final chapter presents more discussions on the results and conclude with some contributions and future direction.

CHAPTER 6: CONCLUSION AND RECOMMENDATIONS

6.1 Summary of Research

This study aims to generate the technical efficiency of five public research universities in Malaysia (PRUM) by firstly establishing the Charnes, Cooper, and Rhodes - Constant-Return-to-Scale (CCR-CRS) Data Envelopment Analysis (DEA) model approach to estimate the relative efficiency of these universities. The technical efficiency of these PRUMs is deemed necessary because it can provide information on the performance of universities based on a set of input and output variables. With decreasing trends of government funds to finance the operational and research expenditures public universities, research universities alike, the government would only channel funds to the public HEIs at levels deemed necessary. At the same time, it is crucial for the PRUMs to increase their research quality and output so, as to increase their international ranking and reputations.

The CCR-CRS DEA model is launched based on three input variables, namely, total number of full-time academic staff, total number of full-time student and percentage of international students; and three output variables, namely, teaching reputation percentage, research reputation percentage and research influence which is the citations percentage for each university. While the secondary data of input variables are gathered from the Ministry of Higher Education Department and respective university websites, all the output variable data are gathered from the "World

University Ranking" website and "QS Top Universities" website for the last four academic years: 2017/2018, 2018/2019, 2019/2020 and 2020/2021. Past studies tend to agree that DEA itself does not provide guidance for the specification of the input and output variables; rather, based on users' own perspectives, discretion, judgment, and expertise to select variables which are more critical to success and contributing to the performance and efficiency in HEIs. In many DEA studies on HEIs, the number of staff (academic and non-academic), number of enrolments (undergraduates and postgraduates) are commonly used as input where the decision making has more control over it. Whereas, with consideration on assessing the research-related output and the international reputation of PRUMs, the relevant QR World Ranking set of indicators are employed as the output variables.

The selection of external output indicators raised the concern over its uncontrollable nature. For this, the concept of fuzzy logic is considered by introducing a fuzzy CCR DEA model with an algorithm established to measure the technical efficiency scores and efficiency status of PRUMs. With the output data considered as crisp data, the data are converted through Triangular Fuzzy Number (TFN) and the Fuzzy DEA efficiency scores of each PRUM is measured for the four consecutive academic years scores and from which the efficiency score for the next following academic year 2021/2022 for each PRUM are estimated.

Since there will be one or more DMUs (universities) set as the referent peer(s), this referent peer should perform at fully efficiency i.e., 100 %. Since there are only 5 public research universities in the whole Malaysia forming the PRUMs, DEA benchmarking solely on PRUMs could be of concern, especially when the PRUMs need to improve the performance in the international arena. So, this study also attempts to benchmark the PRUMs against a selected of Asian public research universities (APRU) which are assumed to be homogenous, in terms of the nature of the operations and the conditions under which they operate as public research universities.

6.2 Summary of Results, Findings and Discussion.

From Research Objective 1 (RO1), the technical efficiency of PRUMs for academic years of 2017/2018, 2018/2019,2019/2020 and 2020/2021 show that only three PRUMs – DMU1, DMU3 and DMU5 achieved full efficiency for each academic year and, also for the average efficiency scores throughout the period. DMU2 did not achieve full efficiency for academic year 2019/2020, while DMU4 was not efficient in 2017/2018 and 2018/2019 academic years. Only DMU1 and DMU5 show full efficiency throughout all the four academic years. Considering the uncontrollable nature of QS World Ranking indictors as the output variables, a fuzzy DEA model proposed by Lertworasirikul et al., (2003b) was adopted with an algorithm based on Triangular Fuzzy Numbers definition to convert crisp data to fuzzy data. measure the technical efficiency scores and efficiency status of PRUMs, under Research Objective

2 (RO2). In Research Objective 3 (RO3), the fuzzy DEA (FDEA) technical efficiency scores computed go through the last steps of dis-fuzzifying in estimating the expected technical efficiency scores for the next academic year of 2021/2022 under uncertainty of the three output variables. The final estimation for fuzzy DEA efficiency scores is made for the following academic year 2021/2022 based on the four preceding academic years, under Research Objective 4. The results indicate all PRUMs are fully efficient. Nonetheless, under Research 5 (RO5), by expanding the FDEA Model to include 5 selected research universities in Asia (APRUs) for benchmarking purposes, the results clearly show only two PRUMs, which are DMU1 and DMU3 are estimated to be fully efficient for academic year 2021/2022.

The Fuzzy DEA (FDEA) has integrated the concept of fuzzy set theory with the traditional DEA by representing imprecise and vague data with fuzzy sets. In this study, the FDEA models in the fuzzy linear programming models (Peykani et al., 2018), utilize the output variable data gathered from QS World Ranking Data which are beyond control of DMUs, i.e., the PRUM. Unlike the input variables – FTE Staff and FTE Students which generally fixed, the number of papers published by each PRUM in the changing academic years (Tavana et al., 2021a). These numbers are usually imprecise or keep on changing that PRUMs can hardly control but directly impacting the research output like research reputation and research influence. Where the fluctuating data are characterized by fuzzy numbers, this present study is in accord with the earliest study on fuzzy CCR model by Sengupta (1992). Fuzzy numbers have been widely used to obtain better results in problems where decision making and analysis are involved (Anand & Bharatraj, 2017). In the case of PRUMs as in this study, these findings can further assist in the financial resources to be allocated. This analysis would allow the decision markers to be aware about the required resources for each DMU and put more control on the budgets and financial allocation for the respective universities. Also, by identifying a set of benchmarks (referent peers), that achieved the full efficiency 100 % score, other DMUs can emulate their referent peers (DMUs) by analyzing its input/output and further improve its own practices (increase/decrease the resources/the outcomes) to increase the efficiency scores.

In addition, to check the consistency with the QS world ranking, the Fuzzy DEA results are scrutinized to identify the nature of returns to scale, the results can indicate if a firm should decrease or increase its scale (or size) in order to minimize the average cost. Even more importantly, the proposed technical efficiency DEA approach take into consideration of the DMUs performance indicators at international level as these are the output variables to the model. These variables are the key drivers of critical to success for the PRUMs (Gökşen, et al., 2015; Avkiran, 2005) as improving the international ranking and global reputations are the aspiration of the National Higher Education Strategic Plan 2007 – 2020 (Sheriff, N.M. &

Abdullah, N., 2017). Therefore, the proposed method can be adapted as the standard performance measures to be set against global standards.

The results and findings are discussed in further detail based on the respective contributions, next.

6.3 Methodological contributions

The DEA methodology of measuring the efficiency of universities has long been researched within higher education institutions like universities. This study compiles most of the previous works on the efficiency measurement of universities or any HEIs as explained in greater detail in chapter 3 and Table 3.1. This study offers several theoretical contributions. First, the study introduces a new approach to technical efficiency measurements for universities, by considering fuzzy variable data for the proposed CCR-DEA. Considerations on the ambiguity and uncontrollable nature of input/output data supports the proposed Fuzzy DEA efficiency measurement model because this benchmarking technique are sensitive to outliers (Guo et al., 2010) that input and output data must be accurately measured (Wen & Li, 2009) whereas nature of real-world data are sometimes imprecise, vague, and uncontrollable (Hatami-Marbini et al., 2010). This study corroborates Fuzzy DEA approach by Mahmudah & Lola (2016) in measuring the efficiency of universities. In addition, this new approach can estimate the technical efficiency not only for a specific academic year but also a series of data over a period time. By converting the input/output data to fuzzy data, this practical FDEA model based on CCR-DEA model (Model 5.7), is more applicable if the data available in a triangular fuzzy form (L, M, U). The decision maker (DM) can choose to select the critical to success data for the variables. Additionally, findings from research can provide useful information to the decision makers which variables exactly should be increased or decreased in order to obtain higher efficiency score.

Secondly, this study provides the R-soft code that be used in converting the crisp to fuzzy data together with the manual algorithms (Figure 5.3 Fuzzy data converter) for small crisp data, how it should be converted to be usable fuzzy data by using a Triangular Fuzzy Number theorem as section 2.7.2 and Figure 2.10.

Lastly, this study provides a full algorithm to convert the fuzzy results to be real results which is termed as dis-fuzzing process and introduced the end results of the estimated technical efficiency score for the next academic year 2021/2022, as shown in this study.

Overall, we can say that in this study the novelty is in using fuzzy DEA with new way fuzzy triangular method with adding R-soft code also, apply this approach to estimate the technical efficiency to any HEI, or any educational system.

6.3.1 Advantages of the research methodology

The Fuzzy Data Envelopment Analysis (DEA) method is a powerful tool for measuring and estimating the technical efficiency of Public Research Universities in Malaysia (PRUM) or any HEI compared to other universities in the region. Fuzzy DEA is advantageous over traditional DEA methods because it allows for the consideration of uncertain and imprecise input and output data. PRUM or any other HEI often face challenges in obtaining accurate and precise data due to the complexity of their operations and the subjective nature of some of their outputs. Fuzzy DEA allows for the incorporation of such imprecision and uncertainty into the analysis, resulting in a more accurate assessment of PRUM's technical efficiency. Fuzzy DEA also allows for the consideration of multiple input and output measures and the possibility of non-linear relationships between them. This feature makes it easier to identify inefficiencies and improve the allocation of resources to achieve optimal outcomes.

Another advantage of Fuzzy DEA is its ability to handle data with missing values or outliers. This is especially important for PRUM, as data collection can be challenging due to limited resources and time constraints.

6.4 Empirical contributions

The empirical contributions of this study can be summarized in three main points. Firstly, this study utilizes the variables that are mostly used in measuring the technical efficiency for higher educational institutions, especially the research higher educational institutions. In particular, DEA approach establishes the relationship between productivity and efficiency that DMUs can be placed in its suitable ranks (Coelli et al., 1998). In evaluating university efficiency, technical efficiency is deemed most suitable as it correlates how much total of output can be achieved from a set of total input. This brings to the second empirical contribution which is potential information for the decision makers on which variables exactly should be increased or decreased for a higher/better the efficiency scores, briefly highlighted earlier.

Secondly, by benchmarking to other research universities like those listed in APRUs, can give guidelines on the next move to the DMUs. Generally, the findings on the DEA efficiency scores support the QS World Rankings for all academic years 2018/2019 to 2020/2021 on which DMU1 is the top list of PRUMs. In the QS World Ranking, DMU1 is ranked in the top 100 throughout the consecutive 4 academic years. Based on the referent DMUs revealed from the efficiency results, the inefficient DMUs can emulate to increase their efficiency scores. Depending on the input/output model orientation, the capacity to change the input/output of a DMU is very much dependent upon the percentage each DMU lacked. For example, the capacity to reduce input so as to increase efficiency scores by 2% means that the DMU should reduce all inputs by 2% in order for it to become efficient and achieve the 100% efficiency score. The correlation analysis between the relevant variables to the world ranking could also assist which input variable to focus based on its significant correlation. For instance, the FTE Student and International Student Ratio of the referent DMUs are significantly correlated to the QS World Ranking. This could further improve their international ranking by emulating the referent DMU(s).

Consequently, the third empirical contribution is in estimating the technical efficiency for the subsequent academic year (2021/2022), based on the six performance indicators of QS world university ranking criteria (Table 1.1), the QS rankings are designed to assess universities in four areas: research, teaching, employability, and internationalization. Each of the six indicators could convey a diverse weighting when calculating the overall efficiency scores. Four of the indicators, namely, Faculty Student Ratio, Citations per faculty from Scopus, International faculty ratio and international student ratio are based on 'hard' data collected as facts with real evidence, and the remaining two Academic Reputation and Employer Reputation is based on major global surveys (TopUniversities, 2022).

Lastly, the technical efficiency scores for PRUM and APRU case for latest 4 academic years can be compared with other universities ranking; Times Higher Education for university ranking, UNISCO U-Multirank Project, RUR Rankings Agency, Webometrics Ranking of World Universities and The Center for World University Rankings (CWUR). Most of these ranking agencies have the same common indicators as QS Ranking agency but, in different weights and different arrangements and percentages. This benchmarking would further help in transforming the

Malaysian Higher Education where the performance of the universities can be assessed as underlined in the Malaysia Education Blueprint (2015-2025).

6.5 Practical implications

The practical implications of this study can be summarized in the following points:

1) The results of the FDEA analysis of the DMUs, such as the efficiency scores of HEIs, can shed light on the the potential areas for improvement particularly in the factor inputs.

2) The Fuzzy DEA method enable vague and uncontrollable nature of data to be considered in the model. The results to benchmark themselves against other institutions.

3) The results and findings could help to support and enhance the policy and decision-making in the sector, particularly on the policy of student intake, especially the international students. By taking into consideration the international ranking indicators, local PRUMs can emulate how the international research universities are operating with regards to the factor inputs. In turn, the results could also be as the guidelines on funding allocation, or how they could improve the overall quality of education provided by HEIs.

4) Finally, recommendations could be proposed on how each HEIs could use FDEA framework for other variables to be considered as variables to assess their efficiency, to suit future strategies for further improvement.

6.6 Managerial implications

The DEA model that used to measure the efficiency called, the control system model which can clearly describe the ability of the DMU to direct the available resources to guarantee achievement the DMU goals or its plans are working in the right way So that from this viewpoint, the understanding of the FDEA model and all its components (objective function, inputs, and outputs) can improve the management performance, and direct the goals of the organization under any new circumstances occur during the academic year (Epstein & Henderson, 1989). This research shares the most related aspect to the management and administration section for any HEI, like detecting the most related variables (input/output) to measure and estimate the technical efficiency and easily can be increased or decreased to raise the efficiency scores.

Also, the study introduces a practical FDEA by expanding model based on CCR-DEA, that can be written in a real visual form in Excel Microsoft, or any other DEA-software and plugs the data and easily can be used by administrators to run and get the efficiency scores for each DMUs. Also, this model can the decision makers can increase or decrease the number of variables. Finally, the research is assessing the long-term viability of DEA for control for example in the attempt to change the input/outputs quantity. This shows the structural properties of FDEA expanding model. It is also can be regarded as a critical managerial tool to test the resources that are chosen to 'plug in and run' in FDEA model. This is called the situational influences that interrelate to cooperatively determine the strengths and limitations of FDEA in a precise decision-making context (Epstein & Henderson, 1989).

6.7 FDEA model sensitivity analysis

The sensitivity analysis for the FDEA model is proposed through the following procedures:

1. Identify the efficient Decision-Making Units (DMUs) in PRUMs and sAPRU using the FDEA model.
2. Select a reference DMU or a target DMU for comparison, by using Win4Deap2 2.1 software.
3. Determine the input and output values of each reference DMU that selected from step 2.
4. Introduce small variations or perturbations to the input and output values of each reference DMU. These perturbations can be positive or negative, representing changes in the input-output space.
5. Recalculate the efficiency scores for the reference DMU and other DMUs using the modified input and output values.
6. Analyze the changes in the efficiency scores to assess the sensitivity of the FDEA model.
7. Repeat steps 4 to 6 for different perturbations to explore the sensitivity across various scenarios.

By this performing sensitivity analysis, it can be observed clearly how the efficiency scores and rankings of the DMUs change when there are slight variations in the input and output values of the reference DMU. This analysis helps in understanding the robustness and sensitivity of the FDEA model to changes in the input-output space. It is important to interpret the results of sensitivity analysis cautiously, by taking into account the magnitude and direction of the changes, as well as the specific context of the problem being analyzed. Sensitivity analysis provides valuable insights into the stability and reliability of the efficiency scores obtained from the FDEA model of this research can help to identify the most influential input and output variables affecting the efficiency of DMUs. It is so important to document and report the results of sensitivity analysis for this FDEA model in order to provide transparency and support the robustness of the findings.

6.8 Novelty of this research

Based on literature review chapter in this study, and up to our current knowledge, this study is among very few studies in HEIs that employ the Fuzzy approach to DEA technique to measure or estimate the technical efficiency of higher education institutions. Moreover, there has been dearth of studies proposing a method and an algorithm to determine the data with linguists (qualitative data) or unclear attribute data. From this viewpoint the novelty of this study is arising and opens a new gate to the researchers to look deeply with more concentration on researching on different types of efficiency of HEIs by employing the approaches introduced in this research.

6.9 Future direction

The findings and the outcomes of this research can be used as guidelines for more future studies on Fuzzy DEA. In general, the concept of "Fuzzy", fuzzy principles and fuzzy theorem can be applied widely in the DEA context within HEI efficiency studies. This application can be in different theoretical models, or the empirical context of technical efficiency measurements, or the other types of efficiency namely allocative efficiency, cost efficiency, price efficiency and others. This study also marks the future risk management studies for any educational initiation, especially in examining the data of a crisis period, like, COVID-19 or any other.

Finally, further studies should emphasize analyzing the efficiency scores of each DMU by emphasizing on the lambda weight of referent DMUs. For this the percentage of lambda could be further scrutinized by considering the managerial decision-making process. Furthermore, studies on risks associated with the use of the fuzzy technique, the sensitivity analysis and model robustness of Fuzzy DEA models can be incorporated for future studies. This would further increase the value of FDEA as the alternative to the conventional DEA models. Such research would increase the effectiveness of FDEA for any education system.

6.10 Limitation of this study

The most common limitation of studies in FDEA, is uncertainty in selecting the most accurate and suitable fuzzy variables in establishing the model. This study also concerns the alternative formulations of DEA model for higher education institutions that FDEA could address some of the risks and limitations identified from the FDEA model established from a set of selected variables only. Furthermore, output maximization orientation was utilized to analyze the model. Presupposing constant return to scale, the FDEA efficiency scored of 5 PRUMs are only evaluated for a series of 4 years only. The evaluation of potential improvement for each university and the inefficient ones are to be investigated based on the scores obtained and the referent DMUs to be emulated.

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