CHAPTER TWO

Some fundamental concepts in quantum optics

2.1 Introduction

Quantum optics is a field which describes the way light is radiated, its interactions with matter and the way in which it is detected; in all these phenomena, the corpuscular character of light plays a central role **[2.1]**. Therefore, it is inevitable that the field of optics evolves along with the quantum-mechanical direction.

Interestingly, quantum optics and quantum physics began at almost the same time or perhaps slightly before since quantum optics initiated quantum physics **[2.2]**. In the history of physics, we can see how the investigation of the black-body spectral distribution and its interpretation with the help of the statistical mechanics of radiation led to the hypothesis of quantization while the notion of the photon was introduced by Einstein in his interpretation of the photoelectric effect . Therefore, we begin this chapter with some aspects of quantum mechanics.

2.2 Formulation of quantum mechanics

The formulation of quantum theory be it for the most elementary or the most complicated system entails the representation of observables and various dynamical variables as linear operators acting as transformations on a Hilbert space of vectors.

2.2.1 Hilbert space

Cosider an infinite orthonormal set of basis vectors $\{\Phi_n : n = 1,2,\ldots\}$. From it, we can construct a linear vector space V by forming all possible finite linear combinations of the basis vectors. Thus V space consists of all vectors of the form $\Psi = \sum_n C_n \Phi_n$ where the sum may contain any finite number of terms. By convergence in a space we mean that the sequence $\{\Psi_i\}$ approaches the limit vector λ as $i \rightarrow \infty$ if and only if

$$\lim_{i\rightarrow\infty} \|\Psi_i - \lambda\| = 0 \qquad (2.2.1)$$

The addition of all such limit vectors to the space V yields a larger space H. For example, the vectors of the form

$$\Psi_i = \sum_{n=1}^{i} C_n \Phi_n \qquad (2.2.2)$$

are members of V for all finite values of i. The limit vector as $i \rightarrow \infty$ is not a member of V but it is a member of H provided $\sum_n |C_n|^2$ is finite. The space H is called a Hilbert space **[2.3]**.

A Hilbert space is in many ways analogous to the familiar three-dimensional Euclidan space of vector analysis. The number of linearly independent vectors determines the dimension D of the Hilbert space which may be finite or infinite. For every pair of vectors $|\Psi\rangle$ and $|\lambda\rangle$ a complex number is defined, denoted by

$$\langle\lambda|\Psi\rangle \tag{2.2.3}$$

and is called the "inner product". The inner product is linear in the vector $|\Psi\rangle$ and obeys the following two conditions

$$\langle\lambda|\Psi\rangle = \langle\Psi|\lambda\rangle^{*} \tag{2.2.4}$$

and

$$\langle\Psi|\lambda\rangle \geq 0 \tag{2.2.5}$$

Equality holds in the last condition (2.2.5) if and only if $|\Psi\rangle = 0$. Further Schwarz's inequality implies that

$$|\langle\lambda|\Psi\rangle|^{2} \leq \langle\lambda|\lambda\rangle\langle\Psi|\Psi\rangle \tag{2.2.6}$$

Equality holding if and only if $|\lambda\rangle = \alpha|\lambda\rangle$ for some complex α. If $|\Psi\rangle = 0, \langle\lambda|\Psi\rangle = 0$ for all $|\Psi\rangle$ and conversely. Every Hilbert space admits a set of vectors $\{|n\rangle\}$, n=1,2,...,D for which $\langle n|m\rangle = \delta_{n,m}$ and which span the Hilbert space. As analogs of an orthonormal basis in conventional vector analysis, the set $\{|n\rangle\}$ gives rise to the expansion

$$|\Psi\rangle = \sum_{n=1}^{D} |n\rangle\langle n|\Psi\rangle \qquad (2.2.7)$$

for every vector $|\Psi\rangle$. In terms of the coefficients in this expansion, the norm-square reads

$$\langle\Psi|\Psi\rangle = \sum_{n=1}^{D} |\Psi_n|^2$$

$$= \sum_{n=1}^{D} \langle\Psi|n\rangle\langle n|\Psi\rangle \qquad (2.2.8)$$

where $\Psi_n = \langle n|\Psi\rangle$

2.2.2 Linear operators

In quantum mechanics, observables and dynamical quantities are represented by linear operators **[2.4]**. Specifically, if A denotes an operator then $A|\Psi\rangle$ denotes the transformed vector and $\langle\lambda|A|\Psi\rangle$ is the corresponding new inner product with $|\lambda\rangle$. It is sufficient to define a linear operator on a set of basis vectors, since every vector can be expressed as a linear combination of the basis vectors. If $|\langle\lambda|A|\Psi\rangle| \leq b < \infty$ for all normed vectors $|\lambda\rangle$ and $|\Psi\rangle$, then the operator A is said to be bounded. In the representation of Hilbert space by sequences $\{\Psi_n\}$, the operator A is represented by a matrix $\{A_{m,n}\}$ with elements

$$A_{m,n} = \langle m|A|n\rangle \qquad (2.2.9)$$

which may be interpreted as the mth element of the vector $A|n\rangle$. In turn, the vector $A|\Psi\rangle$ has the sequence representation $\{(A\Psi)_m\}$ where

$$\begin{aligned}(A\Psi)_m &= \langle m|A|\Psi\rangle \\ &= \sum_{n=1}^{D} \langle m|A|n\rangle\langle n|\Psi\rangle \\ &= \sum_{n=1}^{D} A_{m,n}\Psi_n \qquad (2.2.10)\end{aligned}$$

The adjoint A^{+} of the operator A is defined as that operator for which

$$\langle\lambda|A^{+}|\Psi\rangle = \langle\Psi|A|\lambda\rangle^{*} \qquad (2.2.11)$$

for all $|\lambda\rangle$ and $|\Psi\rangle$. If $AA^{+} = A^{+}A = I$ where

$$I = \sum_{n=1}^{D} |n\rangle\langle n| \qquad (2.2.12)$$

then $A^{+} = A^{-1}$ and A is unitary. If $A^{+} = A$, then A is called Hermitian. In a finite-dimensional Hilbert space, every Hermitian operator has a complete set of orthonormal eigen vectors and real eigen-values such that

$$A|a_n\rangle = a_n|a_n\rangle \qquad (2.2.13)$$

In the basis $|a_n\rangle$, A is diagonal for $\langle a_m|A|a_n\rangle = a_n\delta_{n,m}$. For an infinite-dimensional Hilbert space not every Hermitian operator can be diagonalised. An operator A that is equal to its adjoint A^+ is also called self-adjoint. This means that it satisfies

$$\langle\lambda|A|\Psi\rangle = \langle\Psi|A|\lambda\rangle^* \tag{2.2.14}$$

and that the domain of A coincides with the domain of A^+. An operator that satisfies (2.2.14) is called an Hermitian operator.

2.23 Statistical states in quantum theory

A quantum system is normally composed of a statistical mixture of states. A special state of a statistical mixture is called a pure state. By definition, a state is a pure state if it is described by a well-defined single wave function and this wave function can be used to determine the values of all the physical quantities with respect to the system. A mathematically acceptable pure state operator must satisfy the following three conditions, namely

$$\begin{aligned} &\mathrm{Tr}\rho = 1 \\ &\rho = \rho^+ \\ &\langle u|\rho|u\rangle \geq 0 \end{aligned} \tag{2.2.15}$$

for all $|u\rangle$ where $\rho = |\Psi\rangle\langle\Psi|$ is a density operator.

In quantum theory, the normalized vectors $|\Psi\rangle$ of Hilbert space correspond to pure states of the system. If the Hermitian operator Θ denotes an observable, then the mean observed value for Θ in the state $|\Psi\rangle$ is given by the real number

$$\langle\Theta\rangle = \langle\Psi|\Theta|\Psi\rangle \tag{2.2.16}$$

A pure state can be described by means of the density operator $\rho = |\Psi\rangle\langle\Psi|$ where the vector of unit norm $|\Psi\rangle$ is called a state vector and the density operator is sometimes also called a pure state operator. The rule for computing mean values is

$$\begin{aligned}\langle\Theta\rangle &= \mathrm{Tr}(\rho\Theta)\\ &= \sum_{n=1}^{\infty}\langle n|\Psi\rangle\langle\Psi|\Theta|n\rangle\\ &= \langle\Psi|\Theta|\Psi\rangle\end{aligned} \tag{2.2.17}$$

Clearly ρ is Hermitian, positive and traceable. Moreover

$$\begin{aligned}\rho^2 &= |\Psi\rangle\langle\Psi|\Psi\rangle\langle\Psi|\\ &= |\Psi\rangle\langle\Psi|\\ &= \rho\end{aligned} \tag{2.2.18}$$

since $|\Psi\rangle$ is normalised. The density operators for two physically equivalent pure states are strictly identical i.e.

$$\rho_1 = |\Psi_1\rangle\langle\Psi_1| = |\Psi_2\rangle\langle\Psi_2| = \rho_2 \tag{2.2.19}$$

In conclusion, we note that each pure state of the system can be identified with a positive, Hermitian, density operator ρ for which $\rho^2 = \rho$ and $\mathrm{Tr}(\rho) = 1$.

2.3 Quantization of the harmonic oscillator

A harmonic oscillator plays an important role in quantum optics especially in the quantization of the electromagnetic radiation. The harmonic oscillator provides a model for many kinds of vibrating systems including the electromagnetic field. Basically, it is an object under the influence of a restoring force that is proportional to the displacement from its equilibrium position. The Hamiltonian for such an object whose motion is confined to one dimension is

$$H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 q^2 \tag{2.3.1}$$

where p, q, m and ω are the momentum, position, mass and angular frequency respectively.

The eigen-value spectrum of the Hamiltonian (2.3.1) can be obtained algebraically by using the commutation relation

$$[q,p] = i\hbar \tag{2.3.2}$$

and the self-adjointness of the operators p and q

$$p = p^{+}, \qquad q = q^{+} \tag{2.3.3}$$

Let us introduce the dimensionless position and momentum operators,

$$\hat{q} = \left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}} q \tag{2.3.4}$$

$$\hat{p} = \left(\frac{1}{m\hbar\omega}\right)^{\frac{1}{2}} p \tag{2.3.5}$$

which satisfy the commutation relation

$$[\hat{q},\hat{p}] = i \tag{2.3.6}$$

The Hamiltonian thus becomes

$$H = \frac{1}{2}\hbar\omega\left(\hat{p}^2 + \hat{q}^2\right) \tag{2.3.7}$$

Next, we introduce two more operators

$$a = \frac{\hat{q} + i\hat{p}}{\sqrt{2}} \qquad (2.3.8)$$

$$a^{-} = \frac{\hat{q} - i\hat{p}}{\sqrt{2}} \qquad (2.3.9)$$

These operators are Hermitian conjugates of each other. From (2.3.6), it follows that

$$\left[a, a^{+}\right] = 1 \qquad (2.3.10)$$

The Hamiltonian (2.3.7) the becomes

$$H = \hbar\omega\left(a^{+}a + \frac{1}{2}\right) \qquad (2.3.11)$$

The operator $a^{+}a$ in (2.3.11) is called the number operator for the oscillator and is denoted by

$$\hat{N} = a^{+}a \qquad (2.3.12)$$

The corresponding eigen-value equation is

$$a^{+}a|n\rangle = \hat{N}|n\rangle = n|n\rangle \qquad (2.3.13)$$

where n is the number of quanta in the number state $|n\rangle$ of the harmonic oscillator. The number state can also be treated as the energy eigen state of the harmonic oscillator with eigen value E_n.

Finally, we note that the eigen values and eigen-vectors of the harmonic oscillator Hamiltonian are

$$\begin{aligned} H|n\rangle &= \hbar\omega\left(a^+a + \frac{1}{2}\right)|n\rangle \\ &= \hbar\omega\left(N + \frac{1}{2}\right)|n\rangle \\ &= E_n|n\rangle \qquad (2.3.14) \end{aligned}$$

with

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \qquad (2.3.15)$$

where n = 0, 1, 2,..........

The quantization of the electromagnetic field has to follow certain procedures. Although it looks general and straightforward, the classical variables cannot be quantized directly as it provides a lots of mathematical complication. In field quantization, we have to convert the field observables such as the electric field strength **E**, the magnetic field strength **H** and the vector potential **A** in the algebraic form into the quantum mechanical operators . The relation between **E**, **H** and **A** is given as

$$\mu_0 \mathbf{H} = \nabla \times \mathbf{A} \tag{2.3.16}$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi \tag{2.3.17}$$

where ϕ is the scalar potential and μ_0 is the magnetic permeability of free space.

Let start the quantization by considering a cavity of volume V. A cavity is regarded as a region of space without any real boundaries. The electromagnetic field is treated in a free space and the volume V is taken to be infinite at the end of the calculation. With this preliminary assumptions, the vector potential in the cavity can be in a form of Fourier series expansion which reads

$$\mathbf{A} = \sum \left\{ \mathbf{A}_k \exp(i\mathbf{k} \cdot \mathbf{r}) + \mathbf{A}_k^* \exp(-i\mathbf{k} \cdot \mathbf{r}) \right\} \tag{2.3.18}$$

where **k** is the wave vector of mode **k**, **r** is the position of vector potential and ω_k=ck where c is velocity of light. As well known that, for each mode **k** of the radiation field there are two independent directions of the mode polarization ε_k . From now and on, we will include the mode polarization on **k**, $\mathbf{A}_k(t)$ and $\mathbf{A}_k^*(t)$ which appearing in equation (2.3.18) are the independent Fourier coefficients which can be determined by solving wave equation in free space .

$$\nabla^2 \mathbf{A} = \frac{1}{\mathbf{c}^2}\frac{\partial^2 \mathbf{A}}{\partial \mathbf{t}} \tag{2.3.19}$$

This will satisfy the simple harmonic equation

$$\frac{\partial^2 \mathbf{A}_k(\mathbf{t})}{\partial \mathbf{t}} + \omega_k^2 \mathbf{A}_k(\mathbf{t}) = 0 \tag{2.3.20}$$

Equation (2.3.20) also valid for $\mathbf{A}_k^*$. Their corresponding solutions are as follows

$$\mathbf{A}_k(\mathbf{t}) = \mathbf{A}_k \exp(-\mathbf{i}\omega_k \mathbf{t}) \tag{2.3.21}$$

$$\mathbf{A}_k^*(\mathbf{t}) = \mathbf{A}_k^* \exp(\mathbf{i}\omega_k \mathbf{t}) \tag{2.3.22}$$

Hence, the complete form of vector potential **A** in the cavity is

$$\mathbf{A} = \sum \left\{ \mathbf{A}_k \exp(-\mathbf{i}\omega_k \mathbf{t} + \mathbf{ik}\cdot\mathbf{r}) + \mathbf{A}_k^* \exp(\mathbf{i}\omega_k \mathbf{t} - \mathbf{ik}\cdot\mathbf{r}) \right\} \tag{2.3.23}$$

The electromagnetic field is completely quantized when equation (2.3.23) is transformed to a quantum-mechanical harmonic oscillator equation. This can be done in terms of the creation and annihilation operators $\mathbf{a}_k^+, \mathbf{a}_k$. These operators respectively create and destroy a quantum of energy $\hbar\omega_k$ in the cavity electromagnetic field mode of wavevector **k**. By replacing equations (2.3.21) and (2.3.22)in terms of operators $\mathbf{a}_k^+$ and $\mathbf{a}_k$ will give us

$$A_k = \sqrt{\frac{\hbar}{2\varepsilon_0\omega_k \mathbf{V}}}\hat{\mathbf{a}}_k\varepsilon_\kappa \tag{2.3.24}$$

$$A_k^{*} = \sqrt{\frac{\hbar}{2\varepsilon_0\omega_k \mathbf{V}}}\hat{\mathbf{a}}_k^{+}\varepsilon_\kappa$$

(2.3.25)

where ε_0 is the electric permitivity of free space. On substituting equations (2.3.24) and (2.3.25) into equation (2.3.23) we may readily obtain

$$\mathbf{A} = \sum_{\mathbf{k}}\sqrt{\frac{\hbar}{2\varepsilon_0 \mathbf{V}\omega_k}}\varepsilon_\kappa\left\{\hat{\mathbf{a}}_k\exp\left(-\mathrm{i}\omega_k t + \mathbf{ik}\cdot\mathbf{r}\right) + \hat{\mathbf{a}}_k^{+}\exp\left(\mathrm{i}\omega_k t - \mathbf{ik}\cdot\mathbf{r}\right)\right\} \tag{2.3.26}$$

Finally, we get the quantized electric and magnetic field strengths by inserting equation (2.3.26) into equation (2.3.16) and (2.3.17) and have the form

$$\mathbf{E}_k = \mathbf{i}\sqrt{\frac{\hbar\omega_k}{2\varepsilon_0\mathbf{V}}}\varepsilon_k\left\{\hat{\mathbf{a}}_k\exp\left(-\mathrm{i}\omega_k t + \mathbf{ik}\cdot\mathbf{r}\right) - \hat{\mathbf{a}}_k^{+}\exp\left(\mathrm{i}\omega_k t - \mathbf{ik}\cdot\mathbf{r}\right)\right\} \tag{2.3.27}$$

$$\mathbf{H}_k = \mathbf{i}\sqrt{\frac{\hbar\mathbf{c}^2}{2\mu_0\omega_k\mathbf{V}}}\mathbf{k}\times\varepsilon_k\left\{\hat{\mathbf{a}}_k\exp\left(-\mathrm{i}\omega_k t + \mathbf{ik}\cdot\mathbf{r}\right) - \hat{\mathbf{a}}_k^{+}\exp\left(\mathrm{i}\omega_k t - \mathbf{ik}\cdot\mathbf{r}\right)\right\} \tag{2.3.28}$$

The energy of the single mode k of the quantized electromagnetic field can be proved to have the oscillator form

$$E_k = \hbar\omega_k\left(\mathbf{a}_k^{+}\mathbf{a}_k + \frac{1}{2}\right) \tag{2.3.29}$$

The Hamiltonian for the total electromagnetic field in a cavity then turn out to be an operator and can be shown to take the form

$$\mathbf{H}_{\mathbf{radiation}} = \sum_{\mathbf{k}} \hbar\omega_k\left(\mathbf{n}_k + \frac{1}{2}\right) \tag{2.3.30}$$

where $\mathbf{a}_k^{+}\mathbf{a}_k \equiv \mathbf{n}_k$ is the photon number operator.

In the absence of the radiation field where n=0, the cavity possessing energy of $\frac{1}{2}\hbar\omega$ contributed by background radiation. We call this zero-point energy and the corresponding eigen value equation is

$$H|0\rangle = \frac{1}{2}\hbar\omega|0\rangle \tag{2.3.31}$$

where $|0\rangle$ is the vacuum state responsible for the zero-point fluctuations.

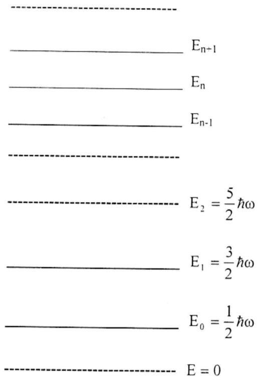

Figure 2.3.1. Energy level for the quantum mechanical harmonic oscillator.

2.4 Equation of motion in quantum mechanics

In quantum mechanics, the state of a system obeys a linear, first-order equation that is the Schrödinger equation. For a particular state $|\Psi\rangle = |\Psi(t)\rangle$, the Schrödinger equation reads

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle = H|\Psi\rangle \tag{2.4.1}$$

where H is the system Hamiltonian operator and assumed to be time independent. For a pure state density operator, $\rho = |\Psi\rangle\langle\Psi|$, it follows that

$$i\hbar \frac{\partial}{\partial t}\rho = [H, \rho] \tag{2.4.2}$$

The solution to equation (2.4.1) is given by

$$|\Psi(t)\rangle = e^{\frac{-itH}{\hbar}} |\Psi(0)\rangle \equiv U(t)|\Psi(0)\rangle \tag{2.4.3}$$

Since the Hamiltonian is Hermitian, the evolution operator U(t), which transforms the initial state $|\Psi(0)\rangle$ into $|\Psi(t)\rangle$ is unitary and so preserves its norm. We can easily determine for a pure-state density operator such that

$$\begin{aligned}\rho(t) &= |\Psi(t)\rangle\langle\Psi(t)| \\ &= U(t)|\Psi(0)\rangle\langle\Psi(0)|U(t)^{-1} \\ &= U(t)\rho(0)U(t)^{-1}\end{aligned} \tag{2.4.4}$$

From linearity properties, it follows that the solution for a general density operator is represented by

$$\rho(t) = U(t)\rho(0)U(t)^{-1} \tag{2.4.5}$$

This view point of the state of the system evolving in time is called the "Schrödinger Picture". However we have an alternative way to account for time evolution is the "Heisenberg Picture". According to this picture, the time-dependent mean value for a general observable Θ is

$$\langle\Theta(t)\rangle \equiv \mathrm{Tr}[\rho(t)\Theta] = \mathrm{Tr}\left[U(t)\rho U(t)^{-1}\Theta\right]$$

$$= \mathrm{Tr}\left[\rho U(t)^{-1}\Theta U(t)\right] \equiv \mathrm{Tr}[\rho\Theta(t)] \qquad (2.4.6)$$

Where we have made use of the invariance of the trace under cyclic operator permutation. Here, we have set $\rho = \rho(0)$ and defined the time-dependent operator observable $\Theta(t)$ by

$$\Theta(t) = U^{-1}(t)\Theta U(t) \qquad (2.4.7)$$

The difference in the two evolution equations i.e. between (2.4.5) and (2.4.6) can be seen below

$$i\hbar\frac{\partial\rho}{\partial t} = [H,\rho] \qquad (2.4.8)$$

$$-i\hbar\frac{\partial\Theta}{\partial t} = [H,\Theta] \qquad (2.4.9)$$

where equations (2.4.8) and (2.4.9) respectively correspond to "Schrödinger and Heisenberg Picture".

2.5 The two level atom.

In quantum optics, we often shine a monochromatic laser light on an atom and study the atom-field interaction e.g. resonance fluoresence. However, to describe the exact process is not a simple task. This is so because an atom possesses an infinity of energy levels according to the atomic theory. Therefore, we introduce the concept of a two-level atom for this type of interaction.

We assume that the atom has only two eigenstates $|1\rangle$ and $|2\rangle$ which are supposed to be non-degenerate:

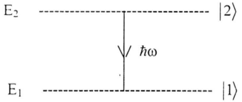

Figure 2.5.1 Energy level diagram of two-level atom

Consider the motion of an electron in an atom which can be represented by the Schrodinger equation:

$$H_o|\Psi\rangle = i\hbar\frac{d}{dt}|\Psi\rangle \tag{2.5.1}$$

where Ψ is the wave function of the system and H_o is the Hamiltonian describing the energy of the unpertubed motion. The general solution of the equation (2.5.1) takes the form of a superposition of the two unpertubed solutions, namely

$$|\Psi\rangle = c_1|\Psi_1\rangle + c_2|\Psi_2\rangle \qquad (2.5.2)$$

The coefficients c_1 and c_2 are time dependent that describe the atomic position probability in the system.

In the theory of atomic structure, energy levels in an atom usually have a number of sublevels that all can contribute to a resonant transition **[2.5]**. This sub-levels is the energy level splittings due to electron coupling and are also responsible for line broadening. However we will ignore the sublevels because they will produce complication when experiment with real atoms is performed **[2.6]**. The Hamiltonian of a two-level atom is defined as **[2.7]**

$$H_A = \hbar\omega S_z \qquad (2.5.3)$$

where S_z is the atomic inversion operator which satisfies the angular momentum commutation relation

$$\left[S^+, S^-\right] = 2S_z \qquad (2.5.4)$$

and $S^{\pm}$ are the atomic dipole operators of the atoms i.e..

$$S^{+} = |2\rangle\langle 1| \tag{2.5.5}$$

$$S^{-} = |1\rangle\langle 2| \tag{2.5.6}$$

Substituting equations (2.5.5) and (2.5.6) into equation (2.5.4) gives

$$S_z = \frac{1}{2}\left(|2\rangle\langle 2| - |1\rangle\langle 1|\right) \tag{2.5.7}$$

So, the Hamiltonian of a two-level atom has the form

$$H_A = \frac{\hbar\omega}{2}\left(|2\rangle\langle 2| - |1\rangle\langle 1|\right) \tag{2.5.8}$$

Operating the Hamiltonian H_A on the appropriate eigenstates, the eigen-values of the levels is given by

$$H_A|2\rangle = \frac{\hbar\omega}{2}|2\rangle \tag{2.5.9}$$

$$H_A|1\rangle = -\frac{\hbar\omega}{2}|1\rangle \tag{2.5.10}$$

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