

PRESERVICE SECONDARY SCHOOL MATHEMATICS TEACHERS'
SUBJECT MATTER KNOWLEDGE OF PERIMETER AND AREA

VOLUME 1

WUN THIAM YEW

UNIVERSITY OF MALAYA

2010

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VOLUME 2

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ORIGINAL LITERARY WORK DECLARATION

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Name of Degree: *Doctor of Philosophy*

Title of Project Paper/Research Report/Dissertation/Thesis ("this Work"): *Preservice secondary school mathematics teachers' subject matter knowledge of perimeter and area.*

Field of Study: *Mathematics Education*

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SYNOPSIS

The purpose of this study was to investigate preservice secondary school mathematics teachers (PSSMTs)' subject matter knowledge (SMK) of perimeter and area. Specifically, this study aimed to investigate PSSMTs' five basic types of knowledge of perimeter and area, namely conceptual knowledge (CK), procedural knowledge (PK), linguistic knowledge (LK), strategic knowledge (SK), and ethical knowledge (EK). This study also aimed to investigate PSSMTs' levels (low, medium, high) of SMK of perimeter and area.

Data of this study was collected using clinical interview technique. Interview sessions were recorded using digital video camera and tape recorder. Subjects of this study consisted of eight PSSMTs enrolled in a Mathematics Teaching Methods course at a public university in Peninsula Malaysia. They were selected based on their majors (mathematics, biology, chemistry, physics) and minors (mathematics, biology, chemistry, physics).

With regard to CK, findings of this study showed that all PSSMTs understand the inverse proportion between the number of units and the unit of measure. Six out of eight PSSMTs knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Nevertheless, most of the PSSMTs did not know that there is no direct relationship between perimeter and area. None of the PSSMTs were able to develop the formula for the area of a rectangle. It was apparent that all of them lack conceptual knowledge underpinning the formula for the area of a rectangle.

With respect to PK, findings of this study depicted that at least half of the PSSMTs had adequate procedural knowledge of converting standard units of area measurement. Most of the PSSMTs had adequate procedural knowledge of calculating perimeter and area of composite figures. Five, two, and three PSSMTs were able to develop the formula for the area of a parallelogram, triangle, and trapezium, respectively.

Concerning the LK, findings of this study demonstrated that most of the PSSMTs used appropriate mathematical symbols to write the formula for the area of a rectangle, parallelogram, triangle, and trapezium. Most of the PSSMTs used appropriate mathematical terms to justify their selection of shapes that have a perimeter and an area. All the PSSMTs understand the general measurement convention that perimeter is measured in linear units while area is measured in square units. Nevertheless, they had limited knowledge about the conventions pertaining to writing and reading of Standard International (SI) area measurement units.

For the SK, findings of this study revealed that three types of strategies were employed by the PSSMTs to compare perimeters as well as areas, namely formal, semi-formal, and informal methods. Two types of strategies used to verify the answers for perimeters and areas were emerged, namely recalculating strategy and alternative method. Three types of strategies used to solve the fencing problem were identified, namely looking for a pattern strategy, trial-and-error strategy, and differentiation method. PSSMTs used the cut and paste strategy, partition strategy, and algebraic method to develop the formula for the area of a parallelogram, triangle, and trapezium, respectively.

With respect to EK, findings of this study exhibited that all PSSMTs had taken the effort to justify the selection of shapes that have a perimeter and an area. Several PSSMTs had attempted to examine the possible pattern of the relationship between perimeter and area, to formulate and test generalization pertaining to the relationship between perimeter and area. However, most of the PSSMTs did not check the correctness of the answers for the perimeters and areas. With regard to the overall level of SMK of perimeter and area, only one of the PSSMTs secured a high level of knowledge, six with medium level and one at low level. The findings of this study lead to the conclusion that many PSSMTs do in fact lack SMK of perimeter and area they are expected to teach.

PENGETAHUAN ISI KANDUNGAN GURU PRAPERKHIIDMATAN MATEMATIK SEKOLAH MENENGAH BAGI PERIMETER DAN LUAS

SINOPSIS

Kajian ini bertujuan untuk menyelidiki pengetahuan isi kandungan (PIK) guru praperkhidmatan matematik sekolah menengah (GPMSM) bagi perimeter dan luas. Secara khusus, kajian ini bertujuan untuk menyelidiki lima jenis pengetahuan asas bagi perimeter dan luas yang dimiliki oleh GPMSM, iaitu pengetahuan konsep (PK), pengetahuan prosedur (PP), pengetahuan linguistik (PL), pengetahuan strategik (PS), dan pengetahuan etika (PE). Kajian ini juga bertujuan untuk menyelidiki peringkat (rendah, sederhana, tinggi) PIK yang dimiliki oleh GPMSM.

Data kajian ini dikumpul melalui teknik temu duga klinikal. Sesi temu duga telah dirakamkan dengan perakam video digital dan perakam audio. Subjek kajian ini terdiri daripada lapan orang GPMSM yang sedang mengikuti kursus Kaedah Mengajar Matematik di sebuah universiti awam di Semenanjung Malaysia. Mereka dipilih berasaskan bidang pengkhususan major (matematik, biologi, kimia, fizik) dan minor (matematik, biologi, kimia, fizik).

Berhubung dengan PK, dapatan kajian ini menunjukkan semua GPMSM memahami perkadaran songsang antara bilangan unit dan unit pengukuran. Enam daripada lapan GPMSM mengetahui hubungan antara unit luas dan unit linear di mana unit luas diterbitkan daripada unit linear berasaskan kuasa dua. Walau bagaimanapun, kebanyakan GPMSM tidak mengetahui bahawa tiada hubungan langsung antara perimeter dan luas. Tiada GPMSM yang dapat menerbitkan rumus bagi luas segiempat tepat. Ini menunjukkan kesemua mereka kekurangan PK di sebalik rumus bagi luas segiempat tepat.

Berkenaan dengan PP, hasil kajian ini menunjukkan sekurang-kurangnya separuh daripada GPMSM mempunyai pengetahuan yang mencukupi untuk menukar unit piawai

ukuran luas. Kebanyakan GPMSM mempunyai pengetahuan yang mencukupi untuk menghitung perimeter dan luas rajah gubahan. Lima, dua, dan tiga orang GPMSM masing-masing dapat menerbitkan rumus bagi luas segiempat selari, segitiga, dan trapezium.

Berkenaan dengan PL, dapatan kajian ini menunjukkan kebanyakan GPMSM menggunakan symbol matematik yang sesuai untuk menulis rumus bagi luas segiempat tepat, segiempat selari, segitiga, dan trapezium. Kebanyakan GPMSM menggunakan istilah matematik yang sesuai untuk memberikan justifikasi ke atas pemilihan bentuk-bentuk yang mempunyai perimeter dan luas. Kesemua GPMSM memahami kelaziman am pengukuran di mana perimeter dan luas masing-masing diukur dalam unit linear dan unit persegi. Walau bagaimanapun, mereka mempunyai pengetahuan yang terhad tentang kelaziman penulisan dan pembacaan unit luas Sistem Antarabangsa.

Bagi PS, hasil kajian ini menunjukkan tiga jenis strategi telah digunakan oleh GPMSM untuk membandingkan perimeter dan juga luas, iaitu kaedah formal, semi-formal, dan informal. Dua jenis strategi yang digunakan untuk menyemak jawapan bagi perimeter dan luas telah muncul, iaitu strategi penghitungan semula dan kaedah alternatif. Tiga jenis strategi yang digunakan untuk menyelesaikan masalah berpagar telah dikenal pasti, iaitu strategi mencari pola, strategi cuba-jaya, dan kaedah pembezaan. GPMSM menggunakan strategi “potong dan tampal”, strategi pembahagian, dan kaedah algebra masing-masing untuk menerbitkan rumus bagi luas segiempat selari, segitiga, dan trapezium.

Berkenaan dengan PE, dapatan kajian ini menunjukkan semua GPMSM telah berusaha untuk member justifikasi ke atas pemilihan bentuk-bentuk yang mempunyai perimeter dan luas. Beberapa orang GPMSM telah mencuba untuk meneliti pola hubungan yang mungkin antara perimeter dan luas, membentuk dan menguji pengitlakan yang berkenaan dengan hubungan antara perimeter dan luas. Namun begitu, kebanyakan GPMSM tidak menyemak jawapan bagi

perimeter dan luas. Pada keseluruhannya, hanya seorang GPMSM mencapai peringkat tinggi untuk PIK bagi perimeter dan luas, enam orang dengan peringkat sederhana dan seorang pada peringkat rendah. Dapatan kajian ini menunjukkan ramai GPMSM kekurangan PIK bagi perimeter dan luas yang akan diajar.

ACKNOWLEDGEMENTS

First and foremost, I wish to express my heartiest appreciation to my supervisors, Associate Professor Dr. Sharifah Norul Akmar Syed Zamri and Dr. Lee Siew Eng, for their invaluable guidance, comments, suggestions, encouragement, and support throughout the process of writing the thesis. Without their guidance, comments, suggestions, encouragement, and support, I would not have been able to complete this study. I also wish to take this opportunity to express my appreciation to my sponsor, University of Science Malaysia, for granted me study leave under the Academic Staff Higher Education Scheme.

I am thankful to Professor Dr. Saedah Siraj, Dean, Faculty of Education, University of Malaya, for her encouragement and support. I am also thankful to Professor Dr. Nik Azis Nik Pa, Department of Mathematics and Science Education, Faculty of Education, University of Malaya, for his constructive comments and suggestions.

I would like to thank Professor Dr. Abdul Rashid Mohamed, Dean, School of Educational Studies, University of Science Malaysia, for his encouragement and support. I would also like to thank my colleagues from the School of Educational Studies, University of Science Malaysia, and my friends, for their encouragement and moral support.

I am grateful to the two university lecturers and a secondary school mathematics teacher for generously gave their valuable of time to validate the tasks used in the clinical interview. I would like to express my gratitude to the university mathematics education lecturer for his scholarly effort to verify the coding of data. I am also grateful to the preservice secondary school mathematics teachers at a public university for their willingness to participate in this study.

I would like to thank my parents and parents-in-law for their love, care, encouragement and support. I would also like to thank my brothers, sister, and relatives, for their moral support. Last but not least, I extend my deepest gratitude to my wife, Lim Hooi Lian, and to our child,

Wun Lim Zhe, for their love, care, encouragement, support, patience, and sacrifices in many ways that have given me the peace of mind to complete this study.

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