CHAPTER ONE

INTRODUCTION

Background of the Study

One cannot teach what one does not know. Teachers must have in-depth knowledge of mathematics they are going to teach. Therefore, it is important that a teacher need to have a comprehensive knowledge of mathematics to enable him or her to organize teaching so that students can learn mathematics meaningfully. Fennema and Franke (1992) advocated that "no one questions the idea that what a teacher know is one of the most important influences on what is done in classroom and ultimately on what students learn" (p. 147). Furthermore, “teachers who do not themselves know a subject well are not likely to help students learn this content.” (Ball, Thames, & Phelps, 2008, p. 404). This applies also to mathematics teacher.

“The amount and organization of the knowledge per se in the mind of the teacher" is referred as subject matter knowledge (SMK) (Shulman, 1986, p. 9). Shulman (1986) referred to the absence of focus on SMK for the research on teaching as the “missing paradigm” (p. 6). It indicated that SMK is an important component of teachers' knowledge. However, there is no consensus on the definition of SMK and its specific components (e.g., Ball, 1988; Fennema & Franke, 1992; Nik Azis, 1996; Shuman, 1986).

Ball (1988) developed a conceptual framework for exploring teachers' SMK of mathematics. She claimed that understanding of mathematics involves both knowledge of mathematics and knowledge about mathematics. Knowledge of mathematics is closely related to Shulman's (1986) dimension of substantive knowledge. It includes both conceptual and procedural knowledge. Knowledge about mathematics is related to Shulman's (1986) dimension of syntactic knowledge. It includes an “understanding of the nature of knowledge in the discipline, namely where it comes from, how it changes, how truth is established, and what it means to know and to do mathematics” (Ball, 1988, p. 163).

According to Fennema and Franke (1992), knowledge of the content of mathematics:

> Includes teachers' knowledge of the concepts, procedures, and problem solving processes within the domain in which they teach, as well as in related content domains. It includes knowledge of the concepts underlying the procedures, the interrelatedness of these concepts, and how these concepts and procedures are used in various types of problem solving. (p. 162)

Nik Azis (1996) suggested that there are five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge. This applies also to SMK. Specifically, SMK encompasses five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge.

The National Council of Teachers of Mathematics's (NCTM) (2000) *Principles and Standards for School Mathematics* documented that:

> Students learn mathematics through the experiences that teachers provide. Thus, students' understanding of mathematics, their ability to use it to solve problems, and their confidence in, and dispositions toward, mathematics are all shaped by the teaching they encountered in school. The implementation of mathematics education for all students requires effective mathematics teaching in all classrooms. To be effective, teachers must know and understand the mathematics they are teaching and be able to draw on that knowledge flexibly in their teaching tasks. (pp. 16-17)

In the *Professional Standards for Teaching Mathematics* document, NCTM (1991) elaborated earlier that "teachers' comfort with, and confidence in, their own knowledge of mathematics affects both what they teach and how they teach it" (p. 132). Thus, it can be concluded that teachers’ SMK is a crucial contributor to effective mathematics teaching.

However, previous research (e.g., Cheah, 2001; Koe, 1992; Ng, 1995) revealed that Malaysian trainee teachers in the teacher training institutes (formerly known as teacher training colleges) were lack of conceptual knowledge as well as procedural knowledge. Do their counterparts in the Malaysian public universities encountered similar problem? Moreover, previous studies (e.g., Ball, 1988; Ramakrishnan, 1998; Reinke, 1997; Ryan & Williams, 2007) demonstrated that preservice teachers had limited SMK of perimeter and area. What are the preservice secondary school mathematics teachers (PSSMTs)' nature and level of SMK of a specific mathematical topic such as perimeter and area? Research work is needed to answer such questions. Furthermore, our secondary school students' performance in the topic of perimeter and area were less satisfactory (Malaysian Examination Syndicate, 1996; 2003). Do the PSSMTs in the Malaysian public universities ready to teach the topic of perimeter and area?

Teacher Education in Malaysia

In Malaysia, preservice teachers are trained at the teacher training institutes (formerly known as teacher training colleges) or public universities (Nik Azis, 2008). At the institute, trainee teachers take a three-years Diploma Teacher Training Course (known as *Kursus Diploma Perguruan Malaysia*' in Malay Language, *KDPM*), majoring in one or two school subjects they plan to teach in schools. A one-year special teacher training course, the Postgraduate Teacher Education Program (known as '*Kursus Perguruan Lepasan Ijazah*' in Malay Language, *KPLI*), is offered to graduates who intend to join the teaching profession. At the university, preservice teachers take a four years course leading to an education degree in science or humanities such as Bachelor of Science with Education (B.Sc.Ed.), Bachelor of Arts with Education (B.A.Ed.), or Bachelor of Education (B. Ed.). They major or minor in one or two subjects they intend to teach in schools.

In general, the teacher education program at the institute and university levels comprises three main components, namely academic, education theory, and practical (Lourdusamy & Tan, 1992). In the academic component, the preservice teachers are required to acquire SMK in one or two school subjects they plan to teach in schools. The theoretical component encompasses: (a) the foundation courses in education such as philosophy, psychology, and sociology; (b) methods courses for the teaching of various school subjects; and (c) supporting courses such as classroom measurement and evaluation, counseling, and management. The practical component consists of teaching practice in schools for a period of between fourteen and eighteen weeks.

Measurement in the Malaysian Mathematics Curriculum

Measurement forms an important part of the Malaysian primary and secondary school mathematics curriculum. Time, length, mass/weight, perimeter, area, and volume/capacity are the measurement concepts in our mathematics curriculum from primary school through secondary school (see Appendix A). The Malaysian students begin to explore the concept of time from Year One at the primary school and this concept is being introduced progressively through Form One at the secondary school. Length measurement is being introduced to the Malaysian students beginning from Year Three at the primary school. They continue to learn this concept in Years Four, Five, and Six at the primary school and Form One at the secondary school (*Kementerian Pendidikan Malaysia* [Ministry of Education Malaysia], 1998a, 1998b, 1998c, 1998d, 1998e, 1998f; Ministry of Education Malaysia, 2003a).

The Malaysian primary school students start to learn the concept of mass/weight formally in Year Four. They proceed to learn this concept in Years Five and Six at the primary school and Form One at the secondary school. Perimeter measurement is being introduced to Year Five students at the primary school. They continue to learn this concept formally in Form One at the secondary school. Measurement of area is being introduced to Year Six students at the primary school. The students continue to learn this concept in Form One at the secondary school. Circumference and area of a circle is being introduced to Form Two students at the secondary school. They also learn the length of an arc and the area of a sector in the Form Two syllabus. The Malaysian students start to learn the measurement concept of volume/capacity in Year Five at the primary school. They continue to study volume/capacity in Year Six, and Forms One, Two, and Three (*Kementerian Pendidikan Malaysia* [Ministry of Education Malaysia], 1998d, 1998e, 1998f; Ministry of Education Malaysia, 2003a, 2003b, 2003c).

Perimeter and Area in the Malaysian Mathematics Curriculum

In the Malaysian mathematics curriculum, students start to learn perimeter concept formally in Year Five (see Appendix B). Perimeter is defined as the total measures around or the boundary of a shape. In this topic, length is measure in units of millimetre, centimetre, and metre. Measurement in centimetre is limited to one decimal place. While measurement in metre may incorporates two decimal places. In Year Five, students learn to determine the perimeters of rectangles, squares, and triangles by measuring each side and find the total measure of each side (*Kementerian Pendidikan Malaysia* [Ministry of Education Malaysia], 1998e).

They also learn to find the perimeters of rectangles, squares, and triangles when its measures are given. At this level, no formula is used for calculating the perimeters. Students also learn to find the perimeter of a circle (known as circumference) by measuring it with a piece of string. Students are encouraged to find different rectangles or triangles that have the same perimeter. Finally, Year Five students learn to solve daily problems involving perimeters such as determining the perimeters of composite figures made up of rectangles, squares, or triangles. They also learn to determine the cost of fencing a region (*Kementerian Pendidikan Malaysia* [Ministry of Education Malaysia], 1998e).

In the Malaysian mathematics curriculum, primary school students start to learn area concept formally in Year Six. It is emphasized in the syllabus that only closed figure has an area and area is defined as the size of region in the closed figure. Initially, counting method is being introduced to determine the area of a shape. Students explore the area of a shape by covering it with square tiles. They also learn to determine the area of rectangle and square drawn on square grid paper by counting the number of units needed to cover the region. In Year Six, the units of area measurement are limited to cm^2 and m^2 (*Kementerian Pendidikan Malaysia* [Ministry of Education Malaysia], 1998f).

Students determine the formula to find the areas of rectangle and squares as "Area = length x width". Through paper folding or cutting, students are guided to develop the formula for finding the area of triangles as "Area of triangle = (base x height) ÷ 2 or $\frac{base \times height}{2}$". Students are encouraged to investigate the areas of rectangles and squares that have the same perimeter. They are also encouraged to investigate the perimeters of rectangles and squares that have the same area. Year Six students learn to find the areas of composite figure made up of rectangles, squares, and triangles. They also learn to solve daily problems involving areas (*Kementerian Pendidikan Malaysia* [Ministry of Education Malaysia], 1998f).

In the Malaysian secondary school mathematics curriculum, perimeter and area is a specific topic (i.e., Topic 11) in the Form One syllabus (see Appendix C). Students learn to identify and find the perimeter of a region. They also investigate and develop formula to find the perimeter of a rectangle. Students learn to find the perimeters of composite figure made up of rectangles, triangles, parallelogram, or trapeziums. They also learn to solve daily problems involving perimeters. For the area concept, Form One students learn to estimate the area of shape by using unit squares, tessellation grids, geoboards, or grid papers. They investigate and develop formula to find the areas of rectangles, triangles, parallelograms, or trapeziums. Students are encouraged to use unit square chips or tiles to investigate, explore, and make generalization about the perimeters of rectangles having the same area and the areas of rectangles having the same perimeter. Students learn to find the areas of composite figure made up of rectangles, triangles, parallelograms, or trapeziums. They also learn to solve daily problems involving areas (Ministry of Education Malaysia, 2003a).

Students continue to learn perimeter and area in Form Two under a specific topic (i.e., Topic 10) called circle. In this topic, they learn to estimate the value of π and derive the formula of the circumference of a circle. They find the circumference of a circle and also solve problems involving circumference of circles. Form Two students explore the relationship between the length of arc and the angle subtended at the centre of a circle, and thus learn to derive the formula of the length of an arc. They find the length of arc and solve problems involving arcs of circle. Next, students learn to derive the formula of the area of a circle. They find the area of a circle and solve problems involving area of circle. Form Two students also explore the relationship between the area of a sector and the angle subtended at the centre of a circle, and thus they learn to derive the formula of the area of a sector. They find the area of a sector and solve problems involving area of sectors (Ministry of Education Malaysia, 2003b).

Statement of the Problem

Even (1990) observed that “interest in teachers' SMK has arisen in recent years” (p. 521). However, she found that most of the studies about teachers' SMK have been general and not topic specific. According to Even (1990), "analyzing what teachers' subject matter knowledge means in general in mathematics, does not inform us of what subject matter knowledge teachers need to have in order to teach a specific piece of mathematics" (p. 522). We need to know more about the specific characteristics of knowledge needed for teaching a specific mathematics topic. Thus, the researcher intended to examine the preservice secondary school mathematics teachers' SMK of a specific mathematical topic, namely perimeter and area.

The researcher focused on the Form One mathematics topic of perimeter and area for the following reasons: First, "measurement is a domain of mathematics that is most closely allied with real-world application" (Baturo & Nason, 1996, p. 236). Second, measurement concepts and processes form a major part of Malaysian primary and secondary school mathematics curriculum (see Appendix A). Third, perimeter and area are taught in Year Five and Year Six at the primary school level, and Form One and Form Two at the secondary school level (see Appendix B).

Fourth, previous studies (e.g., Cavanagh, 2008; Kenney & Kouba, 1997; Lindquist, & Kouba, 1989; Ryan & Williams, 2007; Strutchens, Martin, & Kenny, 2003) suggest that students often have great difficulty in understanding this topic, perimeter and area. In the context of Malaysia, a total of 5577 Malaysian Form Two students took part in the Third International Mathematics and Science Study - Repeat (TIMMS - R) (Ministry of Education Malaysia, 2000). Measurement was one of the five content areas being tested in the TIMMS - R study. Malaysia ranked 16 in the content area of measurement. A total of 5314 Malaysian Form Two students participated in the Trends in International Mathematics and Science Study 2003 (TIMSS 2003) (Ministry of Education Malaysia, 2004). Measurement was one of the five content areas being tested in the TIMMS 2003. However, in the TIMMS 2003, Malaysia's ranking in the content area of measurement had dropped to number 18 (Mullis, Martin, Gonzales, & Chrostowski, 2004).

A total of 4466 Malaysian Form Two students involved in the Trends in International Mathematics and Science Study 2007 (TIMSS 2007) (Martin et al., 2008). In the TIMSS 2007, geometry and measurement were combined as a domain known as geometry shapes and measure. In the TIMMS 2007, Malaysia's ranking in the domain of geometry shapes and measure further dropped to number 24. It was reported that in the TIMMS 2007, Malaysian Form Two students' average scale score (477) in the domain of geometry shapes and measure was significantly lower than TIMSS scale average (500) (Martin et al., 2008). Furthermore, the Malaysian Examination

Syndicate (1996, 2003) reported that the SPM candidates' performance in the topic of perimeter and area were less satisfactory.

Finally, previous studies (Ramakrishnan, 1998; Reinke, 1997) revealed that many preservice elementary teachers have limited knowledge of perimeter and area. However, these studies mainly focused on conceptual knowledge and procedural knowledge of perimeter and area. Thus, these studies offered few insights into the other aspects of the preservice teachers' SMK of perimeter and area, namely linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area.

Ball, Lubienski, and Mewborn (2001) observed that most of the studies related to teachers' knowledge have been conducted with preservice elementary school teachers. Thus, there is a need to conduct similar study with preservice secondary school teachers. There is an assumption that SMK is not a problem for PSSMTs as they specialized in mathematics and thus know their subject matter well. However, previous research revealed the fallacy of this assumption (e.g., Ball, 1988, 1990b; Even, 1993; Ryan & Williams, 2007). It indicated that preservice secondary school teachers do not necessarily know their subject matter well. Furthermore, several studies showed that Malaysian trainee teachers in the teacher training institutes (formerly known as teacher training colleges) had demonstrated a poor understanding of mathematical concepts and a lack of mathematical skills (Cheah, 2001; Koe, 1992; Ng, 1995). Therefore, there is a need to examine the SMK hold by the preservice secondary school mathematics teachers (PSSMTs) in the Malaysian public university in a specific mathematical topic, namely perimeter and area.

Even (1990) suggested that “the teacher's role is to help his or her students achieve understanding of the subject matter. But in order to do so the teachers themselves need to have solid knowledge of the subject matter. A teacher who has solid mathematical knowledge for teaching is more capable of helping his or her students achieve a meaningful understanding of the subject matter" (p. 521). Thus, SMK is an important component of the knowledge of a well prepared teacher and a potential area of study.

In spite of the importance of mathematical knowledge, "no attempt was make to measure what the teachers know about mathematics" (Fennema & Franke, 1992, p. 148). Therefore, teachers' knowledge of mathematics is a valuable area of study. Furthermore, the nature of mathematics has not been adequately considered in many studies of teachers' knowledge of content (e.g., Ramakrishnan, 1998; Reinke, 1997). Thus, it motivated the researcher to investigate the preservice secondary school mathematics teachers' SMK of a specific mathematical topic, namely perimeter and area.

Research Questions

The purpose of this study was to investigate preservice secondary school mathematics teachers' subject matter knowledge (SMK) of measurement, in particular, on the topic of perimeter and area. Specifically, this study aimed to investigate preservice secondary school mathematics teachers' five basic types of knowledge of perimeter and area, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge. This study also aimed to investigate preservice secondary school mathematics teachers' levels (low, medium, high) of subject matter knowledge (SMK) of perimeter and area.

This study attempted to answer the following research questions:

1. What kinds of subject matter knowledge (SMK) of perimeter and area do the preservice secondary school mathematics teachers have?
2. What levels of subject matter knowledge (SMK) of perimeter and area do the preservice secondary school mathematics teachers exhibits?

Definition of Terms

The following definitions were used for the present study:

Preservice secondary school mathematics teachers (PSSMTs) refers to undergraduates in the Bachelor of Science with Education (B.Sc.Ed.) program of a public university who major or minor in mathematics and intend to be secondary school mathematics teachers upon graduation.

Subject matter knowledge (SMK) refers to the amount and organization of knowledge per se in the mind of preservice secondary school mathematics teachers (adapted from Shulman, 1986, p. 9). It encompasses five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge (adapted from Nik Azis, 1996, p. 200).

Conceptual knowledge is "knowledge that is rich in relationships". It consists of "network in which the linking relationships are as prominent as the discrete pieces of information" being linked (Hiebert & Lefevre, 1986, pp. 3-4). In this study, conceptual knowledge of perimeter and area encompasses the following components: (a) notion of perimeter (i.e., the number of linear units it takes to surround a shape), (b) notion of area (i.e., the number of square units it takes to cover a shape), (c) notion of the units of area (square and nonsquare), (d) number of units and unit of measure, (e) inverse relationship/proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units and vice versa, (f) relationship between the standard units of length measurement (linear units) such as 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m, (g) relationship between the standard units of area measurement (square units) such as 1 cm^2 = 100 mm^2, 1 m^2 = 10 000 cm^2, and 1 k m^2 = 1 000 000 m^2, (h) relationship between area units and linear units of measurement: Area units are derived from linear units based on squaring, (i) relationship between perimeter and area, and (j) relationship among area formulae.

Procedural knowledge refers to "the algorithms or rules for completing mathematical tasks" (adapted from Hiebert & Lefevre, 1986, p. 6). In this study, procedural knowledge of perimeter and area encompasses the following components: (a) converting standard units of area measurement, (b) calculating the perimeter of composite figures, (c) calculating the area of composite figures, and (d) developing area formulae.

Linguistic knowledge refers to "formal language, or symbol representation system of mathematics" (adapted from Hiebert & Lefevre, 1986, p. 5). In this study, linguistic knowledge of perimeter and area encompasses the following components: (a) mathematical symbols, (b) mathematical terms, (c) standard unit of length measurement (linear units), (d) standard unit of area measurement (square units), and (e) conventions of writing and reading SI area.

Strategic knowledge refers to "our ability to choose an appropriate strategy to solve a task because it is more effective than alternative strategies" (Henson & Eller, 1999, p. 258). In this study, strategic knowledge of perimeter and area encompasses the following components: (a) strategies for comparing perimeter, (b) strategies for comparing area, (c) strategies for checking answer for perimeter, (d) strategies for checking answer for area, (e) strategies for solving the fencing problem, (f) strategies for checking answer for the fencing problem, and (g) strategies for developing/deriving area formulae.

Ethical knowledge refers to "knowledge of right and wrong, what we are obligated to do, and of values" (Kupperman, 1970, p. 19). There are some good behaviors that the subjects need to follow when dealing with perimeter and area. In this study, ethical knowledge of perimeter and area encompasses the following components: (a) justifies one's mathematical ideas, (b) examines pattern within the domain of perimeter and area measurement, (c) formulates generalization within the domain of perimeter and area measurement, (d) tests generalization within the domain of perimeter and area measurement, (e) develops area formulae, (f) writes units of measurement upon they completed a task, and (g) checks the correctness of their solutions or answers.

Perimeter and area refers to a mathematical topic in Form One of the Malaysian Integrated Curriculum of Secondary School (known as '*Kurikulum Bersepadu Sekolah Menengah, KBSM'* in Malay Language) mathematics. *Perimeter* refers to "the number of linear units it takes to surround a shape" (Rickard, 1996, p. 306). *Area* refers to "the number of square units it takes to cover a shape" (Rickard, 1996, p. 306).

Significance of the Study

The purpose of this study was to investigate preservice secondary school mathematics teachers' subject matter knowledge (SMK) of perimeter and area measurement. Thus, the present study provided some basic information about the readiness of preservice secondary school mathematics teachers (PSSMTs) to teach secondary school mathematics in general, and perimeter and area in particular. PSSMTs themselves had to possess a deep understanding of SMK of secondary school mathematics in general, and perimeter and area in particular, in order to facilitate their students' learning of mathematics.

Through the investigation of preservice secondary school mathematics teachers' SMK of perimeter and area in the context of particular tasks, this study might contribute to the discussion of what might constitutes SMK of perimeter and area, and how might it be assessed. The findings of this study might provide input for the State Education Departments, Teacher Education Division, Teacher Training Institutes, or Curriculum Development Centre to consider inservice courses which would facilitate current mathematics teachers' revisitation on some of the mathematics topics in the secondary school curriculum. Similarly, the findings from this study might also suggest to the mathematics teachers education programs at our public universities or teachers training institutes to provide opportunity for preservice teachers to revisit and reconstruct some of the fundamental mathematical ideas of secondary school mathematics.

Limitations of the Study

This study only involved eight preservice secondary school mathematics teachers. The subjects were drawn from the preservice secondary school mathematics teachers who enrolled in the 4-year Bachelor of Science with Education (B.Sc.Ed.) program in a public university in Peninsula Malaysia. Thus, the findings of this study could not be generalized to other preservice secondary school mathematics teachers enrolled in the 4-year Bachelor of Science with Education (B.Sc.Ed.) program in this public university, in other programs (e.g., Bachelor of Education (B. Ed.), Diploma in Education (Dip.Ed.)), or attending other universities and teacher training institutes.

This study also confined to two measurement concepts, namely perimeter and area. The present study did not examine other measurement concepts such as time, length, mass, surface area, and volume as well as other mathematical topics.

Data for this study was collected using clinical interview technique. The technique for collecting data is not free of limitations. In fact, Ginsburg (1981) pointed out that the technique relies on the verbal reflections of the interviewee, the contingent questioning, and the creativity of the interviewer. Nik Azis (1987) noted that "the technique raised some methodological questions in some circles, because reflection, contingency, and creativity are considered to be unorthodox from the perspective of the standardized, objective, and replicable scientific method" (p. 61). For instance, the clinical interview technique has been criticized for (a) "lack of standardization of procedures, and (b) lack of information for precise replication" (Nik Azis, 1987, p. 61).

The above criticisms were combatted by the standardized interview plans. Each time a task is presented to the preservice teacher, a standardized interview plan was employed as it represents the initial goals of the researcher. However, the freedom to depart from the initial plan is a compensating characteristic of the clinical interview technique, taking into consideration of each preservice teacher's unique path of cognition and action. Moreover, every possible contingency was anticipated in advance in the preparation of the standardized set of interview questions and probes. Nevertheless, precise replication is impossible in any interview (Nik Azis, 1987).

In this study, the researcher employed case study research design to examine, in-depth, preservice secondary school mathematics teachers' subject matter knowledge (SMK) of perimeter and area. Merriam (1998) pointed out that "the special features of case study research that provide the rationale for its selection also present certain limitations in its usage" (p 42). For instance, the product of case study research has been criticized as "too lengthy, too detailed, or too involved for busy policy makers and educators to read and use" (Merriam, 1998, p. 42). This frequent criticism was also observed by Yin (2003).

In qualitative studies, the researcher is the primary instrument of data collection and analysis. Thus, qualitative case studies are also bounded by the integrity and sensitivity of the researcher (Merriam, 1988). Furthermore, "what people think they're doing, what they say they are doing, what they appear to others to be doing, and what in fact they are doing, may be sources of considerable discrepancy" (MacDonald & Walker, 1977, p. 186). Thus, the researcher could only make inferences about what the preservice teachers were thinking based on their verbal and nonverbal behaviours. Case study research has also been criticized "for its lack of representativeness… and its lack of rigor in the collection, construction, and analysis of the empirical materials that give rise to the study" (Hamel, 1993, p. 23). This criticism was also highlighted by Yin (2003).