CHAPTER TWO

REVIEW OF LITERATURE

Introduction

This chapter discusses the review of related literature which includes seven main sections: research on teacher education in Malaysia, teachers' knowledge, subject matter knowledge (SMK), research related to Malaysian preservice teachers' SMK, research related to preservice teachers' SMK of perimeter and area, measurement, and students' performance in perimeter and area. The conceptual framework of the present study was also discussed in this chapter.

Research on Teacher Education in Malaysia

The preservice teacher education programs at the teacher training institute and university levels aim to prepare the future teachers with adequate content knowledge in the subject they plan to teach. The education programs are also design to equip them with some basic pedagogical and management skills. However, Lourdusamy and Tan (1992) revealed that teachers encounter a number of difficult issues and problems. They found that teachers encountered problems in the classroom such as the students' negative attitudes, lack of interest in studies, passiveness, and failure to understand the lesson taught. These problems made it difficult for teachers to carry out their duties, and it affected their feeling of confidence in carrying out their tasks. Furthermore, the Deputy Director of Teacher Education Division pointed out (The News Straits Times, September 30, 1990) that "some teachers feel so intimidated by bright children in their schools that they can find no other resource but to seek transfer to a less challenging environment or adopt authoritarian and dogmatic teaching styles" (Lourdusamy & Tan, 1992, p. 184).

According to Nik Azis (2008), subject matter knowledge (SMK) and mathematical discourse are the important aspects in the preparation of mathematics teachers. Teachers' self confident, comfort, and knowledge about mathematics would influence what they teach and how they teach it. Moreover, teachers' conceptions about mathematics would determine the types of mathematical tasks and problems that they provide to their students, the types of learning environments that they offer, and the ways intellectual discourse conducted in the classroom.

Nevertheless, a few studies have indicated that Malaysian trainee teachers in the teacher training institutes (formerly known as teacher training colleges) had demonstrated a poor understanding of mathematical concepts and a lack of mathematical skills (Cheah, 2001; Koe, 1992; Ng, 1995). Cheah (2001) found that trainee teachers believed that mathematics was mainly procedural and their learning of mathematics focused on procedures, algorithms, and the use of formulae. As a result, their beliefs about teaching mathematics were also focused procedural in nature. The findings of Koe's (1992) study showed that trainee teachers had difficulties in answering mathematics questions taken from the primary six standardized examinations. Ng (1995) also found that trainee teachers were weak in their mathematics content.

Fatimah (1997) had examined eight Form Two mathematics teachers' problem solving schemes through Piaget's clinical interview model. She found that most of the subjects viewed mathematics from two perspectives, namely the pure mathematics and applied mathematics perspectives. Though problem solving was regarded as an important component in school mathematics, Fatimah (1997) noted that most of the subjects used demonstration-practice strategy in their teaching activities which were based on their learning experience as students in schools. They modeled after their previous exemplary teachers. They held the conceptions of mathematics and problem solving that were not in accordance with their actual practices in the classrooms.

Ng (1992) examined teachers' perceptions of the concept, 'understanding in mathematics' by 85 experienced mathematics teachers and 76 mathematics trainee teachers who gave their views on five aspects of understanding. These five aspects were characteristics of understanding, teaching and learning for understanding, how can understanding be assessed, and levels of understanding. Ng (1992) found that these two groups of teachers generally held similar perceptions. They agreed that understanding a topic involved central ideas and consisted of the ability to decide if a given answer was correct or not. In addition, understanding can be determined by knowledge of valid conditions for a conclusion, translation of a conclusion to other form, illustration of a conclusion by concrete materials and description of an observation in one's own language.

In fact, earlier studies had noted similar observations. Ng (1990) stated that the new mathematics for secondary school in Malaysia stressed on a balance between understanding of concepts and mastery of skills which would provide meaningful learning. Nevertheless, he found that many students disliked mathematics because mathematics was hard and boring. These students learnt mathematics through memorization. In another study, Nik Azis and Ng (1990) found that about 23% of the Form One mathematics teachers said that they had not been given any specific training about KBSM mathematics. Thus, the teachers needed specific guidance to teach certain mathematics topics. The trainers were professed in conceptual as well as procedural knowledge. This study also revealed that mathematics textbooks were the main resource for teachers to enhance their knowledge about KBSM mathematics. Nevertheless, the mathematics textbooks were noted to be too prescriptive in nature and most of the mathematics teachers did not know how to use them in a constructive manner. Thus, they need to be given specific guidance and training to overcome this textbooks crisis. This study also found that most of the teachers used traditional teaching methods based on the stimulus-response theory. The 'teaching for examination' has evident.

Seow (1989) investigated the conceptions of mathematics and the teaching of mathematics held by four teacher trainees in primary schools and the relationship between their conceptions and teaching behaviors. The findings of her study showed that there were similarities in the conceptions of the trainees towards mathematics teaching. The trainees preferred a prescriptive approach in their lessons. They perceived correct answers as an important indicator of teaching effectiveness. Her study also revealed that there was a close relationship between the trainees' conceptions and their teaching behavior. Marzita (1998) investigated the extent and nature of mathematics anxiety in primary school teacher trainees in Malaysia, and identified the factors that are associated with it. She found that teachers-students relationship, teachers' teaching style, examination pressure, parental and peer group influences were the main factors contributing to the trainees' mathematics anxiety.

The preceding observations indicated the importance of teachers' knowledge in the subject matter and there is lacking of this area of research. Thus, the researcher would like to explore the preservice secondary school mathematics teachers' SMK of perimeter and area.

Teachers' Knowledge

Fennema and Franke (1992) advocated that "no one questions the idea that what a teacher know is one of the most important influences on what is done in classroom and ultimately on what students learn" (p. 147). Moreover, “teachers who do not themselves know a subject well are not likely to help students learn this content.” (Ball, Thames, & Phelps, 2008, 404). The same goes for mathematics teacher as well.

However, “there is still very little research that looks in depth at what teachers understand about mathematics.” (Brown & Baird, 1993, p. 247). Furthermore, secondary school mathematics is quite complex. Thus, teachers should know secondary school mathematics curriculum well so as to teach it well. Shulman (1986) found that knowledge of subject matter affects the manners in which teachers teach. Teachers with strong mathematical knowledge were more conceptual in their teaching while teachers with weak mathematical knowledge were more rule-bounded or procedural in their teaching.

Shulman (1986) suggested a framework for analyzing teachers' knowledge that differentiated three categories of knowledge, namely subject matter knowledge (SMK), pedagogical content knowledge (PCK), and curricular knowledge. SMK is “the amount and organization of the knowledge per se in the mind of the teacher" (p. 9). PCK includes "the ways of representing and formulating the subject that make it comprehensible to others" and "an understanding of what makes the learning of specific topics easy or difficult, the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" (p. 9). Curricular knowledge refers to knowledge of instructional materials available for teaching various topics and the "sets of characteristics that serves as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances" (p. 10).

Shulman (1986) referred to the absence of focus on subject matter knowledge (SMK) for the research on teaching as the “missing paradigm” (p. 6). This referral suggested that SMK is an important component of teachers' knowledge. Moreover, “subject-matter knowledge is widely accepted as a central component of what teachers need to know” (Ball & McDiarmid, 1990, p. 437). Ball and McDiarmid (1990) pointed out that:

> When teachers possess inaccurate information or conceive of knowledge in narrow ways, they may pass on these ideas to their students. They may fail to challenge students’ misconceptions: they may use texts uncritically or alter them inappropriately. Subtly, teachers’ conceptions of knowledge shape their practice憫he kinds of questions they ask, the ideas they reinforce, the sorts of tasks they assign. (p. 438)

In summary, research on teaching and on teacher knowledge suggests that teachers’ SMK influence their students’ opportunity to learn.

Subject Matter Knowledge

According to Shulman (1986), SMK (also known as content knowledge) is knowledge about a subject (e.g., mathematics) and its structures (i.e., substantive and syntactic structures). Thus, SMK of mathematics encompasses two components, namely knowledge of the basic concepts and principles of mathematics, and knowledge of the ways in which mathematical truth is established (Shulman, 1986). Shulman (1986) pointed out that teachers need to have two types of understanding of the subject matter, namely knowing that and knowing why. He argued that:

> We expect that subject matter content understanding of the teacher be at least equal to that of his or her lay colleague, the mere subject matter major. The teacher need not only understand *that* something is so; the teacher must further understand *why* it is so. (p. 9)

Even and Tirosh (1995) noted that "knowing that" includes “declarative knowledge of rules, algorithms, procedures, and concepts related to specific mathematical topics in the school curriculum” (p. 7). Nevertheless, "knowing that" alone is not adequate. “Knowledge which pertains to the underlying meaning and understanding of why things are the way they are, enables better pedagogical decisions” (Even & Tirosh, 1995, p. 9). Ball (1990a) pointed out that:

> In order for students to develop power and control in mathematics, students must learn to validate their answers. They must have opportunity to make conjectures, justify their claims, and engage in mathematical arguments, all of which depend upon and can extend students' understanding of concepts and procedures. (pp. 457-458)

Thus, teachers themselves should understand mathematics deeply and flexibly so as to facilitate this kind of understanding.

Bromme and Steinbring (1994) stated that research on teaching have pointed out the importance of SMK as one of the variables of effective teaching. They argued that students were able to understand and remember concepts if it made sense to them. SMK is a crucial component of teachers’ knowledge because it “affects both what (the teachers) teach and how they teach it” (NCTM, 1991, p. 132). Nonetheless, “teachers' mathematical knowledge is far from being satisfactory even in terms of the standards for high school mathematics” (Harel, 1994, p. 113).

Olayi (1990) noted that the mathematics content of the mathematics teacher education program in the university now is the mathematics content beyond the level the preservice teachers will teach. Thus, Olayi (1990) suggests that the mathematics content for preservice mathematics teacher education program must encompasses two components: MC1, the mathematics content at the level they will teach, and MC2, the mathematics content beyond the level they will teach. The inclusion of MC1 in the mathematics content of the mathematics teacher education program was suggested for the following reasons (Olayi, 1990):

1. "It has been generally accepted that much of the preparation of a mathematics teacher lies in the task of studying the development of mathematical concepts and skills at the level he or she is going to teach.
2. The preservice teachers were not adequately prepared in secondary school mathematics in the secondary school days due to lack of teachers, time, or the rush to cover the syllabus.
3. Even if we assume that the preservice teachers were taught mathematics properly and adequately in their school days, they had not then reached a mathematical standards adequate for understanding the mathematics taught in secondary schools.
4. Assuming his or her mathematics was adequate and properly taught, and his or her mathematical maturity adequate in his or her school days, the preservice mathematics teacher often fails to realize how he or she has built up his or her concepts over a long period of years.
5. The inclusion of MC1 will improve the graduate's attitude to mathematics and confidence in what he or she does understand, enabling him or her to teach effectively.
6. It will repair the failure in understanding left by his or her previous education.
7. Much of MC2 is at a high level of abstraction, and is apparently totally unrelated to the everyday concerns of those he or she will teach. This will negate his presentation of school mathematics unless he is properly versed in MC1 too." (p. 698)

Hiebert and Lefevre (1986) emphasized that knowledge has two components, namely conceptual and procedural components. Conceptual knowledge is "knowledge that is rich in relationships". It consists of "network in which the linking relationships are as prominent as the discrete pieces of information" being linked (Hiebert & Lefevre, 1986, pp. 3-4). Procedural knowledge refers to "the formal language, or symbol representation system, of mathematics" and "the algorithms or rules for completing mathematical tasks" (Hiebert & Lefevre, 1986, p. 6). Van de Walle (2001) defines conceptual knowledge as "logical relationship constructed internally and existing in the mind as part of a network of ideas" and procedural knowledge as "knowledge of the rules and procedures that are uses in carrying out routine mathematical tasks and also the symbolism that is used to represent mathematics" (p. 31).

Skemp (1978) distinguished the meaning of instrumental and relational understanding. Instrumental understanding is described as "rules without reason" and relational understanding refers to "knowing both what to do and why" (Skemp, 1978, p. 9). Skemp (1978) summarized the difference between instrumental and relational learning as knowing how as compared to knowing how and why. Kinach (2002) argued that "what teachers espousing these two philosophies of mathematics understanding teach about a topic is very different" (p. 54). For instance, teaching division by fraction based on instrumentalist view would mean teaching division algorithm without understand its underlying concepts and principles. By comparison, teachers embrace a relational view would likely expand the instrumentalist's teaching of division by fraction by justifying why the division algorithm works. To the instrumentalist, memorizing rules, facts, and procedures are clear indicator of student achievement. By contrast, for a teacher who espousing a relational view of mathematics, student achievement is much more than memorizing. "Problem posing, critical and contextual thinking, the ability to justify and represent one's thinking mathematically are part of what mathematics achievement means" (Kinach, 2002, p. 54).

Ball (1988, 1990a; 1991b) divided SMK into two components, namely substantive knowledge of mathematics, and knowledge about the nature and discourse of mathematics. Substantive knowledge of mathematics refers to the understanding of particular topics, procedures, concepts, and the relationships among these topics, procedures, and concepts (Ball, 1988, 1990a; 1991b; Ball, Lubienski, & Mewborn, 2001; Davis, 1986; Hiebert & Lefevre, 1986; Skemp, 1978). No one would argue that teachers need substantive knowledge of mathematics of particular concepts and procedures such as perimeter, area, and conversion units of area.

Ball (1988, 1990a, 1991a) outlined three criteria that characterize the kind of substantive knowledge teachers needed. First, teachers' knowledge of concepts and procedures must be correct. Therefore, teachers should be able to draw a triangle, identify the formula for calculating the area of a parallelogram, calculating the area of a trapezium, and so on. Second, teachers should understand the underlying principles and meanings such as "what does 'base' and 'height' mean?" or "why is it 'a half' (referring to the formula for calculating the area of a triangle)?". Finally, teachers should appreciate and understand the relationships among mathematical ideas such as the relationships among measurement of length, area, and volume.

Knowledge about the nature and discourse of mathematics includes “understandings about the nature of knowledge in the discipline惘where it comes from, how it changes, and how truth is established.” It also includes “what it means to know and to do mathematics, what is the relative centrality of different ideas, what is arbitrary or conventional versus what is necessary or logical, and what is key to having a sense of the philosophical debates within the discipline” (Ball, Lubienski, & Mewborn, 2001, p. 444).

In summary, review of literature in this section had shown that the importance of conceptual knowledge and procedural knowledge cannot be denied in developing SMK of mathematics. However, the significance of other three components of SMK, namely linguistic knowledge, strategic knowledge, and ethical knowledge, had been neglected. Nevertheless, Nik Azis (1996) suggested that there are five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge. This applies also to SMK. Specifically, SMK encompasses five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge.

Conceptual knowledge refers to knowledge that is rich in multiple relationships that connects one knowledge scheme to another (Nik Azis, 1996). Procedural knowledge encompasses two components, namely mathematical language or symbol representation system, and algorithm or rule for engaging in mathematical activities. The first component consists of knowledge about symbols for representing mathematical ideas and the awareness about rules for forming equation or write symbol in general accepted format. The second component comprises knowledge about rules, laws, or algorithms that can be used to solve mathematical problem. The main characteristic of this component is that the solution must be carried out step by step in a designated linear sequence. Another feature of the second component is that procedural knowledge has specific structure (Nik Azis, 1996).

Linguistic knowledge refers to the rules of language and mathematical symbols. Some students encountered difficulty in understanding mathematical word problems as they do not understand what they read. Furthermore, they also lack of the ability to translate word problems into equation form or algebraic expressions (Nik Azis, 1996). With regard to strategic knowledge, Nik Azis (1996) stated that the selection of cognitive strategy is influenced by the values and beliefs of an individual. For instance, a student would not use a strategy if the student did not believe that the strategy would help him or her to solve the problem at hand. Strategic knowledge helps students to plan his or her solution. With respect to ethical knowledge, ethical means “a set of rule of behaviors or a set of principles (ideals) that guide an individual to distinguish good behaviors from bad behaviors, and then guide him or her to behave based on that distinction” (Nik Azis, 1996, p. 203). For instance, checking the correctness or reasonableness of one’s answers or solutions is a good behavior in mathematics (Nik Azis, 2007).

Research Related to Subject Matter Knowledge

Even and Tirosh (1995) found that:

> Many teachers do not have a solid understanding of the subject-matter they teach. In fact, serious misunderstandings were found at the level of mere knowledge of rules, procedures, and concepts of almost every topic investigated (i.e., the concept of zero, division, proof, and function). (p. 6)

Furthermore, Toumasis (1992) argued that “many preservice teachers have no opportunity to study in depth some very important concepts which they will teach.” (p. 290). Thus, they need special training in developing a solid understanding of certain school mathematics content.

Ball (1990a) revealed that "many children and adults perform mathematical calculations without understanding the underlying principles or meaning" (p. 458). For instance, almost all the prospective teachers in her study were able to calculate $1\frac{3}{4} \div \frac{1}{2}$ correctly. Nevertheless, only a few of them were able to represent the meaning underlying the procedure they had learned. This finding indicates that prospective teachers lacked explicit understanding of concepts and principles even though they had successfully performed the calculations involved. Ball (1990a) argued that:

> In order to help someone else understand and do mathematics, teachers must not only be able to describe the steps for following an algorithm but also discuss the judgments made and the meanings of and reasons for certain relationships or procedures. Teachers must be able to generate explanations or other representations, often on the spot in response to a student question. (pp. 458-459)

Even (1990) observed that “interest in teachers' SMK has arisen in recent years” (p. 521). However, she found that most of the studies about teachers' SMK have been general and not topic specific. According to Even (1990), "analyzing what teachers' subject matter knowledge means in general in mathematics, does not inform us of what subject matter knowledge teachers need to have in order to teach a specific piece of mathematics" (p. 522). We need to know more about the specific characteristics of knowledge needed for teaching a specific mathematics topic such as perimeter and area.

In summary, studies reviewed in this subsection had discussed the importance of SMK and problems confronted by the preservice teachers pertaining to SMK. However, these studies did not highlight the main components of SMK, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge.

Conceptual knowledge

Conceptual knowledge is "knowledge that is rich in relationships". It consists of "network in which the linking relationships are as prominent as the discrete pieces of information" being linked (Hiebert & Lefevre, 1986, pp. 3-4). This section reviews literature related to the conceptual knowledge of perimeter and area.

Notion of Perimeter and Area

Literally, perimeter means the measurement around. However, numerous definitions of perimeter were provided by the researchers or mathematics educators. Table 2.1 shows some of these definitions. According to Beaumont, Curtis, & Smart (1986), "the study of the concept of perimeter is unified by considering the simple closed curve" (p. 5). A closed but not simple curve has more than one interior. For all simple closed curves, "the perimeter is the total length of the curve - the total distance around it." (Beaumont et al., 1986, p. 5). Perimeter is an extension from the concept of length. Beaumont et al. (1986) emphasized that perimeter is the distance around and measured in linear units (e.g., mm, cm, m, km). The following question has not been answered: What are PSSMTs' notions of perimeter? The present study attempted to answer such question.

Table 2.1

Some of the Definitions of Perimeter

Researchers or mathematics educators	Definition of perimeter
Ball, 1988, p. 170.	The perimeter of a figure or a region is the length of its boundary.
Beaumont, Curtis, & Smart, 1986, p. 5.	The perimeter is the total length of the curve - the total distance around it.
Bennett & Nelson, 2001, p. 658.	The length of the boundary of a region is its perimeter.
Billstein, Liberskind, & Lott, 2006, p. 743.	The perimeter of a simple closed curve is the length of the curve.
Cathcart, Pothier, Vance, & Bezuk, 2006, p. 325.	Perimeter is the total distance around a closed figure.
Haylock, 2001, p. 268.	The perimeter is the length of the boundary.
Kennedy & Tipps, 2000, p. 512.	Perimeter is the measure of the distance around a closed figure.
Long & DeTemple, 2003, p. 771.	The length of a simple closed plane curve is called its perimeter.
O'Daffer, Charles, Cooney, Dossay, & Schielack, 2005, p. 676.	Perimeter is the distance around a figure.
Rickard, 1996, p. 306.	Perimeter is the number of (linear) units required to surround a shape.
Suggate, Davis, & Goulding, 1999, p. 129.	Perimeter is the distance around the edge of a shape.

Similarly, numerous definitions of area were provided by the researchers or mathematics educators. Table 2.2 depicts some of these definitions. Martin and Strutchens (2000) stated that:

> The concept of area is often difficult for students to understand, perhaps due to their initial experiences in which it is tied to a formula (such as area = length × width) rather than more conceptual activities such as counting the number of square units it would take to cover a surface. (p. 223)

Cavanagh (2008) found that 53% of the 43 Year 7 students from two government high schools in Sydney in his study defined area as 'space inside the shape' while 19% referred it as 'length by width'. However, Tierney, Boyd, and Davis (1990) revealed that many prospective primary school teachers from a teachers college in their study thought that area is 'length by width'. When the prospective teachers were asked what they would teach a ten year old child about area, "80% of them drew a rectangle and wrote '*l* × *w*' or '*l* by *w*' near it. Some of these students (prospective teachers) placed arrows aroud a rectangle in a way which denoted perimeter rather than area" (Tierney et al., 1990, pp. 307-308). The remaining 20% of prospective teachers defined area as the space inside a figure. Furthermore, Casa, Spinelli, and Gavin (2006) noticed that many adults thought that area is ‘length by width’. “They understand area as a formula rather than as a concept惘the amount of space covered by the inside boundaries of a two-dimensional figure” (Casa et al., 2006, p. 168).

Baturo and Nason (1996) suggested that area can be viewed from two different perspectives, namely static and dynamic perspectives. From the static perspective, area can be viewed as the amount of surface enclosed within a boundary. If a preservice teacher selected one or more open shapes and explained that the shape(s) had an area of zero, then it indicated that the preservice teacher is having a dynamic perspective of area.

Table 2.2

Some of the Definitions of Area

Researchers or mathematics educators	Definition of area
Ball, 1988, p. 170.	The area is the number of unit squares it takes to cover the figure or region.
Bennett & Nelson, 2001, p. 653.	The number of units it takes to cover a surface (or region) is called its area.
Billstein, Liberskind, & Lott, 2006, p. 750.	Area of a region is the number of nonoverlapping square units that covers the region.
Cathcart, Pothier, Vance, & Bezuk, 2006, p. 330.	Area is the amount of surface enclosed by a curve in the plane.
Haylock, 2001, p. 268.	Area is a measure of the amount of two-dimensional space inside a boundary.
Long & DeTemple, 2003, p. 771.	The number of units required to cover a region in the plane is the area of the region.
Rickard, 1996, p. 306.	Area is represented as the number of square units needed to cover a shape.
Suggate, Davis, & Goulding, 1999, p. 134.	Area-amount of surface.

Baturo and Nason (1996) found that none of the 13 preservice primary school teachers in their study selected open shapes (including the lines) as having an area. It can be inferred that they did not have a dynamic perspective of the notion of area. Furthermore, all of them indicated that these shapes (i.e., open shapes) needed to be closed showing that they had a static perspective of the notion of area. Baturo and Nason (1996) also found that three of the preservice teachers in their study appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). However, the following question remains to be answered: What are the PSSMTs' notions of area?

Notion of the Units of Area

Cavanagh (2008) noted that:

> The basis of area measurement lies in understanding how a specified unit can be iterated until it completely covers a flat surface, without leaving gaps or overlaps. In other words, the region is partitioned into equal-sized units which tessellate the plane. (p. 55)

Similarly, Beaumont et al. (1986) stated that "students are already familiar with linear units of measurement before they introduced to perimeter. But for the concept of area, the concept of units of area must come at the same time" (p. 11). Area is measured in square units while perimeter is measured in linear units. A square unit is a square with each side equal to one unit. For instance, 1 cm^2 (read as 1 square centimetre) is a square unit with each side equal to 1 cm. In the metric system, the basic unit of area is the square metre (m^2).

Beaumont et al. (1986) suggests that one way to extent the concept of area is through the introduction of nonsquare units of area (e.g., rectangular units, triangular units). In reality, any shape or figure that tessellate the plane (i.e., covers the plane without gaps or overlapping) could be chosen as a unit of area (Bennett & Nelson, 2001; Cathcart, Pothier, Vance, & Bezuk, 2006; Long & DeTemple, 2003; O'Daffer, Charles, Cooney, Dossay, & Schielack, 2005). Nevertheless, squares have been found to be the most common and convenient shape for measuring area. The size of the square is arbitrary. In summary, any shape or figure (i.e., square and nonsquare) that tessellate the plane could be chosen as a unit of area. However, the following question has not been answered: What are the PSSMTs' notions of the unit of area?

Inverse Relationship between Number of Units and Unit of Measure

Lehrer (2003) argued that "measurement with different-sized units imply that different quantities can represent the same measure. These quantities will be inversely proportional to the size of the units" (p. 181). Van de Walle (2007) suggests that students should be given opportunity to measure the same attribute with different-sized (varying-sized) units. Such activities would enhance their understanding of the inverse relationship (inverse proportion) between units of measure and number of unit that the larger the unit of measure, the smaller the number of unit required to measure the same attribute, and vice versa. Similarly, Cathcart, Pothier, Vance, and Bezuk (2006) suggest that students should be challenge to predict the effect of using a larger or smaller unit of measure. These activities would lead the students to discover the inverse relationship between the size of the unit (unit of measure) and the number of units.

Lindquist and Kouba (1989) noted that "there are many concepts associated with the role of the unit in measuring that are crucial to understanding measurement. One of these concepts is concerned with the relationship between the unit and the number in a measurement" (p. 36). In the Fourth Mathematics Assessment of the National Assessment of Educational Progress (NAEP), two items were administered to assess students' understanding of this concept. Results from the Fourth Mathematics Assessment of the National Assessment of Educational Progress (NAEP) revealed that almost two-thirds of grade three students knew that it takes more of a smaller unit than a larger unit to fill a box. However, in the other item, more than half of the grade three and grade seven students named the person who used the most units as the one with the largest units. It indicated that they focused on the number of unit rather than the unit of measure when they were comparing the measurement. It also indicated that this concept (i.e., the relationship between the unit and the number in a measurement) was not completely understood by these students.

However, Baturo and Nason (1996) found that all the 13 preservice primary school teachers in their study had an understanding of inverse relationship (indirect proportion) between number of units and unit of measure that the larger the unit of measure, the smaller the number of units, and vice versa. However, the following question remains to be answered: Do the PSSMTs focus on the number of units or unit of measure when comparing perimeters as well as areas with (a) nonstandard units, (b) common nonstandard units, and (c) common standard units? Do the PSSMTs understand the inverse relationship (indirect proportion) between number of units and unit of measure?

Relationship between the Standard Units of Area Measurement

Van de Walle (2007) suggests that "students must not only develop a familiarity with standard units but must also learn appropriate relationships between them" (p. 377). Thus, students must learn appropriate relationships between standard units of area measurement that 1cm^2 = 100 mm^2, 1 m^2 = 10 000 cm^2, and 1 km^2 = 1000 000 m^2. Similarly, they must also know the relationships between standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. Nevertheless, Beaumont et al. (1986) emphasized that "understanding of the relationship should come first and students should be able to make a drawing that will explain the relationship clearly" (p. 15).

Baturo and Nason (1996) found that most of the preservice primary school teachers in their study demonstrated a lack of knowledge about the relationship between the standard units of area measurement. They revealed that 11 of the 13 preservice primary school teachers in their study stated that there were 100 square centimetres in a square metre and thus 128 cm^2 was larger than 1 m^2. However, the following questions have not been answered: Do the PSSMTs know the relationship between the standard units of area measurement? Likewise, do the PSSMTs know the relationship between the standard units of length measurement?

Relationship between Area Units and Linear Units of Measurement

Units of area are derived from corresponding linear units (Cathcart, Pothier, Vance, & Bezuk, 2006). The finding of previous research (Baturo & Nason, 1996) revealed that many preservice primary school teachers in their study demonstrated a lack of knowledge about the relationship between area units and linear units of measurement that they did not know that area units are derived from linear units based on squaring. Thus, they did not know how to derive area units from linear unit. Do the PSSMTs know the relationship between area units and linear units of measurement? The present study attempted to answer such question.

Relationship between Perimeter and Area

Ferrer, Hunter, Irwin, Sheldon, Thompson, and Vistro-Yu (2001) observed that students in many parts of the world encountered difficulty in understanding the concepts of perimeter and area. It is even harder to fully understand the nonconstant relationship between perimeter and area (Ferrer et al., 2001). Bennett and Nelson (2001) pointed out that:

> Intuitively, it may seem that the area of a region should depend on its perimeter. For example, if a person uses more fences to close in a piece of land than another person, it is tempting to assume the first person has enclosed the greater amount of land. However, this is not necessarily true. (p. 658)

Thus, two shapes with the same perimeter could have different areas. Similarly, two shapes with the same area could have different perimeters. For a given perimeter, the dimensions of a shape affect its area. For instance, for a given perimeter of 20 cm, the area of a rectangle could be 9 cm × 1 cm = 9 cm^2, 8 cm × 2 cm = 16 cm^2, and so on.

There is no direct relationship between perimeter and area (Ball, 1988; Haylock, 2001). Nevertheless, previous studies revealed that many students and prospective teachers had a misconception that there is direct relationship between perimeter and area (Arnold, Turner, & Cooney, 1996; Ball, 1988; Chappell, & Thompson, 1999; Tierney, Boyd, & Davis, 1990; Woodward, 1982; Woodward & Byrd, 1983). They thought that two shapes with the same perimeter have the same area, and the shape with the longer perimeter has the larger area. They also thought that as the perimeter of a closed figure increases, the area also increases. Woodward (1982) found that an excellent seventh grade student, Heidi, thought that the garden with the same perimeter have the same area. Woodward and Byrd (1983) revealed that 59% of the 129 eight grade students at a junior high school in Tennessee thought that the garden with the same perimeter have the same area. They also revealed that 63% of the 129 eight grade students at another junior high school in Tennessee thought that the garden with the same perimeter have the same area. Woodward and Byrd (1983) found that prospective elementary teachers took the test with similar results.

Arnold et al. (1996) revealed that most of the middle school and university students in their study thought that when the perimeter of a shape is held constant, its area remains constant. Likewise, Chappell and Thompson (1999) found that only one out of 29 (i.e., 3%) grade six students in their study were able to justify that two shapes with the same area could have different perimeters. None of the 19 grade seven students in their study were able to justify that two shapes with the same area could have different perimeters while three out of 16 (i.e., 19%) grade eight students in their study were able to justify that two shapes with the same area could have different perimeters.

Tierney et al. (1990) noticed that about half of the prospective primary school teachers from a teachers college in their study thought that the shape with the longer perimeter has the larger area. When the prospective teachers were asked to compare areas of two cardboard shapes, "about half of them held the cardboard shapes edge to edge to compare perimeters" (Tierney et al., 1990, p. 311). The prospective teachers were attempting to measure the area of the shapes with linear measure of the edges. They thought that the shape with the longer perimeter has the larger area.

Ball (1988) found that only one of the nine prospective secondary teachers in her study knew that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct. Jon knew that there is no direct relationship between perimeter and area. Five of the prospective secondary teachers in her study thought that there is direct relationship between perimeter and area. Thus, they thought that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was correct. The remaining three prospective secondary teachers were not sure whether the student's "theory" was correct or not.

Ball (1988) also found that only two of the ten prospective elementary teachers in her study knew that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct. They knew that there is no direct relationship between perimeter and area. Three of the prospective elementary teachers in her study thought that there is direct relationship between perimeter and area. Thus, they thought that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was correct. The remaining five prospective elementary teachers were not sure whether the student's "theory" was correct or not. Arnold et al. (1996) also revealed that most of the middle school and university students in their study thought that when the perimeter of a shape increases or decreases, its area also increases or decreases. The following questions have not been answered: Do the Malaysian PSSMTs know that there is no direct relationship between perimeter and area or do they had the similar misconception?

Relationship among Area Formulae

When students engage in developing formulae, they acquire conceptual understanding of the ideas and relationships involved. Thus, there is less possibility that they will confuse perimeter and area or choose the incorrect formula (Van de Walle, 2007). Moreover, "areas of triangles, parallelograms and trapeziums are related to the area of a rectangle. These relationships form the reasoning for the formulae for the areas." (Lim-Teo & Ng, 2008, p. 106). Specifically, formulae for calculating the area of other shapes are developed from the formula for the area of a rectangle (O'Daffer & Clemens, 1992).

However, the findings of previous studies (Baturo & Nason, 1996; Cavanagh, 2008) revealed that high school students as well as preservice teachers demonstrated a limited understanding of the relationship between the areas of triangle and rectangle. Baturo and Nason, (1996) found that only two of the 13 preservice primary school teachers in their study understand the relationship between the formulae for the area of a triangle and rectangle that encloses it. The area of a triangle is half of the area of the rectangle that encloses it.

Similarly, Cavanagh (2008) revealed that high school students in his study demonstrated a limited understanding of the relationship between the areas of triangle and rectangle. They did not make use of the fact that the area of a triangle is half of the area of the rectangle that encloses it. This was apparent when they were asked to calculate the area of a 3-4-5 cm right-angled triangle, which included tick marks at 1 cm intervals along the perpendicular sides. Cavanagh (2008) found that many students in his study chose to construct a grid and attempt to count the squares instead of making used of the rectangle-triangle relationship. Another indication was that some students multiplied the lengths of all three sides (i.e., $3 \times 4 \times 5$) to find the area of the triangle. The following question remain to be answered: Do the Malaysian PSSMTs understand the relationship among area formulae of rectangle, parallelogram, triangle and trapezium?

Procedural knowledge

Procedural knowledge refers to "the algorithms or rules for completing mathematical tasks" (adapted from Hiebert & Lefevre, 1986, p. 6). This section reviews literature related to the procedural knowledge of perimeter and area.

Converting Standard Units of Area Measurement

In the metric system (or System International, SI), the most commonly used units of area are the square millimetre (mm^2), square centimetre (cm^2), square metre (m^2), and square kilometre (km^2). "It is often necessary to convert from one area measure to another within a system" (Billstein, Liberskind, & Lott, 2006, p. 752). As mentioned earlier in the previous section (i.e., conceptual knowledge), Van de Walle (2007) suggests that "students must not only develop a familiarity with standard units but must also learn appropriate relationships between them" (p. 377).

Thus, students must learn appropriate relationships between standard units of area measurement that $1cm^2 = 100\ mm^2$, $1\ m^2 = 10\ 000\ cm^2$, and $1\ km^2 = 1000\ 000\ m^2$. Similarly, they must also know the relationships between standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. Nevertheless, Beaumont et al. (1986) emphasized that "understanding of the relationship should come first and students should be able to make a drawing that will explain the relationship clearly" (p. 15). Likewise, Billstein et al. (2006), and Musser, Burger, and Peterson (2003) also suggest that a drawing be made in order to show the relationship clearly (pictorially).

For instance, when the students convert $3\ cm^2$ to mm^2, $4.7\ m^2$ to cm^2, and $1.25\ km^2$ to m^2, their procedural knowledge of converting standard units of area measurement could be assessed. At the same time, their conceptual knowledge of the relationships between standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m, relationships between standard units of area measurement that 1 cm^2 = 100 mm^2, 1 m^2 = 10 000 cm^2, and 1 km^2 = 1000 000 m^2, and relationships between area units and linear units of measurement that area units are derived from linear units based on squaring, could be determined.

Ryan and Williams (2007) noticed that students encountered difficulty in converting standard units of area measurement. For instance, only 3% of 13 year olds students in their study could correctly find the area in square metres of an A4 paper measuring 210 mm by 297 mm (using a calculator). 12% of them used a conversion factor of 1000:1 while 31 % committed other decimal errors (Ryan &Williams, 2007). Ryan and Williams (2007) argued that students' errors "are indicating that their concept of measurement does not include a recognition of the importance of identifying the unit of measure" (Ryan &Williams, 2007, p. 102).

In summary, the review of literature in this subsection points to the benefit of converting standard units of area measurement as well as the significance of identifying the unit of measure. However, the following questions have not been answered: Do the PSSMTs have adequate procedural knowledge of converting standard units of area measurement? Do the PSSMTs have an understanding of the conceptual knowledge of the relationships between standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m, relationships between standard units of area measurement that 1cm^2 = 100 mm^2, 1 m^2 = 10 000 cm^2, and 1 km^2 = 1000 000 m^2, and relationships between area units and linear units of measurement that area units are derived from linear units based on squaring? The present study attempted to answer such questions.

Calculating Perimeter and area of Composite Figures

Cavanagh (2008) noted that “research has consistently demonstrated that students across all ages can experience difficulties when attempting to find areas of basic two-dimensional shapes” (p. 55). Cavanagh (2008) found that 72% of the 43 Year 7 students from two government high schools in Sydney in his study had successfully calculated the area of a rectangle, 5 cm by 3 cm, which displayed tick marks at 1 cm intervals around its perimeter. However, less than half (i.e., 44%) of the students in his study were able to calculate the area of a 3-4-5 cm right-angled triangle, which included tick marks at 1 cm intervals along the perpendicular sides. Cavanagh (2008) revealed that 12% of the students forgot to divide the product of base and height by two while 12% of them multiplied the lengths of all three sides (i.e., $3 \times 4 \times 5$). Furthermore, only 14% of the students in his study had successfully calculated the area of an L-shaped, rectangular figure. Cavanagh (2008) revealed that 35% of the students in his study attempted to find the perimeter instead of area of the figure while 19% did not attempt the task.

Tsang and Rowland (2005) conducted the SMK survey on 138 mathematics teachers from eight primary schools in Hong Kong. The survey involved ten items. In item 9, Hong Kong primary school mathematics teachers were asked to find the perimeter and area of the parallelogram drawn in the square grid (each square represents a square of length 1 cm). Tsang and Rowland (2005) found that about half of the teachers could recalled Pythagoras’ theorem in finding the length of the slanted side (5 cm) of the parallelogram and correctly calculated its perimeter (i.e., 18 cm). Tsang and Rowland (2005) reported that quite a number of teachers employed the area formula of a trapezium (not parallelogram) in finding the correct area of the parallelogram (i.e., 20 cm^2). It was also reported that a few teachers confused the formula for the area of a triangle with the formula for the area of a parallelogram and gave 10 cm^2 as the area of the parallelogram.

One of the learning outcomes enlisted in the Form One Mathematics Curriculum Specifications is 'find the areas of composite figures made up of rectangle, parallelogram, triangle, or trapezium' (Ministry of Education Malaysia, 2003a). This learning outcome could be extended to perimeter as well. However, previous study (Baturo & Nason, 1996) revealed that preservice primary school teachers in their study had inadequate procedural knowledge of calculating area of the given shapes. Moreover, Cavanagh (2008) and van de Walle (2007) found that students tend to confuse with the slanted side (slanted height) and the height (perpendicular height) of a parallelogram. Do the PSSMTs have adequate procedural knowledge of calculating perimeter and area of composite figures? The present study attempted to answer such question.

Developing Area Formulae

Beaumont et al. (1986) pointed out that "the formulas for perimeter do not compare in important to those for area, some teachers would prefer to defer them. Others see them as opportunity to emphasize calculator applications.…formulas can also be considered applications of algebra" (p. 8). NCTM (2000) suggests that "students should begin to develop formulas for perimeter and area in the elementary grades. Middle grades students should formalize these techniques, as well as develop formulas for the volume and surface area of objects like prisms and cylinders" (p. 46).

Furthermore, the Form One Mathematics Curriculum Specification (Ministry of Education Malaysia, 2003a) recommended that teaching and learning activities in the classroom to provide opportunity for the students to investigate and develop the formula for the area of a rectangle. It also suggested that students be given opportunity to investigate and develop the formulae for the area of triangles, parallelograms, and trapeziums based on the area of a rectangle. The question is, do the PSSMTs able to develop the formulae for the area of a rectangle, parallelogram, triangle, and trapezium? The present study attempted to answer such question.

Linguistic knowledge

Linguistic knowledge refers to “formal language, or symbol representation system of mathematics” (adapted from Hiebert & Lefevre, 1986, p. 5). This section reviews literature related to the linguistic knowledge of perimeter and area.

Mathematical Symbols

Conventionally, the formula for the area of a rectangle is written as ‘$l \times w$’, where l and w represents the length and the width of the rectangle (Beaumont et al., 1986; Billstein, Liberskind, & Lott, 2006; Van de Walle, 2007), or ‘$l \times b$’, where l and b represents the length and the breadth of the rectangle (Cheang, 2002; Chua, Teh, & Ooi, 2002). The formula for the area of a parallelogram is conventionally written as ‘$b \times h$’, where b and h represents the base and the height of the parallelogram (Beaumont et al., 1986; Billstein et al., 2006; Cheang, 2002; Chua et al., 2002; Van de Walle, 2007).

Conventionally, the formula for the area of a triangle is written as ‘$\frac{1}{2} \times b \times h$‘, where b and h represents the base and the height of the triangle (Beaumont et al., 1986; Billstein et al., 2006; Cheang, 2002; Chua et al., 2002; Van de Walle, 2007). The formula for the area of a trapezium is conventionally written as ‘$\frac{1}{2} \times (a + b) \times h$‘, where ($a + b$) and h represents the sum of the parallel sides and the height of the trapezium (Beaumont et al., 1986; Billstein et al., 2006; Cheang, 2002; Chua et al., 2002; Van de Walle, 2007).

In summary, the review of literature in this subsection illustrates the appropriate mathematical symbols used to write the formulae for the area of a rectangle, parallelogram, triangle, and trapezium. However, the following question remains to be answered: Do the PSSMTs use appropriate mathematical symbols to write the formulae for the area of a rectangle, parallelogram, triangle, and trapezium? The present study attempted to answer such question.

Mathematical Terms

Haylock (2001) pointed out that "a major problem for all the student-teachers was that mathematical language seemed to be too technical, too specific to the subject and not reinforced through their language use in everyday life" (p. 6). Nevertheless, the Learning Mathematics for Teaching (LMT) (2006) project identified teacher's use of language (i.e., linguistic knowledge) as one of the major components of teacher's knowledge of mathematics. In the LMT (2006) project, teacher's use of language encompasses three components as follow: (a) conventional notation (mathematical symbols), (b) technical language (mathematical terms and concepts), and (c) general language for expressing mathematical ideas (e.g., borrow). Moreover, Hill, Sleep, Lewis, and Ball (2007) emphasized that "teacher's use of mathematical language is both an indicator and target of growth in knowledge and skill; teachers need, in classrooms, to be able to use mathematical terms accurately and precisely" (p. 137).

Baturo and Nason (1996) found that nine of the 13 preservice primary school teachers in their study used inappropriate mathematical terms to justify the open shapes (including the lines) that do not have an area. These students described the shapes as: "not joined; having a gap; not a fixed area; hasn't been completed; there's a blank" (p. 251). The following questions remain to be answered:

1. Do the PSSMTs use appropriate mathematical terms to justify their selection of shapes that: (a) have a perimeter, (b) do not have a perimeter, (c) have an area, (d) do not have an area, (e) can be used as the unit of area, and (f) cannot be used as the unit of area?

2. Do the PSSMTs use appropriate mathematical terms to state the area formulae or to explain the meaning of the mathematical symbols they employed in the area formulae?

The present study attempted to answer such questions.

Standard Unit of Length Measurement (Linear Units) and Area Measurement (Square Units)

It is a general measurement convention that perimeter and area is measured by linear units (such as mm, cm, m, km) and square units (such as mm^2, cm^2, m^2, km^2) respectively. However, Tierney et al. (1990) noticed that many prospective primary school teachers from a teachers college in their study labelled the area measurements in linear units. Likewise, Baturo and Nason (1996) found that several preservice primary school teachers in their study wrote the calculated area measurement in linear unit such as 128 cm. They did not understand the general measurement convention that area is measured by square units. This behavior was repeated in most of the other items in Task 7 where preservice teachers in their study were asked to calculate the area of the given shapes.

Similarly, Cavanagh (2008) revealed that high school students in his study inappropriately labelled the length of sides in cm^2 or areas in cm on a written test which consisted of five questions. They did not understand the general measurement convention that length is measured in linear units while area is measured in square units. Do the PSSMTs understand the general measurement convention that perimeter and area is measured by linear units (such as mm, cm, m, km) and square units (such as mm^2, cm^2, m^2, km^2), respectively? The present study attempted to answer such question.

Conventions of Writing and Reading SI Area Measurement

The Form One Mathematics Curriculum Specifications (Ministry of Education Malaysia, 2003a) suggests that cm^2 to be read as 'square cm'. Likewise, Cathcart, Pothier, Vance, & Bezuk (2006) also reminded that student need to know the symbol m^2 is read as "square metre" and not "metre square" (p. 333). Nevertheless, Baturo and Nason (1996) found that only one of the 13 preservice primary school teachers in their study was able to correctly read 6 m^2 as 'six square metres'. The rest read it literally as 'six metres squared'. Do the PSSMTs understand the conventions pertaining to the writing and reading of Standard International (SI) area measurement units? The present study attempted to answer such question.

Strategic knowledge

Strategic knowledge refers to "our ability to choose an appropriate strategy to solve a task because it is more effective than alternative strategies" (Henson & Eller, 1999, p. 258). This section reviews literature related to the strategic knowledge of perimeter and area.

Strategies for Comparing Perimeter and Area

Kamii and Kysh (2006) stated that "measurement was invented to make indirect comparisons between two or more objects' (p. 113). In the Task 1 of their study, students were asked to compare the areas of two rectangles (i.e., 3 by 3 and 2 by 4) made on two geobaords. The finding of Kamii and Kysh's (2006) study indicated that 16%, 56%, 41 %, 83%, 53%, and 93% of the fourth, sixth, regular eighth, advanced eighth, regular ninth, and advanced ninth graders in their study employed the strategy of counting squares to compare the areas of the rectangles respectively. The finding of Kamii and Kysh's (2006) study also indicated that 68%, 38%, 59%, 175, 47%, and 7% of the fourth, sixth, regular eighth, advanced eighth, regular ninth, and advanced ninth graders employed the strategy of counting pegs to compare the areas of the rectangles respectively. The remaining 16% of the fourth graders used other strategies (e.g., counting units of length) to compare the areas of the rectangles.

Baturo and Nason (1996) categorized strategies for comparing areas into three types of methods as follow:

1. "Informal: (a) cut-and paste (i.e., cut one shape into pieces and paste on to other), and (b) overlay (i.e., place one shape on top of the other and then make adjustment.
2. Semi-formal: covering both shapes with a common shape or grid.
3. Formal: measuring the side and applying the formula." (p. 246)

Baturo and Nason (1996) found that five of the 13 preservice primary school teachers in their study used informal methods of cut-and-paste to compare areas while only one used informal methods of overlay to compare areas. None of them thought of using semi-formal method to compare areas.

The remaining seven preservice teachers employed the formal method of measuring the side and applying the formula. When probed for alternative methods of comparing areas, most of the preservice teachers could suggest other methods of comparing areas. Three of them thought of using semi-formal method of covering both shapes with a grid (Baturo & Nason, 1996). These preservice teachers demonstrated adequate strategic knowledge for comparing areas that they could generate other methods to compare areas besides the formal method of measuring the side and applying the formula (Baturo & Nason, 1996).

In summary, the review of literature in this subsection has identified three types of strategies for comparing areas (Baturo & Nason, 1996), namely informal, semi-formal, and formal methods. However, the following question remains to be answered: What types of strategies for comparing perimeter as well as area do the PSSMTs have?

Strategies for Checking Answers

Baturo and Nason (1996) revealed that most preservice primary school teachers in their study who attempted to verify their answers did so by recalculating strategy or using the inverse operation. They never think of using an alternative method to verify their answers. The following question remains to be answered: What types of strategies do the PSSMTs use to verify their answers?

Strategies for Solving Problem

The goal of the mathematics curriculum for secondary school in Malaysia is to develop individuals who are able to think mathematically and can apply mathematical knowledge effectively and responsibly in solving problems and making decision (Ministry of Education Malaysia, 2003a). Thus, problem solving is the primary focus of the teaching and learning activities of secondary school mathematics. Various strategies can be used to solve problems.

Among the strategies recommended by the Ministry of Education Malaysia (2003a) to be introduced in the secondary school mathematics curriculum are as follow: "trying a simple case; trial-and-error (also known as guess-and-check); drawing diagrams; identifying patterns; making a table, chart, or systematic list; simulation; using analogies; working backward; logical reasoning; and using algebra" (p. 4). What strategies do PSSMTs employed to solve fencing problem? The present study attempted to answer such question.

Strategies for Developing Area Formulae

Sgroi (2001) suggests that:

> The formula for the area of a rectangle can be developed by having children form many rectangles on a geoboard or dot paper, count up the squares inside, and eventually generalize that the area can be found by multiplying the length of the rectangle by the width. (p. 183)

Similar strategy (i.e., inductive method) was recommended by other mathematics educators or researchers (Billstein, Liberskind, & Lott, 2006; Cathcart, Pothier, Vance, & Bezuk, 2006; Cavanagh, 2008; Chua, Teh, & Ooi, 2002; NCTM, 2000: O'Daffer, Charles, Cooney, Dossey, & Schielack, 2005; van de Walle, 2007) to develop or derive the formula for the area of a rectangle.

Bennett and Nelson (2001) stated that "one of the basic principles in finding area is that a region can be cut into parts and reassembled without changing its area" (p. 659). This principle (area conservation) is beneficial in developing the formula for the area of a parallelogram. The formula for the area of a parallelogram can be developed from the formula for the area of a rectangle using the strategy of cut and paste (decompose and rearrange, i.e., decompose a parallelogram into a triangle and a trapezium and then rearrange these shapes to form a rectangle) (Beaumont, Curtis, & Smart, 1986; Billstein et al., 2006; Cathcart et al., 2006; Cavanagh, 2008; Cheang, 2002; Chua et al., 2002; Lim-Teo & Ng, 2008; NCTM, 2000; O'Daffer et al., 2005; van de Walle, 2007). Similarly, the formula for the area of a triangle can be developed from the formula for the area of a rectangle (Billstein et al., 2006; Cathcart et al., 2006; Cavanagh, 2008; Cheang, 2002; Chua et al., 2002; Lim-Teo & Ng, 2008; O'Daffer et al., 2005) or a parallelogram (Beaumont et al., 1986; Cavanagh, 2008; Lim-Teo & Ng, 2008; NCTM, 2000; van de Walle, 2007) using the strategy of partition (i.e., partition a rectangle or parallelogram along its diagonal into two triangles). The area of a triangle is half of the area of the rectangle or parallelogram that encloses it.

Van de Walle (2007) suggests that there are at least ten different methods of developing the formula for the area of a trapezium. Similarly, the formula for the area of a trapezium can be developed from the formula for the area of a rectangle (Cheang, 2002; NCTM, 2000; van de Walle, 2007), a parallelogram (Billstein et al., 2006; Chua et al., 2002; Lim-Teo & Ng, 2008; NCTM, 2000; O'Daffer et al., 2005; van de Walle, 2007), or a triangle (Beaumont et al., 1986;

Cathcart et al., 2006; van de Walle, 2007). The formula for the area of a trapezium can be developed using the strategies of cut and paste (decompose and rearrange, e.g., decompose an isosceles trapezium into a rectangle and two triangles and then rearrange these shapes to form a rectangle), duplicate (e.g., duplicate the trapezium and arrange the two trapeziums to form a parallelogram), or algebraic method.

In summary, the review of literature in this subsection has identified the strategies for developing area formulae for the area of a rectangle, parallelogram, triangle, and trapezium. However, the following question has not been answered: What types of strategies for developing area formulae for the area of a rectangle, parallelogram, triangle, and trapezium do the PSSMTs have? The present study attempted to answer such question.

Ethical knowledge

Ethical knowledge refers to "knowledge of right and wrong, what we are obligated to do, and of values" (Kupperman, 1970, p. 19). There are some good behaviors that the PSSMTs need to follow when dealing with perimeter and area. Among these good behaviors are as follow: (a) justifies one's mathematical ideas, (b) examines pattern within the domain of perimeter and area measurement, (c) formulates generalization within the domain of perimeter and area measurement, (d) tests generalization within the domain of perimeter and area measurement, (e) attempts to develop area formulae, (f) writes units of measurement upon they completed a task, and (g) checks the correctness of their solutions or answers. This section reviews literature related to the ethical knowledge of perimeter and area.

Justifies One's Mathematical Ideas

Simon and Blume (1996) noticed that in the traditional mathematics classrooms, teacher and textbook were the source of mathematics and evaluators of mathematical validity. However, the shift "toward logic and mathematical evidence as verification—away from the teacher as the sole authority for right answers" (NCTM, 1991, p. 3) is one of the five major shifts recommended by the *Professional Standards for Teaching Mathematics*. Simon and Blume (1996) argued that:

> The shift of authority for verification and validation of mathematical ideas from teacher and textbook to the mathematical community (the class as a whole) is a significant one for several reasons. First, such a change can afford students the possibility of seeing mathematics as created by communities of people based on the goals of that community and its accepted forms of practice. Second, it can give students rich opportunities for understanding mathematics resulting from involvement in the creation and validation of ideas. Finally, it can result in the students' sense that they are capable of creating mathematics and determining its validity. (p. 4)

Moreover, Ball (1991a) pointed out that "in mathematics discourse, justification is as much a part of the answer as is the answer itself" (p. 76). In fact, Ministry of Education Malaysia (2003a) emphasizes the importance of using correct and concise mathematical terms in the mathematics classrooms. Do the PSSMTs attempt to justify their mathematical ideas?

Examines Pattern, Formulates and Test Generalization

Ball (1991a) stated that "mathematics consists of activities such as examining patterns, formulating and testing generalizations, and constructing proofs" (p. 76). Martin and Harel (1989) asked 101 preservice elementary school teachers to judge the mathematical correctness of inductive and deductive verifications of familiar or unfamiliar generalizations. Martin and Harel (1989) reported that more than half of the preservice teachers in their study accepted an inductive argument as a valid mathematical proof. Over 60% of the preservice teachers accepted a correct deductive argument as a valid mathematical proof. It was reported that 38% and 52% of the preservice teachers accepted an incorrect deductive argument as being mathematically correct for the familiar and unfamiliar generalizations respectively.

Martin and Harel (1989) also reported that more than a third of the preservice teachers simultaneously accepted an inductive and a correct deductive argument as being mathematically valid. Martin and Harel (1989) argued that:

> This suggests that the inductive frame, which is constructed at an earlier stage than the deductive frame, is not deleted from memory when students (preservice teachers) acquire the deductive frame. Moreover, the everyday experience of forming and evaluating hypotheses by using evidence to support or refute them serves to reinforce the inductive frame. Thus, as our results indicate, inductive and deductive frame exist simultaneously in many students (preservice teachers). (p. 49)

Even and Lappan (1994) pointed out that “generalization惘starting from specific cases to find a general rule惘is central to doing mathematics” (p. 132). Nevertheless, Even and Lappan (1994) noticed that many prospective elementary school teachers attempt to solve problems by looking for the appropriate formula rather than using inductive reasoning as a tool for solving problems in mathematics. Goulding, Rowland, and Barber (2002) reported that less than half of the preservice elementary school teachers in their study provided correct response to an item devised to assess the ability to express a generalization in words and symbols.

Stylianides and Ball (2008) defined proof as a mathematical argument that fulfills the following three criteria: “set of accepted statements (definitions, axioms, etc.); modes of argumentation (use of logical rules of inference, construction of counterexamples, etc.); and modes of argument representation (pictorial, symbolic, etc.)” (p. 310). Do the PSSMTs examine the possible pattern of the relationship between perimeter and area? Do the PSSMTs formulate and test generalization pertaining to the relationship between perimeter and area?

Develops Area Formulae

Van de Walle (2001) pointed out that:

> Children should never use formulas without participating in the development of those formulas. Formulas for area and volume should all be developed by children. Developing the formulas and seeing how they are connected and interrelated is significantly more important than blindly plugging numbers into formulas, which is primarily computational tedium. (p. 296)

When students develop formulae, they are engaging in one of the real processes of doing mathematics (Van de Walle, 2007). By doing so, students can realize how all area formulae are related to one unifying idea, namely base times height. Moreover, "students who understand where formulas come from do not see them as mysterious, tend to remember them, and are reinforced in the idea that mathematics makes sense" (Van de Walle, 2007, p. 399).

In summary, the review of literature in this subsection pointed to the significance of engaging in developing area formulae. The question is, do the PSSMTs attempt to develop the formulae for the area of a rectangle, parallelogram, triangle, and trapezium?

Writes Units of Measurement upon Completed a Task

It is a general measurement convention that perimeter and area is measured by linear units (such as mm, cm, m, km) and square units (such as mm^2, cm^2, m^2, km^2) respectively. However, Baturo and Nason (1996) revealed that several preservice primary school teachers in their study wrote the calculated area measurement in linear unit such as 128 cm. They did not understand the general measurement convention that area is measured by square units. This behavior was repeated in most of the other items in Task 7 where preservice teachers in their study were asked to calculate the area of the given shapes. Do the PSSMTs write units of measurement upon completed a task?

Check the Correctness of Their Solutions or Answers

Mosenthal and Ball (1992) argued that assessing the reasonableness of one's solutions is a hallmark of understanding. Moreover, checking the correctness or reasonableness of one's answers or solutions is a good behavior in mathematics (Nik Azis, 2007). However, Heaton (1992) reported a case study of a grade five teacher, Sandra, who is lack of SMK of perimeter and area. For instance, when students raised the problem of how to calculate the cost of fencing a park with the dimensions of 300 feet by 200 feet, Sandra told students to multiply 300 × 200 and gave students the answer, "60 000 feet". Sandra did not question the reasonableness of her solutions. It was also reported that students in Sandra's class applied mathematical procedures inappropriately and they did not question the reasonableness of their solutions. If Sandra and her students checked the reasonableness of their solutions, they might have spotted the mistakes in their solutions. Similarly, Baturo and Nason (1996) found that majority of the preservice primary school teachers in their study had to be prodded towards checking their answers. Once getting an answer, they seemed to satisfy that the task was finished. Do the PSSMTs check their answers or solutions (without probed)?

Level of Subject Matter Knowledge

Ramakrishnan (1998) assessed preservice elementary school teachers' understanding of perimeter and area. In his study, the preservice teachers were given four tasks, two tasks to assess their understanding of perimeter and the other two to assess their understanding of area. In Task 1, preservice teachers were asked to set a question to assess student understanding of perimeter. He explained that item that assessed low level of student understanding of perimeter is the item that "assessed a minimum/superficial understanding; generally, such an item could be answered by using a rote-learned procedure, usually using a single step, with minimum understanding of the underlying concept" (p. 362).

Item that assessed medium level of student understanding of perimeter is the item that "assessed a slightly more than superficial, but less than an in-depth, comprehensive understanding; generally, although it required more than a single application or step of a previously learned procedure, it could be answered correctly if sufficient practice in similar items had been given earlier" (p. 362). Item that assessed high level of student understanding of perimeter is the item that "assessed an in-depth and comprehensive understanding; generally, such an item needed the application of related concepts and procedures and could not be answered by a straight-forward application of previously learned procedures'" (p. 362). Ramakrishnan (1998) revealed that 42.5%, 41%, and 11% of the preservice teachers were able to set questions that assess low, medium, and high level of student understanding of perimeter respectively. The other 5.5% of the 54 preservice teachers came up with erroneous or inappropriate question.

Barrett, Clements, Klanderman, Pennisi, and Polaki (2006) developed a theoretical framework to analyze students' levels of understanding of linear measurement based on students' coordination of geometric reasoning and measuring strategies on a fixed perimeter task. They categorized students' levels of understanding of linear measurement as follow:

> Level 1: Visual guessing to assign length (naïve unit strategy); Level 2a: Inconsistent ways of identifying or iterating units: uses salient markers as a counting set for measuring; Level 2b: Consistent identification or iterating of units; Level 3a: Coordinating iterated-unit items, side-lengths, and collections of side lengths to obtain perimeter; Level 3b: Coordinating length attributes, yet with further tendency and ability to relate multiple cases. (p. 197)

A total of 38 grade two through grade ten students were asked to draw and label all the rectangles (triangles) with a fixed perimeter of 24 units. They were also asked to explain their answers. Barrett et al. (2006) found that four, eight, and two Grades 2-3 students exhibited levels of 1, 2a, and 2b respectively. One, five, five, and two Grade 5-6 students exhibited levels of 1, 2a, 2b, and 3a respectively while one, seven, and three Grade 8-10 students exhibited levels of 2b, 3a, and 3b respectively.

The Learning Mathematics for Teaching (LMT) (2006) project which consists of four principal investigators, namely Hill, Ball, Bass, and Schilling, from the School of Education, University of Michigan, had developed a coding rubric for measuring the mathematical quality of instruction. It consists of 83 codes grouped into five sections as follow:

> Section I: Instructional format and content, Section II: Knowledge of mathematical terrain of enacted lesson, Section III: Use of mathematics with students, Section IV: Mathematical features of the curriculum and the teacher's guide, and Section V: Use of mathematics to teach equitably. (p. 6)

"Section II codes the teacher's knowledge of the mathematics entailed in the lesson as revealed by its enactment" (p. 7). It consists of 12 codes grouped into five subsections as follow : "(a) teacher's use of language (3 codes), (b) examples and models used to represent mathematical concepts (4 codes), (c) degrees of mathematical explanation (3 codes), (d) development of mathematical elements (1 code), and (e) computational errors or other mathematical oversight (1 code)" (p. 7). Eventually, "there is a global code used to record the coders' impression on the teacher's level (low, medium, high) of mathematical knowledge. Overall, this section is designed to capture the teacher's understanding of the content being taught and the mathematical resources used during the lesson" (p. 7).

Each mathematical elements in the coding rubric is coded as "present and appropriate (PA)", "present and inappropriate (PI)", "not present and appropriate (NPA)", or "not present and inappropriate (NPI)". In the Learning Mathematics for Teaching (LMT) (2006) project, random pairs of researchers were assigned to code each videotaped lesson. The coders coded each lesson individually and then gave an overall level of the teacher's knowledge of mathematics as low, medium, or high, based on their impression of the teacher's level of mathematical knowledge. They met and reconciled their codes before giving their final level of mathematical knowledge.

What levels of SMK of perimeter and area do the PSSMTs exhibit? In the present study, the researcher determined the PSSMTs' levels of SMK of perimeter and area using coding rubrics adapted from the Learning Mathematics for Teaching (LMT) (2006) project. However, in this study, PSSMTs' levels (low, medium, high) of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area as well as the overall level of SMK of perimeter and area were determined based on the percentage of appropriate mathematical elements of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area as well as the overall percentage of appropriate mathematical elements of SMK of perimeter and area obtained by the PSSMTs.

Research Related to Malaysian Preservice Teachers' Subject Matter Knowledge

Several studies had revealed that Malaysian trainee teachers in the teacher training institutes (formerly known as teacher training colleges) had demonstrated a poor understanding of mathematical concepts and a lack of mathematical skills (Cheah, 2001; Koe, 1992; Ng, 1995). In a multiple case study, Cheah (2001) explored the mathematical beliefs of six trainee teachers who majored in mathematics and science in a Malaysian teachers college. The findings of his study showed that trainee teachers' central belief about mathematics was already formed while they were still schooling. They believed that mathematics was mainly procedural and their learning of mathematics was focused on procedures, algorithms, and the use of formulae. The trainee teachers' beliefs about teaching mathematics were also procedural in nature. Koe (1992) revealed that trainee teachers had difficulties in answering mathematics questions taken from the primary six standardized examinations. Ng (1995) also found that trainee teachers were weak in their mathematics content.

Nik Azis (1995) reported that preservice teachers in his study thought that computational approach is the most appropriate approach to teach school mathematics. For instance, preservice teachers in his study spent much of the instructional time on computational skills that they have no time to guide their students to construct meaningful basic concepts of mathematics. It indicated that these preservice teachers viewed mathematics as a collection of rules to be used to find the correct answer for the given set of questions. Students were not encouraged to use their own strategies to solve mathematical problems. The preservice teachers were uneasy when students question the ways they solve mathematical problems.

Nik Azis (1995) revealed that students were treated as passive receiver of school mathematics, and they were trained to master certain mathematical skills and memorize certain mathematical facts. The preservice teachers in his study employed the demonstration-practice technique to achieve these objectives. Specifically, the preservice teachers demonstrated how to solve certain questions and then students were asked to do certain amount of exercises from textbook or workbook based on the shown examples. If the students were unable to solve the given exercises, the preservice teachers would repeat the cycle of demonstration-practice with other similar example.

Nik Azis (1995) found that school textbook and past year examinations book were the only resources used by the preservice teachers in his study for teaching school mathematics. Moreover, the preservice teachers did not know how to use the textbook effectively. They strictly followed the sequence in the textbook without modification. It was also reported that the preservice teachers lack of confident to teach certain topics of school mathematics (Nik Azis, 1995).

Nik Azis (2003) suggests that there are numerous challenges in securing high standards of mathematics education in Malaysia. One of those challenges is related to teacher education. Nik Azis (2003) asserted that mathematics teachers must know and understand the school mathematics they are teaching. Nik Azis (2003) pointed out that:

> Currently, there is a tacit assumption that, by the time preservice secondary mathematics teachers have completed their university mathematics courses, they will have the understanding of school mathematics subject matter required for teaching the subject matter effectively. However, recent research has shown that this assumption might not be valid. We cannot take for granted that teachers' knowledge of the content of school mathematics is in place by the time they complete their secondary school learning experiences. (p. 6)

Thus, Nik Azis (2003) suggests that mathematics teachers need to be given opportunity to revisit school mathematics topics in the manners that permit them to develop deeper understandings. Nevertheless, Nik Azis (2003) argued that:

> It will be challenging to help teachers understand school mathematics content at a deeper conceptual level, to help them understand the big ideas of mathematics and to be able to present mathematics as a unified discipline, a woven fabric rather a patchwork of discrete topics, and help them developing mathematical reasoning. At the moment, this kind of knowledge is beyond what most mathematics teachers experience in preservice mathematics courses. (pp. 7-8)

Moreover, the success of students depend most of all on the quality of the teacher. Therefore, to be effective mathematics teachers, they must have a good command of both school mathematics and university mathematics, adequate preparation in effective pedagogical practices, and attain high overall academic performance (Nik Azis, 2003).

In summary, previous studies related to Malaysian contexts had identified that the preservice teachers confronted problems of poor understanding of mathematical concepts and a lack of mathematical skills, difficulties in answering mathematics questions, and weaknesses in mathematical content. However, the researchers did not study in-depth the preservice teachers' SMK in a specific topic such as perimeter and area.

Research Related to Preservice Teachers' SMK of Perimeter and Area

Ball (1988) asked the prospective teachers in her study to respond to a hypothetical student who claims that she has discovered that as the perimeter of a closed figure increases, the area of the figure also increases. Ball (1988) found that only two of the ten prospective elementary school teachers knew that the student's claim about the relationship between perimeter and area was not true. Three prospective elementary school teachers thought that the student's claim was true. The remaining five prospective elementary school teachers were not sure whether the student's claim was true or not. Ball (1988) also found that only one of the nine prospective secondary school teachers knew that the student's claim about the relationship between perimeter and area was not true. Five prospective secondary school teachers thought that the student's claim was true. The remaining three prospective secondary school teachers were not sure whether the student's claim was true or not. It can be concluded that most of the prospective teachers in her study had a misconception that there is direct relationship between perimeter and area. Only three of them knew that there is no direct relationship between perimeter and area. They understand that as the perimeter of a closed figure increases, the area of the figure may increases, decreases, or remain the same.

Ball’s (1988) finding is in accordance with the findings of previous studies (Woodward, 1982; Woodward & Byrd, 1983). Woodward (1982) found that an excellent seventh grade student, Heidi, thought that the garden with the same perimeter have the same area. Woodward and Byrd (1983) revealed that 59% of the 129 eight grade students at a junior high School in Tennessee thought that the garden with the same perimeter have the same area. They also revealed that 63% of the 129 eight grade students at another junior high School in Tennessee thought that the garden with the same perimeter have the same area. Woodward and Byrd (1983) found that prospective elementary teachers took the test with similar results.

Ramakrishnan (1998) assessed preservice elementary school teachers' understanding of perimeter and area. In his study, the preservice teachers were given four tasks, two tasks to assess their understanding of perimeter and the other two to assess their understanding of area. In Task 1, preservice teachers were asked to set a question to assess student understanding of perimeter. Findings of this study shows that 42.5%, 41%, and 11% of the preservice teachers were able to set questions that assess low, medium, and high level of student understanding of perimeter respectively. The other 5.5% of the 54 preservice teachers came up with erroneous or inappropriate question.

In Task 2, the preservice teachers were given an L-shaped figure with certain measures. They were asked to decide whether the information was sufficient to calculate the perimeter of the figure. 24% of the preservice teachers stated that there was insufficient information (when actually there was sufficient information) to calculate the perimeter of the figure. In Task 3, a rectangle (without measure) was divided into three triangles (two shaded and one unshaded). The preservice teachers were asked to decide whether the information was sufficient (when actually sufficient) to express the area of the shaded triangle as a fraction of the area of the rectangle. 28% of the preservice teachers believed there was insufficient information to solve the problem.

In Task 4, the preservice teachers were given a figure showing a rectangle and a triangle with an overlapping area. The area of the rectangle and the triangle were given. The preservice teachers were asked to decide whether the information was sufficient (when actually sufficient) to determine the difference in area between two specified regions. 85% of the preservice teachers stated that there was insufficient information to solve this problem. Ramakrishnan (1998) concluded that the preservice teachers seemed to have poor conceptual understanding of perimeter and area. He suggested that teachers with poor conceptual understanding of mathematics tend to feel more comfortable teaching just for procedural understanding. The results of this preliminary study show that the preservice teachers' understanding of perimeter and area is less than satisfactory. Moreover, they are going to teach student mathematics.

Reinke (1997) examined the solutions strategies used by preservice elementary teachers to find the perimeter and area of a shaded geometric figure. The findings of this study show that only 11.8% (9 preservice teachers) of the 76 preservice teachers were able to come up with correct strategies for finding perimeter of the shaded figure. Nevertheless, 52.7% (40 preservice teachers) of them were able to come up with correct strategies for finding area of the shaded figure. Woodward and Byrd (1983) suggested that prospective elementary teachers were just as naïve about area as eight graders. Moreover, Reinke's (1997) findings seemed to indicate that the same can be said for the concept of perimeter.

The *pi*, π, is the most important constant in school mathematics. Nevertheless, there was evidence that many preservice teachers do not really understand what π is (Wong, 1997). These preservice teachers had studied several advanced mathematics courses where π is used frequently. Wong (1997) found that more than half of the 31 preservice teachers in his study thought $\pi = \frac{22}{7}$. They did not know that π is the ratio of the circumference to the diameter of a circle.

Ryan and Williams (2007) found that preservice primary school teachers often exhibit the same misconceptions as their students. Ryan and Williams (2007) argued that:

> This is not surprising as generalist teachers need to have a broad knowledge base and do not claim to be expert in every curriculum area. Primary and preservice primary teachers sometimes suffer, too, from a lack of confidence in mathematics and not uncommonly report a history of ‘trouble with maths’. Surprisingly, secondary specialist mathematics teachers can also exhibit the same misconceptions as their pupils. (p. 137)

Ryan and Williams (2007) concerned that these misconceptions could be transferred to students by their teachers if they are not exposed and attended productively. For instance, a hexagon is drawn on a centimetre square grid. Preservice teachers were asked to determine the perimeter of the hexagon. Ryan and Williams (2007) found that almost two-thirds of the preservice teachers in their study gave incorrect response to the item. The similar item was given to 12, 13, and 14 year olds students decades ago with similar results. It indicated that the preservice teachers had misconceptions about the length of a line drawn between grid lines. It was reported that only 34% of the preservice teachers in their study gave correct response to this item while 36% of them were unable to differentiate between the horizontal side and slanted side of the hexagon.

In summary, previous studies in this section revealed that most of the preservice teachers demonstrated inadequate understanding of the concept of perimeter and area and had a misconception pertaining to the relationship between perimeter and area. However, none of these studies attempted to examine in-depth the five basic types of knowledge of SMK of perimeter and area, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area.

Measurement

In the *Principles and Standards* document, NCTM (2000) organized its content into five standards. Measurement is one of these content standards. It shows the importance and complexity of this domain of mathematics (Van de Walle, 2007). Furthermore, NCTM published a yearbook entitled *Learning and Teaching Measurement: 2003 Yearbook* (Clements & Bright, 2003). Likewise, Lewis and Schad (2006) argued that "measurement is an integral part of all mathematics content as well as the content of most other subjects" (p. 131). They pointed out that the need for more attention to measurement was the reason for selecting measurement as the theme for the October 2006 focus issue of *Teaching Children Mathematics*.

Measurement is pervasive in our daily lives and it also provides opportunities for learning and applying other mathematics, such as number, geometry, statistics, and function. NCTM

(2000) described measurement as "the assignment of a numerical value to an attribute of an object" (p. 44.). NCTM (2000) suggests that students must understand measureable attributes (e.g., length, area, and volume) of objects and the units, systems (metric and customary), and processes of measurement. They must also apply appropriate techniques, instruments (tools), and formulae to determine measurements. However, "past administration of NAEP (National Assessment of Educational Progress) have shown that measurement is a difficult topic for students to master" (Strutchens, Martin, & Kenny, 2003, p. 196).

Unit of Measurement

NCTM (2000) and Van de Walle (2007) suggests that students should begin their learning of measurement by using nonstandard (informal) units, such as paper clips to measure length, square tiles to measure area, and paper cups to measure volume. Long and DeTemple (2003) revealed that working with nonstandard units provides students the opportunity to discover important general principles of the measurement process. Students should also be given opportunity to measure the same attribute with different-sized (varying-sized) units (Van de Walle, 2007). Such activities would enhance their understanding of the inverse relationship (inverse proportion) between units of measure and number of unit that the larger the unit of measure, the smaller the number of unit required to measure the same attribute, and vice versa.

Baturo and Nason (1996) revealed that all the 13 preservice primary school teachers in their study had an understanding of indirect proportion that the larger the unit of measure, the smaller the number of unit required to measure the same attribute, and vice versa. The "standardization" of units would arise when students notice that using one's foot to measure the length of the classroom provides a different length from using other's foot. Such encounters help students see the need and consistency of using standard units. "Students must not only develop a familiarity with standard units but must also learn appropriate relationships between them" (Van de Walle, 2007, p. 377).

Measurement Formulae

NCTM (2000) suggests that "students should begin to develop formulas for perimeter and area in the elementary grades. Middle grades students should formalize these techniques, as well as develop formulas for the volume and surface area of objects like prisms and cylinders" (p. 46). Likewise, the Form One Mathematics Curriculum Specification (Ministry of Education Malaysia, 2003a) recommended that teaching and learning activities in the classroom to provide opportunity for the students to investigate and develop the formula for the area of a rectangle. It also suggested that students be given opportunity to investigate and develop the formulae for the area of triangles, parallelogram, and trapeziums based on the area of a rectangle. Table 2.3 shows the perimeter and area formulae for some of the common figures in the school mathematics curriculum.

The results from the Mathematics Assessment of the National Assessment of Educational Progress (NAEP) consistently showed that students do not have a good understanding of formulae. For instance, only 19% of fourth grade students and 65 % of eight grade students in the Sixth Mathematics Assessment of the NAEP were able to find the area of a carpet 9 feet long and 6 feet wide (Kenney & Kouba, 1997). Kenney and Kouba (1997), and Lindquist and Kouba (1989) revealed that many elementary and middle grades students encountered difficulty with understanding perimeter and area. They seemed to use formulae such as $P = 2l + 2w$ or $A = l \times w$ without understanding how such formulae related to the attribute being measured or the unit of measure being used (NCTM, 2000). Van de Walle (2007) identified two common difficulties in using formulae: (a) students confuse formulae for perimeter and area, and (b) students confuse the slanted side and height of two-and three-dimensional shapes. Thus, some students tended to use the slanted side of a parallelogram as the height of the parallelogram when calculating the area of a parallelogram.

Table 2.3

Some Common Perimeter and Area Formulae

Figure	Formula
1. Rectangle	$P = 2l + 2w$ or $2(l + w)$ $A = l \times w$
2. Square	$P = 4s$ $A = s^2$
3. Triangle	$P = a + b + c$ $A = \frac{1}{2} \times b \times h$
4. Parallelogram	$P = 2a + 2b$ or $2(a + b)$ $A = b \times h$
5. Rhombus	$P = 4s$ $A = b \times h$
6. Trapezium	$P = a + b + c + d$ $A = \frac{1}{2}(a + b)\mathrm{h}$
7. Circle	$C = 2\pi r$ $A = \pi r^2$ Length of arc = $\frac{\theta}{360^0} \times 2\pi r$ Area of sector = $\frac{\theta}{360^0} \times \pi r^2$

Students' Performance in Measurement

Results from the First, Second, Fourth, and Sixth Mathematics Assessment of the National Assessment of Educational Progress (NAEP) indicated that students' performance in perimeter and area were not satisfactory. Results from the First Mathematics Assessment of the

NAEP indicated that the grade seven students seemed to have limited knowledge of basic area concepts. Less than 33% of the grade eleven students could find the area of the resulting region of a rectangle with an interior rectangle removed (Carpenter, Coburn, Reys, & Wilson, 1978). Students also confused between perimeter and area. Similarly, results from the Second Mathematics Assessment of the NAEP confirmed that many students' difficulties came from misconceptions rather than computational errors (Hirstein, 1981). A common misconception was the confusion between perimeter and area. For instance, results from the Second Mathematics Assessment of the NAEP revealed that 23 % of the grade seven and grade eleven students tested gave the perimeter of the requested area of a rectangle.

Lindsquist and Kouba (1989) highlighted that “neither seventh nor eleventh grade students have developed a strong conceptual understanding of area” and “although 75% of the eleventh grade students could find the area of rectangle, fever than half of them could use this skills in related problems” (p. 35). Results from the Fourth Mathematics Assessment of the NAEP showed that many grade seventh students were confusing between perimeter and area. The usual error made on the area items was choosing the perimeter, and vice versa. Lindsquist and Kouba (1989) argued that this confusion was not completely removed by grade eleven. They noted that “there is evidence that the confusion between perimeter and area was not the only misconception that students had about area” (p. 41). There is an item in the Fourth Mathematics Assessment of the NAEP that required the knowledge that the sum of the areas of the part of a rectangle equals the area of the whole rectangle. Lindsquist and Kouba (1989) found that about 25% of the grade seven and grade eleven students stated that the sum of the area of the separated parts could not be ascertained.

Results from the Fourth Mathematics Assessment of the NAEP also revealed that given one side of a square, about 10% of the grade seven students and about 45% of the grade eleven students were able to ascertain the area. Likewise, about 45% of grade eleven students were able to find the area of a square given its perimeter. Lindsquist and Kouba (1989) pointed out that “the students perhaps performed less well on these items because they did not understand a square is a special case of a rectangle or that it is necessary to determine both dimensions” (p. 42). Lindsquist and Kouba (1989) found that less than 30% of the grade eleven students were able to find the area of a figure composed of two rectangles. In another situation, the grade eleven students were told that a photograph had been enlarged by doubling its dimensions. Results from the Fourth Mathematics Assessment of the NAEP revealed that about 45% of grade eleven students had successfully chosen the cost of the enlarged photograph given the price per square inch.

Results from the Sixth Mathematics Assessment of the NAEP shows that 42% of grade four and 66 % grade eight students were able to draw on a square grid, a rectangle with an area of 12 square units. 19% of grade four and 65% of grade eight students were able to select the correct response when asked for the area of a carpet 9 feet long and 6 feet wide. Most of the grade four students selected the sum of the dimensions (15) as their answer (Kenney & Kouba, 1997). Results from the Sixth Mathematics Assessment of the NAEP also shows that students encountered difficulty applying area measurement formulae and they often confused with the formulae for perimeter and area. Only 37% of the grade eight students chose the correct representation for area. Even grade twelve students continued to confuse area and perimeter (Kenney & Kouba, 1997).

Ryan and Williams (2007) stated that students encountered problems with perimeter and area. They found that only 20% of 9 year olds students in their study could correctly find the perimeter of the shape drawn on a grid paper. It was reported that 26% of the students counted the grid squares around the outside of the shape rather than the grid square length. Ryan and

Williams (2007) revealed that 36% of 11 year olds students in their study made an error by matching by area when they were asked to match shapes with the same perimeter.

Students also have problems in measuring the perimeter of shapes with diagonal sides. Ryan and Williams (2007) reported that 13% of 11 year olds students in their study counted the diagonal of a unit square as the same length with the side of the square. Ryan and Williams (2007) also reported that almost one-third of 13 year olds students in their study used the perimeter formula instead of the area formula of a rectangle when finding a missing dimension of one of the rectangle, given that the two rectangles have the same area. It was also reported that 60% of 14 year olds students in their study could correctly calculated the distance a referee ran around a rugby pitch 90 m long and 60 m wide. Nevertheless, 14% of them calculated the area instead of perimeter while 12% simply added 90 and 60. Likewise, Cavanagh (2008) reported that high school students in his study confused area and perimeter.

Jamski (1978) argued that many students just memorized and applied formula without understanding what area is. He concluded that “superficial manipulation of formulas should not be equated with the understanding of the area concept” (p. 37). Woodward (1982) found that Heidi, an excellent seventh grade student, did not understand the concept of area and unable to differentiate between perimeter and area. Heidi thought that all rectangles with the same perimeter had the same size. The findings of Woodward's (1982) study indicated that Heidi's prior experiences with perimeter and area were abstract in nature. She might had been given formulae and was asked to calculate perimeter and area without making sense of it.

Woodward and Byrd (1983) assessed grade eight students and prospective elementary school teachers' understanding of the area concept. They found that only 23% (30) of the 129 grade eight students were able to identify the largest garden given its perimeter is 60 feet while 59% (76) stated that all the gardens were the same size. The same test was administered to another group of 129 grade eight students. Woodward and Byrd (1983) found that only 19% (25) of them answered the question correctly (able to identify the largest garden given that its perimeter is 60 feet) while 63% (81) stated that all the gardens were the same size.

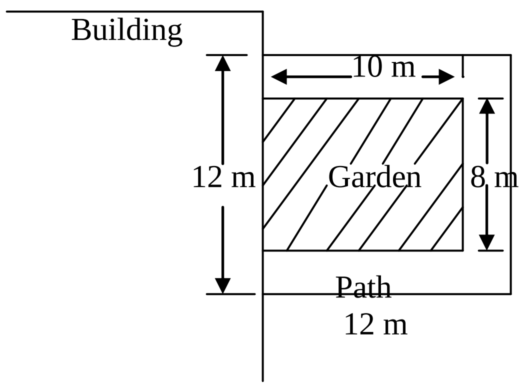
Woodward and Byrd (1983) also found that prospective elementary school teachers took the test with similar results. They also asked the prospective teachers to justify their answers. Here was a typical response: "They are all the same size since the perimeter is 60 ft. The area is arranged differently" (p. 345). Based on these results, Woodward and Byrd (1983) became convinced that area was indeed poorly taught. Woodward and Byrd (1983) concluded that these prospective teachers were just as naïve about area as were the grade eight students.

In summary, the problems of students' and preservice teachers' difficulties in understanding the concepts of perimeter and area, and misconception pertaining to the relationship between perimeter and area pointed to the need and importance of more research on the preservice teachers' SMK of the topic of perimeter and area as teaching play a significant role in developing and shaping the students' understanding of the concepts.

Malaysian Students' Performance in Perimeter and Area

How about Malaysian students' performance in perimeter and area? Malaysia took part in the Third International Mathematics and Science Study - Repeat (TIMSS-R) that organized by the International Association for the Evaluation and Educational Achievement (IEA). In Malaysia, TIMSS-R was administered to Form Two students during October 1998. A total of 5577 of our Form Two students participated in this international study (Ministry of Education Malaysia, 2000). Measurement was one of the five content areas being tested in the TIMSS-R study. Malaysia ranked 16 in the content area of measurement. The content area of measurement included standard and nonstandard units, common measures, perimeter, area, volume, and estimation of measures (Mullis et al., 2000). Two items related to area measurement were released for the public.

A rectangular garden that is next to a building has a path around the other three sides, as shown.

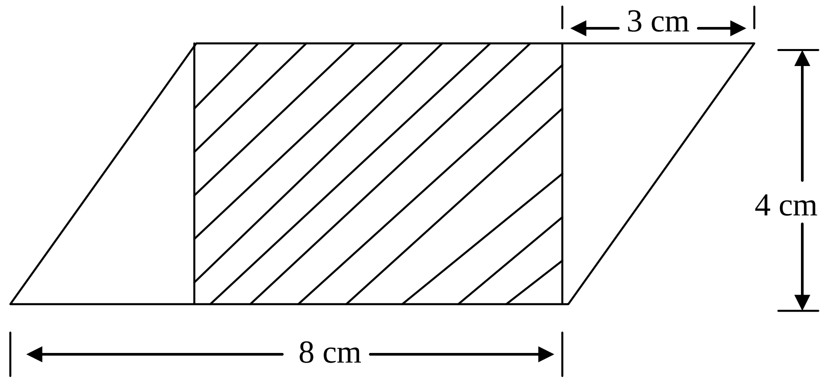


What is the area of the path?
A 144 m^2 B 64 m^2 C 44 m^2 D 16 m^2

Figure 2.1. First item related to measurement that was released for the public.
Source: Mullis et al. (2000), p. 64.

Figure 2.1 shows the first item related to measurement that was released for the public. For this item, slightly more than half (52%) of Malaysian Form Two students who participated in the TIMMS - R study were able to answer it correctly compared with 78% of Singapore students. Figure 2.2 depicts the second item related to measurement that was released for the public. For this item, 56% of Malaysian Form Two students who participated in the TIMMS - R study were able to answer it correctly compared with 83% of their Singapore counterparts.

The figure shows a shaded rectangle inside a parallelogram.

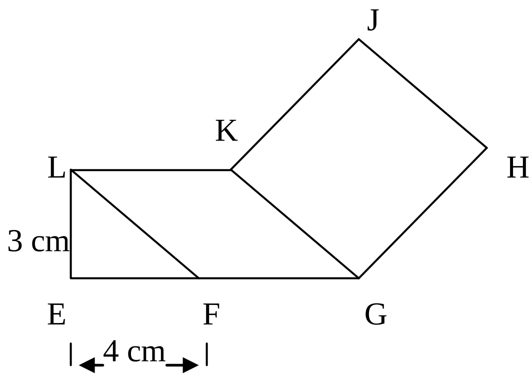


What is the area of the shaded rectangle?

Figure 2.2. Second item related to measurement that was released for the public. Source: Mullis et al. (2000), p. 74.

A total of 5314 Malaysian Form Two students participated in the Trends in International Mathematics and Science Study 2003 (TIMSS 2003) (Ministry of Education Malaysia, 2004). Measurement was one of the five content areas being tested in the TIMMS 2003. However, in the TIMMS 2003, Malaysia's ranking in the content area of measurement had dropped to number 18 (Mullis et al., 2004). A total of 4466 Malaysian Form Two students involved in the Trends in International Mathematics and Science Study 2007 (TIMSS 2007) (Martin et al., 2008). In the TIMSS 2007, geometry and measurement were combined as a domain known as geometry shapes and measure. In the TIMMS 2007, Malaysia's ranking in the domain of geometry shapes and measure further dropped to number 24. It was reported that in the TIMMS 2007, Malaysian Form Two students' average scale score (477) in the domain of geometry shapes and measure was significantly lower than TIMSS scale average (500) (Martin et al., 2008).

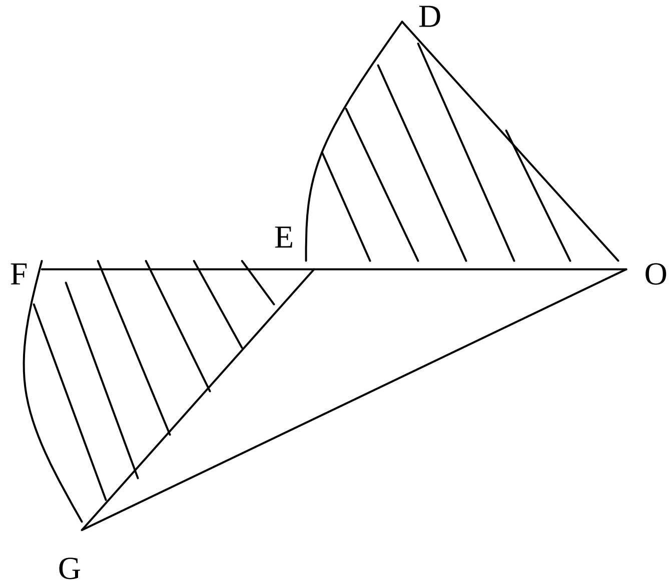
How about Malaysian Form Five students' performance related to perimeter and area in the SPM examination? Figure 2.3 demonstrates an examination question taken from the 1995 SPM Mathematics Papers 1. The Malaysian Examination Syndicate (1996) reported that the SPM candidates performed well in part (a) of the question. Some students gave 42 cm as the answer for part (a) because they did not understand the concept of perimeter. These students mistakenly considered the lengths of LF and KG in the process of calculating the perimeter of the diagram. It was also reported that SPM candidates' performance in part (b) of the question were less satisfactory. Some students assumed that the area of a rhombus equal to the area of a square.



In the above diagram, LFGK is a rhombus and GHJK is a square. EFG is a straight line.
Calculate
(a) the perimeter of the diagram,
(b) the area of the diagram.

Figure 2.3. An examination question taken from the 1995 SPM Mathematics Papers 1.
Source: Malaysian Examination Syndicate (1995)

Figure 2.4 reveals an examination question taken from the 1995 SPM Mathematics Papers 2. The Malaysian Examination Syndicate (1996) reported that the SPM candidates' performance for this question were less satisfactory. There were only about half of the SPM candidates attempted this compulsory question. Some students were using inappropriate formulae for calculating the length of an arc and the area of a sector. Many students were unable to calculate the upright distance from point E to straight line OG. It was also reported that some students calculated the shaded regions without subtracted the area of triangle OEG.

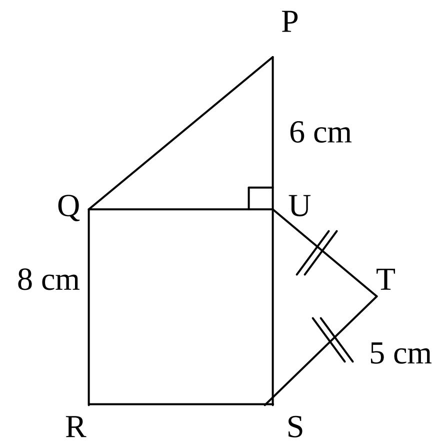


In the above diagram, the arcs of DE and FG respectively subtend an angle of 60^0 and 30^0 at the centre of the circle O. Point E is the midpoint of straight line OF. Given that OD = 7 cm. Assuming $\pi = \frac{22}{7}$, calculate

(i) the perimeter of the whole diagram,
(ii) upright distance from point E to straight line OG,
(iii) the area of the shaded regions.

Figure 2.4. An examination question taken from the 1995 SPM Mathematics Papers 2.
Source: Malaysian Examination Syndicate (1995)

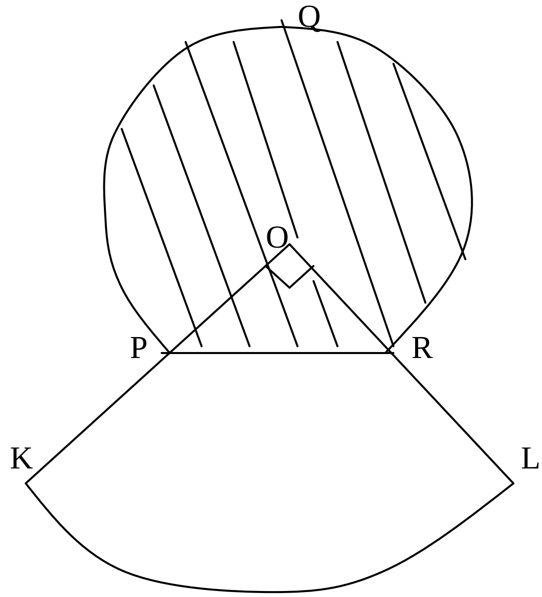
Figure 2.5 exhibits an examination question taken from the 2002 SPM Mathematics Papers 1. In part (a) of the question, The Malaysian Examination Syndicate (2003) reported that some of the SPM candidates either failed to use the Pythagoras' Theorem correctly or they did not know the properties of isosceles triangle. In part (b) of the question, it was reported that many SPM candidates were unable to find the area of isosceles triangle.



In the above diagram, QRSU is a square and PUS is a straight line.
Calculate,
(a) the perimeter of the diagram,
(b) the area of the diagram.

Figure 2.5. An examination question taken from the 2002 SPM Mathematics Papers 1.
Source: Malaysian Examination Syndicate (2002)

Figure 2.6 illustrates an examination question taken from the 2002 SPM Mathematics Papers 2. The Malaysian Examination Syndicate (2003) reported that the SPM candidates were weak in answering this compulsory question. In part (i) of the question, it was reported that students were applying inappropriate formulae or radius to calculate the length of arcs KL or PQR. Similarly, in part (ii), students were also applying inappropriate formulae or radius to calculate the area of sector OPQR. Wilson and Osborne (1992) found that many students who encountered difficulty in memorizing formulae had an inadequate understanding of the basis of the measurement systems.



In the above diagram, PQR and KL are two arcs of circle, respectively centred O. Given that OPK and ORL are straight lines with the conditions that OP = PK = 7 cm.

Assuming $\pi = \frac{22}{7}$, calculate

(i) the perimeter of the whole diagram,

(ii) the area of the shaded regions.

Figure 2.6. An examination question taken from the 2002 SPM Mathematics Papers 2.
Source: Malaysian Examination Syndicate (2002)

In summary, Malaysian Form Two students' performance in the TIMMS-R study were less satisfactory in the two items related to area measurement that were released for the public in comparison with of their Singapore counterparts. Furthermore, Malaysia's ranking in the content area of measurement had dropped from 16 in the TIMSS-R 1999 to 18 in the TIMSS 2003 and 24 (in the domain of geometry shapes and measure) in the TIMSS 2007. Similarly, Malaysian Form Five students' performance in the SPM examination related to the topic of perimeter and area were also less satisfactory (Malaysian Examination Syndicate, 1996, 2003). Thus, more research is needed in the topic of perimeter and area. In particular, preservice secondary school mathematics teeachers' SMK of perimeter and area as they are going to teach our future secondary school students mathematics.

Theoretical Framework of the Study

The present study was guided by the information processing theory (IPT), a major strand in cognitive psychology. According to Slavin (2009), IPT refers to "cognitive theory of learning that describes the processing, storage, and retrieval of knowledge in the mind" (p. 158). Parke and Gauvain (2009) noted that IPT is characterized by the following assumptions: (a) "thinking is information processing, (b) there are mechanisms or processes of change that underlie the processing of information, (c) cognitive development is a self-modifying process, and (d) careful task analysis is crucial" (pp. 314-315).

IPT views human beings in general, participants of this study (i.e., preservice teachers) in particular, as active processors of information, strategy users, organizers and reorganizers of information, and rememberers (or recallers) of information. From the information processing perspective, knowledge in general, mathematical knowledge in this study (i.e., knowledge of perimeter and area) in particular, refers to information that has been processed and stored in the long term memory (LTM). Moreover, there is a fixed body of knowledge to be acquired and prior knowledge influences how information is processed (Woolfolk, 2007).

IPT views teaching as the knowledge transmission processes from the teacher to the students. Teacher guides students toward acquiring more "accurate" and complete knowledge. From the information processing perspective, learning refers to the acquisition of facts, skills, concepts, and strategies. Learning occurs through the effective application of strategies (Woolfolk, 2007).

IPT views problem solving as involves implementing a series of mental computations on mental representations. Meyer and Wittrock (2009) noted that:

> A problem can be represented as a problem space—a representation of the initial state, goal state, and all possible intervening states—and search heuristic—a strategy for moving through the problem space from one state of the problem to the next. (704)

For instance, means-ends analysis is a common heuristic. In the means-ends analysis, the problem solver "seeks to apply an operator that will satisfy the problem solver's current goal; if there is a constraint that blocks the application of the operator, then a goal is set to remove the constraint, and so on" (Meyer & Wittrock, 2009, p. 704).

From the information processing perspective, reasoning refers to "problem solving with a specific task in which the goal is to draw a conclusion from premises using logical rules based on deduction or induction" (Meyer & Wittrock, 2009, p. 703). For example, if students are given the sequence 1, 3, 5, 7, then by inductive reasoning, they can conclude that the next term will be 9.

The question is: What is the nature of values in this study? The environment has its own norms and law of nature. Thus, human behaviors that are in line with the law of nature are considered as good. Similarly, there are some good behaviors that the preservice teachers need to follow when dealing with perimeter and area, such as justifies one's mathematical ideas, examines pattern within the domain of perimeter and area measurement, formulates generalization within the domain of perimeter and area measurement, tests generalization within the domain of perimeter and area measurement, develops area formulae, writes units of measurement upon they completed a task, and checks the correctness of their solutions or answers.

Sherin, Sherin, and Madanes (2000) argued that "to fully understand and appreciate the diversity that exists among research on teacher knowledge, we need to build some understanding of the types of theories that are proposed by various researchers" (p. 366). In the 1980s, new perspectives of teachers' knowledge, Shulman's (1986) theoretical framework in particular, had become prominent that it influences the direction of research on teachers (Ponte & Chapman, 2006).

Shulman (1986) suggested a framework for analyzing teachers' knowledge that differentiated three categories of knowledge, namely subject matter knowledge (SMK), pedagogical content knowledge (PCK), and curricular knowledge. SMK refers to “the amount and organization of the knowledge per se in the mind of the teacher" (Shulman, 1986, p. 9). PCK includes "the ways of representing and formulating the subject that make it comprehensible to others" and "an understanding of what makes the learning of specific topics easy or difficult, the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" (p. 9). Curricular knowledge refers to knowledge of instructional materials available for teaching various topics and the "sets of characteristics that serves as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances" (p. 10).

Shulman (1986) referred to the absence of focus on SMK for the research on teaching as the “missing paradigm” (p. 6). This referral suggested that SMK is an important component of teachers' knowledge. Moreover, “subject-matter knowledge is widely accepted as a central component of what teachers need to know” (Ball & McDiarmid, 1990, p. 437). Shulman's (1986) notion of teachers' knowledge in general, SMK in particular, formed the theoretical background of the present study.

Conceptual Framework of the Study

The literature review and theoretical framework of the study in the preceding sections has provided the basis for the construction of a conceptual framework for this study. The diagram in

Figure 2.7 illustrates this framework. Nik Azis (1996) suggested that there are five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge. This applies also to SMK. Specifically, SMK encompasses five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge.

In the present study, the researcher has adapted Nik Azis's (1996) categorization of knowledge to examine preservice secondary school mathematics teachers' SMK of perimeter and area. Specifically, SMK of perimeter and area encompasses five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area.

Conceptual knowledge is "knowledge that is rich in relationships". It consists of "network in which the linking relationships are as prominent as the discrete pieces of information" being linked (Hiebert & Lefevre, 1986, pp. 3-4). In this study, conceptual knowledge of perimeter and area encompasses the following components: (a) notion of perimeter, (b) notion of area, (c) notion of the units of area, (d) number of units and unit of measure, (e) inverse relationship/proportion between the number of units and the unit of measure, (f) relationship between the standard units of length measurement (linear units), (g) relationship between the standard units of area measurement (square units), (h) relationship between area units and linear units of measurement, (i) relationship between perimeter and area, and (j) relationship among area formulae.

Procedural knowledge refers to "the algorithms or rules for completing mathematical tasks" (adapted from Hiebert & Lefevre, 1986, p. 6). In this study, procedural knowledge of perimeter and area encompasses the following components: (a) converting standard units of area measurement, (b) calculating the perimeter of composite figures, (c) calculating the area of composite figures, and (d) developing area formulae.

Linguistic knowledge refers to “formal language, or symbol representation system of mathematics” (adapted from Hiebert & Lefevre, 1986, p. 5). In this study, linguistic knowledge of perimeter and area encompasses the following components: (a) mathematical symbols, (b) mathematical terms, (c) standard unit of length measurement (linear units), (d) standard unit of area measurement (square units), and (e) conventions of writing and reading SI area.

Strategic knowledge refers to “our ability to choose an appropriate strategy to solve a task because it is more effective than alternative strategies” (Henson & Eller, 1999, p. 258). In this study, strategic knowledge of perimeter and area encompasses the following components: (a) strategies for comparing perimeter, (b) strategies for comparing area, (c) strategies for checking answer for perimeter, (d) strategies for checking answer for area, (e) strategies for solving the fencing problem, (f) strategies for checking answer for the fencing problem, and (g) strategies for developing/deriving area formulae.

Ethical knowledge refers to “knowledge of right and wrong, what we are obligated to do, and of values” (Kupperman, 1970, p. 19). There are some good behaviors that the subjects need to follow when dealing with perimeter and area. In this study, ethical knowledge of perimeter and area encompasses the following components: (a) justifies one’s mathematical ideas, (b) examines pattern within the domain of perimeter and area measurement, (c) formulates generalization within the domain of perimeter and area measurement, (d) tests generalization within the domain of perimeter and area measurement, (e) develops area formulae, (f) writes units of measurement upon they completed a task, and (g) checks the correctness of their solutions or answers.

In this study, PSSMTs’ levels (low, medium, high) of SMK of perimeter and area were determined using coding rubrics (see Appendix L) adapted from the Learning Mathematics for

Teaching (LMT) (2006) project which consists of four principal investigators, namely Hill, Ball, Bass, and Schilling, from the School of Education, University of Michigan. It began by determining the level of PSSMTs' conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area, respectively.

In order to code the mathematical element of conceptual knowledge of perimeter and area, the researcher has to determine whether the mathematical element is present (P) or not present (NP). If the mathematical element is present (P), then mark: (a) appropriate (A) if the PSSMT's use of the mathematical element was mathematically appropriate, accurate, or correct; or mark (b) inappropriate (I) if the PSSMT's use of the mathematical element was mathematically inappropriate, inaccurate, or incorrect. If the mathematical element is not present (NP), then mark: (a) appropriate (A) if the absence of the mathematical element seems appropriate or not problematic; or mark (b) inappropriate (I) if the absence of the mathematical element seems inappropriate or problematic (i.e., the mathematical element should have present) (adapted from LMT, 2006). For the detail of the description of the procedure for determining the overall level of each subject's subject matter knowledge of perimeter and area, see Appendix K. For the sample of coding rubrics for determining overall level of a PSSMT's SMK of perimeter and area, see Appendix M).

In the Learning Mathematics for Teaching (LMT) (2006) project, random pairs of researchers were assigned to code each videotaped lesson. The coders coded each lesson individually and then gave an overall level of the teacher's knowledge of mathematics as low, medium, or high, based on their impression of the teacher's level of mathematical knowledge. They met and reconciled their codes before giving their final level of mathematical knowledge.

In the present study, PSSMTs' levels (low, medium, high) of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area as well as the overall level of SMK of perimeter and area were determined based on the percentage of appropriate mathematical elements of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area as well as the overall percentage of appropriate mathematical elements of SMK of perimeter and area obtained by the PSSMTs. For instance, the percentage of appropriate mathematical elements of conceptual knowledge of perimeter and area obtained by a PSSMT was computed as follow:

Percentage of appropriate mathematical elements of conceptual knowledge obtained by the PSSMT = $\frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\%$, where *fPA, fPI, fNPA,* and, *fNPI* represents the frequency of codes that were coded as "present and appropriate (PA)", "present and inappropriate (PI)", "not present and appropriate (NPA)", and "not present and inappropriate (NPI)", respectively.

In the university where the data of this study was collected, Grade A is assigned to PSSMTs who obtained 80 marks and above in the content as well as the method courses. Grade A− is assigned to PSSMTs who obtained 70 to 79 marks. The passing mark is 40. Thus, in this study, PSSMTs who secured 70% and above of appropriate mathematical elements of conceptual knowledge of perimeter and area were assigned a high level of conceptual knowledge of perimeter and area. PSSMTs who achieved the range from 40% to less than 70% of appropriate mathematical elements of conceptual knowledge of perimeter and area were assigned a medium level of conceptual knowledge of perimeter and area. PSSMTs who gained less than 40% of appropriate mathematical elements of conceptual knowledge of perimeter and area were assigned a low level of conceptual knowledge of perimeter and area.

The same procedure was applied to determine the PSSMTs' levels (low, medium, high) of procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area as well as the overall level of SMK of perimeter and area. PSSMTs' overall level of SMK of perimeter and area were determined by computing the percentage of all the appropriate mathematical elements that appeared in its five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge.

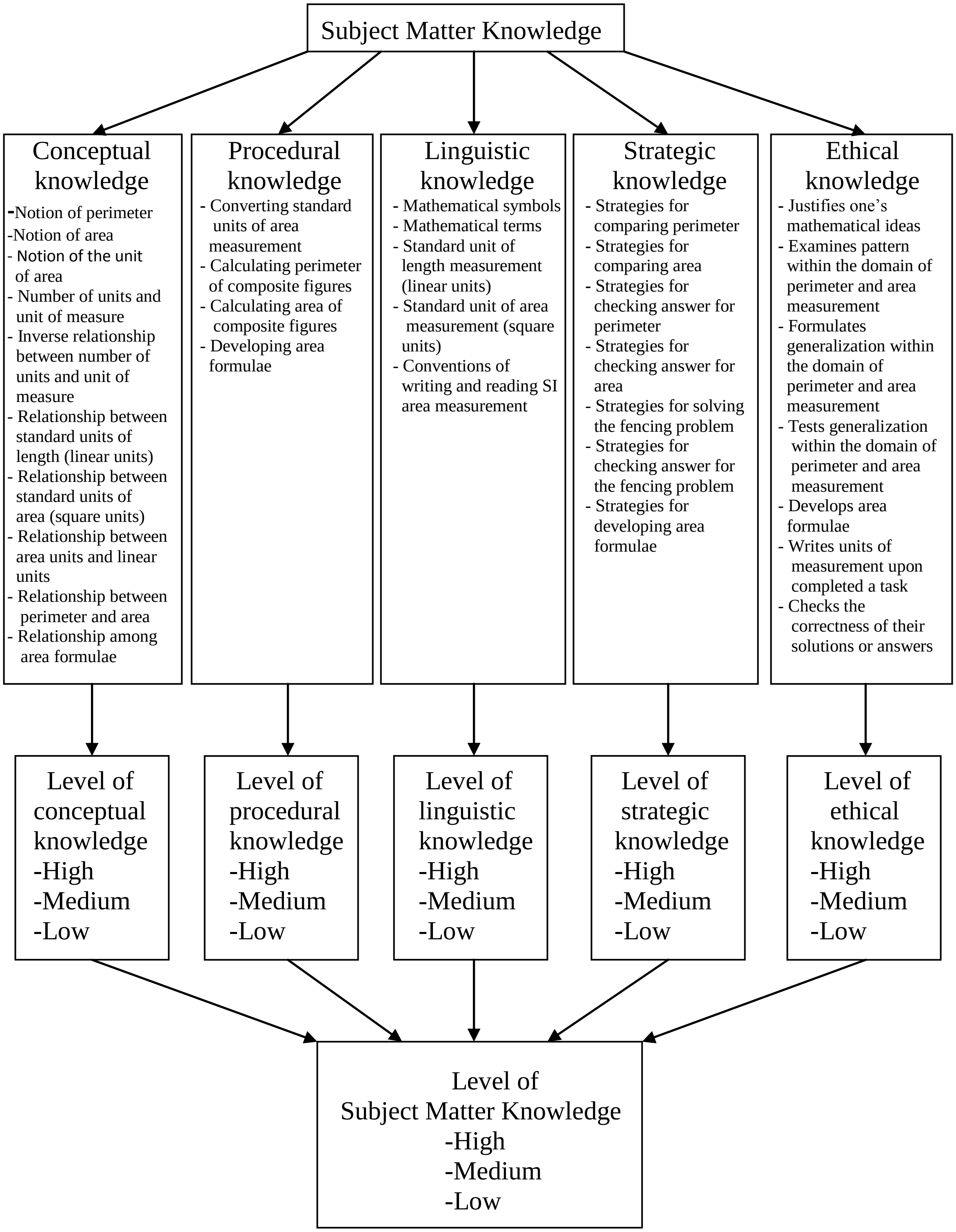


Figure 2.7. Conceptual framework of the study.