CHAPTER FOUR

FINDINGS OF THE STUDY

Introduction

This chapter provides the findings from the across-subjects analysis and descriptions of preservice secondary school mathematics teachers' behaviors when they attempted to solve each of the eight tasks during the clinical interview. The findings from the analysis and description of the behaviors of each subject, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, were presented in case studies in Appendix N.

In Chapter Four, to answer research question one, findings of preservice secondary school mathematics teachers (PSSMTs)' subject matter knowledge of perimeter and area were presented in terms of its five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge that were emerged from the clinical interview. To answer research question two, findings of PSSMTs' levels (low, medium, high) of SMK of perimeter and area were presented in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

Conceptual Knowledge

In this section, findings of PSSMTs' conceptual knowledge of perimeter and area were presented in terms of its components. Table 4.1 shows the components of conceptual knowledge of perimeter and area.

Table 4.1

The Components of Conceptual Knowledge of Perimeter and Area

Type of knowledge	Its components
Conceptual knowledge	1. Notion of perimeter 2. Notion of area 3. Notion of the unit of area 4. Number of units and unit of measure 5. Inverse relationship between number of units and unit of measure 6. Relationship between standard units of length (linear units) 7. Relationship between standard units of area (square units) 8. Relationship between area units and linear units 9. Relationship between perimeter and area 10. Relationship among area formulae

Notion of Perimeter

In Task 1.1, PSSMTs were asked to select the shapes that have a perimeter. Figure 4.1 shows Task 1.1. In Task 1.1, four PSSMTs, namely Liana, Roslina, Tan, and Usha, have successfully selected all the shapes that have a perimeter, namely shapes "A", "C", "D",”F”, "H", "I", “J”, and "K". They have successfully selected all simple closed curves (A, C, H, K) as well as all closed but not simple curves (D, I) that have a perimeter. Liana, Roslina, Tan, and Usha also selected the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that their notion of perimeter was not only limited to simple closed curves, and closed but not simple curves, but also inclusive of 3-dimensional shapes. Table 4.2 shows each PSSMT’s selection of shapes that have a perimeter and their notion of perimeter.

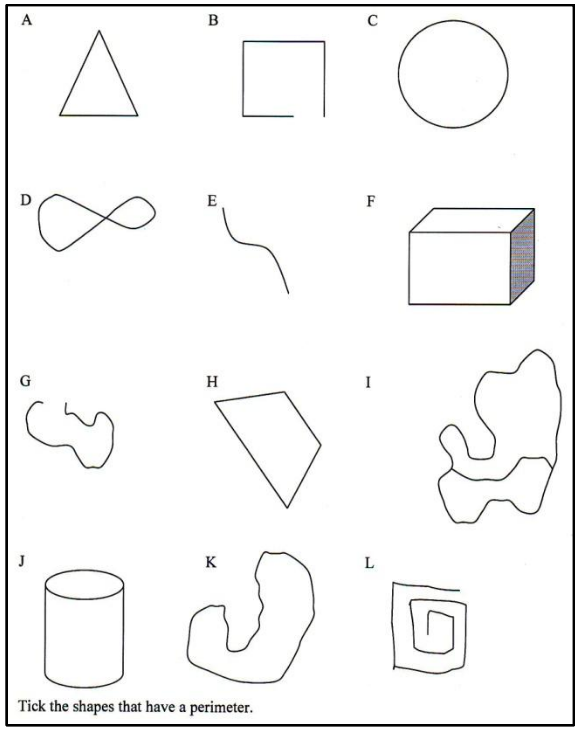


Figure 4.1. Task 1.1.

Table 4.2

PSSMTS' Selection of Shapes That Have a Perimeter and Their Notion of Perimeter

Selection of shapes that have a perimeter	Notion of perimeter	PSSMTs
"A", "C", "D","F", "H", "I", "J", and "K"	Simple closed curves, closed but not simple curves, and 3-dimensional shapes	Liana, Roslina, Tan, Usha
"A", "C", "D", "H", "I", and "K"	Limited to simple closed curves, and closed but not simple curves	Beng, Patrick, Suhana
"A", "C", and "H"	Limited to common simple closed curves (triangle, circle, and trapezium)	Mazlan

Three PSSMTs, namely Beng, Patrick, and Suhana, selected shapes "A", "C", "D", "H", "I", and "K" that have a perimeter. They have selected all simple closed curves (A, C, H, K) as well as all closed but not simple curves (D, I) that have a perimeter. Nevertheless, Beng, Patrick, and Suhana did not select the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that their notion of perimeter was limited to simple closed curves, and closed but not simple curves, exclusive of 3-dimensional shapes.

One PSSMT, namely Mazlan, selected shapes “A”, “C”, and “H” that have a perimeter. He has selected three simple closed curves (A, C, H) that have a perimeter. Nevertheless, Mazlan did not select another simple closed curve (K) and the two closed but not simple curves (D, I) that have a perimeter. He also did not select the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that his notion of perimeter was limited to common simple closed curves (triangle, circle, and trapezium). All the PSSMTs did not select the two simple but not closed curves (B, G) as well as the two 1-dimensional shapes (E, L) that do not have a perimeter. In other words, they did not select an open shape (including the lines) as having a perimeter.

Notion of Area

In Task 1.2, PSSMTs were asked to select the shapes that have an area. Figure 4.2 depicts Task 1.2. In Task 1.2, five PSSMTs, namely Liana, Mazlan, Suhana, Tan, and Usha, have successfully selected all the shapes that have an area, namely shapes "A", "C", "D",”F”, "H", "I", “J”, and "K". They have successfully selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. Liana, Mazlan, Suhana, Tan, and Usha also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that they had a static perspective of the notion of area. Based on this perspective, area can be viewed as the amount of surface enclosed within a boundary. It also indicated that their notion of area was not only limited to 2-dimensional shapes (closed plane shapes), but also inclusive of 3-dimensional shapes. Table 4.3 depicts each PSSMT’s selection of shapes that have an area and their notion of area.

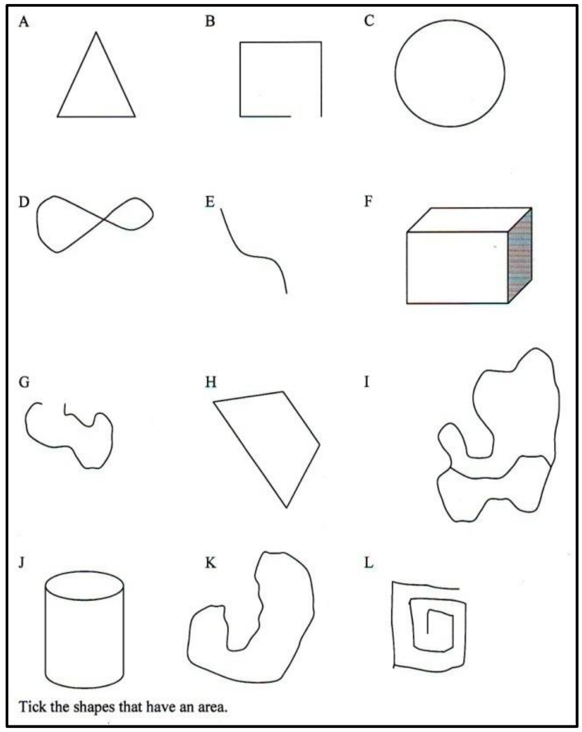


Figure 4.2. Task 1.2.

Table 4.3

PSSMTs' Selection of Shapes That Have an Area and Their Notion of Area

Selection of shapes that have a area	Notion of area	PSSMTs
"A", "C", "D",”F”, "H", "I", “J”, and "K"	A static perspective of the notion of area. Notion of area was not only limited to 2-dimensional shapes, but also inclusive of 3-dimensional shapes.	Liana, Mazlan, Suhana, Tan, Usha
“A”, “C”, “D”, “H”, “I”, and “K”	A static perspective of the notion of area. Notion of area was limited to 2-dimensional shapes.	Beng
“A”, “C”, “F”, “H”, and “J”	Notion of area were limited to regular 2-dimensional shapes (such as triangle, circle, and trapezium) and 3-dimensional shapes (such as cuboid and cylinder), where its area or surface area can be calculated using formula.	Patrick, Roslina

One PSSMT, namely Beng, selected shapes “A”, “C”, “D”, “H”, “I”, and “K” that have an area. She has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. Nevertheless, Beng did not select the two 3-dimensional shapes (F, J) that have an area. It revealed that she had a static perspective of the notion of area. Based on this perspective, area can be viewed as the amount of surface enclosed within a boundary. It also indicated that her notion of area was limited to 2-dimensional shapes (closed plane shapes).

Two PSSMTs, namely Patrick and Roslina, selected shapes “A”, “C”, “F”, “H”, and “J” that have an area. They have selected three of the 2-dimensional shapes (A, C, H) that have an area. Patrick and Roslina also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that their notion of area were limited to regular 2-dimensional shapes (such as triangle, circle, and trapezium) and 3-dimensional shapes (such as cuboid and cylinder), where its area or surface area can be calculated using formula.

All the PSSMTs did not select the two open shapes (B, G) as well as the two 1-dimensional shapes (E, L) that do not have an area. In other words, they did not select an open shape (including the lines) as having an area. It can be inferred that all the PSSMTs did not have a dynamic perspective of area or this knowledge was not accessible to them during the clinical interview.

Notion of the Units of Area

In Task 2, PSSMTs were asked to respond to a scenario where three students were discussing about the units of area. Figure 4.3 demonstrates Task 2.

Ali, Chong, and David are discussing about the units of area. Ali says that we can use a square as the unit of area. Chong says that we can use a rectangle as the unit of area. David says that we can use a triangle as the unit of area. How would you respond to these students?

Figure 4.3. Task 2.

In Task 2, three PSSMTs, namely Beng, Tan, and Usha, have successfully selected all the shapes that can be used as the units of area, namely square, rectangle, and triangle. It indicated that their notion of the units of area was not only limited to square, but also nonsquare (such as rectangle and triangle). Table 4.4 demonstrates each PSSMT's selection of shapes that can be used as the units of area and their notion of the units of area.

Table 4.4

PSSMTs' Selection of Shapes That can be Used as the Units of Area and Their Notion of the Units of Area

Selection of shapes that that can be used as the unit of area	Notion of the units of area	PSSMTs
Square, rectangle, and triangle	Square and nonsquare can be used as the unit of area	Beng, Tan, Usha
Square and rectangle	Notion of the units of area was limited to square and rectangle	Patrick
Square and triangle	Notion of the units of area was limited to square and triangle	Mazlan
Square	Notion of the units of area was limited to square	Roslina, Suhana
None of the square, rectangle, and triangle	None or not accessible to her during the clinical interview	Liana

One PSSMT, namely Patrick, had selected square and rectangle that can be used as the units of area. Patrick was not sure whether a triangle can used as the unit of area measure because there are many types of triangles such as isosceles triangle and equilateral triangle. It indicated that his notion of the unit of area was limited to square and rectangle. One PSSMT, namely Mazlan, had selected square and triangle that can be used as the units of area. He thought that rectangle cannot be used as the unit of area. It indicated that his notion of the unit of area was limited to square and triangle.

Two PSSMTs, namely Roslina and Suhana, had selected square that can be used as the unit of area. They thought that rectangle and triangle cannot be used as the unit of area. It indicated that their notion of the unit of area was limited to square. One PSSMT, namely Liana, selected none of the square, rectangle, and triangle that can be used as the unit of area. She thought that square, rectangle, and triangle that cannot be used as the unit of area. It indicated that Liana did not have any idea about the unit of area or the notion of the unit of area was not accessible to her during the clinical interview.

Number of Units and Unit of Measure

In this subsection, findings of PSSMTs' conceptual knowledge of number of units and unit of measure were presented in terms of: (a) comparing perimeters with nonstandard units, (b) comparing perimeters with common nonstandard units, (c) comparing perimeters with common standard unit, (d) comparing areas with nonstandard units, (e) comparing areas with common nonstandard units, and (f) comparing areas with common standard unit.

Comparing Perimeters with Nonstandard Units

In Task 3.3 (a), PSSMTs were asked to compare, from the measurements given, which shape in Sets 1 has the longer perimeter. Figure 4.4 reveals Task 3.3 (a).

In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 1	25 paper clips	12 sticks

Figure 4.4. Task 3.3 (a).

In Task 3.3 (a), half of the PSSMTs, namely Liana, Patrick, Tan, and Usha, provided the correct response 'unable to determine which shape has the longer perimeter' when they were comparing perimeters in Set 1 with nonstandard units. Liana explained that she was unable to determine which shape has the longer perimeter as there were various sizes of paper clip and stick. Patrick and Tan explained that they were unable to determine which shape has the longer perimeter as the length of each paper clip and stick were not known. Usha explained that she was unable to determine which shape has the longer perimeter as she did not know the size of the paper clip and the stick. It indicated that Liana, Patrick, Tan, and Usha focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. They knew that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. Table 4.5 reveals PSSMTs' responses when comparing perimeters in Set 1 with nonstandard units.

Table 4.5

PSSMTs' Responses When Comparing Perimeters in Set 1 With Nonstandard Units

Responses	PSSMTs
Shape A has the longer perimeter	Beng
Shape B has the longer perimeter	Mazlan, Roslina, Suhana
Unable to determine which shape has the longer perimeter	Liana, Patrick, Tan, Usha

Three PSSMTs, namely Mazlan, Roslina, and Suhana, thought that shape B has the longer perimeter. Mazlan, Roslina, and Suhana explained that shape B has the longer perimeter because they thought that a stick is longer than a paper clip. It indicated that Mazlan, Roslina, and Suhana focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. Nevertheless, they did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters.

Only one PSSMT, namely Beng thought that shape A has the longer perimeter. She made an assumption that a paper clip is almost the size of a stick. Beng argued that there were 25 units of paper clips compared to 12 units of sticks. Therefore, she thought that shape A has the larger area. Beng generalized that the larger the area, the longer the perimeter. Thus, she thought that shape A has the longer perimeter. It indicated that she focused on the number of unit rather than the unit of measure when comparing perimeters in Set 1 with nonstandard units. Beng did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters.

Comparing Perimeters with Common Nonstandard Units

In Task 3.3 (b), PSSMTs were asked to compare, from the measurements given, which shape in Sets 2 has the longer perimeter. Figure 4.5 exhibits Task 3.3 (b).

In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 2	10 paper clips	15 paper clips

Figure 4.5. Task 3.3 (b).

In Task 3.3 (b), three of the PSSMTs, namely Liana, Tan, and Usha, provided the correct response ‘unable to determine which shape has the longer perimeter’ when they were comparing perimeters in Set 2 with common nonstandard units. Liana explained that if the paper clips for shapes A and B were of the varied size, then she was unable to determine which shape has the longer perimeter. Liana also explained that if the paper clips for both shapes A and B were of the same size, then shape B has the longer perimeter.

Tan explained that if the paper clips for shapes A and B were of the different length, then he was unable to determine which shape has the longer perimeter. Tan also explained that if the paper clips for both shapes A and B were of the same length, then shape B has the longer perimeter as it has 15 paper clips compared to 10 paper clips in shape A. Usha explained that she was unable to determine which shape has the longer perimeter as she did not know whether they used the same paper clips for shapes A and B. Usha also explained that shape B has the longer perimeter if the same size of paper clips were used for both shapes. It indicated that Liana, Tan, and Usha focused on the unit of measure when comparing perimeters in Set 2 with common nonstandard unit. They knew that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. Table 4.6 exhibits PSSMTs' responses when comparing perimeters in Set 2 with common nonstandard units.

Table 4.6

PSSMTs' Responses When Comparing Perimeters in Set 2 With Common Nonstandard Units

Responses	PSSMTs
Shape B has the longer perimeter	Beng, Mazlan, Patrick, Roslina, Suhana
Unable to determine which shape has the longer perimeter	Liana, Tan, Usha

Five PSSMTs, namely Beng, Mazlan, Patrick, Roslina, and Suhana, thought that shape B has the longer perimeter. They explained that shape B has the longer perimeter because both shapes A and B used paper clip as the unit of measurement and shape B has 15 paper clips compared to 10 paper clips in shape A. It indicated that Beng, Mazlan, Patrick, Roslina, and Suhana focused on the number of unit rather than the unit of measure when comparing perimeters in Set 2 with common nonstandard unit. They did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters.

Comparing Perimeters with Common Standard Unit

In Task 3.3 (c), PSSMTs were asked to compare, from the measurements given, which shape in Sets 3 has the longer perimeter. Figure 4.6 illustrates Task 3.3 (c).

In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 3	16 cm	13 cm

Figure 4.6. Task 3.3 (c).

In Task 3.3 (c), all the eight PSSMTs provided the correct response 'shape A has the longer perimeter' when they were comparing perimeters in Set 3 with common standard unit. Two of the PSSMTs, namely Tan and Usha, explained that shape A has the longer perimeter because centimetre is a standard unit of length measurement and 16 is larger than 13. Two of the PSSMTs, namely Patrick and Roslina, explained that shape A has the longer perimeter because they used the same unit, namely centimetre, and 16 is larger than 13. Three of the PSSMTs, namely Liana, Mazlan and Suhana, explained that shape A has the longer perimeter because 16 is larger than 13. The remaining PSSMT, namely Beng, explained that the shape with the larger value (larger number of unit) has the longer perimeter. It indicated that all the eight PSSMTs focused on the number of unit when comparing perimeters in Set 3 with common standard unit. They knew that common standard unit (such as cm) is reliable for comparing perimeters.

Comparing Areas with Nonstandard Units

In Task 3.4 (a), PSSMTs were asked to compare, from the measurements given, which shape in Sets 1 has the larger area. Figure 4.7 shows Task 3.4 (a).

In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 1	25 triangles	12 squares

Figure 4.7. Task 3.4 (a).

In Task 3.4 (a), five of the PSSMTs, namely Liana, Patrick, Suhana, Tan, and Usha, provided the correct response ‘unable to determine which shape has the larger area’ when they were comparing areas in Set 1 with nonstandard units. Liana, Suhana and Tan explained that they were unable to determine which shape has the larger area as they did not know the length (of side) of the triangle and square. Patrick explained that he was unable to determine which shape has the larger area as it depends on the area of the square and triangle. Usha explained that she was unable to determine which shape has the larger area as they were different shape, triangle and square, and she did not know the area of each triangle and square. It indicated that Liana, Patrick, Suhana, Tan, and Usha, focused on the unit of measure when comparing area in Set 1 with nonstandard units. They knew that nonstandard units (such as triangle and square) are not reliable for comparing areas. Table 4.7 illustrates PSSMTs' responses when comparing areas in Set 1 with nonstandard units.

Table 4.7

PSSMTs' Responses When Comparing Areas in Set 1 With Nonstandard Units

Responses	PSSMTs
Shape A has the larger area	Beng, Mazlan
Shape B has the larger area	Roslina
Unable to determine which shape has the larger area	Liana, Patrick, Suhana, Tan, Usha

Two PSSMTs, namely Beng and Mazlan, thought that shape A has the larger area. Beng made an assumption that two triangles from shape A would form one square from shape B. She stated that we need 24 triangles from shape A to form 12 square in shape B. Beng explained that shape A has 25 triangles which is one triangle more than the required 24 triangles to form 12 squares in shape B. Thus, she concluded that shape A has the larger area. In reality, two triangles from shape A do not necessarily form one square in shape B.

Mazlan explained that shape A has the larger area because it has more squares, namely 12.5 squares, compared to shape B with 12 squares. He elaborated that a triangle is a half of a square and thus 25 divided by 2 equals to 12.5. In reality, two triangles from shape A do not necessarily form one square in shape B. It indicated that Beng and Mazlan focused on the number of unit rather than the unit of measure when comparing areas in Set 1 with nonstandard units. They did not know that nonstandard units (such as triangle and square) are not reliable for comparing areas.

Only one PSSMT, namely Roslina, thought that shape B has the larger area. Roslina explained that shape B has the larger area because the area of a square is larger compared to the area of a triangle. It indicated that Roslina focused on the unit of measure when comparing areas in Set 1 with nonstandard units. Nevertheless, she did not know that nonstandard units (such as triangle and squares) are not reliable for comparing areas.

Comparing Areas with Common Nonstandard Units

In Task 3.4 (b), PSSMTs were asked to compare, from the measurements given, which shape in Sets 2 has the larger area. Figure 4.8 depicts Task 3.4 (b).

In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 2	10 squares	15 squares

Figure 4.8. Task 3.4 (b).

In Task 3.4 (b), three of the PSSMTs, namely Liana, Tan, and Usha, provided the correct response ‘unable to determine which shape has the larger area’ when they were comparing areas in Set 2 with common nonstandard units. Liana explained that she was unable to determine which shape has the larger area because the squares in shapes A and B might be different (of area). Tan explained that he was unable to determine which shape has the larger area because 10 and 15 are just the quantities of the squares, not the area of the squares. Tan elaborated that if the area of each square were the same, then certainly shape B with 15 squares has the larger area than shape A with 10 squares only. He expressed that if the area of the squares were different, then he would unable to determine which shape has the larger area even though the quantity of 15 is larger than 10.

Usha explained that she was unable to determine which shape has the larger area as she did not know the area of the squares in shapes A and B. It indicated that Liana, Tan, and Usha focused on the unit of measure when comparing areas in Set 2 with common nonstandard unit. They knew that common nonstandard units (such as squares) are not reliable for comparing areas.

Table 4.8 shows PSSMTs' responses when comparing areas in Set 2 with common nonstandard units.

Table 4.8

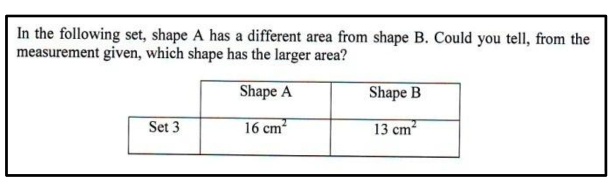
PSSMTs' Responses When Comparing Areas in Set 2 With Common Nonstandard Units

Responses	PSSMTs
Shape B has the larger area	Beng, Mazlan, Patrick, Roslina, Suhana
Unable to determine which shape has the larger area	Liana, Tan, Usha

Five of the PSSMTs, namely Beng, Mazlan, Patrick, Roslina, and Suhana, thought that shape B has the larger area. Patrick and Roslina explained that shape B has the larger area because they used the same unit, namely squares, and 15 is larger than 10. Mazlan and Suhana explained that shape B has the larger area because 15 is larger than 10. Beng assumed that squares from shapes A and B are of equal area. Thus, shape B has the larger area as it has more squares compared to shape A. It indicated that Beng, Mazlan, Patrick, Roslina, and Suhana focused on the number of unit rather than the unit of measure when comparing areas in Set 2 with common nonstandard units. They did not know that common nonstandard units (such as squares) are not reliable for comparing areas.

Comparing Areas with Common Standard Unit

In Task 3.4 (c), PSSMTs were asked to compare, from the measurements given, which shape in Sets 3 has the larger area. Figure 4.9 demonstrates Task 3.4 (c).



In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 3	16 cm^2	13 cm^2

Figure 4.9. Task 3.4 (c).

In Task 3.4 (c), all the eight PSSMTs provided the correct response 'shape A has the larger area' when they were comparing areas in Set 3 with common standard unit. Five of the

PSSMTs, namely Liana, Mazlan, Patrick, Suhana, and Usha, explained that shape A has the larger area because they used the same unit, namely cm^2 and 16 is larger than 13. Beng explained that shape A has a larger value compared to shape B. Therefore, shape A has the larger area.

Roslina explained that shape A has the larger area because they used the same unit, namely cm^2, and shape A has 16 cm^2 compared to shape B with 13 cm^2. Tan explained that shape A has the larger area because they used the same standard unit, namely cm^2, and 16 is larger than 13. It indicated that all the eight PSSMTs focused on the number of unit when comparing areas in Set 3 with common standard units. They knew that common standard unit (such as cm^2) is reliable for comparing areas.

Inverse Proportion between Number of Units and Unit of Measure

In this subsection, findings of PSSMTs' conceptual knowledge of the inverse proportion between the number of units and the unit of measure were presented in terms of: (a) perimeter, and (b) area.

Perimeter

In Task 3.3 (b), in another situation in Set 2 when shapes A and B had the same perimeter, five of the PSSMTs, namely Liana, Mazlan, Roslina, Tan, and Usha explained that the paper clips in shape A is longer than the paper clips in shape B so that they had the same perimeter. Beng explained that the shape with the longer paper clips (shape A) required less number of paper clips to produce the same perimeter as shape B. Patrick explained that the paper clips in shape A is longer than the paper clips in shape B because 10 paper clips of shape A is same length as 15 paper clips of shape B. Suhana explained that the paper clips in shape B is smaller (shorter) than the paper clips in shape A. It indicated that all the eight PSSMTs understand the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length, and vice versa.

Area

In Task 3.4 (b), in another situation in Set 2 when shapes A and B had the same area, six of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Suhana, and Usha, explained that the squares in shape A is larger than the squares in shape B. Roslina explained that the square from shape A is big and the square from shape B is small while Tan explained that the squares in shape A are bigger compare to the squares in shape B. It indicated that all the eight PSSMTs understand the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area, and vice versa.

Relationships between the Standard Units of Length Measurement

In this subsection, findings of PSSMTs' conceptual knowledge of the relationships between the standard units of length measurement were presented in terms of: (a) 1 cm = 10 mm, (b) 1 m = 100 cm, and (c) 1 km = 1000 m.

1 cm = 10 mm

In Task 4, PSSMTs were asked to respond to a scenario where several students encountered difficulty in converting units of area. Figure 4.10 reveals Task 4.

Some Form One teachers noticed that several of their students seemed to multiply by 10, 100, and 1000, respectively when they were converting units of area from cm^2 to mm^2, m^2 to cm^2, and km^2 to m^2:

(a) 3 cm^2 = 3 x 10 mm^2 = 30 mm^2
(b) 4.7 m^2 = 4.7 x 100 cm^2 = 470 cm^2
(c) 1.25 km^2 = 1.25 x 1000 m^2 = 1250 m^2

What would you do if you were teaching Form One and you noticed that several of your students were doing this?

Figure 4.10. Task 4.

In Task 4, five of the PSSMTs, namely Beng, Liana, Patrick, Suhana, and Tan, had successfully converting 3 cm^2 to mm^2. Six of the PSSMTs, namely Beng, Liana, Patrick, Roslina, Suhana, and Tan, knew the relationships between the standard units of length measurement that 1 cm = 10 mm. Table 4.9 depicts PSSMTs who knew and did not know the relationships between the standard units of length measurement that 1 cm = 10 mm.

Table 4.9

PSSMTs who Knew and did not Know the Relationships Between the Standard Units of Length Measurement That 1 cm = 10 mm

Relationships between the standard units of length measurement that 1 cm = 10 mm	PSSMTs
Knew	Beng, Liana, Patrick, Roslina, Suhana, Tan
Did not know	Mazlan, Usha

Beng and Patrick viewed 3 cm^2 as the product of 3 times 1 cm times 1 cm. Beng and Patrick times 10 when they converted 1cm to mm. It indicated that Beng and Patrick knew the relationship between the standard units of length measurement that 1 cm = 10 mm. Liana viewed 3 cm^2 as the product of 3 times 1 cm^2. Liana times ten squared, $(10)^2$, when she converted 3 cm^2 to mm^2. It indicated that she knew the relationship between the standard units of length measurement that 1 cm = 10 mm.

Suhana viewed 3 cm as the product of 3 times 1 cm. Suhana times 10 when she converted 3 cm to mm. It indicated that Suhana knew the relationship between the standard units of length measurement that 1 cm = 10 mm. Tan viewed 3 cm^2 as the product of 1 cm times 3 cm. Tan times 10 twice when he converted 1 cm to mm and 3 cm to mm separately. It indicated that Tan knew the relationship between the standard units of length measurement that 1 cm = 10 mm.

The remaining three PSSMTs, namely Mazlan, Roslina, and Usha, had unsuccessfully converting 3 cm^2 to mm^2. Mazlan has incorrectly converted 3 cm^2 to 3 x 10^{-4} mm^2. Mazlan converted 3 cm^2 to m^2 first and then from m^2, he converted it to mm^2. Mazlan thought that 1 m^2 = 10 cm^2. Thus, Mazlan multiplied 10^{-1} (ten to the power of negative one) when he converted 3 cm^2 to m^2. It indicated that Mazlan did not know the relationship between the standard units of length measurement that 1 cm = 10 mm and 1 m = 100 cm.

Roslina wrote that 1 cm = 10 mm. It indicated that she knew the relationship between the standard units of length measurement that 1 cm = 10 mm. Usha thought that 1 cm^2 = 10 mm^2. It indicated that she did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2. It also indicated that Usha did not know the relationships between the standard units of length measurement that 1 cm = 10 mm.

1 m = 100 cm

In Task 4, half of the PSSMTs, namely Beng, Liana, Patrick, and Tan, had successfully converting 4.7 m^2 to cm^2. Six of the PSSMTs, namely Beng, Liana, Patrick, Roslina, Suhana, and Tan, knew the relationships between the standard units of length measurement that 1 m = 100 cm. Table 4.10 demonstrates PSSMTs who knew and did not know the relationships between the standard units of length measurement that 1 m = 100 cm.

Table 4.10

PSSMTs who Knew and did not Know the Relationships Between the Standard Units of Length Measurement That 1 m = 100 cm

Relationships between the standard units of length measurement that 1 m = 100 cm	PSSMTs
Knew	Beng, Liana, Patrick, Roslina, Suhana, Tan
Did not know	Mazlan, Usha

Beng and Patrick viewed 4.7 m^2 as the product of 4.7 times 1 m times 1 m. Beng and Patrick times 100 when they converted 1 m to cm. It indicated that Beng and Patrick knew the relationship between the standard units of length measurement that 1 m = 100 cm. Liana viewed 4.7 m^2 as the product of 4.7 times 1 m^2. Liana times one hundred squared, namely $(100)^2$, when she converted 4.7 m^2 to cm^2. It indicated that she knew the relationship between the standard units of length measurement that 1 m = 100 cm. Tan viewed 4.7 m^2 as the product of 1 m times 4.7 m. Tan times 100 twice when he converted 1 m to cm and 4.7 m to cm respectively. It indicated that Tan knew the relationship between the standard units of length measurement that 1 m = 100 cm.

The remaining half of the PSSMTs, namely Mazlan, Roslina, Suhana, Usha, had unsuccessfully converting 4.7 m^2 to cm^2. Suhana viewed 4.7 m as the product of 4.7 times 1 m. Thus, she times 100 when she converted 4.7 m to cm. It indicated that Suhana knew the relationship between the standard units of length measurement that 1 m = 100 cm. Mazlan has incorrectly converted 4.7 m^2 to 470 cm^2. He thought that 1 m = 10 cm. Thus, Mazlan multiplied $(10^1 \text{ cm})^2$ or 10^2 cm^2 when he converted 4.7 m^2 to cm^2. It indicated that Mazlan did not know the relationship between the standard units of length measurement that 1 m = 100 cm.

Roslina wrote that 1 m = 100 cm. It indicated that she knew the relationship between the standard units of length measurement that 1 m = 100 cm. Usha thought that 1 m^2 = 100 cm^2. It indicated that she did not know the relationships between the standard units of area measurement that 1 m^2 = 10 000 cm^2. It also indicated that Usha did not know the relationships between the standard units of length measurement that 1 m = 100 cm.

1 km = 1000 m

In Task 4, half of the PSSMTs, namely Beng, Liana, Patrick, and Tan, had successfully converting 1.25 km^2 to m^2. Seven of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, and Tan, knew the relationships between the standard units of length measurement that 1 km = 1000 m. Table 4.11 reveals PSSMTs who knew and did not know the relationships between the standard units of length measurement that 1 km = 1000 m.

Table 4.11

PSSMTs who Knew and did not Know the Relationships Between the Standard Units of Length Measurement That 1 km = 1000 m

Relationships between the standard units of length measurement that 1 km = 1000 m	PSSMTs
Knew	Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan
Did not know	Usha

Beng and Patrick viewed 1.25 km^2 as the product of 1.25 times 1 km times 1 km. Beng and Patrick times 1000 when they converted 1 km to m. It indicated that Beng and Patrick knew the relationship between the standard units of length measurement that 1 km = 1000 m. Liana viewed 1.25 km^2 as the product of 1.25 times 1 km^2. Liana times one thousand squared, namely $(1000)^2$, when she converted 1.25 km^2 to m^2. It indicated that she knew the relationship between the standard units of length measurement that 1 km = 1000 m. Tan viewed 1.25 km^2 as the product of 1 km times 1.25 km. Tan times 1000 twice when he converted 1km to m and 1.25 km to m respectively. It indicated that Tan knew the relationship between the standard units of length measurement that 1 km = 1000 m.

The remaining half of the PSSMTs, namely Mazlan, Roslina, Suhana, and Usha, had unsuccessfully converting 1.25 km^2 to m^2. Suhana viewed 1.25 km as the product of 1.25 times 1 km. Thus, Suhana times 1000 when she converted 1.25 km to m. It indicated that Suhana knew the relationship between the standard units of length measurement that 1 km = 1000 m.

Mazlan has incorrectly converted 1.25 km^2 to 1250 x 10^6 m^2. He multiplied $(10^3\ m)^2$ or 10^6 m^2 when he converted 1.25 km^2 to m^2. It indicated that Mazlan knew the relationship between the standard units of length measurement that 1 km = 1000 m. Roslina wrote that 1 km = 1000

m. It indicated that she knew the relationship between the standard units of length measurement that 1 km = 1000 m. Usha thought that 1 km^2 = 1000 m^2. It indicated that she did not know the relationships between the standard units of area measurement that 1 km^2 = 1000 000 m^2. It also indicated that Usha did not know the relationships between the standard units of length measurement that 1 km = 1000 m.

Summary

In summary, six of the PSSMTs, namely Beng, Liana, Patrick, Roslina, Suhana, and Tan, knew the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. Mazlan did not know the relationship between the standard units of length measurement that 1 cm = 10 mm and 1 m = 100 cm. Nevertheless, he knew that 1 km = 1000 m. Usha did not know the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m.

Relationship between the Standard Units of Area Measurement

In this subsection, findings of PSSMTs' conceptual knowledge of the relationships between the standard units of area measurement were presented in terms of: (a) 1 cm^2 = 100 mm^2, (b) 1 m^2 = 10 000 cm^2, and (c) 1 km^2 = 1 000 000 m^2.

1 cm^2 = 100 mm^2

In Task 4, three of the PSSMTs, namely Mazlan, Roslina, and Usha, did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2. Mazlan has incorrectly converted 3 cm^2 to 3 x 10^{-4} mm^2. Mazlan converted 3 cm^2 to m^2 first and then from m^2, he converted it to mm^2. Mazlan thought that 1 m^2 = 10 cm^2. Thus, Mazlan multiplied 10^{-1}

(ten to the power of negative one) when he converted 3 cm^2 to m^2. It indicated that Mazlan did not know the relationship between the standard units of area measurement that 1 cm^2 = 100 mm^2 and 1 m^2 = 10 000 cm^2.

Roslina knew that 1 cm = 10 mm. Nevertheless, she thought that 1 cm^2 = 10 mm^2. Thus, Roslina times 10 when she converted 3 cm^2 to mm^2. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2. Usha thought that 1 cm^2 = 10 mm^2. It indicated that she did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2.

1 m^2 = 10 000 cm^2

In Task 4, three of the PSSMTs, namely Mazlan, Roslina, and Usha, did not know the relationships between the standard units of area measurement that 1 m^2 = 10 000 cm^2. Mazlan has incorrectly converted 4.7 m^2 to 470 cm^2. He thought that 1 m = 10 cm. Thus, Mazlan multiplied $(10^1 \text{ cm})^2$ or 10^2 cm^2 when he converted 4.7 m^2 to cm^2. It indicated that Mazlan did not know the relationship between the standard units of area measurement that and 1 m^2 = 10 000 cm^2.

Roslina knew that 1 m = 100 cm. Nevertheless, she thought that 1 m^2 = 100 cm^2. Thus, Roslina times 100 when she converted 4.7 m^2 to cm^2. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 m^2 = 10 000 cm^2. Usha thought that 1 m^2 = 100 cm^2. It indicated that She did not know the relationships between the standard units of area measurement that 1 m^2 = 10 000 cm^2.

1 km^2 = 1000 000 m^2

In Task 4, Mazlan knew the relationship between the standard units of area measurement that 1 km^2 = 1 000 000 m^2. Mazlan has incorrectly converted 1.25 km^2 to 1250 x 10^6 m^2. Mazlan multiplied $(10^3 \text{ m})^2$ or 10^6 m^2 when he converted 1.25 km^2 to m^2. It indicated that Mazlan knew the relationship between the standard units of area measurement that 1 km^2 = 1 000 000 m^2.

Two the PSSMTs, namely Roslina and Usha, did not know the relationships between the standard units of area measurement that 1 km^2 = 1000 000 m^2. Roslina knew that 1 km = 1000 m. Nevertheless, she thought that 1 km^2 = 1000 m^2. Thus, Roslina times 1000 when she converted 1.25 km^2 to m^2. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 km^2 = 1000 000 m^2. Usha thought that 1 km^2 = 1000 m^2. It indicated that She did not know the relationships between the standard units of area measurement that 1 km^2 = 1000 000 m^2.

Summary

Mazlan knew the relationship between the standard units of area measurement that 1 km^2 = 1 000 000 m^2. Nevertheless, he did not know the relationship between the standard units of area measurement that 1 cm^2 = 100 mm^2 and 1 m^2 = 10 000 cm^2. Roslina and Usha thought that 1 cm^2 = 10 mm^2, 1 m^2 = 100 cm^2, and 1 km^2 = 1000 m^2. It indicated that they did not know the relationships between the standard units of area measurement such as 1 cm^2 = 100 mm^2, 1 m^2 = 10 000 cm^2, and 1 km^2 = 1000 000 m^2.

Relationship between Area Units and Linear Units of Measurement

In Task 4, Beng and Patrick viewed 3 cm^2 as the product of 3 times 1 cm times 1 cm. It indicated that they viewed 1 cm^2 as 1 cm times 1 cm. It also indicated that Beng and Patrick knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Liana viewed 3 cm^2 as the product of 3 times 1 cm^2. Liana times ten squared, $(10)^2$, when she converted 3 cm^2 to mm^2. It indicated that Liana knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring.

Suhana viewed 3 cm as the product of 3 times 1 cm. Similarly, she viewed 3 cm^2 as the product of 3 times 1 cm^2. Suhana times ten squared, $(10)^2$, when she converted 3 cm^2 to mm^2. It indicated that Suhana knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Tan viewed 3 cm^2 as the product of 1 cm times 3 cm. Tan times 10 twice when he converted 1 cm to mm and 3 cm to mm separately. It indicated that Tan knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring.

Roslina knew that 1 cm = 10 mm. Nevertheless, Roslina thought that 1 cm^2 = 10 mm^2. Thus, Roslina times 10 when she converted 3 cm^2 to mm^2. It indicated that Roslina did not know the relationships between the standard units of area measurement such as 1 cm^2 = 100 mm^2. It also indicated that she did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring.

Usha thought that 1 cm^2 = 10 mm^2. It indicated that she did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2. Usha did not know the relationships between the standard units of length measurement that 1 cm = 10 mm. It also indicated that she did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring.

Beng and Patrick viewed 4.7 m^2 as the product of 4.7 times 1 m times 1 m. It indicated that they viewed 1 m^2 as 1 m times 1 m. It also indicated that Beng and Patrick knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring.

Liana viewed 4.7 m^2 as the product of 4.7 times 1 m^2. Liana times one hundred squared, namely $(100)^2$, when she converted 4.7 m^2 to cm^2. It indicated that Liana knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Tan viewed 4.7 m^2 as the product of 1 m times 4.7 m. Tan times 100 twice when he converted 1 m to cm and 4.7 m to cm respectively. It indicated that Tan knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring.

Suhana viewed 4.7 m as the product of 4.7 times 1 m. Similarly, she viewed 4.7 m^2 as the product of 4.7 times 1 m^2. Suhana times one hundred squared, namely $(100)^2$, when she converted 4.7 m^2 to cm^2. It indicated that Suhana knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Mazlan has incorrectly converted 4.7 m^2 to 470 cm^2. He thought that 1 m = 10 cm. Thus, Mazlan multiplied $(10^1 \text{ cm})^2$ or 10^2 cm^2 when he converted 4.7 m^2 to cm^2. It indicated that Mazlan knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. This can be seen when he squared (10^1 cm) to get 10^2 cm^2.

Roslina knew that 1 m = 100 cm. Nevertheless, Roslina thought that 1 m^2 = 100 cm^2. Thus, Roslina times 100 when she converted 4.7 m^2 to cm^2. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 m^2 = 10 000 cm^2. It also indicated that she did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Usha thought that 1 m^2 = 100 cm^2. It indicated that she did not know the relationships between the standard units of area measurement that 1 m^2 = 10 000 cm^2. Usha did not know the relationships between the standard units of length measurement that 1 m = 100 cm. It also indicated that she did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring.

Beng and Patrick viewed 1.25 km^2 as the product of 1.25 times 1 km times 1 km. It indicated that they viewed 1 km^2 as 1 km times 1 km. It also indicated that Beng and Patrick knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Liana viewed 1.25 km^2 as the product of 1.25 times 1 km^2. Liana times one thousand squared, namely $(1000)^2$, when she converted 1.25 km^2 to m^2. It indicated that Liana knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring.

Tan viewed 1.25 km^2 as the product of 1 km times 1.25 km. Tan times 1000 twice when he converted 1 km to m and 1.25 km to m respectively. It indicated that Tan knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Suhana viewed 1.25 km as the product of 1.25 times 1 km. Similarly, Suhana viewed 1.25 km^2 as the product of 1.25 times 1 km^2. Suhana times one thousand squared, namely $(1000)^2$, when she converted 1.25 km^2 to m^2. It indicated that Suhana knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring.

Mazlan has incorrectly converted 1.25 km^2 to 1250 x 10^6 m^2. He multiplied $(10^3\ m)^2$ or 10^6 m^2 when he converted 1.25 km^2 to m^2. It indicated that Mazlan knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. This can be seen when he squared (10^3 m) to get 10^6 m^2.

Roslina knew that 1 km = 1000 m. Nevertheless, she thought that 1 km^2 = 1000 m^2. Thus, Roslina times 1000 when she converted 1.25 km^2 to m^2. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 km^2 = 1000 000 m^2. It also indicated that she did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring.

Usha thought that 1 km^2 = 1000 m^2. It indicated that She did not know the relationships between the standard units of area measurement that 1 km^2 = 1000 000 m^2. Usha did not know the relationships between the standard units of length measurement that 1 km = 1000 m. It also indicated that she did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring.

Summary

In summary, six of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Suhana, and Tan, knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. The remaining two PSSMTs, namely Roslina and Usha, did not know the relationships between area units and linear units of measurement. Table 4.12 exhibits PSSMTs who knew and did not know the relationships between area units and linear units of measurement.

Table 4.12

PSSMTs who Knew and did not Know the Relationships Between Area Units and Linear Units of Measurement

Relationships between area units and linear units of measurement	PSSMTs
Knew	Beng, Liana, Mazlan, Patrick, Suhana, Tan
Did not know	Roslina, Usha

Relationship between Perimeter and Area

In this subsection, findings of PSSMTs' conceptual knowledge of the relationships between the perimeter and area were presented in terms of: (a) same perimeter, same area?, (b) longer perimeter, larger area?, and (c) perimeter increases, area increases?.

Same Perimeter, Same Area?

In Task 5.1, a Form One student claimed that he found a way to calculate the area of a leaf. The student placed a piece of thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle. PSSMTs were asked how they would respond to this student. Figure 4.11 exhibits Task 5.1.

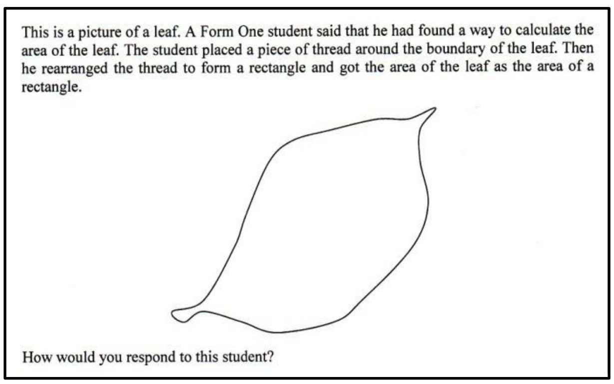


Figure 4.11. Task 5.1.

In Task 5.1, only one of the PSSMTs, namely Suhana, provided the correct response that the student's method of calculating the area of the leaf was not correct. She knew that there is no direct relationship between perimeter and area. Suhana knew that two shapes with the same perimeter can have different areas. Thus, she knew that the student's method of calculating the area of the leaf was not correct.

Suhana generated a counterexample to show that the student's method of calculating the area of the leaf was not correct. She put a 1-cm grid paper on the leaf and then traced its outline. Suhana counted the number of 1-cm grid covered by the leaf, namely 23 1-cm grids. She wrote the area of the leaf as 23 cm^2. Suhana used a piece of thread to measure the perimeter of the traced leaf on 1-cm grid paper and wrote its measurement as 21.5 cm and then rounded it off to 22 cm. She drew a rectangle, labelled its dimensions as 6 by 5 and then calculated its area as 30 cm^2. Suhana found that the area of the rectangle, namely 30 cm^2, is not the same as the area of the leaf which is 23 cm^2 even though they had the same perimeter as 22 cm. Thus, she concluded that the student's method of calculating the area of the leaf was not correct. Suhana had shown that two shapes with the same perimeter can have different areas. Table 4.13 illustrates PSSMTs' responses towards a student's method of calculating the area of a leaf.

Table 4.13

PSSMTs' Responses Towards a Student's Method of Calculating the Area of a Leaf

Responses	PSSMTs
The student's method was correct	Mazlan, Patrick, Roslina, Tan, Usha
The student's method was not correct	Suhana
Not sure whether the student's method was correct or not	Beng, Liana

Five of the PSSMTs, namely Mazlan, Roslina, Patrick, Tan, and Usha, thought that the student's method of calculating the area of the leaf was correct. They did not know that there is no direct relationship between perimeter and area. Mazlan, Roslina, Patrick, Tan, and Usha did not know that two shapes with the same perimeter can have different areas. Thus, they thought that the student's method of calculating the area of the leaf was correct.

Mazlan explained that the thread can also be used to form other shapes such as triangle, square, or circle besides rectangle. He expressed that the area of the leaf same as the area of the triangle, square, or circle formed. Patrick thought that the student's method of calculating the area of the leaf was acceptable because the student placed a piece of thread around the boundary of the leaf and then rearranged the thread to form another shape, namely rectangle (with specific area formula), and got the area of the leaf as the area of a rectangle.

Roslina explained that the student used the perimeter of the leaf to form other shape, namely rectangle, and thus the area of the leaf same as the area of the rectangle. She tried out the student's method by placing a piece of thread around the boundary of the leaf and got the perimeter of the leaf as 24 cm. Roslina drew a rectangle, labelled its dimensions as 8 cm by 4 cm based on the perimeter of the leaf, namely 24 cm, and then calculated its area as 32 cm^2. She reiterated that the student's method works. Tan applauded this student for figuring out the method of calculating the area of the leaf. He explained that the student need not necessarily has to form a rectangle. Usha explained that the student used the perimeter of the leaf to form other shape, namely rectangle, which was easier for him to calculate the area and thus the area of the leaf same as the area of the rectangle.

Two of the PSSMTs, namely Beng and Liana, were not sure whether the student's method of calculating the area of the leaf was correct or not. They did not know that there is no direct relationship between perimeter and area. Beng and Liana did not know that two shapes with the same perimeter can have different areas. Thus, they were not sure whether the student's method of calculating the area of the leaf was correct or not.

Beng explained that she need to verify it first whether the method mentioned by the student was correct or not. Beng explained that she would verify it by covering the surface of the leaf with square units and then compare it with the student's answer. Beng elaborated that she would also seek other people's view to verify it as she never think that the student's method can be used to calculate the area of the leaf. Liana expressed that she need to seek her friends' expertise in science to find out whether the method claimed by the student can be used to determine the area of the leaf. Liana explained that she could not simply say the student's method works or not as she was not sure about the correctness of the method.

Longer Perimeter, Larger Area?

In Task 5.2, PSSMTs were asked to respond to a student, Mary, who claimed that she could determine whose garden has the larger area to plant flowers. Mary claimed that the garden with the longer perimeter has the larger area. Figure 4.12 illustrates Task 5.2.

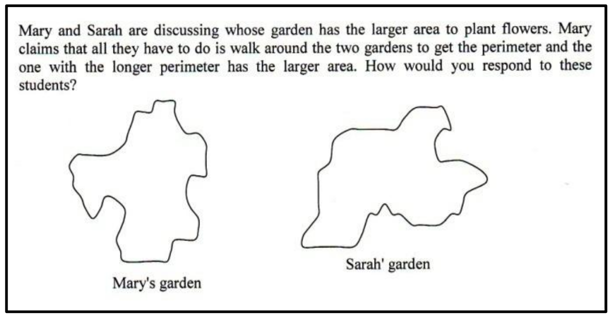


Figure 4.12. Task 5.2.

In Task 5.2, only two of the PSSMTs, namely Beng and Suhana, provided the correct response that Mary's claim that the garden with the longer perimeter has the larger area was not correct. They knew that there is no direct relationship between perimeter and area. Beng and Suhana knew that the garden with the longer perimeter could have a smaller area. Thus, they knew that Mary's claim was not correct. Table 4.14 shows PSSMTs' responses towards Mary's claim that the garden with the longer perimeter has the larger area.

Table 4.14

PSSMTs' Responses Towards Mary's Claim That the Garden With the Longer Perimeter has the Larger Area

Responses	PSSMTs
Mary's claim was correct	Mazlan, Patrick, Roslina , Tan, Usha
Mary's claim was not correct	Beng, Suhana
Not sure whether Mary's claim was correct or not	Liana

Beng made a reflection on Task 3.1 when she approached Task 5.2. From the reflection, she realized that the shape with the longer perimeter may have a smaller area. Beng explained that Mary's method did not work for this situation as these two gardens are of different shape.

She stated that Mary's claim is true only when we are comparing the area of two similar shapes (same shape but different area).

Suhana indicated that Mary's method was not correct because it did not apply to all shapes. Suhana stated that Mary came to the conclusion just based on this situation and was just by luck. Suhana concluded that (the shape with the) longer perimeter does not necessarily has the larger area. She explained that sometimes (the shape with the) shorter perimeter has larger area too compared to (the shape with the) longer perimeter.

Five of the PSSMTs, namely Mazlan, Patrick, Roslina, Tan, and Usha, thought that Mary's claim that the garden with the longer perimeter has the larger area was correct. They did not know that there is no direct relationship between perimeter and area. Mazlan, Patrick, Roslina, Tan, and Usha did not know that the garden with the longer perimeter could have a smaller area. Thus, they thought that Mary's claim was correct.

Mazlan explained that from the perimeters of the garden, the areas of the garden could be obtained. Thus, he elaborated that the garden with the longer perimeter has the larger area. Patrick gave an example where he drew two rectangles and then calculated its perimeter and area. Patrick found that rectangle A with the longer perimeter (22 cm) has the larger area (30 cm^2) compared to rectangle B with the perimeter of 18 cm and the area of 20 cm^2. Thus, he thought that Mary's claim was correct.

Roslina explained that if Mary's garden had the longer perimeter than Sarah's, then Mary's garden has the larger area than Sarah's. When probed further, Roslina generated an example that concurred with Mary's claim. Roslina assumed that the perimeter of Mary's and Sarah's gardens were 24 cm and 12 cm respectively. She used the thread method in the previous task, Task 5.1, to transform the gardens into two rectangles. Roslina drew two rectangles to represent these gardens. She calculated its area as 32 cm^2 and 8 cm^2. Thus, Roslina concluded that (the garden with the) longer perimeter (24 cm) has the larger area (32 cm^2).

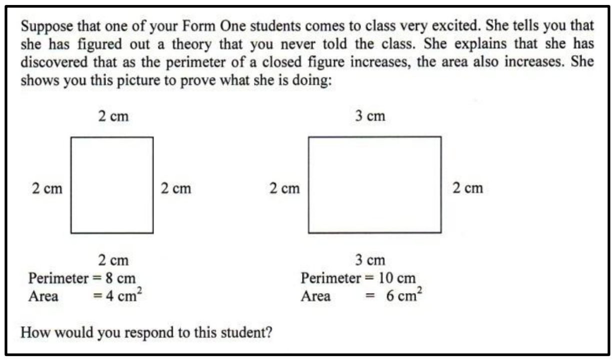
Tan stated that this is one of the ways for Mary to calculate the area (of the gardens) as it was similar to the thread method in the previous task, Task 5.1, where the student rearranged the thread to form a rectangle that was easier to calculate its area. Usha drew two rectangles with the perimeters of 24 cm and 26 cm respectively. She labelled its dimensions as 10 (cm) by 2 (cm) and 10 (cm) by 3 (cm) respectively and then calculates its area as 20 cm^2 and 30 cm^2 respectively. The example generated by Usha showed that the rectangle with the longer perimeter has the larger area. Thus, Usha concluded that the longer the perimeter of a shape, the larger the area of the shape.

The remaining PSSMT, namely Liana, was not sure whether Mary's claim that the garden with the longer perimeter has the larger area was correct or not. Liana did not know that there is no direct relationship between perimeter and area. She did not know that the garden with the longer perimeter could have a smaller area. Thus, Liana was not sure whether Mary's claim was correct or not.

Liana stated that the area of a garden could not be determined by simply measuring the perimeter of the garden as area and perimeter were two different things (concepts). She emphasized that one cannot simply say that the longer the perimeter, the larger the area will be. Liana expressed that this was a wrong "concept" of determining the area (of the gardens). She suggested using other method to calculate the area of the gardens and only then can the larger area (of the garden) be determined. Nevertheless, Liana was not sure whether Mary's claim that the garden with the longer perimeter has the larger area was correct or not. She stated that one cannot simply say whether the method works or not. Liana expressed that she has to do some research (to verify it).

Perimeter Increases, Area Increases?

In Task 5.3, PSSMTs were asked how they would respond to a Form One student's claimed regarding the relationships between perimeter and area of a closed figure. The student claimed that as the perimeter of a closed figure increases, the area also increases. Figure 4.13 shows Task 5.3.


Suppose that one of your Form One students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:

How would you respond to this student?

Figure 4.13. Task 5.3.

In Task 5.3, only two of the PSSMTs, namely Beng and Tan, provided the correct response that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct. They knew that there is no direct relationship between perimeter and area. Beng and Tan knew that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, they knew that the student's "theory" was not correct. Table 4.15 depicts PSSMTs' responses towards the student's "theory" that as the perimeter of a closed figure increases, the area also increases.

Table 4.15

PSSMTs' Responses Towards a Student's "Theory" That as the Perimeter of a Closed Figure Increases, the Area Also Increases

Responses	PSSMTs
The student's "theory" was correct	Liana, Mazlan, Patrick, Roslina, Suhana, Usha
The student's "theory" was not correct	Beng, Tan

Beng explained that the student's 'theory" might be true for this situation as the students can "prove" it with an example. She knew that the student's "theory" might not apply to all the shapes (other situations). When probed further, Beng tried to provide a counterexample to disprove or refute the student's "theory". Nevertheless, she was unable to find a counterexample to refute the student's "theory". The example that Beng generated also suggested that as the perimeter of a closed figure increases, the area also increases. Beng admitted that she was unable to provide a counterexample to refute that the student's "theory" is not correct. Nevertheless, Beng does not think that the student's "theory" is correct even though she was unable to generate a counterexample to refute it.

Tan initially thought that the student's "theory" was correct. He went through the example showed by the student that as the perimeter increases from 8 cm to 10 cm, its area also increases from 4 cm^2 to 6 cm^2. Tan drew an isosceles triangle with the perimeter of 10 cm then calculated its area as 4.472 cm^2. He found that although the rectangle and the triangle have the same perimeter (10 cm), their areas were different, namely 6 cm^2 and 4.472 m^2 respectively. Tan expressed that the triangle has the smaller area even though they had the same perimeter. He realized that increases in perimeter did not guarantee that the area also increases. Subsequently, Tan knew that the student's "theory" was not correct. Nevertheless, Beng and Tan did not know that the student's claim about the relationship between perimeter and area is not a theory. The claim is a conjecture. Beng and Tan also did not know that an example is not a proof and a theory cannot be proved by an example.

The remaining six of the PSSMTs, namely Liana, Mazlan, Patrick, Roslina, Suhana, and Usha, did not know that there is no direct relationship between perimeter and area. They did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Liana, Mazlan, Patrick, Roslina, Suhana, and Usha thought that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was correct. Liana explained that the "theory" was correct because the student has proven it with the picture together with the measurement of perimeter and area that clearly showed that as the perimeter of the figure increases (from 8 cm to 10 cm), the area also increases (from 4 cm^2 to 6 cm^2).

Mazlan praised the student for the good "discovery' because he thought that as the perimeter of a closed figure increases, the area also increases. Mazlan referred to the example generated by the student that indicated that as the perimeter of a closed figure increases from 8 cm to 10 cm, the area also increases from 4 cm^2 to 6 cm^2. Patrick stated that the student's "theory" can be accepted. When probed further, Patrick indicated that the student's "theory" was correct because his example showed that as the perimeter of the triangle increases from 6 cm to 9 cm, its area also increases from 1.7 cm^2 to 3.9 cm^2. The example generated by him concurred with the student's "theory that as the perimeter of a closed figure increases, the area also increases.

Roslina explained that the student has proven it and it was true because when the perimeter (of a shape) is longer compared to other shape, the area also larger. She concurred with the student that when the perimeter (of a closed figure) increases, the area also increases. Suhana explained that when the perimeter increases from 8 (cm) to 10 (cm), the area also increases from 4 (cm^2) to 6 (cm^2). Thus, she concluded that the student's "theory" was correct. Suhana explained that the "theory" was also applied to other figure such as triangle.

Usha explained that when the area of a shape is large, the perimeter that surrounded the outline of the area would be longer. Usha elaborated that a shape with the smaller side would have small perimeter and also small area. Thus, she concluded that a shape with the longer perimeter have the larger area. Nevertheless, Liana, Mazlan, Patrick, Roslina, Suhana, and Usha did not know that the student's claim about the relationship between perimeter and area is not a theory. The claim is a conjecture. They also did not know that an example is not a proof and a theory cannot be proved by an example.

Relationship among Area Formulae

In this subsection, findings of PSSMTs' conceptual knowledge of the relationships among area formulae were presented in terms of: (a) rectangle, (b) parallelogram, (c) triangle, and (d) trapezium.

Rectangle

In Task 8, PSSMTs were asked to show a Form One student the way to develop (derive) area formulae of a rectangle, parallelogram, triangle, and trapezium. Figure 4.14 depicts Task 8.

> Suppose that a Form One student comes to you and says that he does not know how to develop (derive) the formula for calculating the area of the following shapes:
> (a) Rectangle,
> (b) Parallelogram,
> (c) Triangle, and
> (d) Trapezium.
> How would you show him the way to develop (derive) the formula for calculating the area of these shapes?

Figure 4.14. Task 8.

All the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, could recall the formula for the area of a rectangle. Nevertheless, none of the eight PSSMTs were able to develop it. They just memorized the formula. None of the eight PSSMTs attempted to develop the formula, except Tan. Tan had attempted to develop the formula but unsuccessful. It indicated that all of them have no idea how the formula can be developed or derived. They might have rote-learnt the formula. It was apparent that all of them lack of conceptual knowledge underpinning the formula for the area of a rectangle.

Parallelogram

Five of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, and Tan, could recall the formula for the area of a parallelogram. They were able to develop the formula for the area of a parallelogram. Beng, Mazlan, Patrick, and Tan mentally transformed the parallelogram to a rectangle by cutting out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle. Suhana mentally transformed the parallelogram to a rectangle by cutting the parallelogram into two triangles along its diagonal. Suhana mentally moved a triangle from one end of the parallelogram to the other end of the parallelogram to form a rectangle.

It indicated that Beng, Mazlan, Patrick, Suhana, and Tan understand the relationship between the formulae for the area of a parallelogram and rectangle. A parallelogram can always be transformed into a rectangle with the same base, same height, and the same area. Thus, the formula for the area of a parallelogram is exactly the same as the formula for the area of a rectangle, namely 'base times height'. Three of the PSSMTs were unable to develop the formula for the area of a parallelogram. It was apparent that they did not know the relationship between the area of a parallelogram and the area of a rectangle. Had they been known of this relationship, they would know how to develop the formula for the area of a parallelogram.

Triangle

All the PSSMTs could recall the formula for the area of a triangle, except Usha. Only three of them, namely Liana, Suhana, and Tan, attempted to develop the formula. Two of the PSSMTs, namely Liana and Tan, were able to develop the formula for the area of a triangle. Suhana attempted to develop the formula but unsuccessful. Liana developed the formula for the area of a triangle based on the formula for the area of a square. A square is a special case of a rectangle.

Tan developed the formula for the area of a triangle based on the formula for the area of a rectangle. It indicated that they knew the relationship between the formulae for the area of a triangle and rectangle that encloses it. Liana and Tan understand the relationship that the area of a triangle is half of the area of the rectangle that encloses it. Six of the PSSMTs were unable to develop the formula for the area of a triangle. It was quite clear that most of the PSSMTs did not know the relationship between the area of a triangle and the area of the rectangle that encloses it. Had they been known of this relationship, they would know how to develop the formula for the area of a triangle.

Trapezium

Six of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, Tan, and Usha, could recall the formula for the area of a trapezium. Of the six PSSMTs who could recall the formula for the area of a trapezium, five of them, namely Beng, Mazlan, Patrick, Suhana, and Tan, attempted to develop the formula. Of the two PSSMTs who could not recall the formula for the area of a trapezium, one of them, namely Liana, attempted to develop the formula. Of the six PSSMTs who attempted to develop the formula, three of them, namely Beng, Suhana, and Tan, were able to develop the formula for the area of a trapezium. All of them developed the formula using algebraic method.

Beng viewed the area of the trapezium as the different between the area of the large rectangle formed and the area of the triangle formed. Thus, the area of the trapezium equals to 'b x t $-\frac{1}{2}$ (b – a) x t'. She simplified it algebraically to become '$\frac{1}{2}$ $(a + b) \times t$'. Suhana developed the formula for the area of a trapezium from the combination of the formulae for the area of a rectangle and a triangle, namely (a × *tinggi* [*height*]) + [(b – a) × *tinggi* [*height*] × $\frac{1}{2}$] using algebraic method. She correctly simplified it as $\frac{1}{2}$ × *tinggi* [*height*] × (a + b), which is the formula for the area of a trapezium.

Tan developed the formula for the area of a trapezium using the combination of the formula for the area of a triangle and a rectangle or a square. Tan wrote the formula for the total area of a rectangle or a square, and a triangle as '($AB \times AC$) + ($\frac{1}{2} \times BE \times ED$)'. He then used the algebraic method to simplified it as '$\frac{1}{2}\ AC\ (AB + CD)$' which is the formula for the area of a trapezium.

It indicated that Beng, Suhana, and Tan knew that the formula for the area of a trapezium is related to the formulae for the area of a rectangle and triangle. The formula for the area of a trapezium is also related to the formula for the area of a parallelogram. Nevertheless, five of the PSSMTs were unable to develop the formula for the area of a trapezium. It was quite clear that they did not know the relationship between the area formulae of a rectangle, parallelogram, triangle, and trapezium. Had they been known of this relationship, they would know how to develop the formula for the area of a trapezium.

Procedural Knowledge

In this section, findings of PSSMTs' procedural knowledge of perimeter and area were presented in terms of its components. Table 4.16 depicts the components of procedural knowledge of perimeter and area.

Table 4.16

The Components of Procedural Knowledge of Perimeter and Area

Type of knowledge	Its components
Procedural knowledge	11. Converting standard units of area measurement
	12. Calculating perimeter of composite figures
	13. Calculating area of composite figures
	14. Developing area formulae

Converting Standard Units of Area Measurement

In this subsection, findings of PSSMTs' procedural knowledge of converting standard units of area measurement were presented in terms of: (a) converting 3 cm^2 to mm^2, (b) converting 4.7 m^2 to cm^2, and (c) converting 1.25 km^2 to m^2.

Converting 3 cm^2 to mm^2

In Task 4, PSSMTs were asked to respond to a scenario where several students encountered difficulty in converting units of area. Task 4 is shown in Figure 4.10. In Task 4, five of the PSSMTs, namely Beng, Liana, Patrick, Suhana, and Tan, had successfully converting 3 cm^2 to mm^2. Beng and Patrick viewed 3 cm^2 as the product of 3 times 1 cm times 1 cm. They knew the relationship between the standard units of length measurement that 1 cm = 10 mm. Beng and Patrick also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Beng and Patrick times 10 when they converted 1 cm to mm twice. Table 4.17 shows PSSMTs who had successfully and unsuccessfully converting 3 cm^2 to mm^2.

Table 4.17

PSSMTs who had Successfully and Unsuccessfully Converting 3 cm^2 to mm^2

Converting 3 cm^2 to mm^2	PSSMTs
Successful	Beng, Liana, Patrick, Suhana, Tan
Unsuccessful	Mazlan, Roslina, Usha

Liana viewed 3 cm^2 as the product of 3 times 1 cm^2. She knew the relationship between the standard units of length measurement that 1 cm = 10 mm. Liana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Liana times ten squared, $(10)^2$, when she converted 3 cm^2 to mm^2. Suhana viewed 3 cm as the product of 3 times 1 cm. Thus, Suhana times 10 when she converted 3 cm to mm because 1 cm = 10 mm. Similarly, Suhana viewed 3 cm^2 as the product of 3 times 1 cm^2. She knew the relationship between the standard units of length measurement that 1 cm = 10 mm. Suhana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Suhana times ten squared, $(10)^2$, when she converted 3 cm^2 to mm^2.

Tan viewed 3 cm^2 as the product of 1 cm times 3 cm. He knew the relationship between the standard units of length measurement that 1 cm = 10 mm. Tan also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Tan times 10 twice when he converted 1 cm to mm and 3 cm to mm separately.

The remaining three PSSMTs, namely Mazlan, Roslina, and Usha, had unsuccessfully converting 3 cm^2 to mm^2. Mazlan has incorrectly converted 3 cm^2 to 3 x 10^{-4} mm^2. Mazlan converted 3 cm^2 to m^2 first and then from m^2, he converted it to mm^2. Mazlan thought that 1 m^2 = 10 cm^2. Thus, Mazlan multiplied 10^{-1} (ten to the power of negative one) when he converted 3 cm^2 to m^2. Mazlan also thought that 1 m^2 = 10^{-3} mm^2. Therefore, Mazlan multiplied 10^{-3} (ten to the power of negative three) when he converted m^2 to mm^2. It indicated that Mazlan did not know the relationship between the standard units of length measurement that 1 cm = 10 mm and 1 m = 100 cm. It also indicated that he did not know the relationship between the standard units of area measurement that 1 cm^2 = 100 mm^2 and 1 m^2 = 10 000 cm^2.

Roslina thought that the students' method of converting 3 cm^2 to mm^2 was the simplest or easiest way without using calculator. She knew that 1 cm = 10 mm. Nevertheless, Roslina thought that 1 cm^2 = 10 mm^2. Thus, Roslina times 10 when she converted 3 cm^2 to mm^2. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2. She also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Consequently, Roslina did not realize that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2. The students thought that 1 cm^2 = 10 mm^2. Thus, Roslina concluded that the students had correctly converted the unit of area for the first question, namely 3 cm^2 to 30 mm^2, because she thought that 1 cm^2 = 10 mm^2. Roslina stated that she did the same thing as the students did in converting unit of area from 3 cm^2 to mm^2.

Usha did not realize that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2. She thought that 1 cm^2 = 10 mm^2. It indicated that Usha did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2. She did not know the relationships between the standard units of length measurement that 1 cm = 10 mm. Usha also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Consequently, she did not realize that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2. The students also thought that 1 cm^2 = 10 mm^2. Thus, Usha concluded that had correctly converted the unit of area for the first question, namely 3 the students cm^2 to 30 mm^2, because she thought that 1 cm^2 = 10 mm^2.

Converting 4.7 m^2 to cm^2

In Task 4, half of the PSSMTs, namely Beng, Liana, Patrick, and Tan, had successfully converting 4.7 m^2 to cm^2. Beng and Patrick viewed 4.7 m^2 as the product of 4.7 times 1 m times 1 m. They knew the relationship between the standard units of length measurement that 1 m = 100 cm. Beng and Patrick also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Beng and Patrick times 100 when they converted 1 m to cm twice. Table 4.18 depicts PSSMTs who had successfully and unsuccessfully converting 4.7 m^2 to cm^2.

Table 4.18

PSSMTs who had Successfully and Unsuccessfully Converting 4.7 m^2 to cm^2

Converting 4.7 m^2 to cm^2	PSSMTs
Successful	Beng, Liana, Patrick, Tan
Unsuccessful	Mazlan, Roslina, Suhana, Usha

Liana viewed 4.7 m^2 as the product of 4.7 times 1 m^2. She knew the relationship between the standard units of length measurement that 1 m = 100 cm. Liana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Liana times one hundred squared, namely $(100)^2$, when she converted 4.7 m^2 to cm^2. She wrote the answer in the standard form, namely 4.7 x 10^4 cm^2.

Tan viewed 4.7 m^2 as the product of 1 m times 4.7 m. He knew the relationship between the standard units of length measurement that 1 m = 100 cm. Tan also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Tan times 100 twice when he converted 1 m to cm and 4.7 m to cm respectively.

The remaining half of the PSSMTs, namely Mazlan, Roslina, Suhana, Usha, had unsuccessfully converting 4.7 m^2 to cm^2. Suhana has used appropriate algorithm in converting 4.7 m^2 to cm^2. She viewed 4.7 m as the product of 4.7 times 1 m. Thus, Suhana times 100 when she converted 4.7 m to cm because 1 m = 100 cm. Similarly, Suhana viewed 4.7 m^2 as the product of 4.7 times 1 m^2. She knew the relationship between the standard units of length measurement that 1 m = 100 cm. Suhana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Suhana times one hundred squared, namely $(100)^2$, when she converted 4.7 m^2 to cm^2. Nevertheless, Suhana made a mistake when she simplified the product of 4.7 times $(100\ cm)^2$ as 470 000 cm^2. The correct answer should be 47 000 cm^2.

Mazlan has incorrectly converted 4.7 m^2 to 470 cm^2. He thought that 1 m = 10 cm. Thus, Mazlan multiplied $(10^1\ cm)^2$ or $10^2\ cm^2$ when he converted 4.7 m^2 to cm^2. It indicated that Mazlan did not know the relationship between the standard units of length measurement that 1 m = 100 cm. It also indicated that he did not know the relationship between the standard units of area measurement that 1 m^2 = 10 000 cm^2. Nevertheless, Mazlan knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. This can be seen when he squared $(10^1\ cm)$ to get $10^2\ cm^2$.

Roslina knew that 1 m = 100 cm. Nevertheless, Roslina thought that 1 m^2 = 100 cm^2. Thus, Roslina times 100 when she converted 4.7 m^2 to cm^2. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 m^2 = 10 000 cm^2. She also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Consequently, Roslina did not realize that the students made a mistake when they were converting unit of area from 4.7 m^2 to cm^2. The students thought that 1 m^2 = 100 cm^2. Thus, Roslina concluded that the students had correctly converted the unit of area for the second question, namely 4.7 m^2 to 470 cm^2, because she thought that 1 m^2 = 100 cm^2. Roslina did the same thing as the students did in converting unit of area from 4.7 m^2 to cm^2.

Usha thought that 1 m^2 = 100 cm^2. It indicated that she did not know the relationships between the standard units of area measurement that 1 m^2 = 10 000 cm^2. Usha did not know the relationships between the standard units of length measurement that 1 m = 100 cm. She also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Consequently, Usha did not realize that the students made a mistake when they were converting unit of area from 4.7 m^2 to cm^2. The students also thought that 1 m^2 = 100 cm^2. Thus, she concluded that the students had correctly converted the unit of area for the second question, namely 4.7 m^2 to 470 cm^2, because she thought that 1 m^2 = 100 cm^2.

Converting 1.25 km^2 to m^2

In Task 4, half of the PSSMTs, namely Beng, Liana, Patrick, and Tan, had successfully converting 1.25 km^2 to m^2. Beng and Patrick viewed 1.25 km^2 as the product of 1.25 times 1 km times 1 km. They knew the relationship between the standard units of length measurement that 1 km = 1000 m. Beng and Patrick also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, they times 1000 when they converted 1 km to m twice. Table 4.19 demonstrates PSSMTs who had successfully and unsuccessfully converting 1.25 km^2 to m^2.

Table 4.19

PSSMTs who had Successfully and Unsuccessfully Converting 1.25 km^2 to m^2

Converting 1.25 km^2 to m^2	PSSMTs
Successful	Beng, Liana, Patrick, Tan
Unsuccessful	Mazlan, Roslina, Suhana, Usha

Liana viewed 1.25 km^2 as the product of 1.25 times 1 km^2. She knew the relationship between the standard units of length measurement that 1 km = 1000 m. Liana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Liana times one thousand squared, namely $(1000)^2$, in order to get the correct answer when she converted 1.25 km^2 to m^2. She wrote the answer in the standard form, namely $1.25 \times 10^6\ m^2$.

Tan viewed 1.25 km^2 as the product of 1 km times 1.25 km. Tan knew the relationship between the standard units of length measurement that 1 km = 1000 m. He also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Tan times 1000 twice when he converted 1 km to m and 1.25 km to m respectively.

The remaining half of the PSSMTs, namely Mazlan, Roslina, Suhana, and Usha, had unsuccessfully converting 1.25 km^2 to m^2. Suhana has also used appropriate algorithm in converting 1.25 km^2 to m^2. She viewed 1.25 km as the product of 1.25 times 1 km. Thus, Suhana times 1000 when she converted 1.25 km to m because 1 km = 1000 m. Similarly, Suhana viewed 1.25 km^2 as the product of 1.25 times 1 km^2. She knew the relationship between the standard units of length measurement that 1 km = 1000 m. Suhana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Suhana times one thousand squared, namely $(1000)^2$, when she converted 1.25 km^2 to m^2. Nevertheless, Suhana made a mistake when she simplified the product of 1.25 times $(1000\ m)^2$ as 125 000 000 m^2. The correct answer should be 1 250 000 m^2.

Mazlan has incorrectly converted 1.25 km^2 to $1250 \times 10^6\ m^2$. He knew the relationship between the standard units of length measurement that 1 km = 1000 m. Mazlan also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Mazlan multiplied $(10^3\ m)^2$ or $10^6\ m^2$ when he converted 1.25 km^2 to m^2.

Roslina knew that 1 km = 1000 m. Nevertheless, she thought that 1 km^2 = 1000 m^2. Thus, Roslina times 1000 when she converted 1.25 km^2 to m^2. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 km^2 = 1000 000 m^2. She also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Consequently, Roslina did not realize that the students made a mistake when they were converting unit of area from 1.25 km^2 to m^2. The students thought that 1 km^2 = 1000 m^2. Thus, Roslina concluded that the students had correctly converted the unit of area for the third question, namely 1.25 km^2 to 1250 m^2, because she thought that 1 km^2 = 1000 m^2. Roslina did the same thing as the students did in converting unit of area from 1.25 km^2 to m^2.

Usha thought that 1 km^2 = 1000 m^2. It indicated that she did not know the relationships between the standard units of area measurement that 1 km^2 = 1000 000 m^2. Usha did not know the relationships between the standard units of length measurement that 1 km = 1000 m. She also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Consequently, Usha did not realize that the students made a mistake when they were converting unit of area from 1.25 km^2 to m^2. The students also thought that 1 km^2 = 1000 m^2. Thus, she concluded that the students had correctly converted the unit of area for the third question, namely 1.25 km^2 to 1250 m^2, because she thought that 1 km^2 = 1000 m^2.

Calculating Perimeter of Composite Figures

In this subsection, findings of PSSMTs' procedural knowledge of calculating perimeter of composite figures were presented in terms of: (a) calculating perimeter of Diagram 1, and (b) calculating perimeter of Diagram 2.

Calculating Perimeter of Diagram 1

In Task 6.1, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 1) that involved composite figure, namely rectangle and parallelogram/triangle. Figure 4.15 shows Task 6.1.

Suppose that one of your Form One students asks you for help with the following problem:

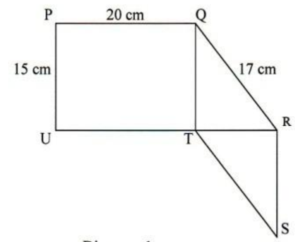


Diagram 1

In Diagram 1, PQTU is a rectangle and QRST is a parallelogram.
UTR is a straight line.
Calculate
(a) the perimeter of the diagram,
(b) the area of the diagram.
How would you solve this problem?

Figure 4.15. Task 6.1.

In Task 6.1, seven of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, and Usha, have successfully calculated the perimeter of Diagram 1 as 104 cm. Table 4.20 reveals PSSMTs who have successfully and unsuccessfully calculated the perimeter of Diagram 1.

Table 4.20
PSSMTs who Have Successfully and Unsuccessfully Calculated the Perimeter of Diagram 1

Calculating the perimeter of Diagram 1	PSSMTs
Successful	Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha
Unsuccessful	Tan

Of the seven PSSMTs who have successfully calculated the perimeter of Diagram 1, five of them, namely Beng, Mazlan, Patrick, Roslina and Usha, used the list all-and-sum algorithm to calculate the perimeter of the diagram. They listed all the length of sides that surrounded the diagram and then summed them up to get the perimeter of the diagram as 104 cm. Table 4.21

exhibits the algorithms used by PSSMTs to calculate the perimeter of Diagram 1. The other two PSSMTs, namely Liana and Suhana, used the doubling-and-sum algorithm to calculate the perimeter of the diagram. They doubled the length of sides UP, PQ, and QR and then summed them up to get the perimeter of the diagram as 104 cm.

Table 4.21

The Algorithms Used by PSSMTs to Calculate the Perimeter of Diagram 1

Algorithms used to calculate the perimeter of Diagram 1	PSSMTs
List all-and-sum	Beng, Mazlan, Patrick, Roslina, Tan, Usha
Doubling-and-sum	Liana, Suhana

Only one PSSMT, namely Tan, have unsuccessfully calculated the perimeter of Diagram 1. Tan mentally cut the triangle TRS of Diagram 1 and pasted it next to the triangle TQR of Diagram 1 so that it formed a rectangle ("TQSR") with the dimension of 15 cm by 8 cm. He used the list all-and-sum algorithm to calculate the perimeter of the diagram, He listed all the length of sides that surrounded the "long" rectangle and then summed them up to get the perimeter of the diagram as 86 cm (the correct answer should be 104 cm). Tan did not know that the "cut and paste" transformation does not conserve the perimeter of a diagram. Thus, he incorrectly calculated the perimeter of the diagram as 86 cm based on the length of sides that surrounded the "long" rectangle formed (20 + 8 + 15 + 20 + 8 + 15) and not based on the length of sides that surrounded Diagram 1 (20 + 17 + 15 + 20 + 17 + 15 = 104).

Calculating Perimeter of Diagram 2

In Task 6.2, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 2) that involved composite figure, namely square and trapezium/triangle. Figure 4.16 depicts Task 6.2.

Suppose that one of your Form One students asks you for help with the following problem:

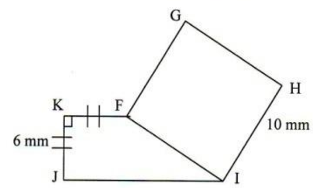


Diagram 2

In Diagram 2, FGHI is a square and FIJK is a trapezium.
Calculate
(a) the perimeter of the diagram,
(b) the area of the diagram.
How would you solve this problem?

Figure 4.16. Task 6.2.

In Task 6.2, all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, have successfully calculated the perimeter of Diagram 2 as 56 mm. Five of the PSSMTs, namely Beng, Mazlan, Patrick, Roslina, and Tan, used the list all-and-sum algorithm to calculate the perimeter of the diagram. Beng, Roslina, and Tan listed all the length of sides that surrounded the diagram and then summed them up to get the perimeter of the diagram as 56 mm. Mazlan and Patrick listed all the length of sides that surrounded the diagram and then summed them up to get the perimeter of the diagram as 56 cm (wrong unit. It should be mm). Table 4.22 illustrates the algorithms used by PSSMTs to calculate the perimeter of Diagram 2.

Two of the PSSMTs, namely Liana and Suhana, used the tripling-and-sum algorithm to calculate the perimeter of the diagram. Liana tripled the length of sides JK and HI, and plus the length of MI. She summed them up to get the perimeter of the diagram as 56 mm. Suhana tripled the length of sides JK and HI, and plus the length of ZI. She summed them up to get the perimeter of the diagram as 56 mm. The remaining PSSMT, namely Usha, used the circle all-and-sum algorithm to calculate the perimeter of the diagram. She circled all the length of sides that surrounded the diagram and then summed them up to get the perimeter of the diagram as 56 mm.

Table 4.22

The Algorithms Used by PSSMTs to Calculate the Perimeter of Diagram 2

Algorithms used to calculate the perimeter of Diagram 2	PSSMTs
List all-and-sum	Beng, Mazlan, Patrick, Roslina, Tan
Tripling-and-sum	Liana, Suhana
Circle all-and-sum	Usha

Calculating Area of Composite Figures

In this subsection, findings of PSSMTs' procedural knowledge of calculating area of composite figures were presented in terms of: (a) calculating area of Diagram 1, and (b) calculating area of Diagram 2.

Calculating Area of Diagram 1

In Task 6.1, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 1) that involved composite figure, namely rectangle and parallelogram/triangle. Task 6.1 is shown in Figure 4.15. In Task 6.1, six of the PSSMTs, namely Beng, Liana, Patrick, Roslina, Suhana, and Tan, have successfully calculated the area of Diagram 1 as 420 cm^2. Table 4.23 shows PSSMTs who have successfully and unsuccessfully calculated the area of Diagram 1.

Table 4.23

PSSMTs who Have Successfully and Unsuccessfully Calculated the Area of Diagram 1

Calculating the area of Diagram 1	PSSMTs
Successful	Beng, Liana, Patrick, Roslina, Suhana, Tan
Unsuccessful	Mazlan, Usha

Of the six PSSMTs who have successfully calculated the area of Diagram 1, five of them, namely Beng, Liana, Patrick, Roslina, and Suhana, used the partition-and-sum algorithm to calculate the area of the diagram. They partitioned Diagram 1 into a rectangle PQTU (labelled as A) and two triangles QRT (labelled as B) and RST (labelled as C). Beng, Liana, Patrick, Roslina, and Suhana calculated the areas of A, B, and C using the area formulae of rectangle and triangles respectively and then summed them up to get the area of the diagram as 420 cm^2. Table 4.24 depicts the algorithms used by PSSMTs to calculate the area of Diagram 1. The other PSSMT, namely Tan, used the “cut and paste” transformation to transform Diagram 1 into a “long” rectangle. He calculated the area of Diagram 1 as the area of the “long” rectangle formed using the area formula of a rectangle where its length and width is 28 cm and 15 cm respectively. Tan got the area of the diagram as 420 cm^2.

Table 4.24

The Algorithms Used by PSSMTs to Calculate the Area of Diagram 1

Algorithms used to calculate the area of Diagram 1	PSSMTs
Partition-and-sum algorithm	Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha
“Cut and paste” transformation	Tan

The remaining two PSSMTs, namely Mazlan and Usha, have unsuccessfully calculated the area of Diagram 1. They used the partition-and-sum algorithm to calculate the area of the diagram. Mazlan partitioned Diagram 1 into a rectangle PQTU and two triangles, namely QRT and RST while Usha partitioned Diagram 1 into a rectangle PQTU and a parallelogram QRST. They correctly calculated the area of the rectangle as 300 cm^2. Mazlan viewed the two triangles as parallelogram QRST. Nevertheless, Mazlan and Usha confused with the slanted side and the height of the parallelogram that they used the slanted side QR as the height (TR = 8 cm) of the parallelogram. Thus, Mazlan and Usha incorrectly calculated the area of the parallelogram as ‘17 x 15 = 255 cm^2' (The area of the parallelogram should be '15 x 8 = 120 cm^2'). Consequently, they got the area of the diagram as 555 cm^2 (The correct answer should be 420 cm^2, not 555 cm^2).

Calculating Area of Diagram 2

In Task 6.2, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 2) that involved composite figure, namely square and trapezium/triangle. Task 6.2 is shown in Figure 4.16. In Task 6.2, all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, have successfully calculated the area of Diagram 2 as 160 mm^2. All of them used the partition-and-sum algorithm to calculate the area of the diagram. Table 4.25 demonstrates the type of partition-and-sum algorithms used by PSSMTs to calculate the area of Diagram 2.

Half of the PSSMTs, namely Beng, Mazlan, Patrick, and Usha, partitioned Diagram 2 into square FGHI and trapezium FIJK. Beng, Patrick, and Usha calculated the area of the square and trapezium separately using the area formulae of square and trapezium respectively and then summed them up to get the area of the diagram as 160 mm^2. Mazlan calculated the area of the trapezium and square separately using the area formulae of trapezium and square respectively and then summed them up to get the area of the diagram as 160 cm^2 (wrong unit. It should be mm^2).

The other half of the PSSMTs, namely Liana, Roslina, Suhana, and Tan, partitioned Diagram 2 into a large square, a triangle and a small square. They calculated the area of the large square, triangle, and small square separately using the area formulae of a square, triangle, and square respectively and then summed them up to get the area of the diagram as 160 mm^2.

Table 4.25

The Type of Partition-And-Sum Algorithms Used by PSSMTs to Calculate the Area of Diagram 2

Type of partition-and-sum algorithms used to calculate the area of Diagram 2	PSSMTs
Partition into square and trapezium-and-sum algorithm	Beng, Mazlan, Patrick, Usha
Partition into two squares and a triangle-and-sum algorithm	Liana. Roslina, Suhana, Tan

Developing Area Formulae

In this subsection, findings of PSSMTs' procedural knowledge of developing area formulae were presented in terms of: (a) rectangle, (b) parallelogram, (c) triangle, and (d) trapezium.

Rectangle

In Task 8, PSSMTs were asked to show a Form One student the way to develop (derive) area formulae of a rectangle, parallelogram, triangle, and trapezium. Task 8 is shown in Figure 4.14. In Task 8, all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, could recall the formula for the area of a rectangle. Beng could recall the formula for the area of a rectangle as '$l \times w$'. Five of the PSSMTs, namely Liana, Mazlan, Patrick, Roslina, and Suhana, could recall the formula for the area of a rectangle as '$a \times b$'.

Tan could recall the formula for the area of a rectangle as 'the horizontal side (refers to the length of the rectangle) times the vertical side' (refers to the width of the rectangle). Usha could recall the formula for the area of a rectangle as 'xy'. Nevertheless, none of the eight PSSMTs were able to develop it. They just memorized the formula. None of the eight PSSMTs attempted to develop the formula, except Tan. Tan had attempted to develop the formula but unsuccessful.

Parallelogram

In Task 8, five of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, and Tan, could recall the formula for the area of a parallelogram. Beng, Mazlan, and Patrick could recall the formula for the area of a parallelogram as '$a \times b$'. Suhana could recall the formula for the area of a parallelogram as 'the *tapak* [base] times the *tinggi* [height]'. Tan could recall the formula for the area of a parallelogram as 'vertical (side) times horizontal (side)'. Table 4.26 reveals PSSMTs who could and could not recall the formula for the area of a parallelogram.

Table 4.26
PSSMTs who Could and Could not Recall the Formula for the Area of a Parallelogram

Recall the formula for the area of a parallelogram	PSSMTs
Could recall	Beng, Mazlan, Patrick, Suhana, Tan
Could not recall	Liana, Roslina, Usha

Three of the PSSMTs, namely Liana, Roslina and Usha, could not recall the formula for the area of a parallelogram. Roslina thought that the formula for the area of a parallelogram is "half times the height times the side bottom it" or $\frac{1}{2}$(h x b). The correct formula for the area of a parallelogram is "base times height or b x h". Usha admitted that she could not recall the formula for the area of a parallelogram. When probed to try, Usha stated that the area of a parallelogram can be found by multiplying the lengths of two adjacent sides of the parallelogram.

Five of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, and Tan, were able to develop the formula for the area of a parallelogram. Of the five PSSMTs, three of them, namely Beng, Mazlan, and Patrick, mentally cuts out a right-angled triangle from one end of the parallelogram and moves it to the other end of the parallelogram to form a rectangle. Thus, the area of the parallelogram equals to the area of the rectangle formed and its area formula is 'a times b' or 'base times height'. Table 4.27 exhibits PSSMTs who were able and unable to develop the formula for the area of a parallelogram.

Table 4.27

PSSMTs who Were Able and Unable to Develop the Formula for the Area of a Parallelogram

Develop the formula for the area of a parallelogram	PSSMTs
Able to develop	Beng, Mazlan, Patrick, Suhana, Tan
Unable to develop	Liana, Roslina, Usha

Suhana mentally cut the parallelogram into two triangles along its diagonal and then she labelled the triangles as "I" and "K". Suhana mentally moved triangle "I" from one end of the parallelogram to the other end of the parallelogram to form a rectangle and wrote its area formula as $a \times b$. She stated the formula for the area of a parallelogram as 'the *tapak* [base] times the *tinggi* [height]'. Tan mentally cut out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle. Thus, the area of the parallelogram equals to the area of the rectangle formed and its area formula is 'vertical (side) times horizontal (side)'. Three of the PSSMTs, namely Liana, Roslina and Usha, were unable to develop the formula for the area of a parallelogram.

Triangle

In Task 8, all the PSSMTs could recall the formula for the area of a triangle, except Usha. Two of the PSSMTs, namely Mazlan and Patrick, could recall the formula for the area of a triangle as '$\frac{1}{2} \times b \times h$'. Beng could recall the formula for the area of a triangle as'$\frac{1}{2} \times b \times t$' while Liana could recall the formula for the area of a triangle as '$\frac{1}{2} \times height \times base$'. Roslina could recall the formula for the area of a triangle as '$\frac{1}{2}\ (h\ \times b)$' while Suhana could recall the formula for the area of a triangle as '$\frac{1}{2} \times t\ \times tapak\ [base]$'. Tan could recall the formula for the area of a triangle as 'half times the vertical (side) times the horizontal (side)'. Table 4.28 illustrates PSSMTs who could and could not recall the formula for the area of a triangle.

Table 4.28

PSSMTs who Could and Could not Recall the Formula for the Area of a Triangle

Recall the formula for the area of a triangle	PSSMTs
Could recall	Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan
Could not recall	Usha

Usha could not recall the formula for the area of a triangle. Initially, she wrote the formula for the area of a triangle as $\frac{1}{2}y \times x$. Later, she shaded the half to become $y \times x$. Usha explained that the formula for the area of a triangle is '$y \times x$', where x and y is the base and the height of the triangle. She elaborated that a triangle is just like a rectangle and we have to multiply the base and the height of the triangle to get its area.

Of the seven PSSMTs who could recall the formula for the area of a triangle, only three of them, namely Liana, Suhana, and Tan, attempted to develop the formula. Usha could not recall the formula for the area of a triangle and she did not attempt to develop the formula. Two of the PSSMTs, namely Liana and Tan, were able to develop the formula for the area of a triangle. Suhana attempted to develop the formula but unsuccessful. Table 4.29 shows PSSMTs who were able and unable to develop the formula for the area of a triangle.

Table 4.29
PSSMTs who Were Able and Unable to Develop the Formula for the Area of a Triangle

Develop the formula for the area of a triangle	PSSMTs
Able to develop	Liana, Tan
Unable to develop	Beng, Mazlan, Patrick, Suhana, Roslina, Usha

Liana and Tan used the partition strategy to develop the formula for the area of a triangle. Liana developed the formula for the area of a triangle based on the formula for the area of a square. She explained that a square can be partitioned into two triangles and thus there is a half in the formula for the area of a triangle. Liana stated that the formula for the area of a square is '$a \times$

b', where a and b represents the height and the base of the square. She then wrote the formula for the area of a triangle as '$\frac{1}{2} \times a \times b$'.

Tan developed the formula for the area of a triangle based on the formula for the area of a rectangle. He mentally cut a rectangle diagonally and then took out a right-angled triangle. Tan stated that the formula for the area of a rectangle is 'the vertical (side) times the horizontal (side)'. He emphasized that it needed to times half in order to get the area of a triangle, namely 'half times the vertical (side) times the horizontal (side)'. Six of the PSSMTs, namely Beng, Mazlan, Patrick, Roslina, Suhana, and Usha, were unable to develop the formula for the area of a triangle.

When probed to develop the formula, Suhana attempted to develop the formula but unsuccessful. She mentally cut an isosceles triangle along its symmetrical line and then rearranged it to be a rectangle. Suhana drew a rectangle and wrote its area formula as '$a \times b$'. When probed further to develop the formula, she mentally cut the rectangle diagonally and then rearranged it to be an isosceles triangle. Suhana drew another triangle (isosceles triangle) and wrote its area formula as '*tinggi* [height] × *tapak* [base] × $\frac{1}{2}$'. When the researcher asked how she got that formula, Suhana expressed that the formula was just like that and she just memorized the formula.

Trapezium

In Task 8, six of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, Tan, and Usha, could recall the formula for the area of a trapezium. Of the six PSSMTs who could recall the formula for the area of a trapezium, two of them, namely Mazlan and Patrick, could recall the formula for the area of a trapezium as '$\frac{1}{2}\ (a+b)h$'. Beng could recall the formula for the area of a trapezium as ‘$\frac{1}{2}\ (a+b)\times t$’ while Suhana could recall the formula for the area of a trapezium as ‘$\frac{1}{2}\times tinggi\ [height]\times(a+b)'$. Tan could recall the formula for the area of a trapezium as ‘$\frac{1}{2}\ AC\ (AB+CD)$’ while Usha could recall the formula for the area of a trapezium as ‘$\frac{1}{2}\times h\ \times (a+b)'$. The remaining two PSSMTs, namely Liana and Roslina, could not recall the formula for the area of a trapezium. Table 4.30 depicts PSSMTs who could and could not recall the formula for the area of a trapezium.

Table 4.30

PSSMTs who Could and Could not Recall the Formula for the Area of a Trapezium

Recall the formula for the area of a trapezium	PSSMTs
Could recall	Beng, Mazlan, Patrick, Suhana, Tan, Usha
Could not recall	Liana, Roslina

Of the six PSSMTs who could recall the formula for the area of a trapezium, five of them, namely Beng, Mazlan, Patrick, Suhana, and Tan, attempted to develop the formula. Usha could recall the formula for the area of a trapezium but she did not attempt to develop the formula for the area of a trapezium. Of the two PSSMTs who could not recall the formula for the area of a trapezium, none of them attempted to develop the formula.

Three of the PSSMTs, namely Beng, Suhana, and Tan, were able to develop the formula for the area of a trapezium. All of them developed the formula for the area of a trapezium using algebraic method. Beng drew dotted lines on the trapezium to form a large rectangle and viewed the area of the trapezium as the different between the area of the large rectangle formed and the area of the triangle formed. Thus, the area of the trapezium equals to ‘b x t $-\frac{1}{2}$ (b – a) x t’. She simplified it algebraically to become ‘$\frac{1}{2}\ (a+b)\times t$’.

Suhana used dotted line to partition the trapezium into a rectangle and a triangle and then wrote the formula for the area of a trapezium as the combination of the formulae for the area of a rectangle and a triangle, namely (a × *tinggi* [*height*]) + [(b – a) × *tinggi* [*height*] × $\frac{1}{2}$]. Suhana developed the formula for the area of a trapezium from the combination of the formulae for the area of a rectangle and a triangle, namely (a × *tinggi* [*height*]) + [(b – a) × *tinggi* [*height*] × $\frac{1}{2}$] using algebraic method. In the first attempt, she had mistakenly simplified (a × *tinggi* [*height*]) + [(b – a) × *tinggi* [*height*] × $\frac{1}{2}$] as *tinggi* [*height*] (a + b – a × $\frac{1}{2}$). Suhana realized her mistake and cancelled it. In the second attempt, she has successfully developed the formula for the area of a trapezium. She correctly simplified (a × *tinggi* [*height*]) + [(b – a) × *tinggi* [*height*] × $\frac{1}{2}$] as follow: (a × *tinggi* [*height*]) + [(b – a) × *tinggi* [*height*] × $\frac{1}{2}$] = *tinggi* [*height*] (a + $\frac{1}{2}b$ – $\frac{1}{2}a$) = *tinggi* [*height*]($\frac{1}{2}a$ + $\frac{1}{2}b$) = $\frac{1}{2}$ × *tinggi* [*height*] × (a + b), which is the formula for the area of a trapezium.

Tan explained that a trapezium is a composite of a triangle and a rectangle or a square. Tan developed the formula for the area of a trapezium using the combination of the formula for the area of a triangle and a rectangle or a square. He wrote the formula for the area of a rectangle or a square, and a triangle as 'AB × AC' and '$\frac{1}{2} \times BE \times ED$' respectively. Tan wrote the formula for the total area of a rectangle or a square, and a triangle as '(AB × AC) + ($\frac{1}{2} \times BE \times ED$)'. He then used the algebraic method to simplified it as '$\frac{1}{2} AC\ (AB + CD)$' which is the formula for the area of a trapezium. Table 4.31 demonstrates PSSMTs who were able and unable to develop the formula for the area of a trapezium.

Table 4.31

PSSMTs who Were Able and Unable to Develop the Formula for the Area of a Trapezium

Develop the formula for the area of a trapezium	PSSMTs
Able to develop	Beng, Suhana, Tan
Unable to develop	Liana, Mazlan, Patrick, Roslina, Usha

Five of the PSSMTs, namely Liana, Mazlan, Patrick, Roslina, and Usha, were unable to develop the formula for the area of a trapezium. Mazlan attempted to develop the formula for the area of a trapezium using algebraic method but unsuccessful. He reiterated that the formula for calculating the area of a trapezium is ‘half times (a + b) times height’. Mazlan partitioned the trapezium into a triangle and a rectangle. He incorrectly wrote the formula for the area of a triangle as ‘$\frac{1}{2}(b \times h)'$ (it should be ‘$\frac{1}{2}(b - a)h'$). Mazlan also incorrectly wrote the formula for the area of a rectangle as ‘$a \times b'$ (it should be ‘$a \times h$’). Consequently, he simplified them algebraically to become ‘$\frac{1}{2}bh + ab$’ which was not equal to the formula for the area of a trapezium, namely ‘$\frac{1}{2}(a + b)h$’.

Patrick admitted that he did not know how to develop it. Nevertheless, Patrick attempted to develop the formula but unsuccessful. Patrick explained that the area of a trapezium can be calculated using the formula for the area of a trapezium itself or using the combination of the formula for the area of a rectangle, namely $a \times b$ (wrong formula. It should be $a \times h$), and a triangle, namely $\frac{1}{2} \times (b - a) \times h$). He moved his hand to indicate that $a \times b$ (wrong formula. It should be $a \times h$) + $\frac{1}{2} \times (b - a) \times h$) equals to $\frac{1}{2} \times (a + b) \times h$). Patrick was unable to show how $a \times b$ (wrong formula. It should be $a \times h$) + $\frac{1}{2} \times (b - a) \times h$) could be simplified as $\frac{1}{2} \times (a + b) \times h$). Patrick moved his head to indicate that he has no idea how to develop the formula.

Linguistic Knowledge

In this section, findings of PSSMTs' linguistic knowledge of perimeter and area were presented in terms of its components. Table 4.32 demonstrates the components of linguistic knowledge of perimeter and area.

Table 4.32
The Components of Linguistic Knowledge of Perimeter and Area

Type of knowledge	Its components
Linguistic knowledge	15. Mathematical symbols 16. Mathematical terms 17. Standard unit of length measurement (linear units) 18. Standard unit of area measurement (square units) 19. Conventions of writing and reading SI area measurement

Mathematical Symbols

In this subsection, findings of PSSMTs' linguistic knowledge of mathematical symbols were presented in terms of: (a) formula for the area of a rectangle, (b) formula for the area of a parallelogram, (c) formula for the area of a triangle, and (d) formula for the area of a trapezium.

Formula for the area of a rectangle

In Task 8, PSSMTs were asked to show a Form One student the way to develop (derive) area formulae of a rectangle, parallelogram, triangle, and trapezium. Task 8 is shown in Figure 4.14.

In Task 8, seven of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, and Usha, used appropriate mathematical symbols to write the formula for the area of a rectangle. Of the seven PSSMTs who used appropriate mathematical symbols to write the formula for the area of a rectangle, five of them, namely Mazlan, Liana, Patrick, Roslina, and Suhana, wrote the formula as '$a \times b$'. Beng wrote the formula as '$l \times w$' while Usha wrote the formula as 'xy'. The remaining PSSMT, namely Tan, did not use any mathematical symbol to write the formula for the area of a rectangle. Conventionally, the formula for the area of a rectangle is written as '$l \times w$', where *l* and *w* represents the length and the width of the rectangle, or '$l \times b$', where *l* and *b* represents the length and the breadth of the rectangle.

Formula for the area of a parallelogram

In Task 8, five PSSMTs, namely Beng, Mazlan, Patrick, Suhana, and Tan used appropriate mathematical symbols to write the formula for the area of a parallelogram. Of the five PSSMTs, four of them, namely Beng, Mazlan, Patrick, and Suhana wrote the formula as '$a \times b$'. Tan wrote the formula as '$AE \times DC$'. The remaining three PSSMTs, namely Liana, Roslina, and Usha, could not recall the formula for the area of a parallelogram. Conventionally, the formula for the area of a parallelogram is written as '$b \times h$', where *b* and *h* represents the base and the height of the parallelogram.

Formula for the area of a triangle

In Task 8, seven of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, and Tan, used appropriate mathematical symbols to write the formula for the area of a triangle. Of the seven PSSMTs, two of hem, namely Mazlan and Patrick wrote the formula as '$\frac{1}{2} \times b \times h$'. Beng wrote the formula as '$\frac{1}{2} \times b \times t$' while Liana wrote the formula as '$\frac{1}{2} \times a \times b$'. Roslina wrote the formula as '$\frac{1}{2}\ (h\ \times b)$' while Suhana wrote the formula as '$\frac{1}{2} \times t\ \times tapak\ [height]$'. Tan indicated the formula as '$\frac{1}{2} \times AB \times BC$'. Usha could not recall the formula for the area of a triangle. Initially, she wrote the formula for the area of a triangle as $\frac{1}{2}\, y \times x$. Later, she shaded the half to become $y \times x$. Usha thought that the formula for the area of a triangle is '$y \times x$', where *x* and y is the base and the height of the triangle. Conventionally, the formula for the area of a triangle is written as '$\frac{1}{2} \times b \times h$', where b and h represents the base and the height of the triangle.

Formula for the area of a trapezium

In Task 8, six of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, Tan, and Usha, used appropriate mathematical symbols to write the formula for the area of a trapezium. Beng wrote the formula as '$\frac{1}{2}\ (a+b) \times t$' while Mazlan wrote the formula as '$\frac{1}{2}\ (a+b)h$'. Patrick wrote the formula as '$\frac{1}{2} \times (a+b) \times h$' while Suhana wrote the formula as $'\frac{1}{2} \times tinggi\ [height] \times (a+b)'$. Tan wrote the formula as '$\frac{1}{2}\ AC\ (AB+CD)$' while Usha wrote the formula as '$\frac{1}{2} \times h \times (a+b)$'. The remaining two PSSMTs, namely Liana and Roslina, could not recall the formula for the area of a trapezium. They did not write the formula. Conventionally, the formula for the area of a trapezium is written as '$\frac{1}{2} \times (a+b) \times h$', where ($a$ + b) and h represents the sum of the parallel sides and the height of the trapezium.

Mathematical Terms

In this subsection, findings of PSSMTs' linguistic knowledge of mathematical terms were presented in terms of: (a) mathematical terms used to justify their selection of shapes that have a perimeter, (b) mathematical terms used to justify their selection of shapes that do not have a perimeter, (c) mathematical terms used to justify their selection of shapes that have an area, (d) mathematical terms used to justify their selection of shapes that do not have an area, (e) mathematical terms used to justify the shapes that can be used as the unit of area, (f) mathematical terms used to justify the shapes that they thought cannot be used as the unit of area, and (g) mathematical terms used to state the area formulae or to explain the meaning of the mathematical symbols that they employed to write the formulae.

Justification of Shapes That Have a Perimeter

In Task 1.1, PSSMTs were asked to select the shapes that have a perimeter. Task 1.1 is shown in Figure 4.1. In Task 1.1, when asked to justify their selection of shapes that have a perimeter, six PSSMTs, namely Liana, Mazlan, Roslina, Suhana, Tan, and Usha, used appropriate mathematical term ‘closed’ to justify their selection of shape “A” that have a perimeter. Liana, Mazlan, Roslina, Suhana, Tan, and Usha explained that they selected shape “A” because it was closed. Beng used appropriate mathematical terms ‘2-dimensional shapes’ and ‘enclosed’ to justify her selection of shape “A” that have a perimeter. Beng explained that she selected shape “A” because it was a 2-dimensional shape and enclosed. Patrick used appropriate mathematical term ‘covered’ to justify his selection of shape “A” that have a perimeter. Patrick explained that he selected shape “A” because it was covered. Table 4.33 shows PSSMTs’ selection of shapes that have a perimeter and the appropriateness of their justification.

Four PSSMTs, namely Mazlan, Roslina, Suhana, and Usha, used appropriate mathematical term ‘closed’ to justify their selection of shape “D” that have a perimeter. Mazlan, Roslina, Suhana, and Usha explained that they selected shape “D” because it was closed. Tan used appropriate mathematical term ‘measure’ to justify his selection of shape “D” that have a perimeter. Tan explained that he selected shape “D” because its perimeter can be measured using thread. It indicated that Tan appeared to associate the notion of perimeter with the measurement of perimeter (i.e., perimeter does not exist until it is measured).

Table 4.33

PSSMTs' Selection of Shapes That Have a Perimeter and the Appropriateness of Their Justification

Selection of shapes that have a perimeter	Justification		PSSMTs
	Appropriate	Inappropriate	
"A"	Closed shape		Liana, Mazlan, Roslina,
	Closed object		Tan
	Closed surface		Usha
	Closed boundary		Suhana
	2-dimensional shape and enclosed		Beng
	Covered		Patrick
"C"	Closed shape		Liana, Mazlan, Roslina
	Closed surface		Usha
	Closed boundary		Suhana
	Its perimeter can be measured (using thread)		Tan
	2-dimensional shape and enclosed		Beng
	Covered		Patrick
"D"	Closed shape		Roslina
	Closed surface		Usha
	Closed boundary		Suhana
	Its perimeter can be measured (using thread)		Tan
	2-dimensional shape and enclosed		Beng
	Covered		Patrick
		All the sides are joined together	Liana
"F" and "J"	Closed shape		Roslina
	Closed surface		Usha
	Its perimeter can be calculated on each surface of the 3-dimensional objects.		Tan
		All the lines are joined together	Liana
"H"	Closed shape		Mazlan, Roslina
	Closed object		Tan
	Closed surface		Usha
	Closed boundary		Suhana
	2-dimensional shape and enclosed		Beng
	Covered		Patrick
		All the lines are joined together	Liana

Table 4.33 (continued)

Selection of shapes that have a perimeter	Justification		PSSMTs
	Appropriate	Inappropriate	
“I” and “K”	Closed shape		Roslina
	Closed surface		Usha
	Closed boundary		Suhana
	Its perimeter can be measured (using thread)		Tan
	2-dimensional shape and enclosed		Beng
	Covered		Patrick
		All the lines are joined together	Liana

Beng used appropriate mathematical terms ‘2-dimensional shapes’ and ‘enclosed’ to justify her selection of shape “D” that have a perimeter. Beng explained that she selected shape “D” because it was a 2-dimensional shape and enclosed. Patrick used appropriate mathematical term ‘covered’ to justify his selection of shape “D” that have a perimeter. Patrick explained that he selected shape “D” because it was covered. Liana used inappropriate words ‘joined together’ to justify her selection of shape “D” that have a perimeter. Liana explained that she selected shape “D” because all the sides are joined together.

Two PSSMTs, namely Roslina and Usha, used appropriate mathematical term ‘closed’ to justify their selection of shapes “F” and “J” that have a perimeter. Roslina and Usha explained that they selected shapes “F” and “J” because they were closed. Tan used appropriate mathematical term ‘calculate’ to justify his selection of shapes “F” and “J” that have a perimeter. Tan explained that he selected shapes “F” and “J” because their perimeter can be calculated on each surface of the 3-dimensional objects. It indicated that Tan appeared to associate the notion of perimeter with the measurement of perimeter (i.e., perimeter does not exist until it is measured). Liana used inappropriate words ‘joined together’ to justify her selection of shapes “F” and “J” that have a perimeter. Liana explained that she selected shapes “F” and “J” because all the lines are joined together.

Five PSSMTs, namely Mazlan, Roslina, Suhana, Tan, and Usha, used appropriate mathematical term ‘closed’ to justify their selection of shape “H” that have a perimeter. Mazlan, Roslina, Suhana, Tan, and Usha explained that they selected shape “H” because it was closed. Beng used appropriate mathematical terms ‘2-dimensional shapes’ and ‘enclosed’ to justify her selection of shape “H” that have a perimeter. Beng explained that she selected shape “H” because it was a 2-dimensional shape and enclosed. Patrick used appropriate mathematical term ‘covered’ to justify his selection of shape “H” that have a perimeter. Patrick explained that he selected shape “H” because it was covered. Liana used inappropriate words ‘joined together’ to justify her selection of shape “H” that have a perimeter. Liana explained that she selected shape “H” because all the lines are joined together.

Three PSSMTs, namely Roslina, Suhana, and Usha, used appropriate mathematical term ‘closed’ to justify their selection of shapes “I” and “K” that have a perimeter. Roslina, Suhana, and Usha explained that they selected shapes “I” and “K” because they were closed. Tan used appropriate mathematical term ‘measure’ to justify his selection of shapes “I” and “K” that have a perimeter. Tan explained that he selected shapes “I” and “K” because their perimeter can be measured using thread. It indicated that Tan appeared to associate the notion of perimeter with the measurement of perimeter (i.e., perimeter does not exist until it is measured). Beng used appropriate mathematical terms ‘2-dimensional shapes’ and ‘enclosed’ to justify her selection of shapes “I” and “K” that have a perimeter. Beng explained that she selected shapes “I” and “K” because they were 2-dimensional shapes and enclosed. Patrick used appropriate mathematical term ‘covered’ to justify his selection of shapes “I” and “K” that have a perimeter. Patrick explained that he selected shapes “I” and “K” because they were covered. Liana used inappropriate words ‘joined together’ to justify her selection of shapes “I” and “K” that have a perimeter. Liana explained that she selected shapes “I” and “K” because all the lines are joined together.

Justification of Shapes That Do Not Have a Perimeter

In Task 1.1, five PSSMTs, namely Beng, Mazlan, Roslina, Suhana, and Usha, used appropriate mathematical term ‘open’ and negation ‘not closed’ or ‘not enclosed’ as their justification for not selecting shape “B” as having a perimeter. Mazlan, Suhana, and Usha explained that they did not select shape “B” because it is not closed. Roslina explained that she did not select shape “B” because it was open whereas Beng explained that she did not select shape “B” because it is not enclosed. Two PSSMTs, namely Liana and Patrick, used inappropriate negation ‘not joined’ as their justification for not selecting shape “B” as having a perimeter. Liana explained that she did not select shape “B” because the line did not join together. Patrick explained that he did not select shape “B” because it is not joined. Tan used inappropriate negation ‘incomplete’ as his justification for not selecting shape “B” as having a perimeter. Tan explained that he did not select shape “B” because it is incomplete to surround to become an object or a shape. Table 4.34 depicts PSSMTs’ appropriateness of justification for not selecting a shape(s) as having a perimeter.

All the PSSMTs selected shapes “D”, “I”, and “K” that have a perimeter, except Mazlan. Mazlan explained that he did not select shapes “D”, “I”, and “K” because they have a curve. ‘Curve’ is an appropriate mathematical term. Nevertheless, it was not the appropriate justification for not selecting shapes “D”, “I”, and “K” that have a perimeter as we still can find perimeter for closed curves.

Table 4.34

PSSMTs' Appropriateness of Justification for not Selecting a Shape(s) as Having a Perimeter

Shapes not selected as having a perimeter	Justification		PSSMTs
	Appropriate	Inappropriate	
"B"	It is not closed		Mazlan, Suhana, Usha
	It was open		Roslina
	It is not enclosed		Beng
		The line did not join together	Liana
		It is not joined	Patrick
		It is incomplete to surround to become an object or a shape	Tan
"D", "I", and "K"		It has a curve	Mazlan
"E"	It is just a line		Liana, Mazlan, Roslina, Suhana, Usha
	It is not enclosed		Beng
	It is not covered		Patrick
		The line is not connected	Tan
"F" and "J"		It is a 3-dimensional shape	Beng, Suhana
		It is a 3-dimensional object	Mazlan, Patrick
"G"	It was open		Mazlan, Roslina
	It is just a line		Usha
	It is not enclosed		Beng
		The line did not join together	Liana
		It is not joined	Patrick
		The line is not connected to each other	Suhana
		It is incomplete to surround to become an object or a shape	Tan
"L"	It is just a line		Roslina, Suhana, Usha,
	It was open		Mazlan
	It is not enclosed		Beng
		The line did not join together	Liana
		It is not joined	Patrick
		It is incomplete to surround certain object	Tan

Five PSSMTs, namely Liana, Mazlan, Roslina, Suhana, and Usha, used appropriate mathematical term 'line' as their justification for not selecting shape "E" as having a perimeter. Liana, Mazlan, Roslina, Suhana, and Usha explained that they did not select shape "E" because it is just a line. Beng used appropriate negation 'not enclosed' as her justification for not selecting shape “E” as having a perimeter. Beng explained that she did not select shape “E” because it is not enclosed. Patrick used appropriate negation ‘not covered’ as his justification for not selecting shape “E” as having a perimeter. Patrick explained that he did not select shape “E” because it is not covered. Tan used inappropriate negation ‘not connected’ as his justification for not selecting shape “E” as having a perimeter. Tan explained that he did not select shape “E” because the line is not connected.

Four PSSMTs, namely Beng, Mazlan, Patrick, and Suhana, explained that they did not select shapes “F” and “J” because they are 3-dimensional shapes or objects. ‘3-dimensional shapes’ or ‘3-dimensional objects’ are appropriate mathematical terms. Nevertheless, they were not the appropriate justification for not selecting shapes “F” and “J” that have a perimeter as we still can find perimeter for the faces of solids.

Three PSSMTs, namely Beng, Mazlan, and Roslina, used appropriate mathematical term ‘open’ and negation ‘not enclosed’ as their justification for not selecting shape “G” as having a perimeter. Mazlan, and Roslina explained that they did not select shape “G” because it was open. Beng explained that she did not select shape “G” because it is not enclosed. Usha used appropriate mathematical term ‘line’ as her justification for not selecting shape “G” as having a perimeter. Usha explained that she did not select shape “G” because it is just a line. Three PSSMTs, namely Liana, Patrick, and, Suhana used inappropriate negation ‘not joined’ or ‘not connected’ as their justification for not selecting shape “G” as having a perimeter. Liana explained that she did not select shape “G” because the line did not join together. Patrick explained that he did not select shape “G” because it is not joined. Suhana explained that she did not select shape “G” because the line is not connected to each other. Tan used inappropriate negation ‘incomplete’ as his justification for not selecting shape “G” as having a perimeter. Tan explained that he did not select shape “G” because it is incomplete to surround to become an object or a shape.

Three PSSMTs, namely Roslina, Suhana, and Usha, used appropriate mathematical term ‘line’ as their justification for not selecting shape “L” as having a perimeter. Roslina, Suhana, and Usha explained that they did not select shape “L” because it is just a line. Two PSSMTs, namely Beng and Mazlan, used appropriate mathematical term ‘open’ and negation ‘not enclosed’ as their justification for not selecting shape “L” as having a perimeter. Mazlan explained that he did not select shape “L” because it was open whereas Beng explained that he did not select shape “L” because it is not enclosed. Two PSSMTs, namely Liana and Patrick, used inappropriate negation ‘not joined’ as their justification for not selecting shape “L” as having a perimeter. Liana explained that she did not select shape “L” because the line did not join together whereas Patrick explained that he did not select shape “L” because it is not joined. Tan used inappropriate negation ‘incomplete’ as his justification for not selecting shape “L” as having a perimeter. Tan explained that he did not select shape “L” because it is incomplete to surround certain object.

Justification of Shapes That Have an Area

In Task 1.2, PSSMTs were asked to select the shapes that have an area. Task 1.2 is shown in Figure 4.2. In Task 1.2, when asked to justify their selection of shapes that have an area, four PSSMTs, namely Mazlan, Suhana, Tan, and Usha, used appropriate mathematical term ‘closed’ to justify their selection of shape “A” that have an area. Mazlan, Suhana, Tan, and Usha explained that they selected shape “A” because it was closed. Beng used appropriate mathematical term ‘enclosed’ to justify her selection of shape “A” that have an area. Beng explained that she selected shape “A” because it was enclosed.

Two PSSMTs, namely Patrick and Roslina, used appropriate mathematical term ‘calculate’ to justify their selection of shape “A” that have an area. Patrick and Roslina explained that they selected shape “A” because its area can be calculated. It indicated that Patrick and Roslina appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Liana used inappropriate word ‘joining’ to justify her selection of shape “A” that has an area. Liana explained that she selected shape “A” because the lines are joining. Table 4.35 demonstrates PSSMTs’ selection of shapes that have an area and the appropriateness of their justification.

Three PSSMTs, namely Mazlan, Suhana, and Usha, used appropriate mathematical term ‘closed’ to justify their selection of shapes “C” and “H” that have an area. Mazlan, Suhana, and Usha explained that they selected shapes “C” and “H” because they were closed. Four PSSMTs, namely Beng, Patrick, Roslina, and Tan, used appropriate mathematical term ‘calculate’ to justify their selection of shapes “C” and “H” that have an area. Beng, Patrick, Roslina, and Tan explained that they selected shapes “C” and “H” because their area can be calculated. It indicated that Beng, Patrick, Roslina, and Tan appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Liana used inappropriate word ‘joining’ to justify her selection of shapes “C” and “H” that have an area. Liana explained that she selected shapes “C” and “H” because the lines are joining.

Three PSSMTs, namely Mazlan, Suhana, and Usha, used appropriate mathematical term ‘closed’ to justify their selection of shape “D” that have an area. Mazlan, Suhana, and Usha explained that they selected shape “D” because it was closed. Two PSSMTs, namely Beng and Tan, used appropriate mathematical term ‘calculate’ to justify their selection of shape “D” that have an area. Beng and Tan explained that they selected shape “D” because its area can be calculated. It indicated that Beng and Tan appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Liana used inappropriate word 'joining' to justify her selection of shape "D" that has an area. Liana explained that she selected shape "D" because the lines are joining.

Table 4.35

PSSMTs' Selection of Shapes That Have an Area and the Appropriateness of Their Justification

Selection of shapes that have an area	Justification		PSSMTs
	Appropriate	Inappropriate	
"A"	Closed		Usha
	Closed shape		Mazlan, Suhana
	Closed length object		Tan
	Enclosed		Beng
	Its area can be calculated		Patrick, Roslina
		The lines are joining	Liana
"C" and "H"	Closed		Usha
	Closed shape		Mazlan, Suhana
	Its area can be calculated		Beng, Patrick, Roslina, Tan
		The lines are joining	Liana
"D"	Closed		Usha
	Closed shape		Mazlan, Suhana
	Its area can be calculated		Beng, Tan
		The lines are joining	Liana
"F" and "J"	3D object		Mazlan
	3D has surface area		Suhana
	Its surface area can be calculated		Patrick, Roslina, Tan, Usha
		The lines are joining	Liana
"I"	Closed shape		Mazlan, Suhana
	Enclosed		Beng
	Its area can be calculated		Tan, Usha
		The lines are joining	Liana
"K"	Closed		Usha
	Closed shape		Mazlan, Suhana
	Enclosed		Beng
	Its area can be calculated		Tan
		The lines are joining	Liana

Two PSSMTs, namely Mazlan and Suhana, used appropriate mathematical symbol ‘3D’ to justify their selection of shapes “F” and “J” that have an area. Mazlan and Suhana explained that they selected shapes “F” and “J” because they were 3D (3-dimensional). Four PSSMTs, namely Patrick, Roslina, Tan, and Usha, used appropriate mathematical term ‘calculate’ to justify their selection of shapes “F” and “J” that have an area. Patrick, Roslina, Tan, and Usha explained that they selected shapes “F” and “J” because their surface area can be calculated. It indicated that Patrick, Roslina, Tan, and Usha appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Liana used inappropriate word ‘joining’ to justify her selection of shapes “F” and “J” that have an area. Liana explained that she selected shapes “F” and “J” because the lines are joining.

Two PSSMTs, namely Mazlan and Suhana, used appropriate mathematical term ‘closed’ to justify their selection of shape “I” that have an area. Mazlan and Suhana explained that they selected shape “I” because it was closed. Beng used appropriate mathematical term ‘enclosed’ to justify her selection of shape “I” that have an area. Beng explained that she selected shape “I” because it was enclosed. Two PSSMTs, namely Tan and Usha, used appropriate mathematical term ‘calculate’ to justify their selection of shape “I” that have an area. Tan and Usha explained that they selected shape “I” because its area can be calculated. It indicated that Tan and Usha appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Liana used inappropriate word ‘joining’ to justify her selection of shape “I” that have an area. Liana explained that she selected shape “I” because the lines are joining.

Three PSSMTs, namely Mazlan Suhana, and Usha used appropriate mathematical term ‘closed’ to justify their selection of shape “K” that have an area. Mazlan Suhana, and Usha explained that they selected shape “K” because it was closed. Beng used appropriate mathematical term ‘enclosed’ to justify her selection of shape “K” that have an area. Beng explained that she selected shape “K” because it was enclosed. Tan used appropriate mathematical term ‘calculate’ to justify his selection of shape “K” that have an area. Tan explained that he selected shape “K” because its area can be calculated. It indicated that Tan appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Liana used inappropriate word ‘joining’ to justify her selection of shape “K” that have an area. Liana explained that she selected shape “K” because the lines are joining.

Justification of Shapes That Do Not Have an Area

In Task 1.2, seven PSSMTs, namely Beng, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, used appropriate mathematical term ‘open’ and negation ‘not closed’, ‘not enclosed’, ‘not covered, or ‘not surrounded’ as their justification for not selecting shape “B” as having an area. Mazlan, Suhana, and Usha explained that they did not select shape “B” because it was open. Roslina explained that she did not select shape “B” because it is not closed. Beng explained that she did not select shape “B” because it is not enclosed. Patrick explained that he did not select shape “B” because it is not covered whereas Tan explained that he did not select shape “B” because it is not surrounded to form an object. Liana used inappropriate negation ‘not joining’ as her justification for not selecting shape “B” as having an area. Liana explained that she did not select shape “B” because the lines are not joining together. Table 4.36 reveals PSSMTs’ appropriateness of justification for not selecting a shape(s) as having an area.

All the PSSMTs selected shapes “D”, “I”, and “K” that have an area, except Patrick and Roslina. Patrick and Roslina explained that they did not select shapes “D”, “I”, and “K” because there is no specific formula that can be used to calculate their area. It indicated that Patrick and Roslina appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Although ‘no specific formula that can be used to calculate their area’ is an appropriate negation but it was not an appropriate justification for not selecting shapes “D”, “I”, and “K” that have an area as their area still exist even though there is no specific formula that can be used to calculate their area.

Table 4.36

PSSMTs' Appropriateness of Justification for not Selecting a Shape(S) as Having an Area

Shapes not selected as having an area	Justification		PSSMTs
	Appropriate	Inappropriate	
“B”	Open		Mazlan, Suhana, Usha
	Not closed		Roslina
	Not enclosed		Beng
	Not covered		Patrick
	Not surrounded to form an object		Tan
		The lines are not joining together	Liana
“D”, “I”, and “K”		No specific formula that can be used to calculate its area	Patrick, Roslina
“E”	It is just a line		Beng, Liana, Mazlan, Roslina, Suhana, Usha
	Not surrounded to form an object		Tan
		Not joined	Patrick
“F” and “J”		3D	Beng
“G”	Open		Mazlan
	It is just a line		Roslina
	Not closed		Suhana, Usha
	Not enclosed		Beng
	Not surrounded to form an object		Tan
		Not joined	Patrick
		The lines are not joining together	Liana
“L”	Open		Mazlan
	It is just a line		Roslina, Suhana, Usha
	Not enclosed		Beng
	Not surrounded to form an object		Tan
		Not joined	Patrick
		The lines are not joining together	Liana

Six PSSMTs, namely Beng, Liana, Mazlan, Roslina, Suhana, and Usha, used appropriate mathematical term ‘line’ as their justification for not selecting shape “E” as having an area. Beng, Liana, Mazlan, Roslina, Suhana, and Usha explained that they did not select shape “E” because it is just a line. Tan used appropriate negation ‘not surrounded’ as his justification for not selecting shape “E” as having an area. Tan explained that he did not select shape “E” because it is not surrounded to form an object. Patrick also used inappropriate negation ‘not joined’ as his justification for not selecting shape “E” as having an area. Patrick explained that he did not select shape “E” because it is just a line and not joined.

All the PSSMTs selected shapes “F” and “J” that have an area, except Beng. Beng explained that she did not select shapes “F” and “J” because they are 3D (3-dimensional). ‘3D’ is an appropriate mathematical symbol. Nevertheless, ‘3D’ is not the appropriate justification for not selecting shapes “F” and “J” that have an area as we still can find area for the faces of solids.

Five PSSMTs, namely Beng, Mazlan, Suhana, Tan, and Usha, used appropriate mathematical term ‘open’ and negation ‘not closed’, ‘not enclosed’, or ‘not surrounded’ as their justification for not selecting shape “G” as having an area. Mazlan explained that he did not select shape “G” because it was open. Suhana and Usha explained that they did not select shape “G” because it is not closed. Beng explained that she did not select shape “G” because it is not enclosed whereas Tan explained that he did not select shape “G” because it is not surrounded to form an object.

Roslina used appropriate mathematical term ‘line’ as her justification for not selecting shape “G” as having an area. Roslina explained that she did not select shape “G” because it is just a line. Patrick used inappropriate negation ‘not joined’ as his justification for not selecting shape “G” as having an area. Patrick explained that he did not select shape “G” because its line not joined. Liana used inappropriate negation ‘not joining’ as her justification for not selecting shape “G” as having an area. Liana explained that she did not select shape “G” because the lines are not joining together.

Three PSSMTs, namely Roslina, Suhana, and Usha, used appropriate mathematical term ‘line’ as their justification for not selecting shape “L” as having an area. Roslina, Suhana, and Usha explained that they did not select shape “L” because it is just a line.

Three PSSMTs, namely Beng, Mazlan, and Tan, used appropriate mathematical term ‘open’ and negation ‘not enclosed’ or ‘not surrounded’ as their justification for not selecting shape “L” as having an area. Mazlan explained that he did not select shape “L” because it was open. Beng explained that she did not select shape “L” because it is not enclosed whereas Tan explained that he did not select shape “L” because it is not surrounded to form an object. Patrick used inappropriate negation ‘not joined’ as his justification for not selecting shape “L” as having an area. Patrick explained that he did not select shape “L” because the lines are not joined. Liana used inappropriate negation ‘not joining’ as her justification for not selecting shape “G” as having an area. Liana explained that she did not select shape “G” because the lines are not joining together.

Justification of Shapes That Can Be Used As the Unit of Area

In Task 2, PSSMTs were asked to respond to a scenario where three students were discussing about the units of area. Task 2 is shown in Figure 4.3. In Task 2, when asked to justify the shapes that can be used as the unit of area, only one PSSMT, namely Patrick, used appropriate mathematical term ‘cover’ to justify that a square can be used as the unit of area. He explained that a square can be used as the unit of area because we can count the numbers of square units it takes to cover a region. It indicated that Patrick knew that a square tessellate a plane and thus can be used as the unit of area measurement. Table 4.37 exhibits each PSSMT’s selection of shapes that can be used as the unit of area and the appropriateness of their justification.

Table 4.37

PSSMTs' Selection of Shapes That can be used as the Unit of Area and the Appropriateness of Their Justification

Selection of shapes that can be used as the unit of area	Justification		PSSMTs
	Appropriate	Inappropriate	
Square	We can count the numbers of square units it takes to cover a region		Patrick
		It has straight lines	Beng
		Its unit is "the power of two" (square unit)	Tan
		It represents the area of a particular shape with some shape	Usha
		The sides of a square have the same length	Mazlan, Roslina, Suhana
Rectangle	We can count the numbers of rectangular units it takes to cover a region		Patrick
		It has straight lines	Beng
		Its unit is "the power of two" (square unit)	Tan
		It represents the area of a particular shape with some shape	Usha
Triangle		It has straight lines	Beng
		Its unit is "the power of two" (square unit)	Tan
		It represents the area of a particular shape with some shape	Usha
		A triangle came from a square that had been partitioned into two triangles	Mazlan

Six of the PSSMTs, namely Beng, Mazlan, Roslina, Suhana, Tan, and Usha, used inappropriate mathematical terms to justify that a square can be used as the unit of area. Mazlan, Roslina, and Suhana used inappropriate mathematical term 'same length' to justify that a square can be used as the unit of area. They explained that a square can be used as the unit of area because the sides of a square have the same length. Beng used inappropriate mathematical term 'straight lines' to justify that a square can be used as the unit of area. She explained that a square can be used as the unit of area because it has straight lines. Tan used inappropriate mathematical term 'its unit is "the power of two" (square unit)' to justify that a square can be used as the unit of area. He explained that a square can be used as the unit of area because its unit is "the power of two" (square unit) such as square centimetre or square metre. Usha used inappropriate mathematical term 'represents' to justify that a square can be used as the unit of area. She explained that a square can be used as the unit of area because it represents the area of a particular shape with some shape.

Only one of the PSSMTs, namely Patrick, had used appropriate mathematical term 'cover' to justify that a rectangle can be used as the unit of area. He explained that a rectangle can be used as the unit of area because we can count the numbers of rectangle units it takes to cover a region. It indicated that Patrick knew that a rectangle tessellate a plane and thus can be used as the unit of area measurement.

Three of the PSSMTs, namely Beng, Tan, and Usha, used inappropriate mathematical terms to justify that a rectangle can be used as the unit of area. Beng used inappropriate mathematical term 'straight lines' to justify that a rectangle can be used as the unit of area. She explained that a rectangle can be used as the unit of area because it has straight lines. Tan used inappropriate mathematical term 'its unit is "the power of two" (square unit)' to justify that a rectangle can be used as the unit of area. He explained that a rectangle can be used as the unit of area because its unit is "the power of two" (square unit) such as square centimetre or square metre. Usha used inappropriate mathematical term 'represents' to justify that a rectangle can be used as the unit of area. She explained that a rectangle can be used as the unit of area because it represents the area of a particular shape with some shape.

All the four PSSMTs, namely Beng, Mazlan, Tan, and Usha, used inappropriate mathematical terms to justify that a triangle can be used as the unit of area. Beng used inappropriate mathematical term 'straight lines' to justify that a triangle can be used as the unit of area. She explained that a triangle can be used as the unit of area because it has straight lines. Mazlan used inappropriate mathematical term 'a triangle came from a square' to justify that a triangle can be used as the unit of area. He explained that a triangle can be used as the unit of area because a triangle came from a square that had been partitioned into two triangles.

Tan used inappropriate mathematical term ‘its unit is “the power of two” (square unit)’ to justify that a triangle can be used as the unit of area. He explained that a triangle can be used as the unit of area because its unit is “the power of two” (square unit) such as square centimetre or square metre. Usha used inappropriate mathematical term ‘represents’ to justify that a triangle can be used as the unit of area. She explained that a triangle can be used as the unit of area because it represents the area of a particular shape with some shape.

Justification of Shapes That They Thought Cannot Be Used As the Unit of Area

In Task 2, only one PSSMT, namely Liana, thought that a square cannot be used as a unit of area measurement. She used inappropriate mathematical term ‘measurement of the side’ to justify that a square cannot be used as the unit of area. Liana explained that a square cannot be used as the unit of area because the measurement of the side that determine the unit of area. Table 4.38 illustrates PSSMTs’ selection of shapes that cannot be used as the unit of area and the appropriateness of their justification.

Four of the PSSMTs, namely Liana, Mazlan, Roslina, and Suhana, thought that a rectangle cannot be used as a unit of area measurement. They used inappropriate mathematical terms to justify that a rectangle cannot be used as the unit of area. Of the four PSSMTs, two of them, namely Mazlan and Roslina, used inappropriate mathematical negation ‘not same length’ to justify that a rectangle cannot be used as the unit of area. Mazlan and Roslina explained that they did not select a rectangle that can be used as the unit of area because the sides of a rectangle are not of the same length.

Table 4.38
PSSMTs' Selection of Shapes That Cannot be Used as the Unit of Area and the Appropriateness of Their Justification

Selection of shapes that cannot be used as the unit of area	Justification		PSSMTs
	Appropriate	Inappropriate	
Square		The measurement of the side that determine the unit of area	Liana
Rectangle		The sides of a rectangle are not of the same length	Mazlan, Roslina
		The measurement of the side that determine the unit of area	Liana
		Difficult to use it	Suhana
Triangle		The measurement of the side that determine the unit of area	Liana
		A triangle has three sides and thus it was impossible to use it as the unit of area measurement	Roslina
		Difficult to use it	Suhana

Liana used inappropriate mathematical term 'measurement of the side' to justify that a rectangle cannot be used as the unit of area. Liana explained that she did not select a rectangle that can be used as the unit of area because the measurement of the side that determine the unit of area. Suhana used inappropriate words 'difficult to use' to justify that a rectangle cannot be used as the unit of area. Suhana explained that a rectangle cannot be used as the unit of area because it was difficult to use it.

Three of the PSSMTs, namely Liana, Roslina, and Suhana, thought that a triangle cannot be used as a unit of area measurement. They used inappropriate mathematical terms to justify that a triangle cannot be used as the unit of area. Liana used inappropriate mathematical term 'measurement of the side' to justify that a triangle cannot be used as the unit of area. Liana explained that she did not select a triangle that can be used as the unit of area because the measurement of the side that determine the unit of area.

Roslina used inappropriate mathematical term 'a triangle has three sides' to justify that a triangle cannot be used as the unit of area. Roslina explained that she did not select a triangle that can be used as the unit of area because a triangle has three sides and thus it was impossible to use it as the unit of area measurement. Suhana used inappropriate words 'difficult to use' to justify that a triangle cannot be used as the unit of area. Suhana explained that a triangle cannot be used as the unit of area because it was difficult to use it.

State the Area Formulae or Explain the Meaning of the Mathematical Symbols

In this subsection, findings of PSSMTs' linguistic knowledge of mathematical terms used to state the area formulae or to explain the meaning of the mathematical symbols that they employed to write the formulae were presented in terms of: (a) formula for the area of a rectangle, (b) formula for the area of a parallelogram, (c) formula for the area of a triangle, and (d) formula for the area of a trapezium.

Formula for the area of a rectangle

In Task 8, PSSMTs were asked to show a Form One student the way to develop (derive) area formulae of a rectangle, parallelogram, triangle, and trapezium. Task 8 is shown in Figure 4.14. In Task 8, seven of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, and Usha, used appropriate mathematical symbols to write the formula for the area of a rectangle. Nevertheless, only two of the PSSMTs, namely Beng and Mazlan, used appropriate mathematical terms to state the formula. They used appropriate mathematical terms 'length', 'times', and 'width' to state the formula for the area of a rectangle.

Beng stated that the formula for the area of a rectangle is"…the length times the width." (Beng/L1311). Mazlan stated that "*Yang kita tahu luas* [We know that the area of] rectangle *panjang darab lebar* [is length times width]*."* (Mazlan/L1361). Table 4.39 shows PSSMTs who used appropriate and inappropriate mathematical terms to state the formula for the area of a rectangle or to explain the meaning of the mathematical symbols that they employed to write the formula for the area of a rectangle.

Table 4.39

PSSMTs who Used Appropriate and Inappropriate Mathematical Terms to State the Formula or to Explain the Meaning of the Mathematical Symbols That They Employed to Write the Formula for the Area of a Rectangle

State the formula or explain the meaning of the mathematical symbols that they employed to write the formula for the area of a rectangle	PSSMTs
Used appropriate mathematical terms	Beng, Mazlan
Used inappropriate mathematical terms	Liana, Patrick, Roslina, Suhana, Tan, Usha

The remaining six PSSMTs, namely Liana, Patrick, Roslina, Suhana, Tan, and Usha, used inappropriate mathematical terms to state the formula for the area of a rectangle or to explain the meaning of the mathematical symbols that they employed to write the formula for the area of a rectangle. Of the six PSSMTs, five of them, namely Liana, Patrick, Roslina, Suhana, and Usha, used inappropriate mathematical terms to explain the meaning of the mathematical symbols that they employed to write the formula for the area of a rectangle.

Liana used inappropriate mathematical terms 'the length of this side' and 'the length of this one' to explain the meaning of the mathematical symbols *a* and *b* that she employed. She explained that *a* represents "…the length of this side." (Liana/L1290) and *b* represents "…the length of this one." (Liana/L1292). Actually, *a* and *b* in her formula represents the width and the length of the rectangle. Patrick used inappropriate mathematical term '*sisi yang berlainan* [different side]' to explain the meaning of the symbols *a* and *b* that he employed. He explained that "…*sisi yang berlainan akan darab.* […different side will be multiplied.]" (Patrick/L1523). Actually, *a* and *b* represents the length and the width of the rectangle. Conventionally, the formula for the area of a rectangle is written as '*l* x *w*', where *l* and *w* represents the length and the width of the rectangle, or ‘*l* × *b*’, where *l* and *b* represents the length and the breadth of the rectangle.

Roslina used inappropriate mathematical terms ‘this length’ and ‘this side’ to explain the meaning of the symbols *a* and *b* that she employed. She explained that “…this length, this side is *a* and this *b*.” (Roslina/L1412). Actually, *a* and *b* represents the length and the width of the rectangle. Suhana used inappropriate mathematical terms ‘longer side’ and ‘shorter side’ to explain the meaning of the symbols *a* and *b* that she employed. She explained that “*a* for the longer side and then *b* is the shorter side.” (Suhana/L1557). Actually, *a* and *b* in her formula represents the width and the length of the rectangle. Conventionally, the formula for the area of a rectangle is written as ‘*l* x *w*’, where *l* and *w* represents the length and the width of the rectangle, or ‘*l* × *b*’, where *l* and *b* represents the length and the breadth of the rectangle.

Usha used inappropriate mathematical terms ‘height’ and ‘width’ to explain the meaning of the symbols *x* and *y* that she employed. Usha explained that “"x" is height. Then "y" is the width, "y" is the width.” (Usha/L1405). Actually, *x* and y in her formula represents the height and the base, or the width and the length of the rectangle. Conventionally, the formula for the area of a rectangle is stated as ‘length times width’ or ‘length times breadth’.

Tan used inappropriate mathematical terms ‘horizontal side’ and ‘vertical side’ to state the formula for the area of a rectangle. He stated the formula as “…the horizontal side (refers to the length of the rectangle) times the vertical side (refers to the width of the rectangle)” (Tan/L1862-1863). Conventionally, the formula for the area of a rectangle is stated as ‘length times width’ or ‘length times breadth’.

Formula for the area of a parallelogram

In Task 8, five PSSMTs, namely Beng, Mazlan, Patrick, Suhana, and Tan used appropriate mathematical symbols to write the formula for the area of a parallelogram. Of the five PSSMTs who used appropriate mathematical symbols to write the formula for the area of a parallelogram, only one of them, namely Suhana, used appropriate mathematical terms '*tapak* [base]', 'times', and '*tinggi* [height]' to state the formula. She stated the formula for the area of a parallelogram as "…the *tapak* [base] times the *tinggi* [height]." (Suhana/L1585). Table 4.40 depicts PSSMTs who used appropriate and inappropriate mathematical terms to state the formula for the area of a parallelogram or to explain the meaning of the mathematical symbols that they employed to write the formula for the area of a parallelogram.

Table 4.40

PSSMTs who Used Appropriate and Inappropriate Mathematical Terms to State the Formula or to Explain the Meaning of the Mathematical Symbols That They Employed to Write the Formula for the Area of a Parallelogram

State the formula or explain the meaning of the mathematical symbols that they employed to write the formula for the area of a parallelogram	PSSMTs
Used appropriate mathematical terms	Suhana
Used inappropriate mathematical terms	Beng, Patrick, Tan

Two of the PSSMTs, namely Beng and Patrick, used inappropriate mathematical terms to explain the meaning of the symbols *a* and *b* that they employed. Beng used inappropriate mathematical terms 'the length here' and 'the line here perpendicular to *a*' to explain the meaning of the symbols *a* and *b* that she employed. Beng explained that "*a* is the length here and *b* is the line here perpendicular to *a*." (Beng/L1332). Patrick used inappropriate mathematical terms 'distance' and 'measurement' to explain the meaning of the mathematical symbols *a* and *b* that he employed. Patrick explained that "*a mewakili jarak, eh ukuran daripada AB dan b mewakili ukuran daripada BC.* …[*a* represents the distance, the measurement of *AB* and *b* represents the measurement of *BC*. …]" (Patrick/L1538-1539). Actually, *a* and *b* represents the base and the height of the parallelogram. Conventionally, the formula for the area of a parallelogram is written as '$b \times h$', where b and h represents the base and the height of the parallelogram.

Mazlan used appropriate mathematical term 'height' to explain the meaning of the mathematical symbol b that he employed. Mazlan explained that "Area, "a *darab* [times] b", height. …" (Mazlan/L1384). Nevertheless, Mazlan did not explain the meaning of the mathematical symbol '*a'* that he employed. Actually, '*a'* represents the base of the parallelogram. Tan used inappropriate mathematical terms 'vertical (side)' and 'horizontal (side)' to state the formula for the area of a parallelogram. He stated the formula as "…vertical (side) times horizontal (side)…" (Tan/L1877). Conventionally, the formula for the area of a parallelogram is stated as 'base times height'.

Three of the PSSMTs, namely Liana, Roslina, and Usha, could not recall the formula for the area of a parallelogram. Liana did not write the formula for the area of a parallelogram. Roslina thought that the formula for the area of a parallelogram is 'half times the height times the side bottom it' or $\frac{1}{2}$(h x b). The correct formula for the area of a parallelogram is "base times height or b x h", not $\frac{1}{2}$(h x b). When probed to try, Usha stated that the area of a parallelogram can be found by multiplying the lengths of two adjacent sides of the parallelogram. Thus, Usha wrote its area formula as 'xy', where x and y in her formula represents the slanted side and the base of the parallelogram.

Formula for the area of a triangle

In Task 8, seven of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, and Tan, used appropriate mathematical symbols to write the formula for the area of a triangle. Of the seven PSSMTs, four of them, namely Mazlan, Patrick, Roslina, and Suhana, used appropriate mathematical terms to state the formula for the area of a triangle. Mazlan used appropriate mathematical terms ‘half’, ‘base’ and ‘height’ to state the formula for the area of a triangle. Mazlan stated that “Triangle, ok half times base times height.” (Mazlan/L1398).

Patrick used appropriate mathematical terms ‘one over two’, ‘base’ and ‘height’ to state the formula for the area of a triangle. Patrick stated that ““‘…*satu perdua darab tapak,* base times height (points to the formula $\frac{1}{2} \times b \times h$). […one over two times base times height (points to the formula $\frac{1}{2} \times b \times h$).]” (Patrick/L1547-1548). Table 4.41 demonstrates PSSMTs who used appropriate and inappropriate mathematical terms to state the formula for the area of a triangle or to explain the meaning of the mathematical symbols that they employed to write the formula for the area of a triangle.

Table 4.41

PSSMTs who Used Appropriate and Inappropriate Mathematical Terms to State the Formula or to Explain the Meaning of the Mathematical Symbols That They Employed to Write the Formula for the Area of a Triangle

State the formula or explain the meaning of the mathematical symbols that they employed to write the formula for the area of a triangle	PSSMTs
Used appropriate mathematical terms	Beng, Liana, Mazlan, Patrick, Roslina, Suhana
Used inappropriate mathematical terms	Tan

Roslina used appropriate mathematical terms ‘half’, ‘times’, ‘height’, and ‘base’ to state the formula for the area of a triangle. Roslina stated that “…the formula is half times the height with the base.” (Roslina/L1402). Suhana used appropriate mathematical terms ‘half’, ‘times’, ‘*tinggi* [height]’, and ‘*tapak* [base]’ to state the formula for the area of a triangle. Suhana stated the formula as “…half times *tinggi* [height] times *tapak* [base].” (Suhana/L1594).

Two of the PSSMTs, namely Beng and Liana, used appropriate mathematical terms ‘base’ and ‘height’ to explain the meaning of the symbols that they employed. Beng used appropriate mathematical terms ‘base’ and ‘height’ to explain the meaning of the symbols that she employed. Beng explained that “*b* stands for the base. Then *t* stands for the height.” (Beng/L1341). Liana used appropriate mathematical terms ‘height’ and ‘base’ to explain the meaning of the symbols that she employed. Liana explained that “…the height will be *a* and the base will be *b*. …” (Liana/L1326-1327).

Tan used inappropriate mathematical terms ‘vertical (side)’ and ‘horizontal (side)’ to state the formula for the area of a triangle. He indicated the formula as half times the vertical (side) times the horizontal (side). Conventionally, the formula for the area of a triangle is stated as ‘half times base times height’. Usha could not recall the formula for the area of a triangle. Initially, she wrote the formula for the area of a triangle as $\frac{1}{2} y \times x$. Later, she shaded the half to become $y \times x$. Usha thought that the formula for the area of a triangle is ‘$y \times x$’, where x and y is the base and the height of the triangle.

Formula for the area of a trapezium

In Task 8, six of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, Tan, and Usha, used appropriate mathematical symbols to write the formula for the area of a trapezium. Of the six PSSMTs who used appropriate mathematical symbols to write the formula for the area of a trapezium, none of them used appropriate mathematical terms to state the formula for the area of a trapezium or to explain the meaning of the symbols that they employed in the formula. Beng and Patrick used inappropriate mathematical terms to explain the meaning of the symbols that they employed in the formula. Beng used inappropriate mathematical terms ‘upper side’ and ‘lower one but parallel to the upper one’, and appropriate mathematical term ‘height’ to explain the meaning of the symbols that she employed in the formula. Beng explained that “"a" stands for the upper side, "b" stands for lower one but parallel to the upper one and the "t" is the height.” (Beng/L1367-1368).

Patrick used appropriate mathematical terms ‘one over two’ and ‘height’ but inappropriate mathematical term ‘base’ to state the formula for the area of a trapezium. Patrick stated that "…one over two times "a", base plus opposite one times "h" (points to the formula $\frac{1}{2} \times (a + b) \times h$)." (Patrick/L1554-1555). Mazlan used appropriate mathematical terms ‘half’ and ‘height’ to state the formula for the area of a trapezium. Nevertheless, Mazlan did not explain the meaning of the mathematical symbols a and b that he employed. Mazlan stated that "Formula trapezium half of "a plus b" height." (Mazlan/L1416).

Suhana did not explain the meaning of the mathematical symbols $(a + b)$ that she employed. Actually, $(a + b)$ in the formula for the area of a trapezium represents the sum of the length of the parallel sides of the trapezium. Two of the PSSMTs, namely Tan and Usha, did not explain the meaning of the mathematical symbols that they employed. Conventionally, the formula for the area of a trapezium is stated as ‘half times the sum of the parallel sides times the height’.

Standard Unit of Length Measurement (Linear Units)

In Task 6.1, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 1) that involved composite figure, namely rectangle and parallelogram/triangle. Task 6.1 is shown in Figure 4.15. In Task 6.1, all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, used the correct standard unit of measurement for perimeter, namely cm, when they wrote the answer for this measurement of Diagram 1. It indicated that they understand the general measurement convention that perimeter is measured by linear unit.

In Task 6.2, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 2) that involved composite figure, namely square and trapezium/triangle. Task 6.2 is shown in Figure 4.16. In Task 6.2, six of the PSSMTs, namely Beng, Liana, Roslina, Suhana, Tan, and Usha, correctly wrote the measurement unit (without probed), namely mm, for the answer of the perimeter of Diagram 2 that they have calculated. Table 4.42 reveals the measurement unit for the answer of the perimeter of Diagram 2 written by PSSMTs.

The remaining two PSSMTs, namely Mazlan and Patrick, incorrectly wrote the measurement unit (without probed) for the answer of the perimeter of Diagram 2 as cm. They might have mixed up with the measurement unit for the answer of the perimeter of Diagram 1 in Task 6.1, namely cm. Nevertheless, they seem to understand the general measurement convention that perimeter is measured by linear units (such as mm, cm, m, km).

Table 4.42

The Measurement Unit for the Answer of the Perimeter of Diagram 2 Written by PSSMTs

Measurement unit for the answer of the perimeter of Diagram 2	PSSMTs
Correctly written as mm	Beng, Liana, Roslina, Suhana, Tan, Usha
Incorrectly written as cm	Mazlan, Patrick

Standard Unit of Area Measurement (Square Units)

In Task 6.1, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 1) that involved composite figure, namely rectangle and parallelogram/triangle. Task 6.1 is shown in Figure 4.15. In Task 6.1, all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, used the correct standard unit of measurement for area, namely cm^2, when they wrote the answer for this measurement of Diagram 1. It indicated that they understand the general measurement convention that area is measured by square unit.

In Task 6.2, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 2) that involved composite figure, namely square and trapezium/triangle. Task 6.2 is shown in Figure 4.16. In Task 6.2, seven of the PSSMTs, namely Beng, Liana, Patrick, Roslina, Suhana, Tan, and Usha, correctly wrote the measurement unit (without probed), namely mm^2, for the answer of the area of Diagram 2 that they have calculated. Table 4.43 reveals the measurement unit for the answer of the area of Diagram 2 written by PSSMTs.

Table 4.43

The Measurement Unit for the Answer of the Area of Diagram 2 Written by PSSMTs

Measurement unit for the answer of the area of Diagram 2	PSSMTs
Correctly written as mm^2	Beng, Liana, Patrick, Roslina, Suhana, Tan, Usha
Incorrectly written as cm^2	Mazlan

The remaining PSSMT, namely Mazlan, incorrectly wrote the measurement unit (without probed) for the answer of the area of Diagram 2 as cm^2. Patrick incorrectly wrote the measurement unit for the answer of area in the second method (alternative method) as cm^2. They might have mixed up with the measurement unit for the answer of the area of Diagram 1, namely cm^2. Nevertheless, they seem to understand the general measurement convention that area is measured by square units (such as mm^2, cm^2, m^2, km^2).

Conventions of Writing and Reading SI Area Measurement

In Task 3.4 (c), PSSMTs were asked to compare, from the measurements given, which shape in Sets 3 has the larger area. Task 3.4 (c) is shown in Figure 4.9. In Task 3.4 (c), four of the PSSMTs, namely Mazlan, Roslina, Suhana, and Usha, read and wrote the area measurement 16 cm^2 literally as ‘sixteen centimeter square’. Beng and Liana read and wrote 16 cm^2 literally as ‘sixteen centimetre square’ and ‘sixthteen centimetre square’ respectively. Patrick and Tan read and wrote 16 cm^2 literally as ’16 centimeter square’ and ‘sixteenth centimetre square’ respectively. Table 4.44 illustrates how PSSMTs read and wrote area measurements 16 cm^2 and 13 cm^2 in English.

Table 4.44
PSSMTs read and wrote area measurements 16 cm^2 and 13 cm^2 in English

Area measurements	Read and wrote as	PSSMTs
16 cm^2	Sixteen centimetre square	Beng
	Sixthteen centimetre square	Liana
	Sixteen centimeter square	Mazlan, Roslina, Suhana, Usha
	16 centimeter square	Patrick
	Sixteenth centimetre square	Tan
13 cm^2	Thirteen centimetre square	Beng
	Thirthteen centimetre square	Liana
	Thirdteen centimeter square	Mazlan, Usha
	13 centimeter square	Patrick
	Thirteen centimeter square	Roslina, Suhana
	Thirteenth centimetre square	Tan

Two of the PSSMTs, namely Mazlan and Usha, read and wrote 13 cm^2 literally as ‘thirdteen centimetre square’. Two of the PSSMTs, namely Roslina and Suhana, read and wrote 13 cm^2 literally as ‘thirteen centimeter square’. Beng and Liana read and wrote 13 cm^2 literally as ‘thirteen centimetre square’ and ‘thirthteen centimetre square’ respectively. Patrick and Tan read and wrote 13 cm^2 literally as ‘13 centimeter square’ and ‘thirteenth centimetre square’ respectively.

The above analysis reveals that none of the eight PSSMTs was able to read and write the area measurements 16 cm^2 and 13 cm^2 correctly in English. The correct answers should be ‘sixteen square centimetres’ and ‘thirteen square centimetres’ respectively. It indicated that all the eight PSSMTs did not know about the conventions pertaining to writing and reading of Standard International (SI) area measurement units.

Strategic Knowledge

In this section, findings of PSSMTs’ strategic knowledge of perimeter and area were presented in terms of its components. Table 4.45 reveals the components of strategic knowledge of perimeter and area.

Table 4.45

The Components of Strategic Knowledge of Perimeter and Area

Type of knowledge	Its components
Strategic knowledge	20. Strategies for comparing perimeter
	21. Strategies for comparing area
	22. Strategies for checking answer for perimeter
	23. Strategies for checking answer for area
	24. Strategies for solving the fencing problem
	25. Strategies for checking answer for the fencing problem
	26. Strategies for developing area formulae

Strategies for Comparing Perimeter

In Task 3.1, PSSMTs were asked to determine whether the given pair of shapes (T-shape and a rectangle) had the same perimeter. Figure 4.17 shows Task 3.1.

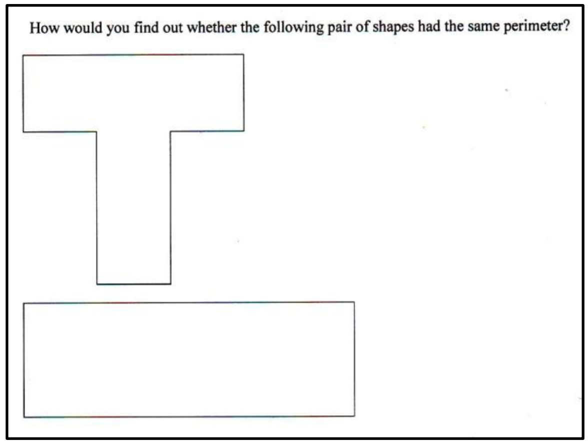


Figure 4.17. Task 3.1.

In Task 3.1, all the PSSMTs used the formal method to determine whether the given pair of shapes had the same perimeter. Table 4.46 shows the types of method used by PSSMTs to compare perimeter. Seven of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, and Usha, used the formal method of measuring the side by ruler and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter. Of the seven PSSMTs who used this method, six of them, namely Beng, Mazlan, Patrick, Roslina, Suhana, and Usha, measured the length of each side of the given T-shape by ruler and then calculated its perimeter. They also measured the length of each side of the given rectangle by ruler and then calculated its perimeter. Liana just measured the length of the top, the bottom, and the left sides of the T-shape by ruler. She doubled the length of the left sides of the T-shape and then plus the length of the top and the bottom parts of the T-shape to get its perimeter. Liana also just measured the length and the width of the rectangle by ruler. She doubled the length and the width of the rectangle to get its perimeter.

Table 4.46

The Types of Method Used by PSSMTs to Compare Perimeter

Types of method	PSSMTs
Measuring the side by ruler and applying definition of perimeter	Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha
Measuring the side by thread and ruler and applying definition of perimeter	Tan

Only one PSSMT, namely Tan, used the formal method of measuring the side by thread and ruler, and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter. He measured the length of each side of the T-shape by a piece of thread and then put it on a ruler to determine its total length (perimeter). Tan also measured the length of each side of the rectangle by a piece of thread and then put it on a ruler to determine its total length (perimeter).

When probed for alternative method of comparing the perimeter, six of the PSSMTs could suggest at least one alternative method to compare the perimeter. Table 4.47 depicts the types of alternative methods used by PSSMTs to compare perimeter.

Table 4.47

The Types of Alternative Methods Used by PSSMTs to Compare Perimeter

Types of methods		PSSMTs
Formal	Measuring the side by ruler and applying area formula	Beng
	Measuring the side by thread and ruler and applying definition of perimeter	Beng, Patrick, Roslina, Usha
	Measuring the side by compass and ruler and applying definition of perimeter	Patrick, Usha
	Measuring the side by paper and ruler and applying definition of perimeter	Patrick
Semi-formal	Measuring the side with a grid paper	Suhana, Tan, Usha
	Measuring the side with a blank paper	Suhana
	Measuring the side with a piece of thread	Suhana
Informal	Cut and paste	Suhana

Suhana suggested four alternative methods while Patrick and Usha suggested three alternative methods to compare the perimeter. Beng suggested two alternative methods while

Roslina and Tan suggested one alternative method to compare the perimeter. Liana and Mazlan could not suggest any alternative method to compare the perimeter or the alternative method was not accessible to them during the clinical interview.

The PSSMTs suggested three types of alternative methods to compare the perimeter, namely formal, semi-formal, and informal methods. Four subtypes of formal methods suggested by them were identified: (a) measuring the side by ruler and applying area formula, (b) measuring the side by thread and ruler and applying definition of perimeter, (c) measuring the side by compass and ruler and applying definition of perimeter, and (d) measuring the side by paper and ruler and applying definition of perimeter. Three subtypes of semi-formal methods were emerged: (a) measuring the side with a grid paper, (b) measuring the side with a blank paper, and (c) measuring the side with a piece of thread. Only one type of informal method was generated, namely cut and paste (i.e., cut one shape into pieces and paste onto the other shape).

The formal method of measuring the side by thread and ruler and applying definition of perimeter and the semi-formal method of using grid paper were the dominant alternative methods suggested by the PSSMTs. Four PSSMTs, namely Beng, Patrick, Roslina, and Usha, used the formal method of measuring the side by thread and ruler and applying definition of perimeter to compare the perimeter. They measured the length of each side of the T-shape by a piece of thread and then put it on a ruler to determine its total length (perimeter). Beng, Patrick, Roslina, and Usha also measured the length of each side of the rectangle by a piece of thread and then put it on a ruler to determine its total length (perimeter).

Three PSSMTs, namely Suhana, Tan, and Usha, used the semi-formal method of using grid paper to compare the perimeter. Suhana put the grid paper on the T-shape and then wrote the length of each side on the grid paper. She calculated the perimeter of the T-shape as 24 grids (it should be 24 cm). Suhana also put the grid paper on the rectangle and then wrote the length of each side on the grid paper. She also calculated the perimeter of the rectangle as 24 grids (it should be 24 cm). Suhana concluded that the given pair of shapes had the same perimeter.

Tan suggested to cut and paste the T-shape on the 1-cm grid paper and then counts the number of unit on each side. He also suggested to cut and paste the rectangle on the 1-cm grid paper and then counts the number of unit on each side. Tan explained that if both the given shapes had the same total length, then they had the same perimeter. Usha traced the rectangle on the 1-cm grid paper and then counted the number of unit on its length and width. She labelled its length and width and then calculated its perimeter as 24 cm. Usha also traced the T-shape on the 1-cm grid paper and then counted the number of unit on each side. She then calculated its perimeter as 24 cm.

Two PSSMTs, namely Patrick and Usha used the formal method of measuring the side by compass and ruler and applying definition of perimeter to compare the perimeter. They measured the length of each side of the T-shape by a compass and then put it on the ruler to determine its length. Patrick and Usha also measured the length of each side of the rectangle by a compass and then put it on the ruler to determine its length.

One PSSMT, namely Beng used the formal method of measuring the side by ruler and applying area formula to compare the perimeter. She partitioned the T-shape into two rectangles. Beng measured its length and widths by ruler respectively and then calculated its area using rectangle area formula. She also measured the length and width of the second diagram by ruler and then calculated its area using rectangle area formula. Beng explained that she measured the areas of each shape to determine whether they were equal and if they were, then the perimeter would be equal. Beng realized her mistake and concluded that this method did not work.

One PSSMT, namely Patrick used the formal method of measuring the side by paper and ruler and applying definition of perimeter to compare the perimeter. He took a piece of A4-sized white blank paper. Patrick marked the length of each side of the T-shape individually with a pencil on the longer side of the blank paper and then put it on the ruler to determine its total length (perimeter). He also marked the length of each side of the rectangle individually with a pencil on the opposite side of the longer side of the blank paper and then put it on the ruler to determine its total length (perimeter).

One PSSMT, namely Suhana used the semi-formal method of using a blank paper to compare the perimeter. She marked the length of each side of the T-shape on the length of the blank paper. Suhana repeated the same for the rectangle and sees whether it ended at the same point. Suhana found that there was a little bit of different of the total length for the T-shape and the rectangle. She thus concluded that this method of comparison was not accurate.

Suhana used another semi-formal method of using a piece of thread to compare the perimeter. She measured the length of each side of the T-shape by a piece of thread. Suhana marked with pencil, the length of each side, on the thread. She cut it and then used the same portion of the thread to measure the total length of the rectangle. Suhana concluded that the given pair of shapes had the same perimeter. Suhana also used the informal method of cut and paste to compare the perimeter. She used the scissors to cut the T-shape along its outline and then superimposed it on the rectangle. Suhana concluded that this method was not accurate as it has errors.

Strategies for Comparing Area

In Task 3.2, PSSMTs were asked to determine whether the given pair of shapes (L-shape and a square) had the same area. Figure 4.18 depicts Task 3.2.

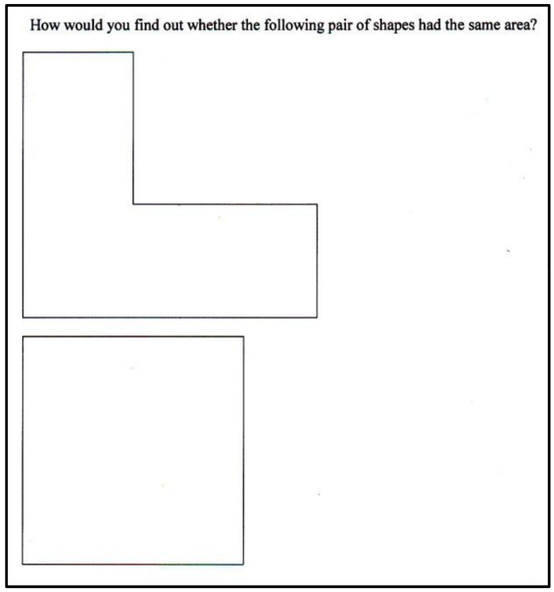


Figure 4.18. Task 3.2.

In Task 3.2, seven of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, and Usha, used the formal method of measuring the side by ruler and applying area formula to determine whether the given pair of shapes had the same area. They partitioned L-shape into two rectangles. Beng, Liana, Mazlan, Patrick, Roslina, Suhana, and Usha measured its lengths and widths by ruler respectively and then calculated its area using rectangle area formulae. They also measured the length of two adjacent sides of the square by ruler and then calculated its area using square area formula. Table 4.48 demonstrates the types of method used by PSSMTs to compare area.

Table 4.48

The Types of Method Used by PSSMTs to Compare Area

Types of method	PSSMTs
Measuring the side by ruler and applying area formula	Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha
Cut and paste	Tan

Only one PSSMT, namely Tan, used the informal method of cut and paste to determine whether the given pair of shapes had the same area. Tan suggested that he would cut the L-shape and then superimposed it on the square. He concluded that if the L-shape covered the square exactly, then they had the same area.

When probed for alternative method of comparing the area, seven of the PSSMTs could suggest at least one alternative method to compare the area. Table 4.49 reveals the types of the alternative methods used by PSSMTs to compare area.

Table 4.49

The Types of Alternative Methods Used by PSSMTs to Compare Area

Types of methods		PSSMTs
Formal	Measuring the side by ruler and applying area formula	Beng, Liana, Tan
	Measuring the side by thread and ruler and applying area formula	Usha
	Measuring the side by compass and ruler and applying area formula	Usha
Semi-formal	Covering both shapes with a grid paper	Patrick, Roslina, Suhana, Tan, Usha
Informal	Cut and paste	Suhana

Usha suggested three alternative methods to compare the area. Beng, Patrick, Suhana, and Tan suggested two alternative methods while Liana and Roslina suggested one alternative method to compare the area. Mazlan could not suggest any alternative method to compare the area or the alternative method was not accessible to him during the clinical interview.

The PSSMTs suggested three types of alternative methods to compare the area, namely formal, semi-formal, and informal methods. Three subtypes of formal methods suggested by them were identified: a) measuring the side by ruler and applying area formula, b) measuring the side by thread and ruler and applying area formula, c) measuring the side by compass and ruler and applying area formula. One type of semi-formal methods was emerged, namely covering both shapes with a grid paper. One type of informal method was generated, namely cut and paste (i.e., cut one shape into pieces and paste onto the other).

The semi-formal method of covering both shapes with a grid paper and the formal method of measuring the side by ruler and applying area formula were the dominant alternative methods suggested by the PSSMTs. Five PSSMTs, namely Patrick, Roslina, Suhana, Tan, and Usha, used the semi-formal method of covering both shapes with a grid paper to compare the area. Patrick, Roslina and Usha traced the L-shape on the 1-cm grid paper. They counted the number of 1-cm square covered by the L-shape. Patrick, Roslina and Usha also traced the square on the 1-cm grid paper. She counted the number of 1-cm square covered by the square.

Suhana traced the square on the 1-cm grid paper. She counted the number of 1-cm grid on the length of the two adjacent sides of the traced square and labelled its lengths as 6 respectively. Suhana multiplied the length of the two adjacent sides to get the area as 36 cm^2. She also traced the L-shape on the grid paper. Suhana partitioned the traced L-shape into two rectangles, labelled as "A" and "B" respectively. She counted the number of 1-cm grid on the length and the width of each rectangle and labelled its length and width as "7 and 3" and "5 and 3" respectively. Suhana multiplied the length and the width respectively to get the area as 21 and 15. She wrote its total area as 36 cm^2.

Tan traced the L-shape on the grid paper. He partitioned the traced L-shape into two rectangles. Tan suggested to count the number of 1-cm grid on the length and the width of each rectangle and then multiplied the length and the width respectively to get its area. He also would trace the square on the 1-cm grid paper. Tan would count the number of 1-cm grid on the length of the two adjacent sides of the traced square. He would multiply the length of the two adjacent sides to get its area. Tan explained that if they had the same value, then they have the same area.

When probed for other method of comparing the area, Patrick generated a similar semi-formal method of covering both shapes with a common grid and then counting the number of the grid required to cover the shapes. He used the scissors to cut a grid with the dimensions of 2 cm by 2 cm. Patrick counted the number of the grids required to cover the L-shape, namely 9 grids. Patrick explained that 9 times 4 cm^2 equal to 36 centimeter square (it should be 36 square centimetres). He also counted the number of the grids required to cover the square, namely 9 grids. Patrick wrote the area of the square as 36 cm^2. He concluded that they had the same area.

Three PSSMTs, namely Beng, Liana, and Tan, used the formal method of measuring the side by ruler and applying area formula to compare the area. Beng formed a large rectangle. She measured the lengths and width of the two rectangles by ruler respectively and then calculated the area of L-shape as the difference between the area of the large (labelled as A) and small (labelled as B) rectangles using area formula of rectangle. She partitioned the given square into two isosceles triangles, labelled as A and B respectively. Beng measured the lengths of each side of the square by ruler and then calculated the area of the given square as the total area of the two triangles using area formula of triangle.

Liana repartitioned the L-shape vertically into two rectangles, labelled as A and B respectively. She measured the length and the width of each rectangle by ruler and then calculated its area using rectangle area formulae. For the square, Liana suggested to use her previous method. Tan partitioned the L-shape horizontally into two rectangles. He suggested to measure the length and the width of each rectangle by ruler and then calculates its area using rectangle area formulae. Tan also suggested to measure the length of the two adjacent sides of the square by ruler and then calculates its area using square area formula. Tan explained that if they had the same value of the final outcome, then they have the same area.

When probed for other method of determining whether the given pair of shapes had the same area, Beng used another formal method of measuring the side by ruler and applying area formula to compare the area. She partitioned L-shape into two trapeziums, labelled as A and B. Beng measured the lengths of each side of the L-shape by ruler and then calculated the area of L-shape as the total area of the two trapeziums using area formula of trapezium. She also measured the lengths of two adjacent sides of the square and then calculated its area using area formula of square. Beng concluded that both the given shapes have the same area, namely 36 (without stating its unit of area measurement).

One PSSMT, namely Usha, used the formal method of measuring the side by thread and ruler and applying area formula to compare the area. She mentally partitioned the L-shape into two rectangles for which area measurement formulae were known. Usha measured the lengths and widths of the rectangles respectively by thread and then put it on the ruler to determine its length. She labelled its measurement on the respective sides and then calculated its area using area formula of a rectangle. Usha also measured the two adjacent lengths of the square respectively and then put it on the ruler to determine its length. She labelled its measurement on the respective sides and then calculated its area using area formula of a square.

Usha used another formal method of measuring the side by compass and ruler and applying area formula to compare the area. Usha suggested that she would measure the lengths and widths of the rectangles respectively by a compass, put it on the ruler to determine its length and then calculated its area using area formula of a rectangle. It indicated that Usha would also measure the two adjacent lengths of the square respectively by a compass, put it on the ruler to determine its length and then calculated its area using area formula of a square.

One PSSMT, namely Suhana, used the informal method of cut and paste (i.e., cut one shape into pieces and paste onto the other) to compare the area. She used the scissors to cut the L-shape along its outline and then superimposed it on the square. Suhana concluded that the area of the L-shape equal to area of the square.

Strategies for Checking Answer for Perimeter

In this subsection, findings of PSSMTs' strategic knowledge of checking answer for perimeter were presented in terms of: (a) checking answer for perimeter of Diagram 1, and (b) checking answer for perimeter of Diagram 2.

Checking Answer for Perimeter of Diagram 1

In Task 6.1, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 1) that involved composite figure, namely rectangle and parallelogram/triangle. Task 6.1 is shown in Figure 4.15.

In Task 6.1, when probed to check the answer for the perimeter of Diagram 1, seven of the PSSMTs, namely Beng, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, suggested that they would use the recalculating strategy to verify the answer. Beng, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha suggested that they would check the answer for perimeter by recalculating strategy that using the same method and calculate again. Table 4.50 exhibits the strategies suggested by PSSMTs to check the answer for the perimeter of Diagram 1.

Table 4.50

Strategies Suggested by PSSMTs to Check the Answer for the Perimeter of Diagram 1

Strategies suggested to check the answer for the perimeter of Diagram 1	PSSMTs
Recalculating strategy	Beng, Mazlan, Patrick, Roslina, Suhana, Tan, Usha
Alternative method	Liana

When probed to check the answer for the perimeter of Diagram 1, Liana suggested that she would use an alternative method to verify the answer. Liana used the doubling-and-sum algorithm to calculate the perimeter of the diagram. She doubled the length of sides UP, PQ, and QR and then summed them up to get the perimeter of the diagram as 104 cm. Liana suggested that she would check the answer for perimeter by using an alternative method, namely list all-and-sum strategy. Liana explained that she could just "15 plus 20 plus 17 plus 15 plus 17 plus 20" (Liana/L991-992) to check the answer for the perimeter.

Checking Answer for Perimeter of Diagram 2

In Task 6.2, subjects were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 2) that involved composite figure, namely square and trapezium/triangle. Task 6.2 is shown in Figure 4.16.

In Task 6.2, when probed to check the answer for the perimeter of Diagram 2, five of the PSSMTs, namely Mazlan, Patrick, Roslina, Suhana, and Usha, suggested that they would use the recalculating strategy to verify the answer. Mazlan, Patrick, Roslina, Suhana, and Usha suggested that they would check the answer for perimeter by recalculating strategy that using the same method and calculate again. Table 4.51 illustrates the strategies suggested by PSSMTs to check the answer for the perimeter of Diagram 2.

Table 4.51

Strategies Suggested by PSSMTs to Check the Answer for the Perimeter of Diagram 2

Strategies suggested to check the answer for the perimeter of Diagram 2	PSSMTs
Recalculating strategy	Mazlan, Patrick, Roslina, Suhana, Usha
Alternative method	Beng, Liana, Tan

When probed to check the answer for the perimeter of Diagram 1, the remaining three PSSMTs, namely Beng, Liana, and Tan, suggested that they would use alternative method to verify the answer. Beng suggested that she would use alternative method, namely the surround-and-measure strategy, to verify the answer. Beng suggested that she would check the answer for the perimeter by surrounding the diagram by thread and then measure its length by ruler. Liana suggested that she would check the answer for perimeter using an alternative method, namely list all-and-sum strategy. Tan suggested that he would use the exact measurement to draw Diagram 2 and then use a piece of thread and ruler to measure its perimeter.

Strategies for Checking Answer for Area

In this subsection, findings of PSSMTs' strategic knowledge of checking answer for perimeter were presented in terms of: (a) checking answer for area of Diagram 1, and (b) checking answer for area of Diagram 2.

Checking Answer for Area of Diagram 1

In Task 6.1, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 1) that involved composite figure, namely rectangle and parallelogram/triangle. Task 6.1 is shown in Figure 4.15.

In Task 6.1, when probed to check the answer for the area of Diagram 1, half of the PSSMTs, namely Mazlan, Roslina, Suhana, and Usha, suggested that they would use the recalculating strategy to verify the answer. Mazlan, Roslina, Suhana, and Usha suggested that they would check the answer for the area by the recalculating strategy that using the same method and calculate again. Table 4.52 shows the strategies suggested by PSSMTs to check the answer for the area of Diagram 1.

Table 4.52

Strategies Suggested by PSSMTs to Check the Answer for the Area of Diagram 1

Strategies suggested to check the answer for the area of Diagram 1	PSSMTs
Recalculating strategy	Mazlan, Roslina, Suhana, Usha
Alternative method	Beng, Liana, Patrick, Tan

When probed to check the answer for the area of Diagram 1, the other half of the PSSMTs, namely Beng, Liana, Patrick, and Tan, used an alternative procedure (alternative method) to generate an answer which could be used to verify their original answer. Beng, Liana, and Patrick used the partition-and-sum algorithm to calculate the area of the diagram. They partitioned Diagram 1 into a rectangle PQTU (labelled as A) and two triangles QRT (labelled as B) and RST (labelled as C). Beng, Liana, and Patrick calculated the area of A, B, and C using the area formulae of rectangle and triangles respectively and then summed them up to get the area of the diagram as 420 cm^2.

Beng, and Liana, checked the answer for area by moving triangle RST under the translation T_{SR} to form a rectangle with the dimensions of 15 cm by 8 cm. Beng drew a large rectangle with the dimension of 28 cm by 15 cm and calculated its area by using area formula of rectangle as 420 cm^2. Liana drew a large rectangle with the dimension of 28 cm by 15 cm. She partitioned the large transformed rectangle into two smaller rectangles with the dimensions of 20 cm by 15 cm and 8 cm by 15 cm and labelled them as A and B respectively. Liana calculated its area by using area formula of rectangle and summed up the area as 420 cm^2. Patrick checked the answer for area by moving triangle RST under the translation T_{SR} to form a rectangle (labelled as new "B") with the dimensions of 15 cm by 8 cm. He calculated area of "B" by using area formula of a rectangle, namely 15 x 8 = 120 cm^2. Patrick explained that both methods gave the same answer, namely 420 cm^2.

When probed to check the answer for the area, Tan used an alternative procedure (alternative method), namely partition-and-sum algorithm to generate an answer which could be used to verify his original answer. Tan used the "cut and paste" transformation to transform Diagram 1 into a "long" rectangle. He calculated the area of Diagram 1 as the area of the "long" rectangle formed using the area formula of a rectangle where its length and width is 28 cm and 15 cm respectively. Tan got the area of the diagram as 420 cm^2.

Tan checked the answer for area using the partition-and-sum algorithm to calculate the area of the diagram. He partitioned Diagram 1 into rectangle PQTU and parallelogram QRST. Tan calculated the area of the rectangle using the area formula of a rectangle as 300 cm^2. He calculated the area of the parallelogram using the area formula of a parallelogram as 120 cm^2. Tan then summed them up to get the area of the diagram as 420 cm^2. Tan explained that both methods gave the same answer, namely 420 cm^2.

Checking Answer for Area of Diagram 2

In Task 6.2, subjects were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 2) that involved composite figure, namely square and trapezium/triangle. Task 6.2 is shown in Figure 4.16.

In Task 6.2, when probed to check the answer for the area of Diagram 2, three of the PSSMTs, namely Roslina, Suhana, and Usha, suggested that they would use the recalculating strategy to verify the answer. Roslina, Suhana, and Usha suggested that they would check the answer for the area by the recalculating strategy that using the same method and calculate again. Table 4.53 depicts the strategies suggested by PSSMTs to check the answer for the area of Diagram 2.

Table 4.53

Strategies Suggested by PSSMTs to Check the Answer for the Area of Diagram 2

Strategies suggested to check the answer for the area of Diagram 2	PSSMTs
Recalculating strategy	Roslina, Suhana, Usha
Alternative method	Beng, Liana, Mazlan, Patrick, Tan

When probed to check the answer for the area of Diagram 2, the remaining five PSSMTs, namely Beng, Liana, Mazlan, Patrick, and Tan, used an alternative procedure (alternative method) to generate an answer which could be used to verify their original answer. Patrick and Tan used the repartition-and-sum strategy to check the answer for the area of Diagram 2 without being probed. Patrick repartitioned Diagram 2 into a large square (FGHI, labelled as A), a triangle (labelled as B), and a small square (labelled as C). He calculated the area of A, B, and C separately using the area formulae of a square, triangle, and square respectively and then summed them up to get the area of the diagram as 160 cm^2 (wrong unit. It should be mm^2). Tan repartitioned Diagram 2 into trapezium KFIJ and square FGHI. He calculated the area of the trapezium and square separately using the area formulae of a trapezium and square respectively and then summed them up to get the area of the diagram as 160 mm^2.

Liana suggested that she would use the repartition-and-sum strategy to check the answer for the area of Diagram 2 without being probed. Liana suggested that she would check the answer for the area by repartitioned Diagram 2 into trapezium FIJK and square FGHI. Nevertheless, Liana stated that she was unable to recall the formula for the area of a trapezium. When probed to check the answer for the area, Mazlan used the repartition-and-sum strategy to check the answer for the area of Diagram 2. Mazlan pointed out that the trapezium can be partitioned into a square and a triangle. He used this alternative method to calculate the area of trapezium FIJK as 60 (mm^2). When probed to check the answer for the area, Beng suggested that she would use an alternative method, namely superimpose method, to verify her original answer. Beng suggested that she would check the answer for the area by superimpose it with square units.

Strategies for Solving the Fencing Problem

In Task 7, PSSMTs were required to help his/her student to solve the fencing problem. Figure 4.19 demonstrates Task 7.

> Suppose that one of your students asks you for help with the following problem:
>
> A gardener has 84 m of fencing to enclose a garden along three sides, with the fourth side of the garden being formed by a wall. (Assume that the wall is perfectly straight). What are the dimensions of a rectangular garden that will yield the largest area being enclosed?
>
> How would you solve this problem?

Figure 4.19. Task 7.

In Task 7, half of the PSSMTs, namely Beng, Patrick, Suhana, and Tan, have successfully solving the fencing problem. Table 4.54 demonstrates PSSMTs who have successfully and unsuccessfully solving the fencing problem. The other half of the PSSMTs, namely Liana, Mazlan, Roslina, and Usha, have unsuccessfully solving the fencing problem.

Table 4.54

PSSMTs who Have Successfully and Unsuccessfully Solving the Fencing Problem

Solving the fencing problem	PSSMTs
Successful	Beng, Patrick, Suhana, Tan
Unsuccessful	Liana, Mazlan, Roslina, Usha

Of the four PSSMTs who have successfully solving the fencing problem, two of them, namely Beng and Patrick, used the looking for a pattern strategy to solve the fencing problem. Suhana used the trial-and-error strategy while Tan used the differentiation method to solve the fencing problem. Of the four PSSMTs who have unsuccessfully solving the fencing problem, two of them, namely Mazlan and Roslina, used the trial-and-error strategy to solve the fencing problem. Usha used the looking for a pattern strategy while Liana used the differentiation method to solve the fencing problem. Table 4.55 reveals the strategies used by PSSMTs to solve the fencing problem.

Table 4.55

Strategies Used by PSSMTs to Solve the Fencing Problem

Strategies used to solve the fencing problem	PSSMTs
Looking for a pattern	Beng, Patrick, Usha
Trial-and-error	Mazlan, Roslina, Suhana
Differentiation method	Liana, Tan

Beng and Patrick have successfully solving the fencing problem using the looking for a pattern strategy. Beng started off with the width and the length of the rectangular garden as 1 m and 82 m respectively and this yielded the smallest area being enclosed, namely 82 m^2. She then increased the width of the rectangular garden, one metre at a time, to 4 m and reduced the length of the rectangular garden accordingly to 76 m. Consequently, the area increased to 304 m^2. Beng saw a pattern that area increases as she increases the width of the rectangular garden while reduces its length accordingly. She increased the width of the rectangular garden to 10 m instead of 5m and reduced its length to 64 m. The area increased to 640 m^2. Subsequently, Beng took half of the 84 m of fencing as length of the rectangular garden and 21 m as its width. The area now increased to 882 m^2.

Beng attempted to verify whether 882 m^2 was the largest area being enclosed. She tested it with two values of the width that were smaller than 21 m, namely 9 m and 8 m respectively. Beng found that the area decreased to 594 m^2 and 544 m^2 respectively. Beng also tested it with two values of the width that were larger than 21 m, namely 22 m and 23 m respectively. Beng found that the area decreased to 880 m^2 and 874 m^2 respectively. Thus, Beng concluded that 882 m^2 is the largest area being enclosed and the dimension of the rectangular garden that yields the largest area being enclosed is 42 m by 21 m.

Patrick started off with the width and the length of the rectangular garden as 1 m and 82 m respectively and this yielded the smallest area being enclosed, namely 82 m^2. He then increased the width of the rectangular garden, one metre at a time, to 10 m and reduced the length of the rectangular garden accordingly to 64 m. Consequently, the area increased to 640 m^2. Patrick saw a pattern that area increases as he increases the width of the rectangular garden while reduces its length accordingly. He increased the width of the rectangular garden to 12 m instead of 11 m and reduced its length to 60 m. The area increased to 720 m^2.

Subsequently, Patrick increased the width of the rectangular garden, four metres at a time, to 20 m and reduced the length of the rectangular garden accordingly to 44 m. Consequently, the area increased to 880 m^2. He then increased the width of the rectangular garden, five metres at a time, to 30 m and reduced the length of the rectangular garden accordingly to 24 m. Consequently, the area decreased to 720 m^2. Patrick realized that the area of the rectangular garden decreasing when he increased the width of the rectangular garden, five metres at a time, from 20 m to 25 m and reduced the length of the rectangular garden accordingly from 44 m to 34 m. Consequently, the area decreased from 880 m^2 to 850 m^2.

Patrick became more cautious and he decided to increase the width of the rectangular garden, one metre at a time, from 20 m to 21 m and reduced the length of the rectangular garden accordingly from 44 m to 42 m. Consequently, the area increased from 880 m^2 to 882 m^2. Patrick continued to increase the width of the rectangular garden, one metre at a time, to 24 m and reduced the length of the rectangular garden accordingly to 36 m. Consequently, the area decreased to 864 m^2. Patrick concluded that 882 m^2 was the largest area being enclosed and 42 m by 21 m is the dimension of the rectangular garden that will yield the largest area being enclosed.

Usha has unsuccessfully solving the fencing problem using the looking for a pattern strategy. Usha started off with the length and the width of the rectangular garden as 70 m and 7 m respectively and this yielded the area being enclosed as 490 m^2. She then reduced the length of the rectangular garden, ten metres at a time, to 60 m and increased the width of the rectangular garden accordingly to 12 m. Consequently, the area increased to 720 m^2. Usha saw a pattern that area increases as she reduces the length of the rectangular garden while increases its width accordingly. Usha noticed that when she reduced the length of the rectangular garden to 50 m and increased its width to 17 m, the area increased to 850 m^2.

Usha initially thought that 50 m by 17 m was the dimension of the rectangular garden that will yield the largest area being enclosed. Subsequently, Usha realized her mistake when she reduced the length of the rectangular garden to 40 m and increased its width to 22 m. She found that the area increased to 880 m^2. Usha concluded that 880 m^2 was the largest area being enclosed. Usha justified her answer that 880 m^2 was the largest area being enclosed by reducing the length of the rectangular garden to 30 m and increasing its width to 27 m. She found that the area decreased to 810 m^2. Usha concluded that 40 m by 22 m was the dimension of the rectangular garden that will yield the largest area being enclosed, namely 880 m^2.

Usha did not aware that 880 m^2 was not the largest area being enclosed and 40 m by 22 m was not the dimension of the rectangular garden that will yield the largest area being enclosed. In fact, 882 m^2 is the largest area being enclosed and 42 m by 21 m is the dimension of the rectangular garden that will yield the largest area being enclosed.

Suhana has successfully solving the fencing problem using the trial-and-error strategy. Suhana drew a diagram to list down the possible factors of 84. Based on the list of factors of 84, she used the trial and error method to solve the fencing problem by identifying the factors that yield the largest area. In the first trial, Suhana viewed 7 as the sum of 2, 3, and 2, and drew a rectangle with the dimension of $2x$ by $3x$. She calculated the area of the rectangle as $2x \times 3x = 24 \times 36 = 864$, where $x = 12$. In the second trial, Suhana viewed 6 as the sum of 1, 4, and 1, and drew a rectangle with the dimension of x by $4x$. She calculated the area of the rectangle as $x \times 4x = 14 \times 56 = 784$, where $x = 14$.

In the third trial, Suhana viewed 4 as the sum of 1, 2, and 1, and drew a rectangle with the dimension of x by $2x$. She calculated the area of the rectangle as $x \times 2x = 21 \times 42 = 882$, where $x = 21$. In the fourth trial, Suhana viewed 7 as the sum of 3, 1, and 3, and drew a rectangle with the dimension of $3x$ by x. She calculated the area of the rectangle as $3x \times x = 36 \times 12 = 432$, where $x = 12$. Suhana compared the areas of the rectangular garden that she had calculated. Suhana indicated that 882 was the largest area among the areas that she had calculated, namely 864, 784, 882, and 432. Thus, Suhana concluded that 882 (m^2) is the largest area being enclosed. She stated that 42 (m) by 21 (m) is the dimension of the rectangular garden that will yield the largest area being enclosed.

Mazlan and Roslina have unsuccessfully solving the fencing problem using the trial-and-error strategy. Mazlan divided the 84 m of fencing into six equal parts, labelled as $6x$, where each part was 14 m, labelled as x. He thought that the length and the width of the rectangular garden was 28 m, labelled as $2x$, and 14 m, labelled as x, respectively. Mazlan calculated the area of the rectangular garden as 28 x 14 = 392 m^2. He thought that the largest area being enclosed was 392 m^2 and 28 m by 14 m was the dimension of the rectangular garden that will yield the largest area being enclosed. When probed to explain why 392 m^2 was the largest area being enclosed, Mazlan was unable to justify it. He just recalculated the area of the rectangular garden and reiterated that "*Em luas ini yang paling besar* [this is the largest area]." (Mazlan/L1309).

Mazlan did not aware that 392 m^2 was not the largest area being enclosed and 28 m by 14 m was not the dimension of the rectangular garden that will yield the largest area being enclosed. In fact, 882 m^2 is the largest area being enclosed and 42 m by 21 m is the dimension of the rectangular garden that will yield the largest area being enclosed.

Roslina drew a rectangular garden with the dimensions of 80 m by 2 m, labelled its dimensions and then calculated its area as 160 m^2. She assumed that the length and the width of the rectangular garden were 80 m and 2 m respectively. Roslina thought that the largest area being enclosed was 160 m^2 and 80 m by 2 m was the dimension of the rectangular garden that will yield the largest area being enclosed.

When probed to explain why 160 m^2 was the largest area being enclosed, Roslina was unable to justify it. She merely explained that 80 m by 2 m was her first choice. Roslina expressed that her other choice would be 82 m by 1 m. Roslina drew another rectangular garden with the dimensions of 82 m by 1 m, labelled its dimensions and then calculated its area as 80 m^2. Roslina reiterated that the largest area being enclosed was 160 m^2 and 80 m by 2 m was the dimension of the rectangular garden that will yield the largest area being enclosed.

Roslina did not aware that 160 m^2 was not the largest area being enclosed and 80 m by 2 m was not the dimension of the rectangular garden that will yield the largest area being enclosed. In fact, 882 m^2 is the largest area being enclosed and 42 m by 21 m is the dimension of the rectangular garden that will yield the largest area being enclosed.

Tan has successfully solving the fencing problem using the differentiation method. He wrote equation ①, namely $84 = 2x + y$, to represent the perimeter of the fencing. Tan wrote equation ②, namely $A = xy$, to represent the area of the rectangular garden. Tan explained that he needed to eliminate one of the variables, namely y, in order to find the derivative ($\frac{dA}{dx}$). Thus, Tan rewrote the equation ① as $y = 84 – 2x$ and labelled it as equation ③. He substituted $y = 84 – 2x$ into equation ② and simplified it as $A = 84x – 2x^2$. After differentiated with respect to x, Tan got the derivative $\frac{dA}{dx} = 84 – 4x$. At the stationary point, $\frac{dA}{dx} = 0$ and he got $x = 21$. Tan substituted the value of x into equation ① and got $y = 42$. Tan elaborated that he needed to find the value of $\frac{d^2A}{dx^2}$ at the stationary point. If $\frac{d^2A}{dx^2} < 0$, then the point is at a maximum. Tan found that $\frac{d^2A}{dx^2} = – 4 < 0$ and thus (21, 42) is a maximum point. Tan concluded that 882 m^2 was the largest area being enclosed and 42 (m) by 21 (m) was the dimension of the rectangular garden that will yield the largest area being enclosed.

Liana had attempted to use differentiation method to solve the fencing problem. Nevertheless, Liana has unsuccessfully solving the fencing problem as she was unable to recall the method. Liana wrote an equation, namely perimeter = $a + 2b$ and area = ab, to represent the perimeter and area of the rectangular garden respectively. She explained that the area for the rectangular garden is 'a times b' whereas the perimeter for the rectangular garden is 'a plus $2b$' as the fourth side of the garden being formed by a wall. She elaborated that $a + 2b = 84$. Liana encountered the problem of how to manipulate the equation of the area, namely area = ab. Liana knew that she needed to manipulate the two equations. Nevertheless, Liana was unable to proceed because she was unable to recall the differentiation method to solve the fencing problem.

Strategies for Checking Answer for the Fencing Problem

In Task 7, three of the PSSMTs, namely Beng, Patrick, and Usha, used the looking for a pattern strategy to check the answer for the fencing problem without being probed. Beng and Patrick attempted to verify whether 882 m^2 was the largest area being enclosed. Beng tested it with two values of the width that were smaller than 21 m, namely 9 m and 8 m respectively. She found that the area decreased to 594 m^2 and 544 m^2 respectively. Beng also tested it with two values of the width that were larger than 21 m, namely 22 m and 23 m respectively. She found that the area decreased to 880 m^2 and 874 m^2 respectively. Thus, Beng concluded that 882 m^2 is the largest area being enclosed and the dimension of the rectangular garden that yields the largest area being enclosed is 42 m by 21 m.

Patrick noticed that when he increased the width of the rectangular garden to 20 m and reduced its length to 44 m, the area increased to 880 m^2. Patrick found that when he further increased the width of the rectangular garden, one metre at a time, from 20 m to 25 m and reduced the length of the rectangular garden accordingly from 44 m to 34 m, he noticed that the area increased from 880 m^2 to 882 m^2, and then kept on decreasing from 882 m^2 to 850 m^2. Thus, Patrick pointed out that it reached its "climax" (the largest area being enclosed) when the width and the length of the rectangular garden is 21 m and 42 m respectively. Patrick reiterated that 882 m^2 was the largest area being enclosed and 42 m by 21 m is the dimension of the rectangular garden that will yield the largest area being enclosed.

Usha attempted to verify whether 880 m^2 was the largest area being enclosed. Usha noticed that when she reduced the length of the rectangular garden to 50 m and increased its width to 17 m, the area increased to 850 m^2. Usha found that the area increased to 880 m^2 when she reduced the length of the rectangular garden to 40 m and increased its width to 22 m. Usha also noticed that when she reduced the length of the rectangular garden to 30 m and increased its width to 27 m, the area decreased to 810 m^2. Thus, Usha concluded that 40 m by 22 m was the dimension of the rectangular garden that will yield the largest area being enclosed, namely 880 m^2. Table 4.56 exhibits the strategies used by PSSMTs to check the answer for the fencing problem.

Suhana used the compare strategy to verify the answer without being probed. She attempted to verify whether 882 m^2 was the largest area being enclosed. Suhana compared the areas of the rectangular garden that she had calculated. Suhana indicated that 882 was the largest area among the areas that she had calculated, namely 864, 784, 882, and 432. Thus, Suhana concluded that 882 (m^2) is the largest area being enclosed. She stated that 42 (m) by 21 (m) is the dimension of the rectangular garden that will yield the largest area being enclosed.

Table 4.56

Strategies Used by PSSMTs to Check the Answer for the Fencing Problem

Strategies used to check the answer for the fencing problem	PSSMTs
Looking for a pattern	Beng, Patrick, Usha
Trial-and-error	Mazlan
Compare	Suhana
List all-and-compare	Roslina
Calculating the value of $\frac{d^2A}{dx^2}$ at the stationary point	Tan

Tan checked the answer of the fencing problem, without being probed, by calculating the value of $\frac{d^2A}{dx^2}$ at the stationary point. Tan found that $\frac{d^2A}{dx^2} = -4 < 0$ and thus (21, 42) is a maximum point. Tan concluded that 882 m^2 was the largest area being enclosed and 42 (m) by 21 (m) was the dimension of the rectangular garden that will yield the largest area being enclosed.

When probed to check the answer for the fencing problem, Mazlan used the same strategy, namely trial and error strategy, to verify the answer. Mazlan explained that 84 minus 4 equal to 80 and 80 divided by 2 equal to 40. The remaining 4 divided by 2 equal to 2. Mazlan took 40 and 2 as the length and the width of the rectangular garden respectively. He found that the area of the rectangular garden was 80 m^2 and reiterated that 392 m^2 was the largest area being enclosed.

When probed to check the answer for the fencing problem, Roslina suggested that she would use the list all-and-compare strategy, to verify the answer. Roslina explained that she would list all the possible answers and then compare them. Roslina emphasized that the total length has to be 84 m of fencing. Roslina concluded that the dimension of 80 m by 2 m will yield the largest area being enclosed.

Strategies for Developing Area Formulae

In this subsection, findings of PSSMTs' strategic knowledge of developing area formulae were presented in terms of: (a) strategies for developing area formula for a rectangle, (b) strategies for developing area formula for a parallelogram, (c) strategies for developing area formula for a triangle, and (d) strategies for developing area formula for a trapezium.

Strategies for Developing Area Formula for a Rectangle

In Task 8, PSSMTs were asked to show a Form One student the way to develop (derive) area formulae of a rectangle, parallelogram, triangle, and trapezium. Task 8 is shown in Figure 4.14. In Task 8, only one PSSMT, namely Tan, attempted to develop the formula for the area of a rectangle but unsuccessful.

Strategies for Developing Area Formula for a Parallelogram

In Task 8, five of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, and Tan, had succeeded in developing the formula for the area of a parallelogram. They used the cut and paste strategy to develop the formula. Beng, Mazlan, Patrick, and Tan mentally cut out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle. Suhana mentally cut the parallelogram into two triangles along its diagonal and then she labelled the triangles as "I" and "K". Suhana mentally moved triangle "I" from one end of the parallelogram to the other end of the parallelogram to form a rectangle and wrote its area formula as $a \times b$.

Strategies for Developing Area Formula for a Triangle

In Task 8, three of the PSSMTs, namely Liana, Suhana, and Tan, had attempted to develop the formula for the area of a triangle. They used the partition strategy to develop the formula. Of the three, two of them, namely Liana and Tan, had succeeded in developing the formula for the area of a triangle. Liana developed the formula for the area of a triangle based on the formula for the area of a square. She explained that a square can be partitioned into two triangles and thus there is a half in the formula for the area of a triangle. Liana stated that the formula for the area of a square is '$a \times b$', where a and b represents the height and the base of the square. She then wrote the formula for the area of a triangle as '$\frac{1}{2} \times a \times b$'.

Tan developed the formula for the area of a triangle from the formula for the area of a rectangle. He stated that the formula for the area of a rectangle is 'the vertical (side) times the horizontal (side)'. Tan mentally cut a rectangle diagonally and then took out a right-angled triangle. He emphasized that it needed to times half in order to get the area of a triangle, namely 'half times the vertical (side) times the horizontal (side).

When probed to develop the formula, Suhana attempted to develop the formula but unsuccessful. She mentally cut an isosceles triangle along its symmetrical line and then rearranged it to be a rectangle. Suhana drew a rectangle and wrote its area formula as '$a \times b$'. When probed further to develop the formula, she mentally cut the rectangle diagonally and then rearranged it to be an isosceles triangle. Suhana drew another triangle (isosceles triangle) and wrote its area formula as '*tinggi* [height] × *tapak* [base] × $\frac{1}{2}$'. When the researcher asked how she got that formula, Suhana expressed that the formula was just like that and she just memorized the formula.

Strategies for Developing Area Formula for a Trapezium

In Task 8, five of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, and Tan, had attempted to develop the formula for the area of a trapezium using algebraic method. Of the five PSSMTs, three of them, namely Beng, Suhana, and Tan, succeeded in developing the formula for the area of a trapezium. Beng used algebraic method to develop the formula for the area of a trapezium. She drew dotted lines on the trapezium to form a large rectangle and viewed the area of the trapezium as the different between the area of the large rectangle formed and the area of the triangle formed. Thus, the area of the trapezium equals to 'b x t $-\frac{1}{2}$ (b – a) x t'. Beng simplified it algebraically to become '$\frac{1}{2}\ (a+b) \times t$'.

Suhana used algebraic method to develop the formula for the area of a trapezium from the combination of the formulae for the area of a rectangle and a triangle, namely (a × *tinggi* [*height*]) + [(b – a) × *tinggi* [*height*] × $\frac{1}{2}$]. In the second attempt, she correctly simplified (a × *tinggi* [*height*]) + [(b – a) × *tinggi* [*height*] × $\frac{1}{2}$] as $\frac{1}{2}$ × *tinggi* [*height*] × (a + b), which is the formula for the area of a trapezium.

Tan had also successfully developed the formula for the area of a trapezium using algebraic method. He developed the formula for the area of a trapezium using the combination of the formula for the area of a triangle and a rectangle or a square. Tan wrote the formula for the total area of a rectangle or a square, and a triangle as '(AB × AC) + ($\frac{1}{2} \times BE\ \times ED$)'. He then used the algebraic method to simplified it as '$\frac{1}{2}\ AC\ (AB + CD)$' which is the formula for the area of a trapezium.

Mazlan and Patrick attempted to develop the formula for the area of a trapezium using algebraic method but unsuccessful. Mazlan partitioned the trapezium into a triangle and a rectangle, and circled the triangle. He incorrectly wrote the formula for the area of a triangle as '$\frac{1}{2}(b \times h)'$ (it should be ' $\frac{1}{2}(b - a)h'$). Mazlan also incorrectly wrote the formula for the area of a rectangle as '$a \times b'$ (it should be '$a \times h$'). Consequently, he simplified them algebraically to become '$\frac{1}{2}bh + ab$' which was not equal to the formula for the area of a trapezium, namely '$\frac{1}{2}(a + b)h$'.

Patrick explained that the area of a trapezium can be calculated using the formula for the area of a trapezium itself or using the combination of the formula for the area of a rectangle, namely $a \times b$ (wrong formula. It should be $a \times h$), and a triangle, namely $\frac{1}{2} \times (b - a) \times h$). He moved his hand to indicate that $a \times b$ (wrong formula. It should be $a \times h$) + $\frac{1}{2} \times (b - a) \times h$) equals to $\frac{1}{2} \times (a + b) \times h$). Patrick was unable to show how $a \times b$ (wrong formula. It should be $a \times h$) + $\frac{1}{2} \times (b - a) \times h$) could be simplified as $\frac{1}{2} \times (a + b) \times h$). He moved his head to indicate that he has no idea how to develop the formula.

Ethical Knowledge

In this section, findings of PSSMTs' ethical knowledge of perimeter and area were presented in terms of its components. Table 4.57 exhibits the components of ethical knowledge of perimeter and area.

Table 4.57

The Components of Ethical Knowledge of Perimeter and Area

Type of knowledge	Its components
Ethical knowledge	27. Justifies one's mathematical ideas 28. Examines pattern within the domain of perimeter and area measurement 29. Formulates generalization within the domain of perimeter and area measurement 30. Tests generalization within the domain of perimeter and area measurement 31. Develops area formulae 32. Writes units of measurement upon completed a task 33. Checks the correctness of their solutions or answers

Justifies One's Mathematical Ideas

In this subsection, findings of PSSMTs' ethical knowledge of justifies one's mathematical ideas were presented in terms of: (a) whether they justify the selection of shapes that have a perimeter, (b) whether they justify the selection of shapes that do not have a perimeter, (c) whether they justify the selection of shapes that have an area, (d) whether they justify the selection of shapes that do not have an area, (e) whether they justify the shapes that can be used as the unit of area, and (f) whether they justify the shapes that they thought cannot be used as the unit of area.

Justify the Selection of Shapes That Have a Perimeter

In Task 1.1, PSSMTs were asked to select the shapes that have a perimeter. Task 1.1 is shown in Figure 4.1. Knowledge and justification of knowledge is an important aspect in any discipline. In Task 1.1, all the PSSMTs had taken the effort to justify the selection of shapes that have a perimeter. All the PSSMTs provided appropriate justification for selecting shapes "A" and "C" that have a perimeter, as shown in Table 4.33. All the PSSMTs who had selected shapes "D", "F" and "J", "H", and "I" and "K" that have a perimeter provided appropriate justification for their selection, except Liana. She had provided inappropriate justification for selecting shapes "D", "F" and "J", "H", and "I" and "K" that have a perimeter.

Justify the Selection of Shapes That Do Not Have a Perimeter

In Task 1.1, all the PSSMTs also had taken the effort to provide justification for not selecting other shapes as having a perimeter. Five PSSMTs, namely Beng, Mazlan, Roslina, Suhana, and Usha, provided appropriate justification for not selecting shape "B" as having a perimeter whereas three PSSMTs, namely Liana, Patrick, and Tan provided inappropriate justification for not selecting shape “B” as having a perimeter, as shown in Table 4.34.

All the PSSMTs selected shapes “D”, “I”, and “K” that have a perimeter, except Mazlan. He provided inappropriate justification for not selecting shapes “D”, “I”, and “K” that have a perimeter. All the PSSMTs provided appropriate justification for not selecting shape “E” as having a perimeter, except Tan. He provided inappropriate justification for not selecting shape “E” as having a perimeter. All the Four PSSMTs, namely Beng, Mazlan, Patrick, and Suhana, who did not select shapes “F” and “J” that have a perimeter provided inappropriate justification for not selecting shapes “F” and “J”.

Four PSSMTs, namely Beng, Mazlan, Roslina, and Usha, provided appropriate justification for not selecting shape “G” as having a perimeter whereas the remaining four PSSMTs, namely Liana, Patrick, Suhana, and Tan, provided inappropriate justification for not selecting shape “G” as having a perimeter. Five PSSMTs, namely Beng, Mazlan, Roslina, Suhana, and Usha, provided appropriate justification for not selecting shape “L” as having a perimeter whereas the remaining three PSSMTs, namely Liana, Patrick, and Tan, provided inappropriate justification for not selecting shape “L” as having a perimeter.

Justify the Selection of Shapes That Have an Area

In Task 1.2, PSSMTs were asked to select the shapes that have an area. Task 1.2 is shown in Figure 4.2. In Task 1.2, all the PSSMTs had taken the effort to justify the selection of shapes that have an area. All the PSSMTs who had selected shapes “A”, “C”, “D”, “F”, “H”, “I”, “J”, and “K” that have an area provided appropriate justification for their selection, except Liana. She had provided inappropriate justification for selecting shapes “A”, “C”, “D”, “F”, “H”, “I”, “J”, and “K” that have an area, as shown in Table 4.35.

Justify the Selection of Shapes That Do Not Have an Area

In Task 1.2, all the PSSMTs also had taken the effort to provide justification for not selecting other shapes as having an area. All the PSSMTs provided appropriate justification for not selecting shape "B" as having an area, except Liana. She had provided inappropriate justification for not selecting shape "B" as having an area, as shown in Table 4.36. All the PSSMTs selected shapes "D", "I", and "K" that have an area, except Patrick and Roslina. Patrick and Roslina provided inappropriate justification for not selecting shapes "D", "I", and "K" that have an area. All the PSSMTs provided appropriate justification for not selecting shape "E" as having an area, except Patrick. He had provided inappropriate justification for not selecting shape "E" as having an area.

All the PSSMTs selected shapes "F" and "J" that have an area, except Beng. She provided inappropriate justification for not selecting shapes "F" and "J" that have an area. Six PSSMTs, namely Beng, Mazlan, Roslina, Suhana, Tan, and Usha, provided appropriate justification for not selecting shapes "G" and "L" as having an area whereas the remaining two PSSMTs, namely Liana and Patrick, provided inappropriate justification for not selecting shapes "G" and "L" as having an area.

Justify the Shapes That Can Be Used As the Unit of Area

In Task 2, PSSMTs were asked to respond to a scenario where three students were discussing about the units of area. Task 2 is shown in Figure 4.3. In Task 2, all the PSSMTs had taken the effort to justify the shapes that can be used as a unit of area measurement. Seven of the PSSMTs, namely Beng, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha stated that a square can be used as the unit of area. Nevertheless, only one of them, namely Patrick, provided appropriate justification for selecting a square that can be used as the unit of area. The other six PSSMTs, namely Beng, Mazlan, Roslina, Suhana, Tan, and Usha, provided inappropriate justification for selecting a square that can be used as the unit of area.

Four of the PSSMTs, namely Beng, Patrick, Tan, and Usha, stated that a rectangle can be used as the unit of area. Nevertheless, only one of them, namely Patrick, provided appropriate justification for selecting a rectangle that can be used as the unit of area. The other three PSSMTs, namely Beng, Tan, and Usha, provided inappropriate justification for selecting a rectangle that can be used as the unit of area. Four of the PSSMTs, namely Beng, Mazlan, Tan, and Usha, stated that a triangle can be used as the unit of area. Nevertheless, none of them provided appropriate justification for selecting a triangle that can be used as the unit of area.

Justify the Shapes That They Thought Cannot Be Used As the Unit of Area

In Task 2, all the PSSMTs also had taken the effort to justify the shapes that they thought cannot be used as a unit of area measurement. Only one PSSMT, namely Liana, thought that a square cannot be used as a unit of area measurement. She provided inappropriate justification for not selecting a square that can be used as the unit of area. Four of the PSSMTs, namely Liana, Mazlan, Roslina, and Suhana, thought that a rectangle cannot be used as a unit of area measurement. They provided inappropriate justification for not selecting a rectangle that can be used as the unit of area. Three of the PSSMTs, namely Liana, Roslina, and Suhana, thought that a triangle cannot be used as a unit of area measurement. They provided inappropriate justification for not selecting a triangle that can be used as the unit of area.

Examines Pattern

In this subsection, findings of PSSMTs' ethical knowledge of examine the possible pattern of the relationship between perimeter and area were presented in terms of: (a) whether they examine the possible pattern that two shapes with the same perimeter have the same area?, and (b) whether they examine the possible pattern that the garden with the longer perimeter has the larger area?

Two Shapes With The Same Perimeter Have The Same Area?

In Task 5.1, a Form One student claimed that he found a way to calculate the area of a leaf. The student placed a piece of thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle. PSSMTs were asked how they would respond to this student. Task 5.1 is shown in Figure 4.11.

In Task 5.1, only two of the PSSMTs, namely Roslina and Suhana, had attempted to examine the possible pattern of the relationship between perimeter and area. Table 4.58 shows PSSMTs who had and had not attempted to examine the possible pattern of the relationship between perimeter and area in Task 5.1.

Table 4.58

PSSMTs who had and had not Attempted to Examine the Possible Pattern of the Relationship Between Perimeter and Area in Task 5.1

Examine the possible pattern of the relationship between perimeter and areain Task 5.1	PSSMTs
Attempted	Roslina, Suhana
Did not attempt	Beng, Liana, Mazlan, Patrick, Tan, Usha

Suhana knew that the student's method of calculating the area of the leaf was not correct. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter have the same area. She had attempted to examine the possible pattern of the relationship between perimeter and area by generated a counterexample that enable her to compare the area of the leaf and rectangle that had the same perimeter. Suhana found that the area of the rectangle, namely 30 cm^2, is not the same as the area of the leaf which is 23 cm^2 even though they had the same perimeter as 22 cm. Thus, Suhana concluded that the student's method of calculating the area of the leaf was not correct.

Roslina thought that the student's method of calculating the area of the leaf was correct. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter have the same area. She had attempted to examine the possible pattern of the relationship between perimeter and area by placing a piece of thread around the boundary of the leaf and got the perimeter of the leaf as 24 cm. Roslina drew a rectangle, labelled its dimensions as 8 cm by 4 cm based on the perimeter of the leaf, namely 24 cm, and then calculated its area as 32 cm^2. Based on this example, she thought that the student's generalization that two shapes with the same perimeter have the same area was correct.

The remaining six PSSMTs, namely Beng, Liana, Mazlan, Patrick, Tan, and Usha, did not attempt to examine the possible pattern of the relationship between perimeter and area. Mazlan, Patrick, Tan, and Usha thought that the student's method of calculating the area of the leaf was correct. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter have the same area. They accepted the student's generalization without attempting to examine the possible pattern of the relationship between perimeter and area. Beng and Liana was not sure whether the student's method of calculating the area of the leaf was correct or not. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter have the same area. They accepted the student's generalization without attempting to examine the possible pattern of the relationship between perimeter and area.

The Garden With The Longer Perimeter Has The Larger Area?

In Task 5.2, PSSMTs were asked to respond to a student, Mary, who claimed that she could determine whose garden has the larger area to plant flowers. Mary claimed that the garden with the longer perimeter has the larger area. Task 5.2 is shown in Figure 4.12.

In Task 5.2, five of the PSSMTs, namely Beng, Patrick, Roslina, Suhana, and Usha, had attempted to examine the possible pattern of the relationship between perimeter and area. Table 4.59 depicts PSSMTs who had and had not attempted to examine the possible pattern of the relationship between perimeter and area in Task 5.2.

Table 4.59

PSSMTs who had and had not Attempted to Examine the Possible Pattern of the Relationship Between Perimeter and Area in Task 5.2

Examine the possible pattern of the relationship between perimeter and area in Task 5.2	PSSMTs
Attempted	Beng, Patrick, Roslina, Suhana, Usha
Did not attempt	Liana, Mazlan, Tan

Beng and Suhana knew that Mary's claim was not correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Beng made a reflection on Task 3.1 when she approached Task 5.2. From the reflection, she realized that the shape with the longer perimeter may have a smaller area. Beng knew that the garden with the longer perimeter could have a smaller area. Beng attempted to examine the possible pattern of the relationship between perimeter and area. Based on her reflection on Task 3.1, she found that the shape with the longer perimeter may have a smaller area. Beng explained that Mary's method did not work for the situation in Task 5.2 as these two gardens are of different shape. She stated that Mary's claim is true only when we are comparing the area of two similar shapes (same shape but different area).

Suhana indicated that Mary's method was not correct because it did not apply to all shapes. Suhana stated that Mary came to the conclusion just based on this situation and was just by luck. Suhana attempted to examine the possible pattern of the relationship between perimeter and area. She had generated two examples to examine the possible pattern of the relationship between perimeter and area. Nevertheless, Suhana found that both of her examples concurred with Mary's generalization that the garden with the longer perimeter has the larger area.

Patrick, Roslina, and Usha thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Patrick attempted to examine the possible pattern of the relationship between perimeter and area. He generated an example where he drew two rectangles, labelled its dimensions, and then calculated its perimeter and area. Patrick found that rectangle A with the longer perimeter (22 cm) has the larger area (30 cm^2) compared to rectangle B with the perimeter of 18 cm and the area of 20 cm^2.

Roslina generated an example to examine the possible pattern of the relationship between perimeter and area. She assumed that the perimeter of Mary's and Sarah's gardens were 24 cm and 12 cm respectively. Roslina used the thread method in the previous task, Task 5.1, to transform the gardens into two rectangles. She drew two rectangles to represent these gardens. Roslina labelled its dimensions and then calculated its area as 32 cm^2 and 8 cm^2. Thus, she concluded that (the garden with the) longer perimeter (24 cm) has the larger area (32 cm^2). The example that Roslina generated concurred with Mary's claim that the garden with the longer perimeter has the larger area.

Usha generated an example to examine the possible pattern of the relationship between perimeter and area. She drew two rectangles with the perimeters of 24 cm and 26 cm respectively. Usha labelled its dimensions as 10 (cm) by 2 (cm) and 10 (cm) by 3 (cm) respectively and then calculates its area as 20 cm^2 and 30 cm^2 respectively. The example generated by Usha showed that the rectangle with the longer perimeter has the larger area. Thus, Usha concluded that the longer the perimeter of a shape, the larger the area of the shape.

The remaining three PSSMTs, namely Liana, Mazlan, and Tan, did not attempt to examine the possible pattern of the relationship between perimeter and area. Mazlan and Tan thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. They accepted the Mary's generalization without attempting to examine the possible pattern of the relationship between perimeter and area.

Liana was not sure whether Mary's claim that the garden with the longer perimeter has the larger area was correct or not. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. She stated that one cannot simply say whether the method works or not. Liana expressed that she has to do some research (to verify it). Nevertheless, Liana accepted Mary's generalization without attempting to examine the possible pattern of the relationship between perimeter and area.

Formulates Generalization

In this subsection, findings of PSSMTs' ethical knowledge of formulate generalization pertaining to the relationship between perimeter and area were presented in terms of: (a) whether they formulate generalization that two shapes with the same perimeter have the same area?, and (b) whether they formulate generalization that the garden with the longer perimeter has the larger area?

Formulate Generalization That Two Shapes With The Same Perimeter Have The Same Area?

In Task 5.1, a Form One student claimed that he found a way to calculate the area of a leaf. The student placed a piece of thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle. PSSMTs were asked how they would respond to this student. Task 5.1 is shown in Figure 4.11.

In Task 5.1, only two of the PSSMTs, namely Roslina and Suhana, had attempted to formulate generalization pertaining to the relationship between perimeter and area. Table 4.60 demonstrates PSSMTs who had and had not attempted to formulate generalization pertaining to the relationship between perimeter and area in Task 5.1.

Table 4.60

PSSMTs who had and had not Attempted to Formulate Generalization Pertaining to the Relationship Between Perimeter and Area in Task 5.1

Formulate generalization pertaining to the relationship between perimeter and area in Task 5.1	PSSMTs
Attempted	Roslina, Suhana
Did not attempt	Beng, Liana,Mazlan, Patrick, Tan, Usha

Based on her counterexample, Suhana knew that the student's generalization that two shapes with the same perimeter have the same area was not correct. Thus, she formulated a generalization pertaining to the relationship between perimeter and area that two shapes with the same perimeter may have the different area. Based on her example, Roslina thought that the student's generalization that two shapes with the same perimeter have the same area was correct. Thus, she formulated a generalization pertaining to the relationship between perimeter and area that two shapes with the same perimeter have the same area that concurred with the student's generalization.

The remaining six PSSMTs, namely Beng, Liana, Mazlan, Patrick, Tan, and Usha, did not attempt to formulate generalization pertaining to the relationship between perimeter and area.

Mazlan, Patrick, Tan, and Usha thought that the student's method of calculating the area of the leaf was correct. They thought that the student's generalization that two shapes with the same perimeter have the same area was correct. Thus, Mazlan, Patrick, Tan, and Usha did not attempt to formulate generalization pertaining to the relationship between perimeter and area. Beng and Liana were not sure whether the student's method of calculating the area of the leaf was correct or not. They were not sure whether the student's generalization that two shapes with the same perimeter have the same area was correct or not. Thus, Beng and Liana did not attempt to formulate generalization pertaining to the relationship between perimeter and area.

Formulate Generalization That the Garden
With The Longer Perimeter Has The Larger Area?

In Task 5.2, PSSMTs were asked to respond to a student, Mary, who claimed that she could determine whose garden has the larger area to plant flowers. Mary claimed that the garden with the longer perimeter has the larger area. Task 5.2 is shown in Figure 4.12.

In Task 5.2, five of the PSSMTs, namely Beng, Patrick, Roslina, Suhana, and Usha, had attempted to formulate generalization pertaining to the relationship between perimeter and area. Table 4.61 reveals PSSMTs who had and had not attempted to formulate generalization pertaining to the relationship between perimeter and area in Task 5.2.

Table 4.61

PSSMTs who had and had not Attempted to Formulate Generalization Pertaining to the Relationship Between Perimeter and Area in Task 5.2

Formulate generalization pertaining to the relationship between perimeter and area in Task 5.2	PSSMTs
Attempted	Beng, Patrick, Roslina, Suhana, Usha
Did not attempt	Liana, Mazlan, Tan

Beng and Suhana knew that Mary's claim was not correct. Based on her reflection on Task 3.1, Beng found that the shape with the longer perimeter may have a smaller area. Thus, she formulated a generalization that the garden with the longer perimeter could have a smaller area. Suhana concluded that the shape with the longer perimeter does not necessarily have the larger area. She explained that sometimes the shape with the shorter perimeter has larger area too compared to the shape with the longer perimeter. It indicated that she formulated a generalization that the garden with the longer perimeter could have a smaller area. Patrick, Roslina, and Usha thought that Mary's claim was correct. Thus, they formulated a generalization pertaining to the relationship between perimeter and area that the garden with the longer perimeter has the larger area that concurred with Mary's generalization.

The remaining three PSSMTs, namely Liana, Mazlan, and Tan, did not attempt to formulate generalization pertaining to the relationship between perimeter and area. Mazlan and Tan thought that Mary's claim that the garden with the longer perimeter has the larger area was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Thus, Mazlan and Tan did not attempt to formulate generalization pertaining to the relationship between perimeter and area. Liana was not sure whether Mary's claim that the garden with the longer perimeter has the larger area was correct or not. Thus, Liana did not formulate generalization pertaining to the relationship between perimeter and area.

Tests Generalization

In this subsection, findings of PSSMTs' ethical knowledge of test generalization pertaining to the relationship between perimeter and area were presented in terms of: (a) whether they test generalization that two shapes with the same perimeter have the same area?, (b) whether they test generalization that the garden with the longer perimeter has the larger area?, and (c) whether they test generalization that as the perimeter of a closed figure increases, the area also increases?

Test Generalization That Two Shapes With The Same Perimeter Have The Same Area?

In Task 5.1, a Form One student claimed that he found a way to calculate the area of a leaf. The student placed a piece of thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle. PSSMTs were asked how they would respond to this student. Task 5.1 is shown in Figure 4.11.

In Task 5.1, only two of the PSSMTs, namely Roslina and Suhana, had attempted to test generalization pertaining to the relationship between perimeter and area. Table 4.62 exhibits PSSMTs who had and had not attempted to test generalization pertaining to the relationship between perimeter and area in Task 5.1.

Table 4.62

PSSMTs who had and had not Attempted to Test Generalization Pertaining to the Relationship Between Perimeter and Area in Task 5.1

Test generalization pertaining to the relationship between perimeter and areain Task 5.1	PSSMTs
Attempted	Roslina, Suhana
Did not attempt	Beng, Liana,Mazlan, Patrick, Tan, Usha

Suhana refuted the student’s generalization that two shapes with the same perimeter have the same area with a counterexample. She knew that a counterexample is sufficient to refute the truth of a generalization. Roslina tested the student’s generalization that two shapes with the same perimeter have the same area with an example. She did not know that an example cannot be used to determine the truth of a generalization. In reality, a counterexample is sufficient to refute the truth of a generalization.

The remaining six PSSMTs, namely Beng, Liana, Mazlan, Patrick, Tan, and Usha, did not attempt to test generalization pertaining to the relationship between perimeter and area. Mazlan, Patrick, Tan, and Usha thought that the student's method of calculating the area of the leaf was correct. They thought that the student's generalization that two shapes with the same perimeter have the same area was correct. Thus, Mazlan, Patrick, Tan, and Usha did not attempt to test generalization pertaining to the relationship between perimeter and area. They never test the student's generalization that two shapes with the same perimeter have the same area.

Beng and Liana were not sure whether the student's method of calculating the area of the leaf was correct or not. They were not sure whether the student's generalization that two shapes with the same perimeter have the same area was correct or not. Thus, Beng and Liana did not attempt to test generalization pertaining to the relationship between perimeter and area. They never test the student's generalization that two shapes with the same perimeter have the same area.

The results of the analysis also exhibits that three of the PSSMTs, namely Beng, Patrick, and Liana, relied on authority, namely other people's view, to verify the correctness of the student's method of calculating the area of the leaf. Patrick thought that the student's method of calculating the area of the leaf was acceptable. Nevertheless, Patrick stated that he would seek his colleagues' view to verify the student's method of calculating the area of the leaf. It indicated that Patrick relied on authority, namely other people's view, to verify the correctness of the student's method of calculating the area of the leaf.

Beng and Liana was not sure whether the student's method of calculating the area of the leaf was correct or not. Thus, Beng and Liana said that they needed to verify it. Beng suggested that she would verify it by covering the surface of the leaf with square units and then compare it with the student's answer. Beng elaborated that she would also seek other people's view to verify it as she never think that the student's method can be used to calculate the area of the leaf. It indicated that Beng relied on authority, namely other people's view, to verify the correctness of the student's method of calculating the area of the leaf. Liana expressed that she need to seek her friends' expertise in science to find out whether the method claimed by the student can be used to determine the area of the leaf. It indicated that Liana relied on authority, namely other people's view, to verify the correctness of the student's method of calculating the area of the leaf.

Test Generalization That the Garden With
The Longer Perimeter Has The Larger Area?

In Task 5.2, PSSMTs were asked to respond to a student, Mary, who claimed that she could determine whose garden has the larger area to plant flowers. Mary claimed that the garden with the longer perimeter has the larger area. Task 5.2 is shown in Figure 4.12.

In Task 5.2, five of the PSSMTs, namely Beng, Patrick, Roslina, Suhana, and Usha, had attempted to test generalization pertaining to the relationship between perimeter and area. Table 4.63 illustrates PSSMTs who had and had not attempted to test generalization pertaining to the relationship between perimeter and area in Task 5.2.

Table 4.63

PSSMTs who had and had not Attempted to Test Generalization Pertaining to the Relationship Between Perimeter and Area in Task 5.2

Test generalization pertaining to the relationship between perimeter and areain Task 5.2	PSSMTs
Attempted	Beng, Patrick, Roslina, Suhana, Usha
Did not attempt	Liana, Mazlan, Tan

Beng and Suhana knew that Mary's claim was not correct. Beng used the example of her first and second method of comparing perimeter in Task 3.1 to test Mary's generalization that the garden with the longer perimeter has the larger area. She found that the shape with the longer perimeter may have a smaller area. Thus, Beng knew that Mary's generalization was not correct. Beng had successfully generated a counterexample to refute Mary's generalization. She knew that a counterexample is sufficient to refute the truth of a generalization. Suhana had generated two examples to test Mary's generalization that the garden with the longer perimeter has the larger area. Nevertheless, Suhana found that both of her examples concurred with Mary's generalization that the garden with the longer perimeter has the larger area. She tried to generate a counterexample to refute Mary's claim but was unsuccessful.

Patrick, Roslina, and Usha thought that Mary's claim was correct. They tested Mary's generalization that the garden with the longer perimeter has the larger area with the example that they generated. Nevertheless, Patrick, Roslina, and Usha did not know that an example could not be used to determine the truth of a generalization. A counterexample can be used to refute the truth of a generalization.

The remaining three PSSMTs, namely Liana, Mazlan, and Tan, did not attempt to test generalization pertaining to the relationship between perimeter and area. Mazlan and Tan thought that Mary's claim that the garden with the longer perimeter has the larger area was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Thus, Mazlan, and Tan, did not attempt to test generalization pertaining to the relationship between perimeter and area. They never tests Mary's generalization that the garden with the longer perimeter has the larger area. Liana was not sure whether Mary's claim that the garden with the longer perimeter has the larger area was correct or not. Thus, she did not attempt to test generalization pertaining to the relationship between perimeter and area. Liana never tests Mary's generalization that the garden with the longer perimeter has the larger area.

The results of the analysis also exhibits that two of the PSSMTs, namely Beng, and Liana, suggested that they would verify Mary's claim that the garden with the longer perimeter has the larger area. Beng knew that Mary's claim was not correct. Nevertheless, Beng suggested that she would use the grid paper to verify Mary's claim. Beng stated that she would calculate the number of units of squares, triangle, or rectangle that cover the surface of the given pictures of the gardens. Liana was not sure whether Mary's claim that the garden with the longer perimeter has the larger area was correct or not. She stated that one cannot simply say whether the method works or not. Liana expressed that she has to do "some research" to verify it.

Test Generalization That as the Perimeter of A Closed Figure Increases, the Area Also Increases?

In Task 5.3, PSSMTs were asked how they would respond to a Form One student's claimed regarding the relationships between perimeter and area of a closed figure. The student claimed that as the perimeter of a closed figure increases, the area also increases. Task 5.3 is shown in Figure 4.13. In Task 5.3, half of the PSSMTs, namely Beng, Liana, Patrick, and Tan, had attempted to test generalization pertaining to the relationship between perimeter and area. Table 4.64 shows PSSMTs who had and had not attempted to test generalization pertaining to the relationship between perimeter and area in Task 5.3.

Table 4.64

PSSMTs who had and had not Attempted to Test Generalization Pertaining to the Relationship Between Perimeter and Area in Task 5.3

Test generalization pertaining to the relationship between perimeter and area in Task 5.3	PSSMTs
Attempted	Beng, Liana, Patrick, Tan
Did not attempt	Mazlan, Roslina, Suhana, Usha

In Task 5.3, the student formulated a generalization that as the perimeter of a closed figure increases, the area also increases. Beng made a reflection on Task 3.1 that the shape with the longer perimeter may has a smaller area. She attempted to test the student's generalization. Nevertheless, Beng was unable to generate a counterexample in Task 5.3 to refute the student's claim that as the perimeter of a closed figure increases, the area also increases. In reality, when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same.

Tan generated an example to test the student's generalization. The example generated by him showed that although the rectangle and the triangle have the same perimeter (10 cm), their areas were different, namely 6 cm^2 and 4.472 m^2 respectively. He explained that the triangle has the smaller area even though they had the same perimeter. Tan realized that increases in perimeter did not guarantee that the area also increases. He knew that the student's "theory" was not correct.

Liana thought that the student's "theory" was correct. She generated two examples to test the student's generalization. The results of her examples concurred with the student's "theory" that as the perimeter of a closed figure increases, the area also increases. Patrick thought that the student's "theory" was correct. He generated an example to test the student's generalization. The example generated by him concurred with the student's "theory that as the perimeter of a closed figure increases, the area also increases. In reality, when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same.

The other half of the preservice secondary school mathematics teachers (PSSMTs), namely, Mazlan, Roslina, Suhana, and Usha, did not attempt to test generalization pertaining to the relationship between perimeter and area. In Task 5.3, the student formulated a generalization that as the perimeter of a closed figure increases, the area also increases. Mazlan and Suhana referred to the example generated by the student that indicated that as the perimeter of a closed figure increases from 8 cm to 10 cm, the area also increases from 4 cm^2 to 6 cm^2. Thus, they concluded that when the perimeter increases, the area also increases.

Roslina explained that when the perimeter (of a closed figure) increases, the area also increases. She expressed that the student has proven it and it was true because when the perimeter (of a shape) is longer compared to other shape, the area also larger. Usha explained that when the area of a shape is large, the perimeter that surrounded the outline of the area would be longer. Usha elaborated that a shape with the smaller side would have small perimeter and also small area. Thus, she concluded that a shape with the longer perimeter have the larger area.

Mazlan, Roslina, Suhana, and Usha thought that the student's "theory" was correct. They did not attempt to test the student's generalization. Mazlan, Roslina, Suhana, and Usha accepted the student's generalization without attempting to generate an example or counterexample to test it. In reality, when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same.

Develops Area Formulae

In this subsection, findings of PSSMTs' ethical knowledge of developing area formulae were presented in terms of: (a) whether they attempt to develop area formula for a rectangle, (b) whether they attempt to develop area formula for a parallelogram, (c) whether they attempt to develop area formula for a triangle, and (d) whether they attempt to develop area formula for a trapezium.

Attempting to develop area formula for a rectangle

In Task 8, PSSMTs were asked to show a Form One student the way to develop (derive) area formulae of a rectangle, parallelogram, triangle, and trapezium. Task 8 is shown in Figure 4.14. In Task 8, all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, could recall the formula for the area of a rectangle. Nevertheless, none of them attempted to develop the formula, except Tan. Tan had attempted to develop the formula but unsuccessful. Table 4.65 depicts PSSMTs who attempted and did not attempt to develop the formula for the area of a rectangle.

Table 4.65

PSSMTs who Attempted and did not Attempt to Develop the Formula for the Area of a Rectangle

Develop the formula for the area of a rectangle	PSSMTs
Attempted	Tan
Did not attempt	Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha

Attempting to develop area formula for a parallelogram

In Task 8, five of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, and Tan, could recall the formula for the area of a parallelogram. They had succeeded in developing the formula. The remaining three PSSMTs, namely Liana, Roslina, and Usha, could not recall the formula for the area of a parallelogram. They did not attempt to develop the formula. Table 4.66 demonstrates PSSMTs who attempted and did not attempt to develop the formula for the area of a parallelogram.

Table 4.66

PSSMTs who Attempted and did not Attempt to Develop the Formula for the Area of a Parallelogram

Develop the formula for the area of a parallelogram	PSSMTs
Attempted	Beng, Mazlan, Patrick, Suhana, Tan
Did not attempt	Liana, Roslina, Usha

Attempting to develop area formula for a triangle

In Task 8, seven of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, and Tan, could recall the formula for the area of a triangle. Of the seven PSSMTs who could recall the formula for the area of a triangle, three of them, namely Liana, Suhana, and Tan, attempted to develop the formula. Liana and Tan had succeeded in developing the formula. Suhana attempted to develop the formula but unsuccessful. The other four PSSMTs, namely Beng, Mazlan, Patrick, and Roslina did not attempt to develop the formula. The remaining PSSMT, namely Usha, could not recall the formula for the area of a triangle. She did not attempt to develop the formula. Table 4.67 reveals PSSMTs who attempted and did not attempt to develop the formula for the area of a triangle.

Table 4.67

PSSMTs who Attempted and did not Attempt to Develop the Formula for the Area of a Triangle

Develop the formula for the area of a triangle	PSSMTs
Attempted	Liana, Suhana, Tan
Did not attempt	Beng, Mazlan, Patrick, Roslina, Usha

Attempting to develop area formula for a trapezium

In Task 8, six of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, Tan, and Usha could recall the formula for the area of a trapezium. Of the six PSSMTs who could recall the formula for the area of a trapezium, five of them, namely Beng, Mazlan, Patrick, Suhana, and Tan, attempted to develop the formula. Beng, Suhana, and Tan succeeded in developing the formula for the area of a trapezium. Mazlan and Patrick attempted to develop the formula for the area of a trapezium but unsuccessful. Usha could recall the formula for the area of a trapezium but she did not attempt to develop the formula. Two of the PSSMTs, namely Liana and Roslina, could not recall the formula for the area of a trapezium. They did not attempt to develop the formula. Table 4.68 exhibits PSSMTs who attempted and did not attempt to develop the formula for the area of a trapezium.

Table 4.68

PSSMTs who Attempted and did not Attempt to Develop the Formula for the Area of a Trapezium

Develop the formula for the area of a trapezium	PSSMTs
Attempted	Beng, Mazlan, Patrick, Suhana, Tan
Did not attempt	Liana, Roslina, Usha

Writes Units of Measurement upon Completed a Task

In this subsection, findings of PSSMTs' ethical knowledge of writing units of measurement upon completed a task were presented in terms of: (a) whether they write unit of measurement for the answer of the perimeter of Diagram 1, (b) whether they write unit of measurement for the answer of the area of Diagram 1, (c) whether they write unit of measurement for the answer of the perimeter of Diagram 2, (d) whether they write unit of measurement for the answer of the area of Diagram 2, (e) whether they write unit of measurement for the largest area being enclosed in the fencing problem, and (f) whether they write unit of measurement for the dimension that would yield the largest area being enclosed in the fencing problem.

Write Unit for Perimeter of Diagram 1

In Task 6.1, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 1) that involved composite figure, namely rectangle and parallelogram/triangle. Task 6.1 is shown in Figure 4.15. In Task 6.1, all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, wrote the measurement unit (without probed), namely cm, for the answer of the perimeter of Diagram 1 that they have calculated.

Write Unit for Area of Diagram 1

In Task 6.1, all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, also wrote the measurement unit (without probed), namely cm^2, for the answer of the area of Diagram 1 that they have calculated.

Write Unit for Perimeter of Diagram 2

In Task 6.2, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 2) that involved composite figure, namely square and trapezium/triangle. Task 6.2 is shown in Figure 4.16. In Task 6.2, all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, wrote the measurement unit (without probed) for the answer of the perimeter of Diagram 2 that they have calculated.

Write Unit for Area of Diagram 2

In Task 6.2, all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, also wrote the measurement unit (without probed) for the answer of the area of Diagram 2 that they have calculated. Nevertheless, Mazlan did not write the measurement unit for the answer of the area of trapezium FIJK in his alternative method.

Write Unit for the Largest Area Being Enclosed

In Task 7, PSSMTs were required to help his/her student to solve the fencing problem. Task 7 is shown in Figure 4.19. In Task 7, five of the PSSMTs, namely Mazlan, Patrick, Roslina, Tan, and Usha, wrote measurement unit for the largest area being enclosed. Patrick and Tan wrote measurement unit for the largest area being enclosed, namely 882 m^2. Usha wrote measurement unit for the area that she thought was the largest area being enclosed, namely 880 m^2.

Mazlan wrote measurement unit for the area that he thought was the largest area being enclosed, namely 392 m^2. Roslina wrote measurement unit for the area that she thought was the largest area being enclosed, namely 160 m^2. Two of the PSSMTs, namely Beng and Suhana, did not write measurement unit for the largest area being enclosed, namely 882. Table 4.69 illustrates PSSMTs who wrote and did not write the measurement unit for the largest area being enclosed.

Table 4.69

PSSMTs who Wrote and did not Write the Measurement Unit for the Largest Area Being Enclosed

Measurement unit for the largest area being enclosed	PSSMTs
Wrote	Mazlan, Patrick, Roslina, Tan, Usha
Did not write	Beng, Suhana

Write Unit for the Dimension That Yield the Largest Area

In Task 7, two of the PSSMTs, namely Roslina and Usha, wrote measurement unit for the dimension that they thought would yield the largest area being enclosed. Usha wrote measurement unit for the dimension that she thought would yield the largest area being enclosed, namely 40 m by 22 m. Roslina wrote measurement unit for the dimension that she thought would yield the largest area being enclosed, namely 80 m by 2 m. Table 4.70 shows PSSMTs who wrote and did not write the measurement unit for the dimension that would yield the largest area being enclosed.

Five of the PSSMTs, namely Beng, Mazlan, Patrick, Suhana, and Tan, did not write measurement unit for the dimension that would yield the largest area being enclosed. Beng, Patrick, Suhana, and Tan, did not write measurement unit for the dimension that would yield the largest area being enclosed, namely 42 by 21. Mazlan did not write measurement unit for the dimension that he thought would yield the largest area being enclosed, namely 28 by 14.

Table 4.70

PSSMTs who Wrote and did not Write the Measurement Unit for the Dimension That Would Yield the Largest Area Being Enclosed

Measurement unit for the dimension that would yield the largest area being enclosed	PSSMTs
Wrote	Roslina, Usha
Did not write	Beng, Mazlan, Patrick, Suhana, Tan

Checks the Correctness of Their Solutions or Answers

In this subsection, findings of PSSMTs' ethical knowledge of checking solutions or answers were presented in terms of: (a) whether they check the answer of the perimeter of Diagram 1, (b) whether they check the answer of the area of Diagram 1, (c) whether they check the answer of the perimeter of Diagram 2, (d) whether they check the answer of the area of Diagram 2, and (e) whether they check the answer for the fencing problem.

Check the Answer of the Perimeter of Diagram 1

In Task 6.1, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 1) that involved composite figure, namely rectangle and parallelogram/triangle. Task 6.1 is shown in Figure 4.15. In Task 6.1, all the PSSMTs have successfully calculated the perimeter of Diagram 1, except Tan. Neverthelsss, none of the PSSMTs checked the correctness of the answer for the perimeter. Tan might has spotted his mistake should he checked the answer for the perimeter of Diagram 1. When probed to check answer, then only all the PSSMTs suggested the strategies that they would use to check the answer for perimeter.

Check the Answer of the Area of Diagram 1

In Task 6.1, all the PSSMTs have successfully calculated the area of Diagram 1, except Mazlan and Usha. Neverthelsss, none of the PSSMTs checked the correctness of the answer for the area. Mazlan and Usha might have spotted their mistake should they checked the answer for the area of Diagram 1. When probed to check answer, then only all the PSSMTs suggested the strategies that they would use to check the answer for area.

Check the Answer of the Perimeter of Diagram 2

In Task 6.2, PSSMTs were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 2) that involved composite figure, namely square and trapezium/triangle. Task 6.2 is shown in Figure 4.16. In Task 6.2, all the PSSMTs have successfully calculated the perimeter of Diagram 2. Neverthelsss, none of the PSSMTs checked the correctness of the answer for the perimeter. When probed to check answer, then only all the PSSMTs suggested the strategies that they would use to check the answer for perimeter.

Check the Answer of the Area of Diagram 2

In Task 6.2, all the PSSMTs have successfully calculated the area of Diagram 2. Nevertheless, only three of the PSSMTs, namely Liana, Patrick and Tan checked the correctness of the answer for the area without being probed. Table 4.71 depicts PSSMTs who checked and did not check the correctness of the answer for the area of Diagram 2 without being probed.

Liana suggested the strategy that she would use to check the answer for area without being probed. Patrick and Tan checked the correctness of the answer for area without being probed. The remaining five PSSMTs, namely Beng, Mazlan, Roslina, Suhana, and Usha, did not check the correctness of the answer for area. When probed to check answers, then only Beng, Mazlan, Roslina, Suhana, and Usha, suggested the strategy that they would use to check the answer for area.

Table 4.71

PSSMTs who had and had not Checked the Correctness of the Answer for the Area of Diagram 2 Without Being Probed

Checked the correctness of the answer for the area of Diagram 2	PSSMTs
Checked	Liana, Patrick, Tan
Did not check	Beng, Mazlan, Roslina, Suhana, Usha

Check the Answer for the Fencing Problem

In Task 7, PSSMTs were required to help his/her student to solve the fencing problem. Task 7 is shown in Figure 4.19. In Task 7, five of the PSSMTs, namely Beng, Patrick, Suhana, Tan, and Usha checked the correctness of the answer for the fencing problem without being probed. Two of the PSSMTs, namely Mazlan and Roslina, did not check the correctness of the answer for the fencing problem. When probed to check answers, then only Mazlan and Roslina suggested the strategy that they would use to check the answer for the fencing problem. Table 4.72 demonstrates PSSMTs who checked and did not check the correctness of the answer for the fencing problem without being probed.

Table 4.72

PSSMTs who had and had not Checked the Correctness of the Answer for the Fencing Problem Without Being Probed

Checked the correctness of the answer for the fencing problem	PSSMTs
Checked	Beng, Patrick, Suhana, Tan, Usha
Did not check	Mazlan, Roslina

Level of Subject Matter Knowledge

In this section, findings of PSSMTs' levels (low, medium, high) of subject matter knowledge of perimeter and area were presented in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

Level of Conceptual Knowledge

Two of the PSSMTs, namely Liana and Tan, secured a high level of conceptual knowledge of perimeter and area when they obtained 72.0% and 88.0% of appropriate mathematical elements of conceptual knowledge of perimeter and area respectively during the clinical interview. Four of the PSSMTs, namely Beng, Patrick, Suhana, and Usha, achieved a medium level of conceptual knowledge of perimeter and area. The remaining two PSSMTs, namely Mazlan and Roslina, gained a low level of conceptual knowledge of perimeter and area. Table 4.73 shows the percentage of appropriate mathematical elements of conceptual knowledge of perimeter and area obtained by PSSMTs and their respective level of conceptual knowledge.

Table 4.73

Percentage of Appropriate Mathematical Elements of Conceptual Knowledge Obtained by PSSMTs and Their Respective Level of Conceptual Knowledge

Percentage of appropriate mathematical elements of conceptual knowledge of perimeter and area obtained by the PSSMTs	Level of conceptual knowledge			PSSMTs
	Low	Medium	High	
64.0		x		Beng
72.0			x	Liana
36.0	x			Mazlan
56.0		x		Patrick
32.0	x			Roslina
68.0		x		Suhana
88.0			x	Tan
44.0		x		Usha

Level of Procedural Knowledge

Four of the PSSMTs, namely Beng, Liana, Patrick, and Tan, secured a high level of procedural knowledge of perimeter and area when they obtained 81.8%, 72.7%, 72.7%, and 81.8% of appropriate mathematical elements of procedural knowledge of perimeter and area respectively during the clinical interview. One of the PSSMTs, namely Suhana, achieved a medium level of procedural knowledge of perimeter and area. The remaining three PSSMTs, namely Mazlan, Roslina, and Usha gained a low level of procedural knowledge of perimeter and area. Table 4.74 depicts the percentage of appropriate mathematical elements of procedural knowledge of perimeter and area obtained by PSSMTs and their respective level of procedural knowledge.

Table 4.74

Percentage of Appropriate Mathematical Elements of Procedural Knowledge Obtained by PSSMTs and Their Respective Level of Procedural Knowledge

Percentage of appropriate mathematical elements of procedural knowledge of perimeter and area obtained by the PSSMTs	Level of procedural knowledge			PSSMTs
	Low	Medium	High	
81.8			x	Beng
72.7			x	Liana
36.4	x			Mazlan
72.7			x	Patrick
36.4	x			Roslina
63.6		x		Suhana
81.8			x	Tan
27.3	x			Usha

Level of Linguistic Knowledge

Only one of the PSSMTs, namely Suhana, secured a high level of linguistic knowledge of perimeter and area when she obtained 72.1% of appropriate mathematical elements of linguistic knowledge of perimeter and area during the clinical interview. Six of the PSSMTs, namely Beng, Mazlan, Patrick, Roslina, Tan, and Usha, achieved a medium level of linguistic knowledge of perimeter and area. Only one of the PSSMTs, namely Liana, gained a low level of linguistic knowledge of perimeter and area. Table 4.75 demonstrates the percentage of appropriate mathematical elements of linguistic knowledge of perimeter and area obtained by PSSMTs and their respective level of linguistic knowledge.

Table 4.75

Percentage of Appropriate Mathematical Elements of Linguistic Knowledge Obtained by PSSMTs and Their Respective Level of Linguistic Knowledge

Percentage of appropriate mathematical elements of linguistic knowledge of perimeter and area obtained by the PSSMTs	Level of linguistic knowledge			PSSMTs
	Low	Medium	High	
69.8		x		Beng
25.6	x			Liana
62.8		x		Mazlan
51.2		x		Patrick
65.1		x		Roslina
72.1			x	Suhana
62.8		x		Tan
69.8		x		Usha

Level of Strategic Knowledge

Five of the PSSMTs, namely Beng, Patrick, Suhana, Tan, and Usha, secured a high level of strategic knowledge of perimeter and area when they obtained 85.7%, 78.6%, 85.7%, 92.9%, and 71.4% of appropriate mathematical elements of strategic knowledge of perimeter and area respectively during the clinical interview. The remaining three PSSMTs, namely Liana, Mazlan, and Roslina, achieved a medium level of strategic knowledge of perimeter and area. Table 4.76 reveals the percentage of appropriate mathematical elements of strategic knowledge of perimeter and area obtained by PSSMTs and their respective level of strategic knowledge.

Table 4.76

Percentage of Appropriate Mathematical Elements of Strategic Knowledge Obtained by PSSMTs and Their Respective Level of Strategic Knowledge

Percentage of appropriate mathematical elements of strategic knowledge of perimeter and area obtained by the PSSMTs	Level of strategic knowledge			PSSMTs
	Low	Medium	High	
85.7			x	Beng
57.1		x		Liana
57.1		x		Mazlan
78.6			x	Patrick
64.3		x		Roslina
85.7			x	Suhana
92.9			x	Tan
71.4			x	Usha

Level of Ethical Knowledge

Seven of the PSSMTs, namely Beng, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, achieved a medium level of ethical knowledge of perimeter and area. Only one of the PSSMTs, namely Liana, gained a low level of ethical knowledge of perimeter and area. Table 4.77 exhibits the percentage of appropriate mathematical elements of ethical knowledge of perimeter and area obtained by PSSMTs and their respective level of ethical knowledge.

Table 4.77

Percentage of Appropriate Mathematical Elements of Ethical Knowledge Obtained by PSSMTs and Their Respective Level of Ethical Knowledge

Percentage of appropriate mathematical elements of ethical knowledge of perimeter and area obtained by the PSSMTs	Level of ethical knowledge			PSSMTs
	Low	Medium	High	
61.2		x		Beng
20.4	x			Liana
53.1		x		Mazlan
46.9		x		Patrick
55.1		x		Roslina
65.3		x		Suhana
63.3		x		Tan
65.3		x		Usha

Overall Level of Subject Matter Knowledge

Only one of the PSSMTs, namely Tan, secured an overall high level of SMK of perimeter and area when he obtained 72.1% of appropriate mathematical elements of SMK of perimeter and area during the clinical interview. Six of the PSSMTs, namely Beng, Mazlan, Patrick, Roslina, Suhana, and Usha, achieved an overall medium level of subject matter knowledge of perimeter and area. They obtained the percentage of appropriate mathematical elements of SMK of perimeter and area ranged from 52.1% to 69.7% during the clinical interview.

Only one of the PSSMTs, namely Liana, gained an overall low level of SMK of perimeter and area when she obtained 38.7% of appropriate mathematical elements of SMK of perimeter and area during the clinical interview. Table 4.78 illustrates the percentage of appropriate mathematical elements of subject matter knowledge (SMK) of perimeter and area obtained by PSSMTs and their respective overall level of subject matter knowledge.

Table 4.78

Percentage of Appropriate Mathematical Elements of Subject Matter Knowledge Obtained by PSSMTs and Their Respective Overall Level of Subject Matter Knowledge

Percentage of appropriate mathematical elements of subject matter knowledge of perimeter and area obtained by the PSSMTs	Overall Level of subject matter knowledge			PSSMTs
	Low	Medium	High	
68.3		x		Beng
38.7	x			Liana
52.1		x		Mazlan
54.9		x		Patrick
53.5		x		Roslina
69.7		x		Suhana
71.8			x	Tan
60.6		x		Usha