# CHAPTER FIVE

# DISCUSSION AND CONCLUSIONS

## Introduction

The purpose of this study was to investigate preservice secondary school mathematics teachers' subject matter knowledge (SMK) of perimeter and area. This study attempted to answer the following research questions:

1. What kinds of subject matter knowledge (SMK) of perimeter and area do the preservice secondary school mathematics teachers have?
2. What levels of subject matter knowledge (SMK) of perimeter and area do the preservice secondary school mathematics teachers exhibits?

This chapter is organized into four main sections: summary of the findings, discussion and conclusions, implications of the findings, and recommendations for further research.

## Summary of the Findings

In this section, to answer research question one, findings of preservice secondary school mathematics teachers (PSSMTs)' subject matter knowledge of perimeter and area are summarized in terms of its five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge that were emerged from the clinical interview. To answer research question two, findings of PSSMTs' levels (low, medium, high) of subject matter knowledge of perimeter and area are summarized in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

## Conceptual Knowledge

In this subsection, findings of PSSMTs' conceptual knowledge of perimeter and area are summarized in terms of its components: (a) notion of perimeter, (b) notion of area, (c) notion of the unit of area, (d) number of units and unit of measure, (e) inverse relationship between number of units and unit of measure, (f) relationship between standard units of length (linear units), (g) relationship between standard units of area (square units), (h) relationship between area units and linear units, (i) relationship between perimeter and area, and (j) relationship among area formulae. Table 5.1 shows a summary of PSSMTs' conceptual knowledge of perimeter and area.

### Notion of Perimeter, Area, and Units of Area

Finding of this study suggests that half of the PSSMTs had the correct notion of perimeter that simple closed curves, closed but not simple curves, and 3-dimensional shapes have a perimeter. Nevertheless, three PSSMTs had the incorrect notion of perimeter that only simple closed curves, and closed but not simple curves have a perimeter. They thought that 3-dimensional shapes do not have a perimeter. Furthermore, one PSSMT had the incorrect notion of perimeter that only common simple closed curves (triangle, circle, and trapezium) have a perimeter.

In this study, five out of eight PSSMTs had the correct notion of area that 2-dimensional and 3-dimensional shapes have an area. However, one PSSMT had the incorrect notion of area that only 2-dimensional shapes have an area. This PSSMT thought that 3-dimensional shapes do not have an area. Moreover, two PSSMTs had the incorrect notion of area that only regular 2-dimensional shapes (i.e., triangle, circle, and trapezium) and 3-dimensional shapes (i.e., cuboid and cylinder) have an area. Finding of this study shows that none of the PSSMTs selected open shapes (including the lines), namely B, E, G, and L, as having an area. Based on these selections, it can be inferred that all the PSSMTs in this study did not have a dynamic perspective of the notion of area. PSSMTs selected only closed shapes as having an area indicated that they had a static perspective of the notion of area.

Finding of this study reveals that three out of eight PSSMTs had the correct notion of unit of area that square and nonsquare (rectangle and triangle) can be used as unit of area. Nevertheless, five PSSMTs had the incorrect notion of unit of area that only square and rectangle, square and triangle, square, or none of the square and nonsquare can be used as unit of area.

**Number of Units and Unit of Measure**

All but one PSSMT in this study had correctly focused on the unit of measure when they were comparing perimeters with nonstandard units. However, only half of the PSSMTs in this study understand that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. Finding of this study suggests that three out of eight PSSMTs had correctly focused on the unit of measure when they were comparing perimeters with common nonstandard units. They understand that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. Nevertheless, all PSSMTs had correctly focused on the number of unit when they were comparing perimeters with common standard units. They understand that common standard unit (such as cm) is reliable for comparing perimeters.

Six out of eight PSSMTs in this study had correctly focused on the unit of measure when they were comparing areas with nonstandard units. However, out of the six, five of them understand that nonstandard units (such as triangle and square) are not reliable for comparing areas. Finding of this study shows that three out of eight PSSMTs had correctly focused on the unit of measure when they were comparing areas with common nonstandard unit. They understand that common nonstandard units (such as squares) are not reliable for comparing areas.

However, all PSSMTs had correctly focused on the number of unit when they were comparing areas with common standard unit. They understand that common standard unit (such as $cm^2$) is reliable for comparing areas.

### Inverse Proportion between Number of Units and Unit of Measure

All PSSMTs in this study understand the inverse proportion between the number of units and the unit of measure. Specifically, they knew that the longer the unit of measure, the smaller the number of units required to get the same length and vice versa. Similarly, they knew that the larger the unit of measure, the smaller the number of units required to get the same area and vice versa.

### Relationships between the Standard Units of Length Measurement

Six out of eight PSSMTs in this study knew the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. One PSSMT did not know that 1 cm = 10 mm and 1 m = 100 cm. Another PSSMT did not know that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m.

### Relationship between the Standard Units of Area Measurement

Three out of eight PSSMTs in this study exhibited a lack of conceptual knowledge about the relationship between the standard units of area measurement. For instance, two PSSMTs thought that 1 $cm^2$ = 10 $mm^2$, 1 $m^2$ = 100 $cm^2$, and 1 $km^2$ = 1000 $m^2$. It indicated that they did not know the relationships between the standard units of area measurement that 1 $cm^2$ = 100 $mm^2$, 1 $m^2$ = 10 000 $cm^2$, and 1 $km^2$ = 1000 000 $m^2$. Another PSSMT did not know the relationships between the standard units of area measurement that 1 $cm^2$ = 100 $mm^2$ and 1 $m^2$ = 10 000 $cm^2$.

**Relationship between Area Units and Linear Units of Measurement**

Six out of eight PSSMTs in this study had adequate conceptual knowledge about the relationship between area units and linear units of measurement. They knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring.

**Relationship between Perimeter and Area**

Most of the PSSMTs in this study had a misconception that there is direct relationship between perimeter and area. Specifically, only one PSSMT knew that two shapes with the same perimeter can have different areas. Thus, she knew that the student's method of calculating the area of the leaf was not correct. Only two PSSMTs knew that the garden with the longer perimeter could have a smaller area. Thus, they knew that Mary's claim that the garden with the longer perimeter has the larger area was not correct. Similarly, only two PSSMTs knew that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, they knew that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct.

**Relationship among Area Formulae**

None of the PSSMTs in this study were able to develop the formula for the area of a rectangle. They might have rote-learnt the formula. It was apparent that all of them lack conceptual knowledge underpinning the formula for the area of a rectangle. However, five PSSMTs were able to develop the formula for the area of a parallelogram. They mentally transformed the parallelogram to a rectangle. It indicated that they understand the relationship between the formulae for the area of a parallelogram and rectangle.

Only two PSSMTs were able to develop the formula for the area of a triangle. They developed the formula for the area of a triangle based on the formula for the area of a rectangle or a square. It indicated that they knew the relationship between the formulae for the area of a triangle and rectangle or square that encloses it. Three PSSMTs were able to develop the formula for the area of a trapezium. They developed the formula for the area of a trapezium from the formulae for the area of a rectangle and a triangle. It indicated that they knew that the formula for the area of a trapezium is related to the formulae for the area of a rectangle and triangle.

Table 5.1

*Summary of PSSMTs' Conceptual Knowledge of Perimeter and Area*

| Component of conceptual knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Notion of perimeter | **Correct notion:** | | |
| | Simple closed curves, closed but not simple curves, and 3-dimensional shapes. | Selected shapes A, C, D, F, H, I, J, and K that have a perimeter. | Liana, Roslina, Tan, Usha |
| | **Incorrect notion:** | | |
| | Limited to simple closed curves, and closed but not simple curves. | Selected shapes A, C, D, H, I, and K that have a perimeter. | Beng, Patrick, Suhana |
| | Limited to common simple closed curves (triangle, circle, and trapezium). | Selected shapes A, C, and H that have a perimeter. | Mazlan |
| Notion of area | **Correct notion:** | | |
| | 2-dimensional and 3-dimensional shapes. | Selected shapes A, C, D, F, H, I, J, and K that have an area. | Liana, Mazlan, Suhana, Tan, Usha |
| | **Incorrect notion:** | | |
| | Limited to 2-dimensional shapes. | Selected shapes A, C, D, H, I, and K that have an area. | Beng |
| | Limited to regular 2-dimensional shapes (i.e., triangle, circle, and trapezium) and 3-dimensional shapes (i.e., cuboid and cylinder), where its area or surface area can be calculated using formula. | Selected shapes A, C, F, H, and J that have an area. | Patrick, Roslina |
| Notion of unit of area | **Correct notion:** | | |
| | Square and nonsquare. | Selected square, rectangle, and triangle that can be used as unit of area. | Beng, Tan, Usha |
| | **Incorrect notion:** | | |
| | Limited to square and rectangle. | Selected square and rectangle that can be used as unit of area. | Patrick |
| | Limited to square and triangle. | Selected square and triangle that can be used as unit of area. | Mazlan |
| | Limited to square. | Selected square that can be used as unit of area. | Roslina, Suhana |
| | None or not accessible to her during the clinical interview. | Stated that square, rectangle, and triangle that cannot be used as unit of area. | Liana |
| Number of units and unit of measure | Comparing perimeters with nonstandard units: | | |
| | **Correct focus:** | | |
| | Focused on the unit of measure and knew that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. | Provided correct response: Unable to determine which shape has the longer perimeter. | Liana, Patrick, Tan, Usha |
| | Focused on the unit of measure but did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. | Stated that shape B has the longer perimeter. | Mazlan, Roslina, Suhana |
| | **Incorrect focus:** | | |
| | Focused on the number of unit and did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. | Stated that shape A has the longer perimeter. | Beng |
| | Comparing perimeters with common nonstandard units: | | |
| | **Correct focus:** | | |
| | Focused on the unit of measure and knew that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. | Provided correct response: Unable to determine which shape has the longer perimeter. | Liana, Tan, Usha |

Table 5.1 (continued)

| Component of conceptual knowledge | Findings | Evidence | PSSMTs |
| --- | --- | --- | --- |
| Number of units and unit of measure | **Incorrect focus:** | | |
| | Focused on the number of unit and did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. | Stated that shape B has the longer perimeter. | Beng, Mazlan, Patrick, Roslina, Suhana |
| | Comparing perimeters with common standard units: | | |
| | **Correct focus:** | | |
| | Focused on the number of unit and knew that common standard unit (such as cm) is reliable for comparing perimeters. | Provided correct response: Shape A has the longer perimeter. | All PSSMTs |
| | Comparing areas with nonstandard units: | | |
| | **Correct focus:** | | |
| | Focused on the unit of measure and knew that nonstandard units (such as triangle and square) are not reliable for comparing areas. | Provided correct response: Unable to determine which shape has the larger area. | Liana, Patrick, Suhana, Tan, Usha |
| | Focused on the unit of measure but did not know that nonstandard units (such as triangle and square) are not reliable for comparing areas. | Stated that shape B has the larger area. | Roslina |
| | **Incorrect focus:** | | |
| | Focused on the number of unit and did not know that nonstandard units (such as triangle and square) are not reliable for comparing areas. | Stated that shape A has the larger area. | Beng, Mazlan |
| | Comparing areas with common nonstandard units: | | |
| | **Correct focus:** | | |
| | Focused on the unit of measure and knew that common nonstandard units (such as squares) are not reliable for comparing areas. | Provided correct response: Unable to determine which shape has the larger area. | Liana, Tan, Usha |
| | **Incorrect focus:** | | |
| | Focused on the number of unit and did not know that common nonstandard units (such as squares) are not reliable for comparing areas. | Stated that shape B has the larger area. | Beng, Mazlan, Patrick, Roslina, Suhana |
| | Comparing areas with common standard units: | | |
| | **Correct focus:** | | |
| | Focused on the number of unit and knew that common standard unit (such as $cm^2$) is reliable for comparing areas | Provided correct response: Shape A has the larger area. | All PSSMTs |
| Inverse proportion between number of units and unit of measure | Understand the inverse proportion between the number of units and the unit of measure. | They knew that the longer the unit of measure, the smaller the number of units required to get the same length and vice versa. | All PSSMTs |
| | | They knew that the larger the unit of measure, the smaller the number of units required to get the same area and vice versa. | All PSSMTs |

Table 5.1 (continued)

| Component of conceptual knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Relationships between the standard units of length measurement | 1 cm = 10 mm: Knew | Times 10 when converted 1 cm to mm. | Beng, Patrick |
| | | Times ten squared, $(10)^2$, when converted 3 $cm^2$ to $mm^2$. | Liana |
| | | Wrote that 1 cm = 10 mm. | Roslina |
| | | Times 10 when converted 3 cm to mm. | Suhana |
| | | Times 10 when converted 1 cm to mm and 3 cm to mm respectively. | Tan |
| | Did not know | Times $10^{-1}$ when converted 3 $cm^2$ to $mm^2$. | Mazlan |
| | | Thought that 1 $cm^2$ = 10 $mm^2$. | Usha |
| | 1 m = 100 cm: Knew | Times 100 when converted 1 m to cm. | Beng, Patrick |
| | | Times hundred squared, $(100)^2$, when converted 4.7 $m^2$ to $cm^2$. | Liana |
| | | Wrote that 1 m = 100 cm. | Roslina |
| | | Times 100 when converted 4.7 m to cm. | Suhana |
| | | Times 100 when converted 1 m to cm and 4.7 m to cm respectively. | Tan |
| | Did not know | Times $(10^1 cm)^2$ or $10^2$ $cm^2$ when converted 4.7 $m^2$ to $cm^2$. | Mazlan |
| | | Thought that 1 $m^2$ = 100 $cm^2$. | Usha |
| | 1 km = 1000 m: Knew | Times 1000 when converted 1 km to m. | Beng, Patrick |
| | | Times thousand squared, $(1000)^2$, when converted 1.25 $km^2$ to $m^2$. | Liana, Mazlan |
| | | Wrote that 1 km = 1000 m. | Roslina |
| | | Times 1000 when converted 1.25 km to m. | Suhana |
| | | Times 1000 when converted 1 km to m and 1.25 km to m respectively. | Tan |
| | Did not know | Thought that 1 $km^2$ = 1000 $m^2$. | Usha |
| Relationship between the standard units of area measurement | 1 $cm^2$ = 100 $mm^2$: Did not know | Times $10^{-1}$ when converted 3 $cm^2$ to $mm^2$. | Mazlan |
| | | Thought that 1 $cm^2$ = 10 $mm^2$. | Roslina, Usha |
| | 1 $m^2$ = 10 000 $cm^2$: Did not know | Times $(10^1 cm)^2$ or $10^2$ $cm^2$ when converted 4.7 $m^2$ to $cm^2$. | Mazlan |
| | | Thought that 1 $m^2$ = 100 $cm^2$. | Roslina, Usha |
| | 1 $km^2$ = 1 000 000 $m^2$: Knew | Times thousand squared, $(1000)^2$, when converted 1.25 $km^2$ to $m^2$. | Mazlan |
| | Did not know | Thought that 1 $km^2$ = 1000 $m^2$. | Roslina, Usha |

Table 5.1 (continued)

| Component of conceptual knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Relationship between area units and linear units of measurement | Knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. | Wrote 3 $cm^2$ as 3 times 1 cm times 1 cm. | Beng, Patrick |
| | | Times ten squared, $(10)^2$, when converted 3 $cm^2$ to $mm^2$. | Liana, Suhana |
| | | Times hundred squared, $(1000)^2$, when converted 1.25 $km^2$ to $m^2$. | Mazlan |
| | | Viewed 3 $cm^2$ as the product of 1 cm times 3 cm. Times 10 twice when converted 1 cm to mm and 3 cm to mm separately. | Tan |
| | Did not know. | Knew that 1 cm = 10 mm but thought that 1 $cm^2$ = 10 $mm^2$. | Roslina |
| | | Thought that 1 $cm^2$ = 10 $mm^2$. | Usha |
| Relationship between perimeter and area | Same perimeter, same area? Knew that there is no direct relationship between perimeter and area. Knew that two shapes with the same perimeter can have different areas. | Provided correct response: The student's method of calculating the area of the leaf was not correct. | Suhana |
| | Did not know that there is no direct relationship between perimeter and area. Did not know that two shapes with the same perimeter can have different areas. | Thought that the student's method of calculating the area of the leaf was correct. | Mazlan, Patrick, Roslina, Tan, Usha |
| | | Not sure whether the student's method of calculating the area of the leaf was correct or not. | Beng, Liana |
| | Longer perimeter, larger area? Knew that there is no direct relationship between perimeter and area. Knew that the garden with the longer perimeter could have a smaller area. | Provided correct response: Mary's claim that the garden with the longer perimeter has the larger area was not correct. | Beng, Suhana |
| | Did not know that there is no direct relationship between perimeter and area. Did not know that the garden with the longer perimeter could have a smaller area. | Thought that Mary's claim that the garden with the longer perimeter has the larger area was correct. | Mazlan, Patrick, Roslina , Tan, Usha |
| | | Not sure whether Mary's claim that the garden with the longer perimeter has the larger area was correct or not. | Liana |
| | Perimeter increases, area increases? Knew that there is no direct relationship between perimeter and area. Knew that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. | Provided correct response: The student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct. | Beng, Tan |
| | Did not know that there is no direct relationship between perimeter and area. Did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. | Thought that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was correct. | Liana, Mazlan, Patrick, Roslina, Suhana, Usha |
| Relationship among area formulae | Rectangle: Did not understand the conceptual knowledge underpinning the formula for the area of a rectangle. | Unable to develop the formula for the area of a rectangle. | All PSSMTs |
| | Parallelogram: Knew the relationship between the formulae for the area of a parallelogram and rectangle. | Able to develop the formula for the area of a parallelogram from rectangle. | Beng, Mazlan, Patrick, Suhana, Tan |
| | Did not know the relationship between the formulae for the area of a parallelogram and rectangle, or a parallelogram and triangle. | Unable to develop the formula for the area of a parallelogram from rectangle or triangle. | Liana, Roslina, Usha |

Table 5.1 (continued)

| Component of conceptual knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Relationship among area formulae | Triangle:<br>Knew the relationship between the formulae for the area of a triangle and rectangle that encloses it. | Able to develop the formula for the area of a triangle from rectangle. | Liana, Tan |
| | Did not know the relationship between the formulae for the area of a triangle and rectangle, or a triangle and parallelogram. | Unable to develop the formula for the area of a triangle from rectangle or parallelogram. | Beng, Mazlan, Patrick, Roslina, Suhana, Usha |
| | Trapezium:<br>Knew that the formula for the area of a trapezium is related to the formulae for the area of a rectangle and triangle. | Able to develop the formula for the area of a trapezium from rectangle and triangle, using algebraic method. | Beng, Suhana, Tan |
| | Did not know the relationship between the formulae for the area of a trapezium and rectangle, a trapezium and parallelogram, or a trapezium and triangle. Also did not know that the formula for the area of a trapezium is related to the formulae for the area of a rectangle and triangle. | Unable to develop the formula for the area of a trapezium from rectangle, parallelogram, or triangle. Also unable to develop the formula for the area of a trapezium from rectangle and triangle, using algebraic method. | Liana, Mazlan, Patrick, Roslina, Usha |

## Procedural Knowledge

In this subsection, findings of PSSMTs' procedural knowledge of perimeter and area are summarized in terms of its components: (a) converting standard units of area measurement, (b) calculating perimeter of composite figures, (c) calculating area of composite figures, and (d) developing area formulae. Table 5.2 depicts a summary of PSSMTs' procedural knowledge of perimeter and area.

### Converting Standard Units of Area Measurement

In this study, five out of eight PSSMTs had successfully converted 3 $cm^2$ to $mm^2$ while half of the PSSMTs had successfully converted 4.7 $m^2$ to $cm^2$ and 1.25 $km^2$ to $m^2$. It indicated that at least half of the PSSMTs had adequate procedural knowledge of converting standard units of area measurement.

**Calculating Perimeter and Area of Composite Figures**

Most of the PSSMTs in this study had adequate procedural knowledge of calculating perimeter and area of composite figures. For instance, all but one PSSMT had successfully calculated the perimeter of Diagram 1 in Task 6.1 as 104 cm while all PSSMTs had successfully calculated the perimeter of Diagram 2 in Task 6.2 as 56 mm. Similarly, six out of eight PSSMTs had successfully calculated the area of Diagram 1 in Task 6.1 as 420 $cm^2$ while all PSSMTs had successfully calculated the area of Diagram 2 in Task 6.2 as 160 $mm^2$.

**Developing Area Formulae**

None of the PSSMTs in this study were able to develop the formula for the area of a rectangle. However, five, two, and three of the PSSMTs were able to develop the formula for the area of a parallelogram, triangle, and trapezium respectively. It indicated that most of the PSSMTs had inadequate procedural knowledge of developing area formulae.

Table 5.2

*Summary of PSSMTs' Procedural Knowledge of Perimeter and Area*

| Components of procedural knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Converting standard units of area measurement | Converting 3 cm² to mm$^2$: Successful | Correctly converted 3 cm² to 300 mm$^2$. | Beng, Liana, Patrick, Suhana, Tan |
| | Unsuccessful | Incorrectly converted 3 cm$^2$ to 3 x $10^{-4}$ mm$^2$. | Mazlan |
| | | Incorrectly converted 3 cm$^2$ to 30 mm$^2$. | Roslina, Usha |
| | Converting 4.7 m² to cm$^2$: Successful | Correctly converted 4.7 m² to 47 000 cm$^2$. | Beng, Liana, Patrick, Tan |
| | Unsuccessful | Incorrectly converted 4.7 m² to 470 cm$^2$. | Mazlan, Roslina, Usha |
| | | Incorrectly converted 4.7 m² to 470 000 cm$^2$. | Suhana |
| | Converting 1.25 km² to m$^2$: Successful | Correctly converted 1.25 k m² to 1 250 000 m$^2$. | Beng, Liana, Patrick, Tan |
| | Unsuccessful | Incorrectly converted 1.25 km$^2$ to 1250 x $10^6$ m$^2$. | Mazlan |
| | | Incorrectly converted 1.25 km$^2$ to 1250 m$^2$. | Roslina, Usha |
| | | Incorrectly converted 1.25 km$^2$ to 125 000 000 m$^2$. | Suhana |
| Calculating perimeter of composite figures | Calculating perimeter of Diagram 1: Successful | Correctly calculated the perimeter as 104 cm. | Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha |
| | Unsuccessful | Incorrectly calculated the perimeter as 86 cm. | Tan |
| | Calculating perimeter of Diagram 2: Successful | Correctly calculated the perimeter as 56 mm. | All PSSMTs |
| Calculating area of composite figures | Calculating area of Diagram 1: Successful | Correctly calculated the area as 420 cm$^2$. | Beng, Liana, Patrick, Roslina, Suhana, Tan |
| | Unsuccessful | Incorrectly calculated the area as 555 cm$^2$. | Mazlan, Usha |
| | Calculating area of Diagram 2: Successful | Correctly calculated the area as 160 mm$^2$. | All PSSMTs |
| Developing area formulae | Developing area formula of rectangle: Unsuccessful | Unable to develop the formula. | All PSSMTs |
| | Developing area formula of parallelogram: Successful | Able to develop the formula. | Beng, Mazlan, Patrick, Suhana, Tan |
| | Unsuccessful | Unable to develop the formula. | Liana, Roslina, Usha |
| | Developing area formula of triangle: Successful | Able to develop the formula. | Liana, Tan |
| | Unsuccessful | Unable to develop the formula. | Beng, Mazlan, Patrick, Suhana, Roslina, Usha |
| | Developing area formula of triangle: Successful | Able to develop the formula. | Beng, Suhana, Tan |
| | Unsuccessful | Unable to develop the formula. | Liana, Mazlan, Patrick, Roslina, Usha |

## Linguistic Knowledge

In this subsection, findings of PSSMTs' linguistic knowledge of perimeter and area are summarized in terms of its components: (a) mathematical symbols, (b) mathematical terms, (c) standard unit of length measurement (linear units), (d) standard unit of area measurement (square units), and (e) conventions of writing and reading SI area measurement. Table 5.3 demonstrates a summary of PSSMTs' linguistic knowledge of perimeter and area.

### Mathematical Symbols

Most of the PSSMTs used appropriate mathematical symbols to write the area formulae. For instance, seven, five, seven, and six of the PSSMTs used appropriate mathematical symbols to write the formula for the area of a rectangle, parallelogram, triangle, and trapezium respectively.

### Mathematical Terms

When asked to justify their selection of shapes that have a perimeter, all PSSMTs used appropriate mathematical terms to justify their selection of shapes A and C that have a perimeter as follow: (a) closed, (b) 2-dimensional shapes and enclosed, (c) covered, or (d) (can be) measured. All the PSSMTs who had selected shapes D, F, H, I, J, and K that have a perimeter used appropriate mathematical terms to justify their selection of shapes that have a perimeter as follow, except Liana: (a) closed, (b) 2-dimensional shapes and enclosed, (c) covered, (d) (can be) measured, or (e) (can be) calculated. Liana used inappropriate words 'joined together' to justify her selection of shapes D, F, H, I, J, and K that have a perimeter. Liana explained that she selected these shapes because all the lines are joined together. It indicated that all but one PSSMT who had selected shapes A, C, D, F, H, I, J, and K that have a perimeter used appropriate mathematical terms to justify their selection of shapes that have a perimeter.

One PSSMT, namely Tan, explained that he selected shapes C, D, I, and K because their perimeters can be measured using thread. Tan also explained that he selected shapes F and J because their perimeters can be calculated on each surface of the 3-dimensional objects. It indicated that Tan appeared to associate the notion of perimeter with the measurement of perimeter (i.e., perimeter does not exist until it is measured).

Five out of eight PSSMTs used appropriate mathematical terms ‘open’, ‘line’, negation ‘not closed’ or ‘not enclosed’ as the justification for not selecting shapes B and L as having a perimeter. All but one PSSMT used appropriate mathematical term ‘line’, negation ‘not enclosed’ or ‘not covered’ as the justification for not selecting shape E as having a perimeter. Half of the PSSMTs used appropriate mathematical terms ‘open’, ‘line’, negation ‘not enclosed’ or ‘not covered’ as the justification for not selecting shape G as having a perimeter. It indicated that at least half of the PSSMTs used appropriate mathematical terms as the justification for not selecting shapes B, E, G, and L as having a perimeter.

Three PSSMTs used inappropriate negation ‘not joined’ or ‘incomplete’ as the justification for not selecting shapes B and L as having a perimeter. Only one PSSMT used inappropriate negation ‘not connected’ as the justification for not selecting shape E as having a perimeter. Half of the PSSMTs used inappropriate negation ‘not joined’, ‘not connected’, or ‘incomplete’ as the justification for not selecting shape G as having a perimeter. Only one PSSMT used inappropriate mathematical term ‘curve’ as the justification for not selecting shapes D, I, and K that have a perimeter. Half of the PSSMTs used inappropriate mathematical terms ‘3-dimensional shapes’ or ‘3-dimensional objects’ as the justification for not selecting shapes F and J that have a perimeter.

When asked to justify their selection of shapes that have an area, all the PSSMTs who had selected shapes A, C, D, F, H, I, J, and K that have an area used appropriate mathematical terms to justify their selection of shapes that have an area as follow, except Liana: (a) closed, (b) enclosed, (c) (can be) calculated, or (d) 3D. Liana used inappropriate word ‘joining’ to justify her selection of shapes A, C, D, F, H, I, J, and K that have an area. Liana explained that she selected these shapes because the lines are joining. Five PSSMTs, namely Beng, Patrick, Roslina, Tan, and Usha, appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured).

All but one PSSMT used appropriate mathematical term ‘open’, negation ‘not closed’, ‘not enclosed’, ‘not covered’, or ‘not surrounded’ as the justification for not selecting shape B as having an area. Similarly, all but one PSSMT used appropriate mathematical term ‘line’ or negation ‘not surrounded’ as the justification for not selecting shape E as having an area. Six out of eight PSSMTs used appropriate mathematical term ‘open’, ‘line’, negation ‘not closed’, ‘not enclosed’, or ‘not surrounded’ as the justification for not selecting shapes G and L as having an area. It indicated that most of the PSSMTs used appropriate mathematical terms as the justification for not selecting shapes B, E, G, and L as having an area.

Only one PSSMT used inappropriate negation ‘not joining’ as the justification for not selecting shape B as having an area. Similarly, only one PSSMT used inappropriate negation ‘not joined’ as the justification for not selecting shape E as having an area. Two PSSMTs used inappropriate negation ‘not joined’ or ‘not joining’ as the justification for not selecting shapes G and L as having an area. Likewise, two PSSMTs used inappropriate negation ‘no specific formula that can be used to calculate its area’ as the justification for not selecting shapes D, I, and K that have an area. Only one PSSMT used inappropriate mathematical term ‘3-dimensional’ as the justification for not selecting shapes F and J that have an area.

When asked to justify the shapes that can be used as the unit of area, only one PSSMT, namely Patrick, used appropriate mathematical term ‘cover’ to justify that a square and rectangle can be used as the unit of area. He explained that a square and rectangle can be used as the unit of area because we can count the numbers of square and rectangular units it takes to cover a region. It indicated that Patrick knew that a square and rectangle tessellate a plane and thus can be used as the unit of area measurement. Nevertheless, none of the PSSMTs used appropriate mathematical term to justify that triangle can be used as the unit of area.

In this study, only two PSSMTs used appropriate mathematical terms ‘length’, ‘times’, and ‘width’ to state the formula for the area of a rectangle. Only one PSSMT used appropriate mathematical terms ‘*tapak* [base]’, ‘times’, and ‘*tinggi* [height]’ to state the formula for the area of a parallelogram. Half of the PSSMTs used appropriate mathematical terms to state the formula for the area of a triangle. Only two PSSMTs used appropriate mathematical terms ‘base’ and ‘height’ to explain the meaning of the symbols that they employed to write the formula for the area of a triangle. None of PSSMTs in this study used appropriate mathematical terms to state the formula for the area of a trapezium or to explain the meaning of the symbols that they employed to write the formula.

**Standard Unit of Length Measurement (Linear Units)
and Area Measurement (Square Units)**

All the PSSMTs in this study understand the general measurement convention that perimeter and area is measured by linear units (such as mm, cm, m, km) and square units (such as $mm^2$, $cm^2$, $m^2$, $km^2$), respectively.

**Conventions of Writing and Reading SI Area Measurement**

None of the PSSMTs in this study was able to read and write the area measurements 16 $cm^2$ and 13 $cm^2$ correctly in English. The correct answers should be 'sixteen square centimetres' and 'thirteen square centimetres' respectively. It indicated that all the PSSMTs in this study had limited linguistic knowledge about the conventions pertaining to writing and reading of Standard International (SI) area measurement units. None of the PSSMTs in this study knew the conventions pertaining to the writing and reading of Standard International (SI) area measurement units.

Table 5.3

*Summary of PSSMTs' Linguistic Knowledge of Perimeter and Area*

| Components of linguistic knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Mathematical symbols | Write area formula of rectangle: Used appropriate mathematical symbols | Wrote the formula as '$l \times w$', where $l$ and $w$ represents the length and the width of the rectangle, or equivalent form. | Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha |
| | Write area formula of parallelogram: Used appropriate mathematical symbols | Wrote the formula as '$b \times h$', where $b$ and $h$ represents the base and the height of the parallelogram, or equivalent form. | Beng, Mazlan, Patrick, Suhana, Tan |
| | Write area formula of triangle: Used appropriate mathematical symbols | Wrote the formula as '$\frac{1}{2} \times b \times h$', where $b$ and $h$ represents the base and the height of the triangle, or equivalent form. | Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan |
| | Write area formula of trapezium: Used appropriate mathematical symbols | Wrote the formula as '$\frac{1}{2} \times (a + b) \times h$', where $(a + b)$ and $h$ represents the sum of the parallel sides and the height of the trapezium, or equivalent form. | Beng, Mazlan, Patrick, Suhana, Tan, Usha |
| Mathematical terms | Justification of shapes that have a perimeter: Used appropriate mathematical terms | Used appropriate mathematical terms to justify the selection of: Shapes A and C that have a perimeter as follow: (a) closed, (b) 2-dimensional shapes and enclosed, (c) covered, or (d) (can be) measured. | All PSSMTs |
| | | Shapes D, I, and K that have a perimeter as follow: (a) closed, (b) 2-dimensional shapes and enclosed, (c) covered, or (d) (can be) measured. | Beng, Patrick, Roslina, Suhana, Tan, Usha |
| | | Shapes F and J that have a perimeter as follow: (a) closed or (b) (can be) calculated. | Roslina, Tan, Usha |
| | | Shape H that has a perimeter as follow: (a) closed, (b) 2-dimensional shapes and enclosed, or (c) covered. | Beng, Mazlan, Patrick, Roslina, Suhana, Tan, Usha |
| | Used inappropriate mathematical terms | Used inappropriate words 'joined together' to justify the selection of shapes D, F, H, I, J, and K that have a perimeter. | Liana |
| | Justification of shapes that do not have a perimeter: Used appropriate mathematical terms | Used appropriate mathematical term 'open', 'line', negation 'not closed', or 'not enclosed' as the justification for not selecting shapes B and L as having a perimeter. | Beng, Mazlan, Roslina, Suhana, Usha |
| | | Used appropriate mathematical term 'line', negation 'not enclosed' or 'not covered' as the justification for not selecting shape E as having a perimeter. | Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha |
| | | Used appropriate mathematical terms 'open', 'line', negation 'not enclosed' or 'not covered' as the justification for not selecting shape G as having a perimeter. | Beng, Mazlan, Roslina, Usha |

Table 5.3 (continued)

| Components of linguistic knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Mathematical terms | Justification of shapes that do not have a perimeter: | | |
| | Used inappropriate mathematical terms | Used inappropriate negation 'not joined' or 'incomplete' as the justification for not selecting shapes B and L as having a perimeter. | Liana, Patrick, Tan |
| | | Used inappropriate negation 'not connected' as the justification for not selecting shape E as having a perimeter. | Tan |
| | | Used inappropriate negation 'not joined', 'not connected', or 'incomplete' as the justification for not selecting shape G as having a perimeter. | Liana, Patrick, Suhana, Tan |
| | | Used inappropriate mathematical term 'curve' as the justification for not selecting shapes D, I, and K that have a perimeter. | Mazlan |
| | | Used inappropriate mathematical terms '3-dimensional shapes' or '3-dimensional objects' as the justification for not selecting shapes F and J that have a perimeter. | Beng, Mazlan, Patrick, Suhana |
| | Justification of shapes that have an area: | | |
| | Used appropriate mathematical terms | Used appropriate mathematical terms to justify the selection of: | |
| | | Shapes A, C, and H that have an area as follow: (a) closed, (b) enclosed, or (c) (can be) calculated. | Beng, Mazlan, Patrick, Roslina, Suhana, Tan, Usha |
| | | Shapes D, I, and K that have an area as follow: (a) closed, (b) enclosed, or (c) (can be) calculated. | Beng, Mazlan, Suhana, Tan, Usha |
| | | Shapes F and J that have an area as follow: (a) 3D, or (b) (can be) calculated. | Mazlan, Patrick, Roslina, Suhana, Tan, Usha |
| | Used inappropriate mathematical terms | Used inappropriate word 'joining' to justify the selection of shapes A, C, D, F, H, I, J, and K that have an area. | Liana |
| | Justification of shapes that do not have an area: | | |
| | Used appropriate mathematical terms | Used appropriate mathematical term 'open', negation 'not closed', 'not enclosed', 'not covered', or 'not surrounded' as the justification for not selecting shape B as having an area. | Beng, Mazlan, Patrick, Roslina, Suhana, Tan, Usha |
| | | Used appropriate mathematical term 'line' or negation 'not surrounded' as the justification for not selecting shape E as having an area. | Beng, Liana, Mazlan, Roslina, Suhana, Tan, Usha |
| | | Used appropriate mathematical term 'open', 'line', negation 'not closed', 'not enclosed', or 'not surrounded' as the justification for not selecting shapes G and L as having an area. | Beng, Mazlan, Roslina, Suhana, Tan, Usha |

Table 5.3 (continued)

| Components of linguistic knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Mathematical terms | Justification of shapes that do not have an area: | | |
| | Used inappropriate mathematical terms | Used inappropriate negation 'not joining' as the justification for not selecting shape B as having an area. | Liana |
| | | Used inappropriate negation 'not joined' as the justification for not selecting shape E as having an area. | Patrick |
| | | Used inappropriate negation 'not joined' or 'not joining' as the justification for not selecting shapes G and L as having an area. | Liana, Patrick |
| | | Used inappropriate negation 'no specific formula that can be used to calculate its area' as the justification for not selecting shapes D, I, and K that have an area. | Patrick, Roslina |
| | | Used inappropriate mathematical term '3-dimensional' as the justification for not selecting shapes F and J that have an area. | Beng |
| | Justification of shapes that can be used as the unit of area: | | |
| | Used appropriate mathematical terms | Used appropriate mathematical term 'cover' to justify that a square and rectangle can be used as the unit of area. | Patrick |
| | Used inappropriate mathematical terms | Used inappropriate mathematical terms 'same length', 'straight lines', 'its unit is "the power of two" (square unit)', or 'represents' to justify that a square can be used as the unit of area. | Beng, Mazlan, Roslina, Suhana, Tan, Usha |
| | | Used inappropriate mathematical terms 'straight lines', 'its unit is "the power of two" (square unit)', or 'represents' to justify that a rectangle can be used as the unit of area. | Beng, Tan, Usha |
| | | Used inappropriate mathematical terms 'straight lines', 'its unit is "the power of two" (square unit)', 'represents' or 'a triangle came from a square' to justify that a triangle can be used as the unit of area. | Beng, Mazlan, Tan, Usha |
| | State the area formulae or explain the meaning of the mathematical symbols: | | |
| | Used appropriate mathematical terms | Used appropriate mathematical terms 'length', 'times', and 'width' to state the formula for the area of a rectangle. | Beng, Mazlan |
| | | Used appropriate mathematical terms '*tapak* [base]', 'times', and '*tinggi* [height]' to state the formula for the area of a parallelogram. | Suhana |
| | | Used appropriate mathematical terms 'half or one over two', 'times', 'base' and 'height' to state the formula for the area of a triangle. | Mazlan, Patrick, Roslina, Suhana |
| | | Used appropriate mathematical terms 'base' and 'height' to explain the meaning of the symbols that they employed to write the formula for the area of a triangle. | Beng, Liana |

Table 5.3 (continued)

| Components of linguistic knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Mathematical terms | State the area formulae or explain the meaning of the mathematical symbols: Used inappropriate mathematical terms | Used inappropriate mathematical terms ‘horizontal side’ and ‘vertical side’ to state the formula for the area of a rectangle. | Tan |
| | | Used inappropriate mathematical terms to explain the meaning of the mathematical symbols that they employed to write the formula for the area of a rectangle. | Liana, Patrick, Roslina, Suhana, Usha |
| | | Used inappropriate mathematical terms ‘vertical (side)’ and ‘horizontal (side)’ to state the formula for the area of a parallelogram. | Tan |
| | | Used inappropriate mathematical terms to explain the meaning of the symbols that they employed to write the formula for the area of a parallelogram. | Beng, Patrick |
| | | Used inappropriate mathematical terms ‘vertical (side)’ and ‘horizontal (side)’ to state the formula for the area of a triangle as half times the vertical (side) times the horizontal (side). | Tan |
| | | Used inappropriate mathematical terms to explain the meaning of the symbols that they employed in the formula for the area of a trapezium. | Beng, Patrick |
| Standard unit of length measurement (linear units) | Understand the general measurement convention that perimeter is measured by linear unit. | Correctly wrote the unit of measurement for perimeter of Diagram 1 as cm. | All PSSMTs |
| | | Correctly wrote the unit of measurement for perimeter of Diagram 2 as mm. | Beng, Liana, Roslina, Suhana, Tan, Usha |
| | | Incorrectly wrote the unit of measurement for perimeter of Diagram 2 as cm. | Mazlan, Patrick |
| Standard unit of area measurement (square units) | Understand the general measurement convention that area is measured by square unit. | Correctly wrote the unit of measurement for area of Diagram 1 as $cm^2$. | All PSSMTs |
| | | Correctly wrote the unit of measurement for area of Diagram 2 as $mm^2$. | Beng, Liana, Patrick, Roslina, Suhana, Tan, Usha |
| | | Incorrectly wrote the unit of measurement for area of Diagram 2 as $cm^2$. | Mazlan |
| Conventions of writing and reading SI area measurement | All the PSSMTs in this study had limited linguistic knowledge about the conventions pertaining to writing and reading of Standard International (SI) area measurement units. | Incorrectly wrote and read 16 $cm^2$ and 13 $cm^2$ as ‘sixteen centimetre square’ and ‘thirteen centimetre square’ respectively, or equivalent form. | All PSSMTs |

## Strategic Knowledge

In this subsection, findings of PSSMTs’ strategic knowledge of perimeter and area are summarized in terms of its components: (a) strategies for comparing perimeter, (b) strategies for comparing area, (c) strategies for checking answer for perimeter, (d) strategies for checking answer for area, (e) strategies for solving the fencing problem, (f) strategies for checking answer for the fencing problem, and (g) strategies for developing area formulae. Table 5.4 reveals a summary of PSSMTs' strategic knowledge of perimeter and area.

**Strategies for Comparing Perimeter**

All the PSSMTs used the formal method to determine whether the given pair of shapes had the same perimeter. Seven out of eight PSSMTs used the formal method of measuring the side by ruler and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter. Only one PSSMT used the formal method of measuring the side by thread and ruler, and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter.

When probed for alternative method of comparing the perimeter, six out of eight PSSMTs could suggest at least one alternative method to compare the perimeter. The PSSMTs suggested three types of alternative methods to compare the perimeter, namely formal, semi-formal, and informal methods. Four subtypes of formal methods suggested by them were identified: (a) measuring the side by ruler and applying area formula, (b) measuring the side by thread and ruler and applying definition of perimeter, (c) measuring the side by compass and ruler and applying definition of perimeter, and (d) measuring the side by paper and ruler and applying definition of perimeter. Three subtypes of semi-formal methods were emerged: (a) measuring the side with a grid paper, (b) measuring the side with a blank paper, and (c) measuring the side with a piece of thread. Only one type of informal method was generated, namely cut and paste (i.e., cut one shape into pieces and paste onto the other). The formal method of measuring the side by thread and ruler and applying definition of perimeter, and the semi-formal method of using grid paper were the dominant alternative methods suggested by the PSSMTs.

**Strategies for Comparing Area**

In this study, seven out of eight PSSMTs used the formal method of measuring the side by ruler and applying area formula to determine whether the given pair of shapes had the same area. Only one PSSMT used the informal method of cut and paste to determine whether the given pair of shapes had the same area. When probed for alternative method of comparing the area, seven out of eight PSSMTs could suggest at least one alternative method to compare the area. The PSSMTs suggested three types of alternative methods to compare the area, namely formal, semi-formal, and informal methods. Three subtypes of formal methods suggested by them were identified: (a) measuring the side by ruler and applying area formula, (b) measuring the side by thread and ruler and applying area formula, and (c) measuring the side by compass and ruler and applying area formula. One type of semi-formal methods was emerged, namely covering both shapes with a grid paper. One type of informal method was generated, namely cut and paste (i.e., cut one shape into pieces and paste onto the other). The semi-formal method of covering both shapes with a grid paper, and the formal method of measuring the side by ruler and applying area formula were the dominant alternative methods suggested by the PSSMTs.

**Strategies for Checking Answer for Perimeter**

When probed to check the answer for the perimeter of Diagram 1, all but one PSSMT suggested that they would use the recalculating strategy to verify the answer. Liana used the doubling-and-sum algorithm to calculate the perimeter of the Diagram 1. Liana suggested that she would check the answer for perimeter by using an alternative method, namely list all-and-sum strategy. Similarly, when probed to check the answer for the perimeter of Diagram 2, five out of eight PSSMTs suggested that they would use the recalculating strategy to verify the answer. The remaining three PSSMTs suggested that they would use alternative method to verify the answer.

### Strategies for Checking Answer for Area

When probed to check the answer for the area of Diagram 1, half of the PSSMTs suggested that they would use the recalculating strategy to verify the answer. The other half of the PSSMTs used an alternative procedure (alternative method) to generate an answer which could be used to verify their original answer. Similarly, when probed to check the answer for the area of Diagram 2, three out of eight PSSMTs suggested that they would use the recalculating strategy to verify the answer. The remaining five PSSMTs used an alternative procedure (alternative method) to generate an answer which could be used to verify their original answer.

### Strategies for Solving the Fencing Problem

Half of the PSSMTs have successfully solving the fencing problem. Of the four PSSMTs who have successfully solving the fencing problem, two of them used the looking for a pattern strategy to solve the fencing problem. One PSSMT used the trial-and-error strategy while another PSSMT used the differentiation method to solve the fencing problem.

### Strategies for Checking Answer for the Fencing Problem

In this study, three out of eight PSSMTs used the looking for a pattern strategy to check the answer for the fencing problem without being probed. One PSSMT used the compare strategy to verify the answer without being probed. One PSSMT checked the answer of the fencing problem, without being probed, by calculating the value of $\frac{d^2A}{dx^2}$ at the stationary point. One PSSMT used the same strategy, namely trial and error strategy, to verify the answer while another PSSMT suggested that she would use the list all-and-compare strategy, to verify the answer.

**Strategies for Developing Area Formulae**

In this study, only one PSSMT attempted to develop the formula for the area of a rectangle but unsuccessful. Five out of eight PSSMTs had succeeded in developing the formula for the area of a parallelogram. They used the cut and paste strategy to develop the formula. Two PSSMTs had succeeded in developing the formula for the area of a triangle. They used the partition strategy to develop the formula. Three PSSMTs succeeded in developing the formula for the area of a trapezium using algebraic method.

Table 5.4

*Summary of PSSMTs' Strategic Knowledge of Perimeter and Area*

| Components of strategic knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Strategies for Comparing Perimeter | Used formal method to determine whether the given pair of shapes had the same perimeter. | Measuring the side by ruler and applying the definition of perimeter. | Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha |
| | | Measuring the side by thread and ruler, and applying the definition of perimeter. | Tan |
| | PSSMTs suggested three types of alternative methods to compare the perimeter, namely formal, semi-formal, and informal methods. | Formal methods:<br>Measuring the side by ruler and applying area formula. | Beng |
| | | Measuring the side by thread and ruler and applying definition of perimeter. | Beng, Patrick, Roslina, Usha |
| | | Measuring the side by compass and ruler and applying definition of perimeter. | Patrick, Usha |
| | | Measuring the side by paper and ruler and applying definition of perimeter. | Patrick |
| | | Semi-formal methods:<br>Tracing the given shapes on a 1-cm grid paper and then counting the number of unit on each side, or equivalent method. | Suhana, Tan, Usha |
| | | Marking the length of each side of the T-shape on the length of a blank paper. Repeated the same for the rectangle and sees whether it ended at the same point. | Suhana |
| | | Measuring the length of each side of the T-shape by a piece of thread. Using the same portion of thread to measure the total length of the rectangle. | Suhana |
| | | Informal method:<br>Cut and paste. | Suhana |
| Strategies for comparing area | Used formal method and informal method to determine whether the given pair of shapes had the same area. | Used formal method of measuring the side by ruler and applying area formula. | Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha |
| | | Used the informal method of cut and paste. | Tan |
| | PSSMTs suggested three types of alternative methods to compare the area, namely formal, semi-formal, and informal methods. | Formal methods:<br>Measuring the side by ruler and applying area formula | Beng, Liana, Tan |
| | | Measuring the side by thread and ruler and applying area formula | Usha |
| | | Measuring the side by compass and ruler and applying area formula | Usha |
| | | Semi-formal method:<br>Covering both shapes with a grid paper | Patrick, Roslina, Suhana, Tan, Usha |
| | | Informal method:<br>Cut and paste. | Suhana |

Table 5.4 (continued)

| Components of strategic knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Strategies for checking answer for perimeter | Recalculating strategy | Using the same method and calculate again to verify the answer for perimeter of Diagram 1. | Beng, Mazlan, Patrick, Roslina, Suhana, Tan, Usha |
| | | Using the same method and calculate again to verify the answer for perimeter of Diagram 2. | Mazlan, Patrick, Roslina, Suhana, Usha |
| | Alternative method | Using other method to verify the answer for perimeter of Diagram 1. | Liana |
| | | Using other method to verify the answer for perimeter of Diagram 2. | Beng, Liana, Tan |
| Strategies for checking answer for area | Recalculating strategy | Using the same method and calculate again to verify the answer for area of Diagram 1. | Mazlan, Roslina, Suhana, Usha |
| | | Using the same method and calculate again to verify the answer for area of Diagram 2. | Roslina, Suhana, Usha |
| | Alternative method | Using other method to verify the answer for area of Diagram 1. | Beng, Liana, Patrick, Tan |
| | | Using other method to verify the answer for area of Diagram 2. | Beng, Liana, Mazlan, Patrick, Tan |
| Strategies for solving the fencing problem | Looking for a pattern:<br>Successful | Able to solve the problem | Beng, Patrick |
| | Unsuccessful | Unable to solve the problem | Usha |
| | Trial-and-error:<br>Successful | Able to solve the problem | Suhana |
| | Unsuccessful | Unable to solve the problem | Mazlan, Roslina |
| | Differentiation method<br>Successful | Able to solve the problem | Tan |
| | Unsuccessful | Unable to solve the problem | Liana |
| Strategies for checking answer for the fencing problem | Looking for a pattern | Using looking for a pattern strategy to verify the answer. | Beng, Patrick, Usha |
| | Trial-and-error | Using trial-and-error strategy to verify the answer. | Mazlan |
| | Compare | Using compare strategy to verify the answer. | Suhana |
| | List all-and-compare | Using list all-and-compare strategy to verify the answer. | Roslina |
| | Calculating the value of $\frac{d^2A}{dx^2}$ at the stationary point | Calculating the value of $\frac{d^2A}{dx^2}$ at the stationary point to verify the answer. | Tan |
| Strategies for developing area formulae | Cut and paste strategy | Using cut and paste strategy to develop the formula for the area of a parallelogram. | Beng, Mazlan, Patrick, Suhana, Tan |
| | Partition strategy | Using partition strategy to develop the formula for the area of a triangle. | Liana, Tan |
| | Using algebraic method | Using algebraic method to develop the formula for the area of a trapezium. | Beng, Suhana, Tan |

## Ethical Knowledge

In this subsection, findings of PSSMTs' ethical knowledge of perimeter and area are summarized in terms of its components: (a) justifies one's mathematical ideas, (b) examines pattern within the domain of perimeter and area measurement, (c) formulates generalization within the domain of perimeter and area measurement, (d) tests generalization within the domain of perimeter and area measurement, (e) develops area formulae, (f) writes units of measurement upon completed a task, and (g) checks the correctness of their solutions or answers. Table 5.5 exhibits a summary of PSSMTs' ethical knowledge of perimeter and area.

### Justifies One's Mathematical Ideas

Justification of knowledge is one of the important activities in mathematics. In this study, all the PSSMTs had taken the effort to justify the selection of shapes that have a perimeter. All the PSSMTs provided appropriate justification for selecting shapes A and C that have a perimeter. All the PSSMTs who had selected shapes D, F, H, I, J, and K that have a perimeter provided appropriate justification for their selection, except Liana. She had provided inappropriate justification for selecting shapes D, F, H, I, J, and K that have a perimeter.

Five out of eight PSSMTs provided appropriate justification for not selecting shapes B and L as having a perimeter. All but one PSSMT provided appropriate justification for not selecting shape E as having a perimeter. Half of the PSSMTs provided appropriate justification for not selecting shape G as having a perimeter. Three PSSMTs provided inappropriate justification for not selecting shapes B and L as having a perimeter. Only one PSSMT provided inappropriate justification for not selecting shape E as having a perimeter. Half of the PSSMTs provided inappropriate justification for not selecting shape G as having a perimeter. Only one PSSMT provided inappropriate justification for not selecting shapes D, I, and K that have a perimeter. Half of the PSSMTs provided inappropriate justification for not selecting shapes F and J that have a perimeter.

All the PSSMTs had taken the effort to justify the selection of shapes that have an area. All the PSSMTs who had selected shapes A, C, D, F, H, I, J, and K that have an area provided appropriate justification for their selection, except Liana. She had provided inappropriate justification for selecting these shapes that have an area.

All but one PSSMT provided appropriate justification for not selecting shape B as having an area. Similarly, all but one PSSMT provided appropriate justification for not selecting shape E as having an area. Six out of eight PSSMTs provided appropriate justification for not selecting shapes G and L as having an area. Only one PSSMT provided inappropriate justification for not selecting shape B as having an area. Similarly, only one PSSMT provided inappropriate justification for not selecting shape E as having an area. Two PSSMTs provided inappropriate justification for not selecting shapes G and L as having an area. Likewise, two PSSMTs provided inappropriate justification for not selecting shapes D, I, and K that have an area. Only one PSSMT provided inappropriate justification for not selecting shapes F and J that have an area.

All the PSSMTs in this study had taken the effort to justify the shapes that can be used as the unit of area measurement. However, only one PSSMT, namely Patrick, provided appropriate justification for selecting a square and rectangle that can be used as the unit of area. Nevertheless, none of the PSSMTs provided appropriate justification for selecting a triangle that can be used as the unit of area.

**Examines Pattern, Formulates and Test Generalization**

Examines pattern, formulates and test generalization are some of the important activities in mathematics. Nevertheless, six out of eight PSSMTs in this study accepted the student's generalization that two shapes with the same perimeter have the same area without attempting to examine the possible pattern of the relationship between perimeter and area. Only one PSSMT generated a counterexample to refute the student's generalization while another PSSMT generated an example that concurred with the student's generalization. Similarly, three out of eight PSSMTs in this study accepted Mary's generalization that the garden with the longer perimeter has the larger area without attempting to examine the possible pattern of the relationship between perimeter and area. Only one PSSMT generated a counterexample to refute Mary's generalization while half of the PSSMTs generated an example that concurred with Mary's generalization.

Finding of this study suggests that six out of eight PSSMTs in this study accepted the student's generalization that two shapes with the same perimeter have the same area without attempting to formulate generalization pertaining to the relationship between perimeter and area. Only one PSSMT formulated a generalization that two shapes with the same perimeter may have the different area while another PSSMT formulated a generalization that concurred with the student's generalization. Similarly, three out of eight PSSMTs in this study accepted Mary's generalization that the garden with the longer perimeter has the larger area without attempting to formulate generalization pertaining to the relationship between perimeter and area. Only two PSSMTs formulated a generalization that the garden with the longer perimeter could have a smaller area while another three PSSMTs formulated a generalization that concurred with Mary's generalization.

Finding of this study also suggests that six out of eight PSSMTs in this study accepted the student's generalization that two shapes with the same perimeter have the same area without attempting to test generalization pertaining to the relationship between perimeter and area. Only one PSSMT tested the student's generalization with a counterexample while another PSSMT tested the student's generalization with an example. Similarly, three out of eight PSSMTs in this study accepted Mary's generalization that the garden with the longer perimeter has the larger area without attempting to test generalization pertaining to the relationship between perimeter and area. Only one PSSMT tested Mary's generalization with a counterexample while half of the PSSMTs tested Mary's generalization with an example. Likewise, half of the PSSMTs in this study accepted the student's generalization that the garden with the longer perimeter has the larger area without attempting to test generalization pertaining to the relationship between perimeter and area. Only one PSSMT tested the student's generalization with a counterexample while another three PSSMTs tested the student's generalization with an example.

**Develops Area Formulae**

Developing area formulae is one of the important activities in mathematics. In this study, only one out of eight PSSMTs had attempted to develop the formula for the area of a rectangle but unsuccessful. However, five PSSMTs had attempted to develop the formula for the area of a parallelogram. They succeeded in developing the formula. Three PSSMTs had attempted to develop the formula for the area of a triangle. Two of them had succeeded in developing the formula. Five PSSMTs had attempted to develop the formula for the area of a trapezium. Three of them succeeded in developing the formula.

**Writes Units of Measurement upon Completed a Task**

In Tasks 6.1 and 6.2, all the PSSMTs wrote the measurement units (without probed) for the answers of the perimeters and areas of Diagrams 1 and 2 that they have calculated. In Task 7, five out of eight PSSMTs wrote measurement unit for the largest area being enclosed. However, only two PSSMTs wrote measurement unit for the dimension that they thought would yield the largest area being enclosed.

**Checks the Correctness of Their Solutions or Answers**

Most of the PSSMTs in this study did not check the correctness of their answers. Once getting an answer, they seemed to satisfy that the task was finished. When probed to check answer, then only they suggested the strategies that they would use to check the answers. For example, none of the PSSMTs checked the correctness of their answers for the perimeter and area of Diagram 1 in Task 6.1, and perimeter of Diagram 2 in Task 6.2. When probed to check answer, then only all the PSSMTs suggested the strategies that they would use to check the answers for perimeters and area. However, in Task 6.2, only three out of eight PSSMTs checked the correctness of the answer for the area of Diagram 2 without being probed. When probed to check answers, then only the other five PSSMTs suggested the strategy that they would use to check the answer for area.

In Task 7, five PSSMTs checked the correctness of their answers for the fencing problem without being probed. Only two PSSMTs did not check the correctness of the answer for the fencing problem. When probed to check answers, then only these two PSSMTs suggested the strategy that they would use to check their answers for the fencing problem.

Table 5.5

*Summary of PSSMTs' Ethical Knowledge of Perimeter and Area*

| Components of ethical knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Justifies one's mathematical ideas | Justify the selection of shapes that have a perimeter: | | |
| | Provided appropriate justification | Provided appropriate justification for selecting: | |
| | | Shapes A and C that have a perimeter. | All PSSMTs |
| | | Shapes D, I, and K that have a perimeter. | Beng, Patrick, Roslina, Suhana, Tan, Usha |
| | | Shapes F and J that have a perimeter. | Roslina, Tan, Usha |
| | | Shape H that has a perimeter. | Beng, Mazlan, Patrick, Roslina, Suhana, Tan, Usha |
| | Provided inappropriate justification | Provided inappropriate justification for selecting shapes D, F, H, I, J, and K that have a perimeter. | Liana |
| | Justify the selection of shapes that do not have a perimeter: | | |
| | Provided appropriate justification | Provided appropriate justification for not selecting: | |
| | | Shapes B and L as having a perimeter. | Beng, Mazlan, Roslina, Suhana, Usha |
| | | Shape E as having a perimeter. | Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha |
| | | Shape G as having a perimeter. | Beng, Mazlan, Roslina, Usha |
| | Provided inappropriate justification | Provided inappropriate justification for not selecting: | |
| | | Shapes B and L as having a perimeter. | Liana, Patrick, Tan |
| | | Shape E as having a perimeter. | Tan |
| | | Shape G as having a perimeter. | Liana, Patrick, Suhana, Tan |
| | | Shapes D, I, and K that have a perimeter. | Mazlan |
| | | Shapes F and J that have a perimeter. | Beng, Mazlan, Patrick, Suhana |
| | Justify the selection of shapes that have an area: | | |
| | Provided appropriate justification | Provided appropriate justification for selecting: | |
| | | Shapes A, C, and H that have an area. | Beng, Mazlan, Patrick, Roslina, Suhana, Tan, Usha |
| | | Shapes D, I, and K that have an area. | Beng, Mazlan, Suhana, Tan, Usha |
| | | Shapes F and J that have an area . | Mazlan, Patrick, Roslina, Suhana, Tan, Usha |
| | Provided inappropriate justification | Provided inappropriate justification for selecting shapes A, C, D, F, H, I, J, and K that have an area. | Liana |

Table 5.5 (continued)

| Components of ethical knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Justifies one's mathematical ideas | Justify the selection of shapes that do not have an area: | | |
| | Provided appropriate justification | Provided appropriate justification for not selecting: | |
| | | Shape B as having an area. | Beng, Mazlan, Patrick, Roslina, Suhana, Tan, Usha |
| | | Shape E as having an area. | Beng, Liana, Mazlan, Roslina, Suhana, Tan, Usha |
| | | Shapes G and L as having an area. | Beng, Mazlan, Roslina, Suhana, Tan, Usha |
| | Provided inappropriate justification | Provided inappropriate justification for not selecting: | |
| | | Shape B as having an area. | Liana |
| | | Shape E as having an area. | Patrick |
| | | Shapes G and L as having an area. | Liana, Patrick |
| | | Shapes D, I, and K that have an area. | Patrick, Roslina |
| | | Shapes F and J that have an area. | Beng |
| | Justify the shapes that can be used as the unit of area: | | |
| | Provided appropriate justification | Provided appropriate justification that a square and rectangle can be used as the unit of area. | Patrick |
| | Provided inappropriate justification | Provided inappropriate justification that: | |
| | | A square can be used as the unit of area. | Beng, Mazlan, Roslina, Suhana, Tan, Usha |
| | | A rectangle can be used as the unit of area. | Beng, Tan, Usha |
| | | A triangle can be used as the unit of area. | Beng, Mazlan, Tan, Usha |
| Examines pattern | Two shapes with the same perimeter have the same area? | | |
| | Attempted and appropriate | Generated a counterexample to refute the student's generalization. | Suhana |
| | Attempted but inappropriate | Generated an example that concurred with the student's generalization. | Roslina |
| | Did not attempt | Accepted the student's generalization without attempting to examine the possible pattern of the relationship between perimeter and area. | Beng, Liana, Mazlan, Patrick, Tan, Usha |
| | The garden with the longer perimeter has the larger area? | | |
| | Attempted and appropriate | Generated a counterexample to refute Mary's generalization. | Beng |
| | Attempted but inappropriate | Generated an example that concurred with Mary's generalization. | Patrick, Roslina, Suhana, Usha |
| | Did not attempt | Accepted Mary's generalization without attempting to examine the possible pattern of the relationship between perimeter and area. | Liana, Mazlan, Tan |

Table 5.5 (continued)

| Components of ethical knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Formulates generalization | Two shapes with the same perimeter have the same area? | | |
| | Attempted and appropriate | Formulated a generalization that two shapes with the same perimeter may have the different area. | Suhana |
| | Attempted but inappropriate | Formulated a generalization that concurred with the student's generalization. | Roslina |
| | Did not attempt | Accepted the student's generalization without attempting to formulate generalization pertaining to the relationship between perimeter and area. | Beng, Liana, Mazlan, Patrick, Tan, Usha |
| | The garden with the longer perimeter has the larger area? | | |
| | Attempted and appropriate | Formulated a generalization that the garden with the longer perimeter could have a smaller area. | Beng, Suhana |
| | Attempted but inappropriate | Formulated a generalization that concurred with Mary's generalization. | Patrick, Roslina, Usha |
| | Did not attempt | Accepted Mary's generalization without attempting to formulate generalization pertaining to the relationship between perimeter and area. | Liana, Mazlan, Tan |
| Tests generalization | Two shapes with the same perimeter have the same area? | | |
| | Attempted and appropriate | Tested the student's generalization with a counterexample. | Suhana |
| | Attempted but inappropriate | Tested the student's generalization with an example. | Roslina |
| | Did not attempt | Accepted the student's generalization without attempting to test generalization pertaining to the relationship between perimeter and area. | Beng, Liana, Mazlan, Patrick, Tan, Usha |
| | The garden with the longer perimeter has the larger area? | | |
| | Attempted and appropriate | Tested Mary's generalization with a counterexample. | Beng |
| | Attempted but inappropriate | Tested Mary's generalization with an example. | Patrick, Roslina, Suhana, Usha |
| | Did not attempt | Accepted Mary's generalization without attempting to test generalization pertaining to the relationship between perimeter and area. | Liana, Mazlan, Tan |
| | As the perimeter of a closed figure increases, the area also increases? | | |
| | Attempted and appropriate | Tested the student's generalization with a counterexample. | Tan |
| | Attempted but inappropriate | Tested the student's generalization with an example. | Beng, Liana, Patrick |
| | Did not attempt | Accepted the student's generalization without attempting to test generalization pertaining to the relationship between perimeter and area. | Mazlan, Roslina, Suhana, Usha |

Table 5.5 (continued)

| Components of ethical knowledge | Findings | Evidence | PSSMTs |
|---|---|---|---|
| Develops area formulae | Develop area formula for a rectangle: | | |
| | Attempted | Attempted to develop the formula. | Tan |
| | Did not attempt | Did not attempt. | Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha |
| | Develop area formula for a parallelogram: | | |
| | Attempted | Attempted to develop the formula. | Beng, Mazlan, Patrick, Suhana, Tan |
| | Did not attempt | Did not attempt. | Liana, Roslina, Usha |
| | Develop area formula for a triangle: | | |
| | Attempted | Attempted to develop the formula. | Liana, Suhana, Tan |
| | Did not attempt | Did not attempt. | Beng, Mazlan, Patrick, Roslina, Usha |
| | Develop area formula for a trapezium: | | |
| | Attempted | Attempted to develop the formula. | Beng, Mazlan, Patrick, Suhana, Tan |
| | Did not attempt | Did not attempt. | Liana, Roslina, Usha |
| Writes units of measurement upon completed a task | Write unit for perimeter of Diagram 1 | Wrote the unit, namely cm (without probed). | All PSSMTs |
| | Write unit for area of Diagram 1 | Wrote the unit, namely $cm^2$ (without probed). | All PSSMTs |
| | Write unit for perimeter of Diagram 2 | Wrote the unit (without probed). | All PSSMTs |
| | Write unit for area of Diagram 2 | Wrote the unit (without probed). | All PSSMTs |
| | Write unit for the largest area being enclosed: | | |
| | Wrote | Wrote the unit, namely $m^2$ (without probed). | Mazlan, Patrick, Roslina, Tan, Usha |
| | Did not write | Did not write. | Beng, Suhana |
| | Write unit for the dimension that yield the largest area: | | |
| | Wrote | Wrote the unit, namely m (without probed). | Roslina, Usha |
| | Did not write | Did not write. | Beng, Mazlan, Patrick, Suhana, Tan |
| Checks the correctness of their solutions or answers | Did not check the answers of the perimeter and area of Diagram 1, and perimeter of Diagram 2 | Did not check the answers. | All PSSMTs |
| | Check the answer of the area of Diagram 2: | | |
| | Checked | Checked the answer (without probed). | Liana, Patrick, Tan |
| | Did not check | Did not check. | Beng, Mazlan, Roslina, Suhana, Usha |
| | Check the answer for the fencing problem: | | |
| | Checked | Checked the answer (without probed). | Beng, Patrick, Suhana, Tan, Usha |
| | Did not check | Did not check. | Mazlan, Roslina |

## Level of Subject Matter Knowledge

With regard to the conceptual knowledge of perimeter and area, only two of the PSSMTs have a high level of knowledge, four with medium level and two at low level. With respect to the procedural knowledge of perimeter and area, four of the PSSMTs have a high level of knowledge, one with medium level and three at low level. For the linguistic knowledge of perimeter and area, only one of the PSSMTs have a high level of knowledge, six with medium level and one at low level. With regard to the strategic knowledge of perimeter and area, five of the PSSMTs have a high level of knowledge while three with medium level. With respect to the ethical knowledge of perimeter and area, seven of the PSSMTs with a medium level of knowledge while only one at low level.

Only one of the PSSMTs, namely Tan, secured an overall high level of SMK of perimeter and area. Six of the PSSMTs, namely Beng, Mazlan, Patrick, Roslina, Suhana, and Usha, achieved an overall medium level of SMK of perimeter and area. They obtained the percentage of appropriate mathematical elements of SMK of perimeter and area ranged from 52.1% to 69.7% during the clinical interview. Only one of the PSSMTs, namely Liana, gained an overall low level of SMK of perimeter and area. Table 5.6 illustrates a summary of PSSMTs' levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area as well as the overall level SMK of perimeter and area.

**Table 5.6**

*Summary of PSSMTs' Levels of Conceptual Knowledge, Procedural Knowledge, Linguistic Knowledge, Strategic Knowledge, and Ethical Knowledge of Perimeter and Area as Well as the Overall Level of SMK of Perimeter and Area*

| Levels of conceptual knowledge | | | Levels of procedural knowledge | | | Levels of linguistic knowledge | | | Levels of strategic knowledge | | | Levels of ethical knowledge | | | Overall level of SMK | | | PSSMTs |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| L | M | H | L | M | H | L | M | H | L | M | H | L | M | H | L | M | H | |
| | x | | | | x | | x | | | | x | | x | | | x | | Beng |
| | | x | | | x | x | | | | x | | x | | | x | | | Liana |
| x | | | x | | | | x | | | x | | | x | | | x | | Mazlan |
| | x | | | | x | | x | | | | x | | x | | | x | | Patrick |
| x | | | x | | | | x | | | x | | | x | | | x | | Roslina |
| | x | | | x | | | | x | | | x | | x | | | x | | Suhana |
| | | x | | | x | | x | | | | x | | x | | | | x | Tan |
| | x | | x | | | | x | | | | x | | x | | | x | | Usha |

Legend: L = Low M = Medium H = High

## Discussion and Conclusions

The finding of this study shows that none of the PSSMTs in this study selected open shapes (including the lines), namely B, E, G, and L, as having an area. Based on these selections, it can be inferred that all the PSSMTs in this study did not have a dynamic perspective of the notion of area. PSSMTs selected only closed shapes as having an area indicated that they had a static perspective of the notion of area. Two PSSMTs did not select irregular closed plane shapes (i.e., D, I, K) that have an area. These PSSMTs provided inappropriate justification that they did not select shapes D, I, and K because there is no specific formula that can be used to calculate their area. For instance, Patrick and Roslina explained that they did not select shape D because:

"…*tak ada satu formula yang khas untuk mengira….* […no specific formula to calculate (its area)]…." (Patrick/L290-291). "So, for D, there is no single formula to calculate this kind of shape in area. Area will be strictly with the formula. So, no, you can not calculate the area for picture D" (Roslina/ L181-183). It indicated that these PSSMTs appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). These findings are concurs with the findings of previous research project regarding area measurement (Baturo & Nason, 1996).

However, the finding of this study depicts that only one PSSMT, namely Beng, did not select 3-dimensional shapes (i.e., F and J) that have an area. Beng provided inappropriate justification that she did not select shapes F and J because they are 3D (3-dimensional). For example, Beng explained that she did not select shape J because "J also same as F. It is a 3D thing. May be we can find volume for it but I don't think we can calculate the area" (Beng/L184-185). This finding is slightly in contrast with the finding of previous research (Baturo & Nason, 1996) which found that all the preservice primary school teachers in their study selected the solid shapes as having an area.

The formal method of 'measuring the side by ruler and applying area formula' was the dominant strategy employed by the PSSMTs in this study to compare the areas of the given pair of shapes (i.e., T-shape and rectangle). Seven out of eight PSSMTs were able to generate at least one alternative method (i.e., semi-formal or informal method) to compare the areas. It indicated that these PSSMTs exhibited adequate strategic knowledge for comparing areas as they could generate method other than applying formula (i.e., formal method) to compare the areas. These findings are in concurrence with the findings of previous research (Baturo & Nason, 1996) which found that the formal method of 'measuring the side by ruler and applying area formula' was the dominant strategy employed by the preservice primary school teachers to compare the areas of the given pair of shapes (i.e., T-shape and square). They also found that most of the preservice teachers in their research project were able to generate at least one alternative method (i.e., semi-formal or informal method) to compare the areas.

None of the PSSMTs in this study was able to correctly write and read 16 $cm^2$ and 13 $cm^2$ as ‘sixteen square centimetres’ and ‘thirteen square centimetres’ respectively. All of them write and read 16 $cm^2$ and 13 $cm^2$ literally as ‘sixteen centimetre square (some with error in spelling)’ and ‘thirteen centimetre square (some with error in spelling)’ respectively (see Table 4.44). It indicated that all the PSSMTs in this study had limited linguistic knowledge about the conventions pertaining to writing and reading of Standard International (SI) area measurement units. None of the PSSMTs in this study knew the conventions pertaining to the writing and reading of Standard International (SI) area measurement units. This finding is in contrast with the recommendations in the Form One Mathematics Curriculum Specifications (Ministry of Education Malaysia, 2003a) which suggests that $cm^2$ to be read as ‘square cm’. This finding is also slightly in contrast with the finding of previous research (Baturo & Nason, 1996) which found that one of the 13 preservice primary school teachers in their study was able to correctly read 6 $m^2$ as ‘six square metres’. The rest read it literally as ‘six metres squared’.

The finding of this study demonstrates that three out of eight PSSMTs in this study exhibited a lack of conceptual knowledge about the relationship between the standard units of area measurement. For instance, Mazlan knew that 1 $km^2$ = 1 000 000 $m^2$. Nevertheless, he did not know that 1 $cm^2$ = 100 $mm^2$ and 1 $m^2$ = 10 000 $cm^2$. Roslina and Usha thought that 1 $cm^2$ = 10 $mm^2$, 1 $m^2$ = 100 $cm^2$, and 1 $km^2$ = 1000 $m^2$. It indicated that they did not know the relationships between the standard units of area measurement that 1 $cm^2$ = 100 $mm^2$, 1 $m^2$ = 10 000 $cm^2$, and 1 $km^2$ = 1000 000 $m^2$. This finding is slightly in contrast with the finding of previous research (Baturo & Nason, 1996) which found that 11 of the 13 preservice primary school teachers in their study stated that there were 100 square centimeters in a square metre and thus 128 $cm^2$ was larger than 1 $m^2$. It indicated that most of the preservice teachers in their study demonstrated a lack of conceptual knowledge about the relationship between the standard units of area measurement.

Six out of eight PSSMTs in this study had adequate conceptual knowledge about the relationship between area units and linear units of measurement. They knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. For example, in Task 4, Liana viewed 3 $cm^2$ as the product of 3 times 1 $cm^2$. Liana times ten squared, $(10)^2$, when she converted 3 $cm^2$ to $mm^2$. It indicated that Liana knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. This finding is in contrast with the finding of previous research (Baturo & Nason, 1996) which found that many preservice primary school teachers in their study demonstrated a lack of conceptual knowledge about the relationship between area units and linear units of measurement that they did not know that area units are derived from linear units based on squaring. Thus, they did not know how to derive area units from linear unit.

All the PSSMTs in this study understand the inverse proportion between the number of units and the unit of measure. They knew that the longer the unit of measure, the smaller the number of units required to get the same length and vice versa. For instance, in Task 3.3 (b), in a situation when shapes A and B had the same perimeter, Suhana explained that "shape B have (sic) the paper clips that is smaller (shorter) than paper clips in shape A. So, 10 paper clips in shape A equal to 15 paper clips in shape B. (Writes the following): 10 A = 15 B" (Suhana/L591-594). It indicated that she understand the inverse proportion between the number of units and the unit of measure that the shorter the unit of measure, the larger the number of units required to get the same length and vice versa. Similarly, all the PSSMTs in this study knew that the larger the unit of measure, the smaller the number of units required to get the same area and vice versa. For example, in Task 3.4 (b), in a situation when shapes A and B had the same area, Usha explained that:

> Shape B. For shape B, they used the smaller squares. So, they need more squares, 15 squares. Then this one is 10 squares, this one is more bigger (sic). The shape, the unit that they used is different *lah*. This one is smaller (points to squares in shape B) and this one is bigger (points to squares in shape A). (Usha/L812-816)

It indicated that she understand the inverse proportion between the number of units and the unit of measure that the smaller the unit of measure, the larger the number of units required to get the same area and vice versa. This finding agrees with the finding of previous study regarding area measurement (Baturo & Nason, 1996) which found that all the preservice primary school teachers in their study had an understanding of the inverse proportion between the number of units and the unit of measure that the larger the unit of measure, the smaller the number of units required to get the same area and vice versa.

The finding of this study suggests that most of the PSSMTs in this study had a misconception that there is direct relationship between perimeter and area. For instance, five out of eight PSSMTs in this study thought that two shapes with the same perimeter have the same area. Thus, in Task 5.1, they thought that the student’s method of calculating the area of the leaf was correct. In Task 5.1, a Form One student claimed that he found a way to calculate the area of a leaf. The student placed a piece of thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle. The student’s method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter have the same area. This finding is in accordance with the findings of previous studies (Arnold, Turner, & Cooney, 1996; Chappell & Thompson, 1999; Woodward, 1982; Woodward & Byrd, 1983).

Woodward (1982) found that an excellent seventh grade student, Heidi, thought that the garden with the same perimeter have the same area. Woodward and Byrd (1983) revealed that 59% of the 129 eight grade students at a junior high school in Tennessee thought that the garden with the same perimeter have the same area. They also revealed that 63% of the 129 eight grade students at another junior high school in Tennessee thought that the garden with the same perimeter have the same area. Woodward and Byrd (1983) found that prospective elementary teachers took the test with similar results.

Arnold et al. (1996) revealed that most of the middle school and university students in their study thought that when the perimeter of a shape is held constant, its area remains constant. Similarly, Chappell and Thompson (1999) found that only one out of 29 (i.e., 3%) grade six students in their study were able to justify that two shapes with the same area could have different perimeters. None of the 19 grade seven students in their study were able to justify that two shapes with the same area could have different perimeters while three out of 16 (i.e., 19%) grade eight students in their study were able to justify that two shapes with the same area could have different perimeters.

Likewise, five out of eight PSSMTs in this study thought that the shape with the longer perimeter has the larger area. Thus, in Task 5.2, they thought that Mary's claim that the garden with the longer perimeter has the larger area was correct. This finding is in concurrence with the finding of previous study (Tierney, Boyd, & Davis, 1990). Tierney et al. (1990) noticed that about half of the prospective primary school teachers from a teachers college in their study thought that the shape with the longer perimeter has the larger area.

Similarly, the finding of this study reveals that only two out of eight PSSMTs in this study knew that in Task 5.3, the student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct. They knew that there is no direct relationship between perimeter and area. Beng and Tan knew that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. The remaining six PSSMTs thought that there is direct relationship between perimeter and area. Thus, they thought that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was correct. This finding is consistent with the finding of previous research studies (Arnold et al., 1996; Ball, 1988) regarding the relationship between perimeter and area. Ball (1988) found that only one of the nine prospective secondary teachers in her study knew that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct. Jon knew that there is no direct relationship between perimeter and area. Five of the prospective secondary teachers in her study thought that there is direct relationship between perimeter and area. Thus, they thought that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was correct. The remaining three prospective secondary teachers were not sure whether the student's "theory" was correct or not.

Ball (1988) also found that only two of the ten prospective elementary teachers in her study knew that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct. They knew that there is no direct relationship between perimeter and area. Three of the prospective elementary teachers in her study thought that there is direct relationship between perimeter and area. Thus, they thought that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was correct. The remaining five prospective elementary teachers were not sure whether the student's "theory" was correct or not. Arnold et al. (1996) also revealed that most of the middle school and university students in their study thought that when the perimeter of a shape increases or decreases, its area also increases or decreases.

This finding also consistent with the finding of Ma's (1999) study regarding the relationship between perimeter and area on the sample of 23 elementary school mathematics teachers from the United States (U.S.). The finding from her study suggests that only one of the 23 elementary school mathematics teachers from the U.S. knew that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct. Ms. Faith knew that there is no direct relationship between perimeter and area. Two of the elementary school mathematics teachers from the U.S. thought that there is direct relationship between perimeter and area. Thus, they thought that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was correct. The remaining 20 elementary school mathematics teachers from the U.S. were not sure whether the student's "theory" was correct or not.

However, this finding is in contrast with the finding of Ma's (1999) study regarding the relationship between perimeter and area on the sample of 72 elementary school mathematics teachers from China. The finding from her study suggests that 50 of the 72 elementary school mathematics teachers from China knew that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct. They knew that there is no direct relationship between perimeter and area. The remaining 22 elementary school mathematics teachers from China thought that there is direct relationship between perimeter and area. Thus, they thought that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was correct.

This finding is also in contrast with Leung and Park's (2002) study regarding the relationship between perimeter and area. Leung and Park (2002) suggests that six of the nine elementary school mathematics teachers from Hong Kong knew that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct. They knew that there is no direct relationship between perimeter and area. The remaining 3 elementary school mathematics teachers from Hong Kong were not sure whether the student's "theory" was correct or not. Leung and Park (2002) also suggests that six of the nine elementary school mathematics teachers from Korea knew that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct. They knew that there is no direct relationship between perimeter and area. One of the elementary school mathematics teachers from Korea thought that there is direct relationship between perimeter and area. Thus, this teacher thought that the student's "theory" that as the perimeter of a closed figure increases, the area also increases was correct. The remaining two elementary school mathematics teachers from Korea were not sure whether the student's "theory" was correct or not.

Most of the PSSMTs in this study had adequate procedural knowledge of calculating perimeter and area of composite figures. For instance, all but one PSSMT had successfully calculated the perimeter of Diagram 1 in Task 6.1 as 104 cm while all PSSMTs had successfully calculated the perimeter of Diagram 2 in Task 6.2 as 56 mm. Tan mentally cut the triangle TRS of Diagram 1 and pasted it next to the triangle TQR of Diagram 1 so that it formed a rectangle ("TQSR") with the dimension of 15 cm by 8 cm. Tan did not know that the "cut and paste" transformation does not conserve the perimeter of a diagram. Thus, he incorrectly calculated the perimeter of the diagram as 86 cm. Similarly, six out of eight PSSMTs had successfully calculated the area of Diagram 1 in Task 6.1 as 420 $cm^2$ while all PSSMTs had successfully calculated the area of Diagram 2 in Task 6.2 as 160 $mm^2$. Mazlan and Usha confused with the slanted side and the height of the parallelogram in Diagram 1 that they used the slanted side QR as the height (TR = 8 cm) of the parallelogram. Thus, Mazlan and Usha incorrectly calculated the area of the diagram as 555 $cm^2$.

This finding is concurs with Cavanagh (2008) and van de Walle (2007) who found that students tend to confuse with the slanted side (slanted height) and the height (perpendicular height) of a parallelogram. However, the finding of this study is in contrast with the findings of Baturo and Nason (1996) and Tsang and Rowland (2005). Baturo and Nason (1996) revealed that preservice primary school teachers in their study had inadequate procedural knowledge of calculating area of the given shapes in their Task 7. Tsang and Rowland (2005) found that about half of the 138 Hong Kong primary school mathematics teachers in their study could recalled Pythagoras' theorem in finding the length of the slanted side (5 cm) of the parallelogram and correctly calculated the perimeter (i.e., 18 cm) of the parallelogram drawn in the square grid (each square represents a square of length 1 cm). Tsang and Rowland (2005) revealed that quite a number of teachers in their study employed the area formula of a trapezium (not parallelogram) in finding the correct area of the parallelogram (i.e., 20 $cm^2$). Moreover, a few teachers confused the formula for the area of a triangle with the formula for the area of a parallelogram and gave 10 $cm^2$ as the area of the parallelogram.

All the PSSMTs in this study understand the general measurement convention that perimeter and area is measured by linear units (such as mm, cm, m, km) and square units (such as $mm^2$, $cm^2$, $m^2$, $km^2$) respectively. These findings are in contrast with the findings of previous studies (Baturo & Nason, 1996; Cavanagh, 2008; Tierney, Boyd, & Davis, 1990). Tierney, Boyd, and Davis (1990) noticed that many prospective primary school teachers from a teachers college in their study labelled the area measurements in linear units. Likewise, Baturo and Nason (1996) found that several preservice primary school teachers in their study wrote the calculated area measurement in linear unit such as 128 cm. They did not understand the general measurement convention that area is measured by square units. Similarly, Cavanagh (2008) revealed that high school students in his study inappropriately labelled the length of sides in $cm^2$ or areas in cm on the written test. They did not understand the general measurement convention that length is measured in linear units while area is measured in square units.

Most of the PSSMTs in this study did not check the correctness of their answers. Once getting an answer, they seemed to satisfy that the task was finished. When probed to check answer, then only they suggested the strategies that they would use to check the answers. For instance, in Tasks 6.1 and 6.2, none of the PSSMTs in this study checked the correctness of the answers for the perimeter and area of Diagram 1, and perimeter Diagram 2. Only three out of eight PSSMTs, namely Liana, Patrick and Tan, checked the correctness of the answer for the area of Diagram 2 without being probed. This finding is in agreement with the finding of previous study (Baturo & Nason, 1996) which found that majority of the preservice primary school teachers in their study had to be prodded towards checking their answers.

The PSSMTs in this study employed two types of strategies to verify their answers for calculating perimeter and area of composite figures in Tasks 6.1 and 6.2, namely recalculating strategy and alternative method. Recalculating strategy refers to strategy using the same method and calculates again while alternative method refers to method that was different from the original method. For instance, Beng used list all-and-sum algorithm to calculate perimeter of Diagram 1 in Task 6.1. When prompted to check her answer, she employed recalculating strategy, namely list all-and-sum algorithm, to verify her answer. Liana used doubling-and-sum algorithm to calculate perimeter of Diagram 1 in Task 6.1. When solicited to check her answer, she employed alternative method, namely list all-and-sum algorithm, to verify her answer. This finding is slightly contrast with the finding of previous study (Baturo & Nason, 1996) which found that most preservice primary school teachers in their study who attempted to verify their answers did so by recalculating strategy or using the inverse operation. They never think of using an alternative method to verify their answers.

The goal of the mathematics curriculum for secondary school in Malaysia is to develop individuals who are able to think mathematically and can apply mathematical knowledge effectively and responsibly in solving problems and making decision (Ministry of Education Malaysia, 2003a). Thus, problem solving is the primary focus of the teaching and learning activities of secondary school mathematics. Various strategies can be used to solve problems. Among the strategies recommended by the Ministry of Education Malaysia (2003a) to be introduced in the secondary school mathematics curriculum are as follow: “trying a simple case; trial-and-error (also known as guess-and-check); drawing diagrams; identifying patterns; making a table, chart, or systematic list; simulation; using analogies; working backward; logical reasoning; and using algebra.” (p. 4).

Similarly, in this study, the fencing problem in Task 7 can be solved using various strategies (e.g., making a chart, looking for a pattern, trial-and-error, differentiation method, quadratic function method). Sgroi (2001) demonstrated how this problem can be solved using the strategy of making a chart (for the detail of her solution, see (Sgroi, 2001, pp. 181-182)). The finding of the present study exhibits that three types of strategies were employed by the PSSMTs in this study to solve the fencing problem, namely looking for a pattern, trial-and-error, and differentiation method. Berinderjeet and Yeap (2008) suggest that “looking for a pattern is a good problem-solving heuristic that enables one to reduce a complex problem to a pattern and then use the pattern to derive a solution” (p. 315). The finding of this study exhibits that half of the PSSMTs in this study had successfully solving the fencing problem. Two of the three PSSMTs who attempted to solve the problem using the strategy of looking for a pattern had successfully solving it. One out of three PSSMTs who attempted to solve it using the strategy of trial-and-error had successfully solving it while one of the two PSSMTs who attempted to solve it using differentiation method had successfully solving it. Nevertheless, the unsuccessful problem solvers of Task 7 (i.e., fencing problem) did not attempt to use alternative method to solve the problem or verify their solutions.

The Form One Mathematics Curriculum Specification (Ministry of Education Malaysia, 2003a) recommended that teaching and learning activities in the classroom to provide opportunity for the students to investigate and develop the formula for the area of a rectangle. It also suggested that students be given opportunity to investigate and develop the formulae for the area of triangles, parallelogram, and trapeziums based on the area of a rectangle.

Sgroi (2001) suggests that “the formula for the area of a rectangle can be developed by having children form many rectangles on a geoboard or dot paper, count up the squares inside, and eventually generalize that the area can be found by multiplying the length of the rectangle by the width“ (p. 183). Similar strategy (i.e., inductive method) was recommended by other mathematics educators or researchers (Billstein, Liberskind, & Lott, 2006; Cathcart, Pothier, Vance, & Bezuk, 2006; Cavanagh, 2008; Chua, Teh, & Ooi, 2002; NCTM, 2000: O’Daffer, Charles, Cooney, Dossey, & Schielack, 2005; van de Walle, 2007) to develop or derive the formula for the area of a rectangle. However, the finding of this study illustrates that none of the PSSMTs in this study were able to develop it even though all of them could recall the formula. Furthermore, none of them attempted to develop the formula, except Tan. Tan had attempted to develop the formula but unsuccessful. It indicated that all of them have no idea how the formula can be developed or derived. They might have rote-learnt the formula. It was apparent that all of them lack of conceptual knowledge underpinning the formula for the area of a rectangle.

Lim-Teo and Ng (2008) suggest that “areas of triangles, parallelograms and trapeziums are related to the area of a rectangle. These relationships form the reasoning for the formulae for the areas” (p. 106). Thus, the formula for the area of a parallelogram can be developed from the formula for the area of a rectangle (Beaumont, Curtis, & Smart, 1986; Billstein et al., 2006; Cathcart et al., 2006; Cavanagh, 2008; Cheang, 2002; Chua et al., 2002; Lim-Teo & Ng, 2008; NCTM, 2000; O’Daffer et al., 2005; van de Walle, 2007). The finding of this study depicts that five of the PSSMTs in this study were able to develop the formula for the area of a parallelogram. They developed it from the formula for the area of a rectangle using the strategy of cut and paste (decompose and rearrange, i.e., decompose a parallelogram into a triangle and a trapezium and then rearrange these shapes to form a rectangle). It indicated that they understand the relationship between the formulae for the area of a parallelogram and rectangle.

Similarly, the formula for the area of a triangle can be developed from the formula for the area of a rectangle (Billstein et al., 2006; Cathcart et al., 2006; Cavanagh, 2008; Cheang, 2002; Chua et al., 2002; Lim-Teo & Ng, 2008; O'Daffer et al., 2005) or a parallelogram (Beaumont et al., 1986; Cavanagh, 2008; Lim-Teo & Ng, 2008; NCTM, 2000; van de Walle, 2007). The finding of this study demonstrates that only two of the PSSMTs in this study were able to develop the formula for the area of a triangle. They developed it from the formula for the area of a rectangle using the strategy of partition (i.e., partition a rectangle along its diagonal into two triangles). It indicated that they knew the relationship between the formulae for the area of a triangle and rectangle that encloses it. Liana and Tan understand the relationship that the area of a triangle is half of the area of the rectangle that encloses it. This finding is in concurrence with the findings of previous studies (Baturo & Nason, 1996; Cavanagh, 2008). Baturo and Nason, (1996) found that only two of the 13 preservice primary school teachers in their study understand the relationship between the formulae for the area of a triangle and rectangle that encloses it. The area of a triangle is half of the area of the rectangle that encloses it.

Likewise, Cavanagh (2008) revealed that high school students in his study demonstrated a limited understanding of the relationship between the areas of triangle and rectangle. They did not make use of the fact that the area of a triangle is half of the area of the rectangle that encloses it. This was apparent when they were asked to calculate the area of a 3-4-5 cm right-angled triangle, which included tick marks at 1 cm intervals along the perpendicular sides. Cavanagh

(2008) found that many students in his study chose to construct a grid and attempt to count the squares instead of making used of the rectangle-triangle relationship. Another indication was that some students multiplied the lengths of all three sides (i.e., $3 \times 4 \times 5$) to find the area of the triangle.

Van de Walle (2007) suggests that there are at least ten different methods of developing the formula for the area of a trapezium. Similarly, the formula for the area of a trapezium can be developed from the formula for the area of a rectangle (Cheang, 2002; NCTM, 2000; van de Walle, 2007), a parallelogram (Billstein et al., 2006; Chua et al., 2002; Lim-Teo & Ng, 2008; NCTM, 2000; O'Daffer et al., 2005; van de Walle, 2007), or a triangle (Beaumont et al., 1986; Cathcart et al., 2006; van de Walle, 2007). The formula for the area of a trapezium can be developed using the strategies of cut and paste (decompose and rearrange, e.g., decompose an isosceles trapezium into a rectangle and two triangles and then rearrange these shapes to form a rectangle), duplicate (e.g., duplicate the trapezium and arrange the two trapeziums to form a parallelogram), or algebraic method. The finding of this study reveals that three of the PSSMTs in this study were able to develop the formula for the area of a trapezium. All of them developed the formula using algebraic method. They developed the formula for the area of a trapezium from the formulae for the area of a rectangle and a triangle. It indicated that they knew that the formula for the area of a trapezium is related to the formulae for the area of a rectangle and triangle.

The finding of this study illustrates that only one out of eight PSSMTs in this study secured an overall high level of SMK of perimeter and area. Six of the PSSMTs achieved an overall medium level of SMK of perimeter and area. The remaining PSSMT gained an overall low level of SMK of perimeter and area. It indicated that most of them were not ready to teach the topic of perimeter and area. This finding is in accordance with the finding of previous study (Ramakrishnan, 1998) which found that only six (11.1%) of the 54 preservice primary school teachers in his study were able construct one question of perimeter that assess a high level of understanding of perimeter.

## Implications of the Findings

Findings of this study have several implications for the preservice mathematics teacher education.

### Implications for Preservice Mathematics Teacher Education

In general, the findings of this study suggest that one can identify three groups of PSSMTs. The first group composed of PSSMTs who appeared to have high level of SMK of perimeter and area. The second group comprised PSSMTs who seen to have medium level of SMK of perimeter and area. The third group encompassed PSSMTs who seemed to have low level of SMK of perimeter and area. The challenge for mathematics teacher educators is to develop a mathematics teacher program that meets the needs of all these groups. This mathematics teacher program would help the PSSMTs to construct their SMK of perimeter and area in terms of its five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge. Eventually, it would help to enhance their level of SMK of perimeter and area. Specifically, such mathematics teacher program aims to help them to understand the importance of these five basic types of knowledge for teaching their future students. It would also facilitate the PSSMTs to include all these five basic types of knowledge appropriately in their lesson plans and teaching.

In this study, clinical interview technique was employed to elicit PSSMTs' subject matter knowledge of perimeter and area. Specifically, such technique had enable the researcher to elicit PSSMTs' five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area. Furthermore, clinical interview technique can also be used to elicit PSSMTs' subject matter knowledge of other mathematical topics (Aida Suraya, 1996; Lim, 2007; Nik Azis, 1987; Sharifah Norul Akmar, 1997; Sutriyono, 1997). It is apparent that this data collection method can be applied by mathematics teacher educators to proffer enriching experience or authentic assessment for their preservice teachers. Similarly, the clinical interview tasks devised for the purpose of this study had facilitated the researcher to determine the PSSMTs' subject matter knowledge of perimeter and area and ultimately, ascertain their level of SMK of perimeter and area. These tasks can be adopted or adapted by mathematics teacher educators to examine the nature and level of their preservice teachers' subject matter knowledge of perimeter and area.

Most of the PSSMTs in this study had a misconception that there is direct relationship between perimeter and area. They thought that two shapes with the same perimeter have the same area, and the garden with the longer perimeter has the larger area. They also thought that as the perimeter of a closed figure increases, the area also increases. The implication of this finding is that mathematics teacher educators need to organize teaching and learning activities that provide opportunity for the preservice mathematics teachers to use unit square chips or tiles to examine the possible pattern of relationship between perimeter and area, formulate and test generalizations pertaining to the relationship between perimeter and area, such as: (a) areas of shapes having the same perimeter and vice versa, (b) areas of shapes having the longer perimeter and vice versa, and (c) areas of shapes as its perimeter increases and vice versa. Through such activities, preservice mathematics teachers would understand that there is no direct relationship between perimeter and area. They would know that two shapes with the same perimeter could have different areas, and the garden with the longer perimeter could have a smaller area. They would also know that as the perimeter of a closed figure increases, the area of the figure may increases, decreases, or remains the same.

This is in line with the recommendations in the Form One Mathematics Curriculum Specifications (Ministry of Education Malaysia, 2003a) which suggests that the mathematics educators need to provide opportunity for their students to use unit square chips or tiles to investigate, explore, and make generalizations about the: (a) “perimeters of rectangles having the same area, and (b) areas of rectangles having the same perimeter” (p. 42).

In the present study, all the PSSMTs have no idea how the formula for the area of a rectangle can be developed or derived. They might have rote-learnt the formula. It was apparent that all of them lack of conceptual knowledge underpinning the formula for the area of a rectangle. Five of the PSSMTs were able to develop the formula for the area of a parallelogram. However, only two and three of the PSSMTs were able to develop the formula for the area of a triangle and trapezium, respectively. It indicated that most of the PSSMTs in this study have no idea how the formula for the area of a triangle and trapezium can be developed or derived from the formula for the area of a rectangle or parallelogram. The relationship among area formulae of parallelogram, triangle, trapezium, and rectangle were absent in their minds. Furthermore, it is recommended in the Form One Mathematics Curriculum Specification (Ministry of Education Malaysia, 2003a) that teaching and learning activities in the classroom to provide opportunity for the students to investigate and develop the formula for the area of a rectangle. It also suggested that students be given opportunity to investigate and develop the formulae for the area of triangles, parallelograms, and trapeziums based on the area of a rectangle. The implication of this finding is that mathematics teacher educators need to organize teaching and learning activities that provide opportunity for their preservice teachers to investigate and develop the formulae for the area of rectangle, triangle, parallelogram, and trapezium in a logical progression and meaningful way.

In summary, the implication for preservice mathematics teacher education is that preservice mathematics teacher's SMK of perimeter and area needs to be enhanced. As mathematics teacher educators, we need to search for appropriate situations that would enable the PSSMTs to discover and construct the different aspects of the topics that they will teach and ultimately, facilitate them to do the same for their future students.

## Recommendations for Further Research

Recommendations for the extension of this study and further research are as follow. This study only involved eight preservice secondary school mathematics teachers. The subjects were drawn from the preservice secondary school mathematics teachers who enrolled in the 4-year Bachelor of Science with Education (B.Sc.Ed.) program in a public university. Thus, it is recommended that the present study be extended to other preservice secondary school mathematics teachers who enrolled in the 4-year Bachelor of Science with Education (B.Sc.Ed.) program in this public university, in other programs (e.g., Bachelor of Education (B.Ed.), Diploma in Education (Dip.Ed.)), or attending other universities and teacher training institutes so as to verify and elaborate the findings of the present study.

The present study focused on preservice secondary school mathematics teachers' subject matter knowledge of perimeter and area. Therefore, it is recommended that further research examine subject matter knowledge of perimeter and area of preservice primary school mathematics teachers as well as primary school and secondary school mathematics teachers. It is also recommended that similar studies might be conducted to examine subject matter knowledge of perimeter and area of primary school and secondary school students. The findings of such studies may contribute to a wider knowledge base of the teachers and students' SMK of perimeter and area.

This study confined to two measurement concepts, namely perimeter and area. The present study did not examine other measurement concepts such as time, length, mass, surface area, and volume as well as other mathematical topics. It is recommended that further research examine preservice secondary school mathematics teachers' subject matter knowledge of other measurement concepts as well as other mathematical topics. The findings of these studies may help in planning more effective preservice mathematics teacher education program.

This study employed clinical interview techniques to collect data. Thus, it is recommended that further research might include other methods of data collection (e.g., observation and document collection) besides clinical interview techniques. Future study might examine preservice secondary school mathematics teachers' SMK of perimeter and area through the observation of a series of the microteachings of the entire topic of perimeter and area and document collection such as the collection of the lesson plans of the entire topic of perimeter and area and the related instructional activities sheets besides carried out the pre- and post-instructional clinical interviews. The findings of such study may provide a wider perspective of the preservice secondary school mathematics teachers' SMK of perimeter and area.

This study examined the nature and levels of preservice secondary school mathematics teachers' subject matter knowledge of perimeter and area. The present study did not examine preservice secondary school mathematics teachers' beliefs about perimeter and area. Therefore, it is recommended that further research examine preservice secondary school mathematics teachers' beliefs about perimeter and area as well as their beliefs about teaching and learning of perimeter and area.