Appendix A
Domain of Measurement in the Malaysian
Primary and Secondary School Mathematics Curriculum

Domain of Measurement	Primary and Secondary School Mathematics Curriculum
Time	Years 1, 2, 3, 4, 5, 6; Form 1
Length	Years 3, 4, 5, 6; Form 1
Mass/Weight	Years 4, 5, 6; Form 1
Perimeter	Year 5
Area	Year 6
Perimeter and Area	Form 1
Circumference and Area of a circle	Form 2
Volume/capacity	Years 5, 6; Forms 1, 2, 3

Sources: *Kementerian Pendidikan* [Ministry of Education] Malaysia (1998a, 1998b, 1998c, 1998d, 1998e, 1998f); Ministry of Education Malaysia (2003a, 2003b, 2003c).

Appendix B
Summary of the Learning Outcomes/Activities Related to Perimeter and Area in the Malaysian Primary and Secondary School Mathematics Curriculum

Measurement Concepts	Year/From	Summary of Learning Outcomes/Activities
Perimeter	Year 5	1. Determine the perimeters of rectangles, squares, triangles, and circles. Note: The perimeter of a circle also known as circumference. 2. Solve daily problems involving perimeters.
Area	Year 6	1. Determine the areas of rectangles, squares, and triangles. 2. Solve daily problems involving areas.
Perimeter and Area	Form 1	1. Identify and find the perimeter of a region. 2. Investigate and develop formula to find the perimeter of a rectangle. 3. Solve problems involving perimeters. 4. Estimate the area of a shape. 5. Find the areas of rectangles, triangles, parallelograms, and trapeziums. 6. Investigate and develop formula to find the areas of rectangles, triangles, parallelograms, and trapeziums. 7. Solve problems involving areas. 8. Investigate, explore, and make generalizations about the: (a) Perimeters of rectangles having the same area. (b) Areas of rectangles having the same perimeter.
Circumference and Area of a circle	Form 2	1. Estimate the value of π. 2. Derive the formula of the circumference of a circle. 3. Find the circumference of a circle. 4. Solve problems involving circumference of circles. 5. Derive the formula of the length of an arc. 6. Find the length of an arc 7. Solve problems involving arcs of circles. 8. Derive the formula of the area of a circle. 9. Find the area of a circle. 10. Solve problems involving area of circles. 11. Derive the formula of the area of a sector. 12. Find the area of a sector. 13. Solve problems involving area of sectors.

Sources: *Kementerian Pendidikan* [Ministry of Education] Malaysia (1998e, 1998f); Ministry of Education Malaysia (2003a, 2003b).

Appendix C
Mathematics Content Courses for B.Sc.Ed. Program Students Who Majored or Minored in Mathematics

Table C1

Mathematics Content Courses for B.Sc.Ed. Program Students Who Majored in Mathematics

Course Code	Course Title
Compulsory Courses (36 units)	
1. MAA101/4	Calculus for Science Students I
2. MAA111/4	Algebra for Science Students
3. MAA161/4	Statistics for Science Students
4. MAA102/4	Calculus for Science Students II
5. MAT122/4	Differential Equations I
6. MAT181E/4	Programming for Scientific Applications
7. MAT202/4	Introduction to Analysis
8. MAT203/4	Vector Calculus
9. MSS318/4	Discrete Mathematics
Optional Courses (8 units)	
1. MSS211/4	Modern Algebra
2. MAT222/4	Differential Equations II
3. MAT263/4	Theory of Probability
4. MSS212/4	Advanced Linear Algebra
5. MSS301/4	Complex Analysis
6. MAT363/4	Statistical Inferences
7. MSS391/4	Specific Topic Studies

Table C2

Mathematics Content Courses for B.Sc.Ed. Program Students Who Minored In Mathematics

Course Code	Course Title
Select 20 units from the following courses:	
1. MAA101/4	Calculus for Science Students I
2. MAA111/4	Algebra for Science Students
3. MAA161/4	Statistics for Science Students
4. MAA102/4	Calculus for Science Students II
5. MAT122/4	Differential Equations I
6. MAT203/4	Vector Calculus
7. MAT263/4	Theory of Probability
8. MSS211/4	Modern Algebra
9. MSG162/4	Applied Statistics Methods
10. MSG262/4	Quality Control

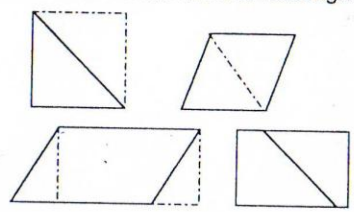
Appendix D
Topic 11: Perimeter and Area

11. LEARNING AREA: PERIMETER AND AREA

LEARNING OBJECTIVES	SUGGESTED TEACHING AND LEARNING ACTIVITIES	LEARNING OUTCOMES	POINTS TO NOTE	VOCABULARY
Students will be taught to:		*Students will be able to:*		
11.1 Understand the concept of perimeter to solve problems.	• Use square chips, tessellation grids, geo-boards, grid-papers or computer software to explore the concept of perimeter. • Investigate and develop formula to find the perimeter of a rectangle.	i. Identify the perimeter of a region. ii. Find the perimeter of a region enclosed by straight lines. iii. Solve problems involving perimeters.	Shapes enclosed by straight lines and curves. Limit to straight lines.	perimeter formula measure figure area square unit region enclosed
11.2 Understand the concept of area of rectangles to solve problems.	• Use unit squares, tessellation grids, geo-boards, grid-papers or computer software to explore the concept of area. • Investigate and develop formula to find the area of a rectangle. • Use unit square chips or tiles to investigate, explore and make generalisations about the: a) Perimeters of rectangles having the same area. b) Areas of rectangles having the same perimeter.	i. Estimate the area of a shape. ii. Find the area of a rectangle. iii. Solve problems involving areas.	cm^2 read as: "square cm". The area of a unit square is 1 square unit. Area of a right-angled triangle $= \frac{1}{2}$ of the area of a rectangle.	

42

LEARNING AREA: PERIMETER AND AREA

LEARNING OBJECTIVES	SUGGESTED TEACHING AND LEARNING ACTIVITIES	LEARNING OUTCOMES	POINTS TO NOTE	VOCABULARY
Students will be taught to:		*Students will be able to:*		
11.3 Understand the concept of area of triangles, parallelograms and trapezium to solve problems.	• Investigate and develop formulae to find the areas of triangles, parallelograms and trapeziums based on the area of rectangle.  • Solve problems such as finding the height or base of a parallelogram.	i. Identify the heights and bases of triangles, parallelograms and trapeziums. ii. Find the areas of triangles, parallelograms and trapeziums. iii. Find the areas of figures made up of triangles, rectangles, parallelograms or trapeziums. iv. Solve problems involving the areas of triangles, rectangles, parallelograms and trapeziums.		area triangle trapezium parallelogram height base rectangle figure

43

Source: From the Form One Mathematics Curriculum Specifications (Ministry of Education Malaysia, 2003a, pp. 42-43).

Appendix E

Distribution of the Components of Each Type of Knowledge That Were Assessed During the Clinical Interview

Table E1

Distribution of the Components of Conceptual Knowledge in the 8 Tasks

	Conceptual knowledge									
Task	1. Notion of perimeter	2. Notion of area	3. Notion of the unit of area (square and nonsquare)	4. Number of units and unit of measure	5. Inverse relationship between number of units and unit of measure	6. Relationship between standard units of length (linear units)	7. Relationship between standard units of area (square units)	8. Relationship between area units and linear units	9. Relationship between Perimeter and area	10. Relationship among area formulae
1.1	x									
1.2		x								
2			x							
3.1										
3.2										
3.3				x	x					
3.4				x	x					
4						x	x	x		
5.1									x	
5.2									x	
5.3									x	
6.1										
6.2										
7										
8										x

Table E2

Distribution of the Components of Procedural Knowledge in the 8 Tasks

Task	Procedural knowledge			
	1. Converting standard units of area measurement	2. Calculating perimeter of composite figures	3. Calculating area of composite figures	4. Developing area formulae
1.1				
1.2				
2				
3.1				
3.2				
3.3				
3.4				
4	x			
5.1				
5.2				
5.3				
6.1		x	x	
6.2		x	x	
7				
8				x

Table E3

Distribution of the Components of Linguistic Knowledge in the 8 Tasks

Task	Linguistic knowledge				
	1. Mathematical symbols	2. Mathematical terms	3. Standard unit of length measurement (linear units)	4. Standard unit of area measurement (square units)	5. Conventions of writing and reading SI area measurement
1.1		x			
1.2		x			
2		x			
3.1					
3.2					
3.3					
3.4					x
4					
5.1					
5.2					
5.3					
6.1			x	x	
6.2			x	x	
7					
8	x	x			

Table E4

Distribution of the Components of Strategic Knowledge in the 8 Tasks

Task	Strategic knowledge						
	1. Strategies for comparing perimeter	2. Strategies for comparing area	3. Strategies for checking answer for perimeter	4. Strategies for checking answer for area	5. Strategy for solving the fencing problem	6. Strategy for checking answer for the fencing problem	7. Strategy for developing the formulae
1.1							
1.2							
2							
3.1	x						
3.2		x					
3.3							
3.4							
4							
5.1							
5.2							
5.3							
6.1			x	x			
6.2			x	x			
7					x	x	
8							x

Table E5

Distribution of the Components of Ethical Knowledge in the 8 Tasks

Task	Ethical knowledge						
	1. Justifies one's mathematical ideas	2. Examines patterns within the domain of perimeter and area measurement	3. Formulates generalization within the domain of perimeter and area measurement	4. Tests generalization within the domain of perimeter and area measurement	5. Develops area formulae	6. Writes units of measurement upon completed a task	7. Checks the correctness of their solutions/answers
1.1	x						
1.2	x						
2	x						
3.1							
3.2							
3.3							
3.4							
4							
5.1		x	x	x			
5.2		x	x	x			
5.3				x			
6.1						x	x
6.2						x	x
7						x	x
8					x		

Appendix F
Background Information Form

Please tick □ the appropriate space:

1. Name: ______________________________________

   Contact number:
   Tel.: _________________
   Handphone: _____________________

2. Gender:
   □ Male
   □ Female

3. Age: _______ years ______ month(s)

4. National examinations mathematics results:

Year	Level	Subject	Grade
	SPM	Mathematics Additional Mathematics	
	STPM	Mathematics T Further Mathematics T Others, please specify: ___________________________	
	Matriculation	Mathematics T Further Mathematics T Others, please specify: ___________________________	

5. Program:
   □ B.Sc. Ed.
   □ B. Ed.
   □ Others, please specify: _________________________

6. Major:
   □ Mathematics
   □ Biology
   □ Chemistry
   □ Physics
   □ Others, please specify: ________________________

7. Minor:
   □ Mathematics
   □ Biology
   □ Chemistry
   □ Physics
   □ Others, please specify: ________________________

8. Please tick □ the university mathematics content courses that you have completed and write down the grades attained respectively:

Courses	**Grades**
□ Calculus for Science Students I	______
□ Algebra for Science Students	______
□ Statistics for Science Students	______
□ Calculus for Science Students II	______
□ Differential Equations I	______
□ Programming for Scientific Applications	______
□ Introduction to Analysis	______
□ Vector Calculus	______
□ Discrete Mathematics	______
□ Modern Algebra	______
□ Differential Equations II	______
□ Theory of Probability	______
□ Advanced Linear Algebra	______
□ Complex Analysis	______
□ Statistical Inferences	______
□ Applied Statistics Methods	______
□ Quality Controls	______
□ Others, please specify: ________________________	______

9. Cumulative Grade Point Average (CGPA)/*Purata Nilai Gred Kumulatif* (*PNGK*): ________________

10. Mathematics teaching experience as a temporary teacher:
- ☐ None
- ☐ Primary school: ______ year(s) ______ month(s)
- ☐ Secondary school: ______ year(s) ______ month(s)
- ☐ Others, please specify: ________________________ :
  ______ year(s) ______ month(s)

11. Mathematics teaching experience as a trained teacher:
- ☐ None
- ☐ Primary school: ______ year(s) ______ month(s)
- ☐ Secondary school: ______ year(s) ______ month(s)
- ☐ Others, please specify: ________________________ :
  ______ year(s) ______ month(s)

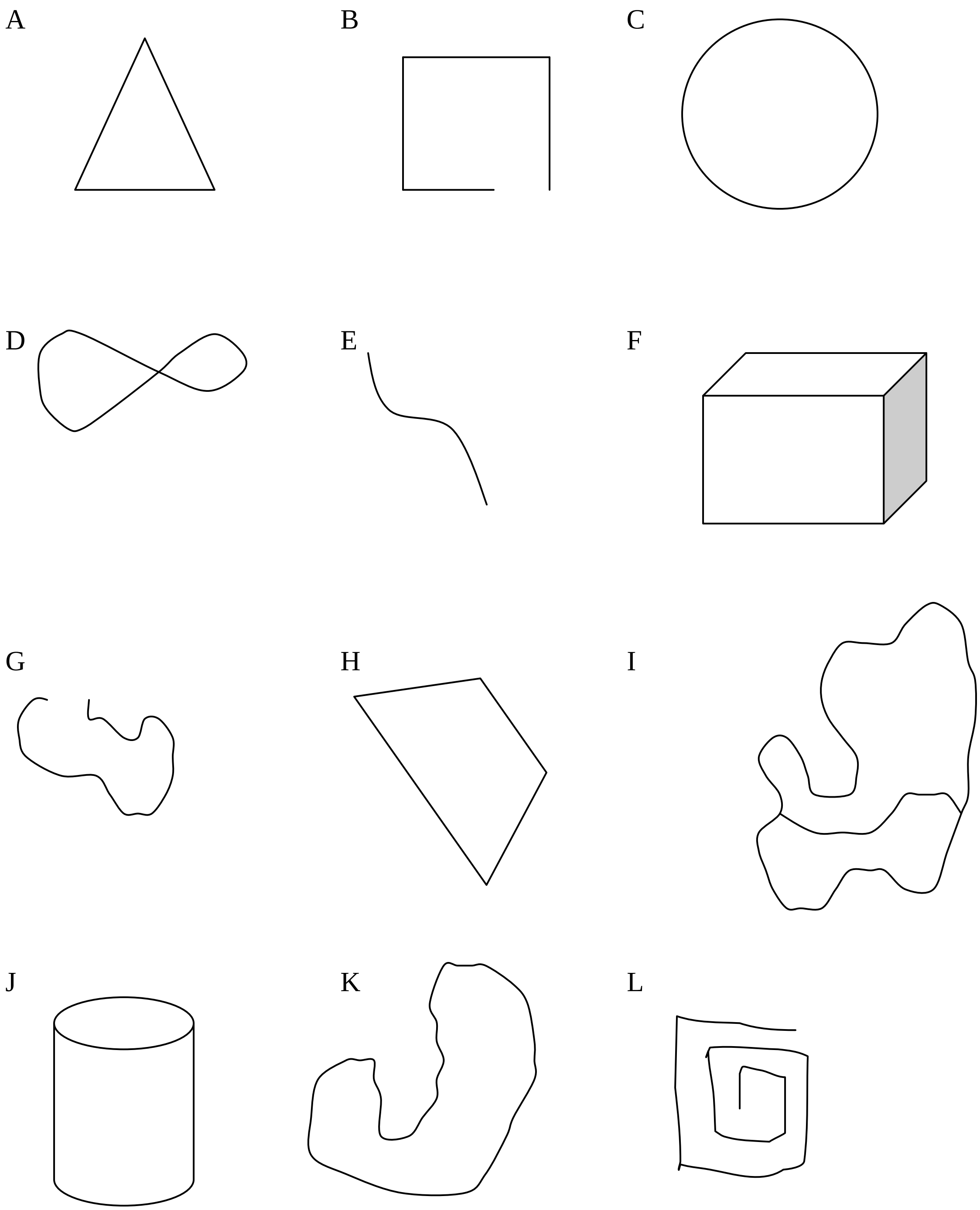
Appendix G

The Tasks

Interview 1

Task 1: Notion of perimeter and area (*Adapted from Baturo & Nason, 1996, p. 245*)
Task 1.1: Notion of perimeter

(Puts a handout comprise 12 shapes in front of the subject).



Tick the shapes that have a perimeter.

Probes:
What do you mean by ____ ?
Could you tell me more about it?

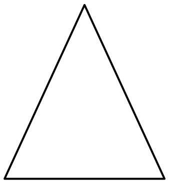
Why did you select this shape?

Why didn't you select this shape?

Task 1.2: Notion of area

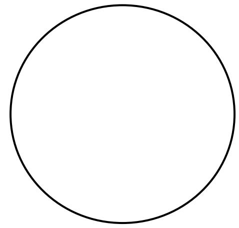
(Puts a handout comprise 12 shapes in front of the subject).

A

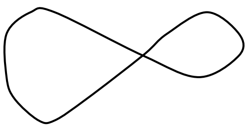


B

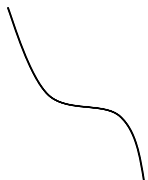
C



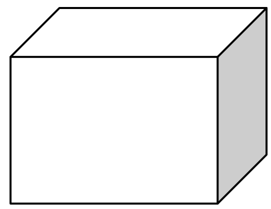
D



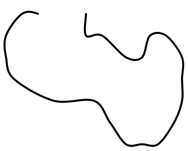
E



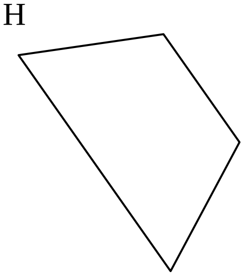
F



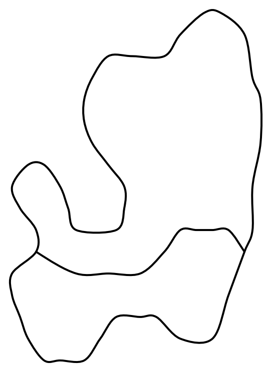
G



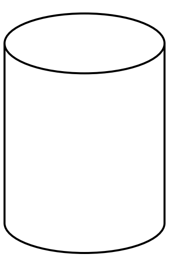
H



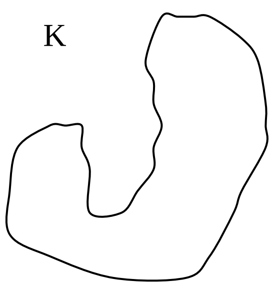
I



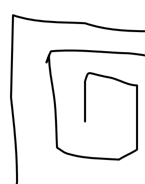
J



K



L



Tick the shapes that have an area.

Probes:
What do you mean by ____ ?
Could you tell me more about it?

Why did you select this shape?

Why didn't you select this shape?

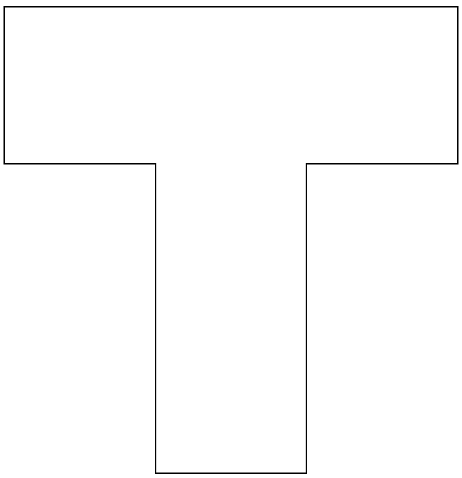
Task 2: Notion of the units of area

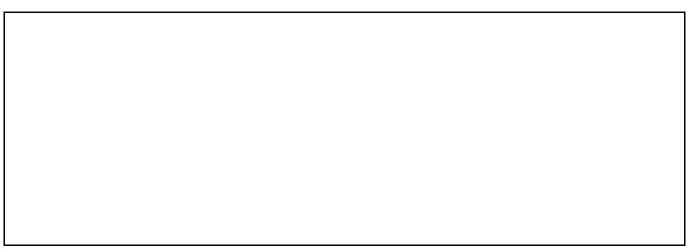
(Puts a handout written the following scenario in front of the subject). Ali, Chong, and David are discussing about the units of area. Ali says that we can use a square as the unit of area. Chong says that we can use a rectangle as the unit of area. David says that we can use a triangle as the unit of area. How would you respond to these students?

Probes:
What do you mean by ____ ?
Could you tell me more about it?

Interview 2
Task 3: Comparing perimeter and area (*Adapted from Baturo & Nason, 1996, p. 246, 266*)
Task 3.1: Comparing perimeter (No dimension was given)

(Puts the following pair of shapes in front of the subject). How would you find out whether the following pair of shapes had the same perimeter?



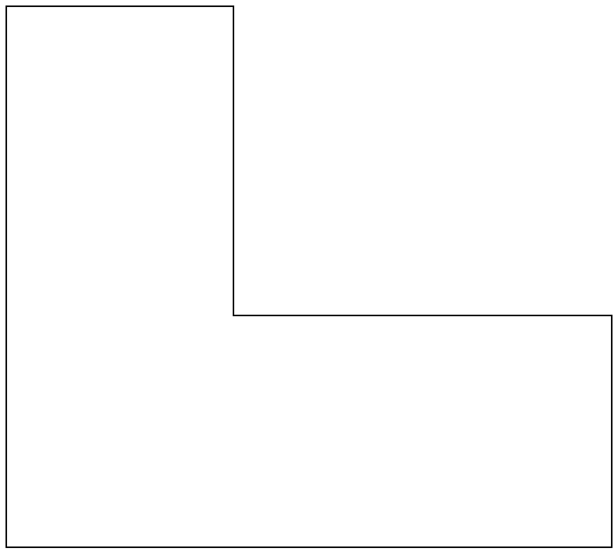


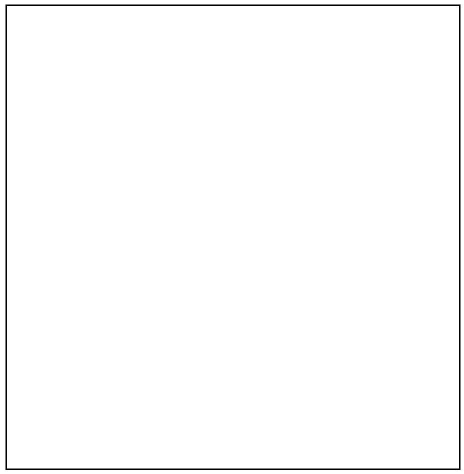
Probe:
What do you mean by ____ ?
Could you tell me more about it?
Could you show me how it is?

Could you think of other way of finding out whether they had the same perimeter?
(Repeat this question until the subject can not produce any more method).

Task 3.2: Comparing area (No dimension was given)

(Puts the following pair of shapes in front of the subject). How would you find out whether the following pair of shapes had the same area?





Probe:
What do you mean by _____ ?
Could you tell me more about it?
Could you show me how it is?

Could you think of other way of finding out whether they had the same area?
(Repeat this question until the subject can not produce any more method).

Task 3.3: Comparing perimeter (Nonstandard and standard units)

3.3 (a) (Puts the following table in front of the subject). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 1	25 paper clips	12 sticks

Probes:
What do you mean by ____ ?
Could you tell me more about it?

If subject says, "shape A has the longer perimeter", ask:
Why is it?

If subject says, "I can't tell", ask:
Why is it?

3.3 (b) (Puts the following table in front of the subject). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 2	10 paper clips	15 paper clips

Probes:
What do you mean by ____ ?
Could you tell me more about it?

If subject says, "shape B has the longer perimeter", ask:
Why is it?

If subject says, "I can't tell", ask:
Why is it?

If shapes A and B had the same perimeter, what would you tell about their units of measurement?
Why is it?

3.3 (c) (Puts the following table in front of the subject). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 3	16 cm	13 cm

Probes:
What do you mean by ____ ?
Could you tell me more about it?

If subject says, "shape A has the longer perimeter", ask:
Why is it?

Task 3.4: Comparing area (Nonstandard and standard units)

3.4 (a) (Puts the following table in front of the subject). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 1	25 triangles	12 squares

Probes:
What do you mean by ____ ?
Could you tell me more about it?

If subject says, "shape A has the larger area", ask:
Why is it?

If subject says, "I can't tell", ask:
Why is it?

3.4 (b) (Puts the following table in front of the subject). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 2	10 squares	15 squares

Probes:
What do you mean by _____ ?
Could you tell me more about it?

If subject says, "shape B has the larger area", ask:
Why is it?

If subject says, "I can't tell", ask:
Why is it?

If shapes A and B had the same area, what would you tell about their units of measurement?
Why is it?

3.4 (c) (Puts the following table in front of the subject). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 3	16 cm^2	13 cm^2

Probes:
What do you mean by _____ ?
Could you tell me more about it?

If subject says, "shape A has the larger area", ask:
Why is it?

(Puts a blank paper, written the following measurements, in front of the subject).
16 cm^2
13 cm^2
How would you write these measurements in English words?

Interview 3
Task 4: Converting units of area

(Puts a handout written the following scenario in front of the subject). Some Form One teachers noticed that several of their students seemed to multiply by 10, 100, and 1000, respectively when they were converting units of area from cm^2 to mm^2, m^2 to cm^2, and km^2 to m^2:

(a) $3\ cm^2 = 3 \times 10\ mm^2 = 30\ mm^2$
(b) $4.7\ m^2 = 4.7 \times 100\ cm^2 = 470\ cm^2$
(c) $1.25\ km^2 = 1.25 \times 1000\ m^2 = 1250\ m^2$

What would you do if you were teaching Form One and you noticed that several of your students were doing this?

Probes:
What do you mean by ____ ?
Could you tell me more about it?

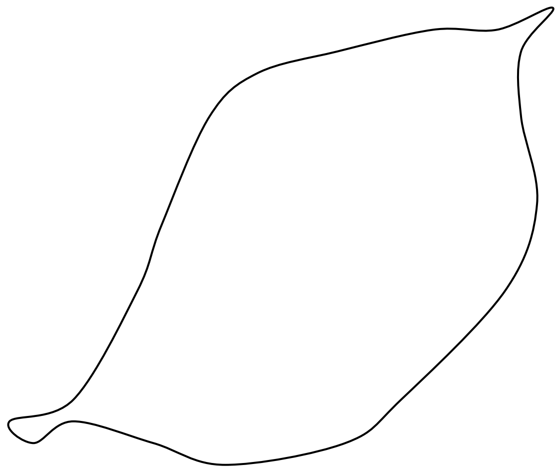
If subject says, "$1\ cm^2 = 100\ mm^2$", ask:
How do you get that?

If subject says, "$1\ m^2 = 10\ 000\ cm^2$", ask:
How do you get that?

If subject says, "$1\ km^2 = 1\ 000\ 000\ m^2$", ask:
How do you get that?

Task 5: Relationship between perimeter and area
Task 5.1: Same perimeter, same area? (*Adapted from Wilson & Chavarria, 1993, pp. 139-140*)

(Puts a handout written the following scenario in front of the subject). This is a picture of a leaf. A Form One student said that he had found a way to calculate the area of the leaf. The student placed a piece of thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle.



How would you respond to this student?

Probes:
What do you mean by ____ ?
Could you tell me more about it?

Would this method works?
Why is it?

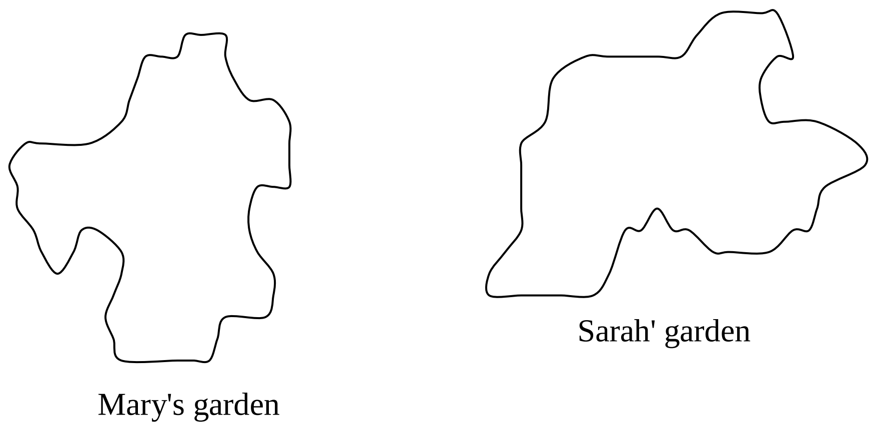
If subject says, "this method works/is correct", ask:
Why is it?

If subject says, "they have the same area ", ask:
Why is it?

If subject says, "this method is wrong/inaccurate", ask:
Why is it?

Task 5.2: Longer perimeter, larger area? (*Adapted from Billstein et al., 2006, p. 821*)

(Puts a handout written the following scenario in front of the subject). Mary and Sarah are discussing whose garden has the larger area to plant flowers. Mary claims that all they have to do is walk around the two gardens to get the perimeter and the one with the longer perimeter has the larger area. How would you respond to these students?



Probes:
What do you mean by ____ ?
Could you tell me more about it?

Would this method works?
Why is it?

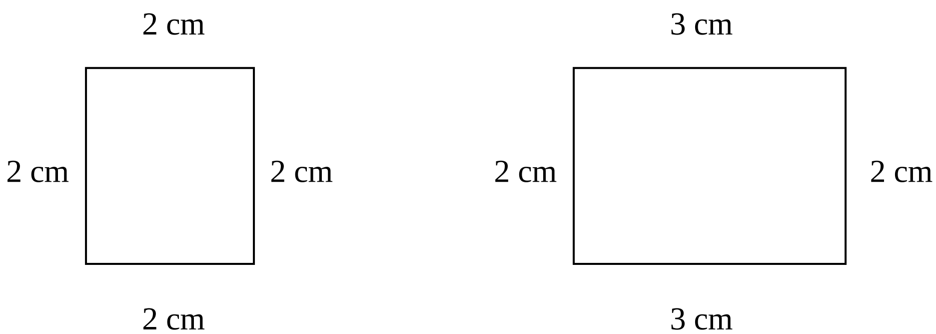
If subject says, "this method is correct", ask:
Why is it?

If subject says, "the garden with the longer perimeter has the larger area", ask:
Why is it?

If subject says, "this method is wrong/inaccurate", ask:
Why is it?

Task 5.3: Perimeter increases, area increases? (*Adapted from Ball, 1988, p. 399*)

(Puts a handout written the following scenario in front of the subject). Suppose that one of your Form One students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:



Perimeter = 8 cm
Area = 4 cm^2

Perimeter = 10 cm
Area = 6 cm^2

How would you respond to this student?

Probes:
What do you mean by ____ ?
Could you tell me more about it?

If subject says, "this is correct/true", ask:
Why is it?

If subject says, "its area increases as its perimeter increases", ask:
Why is it?

If subject says, "this is wrong/false", ask:
Why is it?

If subject says, "this is not a proof", ask:
Why is it?

If subject says, "this is not a theory", ask:
Why is it?

Interview 4

Task 6: Calculating perimeter and area

Task 6.1: Rectangle and parallelogram/triangle

(Puts a handout written the following problem in front of the subject). Suppose that one of your Form One students asks you for help with the following problem:

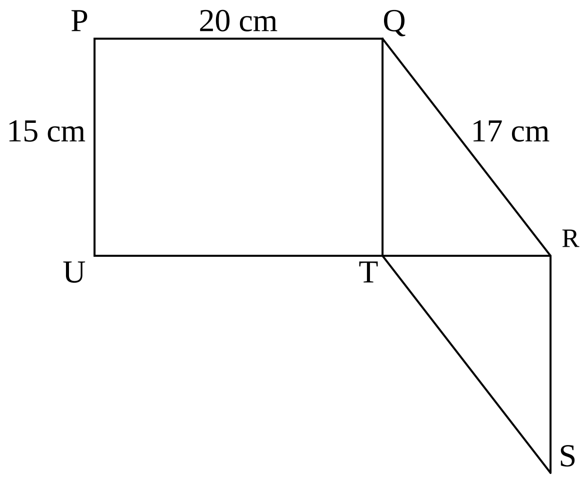


Diagram 1

In Diagram 1, PQTU is a rectangle and QRST is a parallelogram.
UTR is a straight line.
Calculate
(a) the perimeter of the diagram,
(b) the area of the diagram.

How would you solve this problem?

Probes:
What do you mean by ____ ?
Could you tell me more about it?

Could you explain your solution?
How did you get that answer?

How would you check your answer?

Task 6.2: Square and trapezium/triangle

(Puts a handout written the following problem in front of the subject). Suppose that one of your Form One students asks you for help with the following problem:

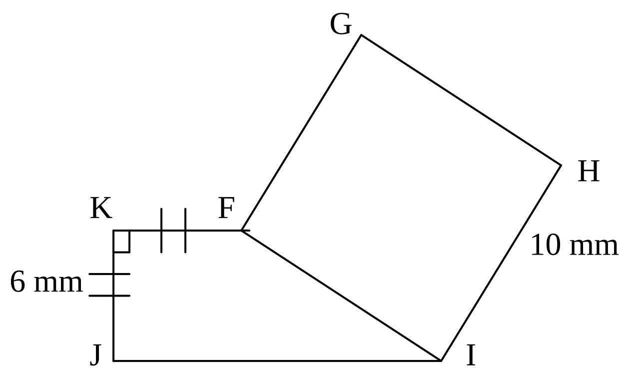


Diagram 2

In Diagram 2, FGHI is a square and FIJK is a trapezium.
Calculate
(a) the perimeter of the diagram,
(b) the area of the diagram.

How would you solve this problem?

Probes:
What do you mean by ____ ?
Could you tell me more about it?

Could you explain your solution?
How did you get that answer?

How would you check your answer?

Task 7: Fencing problem (*Adapted from Sgroi, 2001, p. 181*)

(Puts a handout written the following problem in front of the subject). Suppose that one of your students asks you for help with the following problem:

> A gardener has 84 m of fencing to enclose a garden along three sides, with the fourth side of the garden being formed by a wall. (Assume that the wall is perfectly straight). What are the dimensions of a rectangular garden that will yield the largest area being enclosed?

How would you solve this problem?

Probes:
What do you mean by ____ ?
Could you tell me more about it?

Could you explain your solution?
How did you get that answer?

How do you know the dimensions ________ maximize the area?

How would you check your answer?

Task 8: Developing area formulae

(Puts a handout written the following scenario in front of the subject). Suppose that a Form One student comes to you and says that he does not know how to develop (derive) the formula for calculating the area of the following shapes:

(a) Rectangle,
(b) Parallelogram,
(c) Triangle, and
(d) Trapezium.

How would you show him the way to develop (derive) the formula for calculating the area of these shapes?

Probes:
What do you mean by ____ ?
Could you tell me more about it?

Note: Area formulae for rectangle, parallelogram, triangle, or trapezium will be given to the subjects if they can not recall it.

Appendix H
Task Relevance Judgment Form

1. Kindly judge each of the tasks in terms of task relevance to the topic of perimeter and area.
2. Please give your judgment based on the following judgment scales. Your comments and suggestions are invaluable and very much appreciated. Thank you very much in anticipation.
   5 - most relevant
   4 - quite relevant
   3 - relevant
   2 - less relevant
   1 - not relevant

Task	Judgment Scale	Comments/Suggestions
1.1		
1.2		
2		
3.1		
3.2		
3.3 (a)		
3.3 (b)		
3.3 (c)		
3.4 (a)		

Task	Judgment Scale	Comments/Suggestions
3.4 (b)		
3.4 (c)		
4		
5.1		
5.2		
5.3		
6.1		
6.2		
7		
8		

Overall comments/suggestions: ____

____

____

Judge's signature: ______________________

Judge's name: _______________________________________

Judge's position: _______________________________________

Appendix I
Content Coverage Judgment Form

1. Kindly judge each of the tasks in terms of content coverage to the topic of perimeter and area.
2. Please give your judgment based on the following judgment scales. Your comments and suggestions are invaluable and very much appreciated. Thank you very much in anticipation.
   - 5 - most comprehensive
   - 4 - quite comprehensive
   - 3 - comprehensive
   - 2 - less comprehensive
   - 1 - not comprehensive

Task	Judgment Scale	Comments/Suggestions
1.1		
1.2		
2		
3.1		
3.2		
3.3 (a)		
3.3 (b)		
3.3 (c)		
3.4 (a)		

Task	Judgment Scale	Comments/Suggestions
3.4 (b)		
3.4 (c)		
4		
5.1		
5.2		
5.3		
6.1		
6.2		
7		
8		

Overall comments/suggestions: ____

____

____

Judge's signature: ____________________

Judge's name: _________________________________

Judge's position: _________________________________

Appendix J
Coding Rubrics for Determining Subject Matter Knowledge of Perimeter and Area

Task	Response	Code
1.1	Correct response: A, C, D, F, H, I, J, and K.	**Conceptual knowledge: Notion of perimeter:** Simple closed curves, closed but not simple curves, and 3-dimensional shapes. [**SCC, CNSC, & 3D**]
	Incorrect response: e.g., A, C, D, H, I, and K.	Limited to simple closed curves, and closed but not simple curves. [**SCC & CNSC**]
	Justification of shapes that have a perimeter: Appropriate response: Closed, covered, enclosed, (can be) measured/calculated, or equivalent mathematical terms.	**Linguistic knowledge: Mathematical term:** Appropriate mathematical term. [**AMT**] **Ethical knowledge: Justifies one's mathematical ideas:** Provided appropriate justification. [**AJ**]
	Inappropriate response: e.g., joined, connected, completed.	Inappropriate mathematical term. [**IMT**] Provided inappropriate justification. [**IJ**]
	Justification of shapes that do not have a perimeter (B, E, G, L): Appropriate response: open, not closed, not covered, not enclosed, (it is just) a line, or equivalent mathematical terms.	Appropriate mathematical term. [**AMT**] Provided appropriate justification. [**AJ**]
	Inappropriate response: e.g., not joined, not connected, incomplete, a curve, 3-dimensional shape/object (F, J).	Inappropriate mathematical term. [**IMT**] Provided inappropriate justification. [**IJ**]
1.2	Correct response: A, C, D, F, H, I, J, and K.	**Conceptual knowledge: Notion of area:** Notion of area was not only limited to 2-dimensional shapes, but also inclusive of 3-dimensional shapes. [**2D & 3D**]
	Incorrect response: e.g., A, C, D, H, I, and K.	Notion of area was limited to 2-dimensional shapes. [**2D**]
	Justification of shapes that have an area: Appropriate response: Closed, covered, enclosed, (can be) measured/calculated, or equivalent mathematical terms.	**Linguistic knowledge: Mathematical term:** Appropriate mathematical term. [**AMT**] **Ethical knowledge: Justifies one's mathematical ideas:** Provided appropriate justification. [**AJ**]
	Inappropriate response: e.g., joined, connected, completed.	Inappropriate mathematical term. [**IMT**] Provided inappropriate justification. [**IJ**]
	Justification of shapes that do not have an area(B, E, G, L): Appropriate response: open, not closed, not covered, not enclosed, (it is just) a line, or equivalent mathematical terms.	Appropriate mathematical term. [**AMT**] Provided appropriate justification. [**AJ**]
	Inappropriate response: e.g., not joined, not connected, incomplete, a curve, 3-dimensional shape/object (F, J).	Inappropriate mathematical term. [**IMT**] Provided inappropriate justification. [**IJ**]
2	Correct response: Square, rectangle, and triangle can be used as the unit of area.	**Conceptual knowledge: Notion of the unit of area:** Square and nonsquare can be used as the unit of area. [**SNS**]
	Incorrect response: e.g., Square and rectangle. None of the square, rectangle, and triangle.	Notion of the units of area was limited to square and rectangle. [**SR**] None or not accessible to her during the clinical interview. [**N**]
	Justification of shapes that can be used as the unit of area: Appropriate response: tessellate, cover or equivalent mathematical terms.	**Linguistic knowledge: Mathematical term:** Appropriate mathematical term. [**AMT**] **Ethical knowledge: Justifies one's mathematical ideas:** Provided appropriate justification. [**AJ**]
	Inappropriate response: e.g., straight lines, same length.	Inappropriate mathematical term. [**IMT**] Provided inappropriate justification. [**IJ**]

Task	Response	Code
3.1	Appropriate response: Generate an appropriate strategy for comparing perimeter. Inappropriate response: Generate an inappropriate strategy for comparing perimeter (e.g., measuring the side by ruler and applying area formula, and then generalized that the shape with the larger area has the longer perimeter), or no response. **Alternative strategies for comparing perimeter.** Appropriate response: Generate appropriate alternative strategy/strategies for comparing perimeter. Inappropriate response: Generate inappropriate alternative strategy/strategies for comparing perimeter (e.g., measuring the side by ruler and applying area formula, and then generalized that the shape with the larger area has the longer perimeter), or no response. Unable to generate alternative strategy.	**Staretegic knowledge: Strategies for comparing perimeter:** Appropriate strategy. **Formal methods:** (a) Measuring the side by ruler and applying definition of perimeter. [**F(a)**] (b) Measuring the side by thread and ruler and applying definition of perimeter. [**F(b)**] (c) Measuring the side by compass and ruler and applying definition of perimeter. [**F(c)**] (d) Measuring the side by paper and ruler and applying definition of perimeter. [**F(d)**] (e) Other equivalent methods. [**F(e), please specify**] **Semi-formal methods:** (a) Putting or tracing the shapes on a **grid paper** and then counting the number of units on each side. [**S(a)**] (b) Marking the length of each side of the shapes on a **blank paper** and see whether they ended at he same point. [**S(b)**] (c) Using a piece of **thread** to surround the shapes and then compare. [**S(c)**] (d) Other equivalent methods. [**S(d), please specify**] **Informal methods:** (a) Cut and paste. [**I(a)**] (b) Other equivalent methods. [**I(b), please specify**] Inappropriate strategy. [**IS**] No alternative strategy. [**NAS**]
3.2	Appropriate response: Generate an appropriate strategy for comparing area. Inappropriate response: Generate an inappropriate strategy for comparing area (e.g., measuring the side by ruler and applying definition of perimeter, and then generalized that the shape with the longer perimeter has the larger area), or no response. **Alternative strategies for comparing area.** Appropriate response: Generate appropriate alternative strategy/strategies for comparing area. Inappropriate response: Generate inappropriate alternative strategy/strategies for comparing area (e.g., measuring the side by ruler and applying definition of perimeter, and then generalized that the shape with the longer perimeter has the larger area), or no response. Unable to generate alternative strategy.	**Staretegic knowledge: Strategies for comparing area:** Appropriate strategy. **Formal methods:** (a) Measuring the side by ruler and applying area formula. [**F(a)**] (b) Measuring the side by thread and ruler and applying area formula. [**F(b)**] (c) Measuring the side by compass and ruler and applying area formula. [**F(c)**] (d) Measuring the side by paper and ruler and applying area formula. [**F(d)**] (e) Other equivalent methods. [**F(e), please specify**] **Semi-formal methods:** (a) Covering both shapes with a grid paper. [**S(a)**] (b) Other equivalent methods. [**S(b), please specify**] **Informal methods:** (a) Cut and paste (i.e., cut one shape into pieces and paste on to the other shape). [**I(a)**] (b) Superimpose/overlay (i.e., place one shape on top of the other shape). [**I(b)**] (a) Other equivalent methods. [**I(c), please specify**] Inappropriate strategy. [**IS**] No alternative strategy. [**NAS**]
3.3 (a)	Correct response: Unable to determine which shape has the longer perimeter, or equivalent responses. Incorrect response: e.g.: (a) Shape A has the longer perimeter. (b) Shape B has the longer perimeter.	**Conceptual knowledge: Number of units and unit of measure:** **Focus** on **unit** of measure. **Knew** that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. [**FUK**] (a) **Focused** on the **number** of unit. Did **not know** that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. [**FNNK**] (b) **Focus** on **unit** of measure. Did **not know** that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. [**FUNK**]

Task	Response	Code
3.3 (b)	Correct response: Unable to determine which shape has the longer perimeter, or equivalent responses.	**Conceptual knowledge: Number of units and unit of measure:** **Focus** on **unit** of measure. **Knew** that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. [**FUK**]
	Incorrect response: e.g., Shape B has the longer perimeter.	**Focus** on **unit** of measure. Did **not know** that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. [**FUNK**]
	If shapes A and B had the same perimeter: Correct response: e.g.: (a) Unit of measurement of shape A is longer. (b) Paper clip in shape A is longer. (c) Unit of measurement of shape B is shorter. (d) Paper clip in shape B is shorter. (e) Other equivalent response.	**Conceptual knowledge: Inverse proportion between the number of units and the unit of measure:** **Understand** the **inverse proportion** between the number of units and the unit of measure (i.e., the longer the unit of measure, the smaller the number of units required to get the same length). [**UIP**]
	Incorrect response: e.g.: (a) Unit of measurement of shape B is longer. (b) Paper clip in shape B is longer.	Did **not understand** the **inverse proportion** between the number of units and the unit of measure. [**NUIP**]
3.3 (c)	Correct response: Shape A has the longer perimeter, or equivalent responses. Incorrect response: e.g., unable to determine which shape has the longer perimeter.	**Conceptual knowledge: Number of units and unit of measure:** **Focus** on the **number** of unit. **Knew** that common standard unit (such as cm) is reliable for comparing perimeters. [**FNK**]
3.4 (a)	Correct response: Unable to determine which shape has the larger area, or equivalent responses.	**Conceptual knowledge: Number of units and unit of measure:** **Focus** on **unit** of measure. **Knew** that nonstandard units (such as triangle and square) are not reliable for comparing areas. [**FUK**]
	Incorrect response: e.g.: (a) Shape A has the larger area. (b) Shape B has the larger area.	 (a) **Focused** on the **number** of unit. Did **not know** that nonstandard units (such as triangle and square) are not reliable for comparing areas. [**FNNK**] (b) **Focus** on **unit** of measure. Did **not know** that nonstandard units (such as triangle and square) are not reliable for comparing areas. [**FUNK]**
3.4 (b)	Correct response: Unable to determine which shape has the larger area, or equivalent responses.	**Conceptual knowledge: Number of units and unit of measure:** **Focus** on **unit** of measure. **Knew** that common nonstandard units (such as squares) are not reliable for comparing areas. [**FUK**]
	Incorrect response: e.g., Shape B has the larger area.	**Focus** on **unit** of measure. Did **not know** that common nonstandard units (such as squares) are not reliable for comparing areas. [**FUNK**]
	If shapes A and B had the same area: Correct response: e.g.: (f) Unit of measurement of shape A is larger. (g) Square in shape A is larger. (h) Unit of measurement of shape B is smaller. (i) Square in shape B is smaller. (j) Other equivalent response.	**Conceptual knowledge: Inverse proportion between the number of units and the unit of measure:** **Understand** the **inverse proportion** between the number of units and the unit of measure (i.e., the larger the unit of measure, the smaller the number of units required to get the same area). [**UIP**]
	Incorrect response: e.g.: (a) Unit of measurement of shape B is larger. (b) Square in shape B is larger.	Did **not understand** the **inverse proportion** between the number of units and the unit of measure. [**NUIP**]

Task	Response	Code
3.4 (c)	Correct response: Shape A has the larger area, or equivalent responses. Incorrect response: e.g., unable to determine which shape has the larger area.	**Conceptual knowledge: Number of units and unit of measure: Focus** on the **number** of unit. **Knew** that common standard unit (such as cm^2) is reliable for comparing areas. [**FNK**]
	Writing SI area measurement: Correct response: 'sixteen square centimetres' and 'thirteen square centimetres' respectively, or equivalent responses.	**Linguistic knowledge: conventions of writing and reading SI area measurement:** **Knew** the convention of writing and reading SI area measurement. [**K**]
	Incorrect response: e.g., 'sixteen centimetres square' and 'thirteen centimetres square' respectively.	Did **not know** the convention of writing and reading SI area measurement. [**NK**]
4	Correct response: 300 mm^2, 47 000 cm^2, and 1 250 000 m^2 respectively, or equivalent responses.	**Procedural knowledge: Converting standard units of area measurement:** Successful. [**S**]
	Incorrect response: e.g., 30 mm^2, 470 cm^2, and 1250 m^2 respectively.	Unsuccessful. [**U**]
	Correct response (demonstrated/indicated explicitly/implicitly): (a) Wrote that 1 cm = 10 mm. (b) Times 10 when converted 1 cm to mm. (c) Times 10 when converted 3 cm to mm. (d) Times ten squared, $(10)^2$, when converted 3 cm^2 to mm^2. (e) Other equivalent responses.	**Conceptual knowledge: relationships between the standard units of length measurement:** (i) 1 cm = 10 mm. Knew. [**K**]
	Incorrect response (demonstrated/indicated explicitly/implicitly): e.g.: (a) Times 10^{-1} when converted 3 cm^2 to mm^2. (b) Thought that 1 cm^2 = 10 mm^2.	Did not know. [**NK**]
	Correct response (demonstrated/indicated explicitly/implicitly): (a) Wrote that 1 m = 100 cm. (b) Times 100 when converted 1 m to cm. (c) Times 100 when converted 4.7 m to cm. (d) Times hundred squared, $(100)^2$, when converted 4.7 m^2 to cm^2. (e) Other equivalent responses.	(ii) 1 m = 100 cm. Knew. [**K**]
	Incorrect response (demonstrated/indicated explicitly/implicitly): e.g.: (a) Times $(10^1 cm)^2$ or 10^2 cm^2 when converted 4.7 m^2 to cm^2. (b) Thought that 1 m^2 = 100 cm^2.	Did not know. [**NK**]
	Correct response (demonstrated/indicated explicitly/implicitly): (a) Wrote that 1 km = 1000 m. (b) Times 1000 when converted 1 km to m. (c) Times 1000 when converted 1.25 km to m. (d) Times thousand squared, $(1000)^2$, when converted 1.25 km^2 to m^2. (e) Other equivalent responses.	(iii) 1 km = 1000 m. Knew. [**K**]
	Incorrect response (demonstrated/indicated explicitly/implicitly): e.g.: (a) Thought that 1 km^2 = 1000 m^2.	Did not know. [**NK**]

Task	Response	Code
4	Correct response (demonstrated/indicated explicitly/implicitly): (a) Wrote that 1 cm^2 = 100 mm^2. (b) Times ten squared, $(10)^2$, when converted 3 cm^2 to mm^2. (c) Other equivalent responses.	**Conceptual knowledge: relationships between the standard units of area measurement:** (j) 1 cm^2 = 100 mm^2. Knew. [**K**]
	Incorrect response (demonstrated/indicated explicitly/implicitly): e.g.: (a) Times 10^{-1} when converted 3 cm^2 to mm^2. (b) Thought that 1 cm^2 = 10 mm^2.	Did not know. [**NK**]
	Correct response (demonstrated/indicated explicitly/implicitly): (a) Wrote that 1 m^2 = 10 000 cm^2. (b) Times hundred squared, $(100)^2$, when converted 4.7 m^2 to cm^2. (c) Other equivalent responses.	(ii) 1 m^2 = 10 000 cm^2. Knew. [**K**]
	Incorrect response (demonstrated/indicated explicitly/implicitly): e.g.: (a) Times $(10^1 cm)^2$ or 10^2 cm^2 when converted 4.7 m^2 to cm^2. (b) Thought that 1 m^2 = 100 cm^2.	Did not know. [**NK**]
	Correct response (demonstrated/indicated explicitly/implicitly): (a) Wrote that 1 km^2 = 1 000 000 m^2. (b) Times thousand squared, $(1000)^2$, when converted 1.25 km^2 to m^2. (c) Other equivalent responses.	(iii) 1 km^2 = 1 000 000 m^2. Knew. [**K**]
	Incorrect response (demonstrated/indicated explicitly/implicitly): e.g.: (a) Thought that 1 km^2 = 1000 m^2.	Did not know. [**NK**]
	Correct response (demonstrated/indicated explicitly/implicitly): (a) Wrote 3 cm^2 as 3 times 1 cm times 1 cm. (b) Wrote 1 cm^2 as 1 cm times 1 cm. (c) Times ten squared, $(10)^2$, when converted 3 cm^2 to mm^2. (d) Wrote 4.7 m^2 as 4.7 times 1 m times 1 m. (e) Wrote 1 m^2 as 1 m times 1 m. (f) Times hundred squared, $(100)^2$, when converted 4.7 m^2 to cm^2. (g) Wrote 1.25 km^2 as 1.25 times 1 km times 1 km. (h) Wrote 1 km^2 as 1 km times 1 km. (i) Times hundred squared, $(1000)^2$, when converted 1.25 km^2 to m^2. (j) Other equivalent responses.	**Conceptual knowledge: Relationship between area units and linear units of measurement:** **Knew** the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. [**K**]
	Incorrect response (demonstrated/indicated explicitly/implicitly): e.g.: (a) Wrote or thought that 1 cm^2 = 10 mm^2, 1 m^2 = 100 cm^2, or 1 km^2 = 1000 m^2.	Did not know. [**NK**]

Task	Response	Code
5.1	Correct response: The student's method of calculating the area of the leaf was not correct, or equivalent responses.	**Conceptual knowledge: Relationships between perimeter and area:** **Knew** that there is no direct relationship between perimeter and area (i.e., knew that two shapes with the same perimeter can have different areas). [**K**]
	Incorrect response: e.g.: (a) The student's method was correct. (b) Not sure whether the student's method was correct or not.	Did **not know** that there is no direct relationship between perimeter and area (i.e., did not know that two shapes with the same perimeter can have different areas). [**NK**]
	Examines Pattern: Appropriate response: Generated a counterexample to refute the student's generalization.	**Ethical knowledge: Examines Pattern:** **Attempted** to examine the possible pattern of the relationship between perimeter and area, **and appropriate.** [**AA**]
	Inappropriate response: (a) Generated an example that concurred with the student's generalization.	Attempted but inappropriate. [**AI**]
	(b) Accepted the student's generalization without attempting to examine the possible pattern of the relationship between perimeter and area.	Did not attempt. [**NA**]
	Formulates generalization: Appropriate response: Formulated a generalization pertaining to the relationship between perimeter and area that two shapes with the same perimeter may have the different area.	**Ethical knowledge: Formulates generalization:** **Attempted** to formulate generalization pertaining to the relationship between perimeter and area, **and appropriate.** [**AA**]
	Inappropriate response: (a) Formulated a generalization that concurred with the student's generalization.	Attempted but inappropriate. [**AI**]
	(b) Accepted the student's generalization without attempting to formulate generalization pertaining to the relationship between perimeter and area.	Did not attempt. [**NA**]
	Tests generalization: Appropriate response: Tested the student's generalization with a counterexample.	**Ethical knowledge: Tests generalization:** **Attempted** to test generalization pertaining to the relationship between perimeter and area, **and appropriate.** [**AA**]
	Inappropriate response: (a) Tested the student's generalization with an example.	Attempted but inappropriate. [**AI**]
	(b) Accepted the student's generalization without attempting to test generalization pertaining to the relationship between perimeter and area.	Did not attempt. [**NA**]
5.2	Correct response: Mary's claim that the garden with the longer perimeter has the larger area was not correct, or equivalent responses.	**Conceptual knowledge: Relationships between perimeter and area:** **Knew** that there is no direct relationship between perimeter and area (i.e., knew that the garden with the longer perimeter could have a smaller area). [**K**]
	Incorrect response: e.g.: (a) Mary's claim was correct. (b) Not sure whether Mary's claim was correct or not.	Did **not know** that there is no direct relationship between perimeter and area (i.e., Did not know that the garden with the longer perimeter could have a smaller area). [**NK**]

Task	Response	Code
5.2	**Examines Pattern:** Appropriate response: Generated a counterexample to refute Mary's generalization.	**Ethical knowledge: Examines Pattern:** **Attempted** to examine the possible pattern of the relationship between perimeter and area, **and appropriate.** [**AA**]
	Inappropriate response: (a) Generated an example that concurred with Mary's generalization.	Attempted but inappropriate. [**AI**]
	(b) Accepted Mary's generalization without attempting to examine the possible pattern of the relationship between perimeter and area.	Did not attempt. [**NA**]
	Formulates generalization: Appropriate response: Formulated a generalization pertaining to the relationship between perimeter and area that the garden with the longer perimeter could have a smaller area.	**Ethical knowledge: Formulates generalization:** **Attempted** to formulate generalization pertaining to the relationship between perimeter and area, **and appropriate.** [**AA**]
	Inappropriate response: (a) Formulated a generalization that concurred with Mary's generalization.	Attempted but inappropriate. [**AI**]
	(b) Accepted Mary's generalization without attempting to formulate generalization pertaining to the relationship between perimeter and area.	Did not attempt. [**NA**]
	Tests generalization: Appropriate response: Tested Mary's generalization with a counterexample.	**Ethical knowledge: Tests generalization:** **Attempted** to test generalization pertaining to the relationship between perimeter and area, **and appropriate.** [**AA**]
	Inappropriate response: (a) Tested Mary's generalization with an example.	Attempted but inappropriate. [**AI**]
	(b) Accepted Mary's generalization without attempting to test generalization pertaining to the relationship between perimeter and area.	Did not attempt. [**NA**]
5.3	Correct response: The student's "theory" that as the perimeter of a closed figure increases, the area also increases was not correct.	**Conceptual knowledge: Relationships between perimeter and area:** **Knew** that there is no direct relationship between perimeter and area (i.e., knew that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same). [**K**]
	Incorrect response: e.g.: (a) The student's "theory" was correct. (b) Not sure whether the student's "theory" was correct or not.	Did **not know** that there is no direct relationship between perimeter and area (i.e., Did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same). [**NK**]
	Tests generalization: Appropriate response: Tested the student's generalization with a counterexample.	**Ethical knowledge: Tests generalization:** **Attempted** to test generalization pertaining to the relationship between perimeter and area, **and appropriate.** [**AA**]
	Inappropriate response: (a) Tested the student's generalization with an example.	Attempted but inappropriate. [**AI**]
	(b) Accepted the student's generalization without attempting to test generalization pertaining to the relationship between perimeter and area.	Did not attempt. [**NA**]

Task	Response	Code
6.1	Perimeter: Correct response: 104 cm.	**Procedural knowledge: Calculating perimeter of composite figures:** Successful. [**S**]
	Incorrect response: e.g., 86 cm, 127 cm.	Unsuccessful. [**U**]
	Area: Correct response: 420 cm^2.	**Procedural knowledge: Calculating area of composite figures:** Successful. [**S**]
	Incorrect response: e.g., 555 cm^2.	Unsuccessful. [**U**]
	Unit of perimeter measurement: Correct response: cm.	**Linguistic knowledge: Standard unit of length measurement (linear units):** Used correct unit. [**CU**]
	Incorrect response: e.g., cm^2, mm.	Used incorrect unit. [**IU**]
	Unit of area measurement: Correct response: cm^2.	**Linguistic knowledge: Standard unit of area measurement (square units):** Used correct unit. [**CU**]
	Incorrect response: e.g., cm, mm^2.	Used incorrect unit. [**IU**]
	Strategies for checking answer for perimeter: Used the same method and calculate again.	**Strategic knowledge: Strategies for checking answer for perimeter:** Recalculating strategy. [**RS**]
	Used different method to check the answer.	Alternative strategy. [**AS**]
	Strategies for checking answer for area: Used the same method and calculate again.	**Strategic knowledge: Strategies for checking answer for area:** Recalculating strategy. [**RS**]
	Used different method to check the answer.	Alternative strategy. [**AS**]
	Write unit for perimeter: Appropriate response: Wrote the unit (without probed).	**Ethical knowledge: Writes units of measurement upon completed a task:** Wrote. [**W**]
	Inappropriate response: Did not write the unit (without probed).	Did not write. [**NW**]
	Write unit for area: Appropriate response: Wrote the unit (without probed).	Wrote. [**W**]
	Inappropriate response: Did not write the unit (without probed).	Did not write. [**NW**]
	Check the answer for perimeter: Appropriate response: Checked the answer (without probed).	**Ethical knowledge: Checks the correctness of their solutions or answers:** Checked. [**C**]
	Inappropriate response: Did not check the answer (without probed).	Did not check. [**NC**]
	Check the answer for area: Appropriate response: Checked the answer (without probed).	Checked. [**C**]
	Inappropriate response: Did not check the answer (without probed).	Did not check. [**NC**]

Task	Response	Code
6.2	Perimeter: Correct response: 56 mm.	**Procedural knowledge: Calculating perimeter of composite figures:** Successful. [**S**]
	Incorrect response: e.g., 66 mm.	Unsuccessful. [**U**]
	Area: Correct response: 160 mm^2.	**Procedural knowledge: Calculating area of composite figures:** Successful. [**S**]
	Incorrect response: e.g., 136 mm^2.	Unsuccessful. [**U**]
	Unit of perimeter measurement: Correct response: mm.	**Linguistic knowledge: Standard unit of length measurement (linear units):** Used correct unit. [**CU**]
	Incorrect response: e.g., mm^2, cm.	Used incorrect unit. [**IU**]
	Unit of area measurement: Correct response: mm^2.	**Linguistic knowledge: Standard unit of area measurement (square units):** Used correct unit. [**CU**]
	Incorrect response: e.g., mm, cm^2.	Used incorrect unit. [**IU**]
	Strategies for checking answer for perimeter: Used the same method and calculate again.	**Strategic knowledge: Strategies for checking answer for perimeter:** Recalculating strategy. [**RS**]
	Used different method to check the answer.	Alternative strategy. [**AS**]
	Strategies for checking answer for area: Used the same method and calculate again.	**Strategic knowledge: Strategies for checking answer for area:** Recalculating strategy. [**RS**]
	Used different method to check the answer.	Alternative strategy. [**AS**]
	Write unit for perimeter: Appropriate response: Wrote the unit (without probed).	**Ethical knowledge: Writes units of measurement upon completed a task:** Wrote. [**W**]
	Inappropriate response: Did not write the unit (without probed).	Did not write. [**NW**]
	Write unit for area: Appropriate response: Wrote the unit (without probed).	Wrote. [**W**]
	Inappropriate response: Did not write the unit (without probed).	Did not write. [**NW**]
	Check the answer for perimeter: Appropriate response: Checked the answer (without probed).	**Ethical knowledge: Checks the correctness of their solutions or answers:** Checked. [**C**]
	Inappropriate response: Did not check the answer (without probed).	Did not check. [**NC**]
	Check the answer for area: Appropriate response: Checked the answer (without probed).	Checked. [**C**]
	Inappropriate response: Did not check the answer (without probed).	Did not check. [**NC**]

Task	Response	Code
7	**Solving the fencing problem:** Correct response: Largest area, 882 m^2; Dimension, 42 m by 21 m.	**Solving the fencing problem:** Successful. [**S**]
	Incorrect response: e.g., Largest area, 880 m^2; Dimension, 40 m by 22 m.	Unsuccessful. [**U**]
	Strategies for solving the fencing problem.	**Strategic knowledge: Strategies for solving the fencing problem:** (a) Looking for a pattern. [**LP**] (b) Trial-and-error. [**TE**] (c) Differentiation method. [**DM**] (d) Other appropriate strategies. [**Please specify**]
	Strategies for checking answer.	**Strategic knowledge: Strategies for checking answer for the fencing problem:** (a) Looking for a pattern. [**LP**] (b) Trial-and-error. [**TE**] (c) Calculating the value of $\frac{d^2A}{dx^2}$ at the stationary point. [**CV**] (d) Other appropriate strategies. [**Please specify**]
	Write unit for the largest area being enclosed: Appropriate response: Wrote the unit (without probed).	**Ethical knowledge: Writes units of measurement upon completed a task:** Wrote. [**W**]
	Inappropriate response: Did not write the unit (without probed).	Did not write. [**NW**]
	Write unit for the dimension that yield the largest area: Appropriate response: Wrote the unit (without probed).	Wrote. [**W**]
	Inappropriate response: Did not write the unit (without probed).	Did not write. [**NW**]
	Check the answer for the fencing problem: Appropriate response: Checked the answer (without probed).	**Ethical knowledge: Checks the correctness of their solutions or answers:** Checked. [**C**]
	Inappropriate response: Did not check the answer (without probed).	Did not check. [**NC**]
8	**Developing area formulae:** Rectangle:	**Procedural knowledge: Developing area formulae:**
	Appropriate response: Able to develop the formula.	Successful (Able to develop). [**S**]
	Inappropriate response: Unable to develop the formula.	Unsuccessful (Unable to develop). [**U**]
	Parallelogram: Appropriate response: Able to develop the formula.	Successful (Able to develop). [**S**]
	Inappropriate response: Unable to develop the formula.	Unsuccessful (Unable to develop). [**U**]
	Triangle: Appropriate response: Able to develop the formula.	Successful (Able to develop). [**S**]
	Inappropriate response: Unable to develop the formula.	Unsuccessful (Unable to develop). [**U**]

Task	Response	Code
8	**Developing area formulae:**	**Procedural knowledge: Developing area formulae:**
	Trapezium:	
	Appropriate response: Able to develop the formula.	Successful (Able to develop). [**S**]
	Inappropriate response: Unable to develop the formula.	Unsuccessful (Unable to develop). [**U**]
	Relationship among area formulae:	**Conceptual knowledge: Relationship among area formulae:**
	Rectangle:	
	Appropriate response: Able to develop the formula.	**Understood** the **conceptual knowledge** underpinning the formula for the area of a rectangle. [**UCK**]
	Inappropriate response: Unable to develop the formula.	Did not understand. [**NUCK**]
	Parallelogram:	
	Appropriate response: Able to develop the formula from:	
	(a) Rectangle.	**Knew** the relationship between the formulae for the area of a parallelogram and rectangle. [**K1**]
	(b) Triangle.	**Knew** the relationship between the formulae for the area of a parallelogram and triangle. [**K2**]
	Inappropriate response: Unable to develop the formula.	Did not know. [**NK**]
	Triangle:	
	Appropriate response: Able to develop the formula from:	
	(a) Rectangle.	**Knew** the relationship between the formulae for the area of a triangle and rectangle that encloses it. [**K1**]
	(b) Parallelogram.	**Knew** the relationship between the formulae for the area of a triangle and parallelogram. [**K2**]
	Inappropriate response: Unable to develop the formula.	Did not know. [**NK**]
	Trapezium:	
	Appropriate response: Able to develop the formula from:	
	(a) Rectangle.	**Knew** the relationship between the formulae for the area of a trapezium and rectangle. [**K1**]
	(b) Triangle.	**Knew** the relationship between the formulae for the area of a trapezium and triangle. [**K2**]
	(c) Parallelogram.	**Knew** the relationship between the formulae for the area of a trapezium and parallelogram. [**K3**]
	(d) Rectangle and triangle, using algebraic method.	**Knew** that the formula for the area of a trapezium is related to the formulae for the area of a rectangle and triangle. [**K4**]
	Inappropriate response: Unable to develop the formula.	Did not know. [**NK**]
	Write the formula for the area of:	**Linguistic knowledge: Mathematical symbols:**
	Rectangle.	
	Appropriate response: (a) $l \times w$, where l and w represents the length and the width of the rectangle. (b) $l \times b$, where l and b represents the length and the breadth of the rectangle. (c) Other equivalent mathematical symbols.	Used **appropriate mathematical symbols** to write the formula. [**AMS**]
	Inappropriate response: e.g., $\frac{1}{2} \times p \times q$, where p and q represents the length and the width of the rectangle.	Used **inappropriate mathematical symbols** to write the formula. [**IMS**]

Task	Response	Code
8	**Write the formula for the area of:** Parallelogram. Appropriate response: (a) $b \times h$, where b and h represents the base and the height of the parallelogram. (b) Other equivalent mathematical symbols.	**Linguistic knowledge: Mathematical symbols:** Used **appropriate mathematical symbols** to write the formula. [**AMS**]
	Inappropriate response: e.g., $p \times q$, where p and q represents the base and the slanted side of the parallelogram.	Used **inappropriate mathematical symbols** to write the formula. [**IMS**]
	Triangle. Appropriate response: (a) $\frac{1}{2} \times b \times h$, where b and h represents the base and the height of the triangle. (b) Other equivalent mathematical symbols.	Used **appropriate mathematical symbols** to write the formula. [**AMS**]
	Inappropriate response: e.g., $b \times h$, where b and h represents the base and the height of the triangle.	Used **inappropriate mathematical symbols** to write the formula. [**IMS**]
	Trapezium. Appropriate response: (a) $\frac{1}{2} \times (a + b) \times h$, where $(a + b)$ and h represents the sum of the parallel sides and the height of the trapezium. (b) Other equivalent mathematical symbols.	Used **appropriate mathematical symbols** to write the formula. [**AMS**]
	Inappropriate response: e.g., $(a + b) \times h$, where $(a + b)$ and h represents the sum of the parallel sides and the height of the trapezium.	Used **inappropriate mathematical symbols** to write the formula. [**IMS**]
	State the area formulae or explain the meaning of the mathematical symbols: Rectangle. Appropriate response: (a) Length times width. (b) Length times breadth. (c) Other equivalent terms.	**Linguistic knowledge: Mathematical terms:** Used appropriate mathematical terms. [**AMT**]
	Inappropriate response: e.g.: (a) Length of this side times length of this side. (b) Horizontal side times vertical side.	Used inappropriate mathematical terms. [**IMT**]
	Parallelogram. Appropriate response: (a) Base times height. (b) Other equivalent terms.	Used appropriate mathematical terms. [**AMT**]
	Inappropriate response: e.g., this length times this length.	Used inappropriate mathematical terms. [**IMT**]
	Triangle. Appropriate response: (a) Half times base times height. (b) Other equivalent terms.	Used appropriate mathematical terms. [**AMT**]
	Inappropriate response: e.g., half times horizontal side times vertical side.	Used inappropriate mathematical terms. [**IMT**]

Task	Response	Code
8	**State the area formulae or explain the meaning of the mathematical symbols:**	**Linguistic knowledge: Mathematical terms:**
	Trapezium.	
	Appropriate response: (a) Half times the sum of the parallel sides times the height. (b) Other equivalent terms.	Used appropriate mathematical terms. [**AMT**]
	Inappropriate response: e.g., Half times the sum of the upper side and lower side times the height.	Used inappropriate mathematical terms. [**IMT**]
	Strategies for developing area formulae:	**Strategic knowledge: Strategies for developing area formulae:**
	Rectangle.	Rectangle. (a) Looking for a pattern (inductive method). [**LP**] (b) Other equivalent strategies. [**Please specify**]
	Parallelogram.	Parallelogram. (a) **Cut and paste**/decompose and rearrange (e.g., decompose a parallelogram into a triangle and a trapezium and then rearrange these shapes to form a rectangle). [**CP**] (b) Other equivalent strategies. [**Please specify**]
	Triangle.	Triangle. (a) **Partition** (e.g., partition a rectangle/parallelogram along its diagonal into two triangles). [**P**] (b) **Duplicate** (e.g., duplicate the triangle and arrange the two triangles to form a parallelogram). [**D**] (c) Other equivalent strategies. [**Please specify**]
	Trapezium.	Trapezium. (a) **Cut and paste**/decompose and rearrange (e.g., decompose an isosceles trapezium into a rectangle and two triangles and then rearrange these shapes to form a rectangle). [**CP**] (b) **Duplicate** (e.g., duplicate the trapezium and arrange the two trapeziums to form a parallelogram). [**D**] (c) Algebraic method. [**AM**] (d) Other equivalent strategies. [**Please specify**]
	Attempting to develop area formulae:	**Ethical knowledge: Develops area formulae:**
	Rectangle.	
	Appropriate response: Attempted to develop the formula.	Attempted. [**A**]
	Inappropriate response: Did not attempt to develop the formula.	Did not attempt. [**NA**]
	Parallelogram.	
	Appropriate response: Attempted to develop the formula.	Attempted. [**A**]
	Inappropriate response: Did not attempt to develop the formula.	Did not attempt. [**NA**]
	Triangle.	
	Appropriate response: Attempted to develop the formula.	Attempted. [**A**]
	Inappropriate response: Did not attempt to develop the formula.	Did not attempt. [**NA**]
	Trapezium.	
	Appropriate response: Attempted to develop the formula.	Attempted. [**A**]
	Inappropriate response: Did not attempt to develop the formula.	Did not attempt. [**NA**]

Appendix K
Procedure for Determining the Overall Level of Preservice Secondary School Mathematics Teachers' Subject Matter Knowledge of Perimeter and Area

To answer research question two, PSSMTs' levels (low, medium, high) of SMK of perimeter and area was analyzed in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

For the purpose of discussion, the researcher would give an example of how to determine the overall level of Beng's SMK of perimeter and area. It began by determining the level of Beng's conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area, using coding rubrics adapted from the Learning Mathematics for Teaching (LMT) (2006) project which consists of four principal investigators, namely Hill, Ball, Bass, and Schilling, from the School of Education, University of Michigan.

In order to code the mathematical element of conceptual knowledge of perimeter and area, the researcher has to determine whether the mathematical element is present (P) or not present (NP). If the mathematical element is present (P), then mark: (a) appropriate (A) if the PSSMT's use of the mathematical element was mathematically appropriate, accurate, or correct; or mark (b) inappropriate (I) if the PSSMT's use of the mathematical element was mathematically inappropriate, inaccurate, or incorrect. If the mathematical element is not present (NP), then mark: (a) appropriate (A) if the absence of the mathematical element seems appropriate or not problematic; or mark (b) inappropriate (I) if the absence of the mathematical element seems inappropriate or problematic (i.e., the mathematical element should have present) (adapted from LMT, 2006). Figure K1 shows the coding decision tree that represents the thought process involved (adapted from LMT, 2006, p. 13).

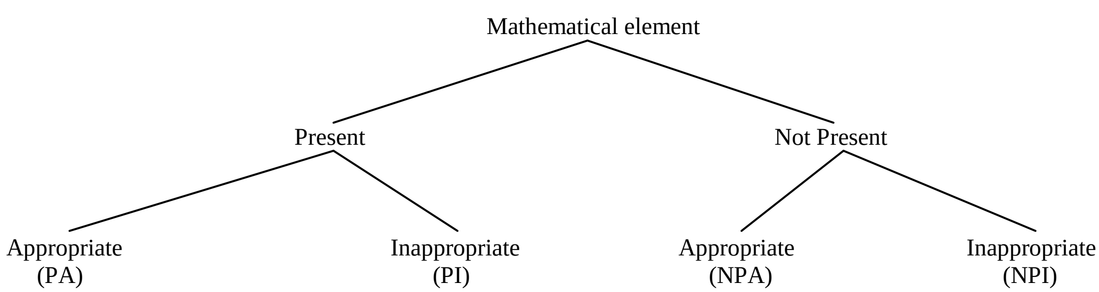


Figure K1. Coding decision tree.

In Task 1.1, Beng's notion of perimeter was limited to simple closed curves, and closed but not simple curves, exclusive of 3-dimensional shapes. Her notion of perimeter was inappropriate. Thus, the researcher coded the mathematical element of conceptual knowledge of perimeter and area, notion of perimeter, as "present and inappropriate (PI)" (see Appendix M). The same code goes for Beng's notion of area in Task 1.2. In Task 2, Beng's notion of the unit of area was not only limited to square, but also nonsquare (such as rectangle and triangle). In this case, her notion of the unit of area was appropriate. Thus, the researcher coded the mathematical element, notion of the unit of area, as "present and appropriate (PA)".

In Task 3.3 (a), Beng focused on the number of unit rather than the unit of measure when comparing perimeters in Set 1 with nonstandard units. She did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. Thus, this mathematical element, comparing perimeters in Set 1 with nonstandard units, was coded as "present and inappropriate (PI)" (see Appendix M). The same code goes for Tasks 3.3 (b), 3.4 (a), and 3.4 (b). In Task 3.3 (c), Beng focused on the number of unit when comparing perimeters in Set 3 with common standard unit. She knew that common standard unit (such as cm) is reliable for comparing perimeters.

Thus, this mathematical element, comparing perimeters in Set 3 with common standard unit, was coded as "present and appropriate (PA)". The same code goes for Tasks 3.4 (c).

In another situation in Tasks 3.3 (b), Beng understands the inverse proportion between the number of units and the unit of measure that the longer the unit of measure, the smaller the number of units required to get the same length. Thus, this mathematical element, the inverse proportion between the number of units and the unit of measure, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for Tasks 3.4 (b).

In Task 4, Beng times 10 when she converted 1cm to mm. It indicated that Beng knew the relationship between the standard units of length measurement that 1 cm = 10 mm. Thus, this mathematical element, the relationship between the standard units of length measurement that 1 cm = 10 mm, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for the relationship between the standard units of length measurement that 1 m = 100 cm and 1 km = 1000 m respectively in Tasks 4.

In Task 4, Beng's knowledge of the relationship between the standard units of area measurement that 1 cm^2 = 100 mm^2 was not present (absent) during the clinical interview. Nevertheless, the absence of this mathematical element, the relationship between the standard units of area measurement that 1 cm^2 = 100 mm^2, did not hinder her from successfully converted 3 cm^2 to mm^2. Thus, this mathematical element, the relationship between the standard units of area measurement that 1 cm^2 = 100 mm^2, was coded as "not present and appropriate (NPA)" (see Appendix M). The same code goes for the relationship between the standard units of area measurement that 1 m^2 = 10 000 cm^2 and 1 km^2 = 1 000 000 m^2 respectively in Tasks 4.

In Task 4, Beng viewed 3 cm^2 as the product of 3 times 1 cm times 1 cm. It indicated that she viewed 1 cm^2 as 1 cm times 1 cm. It also indicated that Beng knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, this mathematical element, the relationship between area units and linear units of measurement, was coded as "present and appropriate (PA)" (see Appendix M).

In Task 5.1, Beng did not know that there is no direct relationship between perimeter and area. She did not know that two shapes with the same perimeter can have different areas. Thus, Beng was not sure whether the student's method of calculating the area of the leaf was correct or not. In this case, this mathematical element, relationship between perimeter and area: Same perimeter, same area?, was coded as "present and inappropriate (PI)" (see Appendix M).

In Task 5.2, Beng provided the correct response that Mary's claim that the garden with the longer perimeter has the larger area was not correct. Beng knew that there is no direct relationship between perimeter and area. Beng knew that the garden with the longer perimeter could have a smaller area. Thus, she knew that Mary's claim was not correct. In this case, this mathematical element, relationship between perimeter and area: Longer perimeter, larger area?, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for the relationship between perimeter and area: Perimeter increases, area increases?, in Task 5.3.

In Task 8, Beng has no idea how the formula for the area of a rectangle can be developed or derived. She might has rote-learnt the formula. It was apparent that Beng lack of conceptual knowledge underpinning the formula for the area of a rectangle. The absence of this mathematical element, the relationship among area formulae: Rectangle, has hindered her from attempting to develop the formula for the area of a rectangle. Thus, this mathematical element, the relationship among area formulae: Rectangle, was coded as "not present and inappropriate (NPI)" (see Appendix M). The same code goes for the relationship among area formulae: Triangle, in Task 8.

In Task 8, Beng was able to develop the formula for the area of a parallelogram. She mentally transformed the parallelogram to a rectangle by cutting out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle. It indicated that Beng understands the relationship between the formulae for the area of a parallelogram and rectangle. Thus, this mathematical element, the relationship among area formulae: Parallelogram, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for the relationship among area formulae: Trapezium, in Task 8.

From the Appendix M, it can be seen that there were 25 codes of the mathematical elements of conceptual knowledge of perimeter and area. Out of the total of 25 codes, 16 of them were coded as "present and appropriate (PA)" or "not present and appropriate (NPA)". Thus, the percentage of appropriate mathematical elements of conceptual knowledge of perimeter and area obtained by Beng was computed as follow:

Percentage of appropriate mathematical elements of conceptual knowledge obtained by Beng

$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% = \frac{16}{25} \times 100\% = 64.0\%$

In the Learning Mathematics for Teaching (LMT) (2006) project, random pairs of researchers were assigned to code each videotaped lesson. The coders coded each lesson individually and then gave an overall level of the teacher's knowledge of mathematics as low, medium, or high, based on their impression of the teacher's level of mathematical knowledge. They met and reconciled their codes before giving their final level of mathematical knowledge.

In this study, PSSMTs' levels (low, medium, high) of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area as well as the overall level of SMK of perimeter and area were determined based on the percentage of appropriate mathematical elements of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area as well as the overall percentage of appropriate mathematical elements of SMK of perimeter and area obtained by the PSSMTs.

In the university where the data of this study was collected, Grade A is assigned to PSSMTs who obtained 80 marks and above in the content as well as the method courses. Grade A− is assigned to PSSMTs who obtained 70 to 79 marks. The passing mark is 40. Thus, in this study, PSSMTs who secured 70% and above of appropriate mathematical elements of conceptual knowledge of perimeter and area were assigned a high level of conceptual knowledge of perimeter and area. PSSMTs who achieved the range from 40% to less than 70% of appropriate mathematical elements of conceptual knowledge of perimeter and area were assigned a medium level of conceptual knowledge of perimeter and area. PSSMTs who gained less than 40% of appropriate mathematical elements of conceptual knowledge of perimeter and area were assigned a low level of conceptual knowledge of perimeter and area. Table K1 summarizes the range of percentage of appropriate mathematical elements of conceptual knowledge of perimeter and area obtained by the PSSMTs and their respective levels of conceptual knowledge of perimeter and area. The same goes for other components of SMK as well as SMK of perimeter and area.

Table K1

Range of Percentage of Appropriate Mathematical Elements of Conceptual Knowledge Obtained by the PSSMTs and Their Respective Levels of Conceptual Knowledge

Percentage of appropriate mathematical elements of conceptual knowledge obtained by the PSSMTs	Levels of conceptual knowledge
$0 \leq x < 40$	low
$40 \leq x < 70$	medium
$70 \leq x < 100$	high

Beng achieved 64.0% of appropriate mathematical elements of conceptual knowledge of perimeter and area. Thus, she was assigned a medium level of conceptual knowledge of perimeter and area (see Appendix M). The same procedure was applied to determine the PSSMTs' levels (low, medium, high) of procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area as well as the overall level of SMK of perimeter and area.

For the procedural knowledge of perimeter and area, Beng had successfully converting 3 cm^2 to mm^2 in Task 4. Thus, the researcher coded the mathematical element of procedural knowledge of perimeter and area, converting standard units of area measurement: 3 cm^2 to mm^2, as "present and appropriate (PA)" (see Appendix M). The same code goes for converting standard units of area measurement: 4.7 m^2 to cm^2 as well as 1.25 km^2 to m^2 in Task 4.

In Task 6.1, Beng has successfully calculated the perimeter of Diagram 1 as 104 cm. Thus, this mathematical element, calculating perimeter of composite figures: Diagram 1, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for calculating perimeter of composite figures: Diagram 2, in Tasks 6.2. In Task 6.1, Beng has successfully calculated the area of Diagram 1 as 420 cm^2. Thus, this mathematical element, calculating area of composite figures: Diagram 1, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for calculating area of composite figures: Diagram 2, in Tasks 6.2.

In Task 8, Beng was unable to develop the formula for the area of a rectangle. She did not attempt to develop the formula. Thus, this mathematical element, developing area formulae: Rectangle, was coded as “not present and inappropriate (NPI)” (see Appendix M). The same code goes for developing area formulae: Triangle, in Task 8. In Task 8, Beng was able to develop the formula for the area of a parallelogram. Thus, this mathematical element, developing area formulae: Parallelogram, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for developing area formulae: Trapezium, in Task 8.

From the Appendix M, it can be seen that there were 11 codes of the mathematical elements of procedural knowledge of perimeter and area. Out of the total of 11 codes, 9 of them were coded as “present and appropriate (PA)” or “not present and appropriate (NPA)”. Thus, the percentage of appropriate mathematical elements of procedural knowledge of perimeter and area obtained by Beng was computed as follow:

Percentage of appropriate mathematical elements of procedural knowledge obtained by Beng

$$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% = \frac{9}{11} \times 100\% = 81.8\%$$

Beng secured 81.8% of appropriate mathematical elements of procedural knowledge of perimeter and area. Thus, she was assigned a high level of procedural knowledge of perimeter and area (see Appendix M).

For the linguistic knowledge of perimeter and area, Beng used appropriate mathematical symbols to write the formula for the area of a rectangle, in Task 8. She wrote the formula as ‘$l \times w$’. Thus, the researcher coded the mathematical element of linguistic knowledge of perimeter and area, mathematical symbols: formula for the area of a rectangle, as “present and appropriate (PA)” (see Appendix M). The same code goes for mathematical symbols: formula for the area of a parallelogram, triangle, and trapezium, in Task 8.

In Task 1.1, Beng used appropriate mathematical terms ‘2-dimensional shapes’ and ‘enclosed’ to justify her selection of shape A that have a perimeter. Beng explained that she selected shape A because it was a 2-dimensional shape and enclosed. Thus, this mathematical element, mathematical terms: Justification of shape A that have a perimeter, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for mathematical terms: Justification of shapes C, D, H, I, and K that have a perimeter, in Task 1.1.

In Task 1.1, Beng used appropriate mathematical negation ‘not enclosed’ as her justification for not selecting shape B as having a perimeter. Beng explained that she did not select shape B because it is not enclosed. Thus, this mathematical element, mathematical terms: Justification for not selecting shape B that does not have a perimeter, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for mathematical terms: Justification for not selecting shapes E, G, and L that do not have a perimeter, in Task 1.1.

In Task 1.1, Beng explained that she did not select shape F because it is a 3-dimensional shape. '3-dimensional shape' is an appropriate mathematical term. Nevertheless, it was not the appropriate justification for not selecting shape F that have a perimeter as we still can find perimeter for the faces of solids. Thus, this mathematical element, mathematical terms: Justification for not selecting shape F that have a perimeter, was coded as "present and inappropriate (PI)" (see Appendix M). The same code goes for mathematical terms: Justification for not selecting shape J that have a perimeter, in Task 1.1.

In Task 1.2, Beng used appropriate mathematical term 'enclosed' to justify her selection of shape A that have an area. Beng explained that she selected shape A because it was enclosed. Thus, this mathematical element, mathematical terms: Justification of shape A that have an area, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for mathematical terms: Justification of shapes C, D, H, I, and K that have an area, in Task 1.2.

In Task 1.2, Beng used appropriate mathematical negation 'not enclosed' as her justification for not selecting shape B as having an area. Beng explained that she did not select shape B because it is not enclosed. Thus, this mathematical element, mathematical terms: Justification for not selecting shape B that does not have an area, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for mathematical terms: Justification for not selecting shapes E, G, and L that do not have an area, in Task 1.2.

In Task 1.2, Beng explained that she did not select shape F because it is a 3D (3-dimensional). '3D' is an appropriate mathematical symbol. Nevertheless, '3D' is not the appropriate justification for not selecting shape F that have an area as we still can find area for the faces of solids. Thus, this mathematical element, mathematical terms: Justification for not selecting shape F that have an area, was coded as "present and inappropriate (PI)" (see Appendix M). The same code goes for mathematical terms: Justification for not selecting shape J that have an area, in Task 1.2.

In Task 2, Beng used inappropriate mathematical term 'straight lines' to justify that a square can be used as the unit of area. She explained that a square can be used as the unit of area because it has straight lines. Thus, this mathematical element, mathematical terms: Justification of shape that can or cannot be used as the unit of area: Square, was coded as "present and inappropriate (PI)" (see Appendix M). The same code goes for mathematical terms: Justification of shape that can or cannot be used as the unit of area: Rectangle and triangle, in Task 1.2.

In Task 8, Beng used appropriate mathematical terms 'length', 'times', and 'width' to state the formula for the area of a rectangle. She stated the formula as 'length times width'. Thus, this mathematical element, state the area formula or explain the meaning of the mathematical symbols employed in the formula: Rectangle, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for the mathematical element, state the area formula or explain the meaning of the mathematical symbols employed in the formula: Triangle, in Task 8.

In Task 8, Beng used inappropriate mathematical terms to explain the meaning of the symbols a and b that she employed in the formula for the area of a parallelogram. Beng used inappropriate mathematical terms 'the length here' and 'the line here perpendicular to a' to explain the meaning of the symbols a and b that she employed. Beng explained that "a is the length here and b is the line here perpendicular to a." (Beng/L1332). Actually, a and b represents the base and the height of the parallelogram. Conventionally, the formula for the area of a parallelogram is written as '$b \times h$', where b and h represents the base and the height of the parallelogram. Thus, this mathematical element, state the area formula or explain the meaning of the mathematical symbols employed in the formula: Parallelogram, was coded as "present and inappropriate (PI)" (see Appendix M). The same code goes for the mathematical element, state the area formula or explain the meaning of the mathematical symbols employed in the formula: Trapezium, in Task 8.

In Task 6.1, Beng used the correct standard unit of measurement for perimeter, namely cm, when she wrote the answer for this measurement of Diagram 1. It indicated that Beng understands the general measurement convention that perimeter is measured by linear unit. Thus, this mathematical element, standard unit of length measurement (linear units): Perimeter of Diagram 1, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for the mathematical element, standard unit of length measurement (linear units): Perimeter of Diagram 2, in Task 6.2.

In Task 6.1, Beng used the correct standard unit of measurement for area, namely cm^2, when she wrote the answer for this measurement of Diagram 1. It indicated that Beng understands the general measurement convention that area is measured by square unit. Thus, this mathematical element, standard unit of area measurement (square units): Area of Diagram 1, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for the mathematical element, standard unit of area measurement (square units): Area of Diagram 2, in Task 6.2.

In Task 3.4, Beng wrote 16 cm^2 literally as 'sixteen centimetre square'. The correct answer should be 'sixteen square centimetres'. Thus, this mathematical element, conventions of writing SI area measurement: 16 cm^2, was coded as "present and inappropriate (PI)" (see Appendix M). The same code goes for the mathematical element, conventions of writing SI area measurement: 13 cm^2, in Task 3.4.

In Task 3.4, Beng read 16 cm^2 literally as 'sixteen centimetre square'. The correct answer should be 'sixteen square centimetres'. Thus, this mathematical element, conventions of reading SI area measurement: 16 cm^2, was coded as "present and inappropriate (PI)" (see Appendix M). The same code goes for the mathematical element, conventions of reading SI area measurement: 13 cm^2, in Task 3.4.

From the Appendix M, it can be seen that there were 43 codes of the mathematical elements of linguistic knowledge of perimeter and area. Out of the total of 43 codes, 30 of them were coded as "present and appropriate (PA)" or "not present and appropriate (NPA)". Thus, the percentage of appropriate mathematical elements of linguistic knowledge of perimeter and area obtained by Beng was computed as follow:

Percentage of appropriate mathematical elements of linguistic knowledge obtained by Beng

$$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% = \frac{30}{43} \times 100\% = 69.8\%$$

Beng achieved 69.8% of appropriate mathematical elements of linguistic knowledge of perimeter and area. Thus, she was assigned a medium level of linguistic knowledge of perimeter and area (see Appendix M).

For the strategic knowledge of perimeter and area, in Task 3.1, Beng used the formal method of measuring the side by ruler and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter. She measured the length of each side of the given T-shape by ruler and then calculated its perimeter. Beng also measured the length of each side of the given rectangle by ruler and then calculated its perimeter. She used appropriate strategy to determine whether the given pair of shapes had the same perimeter. Thus, the researcher coded the mathematical element of strategic knowledge of perimeter and area, strategy for comparing perimeter, as "present and appropriate (PA)" (see Appendix M). The same code goes for the mathematical element, strategy for comparing area, in Task 3.2.

In Task 3.1, when probed for alternative method of comparing the perimeter, Beng could suggest at least one appropriate alternative method to compare the perimeter (see Table 4.47). Thus, this mathematical element, alternative method for comparing perimeter, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for the mathematical element, alternative method for comparing area, in Task 3.2.

In Task 6.1, when probed to check the answer for the perimeter of Diagram 1, Beng suggested that she would use the recalculating strategy to verify the answer. Beng suggested that she would check the answer for perimeter by recalculating strategy that using the same method and calculate again. She used the appropriate strategy to check the answer for the perimeter of Diagram 1. Thus, this mathematical element, strategy for checking the answer for the perimeter of Diagram 1, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for the mathematical element, strategy for checking the answer for the perimeter of Diagram 2, in Task 6.2.

In Task 6.2, when probed to check the answer for the area of Diagram 1, Beng, used an alternative procedure (alternative method) to generate an answer which could be used to verify the original answer. She checked the answer for area by moving triangle RST under the translation T_{SR} to form a rectangle with the dimensions of 15 cm by 8 cm. Beng drew a large rectangle with the dimension of 28 cm by 15 cm and calculated its area by using area formula of rectangle as 420 cm^2. She used the appropriate strategy to check the answer for the area of Diagram 1. Thus, this mathematical element, strategy for checking the answer for the area of Diagram 1, was coded as "present and appropriate (PA)" (see Appendix M). The same code goes for the mathematical element, strategy for checking the answer for the area of Diagram 2, in Task 6.2.

In Task 7, Beng had successfully solving the fencing problem. She used the looking for a pattern strategy to solve the fencing problem. Beng used the appropriate strategy to solve the fencing problem. Thus, this mathematical element, strategy for solving the fencing problem, was coded as "present and appropriate (PA)" (see Appendix M).

In Task 7, Beng used the looking for a pattern strategy to check the answer for the fencing problem without being probed. Beng attempted to verify whether 882 m^2 was the largest area being enclosed. Beng tested it with two values of the width that were smaller than 21 m, namely 9 m and 8 m respectively. She found that the area decreased to 594 m^2 and 544 m^2 respectively. Beng also tested it with two values of the width that were larger than 21 m, namely 22 m and 23 m respectively. She found that the area decreased to 880 m^2 and 874 m^2 respectively. Thus, Beng concluded that 882 m^2 is the largest area being enclosed and the dimension of the rectangular garden that yields the largest area being enclosed is 42 m by 21 m. Beng used the appropriate strategy for checking the answer for the fencing problem. Thus, this mathematical element, strategy for checking the answer for the fencing problem, was coded as “present and appropriate (PA)” (see Appendix M).

In Task 8, Beng did not attempt to develop the formula for the area of a rectangle. Thus, this mathematical element, strategy for developing area formula for a rectangle, was coded as “not present and inappropriate (NPI)” (see Appendix M). The same code goes for the mathematical element, strategy for developing area formula for a triangle, in Task 8.

In Task 8, Beng had succeeded in developing the formula for the area of a parallelogram. She used the cut and paste strategy to develop the formula. Beng mentally cut out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle. She used the appropriate strategy to develop the formula for the area of a parallelogram. Thus, this mathematical element, strategy for developing area formula for a parallelogram, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for the mathematical element, strategy for developing area formula for a trapezium, in Task 8.

From the Appendix M, it can be seen that there were 14 codes of the mathematical elements of strategic knowledge of perimeter and area. Out of the total of 14 codes, 12 of them were coded as “present and appropriate (PA)” or “not present and appropriate (NPA)”. Thus, the percentage of appropriate mathematical elements of strategic knowledge of perimeter and area obtained by Beng was computed as follow:

Percentage of appropriate mathematical elements of strategic knowledge obtained by Beng

$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% = \frac{12}{14} \times 100\% = 85.7\%$

Beng secured 85.7% of appropriate mathematical elements of strategic knowledge of perimeter and area. Thus, she was assigned a high level of strategic knowledge of perimeter and area (see Appendix M).

For the ethical knowledge of perimeter and area, in Task 1.1, Beng had provided appropriate justification for selecting shape A that has a perimeter. Thus, the researcher coded the mathematical element of ethical knowledge of perimeter and area, justifies the selection of shape A that has a perimeter, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for the mathematical elements, justifies the selection of shapes C, D, H, I, and K that has a perimeter, in Task 1.1.

In Task 1.1, Beng had provided appropriate justification for not selecting shape B as having a perimeter. Thus, this mathematical element, justification for not selecting shape B as having a perimeter, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for the mathematical elements, justification for not selecting shapes E, G, and L as having a perimeter, in Task 1.1.

In Task 1.1, Beng had provided inappropriate justification for not selecting shape F that has a perimeter. Thus, this mathematical element, justification for not selecting shape F that has a perimeter, was coded as “present and inappropriate (PI)” (see Appendix M). The same code goes for the mathematical element, justification for not selecting shape J that has a perimeter, in Task 1.1.

In Task 1.2, Beng had provided appropriate justification for selecting shape A that has an area. Thus, this mathematical element, justifies the selection of shape A that has an area, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for the mathematical elements, justifies the selection of shapes C, D, H, I, and K that has an area, in Task 1.2.

In Task 1.2, Beng had provided appropriate justification for not selecting shape B as having an area. Thus, this mathematical element, justification for not selecting shape B as having an area, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for the mathematical elements, justification for not selecting shapes E, G, and L as having an area, in Task 1.2.

In Task 1.2, Beng had provided inappropriate justification for not selecting shape F that has an area. Thus, this mathematical element, justification for not selecting shape F that has an area, was coded as “present and inappropriate (PI)” (see Appendix M). The same code goes for the mathematical element, justification for not selecting shape J that has an area, in Task 1.2.

In Task 2, Beng had provided inappropriate justification for selecting a square that can be used as the unit of area. Thus, this mathematical element, justifies the shape that can or cannot be used as the unit of area: Square, was coded as “present and inappropriate (PI)” (see Appendix M). The same code goes for the mathematical elements, justifies the shape that can or cannot be used as the unit of area: Rectangle and triangle, in Task 2.

In Task 5.1, Beng did not attempt to examine the possible pattern of the relationship between perimeter and area. Thus, this mathematical element, examines pattern: Two shapes with the same perimeter have the same area?, was coded as “not present and inappropriate (NPI)” (see Appendix M). The same code goes for the mathematical elements, “formulates generalization that two shapes with the same perimeter have the same area?” and “tests generalization that two shapes with the same perimeter have the same area?”, in Task 5.1.

In Task 5.2, Beng had attempted to examine the possible pattern of the relationship between perimeter and area. Based on her reflection on Task 3.1, she found that the shape with the longer perimeter may have a smaller area. Thus, this mathematical element, examines pattern: The garden with the longer perimeter has the larger area?, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for the mathematical elements, “formulates generalization that the garden with the longer perimeter has the larger area?” and “tests generalization that the garden with the longer perimeter has the larger area?”, in Task 5.2. In Task 5.3, Beng had attempted to test generalization pertaining to the relationship between perimeter and area with an example (not counterexample). Thus, this mathematical element, tests generalization that as the perimeter of a closed figure increases, the area also increases?, was coded as “present and inappropriate (PI)” (see Appendix M).

In Task 8, Beng did not attempt to develop the formula for the area of a rectangle. Thus, this mathematical element, attempts to develop area formula for a rectangle, was coded as “not present and inappropriate (NPI)” (see Appendix M). The same code goes for the mathematical element, attempts to develop area formula for a triangle, in Task 8. In Task 8, Beng had succeeded in developing the formula for the area of a parallelogram. Thus, this mathematical element, attempts to develop area formula for a parallelogram, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for the mathematical element, attempts to develop area formula for a trapezium, in Task 8.

In Task 6.1, Beng wrote the measurement unit (without probed), namely cm, for the answer of the perimeter of Diagram 1 that she has calculated. Thus, this mathematical element, writes unit for perimeter of Diagram 1, was coded as “present and appropriate (PA)” (see Appendix M). The same code goes for the mathematical elements, “writes unit for area of Diagram 1” in Task 6.1, “writes unit for perimeter of Diagram 2” and “writes unit for area of Diagram 2” in Task 6.2.

In Task 7, Beng did not write measurement unit for the largest area being enclosed. Thus, this mathematical element, writes unit for the largest area being enclosed, was coded as “not present and inappropriate (NPI)” (see Appendix M). The same code goes for the mathematical element, writes unit for the dimension that yields the largest area, in Task 7.

In Task 6.1, Beng did not check the correctness of the answer for the perimeter of Diagram 1. When probed to check answer, then only Beng suggested the strategies that she would use to check the answer for perimeter of Diagram 1. Thus, this mathematical element, checks the answer of the perimeter of Diagram 1, was coded as “not present and inappropriate (NPI)” (see Appendix M). The same code goes for the mathematical elements, “checks the answer of the area of Diagram 1” in Task 6.1, “checks the answer of the perimeter of Diagram 2” and “checks the answer of the area of Diagram 2” in Task 6.2. In Task 7, Beng had checked the correctness of the answer for the fencing problem without being probed. Thus, this mathematical element, checks the answer for the fencing problem, was coded as “present and appropriate (PA)” (see Appendix M).

From the Appendix M, it can be seen that there were 49 codes of the mathematical elements of ethical knowledge of perimeter and area. Out of the total of 49 codes, 30 of them were coded as “present and appropriate (PA)” or “not present and appropriate (NPA)”. Thus, the percentage of appropriate mathematical elements of ethical knowledge of perimeter and area obtained by Beng was computed as follow:

Percentage of appropriate mathematical elements of ethical knowledge obtained by Beng

$$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% = \frac{30}{49} \times 100\% = 61.2\%$$

Beng achieved 61.2% of appropriate mathematical elements of ethical knowledge of perimeter and area. Thus, she was assigned a medium level of ethical knowledge of perimeter and area (see Appendix M).

From the Appendix M, it can be seen that there were 25, 11, 43, 14, and 49 codes of the mathematical elements of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area, respectively. Out of the total of 142 codes, 16, 9, 30, 12, and 30 codes of the mathematical elements of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of perimeter and area were coded as “present and appropriate (PA)” or “not present and appropriate (NPA)”, respectively. The total number of appropriate mathematical elements obtained by Beng was 97. Thus, the percentage of appropriate mathematical elements of SMK of perimeter and area obtained by Beng was computed as follow:

Percentage of appropriate mathematical elements of SMK obtained by Beng

$$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% = \frac{16 + 9 + 30 + 12 + 30}{25 + 11 + 43 + 14 + 49} \times 100\% = \frac{97}{142} \times 100\% = 68.3\%$$

Beng achieved 68.3% of appropriate mathematical elements of SMK of perimeter and area. Thus, she was assigned an overall medium level of SMK of perimeter and area (see Appendix M). The same procedure was applied to determine the overall level of SMK of perimeter and area of other PSSMTs, namely Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha.

Appendix L
Coding Rubrics for Determining Overall Level of …………'s SMK

Coding Rubrics for Determining Level of …………'s Conceptual Knowledge

Task	Conceptual Knowledge																																							
	1. Notion of perimeter				2. Notion of area				3. Notion of the unit of area (square and nonsquare)				4.1. Number of units and unit of measure: Comparing perimeter with nonstandard units				4.2. Number of units and unit of measure: Comparing perimeter with common nonstandard units				4.3. Number of units and unit of measure: Comparing perimeter with common standard unit				4.4. Number of units and unit of measure: Comparing area with nonstandard units				4.5. Number of units and unit of measure: Comparing area with common nonstandard units				4.6. Number of units and unit of measure: Comparing area with common standard unit				5.1. Inverse relationship between number of units and unit of measure: Perimeter			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of ………….’s Conceptual Knowledge (continued)

Task	Conceptual Knowledge																																							
	5.2. Inverse relationship between number of units and unit of measure: Area				6.1. Relationship between standard units of length (linear units): 1 cm = 10 mm				6.2. Relationship between standard units of length (linear units): 1 m = 100 cm				6.3. Relationship between standard units of length (linear units): 1 km = 1000 m				7.1. Relationship between standard units of area (square units): 1 cm^2 = 100 mm^2				7.2. Relationship between standard units of area (square units): 1 m^2 = 10 000 cm^2				7.3. Relationship between standard units of area (square units): 1 km^2 = 1 000 000 m^2				8. Relationship between area units and linear units				9.1. Relationship between Perimeter and area: Same perimeter, same area?				9.2. Relationship between Perimeter and area: Longer perimeter, larger area?			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of …………'s Conceptual Knowledge (continued)

Task	Conceptual Knowledge																																							
	9.3. Relationship between Perimeter and area: Perimeter increases, area increases?				10.1. Relationship among area formulae: Rectangle				10.2. Relationship among area formulae: Parallelogram				10.3. Relationship among area formulae: Triangle				10.4. Relationship among area formulae: Trapezium																							
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Percentage of appropriate mathematical element(s) of conceptual knowledge obtained by the subject

$$= \frac{f\,(PA+NPA)}{f\,(PA+PI+NPA+NPI)} \times 100\% =$$

Overall level of subject's conceptual knowledge		
Low ($0 \le x < 40$)	Medium ($40 \le x < 70$)	High ($70 \le x < 100$)

Coding Rubrics for Determining Level of …………'s Procedural Knowledge

Task	Procedural Knowledge																																							
	1.1. Converting standard units of area measurement: 3 cm^2 to mm^2				1.2. Converting standard units of area measurement: 4.7 m^2 to cm^2				1.3. Converting standard units of area measurement: 1.25 km^2 to m^2				2.1. Calculating perimeter of composite figures: Diagram 1				2.2. Calculating perimeter of composite figures: Diagram 2				3.1. Calculating area of composite figures: Diagram 1				3.2. Calculating area of composite figures: Diagram 2				4.1. Developing area formula: Rectangle				4.2. Developing area formula: Parallelogram				4.3. Developing area formula: Triangle			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of …………’s Procedural Knowledge (continued)

Task	Procedural Knowledge																																						
	4.4. Developing area formula: Trapezium																																						
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Percentage of appropriate mathematical element(s) of procedural knowledge obtained by the subject

$$= \frac{f\,(PA+NPA)}{f\,(PA+PI+NPA+NPI)} \times 100\% =$$

Overall level of subject’s procedural knowledge		
Low ($0 \leq x < 40$)	Medium ($40 \leq x < 70$)	High ($70 \leq x < 100$)

Coding Rubrics for Determining Level of …………’s Linguistic Knowledge

Task	Linguistic Knowledge																																							
	1.1. Mathematical symbols: Formula for the area of a rectangle				1.2. Mathematical symbols: Formula for the area of a parallelogram				1.3. Mathematical symbols: Formula for the area of a triangle				1.4. Mathematical symbols: Formula for the area of a trapezium				2.1.1. Mathematical terms: Justification of shape A that have or do not have a perimeter				2.1.2. Mathematical terms: Justification of shape B that have or do not have a perimeter				2.1.3. Mathematical terms: Justification of shape C that have or do not have a perimeter				2.1.4. Mathematical terms: Justification of shape D that have or do not have a perimeter				2.1.5. Mathematical terms: Justification of shape E that have or do not have a perimeter				2.1.6. Mathematical terms: Justification of shape F that have or do not have a perimeter			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of …………’s Linguistic Knowledge (continued)

Task	Linguistic Knowledge																																							
	2.1.7. Mathematical terms: Justification of shape G that have or do not have a perimeter				2.1.8. Mathematical terms: Justification of shape H that have or do not have a perimeter				2.1.9. Mathematical terms: Justification of shape I that have or do not have a perimeter				2.1.10. Mathematical terms: Justification of shape J that have or do not have a perimeter				2.1.11. Mathematical terms: Justification of shape K that have or do not have a perimeter				2.1.12. Mathematical terms: Justification of shape L that have or do not have a perimeter				2.2.1. Mathematical terms: Justification of shape A that have or do not have an area				2.2.2. Mathematical terms: Justification of shape B that have or do not have an area				2.2.3. Mathematical terms: Justification of shape C that have or do not have an area				2.2.4. Mathematical terms: Justification of shape D that have or do not have an area			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of …………'s Linguistic Knowledge (continued)

Task	Linguistic Knowledge																																							
	2.2.5. Mathematical terms: Justification of shape E that have or do not have an area				2.2.6. Mathematical terms: Justification of shape F that have or do not have an area				2.2.7. Mathematical terms: Justification of shape G that have or do not have an area				2.2.8. Mathematical terms: Justification of shape H that have or do not have an area				2.2.9. Mathematical terms: Justification of shape I that have or do not have an area				2.2.10. Mathematical terms: Justification of shape J that have or do not have an area				2.2.11. Mathematical terms: Justification of shape K that have or do not have an area				2.2.12. Mathematical terms: Justification of shape L that have or do not have an area				2.3.1. Mathematical terms: Justification of shape that can or cannot be used as the unit of area: Square				2.3.2. Mathematical terms: Justification of shape that can or cannot be used as the unit of area: Rectangle			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of …………’s Linguistic Knowledge (continued)

Task	Linguistic Knowledge																																							
	2.3.3. Mathematical terms: Justification of shape that can or cannot be used as the unit of area: Triangle				2.4.1. State the area formula or explain the meaning of the mathematical symbols employed in the formula: Rectangle				2.4.2. State the area formula or explain the meaning of the mathematical symbols employed in the formula: Parallelogram				2.4.3. State the area formula or explain the meaning of the mathematical symbols employed in the formula: Triangle				2.4.4. State the area formula or explain the meaning of the mathematical symbols employed in the formula: Trapezium				3.1. Standard unit of length measurement (linear unit): Perimeter of Diagram 1				3.2. Standard unit of length measurement (linear unit): Perimeter of Diagram 2				4.1. Standard unit of area measurement (square unit): Area of Diagram 1				4.2. Standard unit of area measurement (square unit): Area of Diagram 2				5.1. Conventions of writing SI area measurement: 16 cm^2			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of …………'s Linguistic Knowledge (continued)

Task	Linguistic Knowledge																																							
	5.2. Conventions of reading SI area measurement: 16 cm^2				5.3. Conventions of writing SI area measurement: 13 cm^2				5.4. Conventions of reading SI area measurement: 13 cm^2																															
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Percentage of appropriate mathematical element(s) of linguistic knowledge obtained by the subject

$$= \frac{f\,(PA+NPA)}{f\,(PA+PI+NPA+NPI)} \times 100\% =$$

Overall level of subject's linguistic knowledge		
Low ($0 \le x < 40$)	Medium ($40 \le x < 70$)	High ($70 \le x < 100$)

Coding Rubrics for Determining Level of …………'s Strategic Knowledge

Task	Strategic Knowledge																																							
	1.1. Strategy for comparing perimeter				1.2. Alternative method for comparing perimeter				2.1. Strategy for comparing area				2.2. Alternative method for comparing area				3.1. Strategy for checking answer for perimeter of Diagram 1				3.2. Strategy for checking answer for perimeter of Diagram 2				4.1. Strategy for checking answer for area of Diagram 1				4.2. Strategy for checking answer for area of Diagram 2				5. Strategy for solving the fencing problem				6. Strategy for checking answer for the fencing problem			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of …………'s Strategic Knowledge (continued)

Task	Strategic Knowledge																																							
	7.1. Strategy for developing area formula for a rectangle				7.2. Strategy for developing area formula for a parallelogram				7.3. Strategy for developing area formula for a triangle				7.4. Strategy for developing area formula for a trapezium																											
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Percentage of appropriate mathematical element(s) of strategic knowledge obtained by the subject

$$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% =$$

Overall level of subject's strategic knowledge		
Low ($0 \le x < 40$)	Medium ($40 \le x < 70$)	High ($70 \le x < 100$)

Coding Rubrics for Determining Level of …………’s Ethical Knowledge

Task	Ethical Knowledge																																							
	1.1.1. Justifies the selection of shape A that have or do not have a perimeter				1.1.2. Justifies the selection of shape B that have or do not have a perimeter				1.1.3. Justifies the selection of shape C that have or do not have a perimeter				1.1.4. Justifies the selection of shape D that have or do not have a perimeter				1.1.5. Justifies the selection of shape E that have or do not have a perimeter				1.1.6. Justifies the selection of shape F that have or do not have a perimeter				1.1.7. Justifies the selection of shape G that have or do not have a perimeter				1.1.8. Justifies the selection of shape H that have or do not have a perimeter				1.1.9. Justifies the selection of shape I that have or do not have a perimeter				1.1.10. Justifies the selection of shape J that have or do not have a perimeter			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of …………'s Ethical Knowledge (continued)

Task	Ethical Knowledge																																							
	1.1.11. Justifies the selection of shape K that have or do not have a perimeter				1.1.12. Justifies the selection of shape L that have or do not have a perimeter				1.2.1. Justifies the selection of shape A that have or do not have an area				1.2.2. Justifies the selection of shape B that have or do not have an area				1.2.3. Justifies the selection of shape C that have or do not have an area				1.2.4. Justifies the selection of shape D that have or do not have an area				1.2.5. Justifies the selection of shape E that have or do not have an area				1.2.6. Justifies the selection of shape F that have or do not have an area				1.2.7. Justifies the selection of shape G that have or do not have an area				1.2.8. Justifies the selection of shape H that have or do not have an area			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of …………'s Ethical Knowledge (continued)

Task	Ethical Knowledge																																							
	1.2.9. Justifies the selection of shape I that have or do not have an area				1.2.10. Justifies the selection of shape J that have or do not have an area				1.2.11. Justifies the selection of shape K that have or do not have an area				1.2.12. Justifies the selection of shape L that have or do not have an area				1.3.1. Justifies the shape that can or cannot be used as the unit of area: Square				1.3.2. Justifies the shape that can or cannot be used as the unit of area: Rectangle				1.3.3. Justifies the shape that can or cannot be used as the unit of area: Triangle				2.1. Examines pattern: Two shapes with the same perimeter have the same area?				2.2. Examines pattern: The garden with the longer perimeter has the larger area?				3.1. Formulates generalization that two shapes with the same perimeter have the same area?			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of …………'s Ethical Knowledge (continued)

Task	Ethical Knowledge																																							
	3.2. Formulates generalization that the garden with the longer perimeter has the larger area?				4.1. Tests generalization that two shapes with the same perimeter have the same area?				4.2. Tests generalization that the garden with the longer perimeter has the larger area?				4.3. Tests generalization that as the perimeter of a closed figure increases, the area also increases?				5.1. Attempts to develop area formula for a rectangle				5.2. Attempts to develop area formula for a parallelogram				5.3. Attempts to develop area formula for a triangle				5.4. Attempts to develop area formula for a trapezium				6.1. Writes unit for perimeter of Diagram 1				6.2. Writes unit for area of Diagram 1			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of …………'s Ethical Knowledge (continued)

Task	Ethical Knowledge																																							
	6.3. Writes unit for perimeter of Diagram 2				6.4. Writes unit for area of Diagram 2				6.5. Writes unit for the largest area being enclosed				6.6. Writes unit for the dimension that yield the largest area				7.1. Checks the answer of the perimeter of Diagram 1				7.2. Checks the answer of the area of Diagram 1				7.3. Checks the answer of the perimeter of Diagram 2				7.4. Checks the answer of the area of Diagram 2				7.5. Checks the answer for the fencing problem							
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f																																								

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Percentage of appropriate mathematical element(s) of ethical knowledge obtained by the subject

$$= \frac{f\,(PA+NPA)}{f\,(PA+PI+NPA+NPI)} \times 100\% =$$

Overall level of subject's ethical knowledge		
Low ($0 \le x < 40$)	Medium ($40 \le x < 70$)	High ($70 \le x < 100$)

The overall level of ……………..'s SMK:

Percentage of appropriate mathematical element(s) of SMK obtained by the subject

$$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% =$$

Overall level of subject's SMK		
Low ($0 \leq x < 40$)	Medium ($40 \leq x < 70$)	High ($70 \leq x < 100$)

Appendix M
Sample of Coding Rubrics for Determining Overall Level of Beng's SMK

Coding Rubrics for Determining Level of Beng's Conceptual Knowledge

Task	Conceptual Knowledge																																							
	1. Notion of perimeter				2. Notion of area				3. Notion of the unit of area (square and nonsquare)				4.1. Number of units and unit of measure: Comparing perimeter with nonstandard units				4.2. Number of units and unit of measure: Comparing perimeter with common nonstandard units				4.3. Number of units and unit of measure: Comparing perimeter with common standard unit				4.4. Number of units and unit of measure: Comparing area with nonstandard units				4.5. Number of units and unit of measure: Comparing area with common nonstandard units				4.6. Number of units and unit of measure: Comparing area with common standard unit				5.1. Inverse relationship between number of units and unit of measure: Perimeter			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1		x																																						
1.2						x																																		
2									x																															
3.1																																								
3.2																																								
3.3														x				x			x																			
3.4																										x				x			x							
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f		1				1			①					1				1			①					1				1			①				①			

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng’s Conceptual Knowledge (continued)

Task	Conceptual Knowledge: 5.2. Inverse relationship between number of units and unit of measure: Area				6.1. Relationship between standard units of length (linear units): 1 cm = 10 mm				6.2. Relationship between standard units of length (linear units): 1 m = 100 cm				6.3. Relationship between standard units of length (linear units): 1 km = 1000 m				7.1. Relationship between standard units of area (square units): 1 cm^2 = 100 mm^2				7.2. Relationship between standard units of area (square units): 1 m^2 = 10 000 cm^2				7.3. Relationship between standard units of area (square units): 1 km^2 = 1 000 000 m^2				8. Relationship between area units and linear units				9.1. Relationship between Perimeter and area: Same perimeter, same area?				9.2. Relationship between Perimeter and area: Longer perimeter, larger area?			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4	x																																							
4					x				x				x						x				x				x		x											
5.1																																		x						
5.2																																					x			
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f	①				①				①				①						①				①				①		①						1				①			

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng’s Conceptual Knowledge (continued)

Task	Conceptual Knowledge																																							
	9.3. Relationship between Perimeter and area: Perimeter increases, area increases?				10.1. Relationship among area formulae: Rectangle				10.2. Relationship among area formulae: Parallelogram				10.3. Relationship among area formulae: Triangle				10.4. Relationship among area formulae: Trapezium																							
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3	x																																							
6.1																																								
6.2																																								
7																																								
8								x	x							x	x																							
f	①							1	①							1	①																							

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Percentage of appropriate mathematical elements of conceptual knowledge obtained by Beng

$$= \frac{f\,(PA+NPA)}{f\,(PA+PI+NPA+NPI)} \times 100\% = \frac{16}{25} \times 100\% = 64.0\%$$

Overall level of Beng’s conceptual knowledge		
Low ($0 \leq x < 40$)	Medium ($40 \leq x < 70$)	High ($70 \leq x < 100$)
	x	

Coding Rubrics for Determining Level of Beng's Procedural Knowledge

Task	Procedural Knowledge																																							
	1.1. Converting standard units of area measurement: 3 cm^2 to mm^2				1.2. Converting standard units of area measurement: 4.7 m^2 to cm^2				1.3. Converting standard units of area measurement: 1.25 km^2 to m^2				2.1. Calculating perimeter of composite figures: Diagram 1				2.2. Calculating perimeter of composite figures: Diagram 2				3.1. Calculating area of composite figures: Diagram 1				3.2. Calculating area of composite figures: Diagram 2				4.1. Developing area formula: Rectangle				4.2. Developing area formula: Parallelogram				4.3. Developing area formula: Triangle			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4	x				x				x																															
5.1																																								
5.2																																								
5.3																																								
6.1													x								x																			
6.2																	x								x															
7																																								
8																																x	x							x
f	①				①				①				①				①				①				①							1	①							1

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng’s Procedural Knowledge (continued)

Task	Procedural Knowledge																																							
	4.4. Developing area formula: Trapezium																																							
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8	x																																							
f	①																																							

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Percentage of appropriate mathematical elements of procedural knowledge obtained by Beng

$$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% = \frac{9}{11} \times 100\% = 81.8\%$$

Overall level of Beng’s procedural knowledge		
Low ($0 \leq x < 40$)	Medium ($40 \leq x < 70$)	High ($70 \leq x < 100$)
		x

Coding Rubrics for Determining Level of Beng's Linguistic Knowledge

Task	Linguistic Knowledge																																							
	1.1. Mathematical symbols: Formula for the area of a rectangle				1.2. Mathematical symbols: Formula for the area of a parallelogram				1.3. Mathematical symbols: Formula for the area of a triangle				1.4. Mathematical symbols: Formula for the area of a trapezium				2.1.1. Mathematical terms: Justification of shape A that have or do not have a perimeter				2.1.2. Mathematical terms: Justification of shape B that have or do not have a perimeter				2.1.3. Mathematical terms: Justification of shape C that have or do not have a perimeter				2.1.4. Mathematical terms: Justification of shape D that have or do not have a perimeter				2.1.5. Mathematical terms: Justification of shape E that have or do not have a perimeter				2.1.6. Mathematical terms: Justification of shape F that have or do not have a perimeter			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																	x				x				x				x				x					x		
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8	x				x				x				x																											
f	①				①				①				①				①				①				①				①				①					1		

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng's Linguistic Knowledge (continued)

Task	Linguistic Knowledge																																							
	2.1.7. Mathematical terms: Justification of shape G that have or do not have a perimeter				2.1.8. Mathematical terms: Justification of shape H that have or do not have a perimeter				2.1.9. Mathematical terms: Justification of shape I that have or do not have a perimeter				2.1.10. Mathematical terms: Justification of shape J that have or do not have a perimeter				2.1.11. Mathematical terms: Justification of shape K that have or do not have a perimeter				2.1.12. Mathematical terms: Justification of shape L that have or do not have a perimeter				2.2.1. Mathematical terms: Justification of shape A that have or do not have an area				2.2.2. Mathematical terms: Justification of shape B that have or do not have an area				2.2.3. Mathematical terms: Justification of shape C that have or do not have an area				2.2.4. Mathematical terms: Justification of shape D that have or do not have an area			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1	x				x				x					x			x				x																			
1.2																									x				x				x				x			
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f	①				①				①					1			①				①				①				①				①				①			

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng’s Linguistic Knowledge (continued)

Task	Linguistic Knowledge																																							
	2.2.5. Mathematical terms: Justification of shape E that have or do not have an area				2.2.6. Mathematical terms: Justification of shape F that have or do not have an area				2.2.7. Mathematical terms: Justification of shape G that have or do not have an area				2.2.8. Mathematical terms: Justification of shape H that have or do not have an area				2.2.9. Mathematical terms: Justification of shape I that have or do not have an area				2.2.10. Mathematical terms: Justification of shape J that have or do not have an area				2.2.11. Mathematical terms: Justification of shape K that have or do not have an area				2.2.12. Mathematical terms: Justification of shape L that have or do not have an area				2.3.1. Mathematical terms: Justification of shape that can or cannot be used as the unit of area: Square				2.3.2. Mathematical terms: Justification of shape that can or cannot be used as the unit of area: Rectangle			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2	x					x			x				x				x					x			x				x											
2																																		x				x		
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f	①					1			①				①				①					1			①				①					1				1		

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng's Linguistic Knowledge (continued)

Task	Linguistic Knowledge																																							
	2.3.3. Mathematical terms: Justification of shape that can or cannot be used as the unit of area: Triangle				2.4.1. State the area formula or explain the meaning of the mathematical symbols employed in the formula: Rectangle				2.4.2. State the area formula or explain the meaning of the mathematical symbols employed in the formula: Parallelogram				2.4.3. State the area formula or explain the meaning of the mathematical symbols employed in the formula: Triangle				2.4.4. State the area formula or explain the meaning of the mathematical symbols employed in the formula: Trapezium				3.1. Standard unit of length measurement (linear unit): Perimeter of Diagram 1				3.2. Standard unit of length measurement (linear unit): Perimeter of Diagram 2				4.1. Standard unit of area measurement (square unit): Area of Diagram 1				4.2. Standard unit of area measurement (square unit): Area of Diagram 2				5.1. Conventions of writing SI area measurement: 16 cm^2			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2		x																																						
3.1																																								
3.2																																								
3.3																																								
3.4																																						x		
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																					x								x											
6.2																									x								x							
7																																								
8					x					x			x					x																						
f		1			①					1			①					1			①				①				①				①					1		

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng’s Linguistic Knowledge (continued)

Task	Linguistic Knowledge																																							
	5.2. Conventions of reading SI area measurement: 16 cm^2				5.3. Conventions of writing SI area measurement: 13 cm^2				5.4. Conventions of reading SI area measurement: 13 cm^2																															
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4		x				x				x																														
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f		1				1				1																														

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Percentage of appropriate mathematical elements of linguistic knowledge obtained by Beng

$$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% = \frac{30}{43} \times 100\% = 69.8\%$$

Overall level of Beng’s linguistic knowledge		
Low ($0 \le x < 40$)	Medium ($40 \le x < 70$)	High ($70 \le x < 100$)
	x	

Coding Rubrics for Determining Level of Beng's Strategic Knowledge

Task	Strategic Knowledge																																							
	1.1. Strategy for comparing perimeter				1.2. Alternative method for comparing perimeter				2.1. Strategy for comparing area				2.2. Alternative method for comparing area				3.1. Strategy for checking answer for perimeter of Diagram 1				3.2. Strategy for checking answer for perimeter of Diagram 2				4.1. Strategy for checking answer for area of Diagram 1				4.2. Strategy for checking answer for area of Diagram 2				5. Strategy for solving the fencing problem				6. Strategy for checking answer for the fencing problem			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1	x				x																																			
3.2									x				x																											
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																	x								x															
6.2																					x								x											
7																																	x				x			
8																																								
f	①				①				①				①				①				①				①				①				①				①			

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng's Strategic Knowledge (continued)

Task	Strategic Knowledge																																							
	7.1. Strategy for developing area formula for a rectangle				7.2. Strategy for developing area formula for a parallelogram				7.3. Strategy for developing area formula for a triangle				7.4. Strategy for developing area formula for a trapezium																											
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8				x	x							x	x																											
f				1	①							1	①																											

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Percentage of appropriate mathematical elements of strategic knowledge obtained by Beng

$$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% = \frac{12}{14} \times 100\% = 85.7\%$$

Overall level of Beng's strategic knowledge		
Low ($0 \leq x < 40$)	Medium ($40 \leq x < 70$)	High ($70 \leq x < 100$)
		x

Coding Rubrics for Determining Level of Beng's Ethical Knowledge

Task	Ethical Knowledge																																							
	1.1.1. Justifies the selection of shape A that have or do not have a perimeter				1.1.2. Justifies the selection of shape B that have or do not have a perimeter				1.1.3. Justifies the selection of shape C that have or do not have a perimeter				1.1.4. Justifies the selection of shape D that have or do not have a perimeter				1.1.5. Justifies the selection of shape E that have or do not have a perimeter				1.1.6. Justifies the selection of shape F that have or do not have a perimeter				1.1.7. Justifies the selection of shape G that have or do not have a perimeter				1.1.8. Justifies the selection of shape H that have or do not have a perimeter				1.1.9. Justifies the selection of shape I that have or do not have a perimeter				1.1.10. Justifies the selection of shape J that have or do not have a perimeter			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1	x				x				x				x				x					x			x				x				x					x		
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f	①				①				①				①				①					1			①				①				①					1		

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng's Ethical Knowledge (continued)

Task	Ethical Knowledge																																							
	1.1.11. Justifies the selection of shape K that have or do not have a perimeter				1.1.12. Justifies the selection of shape L that have or do not have a perimeter				1.2.1. Justifies the selection of shape A that have or do not have an area				1.2.2. Justifies the selection of shape B that have or do not have an area				1.2.3. Justifies the selection of shape C that have or do not have an area				1.2.4. Justifies the selection of shape D that have or do not have an area				1.2.5. Justifies the selection of shape E that have or do not have an area				1.2.6. Justifies the selection of shape F that have or do not have an area				1.2.7. Justifies the selection of shape G that have or do not have an area				1.2.8. Justifies the selection of shape H that have or do not have an area			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1	x				x																																			
1.2									x				x				x				x				x					x			x				x			
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f	①				①				①				①				①				①				①					1			①				①			

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng’s Ethical Knowledge (continued)

Task	Ethical Knowledge																																							
	1.2.9. Justifies the selection of shape I that have or do not have an area				1.2.10. Justifies the selection of shape J that have or do not have an area				1.2.11. Justifies the selection of shape K that have or do not have an area				1.2.12. Justifies the selection of shape L that have or do not have an area				1.3.1. Justifies the shape that can or cannot be used as the unit of area: Square				1.3.2. Justifies the shape that can or cannot be used as the unit of area: Rectangle				1.3.3. Justifies the shape that can or cannot be used as the unit of area: Triangle				2.1. Examines pattern: Two shapes with the same perimeter have the same area?				2.2. Examines pattern: The garden with the longer perimeter has the larger area?				3.1. Formulates generalization that two shapes with the same perimeter have the same area?			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2	x					x			x				x																											
2																		x				x				x														
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																x								x
5.2																																	x							
5.3																																								
6.1																																								
6.2																																								
7																																								
8																																								
f	①					1			①				①					1				1				1						1	①							1

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng’s Ethical Knowledge (continued)

Task	Ethical Knowledge																																							
	3.2. Formulates generalization that the garden with the longer perimeter has the larger area?				4.1. Tests generalization that two shapes with the same perimeter have the same area?				4.2. Tests generalization that the garden with the longer perimeter has the larger area?				4.3. Tests generalization that as the perimeter of a closed figure increases, the area also increases?				5.1. Attempts to develop area formula for a rectangle				5.2. Attempts to develop area formula for a parallelogram				5.3. Attempts to develop area formula for a triangle				5.4. Attempts to develop area formula for a trapezium				6.1. Writes unit for perimeter of Diagram 1				6.2. Writes unit for area of Diagram 1			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1								x																																
5.2	x								x																															
5.3														x																										
6.1																																	x				x			
6.2																																								
7																																								
8																				x	x							x	x											
f	①							1	①					1						1	①							1	①				①				①			

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Coding Rubrics for Determining Level of Beng’s Ethical Knowledge (continued)

Task	Ethical Knowledge																																							
	6.3. Writes unit for perimeter of Diagram 2				6.4. Writes unit for area of Diagram 2				6.5. Writes unit for the largest area being enclosed				6.6. Writes unit for the dimension that yield the largest area				7.1. Checks the answer of the perimeter of Diagram 1				7.2. Checks the answer of the area of Diagram 1				7.3. Checks the answer of the perimeter of Diagram 2				7.4. Checks the answer of the area of Diagram 2				7.5. Checks the answer for the fencing problem							
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
1.1																																								
1.2																																								
2																																								
3.1																																								
3.2																																								
3.3																																								
3.4																																								
4																																								
5.1																																								
5.2																																								
5.3																																								
6.1																				x				x																
6.2	x				x																							x				x								
7												x				x																	x							
8																																								
f	①				①							1				1				1				1				1				1	①							

Legend: P = Present NP = Not Present A = Appropriate I = Inappropriate

Percentage of appropriate mathematical elements of ethical knowledge obtained by Beng

$$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% = \frac{30}{49} \times 100\% = 61.2\%$$

Overall level of Beng’s ethical knowledge		
Low ($0 \leq x < 40$)	Medium ($40 \leq x < 70$)	High ($70 \leq x < 100$)
	x	

The overall level of Beng's SMK:

Percentage of appropriate mathematical elements of SMK obtained by Beng

$$= \frac{f(PA+NPA)}{f(PA+PI+NPA+NPI)} \times 100\% = \frac{16+9+30+12+30}{25+11+43+14+49} \times 100\% = \frac{97}{142} \times 100\% = 68.3\%$$

Overall level of Beng's SMK		
Low ($0 \leq x < 40$)	Medium ($40 \leq x < 70$)	High ($70 \leq x < 100$)
	x	

Appendix N
Case Studies

Beng

Beng lives in Kuching, Sarawak. Beng is 22 years 9 months old when she was interviewed. Currently, she is pursuing a 4-year Bachelor of Science with Education (B.Sc.Ed.) program at a public university. She majored and minored in mathematics and physics respectively. She obtained grade 1A in Mathematics and Additional Mathematics in her 2002 SPM examination (equivalent to O level examination). She also scored A in Mathematics T in the 2004 STPM examination (equivalent to A level examination). Beng performed excellently in her mathematics content courses at the university level when she secured six A, and one A− in seven mathematics content courses she had completed during the first and second year of her studies. The detail of her performance is shown in Table N1.

Table N1

Mathematics Performance of Beng

Courses	Grades
1. Calculus for Science Students I	A−
2. Algebra for Science Students	A
3. Statistics for Science Students	A
4. Calculus for Science Students II	A
5. Differential Equation I	A
6. Programming for Scientific Applications	A
7. Vector Calculus	A

At the time of data collection, Beng was in her second semester of third year studies. She attained 3.82 in the Cumulative Grade Point Average (CGPA) for her first two years of studies at the public university. She does not have any teaching experience prior to this interview.

Notion of Perimeter

Conceptual Knowledge

Beng has selected shapes “A”, “C”, “D”, “H”, “I”, and “K” as having a perimeter. Excerpt B1 shows her choice of shapes that have a perimeter (Beng/L94-96).

Excerpt B1

R: (Puts a handout comprises 12 shapes in front of Beng). Tick the shapes that have a perimeter.
S: (Circles shapes "A", "C", "D", "H", "I", and "K", as shown in Figure N1).

In Excerpt B1, Beng has selected all simple closed curves (A, C, H, K) as well as all closed but not simple curves (D, I) that have a perimeter. Nevertheless, she did not select the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that her notion of perimeter was limited to simple closed curves, and closed but not simple curves, exclusive of 3-dimensional shapes.

Beng also did not select the two simple but not closed curves (B, G) as well as the two 1-dimensional shapes (E, L) that do not have a perimeter. In other words, Beng did not select an open shape (including the lines) as having a perimeter.

When asked to justify her selection, Beng explained that she selected shapes “A”, “C”, “D”, “H”, “I”, and “K” because they are 2-dimensional shapes and all these shapes are enclosed. Excerpt B2 depicts her justification of selecting each of these shapes (Beng/L102-114).

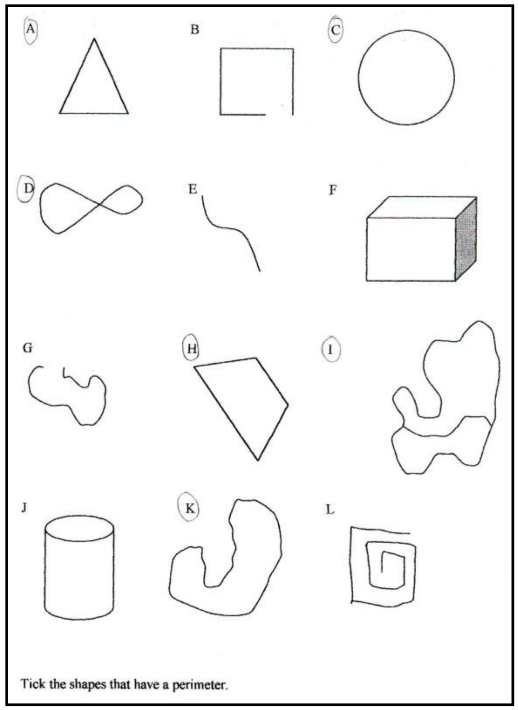


Figure N1. Beng’s selection of shapes that have a perimeter.

Excerpt B2

R: Why did you select shape "A"?
S: Because in my opinion the circumference need to have a two dimension shape and all the shapes should be enclosed.
R: Why did you select shape "C"?
S: "C", it is two dimension and it is enclosed.
R: Why did you select shape "D"?
S: Same reason this one. Enclosed and it is 2D.
R: Why did you select shape "H"?
S: "H", two dimension and enclosed.
R: Why did you select shape "I"?
S: Also two dimension and enclosed.
R: Why did you select shape "K"?
S: Two dimension and enclosed.

Beng explained that she did not select shapes “F” and “J” because they are 3-dimensional shapes. She elaborated that perimeter is limited to 2-dimensional shapes. Excerpt B3 demonstrates her justification for not selecting shapes “F” and “J” as having a perimeter (Beng/L126-128, L132-133).

Excerpt B3

R: Why didn't you select shape "F"?
S: Because "F" is a 3D thing. Because I think, I'm not sure but I think that in order to calculate perimeter, may be it should be 2D only.
.
.
.
R: Why didn't you select shape "J"?
S: "J" because it is a 3D, it is 3D.

Beng explained that she did not select shapes “B” and “G” because they are not enclosed. Excerpt B4 reveals her justification for not selecting shapes “B” and “G” as having a perimeter (Beng/L115-117, L129-131).

Excerpt B4

R: Why didn't you select shape "B"?
S: Because it is not enclosed here (points to the gap on shape "B" as shown in Figure N1).
.
.
.
R: Why didn't you select shape "G"?
S: Because it is not enclosed here (points to the gap on shape "G" as shown in Figure N1), no line here. So, it is not an enclosed object.

Beng explained that she did not select shapes “E” and “L” because these shapes are not enclosed. She elaborated that shape “E” looks like a line. Beng stated that it takes at least three lines to form an enclosed of length. Excerpt B5 exhibits her justification for not selecting shapes “E” and “L” as having a perimeter (Beng/L118-125, L134-135).

Excerpt B5

R: Why didn't you select shape "E"?
S: It is just a line. In my opinion, the shape, it should be at least three lines. For example, triangle. That's why I didn't select it.
R: Just now you mentioned "at least three lines". Could you tell me more above it?
S: In order to form an enclosed of length, I think we need at least three lines to form an enclosed. If two lines, it can just become a parallel line or a perpendicular line.
.
.
.
R: Why didn't you select shape "L"?
S: It is not enclosed.

Summary

In summary, Beng has selected all simple closed curves (A, C, H, K) and all closed but not simple curves (D, I) that have a perimeter. It indicated that her notion of perimeter was limited to simple closed curves, and closed but not simple curves, exclusive of 3-dimensional shapes. She justified her selection by explaining that they are all 2-dimensional shapes and all these shapes are enclosed.

Linguistic Knowledge

Beng used appropriate mathematical terms ‘2-dimensional shapes’ and ‘enclosed’ to justify her selection of shapes that have a perimeter. Beng explained that she selected shapes “A”, “C”, “D”, “H”, “I”, and “K” because they are 2-dimensional shapes and all these shapes are enclosed, as shown in Excerpt B2.

Beng used appropriate mathematical symbol '3D' to represents 3-dimensional shapes, "F" and "J", as shown in Excerpt B3. Nevertheless, '3-dimensional shapes' was not the appropriate justification for not selecting shapes "F" and "J" that have a perimeter as we still can find perimeter for the faces of solids.

Beng used appropriate negation 'not enclosed' as her justification for not selecting shapes "B" and "G" as having a perimeter. Beng explained that she did not select shapes "B" and "G" because they are not enclosed, as shown in Excerpt B4. Beng used the same negation 'not enclosed' as her justification for not selecting shapes "E" and "L" as having a perimeter. Beng explained that she did not select shapes "E" and "L" because these shapes are not enclosed, as shown in Excerpt B5. Beng also used appropriate mathematical term 'line' to describe shape "E". She elaborated that shape "E" looks like a line.

The perimeter of a circle is given a specific name known as circumference. Beng inappropriately used the mathematical term 'circumference' to include perimeters of noncircular shapes, as shown in Excerpt B2. She used appropriate mathematical symbol '2D' to represents 2-dimensional shapes, as shown in Excerpts B2 and B3.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Beng had taken the effort to justify the selection of shapes that have a perimeter, as shown in Excerpt B2. She provided appropriate justification for selecting shapes "A", "C", "D", "H", "I", and "K" that have a perimeter. Beng also provided justification for not selecting shapes "F" and "J" that have a perimeter, as shown in Excerpt B3. Nevertheless, '3-dimensional shapes' was not the appropriate justification for not selecting shapes "F" and "J" that have a perimeter as we still can find perimeter for the faces of solids.

Beng also had taken the effort to provide justification for not selecting other shapes that do not have a perimeter. She provided appropriate justification for not selecting shapes "B" and "G" as having a perimeter, as shown in Excerpt B4. Beng also provided appropriate justification for not selecting shapes "E" and "L" as having a perimeter, as shown in Excerpt B5.

Notion of Area

Conceptual Knowledge

Beng has selected shapes "A", "C", "D", "H", "I", and "K" as having an area. Excerpt B6 shows her choice of shapes that have an area (Beng/L149-151).

Excerpt B6

R: (Puts a handout comprises 12 shapes in front of Beng). Tick the shapes that have an area.
S: (Circles shapes "A", "C", "D", "H", "I", and "K", as shown in Figure N2).

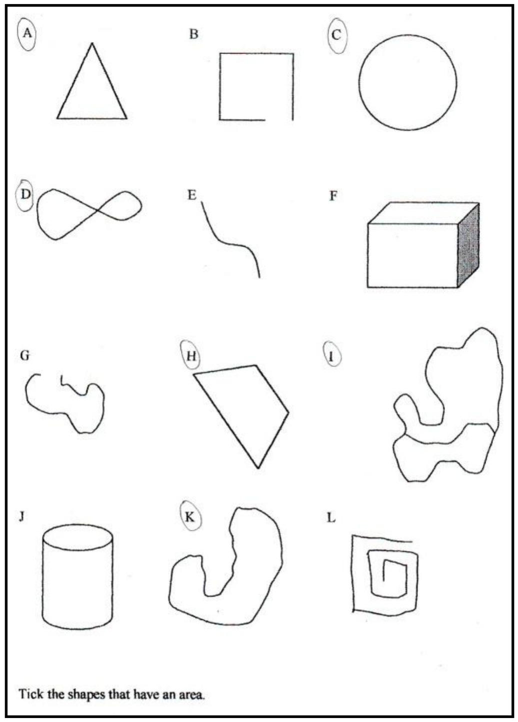


Figure N2. Beng's selection of shapes that have an area.

In Excerpt B6, Beng has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. Nevertheless, she did not select the two 3-dimensional shapes (F, J) that have an area. It revealed that Beng had a static perspective of the notion of area. Based on this perspective, area can be viewed as the amount of surface enclosed within a boundary. It also indicated that her notion of area was limited to 2-dimensional shapes (closed plane shapes). Beng also did not select the two open shapes (B, G) as well as the two 1-dimensional shapes (E, L) that do not have an area. In other words, Beng did not select an open shape (including the lines) as having an area. It can be inferred that she did not has a dynamic perspective of area or, at least, this knowledge was not accessible to her during the clinical interview.

When asked to justify her selection, Beng explained that she selected shapes "A", "I", and "K" because they are enclosed and thus their area can be calculated. Beng also explained that she selected shapes "C", "D", and "H" because their area can be calculated. It indicated that Beng appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Excerpt B7 depicts her justification of selecting each of these shapes (Beng/L158-171).

Excerpt B7

R: Why did you select shape "A"?
S: "A" because it is enclosed. So, I can calculate the area using the formula given.
R: Why did you select shape "C"?
S: "C" also. I can use the formula to calculate it.
R: Why did you select shape "D"?
S: For "D", I'm not sure whether there is exist a formula to calculate it. But in my opinion, I think we should be able to calculate its area.
R: Why did you select shape "H"?
S: "H", it is a trapezium. So, I have formula to calculate its area.
R: Why did you select shape "I"?
S: It is enclosed. I think should be able to calculate its area.
R: Why did you select shape "K"?

S: Em also enclosed and I think it is possible to calculate its area.

Beng explained that she did not select shapes “F” and “J” because they are 3D (3-dimensional). She elaborated that we could find volume for it but we can not calculate area for it. Excerpt B8 demonstrates her justification for not selecting shapes “F” and “J” as having an area (Beng/L178-180, L183-185).

Excerpt B8

R: Why didn't you select shape "F"?
S: "F" is a 3D and it is a combination of area. So, I think may be we can't calculate the exact one area for it.
.
.
.
R: Why didn't you select shape "J"?
S: "J" also same as "F". It is a 3D thing. May be we can find volume for it but I don't think we can calculate the area.

Beng explained that she did not select shapes “B” and “G” because they are not enclosed and therefore we cannot measure the exact area of it. Excerpt B9 reveals her justification for not selecting shapes “B” and “G” as having an area (Beng/L172-175, L181-182).

Excerpt B9

R: Why didn't you select shape "B"?
S: "B" because here (points to the gap on shape "B" as shown in Figure N2) it is not enclosed. So, I am not sure. I think we can't measure the exact area of it.
.
.
.
R: Why didn't you select shape "G"?
S: "G" is not enclosed.

Beng explained that she did not select shape “E” because it is just a line and she did not think a line has an area. Beng also explained that she did not select shape “L” as it is not enclosed. Excerpt B10 exhibits her justification for not selecting shapes “E” and “L” as having an area (Beng/L176-177, L186-187).

Excerpt B10

R: Why didn't you select shape "E"?
S: "E" is just a line. I don't think a line has an area.
.
.
.
R: Why didn't you select shape "L"?
S: It is not enclosed.

Summary

In summary, Beng has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. Nevertheless, she did not select the two 3-dimensional shapes (F, J) that have an area. It revealed that Beng had a static perspective of the notion of area. Her notion of area was limited to 2-dimensional shapes (closed plane shapes). Beng justified her selection by explaining that she selected shapes “A”, “I”, and “K” because they are enclosed and thus their area can be calculated. Beng also explained that she selected shapes “C”, “D”, and “H” because their area can be calculated. It indicated that Beng appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured).

Linguistic Knowledge

Beng used appropriate mathematical term ‘enclosed’ to justify her selection of shapes “A”, “I”, and “K” that have an area. Beng explained that she selected shapes “A”, “I”, and “K” because they are enclosed and thus their area can be calculated. Beng used appropriate mathematical term ‘calculate’ to justify her selection of shapes “C”, “D”, and “H” that have an area. Beng explained that she selected shapes “C”, “D”, and “H” because their area can be calculated, as shown in Excerpt B7

Beng used appropriate mathematical symbol ‘3D’ to represents 3-dimensional, “F” and “J”, as shown in Excerpt B8. Nevertheless, ‘3D’ was not the appropriate justification for not selecting shapes “F” and “J” that have an area as we still can find area for the faces of solids.

She used appropriate negation ‘not enclosed’ as her justification for not selecting shapes “B” and “G” as having an area. Beng explained that she did not select shapes “B” and “G” because they are not enclosed and therefore we cannot measure the exact area of it, as shown in Excerpt B9. Beng used the same negation ‘not enclosed’ as her justification for not selecting shape “L” as having an area. Beng explained that she did not select shapes “L” as it is not enclosed, as shown in Excerpt B10. Beng used appropriate mathematical term ‘line’ as her justification for not selecting shapes “E” as having an area. Beng explained that she did not select shape “E” because it is just a line and she did not think a line has an area, as shown in Excerpt B10.

Beng used appropriate mathematical term ‘volume’ to refer to a measurement that we can find in 3-dimensional shape, “J”, as shown in Excerpt B8.

Ethical Knowledge

Beng had taken the effort to justify the selection of shapes that have an area, as shown in Excerpt B7. She provided appropriate justification for selecting shapes “A”, “C”, “D”, “H”, “I”, and “K” that have a perimeter. Beng also provided justification for not selecting shapes “F” and “J” that have an area, as shown in Excerpt B8. Nevertheless, ‘3-dimensional shapes’ was not the appropriate justification for not selecting shapes “F” and “J” that have an area as we still can find area for the faces of solids.

Beng also had taken the effort to provide justification for not selecting other shapes that do not have a perimeter. She provided appropriate justification for not selecting shapes “B” and “G” as having an area, as shown in Excerpt B9. Beng also provided appropriate justification for not selecting shapes “E” and “L” as having a perimeter, as shown in Excerpt B10.

Notion of the Units of Area

Conceptual Knowledge

Initially, Beng thought that square can only be used as the unit of area measurement to measure the area of square shapes. She explained that it is possible for us to locate all the small squares inside a square shape in order to measure its area. Beng gave an example of a large square where its length of side is five times the length of square to be used as the unit of area measurement. She explained that it is possible for us to locate the small squares inside the large square and thus we are able to measure the area of the large square.

Subsequently, Beng gave another example of a trapezium. She stated that it is not possible to measure the area of the trapezium by using the square as the unit of area measurement. Beng explained that it is impossible to use square to measure the area of the ‘triangle’ part of the trapezium. She concluded that square can only be used as unit of measurement to measure the area of square shape. Excerpt B11 shows her behavior in explaining why square could only be used as unit of area measurement to measure the area of square shape (Beng/L203-228).

Excerpt B11

R: (Puts a card written the following scenario in front of Beng). Ali, Chong, and David are discussing about the units of area. Ali says that we can use a square as the unit of area. Chong says that we can use a rectangle as the unit of area. David says that we can use a triangle as the unit of area. How would you respond to these students?

S: …(Silent for a while) I will tell Ali that it is possible for us to use square as a unit of area but need to depend on the situation. If the shapes given are in square, So, it is possible for us to locate all the small square inside it to calculate the area.

R: Could you tell me more about it?

S: (Draws a large square, as shown in Figure N3). Em for example, if the shape is big square like this. Then it is possible for us to locate many many (sic) small squares inside it in order to calculate the area. For example, the small square is 2 times 2. If here is a five square, then we are able to calculate the area of it. (Draws a trapezium, as shown in Figure N4). But if in case of the trapezium, it is not possible to calculate it because of this area (points to the "triangle" part of the trapezium). So, we can use the square as a unit of area when we calculate, when we measure the area of a square.

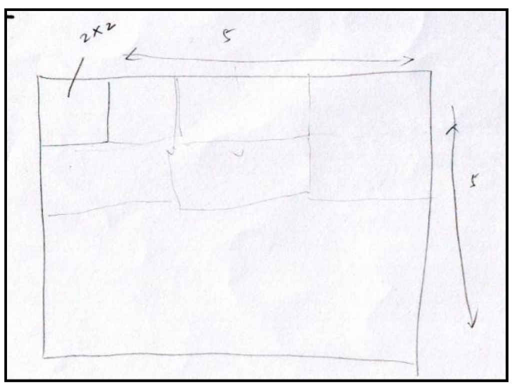


Figure N3. Beng draws a large square.

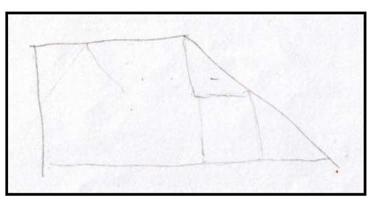


Figure N4. Beng draws a trapezium.

Beng explained that rectangle can be used as the unit of area to measure the area of a large rectangle. She also thought that rectangle can be used to measure the area of other shape such as square. After confirmed with the researcher that we need not necessary used the whole rectangle to determine the area of a shape. Beng explained that we can also used rectangle as the unit of area to measure the area of other shape such as trapezium. Excerpt B12 depicts her behavior in explaining why rectangle could be used as the unit of area to measure the area of a large rectangle as well as the area of other shapes such as square and trapezium (Beng/L229-241).

Excerpt B12

R: Chong says that we can use a rectangle as the unit of area. How would you respond to this student?
S: Rectangle. I think it is possible to use the rectangle to calculate a big rectangle.
R: Can we use the rectangle to measure the area of other shape?
S: …(Silent for a while) yeah I think we can.
R: How?
S: For example, for the square. I think the rectangle can be used to calculate the square as well.
R: Can we use the rectangle to measure the area of other shape?
S: Is it possible we no need to use exactly the rectangle itself, half?
R: Not necessary.
S: If like this, I think trapezium also, we can use it to calculate.

Beng explained that triangle can be used as the unit of area to measure the area of the large triangle as well as the area of other shapes such rectangle and square. Beng stated that she wanted to change her initial thought that square could only be used as the unit of area measurement to measure the area of a large square. She elaborated that square can also be used as the unit of area measurement to measure the area of other shapes as we can also just use part of the square to determine the area of a shape. Beng concluded that square, rectangle and triangle can also be used as the unit of area to measure the area of other shapes such as trapezium and parallelogram. It indicated that her notion of the unit of area was not only limited to square, but also nonsquare (such as rectangle and triangle). Excerpt B13 demonstrates her behavior in explaining why the given three shapes, namely square, rectangle and triangle can be used as the unit of area (Beng/L242-256).

Excerpt B13

R: David says that we can use a triangle as the unit of area. How would you respond to this student?
S: For triangle, I think it is possible to calculate other size as well.
R: What do you men by "other size"?
S: Not necessary the big triangle. Maybe the rectangle, the square, or the others. So, I want to change. I think the square should be able also because we can use just half the square, not whole square.
R: Ali says that we can use a square as the unit of area. How would you respond to this student?
S: Other shape also. I want to conclude that the given shapes; square, rectangle, and triangle is able to calculate, is able to use as a unit of area to others area diagrams.
R: What do you mean by "others area diagrams"?
S: For example, the square, rectangle, trapezium, triangle. Even the parallelogram.

When probed further, Beng explained that square, rectangle, and triangle can be used as a unit of area measurement because they have straight lines. It indicated that she was unable to provide the appropriate justification that any shape that tessellates a plane can be used as a unit of area measurement. Excerpt B14 is illustrative (Beng/L285-289).

Excerpt B14

R: Why is it that a square, rectangle, and triangle can be used as a unit of area?
S: Em…(silent for a while) they have straight lines.
R: Is there other reason?
S: Em…(silent for a while) I can't think, I can't think.

Summary

In summary, Beng initially thought that square can only be used as the unit of area measurement to measure the area of square shapes. Subsequently, she realized that square can also be used to measure the area of other shapes as well. Beng expressed that nonsquare, such as rectangle and triangle, can also be used as the unit of area measurement. It indicated that her notion of the unit of area was not only limited to square, but also nonsquare (such as rectangle and triangle). Beng explained that square, rectangle, and triangle can be used as a unit of area measurement because they have straight lines. It indicated that she was unable to provide the appropriate justification that any shape that tessellates a plane can be used as a unit of area measurement.

Linguistic knowledge

Beng used inappropriate mathematical term ‘straight lines’ to justify that a square, rectangle, and triangle can be used as the unit of area. She explained that a square, rectangle, and triangle can be used as a unit of area measurement because they have straight lines, as shown in Excerpt B14.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Beng had taken the effort to justify the shapes that can be used as a unit of area measurement. Nevertheless, she was unable to provide an appropriate justification for the shapes that can be used as a unit of area measure. This can be seen in Excerpt B14. In reality, any shape that tessellates a plane can be used as a unit of area measurement.

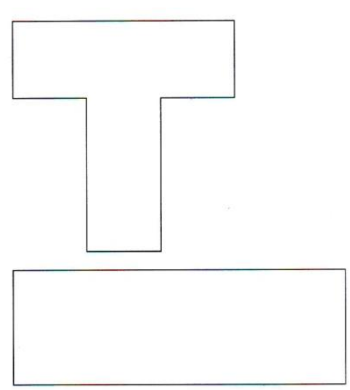
Comparing Perimeter (No Dimension Given)

Strategic Knowledge

Beng used the formal method of measuring the side and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter. Excerpt B15 shows the formal method that she used to compare the perimeter of the given pair of shapes (Beng/L361-374).

Excerpt B15

R: (Puts the following pair of shape in front of Beng). How would you find out whether the following pair of shapes had the same perimeter?



S: I’ll calculate this (points to the given T-shape) first and then calculate this (points to the given rectangle) and compare the answer. I'll calculate their perimeter or even area also can.

R: How would you calculate it?
S: So, I'll calculate it. (Measures the length of each side by ruler and then calculate its perimeter respectively, as shown in Figure N5).
R: What do you get?
S: Both of them are not same. The perimeter (sic) are not the same.
R: Which one is longer?
S: The first one is longer (points to the given T-shape).

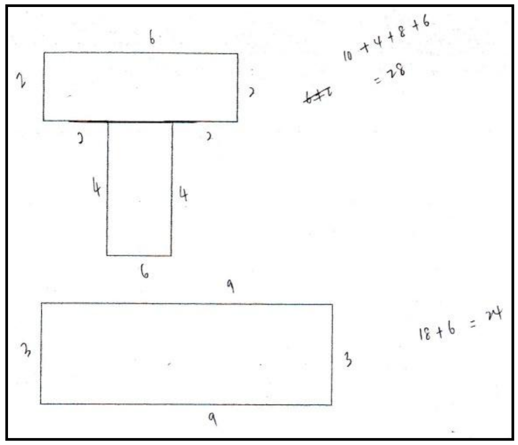


*Figure N*5. Beng measures the length of each side by ruler and then calculates its perimeter respectively.

In Excerpt B15, Beng measured the length of each side of the given T-shape by ruler. She mistakenly labelled the lowest side of the T-shape as 6 units instead of 2 units. Consequently, Beng got the wrong answer when she summed up the length of each side as 28 units. The correct perimeter of the given T-shape is 24 units. She measured the length of each side of the given rectangle by ruler and then calculated its perimeter correctly as 24 units (it should be 24 cm), as shown in Figure N5. Beng did not realize the mistake and thus concluded that the given T-shape had the longer perimeter.

When probed for alternative method of comparing the perimeter, Beng partitioned T-shape into two rectangles for which area measurement formula was known. Excerpt B16 depicts the formal method of measuring the side and applying the area (sic) formula that she used to compare the perimeter of the given pair of shape (Beng/L375-393).

Excerpt B16

R: Could you think of other way of finding out whether they had the same perimeter?
S: (Partitions T-shape into two rectangles. Measures its lengths and widths respectively and then calculates its area using rectangle area formula, as shown in Figure N6. Measures the length and width of the second given diagram and then calculates its area using rectangle area formula, as shown in Figure N6). I calculating the area. If area are larger, it has a longer perimeter, not necessary (realizes her mistake).
R: Could you tell me more about it?
S: Just now I tried to use area to calculate it because in my opinion, it may be if area is larger, the perimeter will be larger. But from the conclusion I made (refers to the first method and second method: Area of T-shape = 20, its perimeter = 28; Area of rectangle = 27, its perimeter = 24), I found that I am wrong. The larger area not necessary the longer perimeter. So, I think this method can not be used.

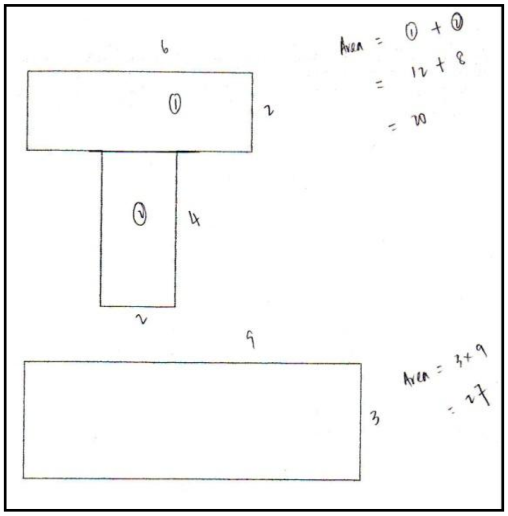


*Figure N*6. Beng calculates the area of each given shape.

In Excerpt B16, Beng partitioned the T-shape into two rectangles. She measured its length and widths by ruler respectively and then calculated its area using rectangle area formula, as shown in Figure N6. Beng also measured the length and width of the second diagram by ruler and then calculated its area using rectangle area formula, as shown in Figure N6.

Beng explained that she would measure the areas of each shape to determine whether they were equal and if they were, then the perimeter would be equal. Beng realized her mistake that the shape with the larger area may not necessarily has the longer perimeter. She concluded that this method did not work based on the perimeters and areas that she had calculated from the first method (perimeter of T-shape = 28 and perimeter of the rectangle = 24) and the second method (area of T-shape = 20 and area of rectangle = 27) respectively.

When probed further for other method of determining whether the given pair of shapes had the same perimeters, Beng used another formal method of measuring the side by thread and ruler. Excerpt B17 demonstrates how she used thread and ruler to determine each perimeter and then compare their measurement (Beng/L 394-408).

Excerpt B17

R: Could you think of other way of finding out whether they had the same perimeter?
S: Using a thread (and ruler).
R: Could you show me how it is?
S: (Measures the length of each side of the T-shape by a thread and then puts it on a ruler to determine its total length. Measures the length of each side of the rectangle by a thread and then puts it on a ruler to determine its total length). Em I found something very rare because I used the thread, I found that both are the same length. But I don't know why.
R: What do you get?
S: 23 roughly.
R: Could you think of other way of finding out whether they had the same perimeter?
S: Other method. No, no.

In Excerpt B17, Beng measured the length of each side of the T-shape by thread and then put it on a ruler to determine its total length (perimeter). She also measured the length of each side of the rectangle by thread and then put it on a ruler to determine its total length (perimeter). Beng expressed that she found something very rare as she got the same length (roughly 23 units) by using the thread and ruler, whereas in the first method, she got different perimeter, 28 units and 24 units respectively).

Beng did not know the reason behind it as she did not realize that she had made a mistake in the first method when she mistakenly labelled the lowest side of the T-shape as 6 units.

Summary

In summary, Beng produced three formal methods of determining whether the given pair of shape had the same perimeter. In the first method, Beng measured the length of sides by ruler and applied the definition of perimeter. In the second method, she measured the length of sides by ruler and applied the area formula of rectangle. Beng realized that this method did not work because the shape with the larger area may not necessarily have the longer perimeter. In the third method, Beng measured the length of sides by thread and then put it on a ruler to determine its total length.

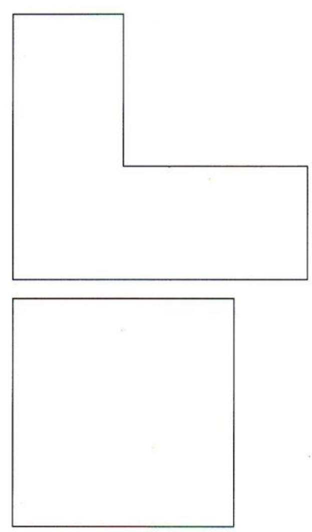
Comparing Area (No Dimension Given)

Strategic Knowledge

Beng partitioned L-shape into two rectangles for which area measurement formulae were known. Excerpt B18 shows the formal method of measuring the side and applying the area formula that she used to compare the area of the given pair of shape (Beng/L461-480).

Excerpt B18

R: (Puts the following pair of shape in front of Beng). How would you find out whether the following pair of shapes had the same area?



S: (Measures the lengths of each side of the L-shape by ruler. Partitions the L-shape into two rectangles, labelled as A and B respectively, as shown in Figure N7. Then calculates the total area using area formula of rectangle). Measures the lengths of two adjacent sides of the square by ruler. Then calculates the area using area formula of square, as shown in Figure N8).

R: Could you explain your solution?

S: First one I will calculate all the sides, the value of the sides. Then I will divide it into two pieces for me to calculate. Let's say this one is the A. This one is the B. So, the area is just A plus B and A is a rectangle, B is a rectangle. 7 times 3 and B, 5 times 3. Same goes to this one (point to the given square).

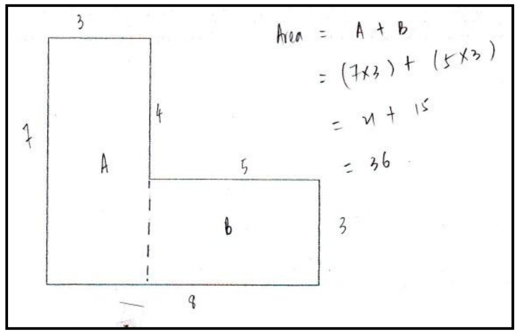


Figure N7. Beng measures the length of each side by ruler and then calculates its area.

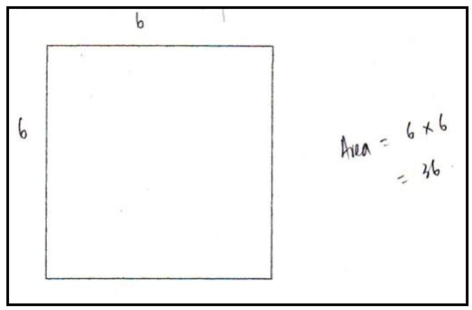


Figure N8. Beng measures the length of two adjacent sides by ruler and then calculates its area.

In Excerpt B18, Beng partitioned L-shape into two rectangles, labelled as A and B respectively. She measured its lengths and widths by ruler respectively and then calculated its area using rectangle area formulae, as shown in Figure N7. Beng also measured the length of two adjacent sides of the square by ruler and then calculated its area using square area formula, as shown in Figure N8.

When probed for alternative method of comparing the area, Beng formed a large rectangle, labelled as A, for which area measurement formula was known. Excerpt B19 depicts the formal method of measuring the side and applying the area formula that she used to compare the area of the given pair of shapes (Beng/L481-506).

Excerpt B19

R: Could you think of other way of finding out whether they had the same area?

S: I will ask them to form a big square. (Forms a large rectangle, as shown in Figure N9. Measures the lengths and widths of the large (labelled as A) and small (labelled as B) rectangles respectively using ruler. Calculates the area of L-shape by finding the difference between the area of the large and small rectangle). Since the value here is 7, 3. This one is 4, this one is 4. So, all total 7 as well. Here is 5 and the big square outside I will consider it as A and the small square here I will consider it as B. Then the area is just the big A minus, the big A cut off this (refers to B) and I will get this (refers to the L-shape). The A is 7 times 8 and the B is 4 times 5. (Partitions the given square into two isosceles triangles, labelled as A and B respectively, as shown in Figure B10. Measures the lengths of each side of the square using ruler and then calculates the total area of the two triangles using area formula of triangles). For B it is just the same wise as A (points to the given square, as shown in Figure N10). May be I can ask them to form, get two triangles. So, the area is combination of two triangles A and B.

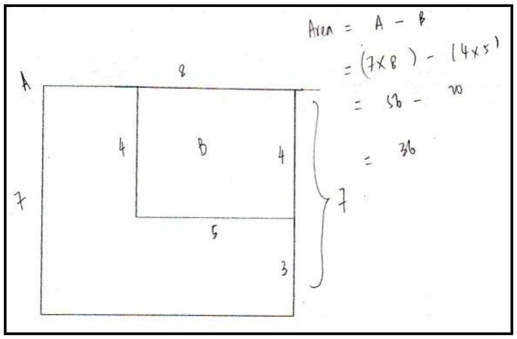


Figure N9. Beng calculates the area of L-shape as the different between the area of the large and the small rectangle.

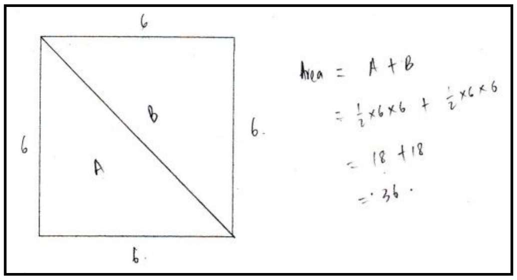


Figure N10. Beng calculates the area of the given square as the area of two isosceles triangles.

In Excerpt B19, Beng formed a large rectangle. She measured the lengths and width of the two rectangles by ruler respectively and then calculated the area of L-shape as the difference between the area of the large (labelled as A) and small (labelled as B) rectangles using area formula of rectangle, as shown in Figure N9. She partitioned the given square into two isosceles triangles, labelled as A and B respectively, as shown in Figure N10. Beng measured the lengths of each side of the square by ruler and then calculated the area of the given square as the total area of the two triangles using area formula of triangle. Area of B equals to area of A as they are identical (two isosceles triangles).

When probed for other method of determining whether the given pair of shapes had the same area, Beng used another formal method of calculating the area of L-shape. Excerpt B20 demonstrates how she used another method to compare the area of the given pair of shapes (Beng/L507-526).

Excerpt B20

R: Could you think of other way of finding out whether they had the same area?

S: Form two trapeziums. (Partitions the L-shape into two trapeziums, as shown in Figure N11. Measures the lengths of each side of the L-shape by ruler. Calculates the total area by using area formula of trapezium). Measures the lengths of two adjacent sides of the square and then calculates its area, as shown in Figure N12). This one (points to the area of the given square, as shown in Figure N12) same, 36 (compared to the area of the L-shape, as shown in Figure N11).

R: Could you think of other way of finding out whether they had the same area?

S: …(Silent for a while) I think that's all.

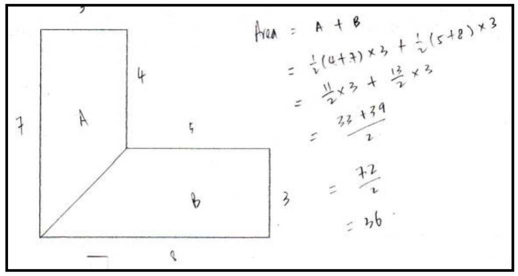


Figure N11. Beng partitions L-shape into two trapeziums and then calculates its total area.

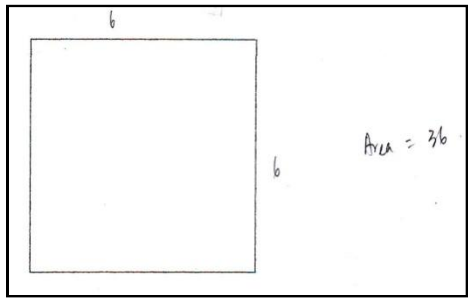


Figure N12. Beng calculates the area of the given square.

In Excerpt B20, Beng partitioned L-shape into two trapeziums, labelled as A and B. She measured the lengths of each side of the L-shape by ruler and then calculated the area of L-shape as the total area of the two trapeziums using area formula of trapezium, as shown in Figure N11. Beng also measured the lengths of two adjacent sides of the square and then calculated its area using area formula of square, as shown in Figure N12. She concluded that both the given shapes have the same area, namely 36 (without stating its unit of area measurement).

Summary

In summary, Beng produced three similar formal methods of determining whether the given pair of shapes had the same area, namely measuring the length of side by ruler and applying area formulae. In the first method, Beng partitioned L-shape into two rectangles. In the second method, she formed a large rectangle from the L-shape and partitioned the given square into two isosceles triangles. In the third method, Beng partitioned the L-shape into two trapeziums.

Comparing Perimeter (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Beng stated that shape A has the longer perimeter compared to shape B. Excerpt B21 shows her choice of shape that has the longer perimeter and the justification that she made (Beng/L548-563).

Excerpt B21

R: (Puts the following table in front of Beng). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 1	25 paper clips	12 sticks

S: Em shape A.
R: Why?
S: If I assume because I'm not sure, if I assume the paper clip is almost the same size of the stick, so the more paper clip, the more bigger the area is. That's my opinion. That's why I chose A, shape A as the bigger area. My assumption is the more things that we put in a shape, in a figure, the bigger the figure is.
R: So, which shape has the longer perimeter?
S: Perimeter, shape A.
R: Why?
S: The bigger the area, the longer the perimeter.

In Excerpt B21, Beng made an assumption that a paper clip is almost the size of a stick. She argued that there were 25 units of paper clips compared to 12 units of sticks. Therefore, she thought that shape A has the larger area. Beng generalized that the larger the area, the longer the perimeter. Thus, she thought that shape A has the longer perimeter. It indicated that Beng focused on the number of unit rather than the unit of measure when comparing perimeters in Set 1 with nonstandard units. She did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters.

In Set 2, Beng stated that shape B has the longer perimeter compared to shape A. Excerpt B22 depicts her choice of shape that has the longer perimeter and the justification that she made (Beng/L590-598).

Excerpt B22

R: (Puts the following table in front of Beng). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 2	10 paper clips	15 paper clips

S: Shape B.
R: Why?
S: Em both of them are form from the same thing, paper clips. So, I will choose the more paper clips one to be the longer perimeter.

In Excerpt B22, Beng stated that both shapes were measured with the common unit, namely paper clips. Thus, she chose the shape (shape B) that has more paper clips (more units of measurement) to be the longer perimeter. It indicated that Beng focused on the number of unit rather than the unit of measure when comparing perimeters in Set 2 with common nonstandard unit. She did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters.

In another situation when shapes A and B had the same perimeter, Beng explained that the paper clips in shape A is longer than the paper clips in shape B. Excerpt B23 demonstrates her justification about their units of measurement (Beng/L599-604).

Excerpt B23

R: If shapes A and B had the same perimeter, what would you tell about their units of measurement?
S: The paper clips in shape A is longer than the paper clips in shape B.
R: Why?
S: Because I think the longer paper clips need less to get the same perimeter as B.

In Excerpt B23, Beng explained that the shape with the longer paper clips (shape A) required less number of paper clips to produce the same perimeter as shape B. It indicated that Beng understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length.

In Set 3, Beng stated that shape A has the longer perimeter. Excerpt B24 reveals her choice of shape that has the longer perimeter and the justification that she made (Beng/L632-648).

Excerpt B24

R: (Puts the following table in front of Beng). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 3	16 cm	13 cm

S: Shape A.
R: Why?
S: Because this value is the perimeter value. The larger the value of the perimeter, I think it has a bigger perimeter.

In Excerpt B24, Beng explained that the shape with the larger value of the perimeter (larger number of unit), has the longer perimeter. It indicated that Beng focused on the number of unit when comparing perimeters in Set 3 with common standard unit. She knew that common standard unit (such as cm) is reliable for comparing perimeters.

Summary

In summary, Beng focused on the number of unit when comparing perimeter in Set 1 with nonstandard units. She did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. Beng focused on the number of unit when comparing perimeters in Set 2 with common nonstandard unit. She did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. Beng understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length. She focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Beng knew that common standard unit (such as cm) is reliable for comparing perimeters.

Comparing Area (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Beng stated that shape A has the larger area compared to shape B. Excerpt B25 shows her choice of shape that has the larger area and the justification that she made (Beng/L674-686).

Excerpt B25

R: (Puts the following table in front of Beng). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 1	25 triangles	12 squares

S: In my opinion, I think shape A.
R: Why?
S: In this case I made some assumption. I think my assumption is the triangle, two triangles will form one square. Two triangles from shape A will form one square from the shape B. So, in order to form 12 squares in the shape B, we need 24 triangles to form it. But the shape A has 25 triangles which is more, one more triangle than shape B. So, I conclude that shape A will be the larger area.

In Excerpt B25, Beng made an assumption that two triangles from shape A would from one square from shape B. She stated that we need 24 triangles from shape A to form 12 square in shape B. Beng explained that shape A has 25 triangles which is one triangle more than the required 24 triangles to form 12 squares in shape B. Thus, she concluded that shape A has the larger area. In reality, two triangles from shape A do not necessarily form one square in shape B. It indicated that Beng focused on the number of unit rather than the unit of measure when comparing areas in Set 1 with nonstandard units. She did not know that nonstandard units (such as triangle and squares) are not reliable for comparing areas.

In Set 2, Beng stated that shape B has the larger area compared to shape A. Excerpt B26 depicts her choice of shape that has the larger area and the justification that she made (Beng/L716-725).

Excerpt B26

R: (Puts the following table in front of Beng). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 2	10 squares	15 squares

S: I think shape B
R: Why?
S: I assumed that both the squares from the shape A and the shape B is of the same size. So, the shape B has more squares compared to shape A. So, shape B should have larger area.

In Excerpt B26, Beng assumed that squares from shapes A and B are of equal area. Thus, shape B has the larger area as it has more squares compared to shape A. It indicated that she focused on the number of unit rather than the unit of measure when comparing areas in Set 2 with common nonstandard units. Beng did not know that common nonstandard units (such as squares) are not reliable for comparing areas.

In another situation when shape A and B had the same area, Beng expressed that the squares from shapes A and B are of different area. Excerpt B27 demonstrates her justification about their units of measurement (Beng/L762-731).

Excerpt B27

R: If shapes A and B had the same area, what would you tell about their units of measure?
S: The squares from shape A and shape B is of the different size.
R: Could you tell me more about it?
S: The squares from shape A is bigger than the squares from shape B to get the same area.

In Excerpt B27, Beng explained that the square from shape A is larger than the square from shape B in order to get the same area. It indicated that Beng understand the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area.

In Set 3, Beng stated that shape A has the larger area. Excerpt B28 reveals her choice of shape that has the larger area and the justification that she made (Beng/L758-766).

Excerpt B28

R: (Puts the following table in front of Beng). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 3	16 cm^2	13 cm^2

S: By looking at the measurement, shape A has a larger value compared to shape B. So, I will chose shape A which has the larger area.
R: Why?
S: Because its value is bigger.

In Excerpt B28, Beng explained that shape A has a larger value compared to shape B. Therefore, shape A has the larger area. It indicated that Beng focused on the number of unit when comparing areas in Set 3 with common standard units. She knew that common standard unit (such as cm^2) is reliable for comparing areas.

Summary

In summary, Beng focused on the number of unit when comparing area in Set 1 with nonstandard units. She did not know that nonstandard units (such as triangle and square) are not reliable for comparing areas. Beng focused on the number of unit when comparing areas in Set 2 with common nonstandard units. She did not know that common nonstandard units are not reliable for comparing areas. Beng understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area. She focused on the number of unit when comparing areas in Set 3 with common standard units. Beng knew that common standard unit (such as cm^2) is reliable for comparing areas.

Linguistic Knowledge

In another situation in Set 3, Excerpt B29 exhibits how Beng wrote 16 cm^2 and 13 cm^2 in English words (Beng/L767-773).

Excerpt B29

R: (Puts a blank paper written the following measurements in front of Beng).
16 cm^2
13 cm^2
How would you write these measurements in English words?
S: (Writes the following).

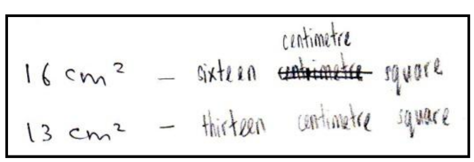


Figure N13. Beng writes 16 cm^2 and 13 cm^2 in English words.

In Excerpt B29, Beng wrote 16 cm^2 and 13 cm^2 literally as 'sixteen centimetre square' and 'thirteen centimetre square', as shown in Figure N13. The correct answer should be 'sixteen square centimetres' and 'thirteen square centimetres'. It indicated that she did not know about the conventions pertaining to writing and reading of Standard International (SI) area measurement units.

Converting Standard Units of Area Measurement

Procedural Knowledge

Beng realized that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2. Excerpt B30 shows the algorithms that Beng used when she was converting 3 cm^2 to mm^2 (Beng/L792-812).

Excerpt B30

R: (Puts a handout written the following scenario in front of Beng). Some Form One teachers noticed that several of their students seemed to multiply by 10, 100, and 1000, respectively when they were converting units of area from cm^2 to mm^2, m^2 to cm^2, and km^2 to m^2:

$3\ cm^2 = 3 \times 10\ mm^2 = 30\ mm^2$
$4.7\ m^2 = 4.7 \times 100\ cm^2 = 470\ cm^2$
$1.25\ km^2 = 1.25 \times 1000\ m^2 = 1250\ m^2$

What would you do if you were teaching Form One and you noticed that several of your students were doing this?

S: I'll tell them. For example, I tell them this one (converts 3 cm^2 to mm^2, as shown in Figure N14).

R: Could you explain your solution?

S: 3 cm^2 (misreads 3 cm^2 as 3 centimetre square) is a combination of 3 times cm times cm and 3 is a constant. So, if you want to convert cm to mm, we need to times 10 mm and same goes to this as well. Then we times the constant, we will get 300 mm^2 (misreads 300 mm^2 as 300 millimetre square). So, I will tell them we need to convert it using this way.

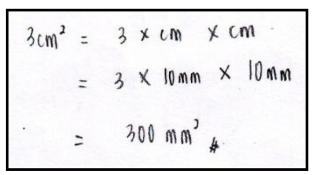


Figure N14. Beng converts 3 cm^2 to mm^2.

In Excerpt B30, Beng has successfully converted 3 cm^2 to mm^2. She viewed 3 cm^2 as the product of 3 times 1 cm times 1 cm. Beng knew the relationship between the standard units of length measurement that 1 cm = 10 mm. She also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Beng times 10 when she converted 1cm to mm twice, as shown in Figure N14.

Beng found that the students made a mistake when they were converting unit of area from 4.7 m^2 to cm^2. Excerpt B31 depicts the algorithms that Beng used when she was converting 4.7 m^2 to cm^2 (Beng/L813-822).

Excerpt B31

R: What about the second one (points to 4.7 m^2)?

S: Same goes to second as well. (Converts 4.7 m^2 to cm^2, as shown in Figure N15).

R: Could you explain your solution?

S: First I will explain the same way. 4.7 m^2 (misreads 4.7 m^2 as 4.7 metre square) is combination of 4.7 times metre times metre. One metre is equal to 100 cm. They should know this. So, I just write and we times the constant and as well as the unit and we will get the answer.

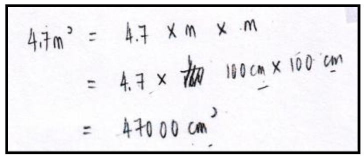


Figure N15. Beng converts 4.7 m^2 to cm^2.

In Excerpt B31, Beng has successfully converted 4.7 m^2 to cm^2. She viewed 4.7 m^2 as the product of 4.7 times 1 m times 1 m. Beng knew the relationship between the standard units of length measurement that 1 m = 100 cm. She also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Beng times 100 when she converted 1 m to cm twice, as shown in Figure N15.

Beng also found that the students made a mistake when they were converting unit of area from 1.25 km^2 to m^2. Excerpt B32 demonstrates the algorithms that Beng used when she was converting 1.25 km^2 to m^2 (Beng/L823-831).

Excerpt B32

R: What about the third one (points to 1.25 km^2)?
S: (converts 1.25 km^2 to m^2, as shown in Figure N16).
R: Could you explain your solution?
S: For the question three, 1.25 km^2 (misreads 1.25 km^2 as 1.25 kilometre square) just write 1.25 times km times km and 1 km is 1000 m. So, we times it. We will get the final answer.

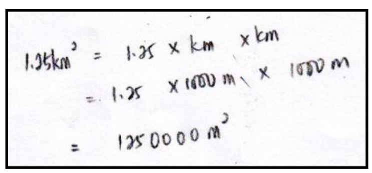


Figure N16. Beng converts 1.25 km^2 to m^2.

In Excerpt B32, Beng has successfully converted 1.25 km^2 to m^2. She viewed 1.25 km^2 as the product of 1.25 times 1 km times 1 km. Beng knew the relationship between the standard units of length measurement that 1 km = 1000 m. She also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Beng times 1000 when she converted 1 km to m twice, as shown in Figure N16.

Summary

In summary, Beng realized that the students made mistakes when they were converting 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2 respectively. She knew the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. Beng also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. She viewed 3 cm^2 as the product of 3 times 1 cm times 1 cm, 4.7 m^2 as the product of 4.7 times 1 m times 1 m, and 1.25 km^2 as the product of 1.25 times 1 km times 1 km. Thus, Beng times 10, 100, and 1000 respectively when she converted 1 cm to mm, 1 m to cm, and 1 km to m twice, as shown in Figures N14, N15, and N16.

Conceptual Knowledge

Beng knew the relationships between the standard units of length measurement such as 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. These can be seen in Figures N14, N15, and N16. She also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring, as shown in Figures N14, N15, and N16.

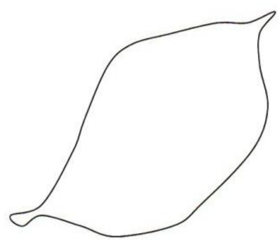
Relationship between Perimeter and Area
(Same Perimeter, Same Area?)

Conceptual Knowledge

Beng did not know that there is no direct relationship between perimeter and area. She did not know that two shapes with the same perimeter can have different areas. Thus, Beng was not sure whether the student's method of calculating the area of the leaf was correct or not. Excerpt B33 shows Beng's responses to the Form One student (Beng/L858-886).

Excerpt B33

R: (Puts a handout written the following scenario in front of Beng). This is a picture of a leaf. A Form One student said that he had found a way to calculate the area of the leaf. The student placed a piece of thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle.



How would you respond to this student?

S: I will tell the student this is, this might be one way to calculate the area of the leaf but I need to confirm it first whether it is correct or not because I never think that we can use this way to calculate the area of the leaf.

R: Would this method works?

S: Most properly, it might. I think it will work.

In Excerpt B33, Beng was not sure whether the student's method of calculating the area of the leaf was correct or not. When probed further, she stated that this method most properly works. Nevertheless, Beng explained that she need to verify it first whether the method mentioned by the student was correct or not. Excerpt B34 depicts how she would verify it (Beng/L887-893).

Excerpt B34

R: Just now you mentioned that you need to confirm it first. So, how would you confirm it?

S: May be I will put it on a way, put all the square inside it and then use the square as the unit of area to calculate it and compare the answer with the student's answer to confirm it. I will try to search, seek for other people's opinion first before I confirm it because I never think that we can use this method to calculate it.

In Excerpt B34, Beng explained that she would verify it by covering the surface of the leaf with square units and then compare it with the student's answer. Beng elaborated that she would also seek other people's view to verify it as she never think that the student's method can be used to calculate the area of the leaf.

Summary

In summary, Beng did not know that there is no direct relationship between perimeter and area. She did not know that two shapes with the same perimeter can have different areas. Thus, Beng was not sure whether the student's method of calculating the area of the leaf was correct or not.

Ethical Knowledge

In Task 5.1, Beng was not sure whether the student's method of calculating the area of the leaf was correct or not, as shown in Excerpt B33. Thus, Beng said that she needed to verify it. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter has the same area. Beng did not attempt to examine the possible pattern of the relationship between perimeter and area.

Beng suggested that she would verify it by covering the surface of the leaf with square units and then compare it with the student's answer. Beng elaborated that she would also seek other people's view to verify it as she never think that the student's method can be used to calculate the area of the leaf, as shown in Excerpt B34. It indicated that Beng relied on authority, namely other people's view, to verify the correctness of the student's method of calculating the area of the leaf. She did not attempt to formulate generalization pertaining to the relationship between perimeter and area. Beng never tests the student's generalization that two shapes with the same perimeter have the same area.

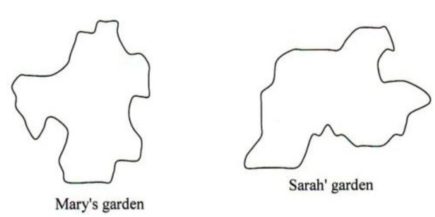
Relationship between Perimeter and Area
(Longer Perimeter, Larger Area?)

Conceptual Knowledge

Beng made a reflection on Task 3.1 when she approached Task 5.2. From the reflection, she realized that the shape with the longer perimeter may have a smaller area. Beng knew that there is no direct relationship between perimeter and area. She knew that the garden with the longer perimeter could have a smaller area. Thus, Beng knew that Mary's claim was not correct. Excerpt B35 shows Beng's responses to the claim made by Mary that the garden with the longer perimeter has the larger area (Beng/L904-930).

Excerpt B35

R: (Puts a handout written the following scenario in front of Beng). Mary and Sarah are discussing whose garden has the larger area to plant flowers. Mary claims that all they have to do is walk around the two gardens to get the perimeter and the one with the longer perimeter has the larger area. How would you respond to these students?



S: I will tell them this method may not be works because the area, these two gardens is of the different shape. Just now the task, one of the task that I did (refers to Task 3.1 just now for the T (shape) and the rectangle one. The T (shape) one has the longer perimeter but has a smaller area compared to the other one. So, will encourage them to calculate it first by not using the perimeter. I think the, using this concept, this statement "longer perimeter has the larger area" is suitable when we comparing two shapes, two equal shapes but with different length.

In Excerpt B35, Beng knew that there is no direct relationship between perimeter and area upon reflection on Task 3.1. In Task 3.1, she found that the shape with the longer perimeter may have a smaller area. Thus, Beng knew that the garden with the longer perimeter may have a smaller area. Beng explained that Mary's method did not work for this situation as these two gardens are of different shape. She stated that Mary's claim is true only when we are comparing the area of two similar shapes (same shape but different area).

When probed further for the meaning of her statement 'two equal shapes but with different length', Beng drew two shapes as depicted in Excerpt B36 (Beng/L931-945).

Excerpt B36

R: What do you mean by "two equal shapes but with different length."?

S: (Draws two squares and labels them as A and B respectively, as shown in Figure N17). For example, the squares here (points to the squares that she has drawn, as shown in Figure N17). Both I labelled it as A and B. Both A and B are square but they are, both are plotted out of the same shape but they are of different size. So, I think this method, this longer perimeter has larger area can be applied in this situation, but not this picture here (points to the given pictures of the two gardens, as shown above) because they are of different shape. Besides that we can't measure exactly the area of this shape. So, it's better for us to measure it using the ordinary way to confirm it.

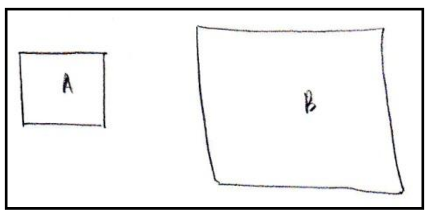


Figure N17. Beng draws two squares and labels them as A and B respectively.

In Excerpt B36, Beng drew two squares with different length of sides. She pointed out that Mary's method works in a situation where the two shapes being compared are of similar shape. She reiterated that Mary's claim is not true for the situation in Task 5.2 as the given pairs of garden are of different shape. Beng stated that the area of these shapes (irregular shapes) could not be measured exactly. She suggested conventional method to verify Mary's claim.

When probed further, Beng stated that she would use the grid paper to verify Mary's claim. She would calculate the number of units of squares, triangle, or rectangle that cover the surface of the given pictures of the gardens. This is illustrative in Excerpt B37 (Beng/L946-954).

Excerpt B37

R: How would you confirm it?

S: Using the grid. Put all the squares or triangle or rectangle as a unit of area and calculate the square inside it.

R: Would Mary's method works?

S: This method I am not sure but I don't think so because of just now, of the example just now (refers to Task 3.1. I just realize that not necessary.

R: Which example are you referring to?

S: Yeah, the T (shape) (refers to Task 3.1.).

In Excerpt B37, initially Beng stated that she was not sure whether Mary's method works. Later, Beng thought that Mary's methods does not work after she made a reflection on Task 3.1 that the shape with the longer perimeter may has a smaller area.

Summary

In summary, Beng knew that there is no direct relationship between perimeter and area. She knew that the garden with the longer perimeter could have a smaller area. Thus, Beng knew that Mary's claim was not correct.

Ethical Knowledge

In Task 5.2, Beng made a reflection on Task 3.1 when she approached Task 5.2. From the reflection, she realized that the shape with the longer perimeter may have a smaller area. Beng knew that the garden with the longer perimeter could have a smaller area. Thus, she knew that Mary's claim was not correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area.

Beng attempted to examine the possible pattern of the relationship between perimeter and area, as shown in Excerpt B35. Based on her reflection on Task 3.1, she found that the shape with the longer perimeter may have a smaller area. Beng explained that Mary's method did not work for the situation in Task 5.2 as these two gardens are of different shape. She stated that Mary's claim is true only when we are comparing the area of two similar shapes (same shape but different area).

Based on her reflection on Task 3.1, she found that the shape with the longer perimeter may have a smaller area. Thus, Beng formulated a generalization that the garden with the longer perimeter could have a smaller area. Beng used the example of her first and second method of comparing perimeter in Task 3.1 to test Mary's generalization that the garden with the longer perimeter has the larger area. She found that the shape with the longer perimeter may have a smaller area. Thus, Beng knew that Mary's generalization was not correct. Beng had successfully generated a counterexample to refute Mary's generalization. She knew that a counterexample is sufficient to refute the truth of a generalization. Beng suggested that she would use the grid paper to verify Mary's claim. She would calculate the number of units of squares, triangle, or rectangle that cover the surface of the given pictures of the gardens. This is illustrative in Excerpt B37.

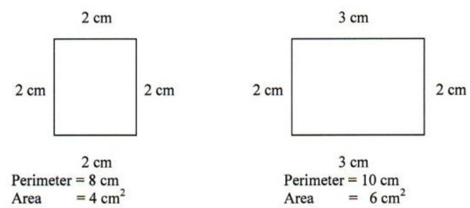
Relationship between Perimeter and Area
(Perimeter Increases, Area Increases?)

Conceptual Knowledge

Beng knew that there is no direct relationship between perimeter and area. She knew that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Beng knew that the student's "theory" was not correct. This is shown in Excerpt B38 (Beng/L984-1006).

Excerpt B38

R: Puts a handout written the following scenario in front of Beng). Suppose that one of your Form One students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:



How would you respond to this student?

S: …(Silent for a while) em I will tell the student that his theory might be right for this situation because she can prove it but it might not apply to all the shapes.

In Excerpt B38, Beng explained that the student's 'theory" might be true for this situation as the students can "prove" it with an example, as shown in Excerpt B38. Beng knew that the student's "theory" might not apply to all the shapes (other situations). Nevertheless, Beng did not know that the student's claim about the relationship between perimeter and area is not a theory. The claim is a conjecture. Beng also did not know that an example is not a proof and a theory cannot be proved by an example.

When probed to tell more about it, Beng attempted to provide a counterexample to disprove or refute the student's "theory". Nevertheless, she was unable to find a counterexample to refute the student's conjecture as depicted in Excerpt B39 (Beng/L1007-1015).

Excerpt B39

R: Could you tell me more about it?
S: But till now I can't find any suitable example to illustrate.
R: So, what's your conclusion?
S: The longer perimeter not necessarily to be the larger area.
R: Why?
S: Em…(silent for a while) may be because of the example that I did just now (refers to the T-shape in Task 3.1.
R: Which example are you referring to?
S: The T (shape) one.

In Excerpt B39, Beng concluded that the figure with the longer perimeter does not necessarily have the larger area. Beng came to this conclusion based on a reflection that she made on Task 3.1 that the shape with the longer perimeter may have a smaller area. When the researcher asked Beng whether the student's "theory" correct, she attempted to find a counterexample to refute the "theory" by comparing the perimeters and areas of the trapezium and rectangle that she has drawn, as shown in Figure N18 in Excerpt B40 (Beng/L 1020-1033).

Excerpt B40

R: Does her "theory" correct?
S: (Calculates the perimeters and area of trapezium and rectangle that she has drawn, as shown in Figure N18). (For the trapezium, perimeter = 16 cm (not 14 cm, as shown in Figure N18) and area = 14 cm^2 (not 10 cm^2, as shown in Figure N18). For the rectangle, perimeter = 16 cm and area = 15 cm^2). Since I can't find any example to prove that this theory is not correct, I won't say it is incorrect but I don't think it correct hundred percent. So, may be more research in this field to confirm it.
R: Could you tell me more about it?
S: I try to prove whether there is example for different perimeter, eh longer perimeter but with smaller area but I can't prove it.

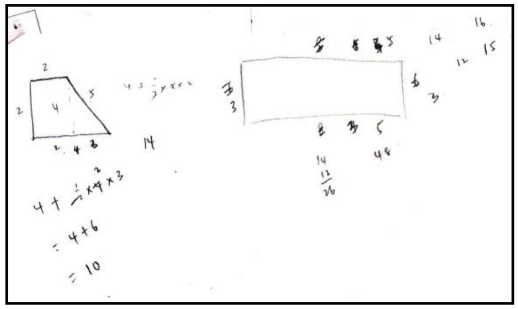


Figure N18. Beng calculates the perimeters and area of trapezium and rectangle that she has drawn.

In Excerpt B40, Beng correctly calculated the perimeter and area of the rectangle as 16 and 15 respectively but incorrectly calculated the perimeter and area of the trapezium as 14 and 10 respectively (It should be 16 cm and 14 cm^2 respectively). The example that Beng generated did not help her to refute that the "theory" is not correct. The example that she gave suggested that as the perimeter of a closed figure increases, the area also increases. Beng admitted that she was unable to provide a counterexample to refute that the student's "theory" is not correct. Nevertheless, Beng does not think that the student's "theory" is correct even though she was unable to generate a counterexample to refute it. Thus, Beng stated that she need more research in this field to verify it.

Summary

In summary, Beng knew that there is no direct relationship between perimeter and area. She knew that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Beng knew that the student's "theory" was not correct. Nevertheless, she was unable to generate a counterexample to refute the student's claim that as the perimeter of a closed figure increases, the area also increases.

Ethical Knowledge

In Task 5.3, the student formulated a generalization that as the perimeter of a closed figure increases, the area also increases. Beng made a reflection on Task 3.1 that the shape with the longer perimeter may has a smaller area, as shown in Excerpt B39. She attempted to test the student's generalization, as shown in Excerpt B40. Nevertheless, Beng was unable to generate a counterexample in Task 5.3 to refute the student's claim that as the perimeter of a closed figure increases, the area also increases. In reality, when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same.

Calculating Perimeter and Area
(Rectangle and Parallelogram/Triangle)

Procedural knowledge

After read through Task 6.1, Beng labelled the missing sides of Diagram 1 that required for calculating the perimeter and area of the diagram, as shown in Figure B19 in Excerpt B41 (Beng/L1053-1086, 1091-1096).

Excerpt B41

R: (Puts a handout written the following problem in front of Beng). Suppose that one of your Form One students asks you for help with the following problem:

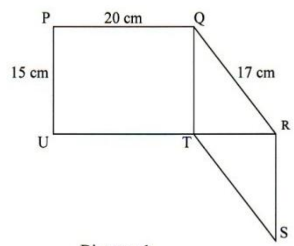


Diagram 1

In Diagram 1, PQTU is a rectangle and QRST is a parallelogram. UTR is a straight line.
Calculate
(a) the perimeter of the diagram,
(b) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 1, as shown in Figure N19). I just ask the student to list down what she knows. For example, this is a rectangle. Here PU is 15 cm. Same goes to QT as well, 15 cm and PQ is 20. UT also 20. …So, now the problem comes because that we don't know the length of the TR and I don't think Form One (student) already study the Pythagoras' theorem. So, in this case, I will ask them to put the RTS here (moves triangle RST under the translation T_{SR} to form a rectangle with the dimensions of 15 cm by 8 cm as shown in Figure N19).

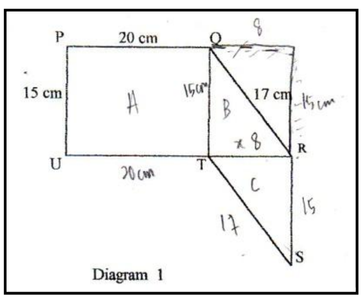


Figure N19. Beng labels the missing sides of Diagram 1.

In Excerpt B41, Beng labelled QT, TU, RS, and ST as 15 cm, 20 cm, 15, and 17 respectively on Diagram 1. Beng realized that she needed to find the length of TR in order to calculate the area of parallelogram QRST. Beng knows that Form One students have not study the Pythagoras' theorem. Thus, she moves triangle RST under the translation T_{SR} to form a large rectangle with the dimension of 15 cm by 8 cm, as shown in Figure N19.

As the researcher assessing the algorithms that accessible to the subject during the clinical interview to calculate the perimeter and area of Diagram 1, Beng was given the freedom to use Pythagoras' theorem to find the length of TR by assuming that the student has studied it. Excerpt B42 shows that Beng has successfully calculated the length of TR using Pythagoras' theorem (Beng/L1097-1105).

Excerpt B42

R: How would you solve this problem?
S: Form One students haven't study the Pythagoras' theorem right?
R: Never mind. You can assumed that they have studied it.
S: Then I will ask them to use the Pythagoras' theorem to calculate it. (Writes the following).
$17^2 = 15^2 + x^2$
$x^2 = 17^2 - 15^2$
$x = 8$
TR is x, here is x.

Excerpt B43 depicts how Beng has successfully calculated the perimeter of Diagram 1 (Beng/L1116-1123).

Excerpt B43

R: How would you calculate the perimeter of the diagram?
S: (Calculates the perimeter of Diagram 1, as shown in Figure N20).
R: What do you get?
S: 104.

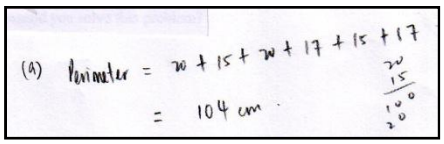


Figure N20. Beng calculates the perimeter of Diagram 1.

In Excerpt B43, Beng used the list all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N20. She listed all the length of sides that surrounded the diagram and then summed them up to get the perimeter of the diagram as 104 cm.

Excerpt B44 demonstrates how Beng has successfully calculated the area of Diagram 1 (Beng/L1128-1134).

Excerpt B44

R: How would you calculate the area of the diagram?
S: (Calculates the area of Diagram 1, as shown in Figure N21).
R: What do you get?
S: 420.

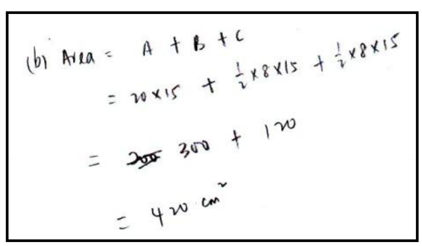


Figure N21. Beng calculates the area of Diagram 1.

In Excerpt B44, Beng used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N21. She partitioned Diagram 1 into a rectangle PQTU (labelled as A) and two triangles QRT (labelled as B) and RST (labelled as C). Beng calculated the area of A, B, and C using the area formulae of rectangle and triangles respectively and then summed them up to get the area of the diagram as 420 cm^2.

Summary

In summary, Beng has successfully calculated the perimeter of Diagram 1 using the list all-and-sum algorithm. She has also correctly calculated the area of Diagram 1 using the partition-and-sum algorithm to calculate the area of the diagram.

Linguistic knowledge

Beng used the correct standard units of measurement for perimeter (cm) and area (cm^2) when she wrote the answers for these measurements, as shown in Figures N20, N21, and N22.

Strategic Knowledge

When probed to check the answer for the perimeter, Beng suggested that she would use the recalculating strategy to verify the answer. Excerpt B45 is illustrative (Beng/L1124-1127).

Excerpt B45

R: How would you check your answer for the perimeter?
S: Try to do it again.
R: How would you do it again?
S: Use the same method and calculate again.

In Excerpt B45, Beng suggested that she would check the answer for perimeter by recalculating strategy that using the same method and calculate again.

When probed to check the answer for the area, Beng used an alternative procedure (alternative method) to generate an answer which could be used to verify her original answer. Excerpt B46 is illustrative (Beng/L1135-1149).

Excerpt B46

R: How would you check your answer for the area?
S: Em I will ask them to put this TR here (moves triangle RST under the translation T_{SR} to form a rectangle with the dimensions of 15 cm by 8 cm, as shown in Figure N19) to form a rectangle and here is 15 cm (refers to the width of the new rectangle just formed), 28 cm (refers to the length of the new rectangle just formed). (Draws a rectangle and then calculates its area using area formula of rectangle, as shown in Figure N22).
R: What do you get?
S: Then it will form a great rectangle like this (points to the new rectangle that she has just formed, as shown in Figure N22). Then using the formula of the rectangle to calculate the area, to confirm the answer.

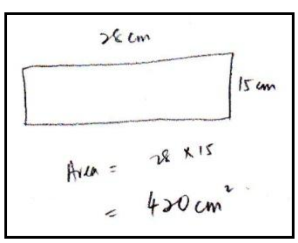


Figure N22. Beng uses alternative method to calculate the area of Diagram 1.

In Excerpt B46, Beng checked the answer for area by moving triangle RST under the translation T_{SR} to form a rectangle with the dimensions of 15 cm by 8 cm, as shown in Figure N19. She then drew a large rectangle, as shown in Figure N22 and calculated its area by using area formula of rectangle. Both methods come to the same answer, namely 420 cm^2. Thus, it confirms that the answer is correct.

Ethical Knowledge

Beng has successfully calculated the perimeter and area of Diagram 1. Nevertheless, she did not check the correctness of the answers for perimeter as well as area. When probed to check answers, then only Beng suggested the strategies that she would use to check the answers for perimeter and area. Beng wrote the measurement units (without probed) for the answers of the perimeter and area that she has calculated, as shown in Figures N20, N21, and N22.

Calculating Perimeter and Area
(Square and Trapezium/Triangle)

Procedural Knowledge

After read through Task 6.2, Beng labelled the missing sides of Diagram 2 that required for calculating the perimeter and area of the diagram, as shown in Figure N23 in Excerpt B47 (Beng/L1183-1222).

Excerpt B47

R: (Puts a handout written the following problem in front of Beng). Suppose that one of your Form One students asks you for help with the following problem:

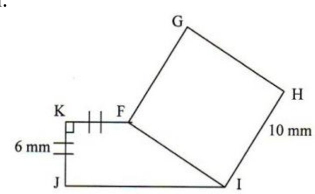


Diagram 2

In Diagram 2, FGHI is a square and FIJK is a trapezium.
Calculate
(a) the perimeter of the diagram,
(b) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 2, as shown in Figure N23). First I will ask the student to find out what is given. This means here is 6 mm (points to KJ), here is 6 mm as well (points to KF) and then FGHI is a square. So, all the lengths have, all the sides have the same length. So, just list out (labels 10 mm respectively on the other three sides of the square FGHI). Now, the only thing that we can't find is the JI. So, I'll ask the student to split it out into two (partitions FIJK into a square and a triangle, as shown in Figure N23). One is the square and the other one is a triangle. I'll tell them this is a square because the JK is same as KF. So, let's say I put a "A" here. JA is also 6 mm as well as FA. Then now for FAI (refers to triangle FAI), we need to find the AI. We use the Pythagoras' theorem to find it (writes the following).
$AI^2 = 10^2 - 6^2$
And we will get the answer as 8.

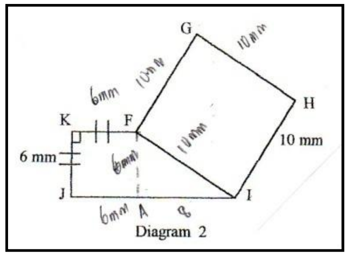


Figure N23. Beng labels the missing sides of Diagram 2.

In Excerpt B47, Beng labelled KF, FG, GH, and IF as 6 mm, 10 mm, 10 mm, and 10 mm respectively on Diagram 2. Beng realized that she needed to find the length of JI. Beng partitioned trapezium FIJK into a square and a triangle, as shown in Figure N23. She labelled FA and AJ as 6 mm on Diagram 2. Beng has successfully calculated the length of AI as 8 (mm) by using Pythagoras' theorem. Excerpt B48 depicts how Beng has successfully calculated the perimeter of Diagram 2 (Beng/L1222-1227).

Excerpt B48

S: **…**So, to calculate the perimeter, just add up all the value outside the figure (refers to Diagram 2, as shown in Figure B23). (Calculates the perimeter of Diagram 2, as shown in Figure N24).

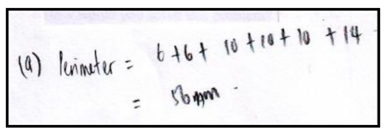


Figure N24. Beng calculates the perimeter of Diagram 2.

In Excerpt B48, Beng used the list all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N24. She listed all the length of sides that surrounded the diagram and then summed them up to get the perimeter of the diagram as 56 mm.

Excerpt B49 demonstrates how Beng has successfully calculated the area of Diagram 2 (Beng/L1231-1239).

Excerpt B49

R: How would you calculate the area of the diagram?
S: Em area. This is a combination of a square and trapezium. So, we just use the formula of square and trapezium to calculate the area (calculates the area of Diagram 2, as shown in Figure N25).
R: What do you get?
S: 160.

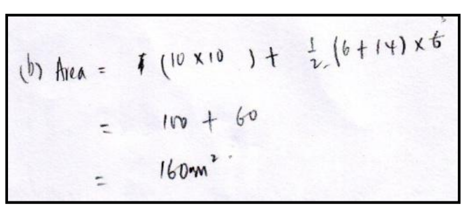


Figure N25. Beng calculates the area of Diagram 2.

In Excerpt B49, Beng used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N25. She partitioned Diagram 2 into square FGHI and trapezium FIJK. Beng calculated the area of the square and trapezium separately using the area formulae of square and trapezium respectively and then summed them up to get the area of the diagram as 160 mm^2.

Summary

In summary, Beng has successfully calculated the perimeter of Diagram 2 using the list-and-sum algorithm. She has also correctly calculated the area of Diagram 2 using the partition-and-sum algorithm.

Linguistic Knowledge

Beng used the correct standard units of measurement for perimeter (mm) and area (mm^2) when she wrote the answer of these measurements, as shown in Figures N24 and N25. Initially, Beng mistakenly used the incorrect units of measurement for perimeter (cm) and area (cm^2) when she wrote the answer of these measurements. She realized the mistake and then wrote the correct units.

Strategic Knowledge

When probed to check the answer for the perimeter, Beng suggested that she would use alternative method, namely the surround-and-measure strategy, to verify the answer. Excerpt B50 is illustrative (Beng/L1228-1230).

Excerpt B50

R: How would you check your answer for the perimeter?
S: Em…(silent for a while) may be using the thread to measure it and measure it from the ruler.

In Excerpt B50, Beng suggested that she would check the answer for the perimeter by surrounding the diagram by thread and then measure its length by ruler.

When probed to check the answer for the area, Beng suggested that she would use an alternative method, namely superimpose method, to verify her original answer. Excerpt B51 is illustrative (Beng/L1240-1242).

Excerpt B51

R: How would you check your answer for the area?
S: May be I'll ask them to use the unit of area, may be square. Put (superimpose) it here (points to FGHI) and here (points to FIJK) to confirm the answer.

In Excerpt B51, Beng suggested that she would check the answer for the area by superimpose it with square units.

Ethical Knowledge

Beng has successfully calculated the perimeter and area of Diagram 2. Nevertheless, she did not check the correctness of the answers for perimeter and area. When probed to check answers, then only Beng suggested the strategy that she would use to check the answers for perimeter and area. Beng wrote the measurement units (without probed) for the answers of perimeter and area, as shown in Figures N24 and N25.

Fencing Problem

Strategic Knowledge

Beng used looking for a pattern strategy to solve the fencing problem. Excerpt B52 is illustrative (Beng/L1244-1258).

Excerpt B52

R: (Puts a card written the following problem in front of Beng). Suppose that one of your students asks you for help with the following problem:

A gardener has 84 m of fencing to enclose a garden along three sides, with the fourth side of the garden being formed by a wall. (Assume that the wall is perfectly straight). What are the dimensions of a rectangular garden that will yield the largest area being enclosed?

How would you solve this problem?

S: (Uses looking for a pattern strategy to solve this problem. Draws the following diagram, as shown in Figure N26).

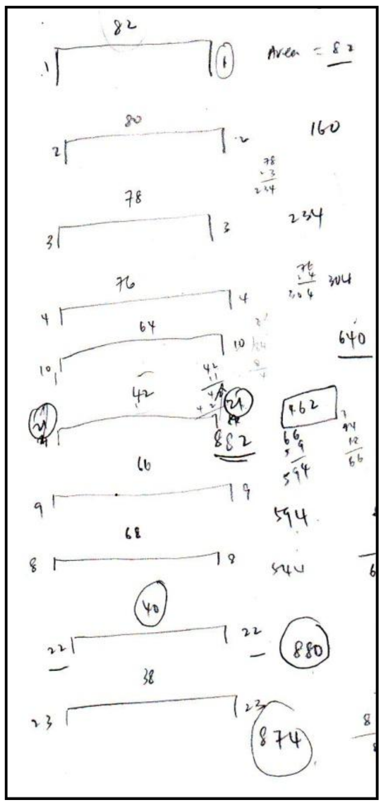


Figure N26. Beng uses looking for a pattern strategy to solve the fencing problem.

Excerpt B53 further illustrates how Beng used this strategy to solve the fencing problem (Beng/L1259-1270).

Excerpt B53

R: Could you explain your solution?

S: I'll ask them to try first. If here is 1, 1, here is 82 and the area is 82 and will be increasing. So, I just ask them to take the half of the value. 84, half is 42 and this one will be 21 and 21. So, the value they get will be 882. Then I need to test again. Ask them to use, increase this number. 22, 22, this one 40 and the value get is 880. Continue with 23, 23, 38. The value decreasing (draws the pattern of the area values, as shown in Figure N27). So, the trend is increasing up to this point and then decreasing. So, the dimension for the largest area will be 42 times 21.

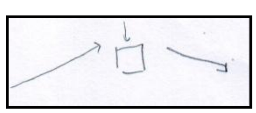


Figure N27. Beng draws the pattern of the area values.

In Excerpt B53, Beng started off with the width and the length of the rectangular garden as 1 m and 82 m respectively and this yielded the smallest area being enclosed, namely 82 m^2. She then increased the width of the rectangular garden, one metre at a time, to 4 m and reduced the length of the rectangular garden accordingly to 76 m. Consequently, the area increased to 304 m^2. Beng saw a pattern that area increases as she increases the width of the rectangular garden while reduces its length accordingly. She increased the width of the rectangular garden to 10 m instead of 5m and reduced its length to 64 m. The area increased to 640 m^2. Subsequently, Beng took half of the 84 m of fencing as length of the rectangular garden and 21 m as its width. The area now increased to 882 m^2.

Beng attempted to verify whether 882 m^2 was the largest area being enclosed. She tested it with two values of the width that were smaller than 21 m, namely 9 m and 8 m respectively. Beng found that the area decreased to 594 m^2 and 544 m^2 respectively. Beng also tested it with two values of the width that were larger than 21 m, namely 22 m and 23 m respectively. Beng found that the area decreased to 880 m^2 and 874 m^2 respectively. Thus, Beng concluded that 882 m^2 is the largest area being enclosed and the dimension of the rectangular garden that yields the largest area being enclosed is 42 m by 21 m. Table N2 summarizes the dimensions of the rectangular garden and its area that Beng has figured out. Figure N27 shows the pattern of the area values that Beng has drawn.

When probed to verify the dimension of the rectangular garden that yields the largest area being enclosed, Beng made a reflection on the answers and solutions that she has figured out, as shown in Figure N26. Beng explained that she started off with the smallest value of the width of the rectangular garden, namely 1 m, the length of the garden as 82 m. Beng explained that its area increased to 882 m^2 as she increased the width to 21 m and reduced the length of the rectangular garden to 42 m. Beng realized that its area keep decreases after that when she increases the width of the rectangular garden to 22 m and 23 m respectively and reduced its length accordingly. Thus, Beng reiterated that the maximum area, 882 m^2, occurred at the "center" when its dimension is 42 m by 21 m. Excerpt B54 is illustrative (Beng/L1271-1282).

Excerpt B54

R: How do you know that the dimensions will give you the largest area?

S: Em I just use sequence because I test from the largest one here (points to diagram with the value of 21, 42, 21, as shown in Figure N26). Start here with the smallest one (points to diagram with the value of 1, 82, 1, as shown in Figure N26) and it keeps increasing. (Draws the fence with the shortest length (2 m) and the longest width (41 m), as shown in Figure N28). I think when it goes back, dwell down, finally here will become two and the value is smaller and smaller. So, the maximum value will be at the center. That's why I try the center.

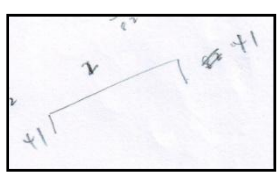


Figure N28. Beng draws the fence with the shortest length (2 m) and the longest width (41 m).

Table N2

Dimensions of Rectangular Garden and its Area That Beng has Figured out

Length (cm)	Width (cm)	Width (cm)	Area (cm^2)
82	1	1	82
80	2	2	160
78	3	3	234
76	4	4	304
64	10	10	640
42	21	21	882
66	9	9	594
68	8	8	544
40	22	22	880
38	23	23	874

Summary

In summary, Beng has successfully solved the fencing problem using the looking for a pattern strategy. Beng used the same strategy, namely the looking for a pattern strategy, to check the answer for the fencing problem without being probed.

Ethical Knowledge

Beng used the same strategy, namely the looking for a pattern strategy, to check the answer for the fencing problem without being probed. This can be seen in Excerpt B57. Nevertheless, Beng did not write any measurement units throughout Task 7. Excerpts B52 and B53 are illustrative.

Developing Area Formulae

Procedural Knowledge

Beng could recall the formula for the area of a rectangle, namely '$l \times w$', as shown in Figure B29. Nevertheless, she was unable to develop it. Excerpt B55 is illustrative (Beng/L1297-1314).

Excerpt B55

R: (Puts a handout written the following scenario in front of Beng). Suppose that a Form One student comes to you and says that he does not know how to develop the formula for the area of the following shapes:
(a) Rectangle,
(b) Parallelogram,
(c) Triangle, and
(d) Trapezium.
How would you show him the way to develop the formula for the area of these shapes? Let's start with rectangle.
S: (Draws a rectangle and then writes its area formula, as shown in Figure N29). Area just the length times the width.
R: How would you develop the formula for the area of a rectangle?
S: …(Silent for a while) I really don't know how to develop it.

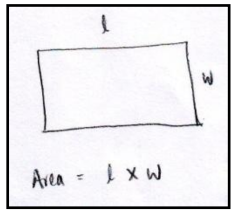


Figure N29. Beng draws a rectangle and writes its area formula.

In Excerpt B55, Beng stated that the formula for the area of a rectangle is 'length times width'. Nevertheless, she was unable to develop it. Beng just memorized the formula. She did not attempt to develop the formula.

Beng could recall the formula for the area of a parallelogram as '$a \times b$', as shown in Figure N30. She knew how to develop the formula for the area of a parallelogram. Excerpt B56 is illustrative (Beng/L1315-1332).

Excerpt B56

R: How would you develop the formula for the area of a parallelogram?
S: (Draws a parallelogram and then writes its area formula, as shown in Figure N30).
(Draws the diagrams, as shown in Figure N31, to show how she develops the formula).
R: Could you explain your solution?
S: If I assume that the student knew the formula for rectangle, so the parallelogram is roughly like this (points to the parallelogram that she has drawn). So, I move this triangle over here to form rectangle. So, it will become a rectangle and this one is still "a". Em this is "b". So, it is just a formula for rectangle.
R: What does the "a" mean?
S: "a" is the length here and "b" is the line here perpendicular to "a".

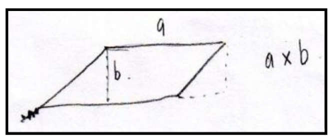


Figure N30. Beng draws a parallelogram and writes its area formula.

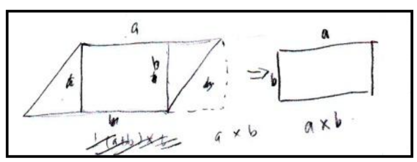


Figure N31. Beng develops the formula for the area of a parallelogram.

In Excerpt B56, Beng mentally cuts out a right-angled triangle from one end of the parallelogram and moves it to the other end of the parallelogram to form a rectangle, as shown in Figure N31. Thus, the area of the parallelogram equals to the area of the rectangle formed and its area formula is 'a times b' or 'base times height'.

Beng could recall the formula for the area of a triangle, namely '$\frac{1}{2} \times b \times t$', as shown in Figure N32. Nevertheless, she was unable to develop it. Excerpt B57 is illustrative (Beng/L1333-1341).

Excerpt B57

R: How would you develop the formula for the area of a triangle?
S: (Draws a triangle and then writes its area formula, as shown in Figure N32). Em I have no idea how to develop the formula.
R: What does the "b" mean?
S: "b" stands for the base. Then "t" stands for the height.

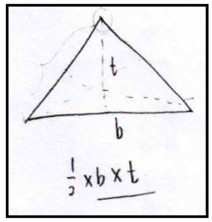


Figure N32. Beng draws a triangle and writes its area formula.

In Excerpt B57, Beng explained that b and t represents the base and the height of the triangle. She could recall the formula for the area of a triangle. Nevertheless, Beng was unable to develop it. Beng admitted that she has no idea of how to develop the formula. Beng just memorized the formula. She did not attempt to develop the formula.

Beng could recall the formula for the area of a trapezium, namely '$\frac{1}{2}(a+b) \times t$', as shown in Figure N33. Excerpt B58 is illustrative (Beng/L1360-1368).

Excerpt B58

R: How would you develop the formula for the area of a trapezium?
S: (Draws a trapezium and then writes its area formula, as shown in Figure N33).
R: What do you mean by "a", "b", and "t" here?
S: "a" stands for the upper side, "b" stands for lower one but parallel to the upper one and the "t" is the height.

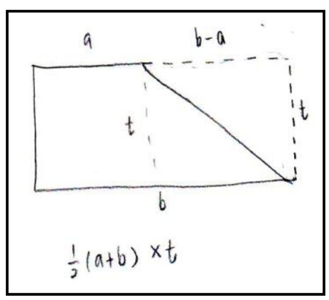


Figure N33. Beng draws a trapezium and writes its area formula.

Beng knew how to develop the formula for the area of a trapezium. Excerpt B59 is illustrative (Beng/L1369-1382).

Excerpt B59

R: How would you develop the formula for the area of a trapezium?
S: (Develops the formula for the area of a trapezium, as shown in Figure N34).
R: Could you explain your solution?
S: In order to get a trapezium, we assumed that it is a big rectangle minus the triangle here. For the big rectangle, it is "b times t" and for the triangle, this one is "b minus a", this one is "t". So, we minus half, this is the formula for the triangle (points to $\frac{1}{2}$(b - a) x t). Then we just take out the "t" (points to t (b - $\frac{1}{2}$b + $\frac{1}{2}$a) = t ($\frac{1}{2}$a + $\frac{1}{2}$b) = $\frac{1}{2}$(a + b) x t). Then we will get the formula for trapezium (points to $\frac{1}{2}$(a + b) x t).

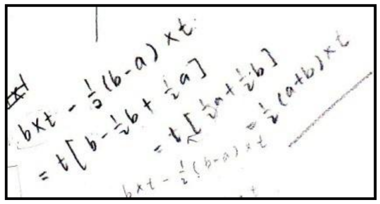


Figure N34. Beng develops the formula for the area of a trapezium.

In Excerpt B59, Beng developed the formula for the area of a trapezium using algebraic method, as shown in Figure N34. She drew dotted lines on Figure N33 to form a large rectangle and viewed the area of the trapezium as the different between the area of the large rectangle formed and the area of the triangle formed. Thus, the area of the trapezium equals to ‘b x t $-\frac{1}{2}$ (b − a) x t’. Beng simplified it algebraically to become ‘$\frac{1}{2}\ (a + b) \times t$’, as shown in Figure N34.

Summary

In summary, Beng could recall the formula for the area of a rectangle, parallelogram, triangle, and trapezium. Nevertheless, she was only able to develop the formulae for the area of a parallelogram and trapezium. Beng did not attempt to develop the formulae for the area of a rectangle and triangle.

Conceptual Knowledge

Beng could recall the formula for the area of a rectangle. Nevertheless, she was unable to develop the formula. It was apparent that Beng lack of conceptual knowledge underpinning the formula for the area of a rectangle.

Beng could recall the formula for the area of a parallelogram. She was able to develop the formula. Beng mentally transformed the parallelogram to a rectangle by cutting out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle. It indicated that she understands the relationship between the formula for the area of a parallelogram and rectangle. A parallelogram can always be transformed into a rectangle with the same base, same height, and the same area. Thus, the formula for the area of a parallelogram is exactly the same as the formula for the area of a rectangle, namely ‘base times height’.

Beng could recall the formula for the area of a triangle. Nevertheless, she was unable to develop the formula. Beng did not know the relationship between the area of a triangle and the area of the rectangle that encloses it. Had she been known of this relationship, Beng would know how to develop the formula for the area of a triangle.

Beng could recall the formula for the area of a trapezium. She was able to develop the formula. Beng developed the formula using algebraic method. She viewed the area of the trapezium as the different between the area of the large rectangle formed and the area of the triangle formed. Thus, the area of the trapezium equals to ‘b x t $-\frac{1}{2}$ (b − a) x t’. Beng simplified it algebraically to become '$\frac{1}{2}$ $(a + b) \times t$'. It indicated that Beng knew that the formula for the area of a trapezium is related to the formulae for the area of a rectangle and triangle.

Linguistic Knowledge

Beng used appropriate mathematical symbols to write the formula for the area of a rectangle, namely 'l x w', as shown in Figure N29. She also used appropriate mathematical terms 'length', 'times', and 'width' to state the formula for the area of a rectangle. Beng stated that "…the length times the width." (Beng/L1311).

Beng used appropriate mathematical symbols to write the formula for the area of a parallelogram, namely '$a \times b$', as shown in Figure N30. Nevertheless, Beng used inappropriate mathematical terms 'the length here' and 'the line here perpendicular to a' to explain the meaning of the symbols *a* and *b* that she employed. Beng explained that "a is the length here and *b* is the line here perpendicular to a." (Beng/L1332). Actually, *a* and *b* represents the base and the height of the parallelogram. Conventionally, the formula for the area of a parallelogram is written as 'b x h', where b and h represents the base and the height of the parallelogram.

Beng used appropriate mathematical symbols to write the formula for the area of a triangle, namely '$\frac{1}{2} \times b \times t$', as shown in Figure N32. She also used appropriate mathematical terms 'base' and 'height' to explain the meaning of the symbols that she employed. Beng explained that "*b* stands for the base. Then *t* stands for the height." (Beng/L1341).

Beng used appropriate mathematical symbols to write the formula for the area of a trapezium, namely '$\frac{1}{2}$ $(a + b) \times t$', as shown in Figure N33. Beng used inappropriate mathematical terms 'upper side' and 'lower one but parallel to the upper one', and appropriate mathematical term 'height' to explain the meaning of the symbols that she employed in the formula. Beng explained that ""a" stands for the upper side, "b" stands for lower one but parallel to the upper one and the "t" is the height." (Beng/L1367-1368).

Strategic Knowledge

Beng used the cut and paste strategy to develop the formula for the area of a parallelogram. She mentally cuts out a right-angled triangle from one end of the parallelogram and moves it to the other end of the parallelogram to form a rectangle, as shown in Figure N31.

Beng used algebraic method to develop the formula for the area of a trapezium, as shown in Figure N34. She drew dotted lines on Figure N33 to form a large rectangle and viewed the area of the trapezium as the different between the area of the large rectangle formed and the area of the triangle formed. Thus, the area of the trapezium equals to 'b x t $-\frac{1}{2}$ (b $-$ a) x t'. Beng simplified it algebraically to become '$\frac{1}{2}$ $(a + b) \times t$', as shown in Figure N34.

Ethical Knowledge

Beng could recall the formula for the area of a rectangle but she did not attempt to develop the formula, as shown in Excerpt B55. Beng had succeeded in developing the formula for the area of a parallelogram, as shown in Excerpt B56. Beng could recall the formula for the area of a triangle but she did not attempt to develop the formula, as shown in Excerpt B57. Beng also succeeded in developing the formula for the area of a trapezium, as shown in Excerpt B59.

Level of Subject Matter Knowledge

In this section, Beng' levels (low, medium, high) of subject matter knowledge of perimeter and area was analyzed in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

Beng achieved a medium level of conceptual knowledge of perimeter and area when she obtained 64.0% of appropriate mathematical elements of conceptual knowledge of perimeter and area during the clinical interview. Beng secured a high level of procedural knowledge of perimeter and area when she obtained 81.8% of appropriate mathematical elements of procedural knowledge of perimeter and area. Beng achieved a medium level of linguistic knowledge of perimeter and area when she obtained 69.8%% of appropriate mathematical elements of linguistic knowledge of perimeter and area. Beng secured a high level of strategic knowledge of perimeter and area when she obtained 85.7% of appropriate mathematical elements of strategic knowledge of perimeter and area. Beng achieved a medium level of ethical knowledge of perimeter and area when she obtained 61.2% of appropriate mathematical elements of ethical knowledge of perimeter and area. Beng achieved an overall medium level of subject matter knowledge of perimeter and area when she obtained 68.3% of appropriate mathematical elements of subject matter knowledge of perimeter and area (see Appendix M).

Liana

Liana lives in Sungai Nibong, Penang. Liana is 21 years 5 months old when she was interviewed. Currently, she is pursuing a 4-year Bachelor of Science with Education (B.Sc.Ed.) program at a public university. She majored and minored in chemistry and mathematics respectively. She obtained grade 1A in Mathematics and 3B in Additional Mathematics in her 2003 SPM examination (equivalent to O level examination). She scored B+ in Mathematics in the 2004 Matriculation examination (equivalent to A level examination). Liana performed moderately in her mathematics content courses at the university level when she secured two B, one B−, and one C− in four mathematics content courses she had completed during the first and second year of her studies. The detail of her performance is shown in Table N3.

Table N3

Mathematics Performance of Liana

Courses	Grades
1. Calculus for Science Students I	B
2. Algebra for Science Students	B−
3. Calculus for Science Students II	C−
4. Vector Calculus	B

At the time of data collection, Liana was in her second semester of third year studies. She attained 2.77 in the Cumulative Grade Point Average (CGPA) for her first two years of studies at the public university. She does not have any teaching experience prior to this interview.

Notion of Perimeter

Conceptual Knowledge

Liana has successfully selected all the shapes that have a perimeter, namely "A", "C", "D",”F”, "H", "I", “J”, and "K". Excerpt L1 shows her choice of shapes that have a perimeter (Liana/L75-78).

Excerpt L1

R: (Puts a handout comprises 12 shapes in front of Liana). Tick the shapes that have a perimeter.
S: (Ticks shapes "A", "C", "D",”F”, "H", "I", “J”, and "K", as shown in Figure N35).

In Excerpt L1, Liana has selected all simple closed curves (A, C, H, K) as well as all closed but not simple curves (D, I) that have a perimeter. She also selected the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that her notion of perimeter was not only limited to simple closed curves, and closed but not simple curves, but also inclusive of 3-dimensional shapes. Liana did not select the two simple but not closed curves (B, G) as well as the two 1-dimensional shapes (E, L) that do not have a perimeter. In other words, Liana did not select an open shape (including the lines) as having a perimeter.

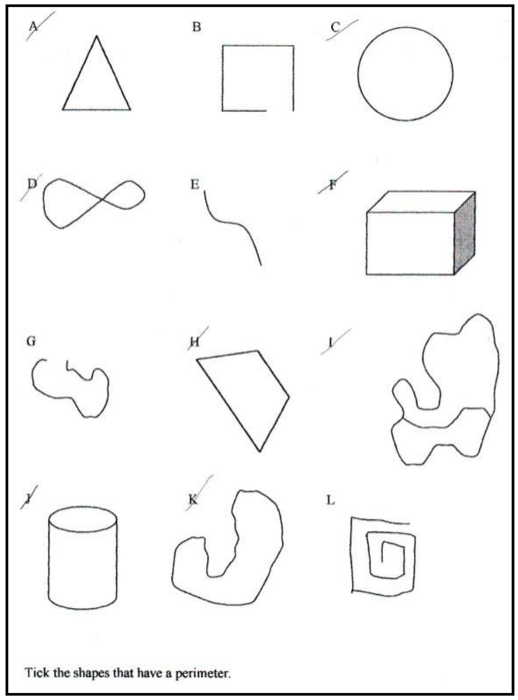


Figure N35. Liana's selection of shapes that have a perimeter.

Excerpt L2 depicts her justification of selecting each of these shapes (Liana/L84-104).

Excerpt L2

R: Why did you select shape "A"?
S: Because they are triangle (laughs) and it is closed. It made up by three straight lines.
R: Why did you select shape "C"?
S: "C" because it is a round shape and it is also a closed shape.
R: Why did you select shape "D"?
S: "D" because all the sides are join together. So, that is a complete shape.
R: Why did you select shape "F"?
S: Because it is a complete shape that can form a, something, all the lines are connected to each other.
R: Could you tell me more about it?
S: (Laughs) the lines are contact with each other. So, the square are making a cuboid.
R: Is there other reason why you chose shape "F"?
S: No.
R: Why did you select shape "H"?
S: Em well I think the answer for actually "H", "I", "J", "K" is the same because all lines are making a space that is lines join together. That means no free size thing and then all area making some shape whereby it can be measured.

In Excerpt L2, Liana explained that she selected shapes "A" and "C", because they are closed shapes. Liana explained that she selected shape "D" because all the sides are joined together. Liana explained that she selected shapes "F", "H", "I", "J", and "K" because all the lines are joined together.

Liana explained that she did not select shapes "B", "G", and "L" because the lines did not join together. Liana also explained that she did not select shape "E" because it is just a line. Excerpt L3 reveals her justification for not selecting shapes "B", "E", "G", and "L" as having a perimeter (Liana/L105-109).

Excerpt L3

R: Why didn't you select shape "B"?
S: "B", for "B", "G","L", I think "B", "G","L" is because the line is didn't join together and didn't form any shape.
R: Why didn't you select shape "E"?
S: "E" because it is only a line, that just only a line, is not anything.

Summary

In summary, Liana has selected all simple closed curves (A, C, H, K) and all closed but not simple curves (D, I) that have a perimeter. She also selected the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that her notion of perimeter was not only limited to simple closed curves, and closed but not simple curves, but also inclusive of 3-dimensional shapes. Liana justified her selection by explaining that she selected shapes "A" and "C", because they are closed shapes. Liana explained that she selected shape "D" because all the sides are joined together. Liana explained that she selected shapes “F”, "H", "I", "J", and "K" because all the lines are joined together.

Linguistic Knowledge

Liana used appropriate mathematical term ‘closed’ to justify her selection of shapes “A” and “C” that have a perimeter. Liana explained that she selected these shapes because they are closed. Nevertheless, she used inappropriate words ‘joined together’ to justify her selection of shapes “D”, “F”, "H", "I", "J", and "K" that have a perimeter. Liana explained that she selected shape “D” because all the sides are joined together. Liana also explained that she selected shapes “F”, "H", "I", "J", and "K" because all the lines are joined together, as shown in Excerpt L2.

Liana used inappropriate negation ‘not joined’ as her justification for not selecting shapes “B”, “G”, and “L” as having a perimeter. Liana explained that she did not select shapes “B”, “G”, and “L” because the lines did not join together, as shown in Excerpt L3. Nevertheless, Liana used appropriate mathematical term ‘line’ as her justification for not selecting shape “E” as having a perimeter. Liana explained that she did not select shape “E” because it is just a line, as shown in Excerpt L3.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Liana had taken the effort to justify the selection of shapes that have a perimeter, as shown in Excerpt L2. She provided appropriate justification for selecting shapes “A” and “C” that have a perimeter. Nevertheless, Liana provided inappropriate justification for selecting shapes “D”, ”F”, “H”, “I”, “J”, and “K” that have a perimeter.

Liana also had taken the effort to provide justification for not selecting other shapes that do not have a perimeter. She provided inappropriate justification for not selecting shapes “B”, “G”, and “L” as having a perimeter, as shown in Excerpt L3. Nevertheless, Liana provided appropriate justification for not selecting shape “E” as having a perimeter, as shown in Excerpt L3.

Notion of Area

Conceptual Knowledge

Liana has successfully selected all the shapes that have an area, namely "A", "C", "D",”F”, "H", "I", “J”, and "K". Excerpt L4 shows her choice of shapes that have an area (Liana/L130-133).

Excerpt L4

R: (Puts a handout comprises 12 shapes in front of Liana). Tick the shapes that have an area.
S: (Ticks shapes "A", "C", "D",”F”, "H", "I", “J”, and "K", as shown in Figure N36).

In Excerpt L4, Liana has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. She also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that Liana had a static perspective of the notion of area. Based on this perspective, area can be viewed as the amount of surface enclosed within a boundary. It also indicated that her notion of area was not only limited to 2-dimensional shapes (closed plane shapes), but also inclusive of 3-dimensional shapes. Liana also did not select the two open shapes (B, G) as well as the two 1-dimensional shapes (E, L) that do not have an area. In other words, Liana did not select an open shape (including the lines) as having an area. It can be inferred that she did not has a dynamic perspective of area or, at least, this knowledge was not accessible to her during the clinical interview.

When asked to justify her selection, Liana explained that she selected shapes "A", "C", "D",”F”, "H", "I", “J”, and "K" because the lines are joining. Thus, Liana elaborated that they form a shape where their area can be calculated using some specific formulae. Excerpt L5 depicts her justification of selecting each of these shapes (Liana/L139-142).

Excerpt L5

S: "A", "C", "D", "F", "H", "I", "J", "K".
R: Why did you select those shapes?
S: I think the answer for these shapes is the same because the lines are joining. And then it forming a shape that we can actually calculate the area by using some specific formula.

Liana explained that she did not select shapes “B”, “G”, and “L” because the lines are not joining together. Liana elaborated that they do not occupy any space and thus no area. Liana explained that she did not select shape “E” because it is just a line.

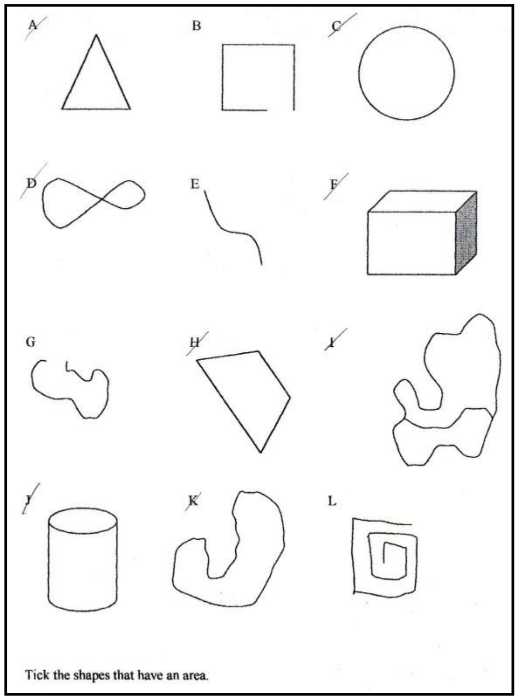


Figure N36. Liana's selection of shapes that have an area.

Excerpt L6 reveals her justification for not selecting shapes "B", "E", "G", and "L" as having an area (Liana/L143-148).

Excerpt L6

R: Why didn't you select shape "B"?
S: For "B", for "B", "G", "L", the lines are not joining together. So, it doesn't occupy any space, em like area. So, we can not calculate the area there.
R: Why didn't you select shape "E"?
S: For the same reason endorsed. It's only a line.

Summary

In summary, Liana has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. She also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that Liana had a static perspective of the notion of area. Her notion of area was not only limited to 2-dimensional shapes (closed plane shapes), but also inclusive of 3-dimensional shapes. Liana justified her selection by explaining that the lines of these shapes are joining.

Linguistic Knowledge

Liana used inappropriate word 'joining' to justify her selection of shapes that have an area. Liana explained that she selected shapes "A", "C", "D", "F", "H", "I", "J", and "K" because the lines of these shapes are joining, as shown in Excerpt L5.

Liana used inappropriate negation 'not joining' as her justification for not selecting shapes "B", "G", and "L" as having an area. Liana explained that she did not select shapes "B", "G", and "L" because the lines are not joining together, as shown in Excerpt L6. Liana used appropriate mathematical term 'line' as her justification for not selecting shape "E" as having an area. Liana explained that she did not select shape "E" because it is just a line, as shown in Excerpt L6.

Ethical Knowledge

Liana had taken the effort to justify the selection of shapes that have an area, as shown in Excerpt L5. She provided inappropriate justification for selecting shapes “A”, “C”, “D”, ”F”, “H”, “I”, “J”, and “K” that having an area.

Liana also had taken the effort to provide justification for not selecting other shapes that do not have an area. She provided inappropriate justification for not selecting shapes “B”, “G”, and “L” as having an area, as shown in Excerpt L6. Nevertheless, Liana provided appropriate justification for not selecting shape “E”, as shown in Excerpt L6.

Notion of the Units of Area

Conceptual Knowledge

Liana stated that the unit of area depends on the measurements that were made. She explained that if the measurement is in cm, then its area will be in cm^2. If the measurement is in m, then its area will be in m^2. Liana expressed that the unit of area does not depend on the shape used as the unit of area. Excerpt L7 is illustrative (Liana/L185-194).

Excerpt L7

R: (Puts a card written the following scenario in front of Liana). Ali, Chong, and David are discussing about the units of area. Ali says that we can use a square as the unit of area. Chong says that we can use a rectangle as the unit of area. David says that we can use a triangle as the unit of area. How would you respond to these students?

S: I will say that actually it is depends on the measurement that you made. If your measurement in cm, the area will be cm^2 (misreads cm^2 as cm square). And if it is in m, that will be m^2 (misreads m^2 as m square). It is not depends on the shape but it is really depends on the measurement actually.

When probed for the idea suggested by Ali, Liana initially stated that a square can be used as the unit of area measurement because it involved the measurement of the sides. Subsequently, she changed her mind and explained that a square cannot be used as the unit of area measurement because the measurement of the side that determine the unit of area. Liana elaborated that a square cannot be the unit of area but the length of that square with specific formula that can give the answer in the unit of area. Excerpt L8 is illustrative (Liana/L207-216).

Excerpt L8

R: Can we use a square as the unit of area?

S: Well, I think so because all the shape is going to give the measurement of the side. So, we can get the unit of the area.

R: Can we use a square as the unit of area?

S: No.

R: Why?

S: Because as I told just now is actually the measurement of the side that determine the unit of area. Square can not be the unit of area but the length of that square with specific formula that can give answer in the unit of area.

. When probed for the idea suggested by Chong, Liana stated that a rectangle cannot be used as the unit of area measurement Excerpt L9 is illustrative (Liana/L217-219).

Excerpt L9

R: Chong says that we can use a rectangle as the unit of area. How would you respond to Chong?

S: No.

When probed for the idea suggested by David, Liana stated that a triangle cannot be used as the unit of area measurement. The unit of area measurement depends on the measurement of the length of that object. Excerpt L10 is illustrative (Liana/L220-223).

Excerpt L10

R: David says that we can use a triangle as the unit of area. How would you respond to David?
S: No. The answer is the same. It depends on the measurement of the length of that object.

Summary

In summary, Liana thought that a square, rectangle and triangle cannot be used as the unit of area measurement. It indicated that Liana did not have any idea about the unit of area or the notion of the unit of area was not accessible to her during the clinical interview. She explained that a square, rectangle and triangle cannot be used as the unit of area measurement because the measurement of the side that determined the unit of area. It indicated that she was unable to provide the appropriate justification that any shape that tessellates a plane can be used as a unit of area measurement.

Linguistic knowledge

Liana used inappropriate mathematical term 'measurement of the side' to justify that a square, rectangle, and triangle cannot be used as the unit of area. She explained that a square, rectangle and triangle cannot be used as the unit of area measurement because the measurement of the side that determined the unit of area, as shown in Excerpts L7. L8, and L10.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Liana had taken the effort to justify the shapes that she thought cannot be used as a unit of area measurement. Nevertheless, Liana was unable to provide an appropriate justification for the shapes that she thought cannot be used as a unit of area measure. This can be seen in Excerpts L7, L8, and L10. In reality, any shape that tessellates a plane can be used as a unit of area measurement.

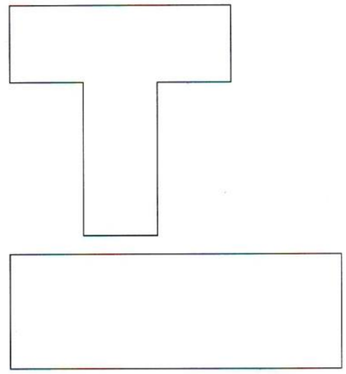
Comparing Perimeter (No Dimension Given)

Strategic Knowledge

Liana used the formal method of measuring the side and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter. Excerpt L11 shows the formal method that she used to compare the perimeter of the given pair of shapes (Liana/L256-273).

Excerpt L11

R: (Puts the following pair of shape in front of Liana). How would you find out whether they had the same perimeter?



S: Just measure the length of each side and then plus all together.
R: Could you show me how it is?
S: (Measures the length of each side of the T-shape by ruler and then calculates its perimeter, as shown in Figure N37). (Measures the length and width of the rectangle by ruler and then calculates its perimeter, as shown in Figure N38).
R: What do you get?
S: For the T-shape, I have, I get 24.2 cm. The perimeter for the rectangle here I have 24.
R: So, what's your conclusion?
S: Both shape doesn't have the same perimeter.

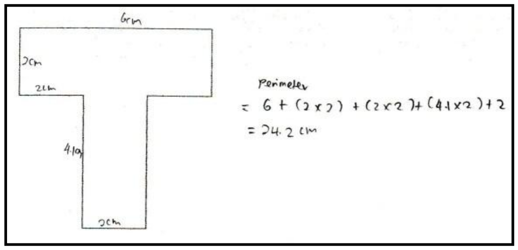


Figure N37. Liana measures the length of the top, the bottom, and the left sides of the T-shape by ruler and then calculates its perimeter.

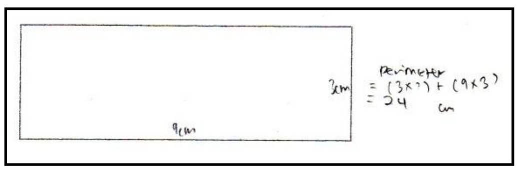


Figure N38. Liana measures the length and the width of the rectangle by ruler and then calculates its perimeter.

In Excerpt L11, Liana just measured the length of the top, the bottom, and the left sides of the T-shape by ruler. She doubled the length of the left sides of the T-shape and then plus the length of the top and the bottom parts of the T-shape to get its perimeter as 24.2 cm, as shown in Figure N37. Liana also just measured the length and the width of the rectangle by ruler. She doubled the length and the width of the rectangle to get its perimeter as 24 cm, as shown in Figure N38. Liana concluded that the given pair of shapes did not have the same perimeter. When probed for alternative method of comparing the perimeter, Liana was unable to provide other method of comparing the perimeter. Excerpt L12 is illustrative (Liana/L274-276).

Excerpt L12

R: Could you think of other way of finding out whether they had the same perimeter?
S: Yeah, I think that's all the method that I know.

Summary

In summary, Liana produced one method of determining whether the given pair of shapes had the same perimeter. In this formal method, she measured the length of sides by ruler and applied the definition of perimeter.

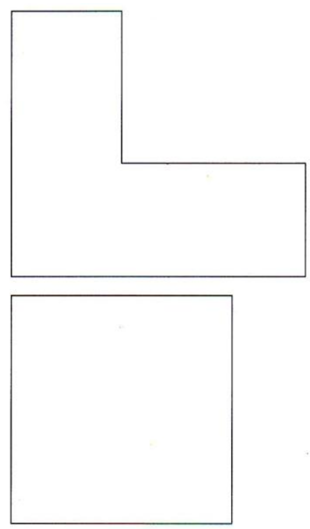
Comparing Area (No Dimension Given)

Strategic Knowledge

Liana partitioned L-shape into two rectangles for which area measurement formulae were known. Excerpt L13 shows the formal method of measuring the side and applying the area formula that she used to compare the area of the given pair of shape (Liana/L297-319).

Excerpt L13

R: (Puts the following pair of shape in front of Liana). How would you find out whether they had the same area?



S: By using the formula of an area.
R: Could you show me how it is?
S: (Partitions the L-shape into two rectangles, as shown in Figure N39. Measures the lengths and the widths of each rectangle using ruler and then calculates its area using formula of rectangle). (Measures the length of the two adjacent sides of the square using ruler and then calculates its area using formula of square).
R: Could you explain your solution?
S: The solution for L-shape I separate to area, to shape in two areas, where area A plus area B and the result is 627.34 cm^2 (Based on her measurement, it should be 37.1 cm^2, not 627.34 cm^2). And the shape of the square is actually using the formula "a times b" and then I get 36.6 cm^2 (misreads 36.6 cm^2 as 36.6 cm square).
R: What's your conclusion?
S: Conclusion is both shape doesn't have the same area.

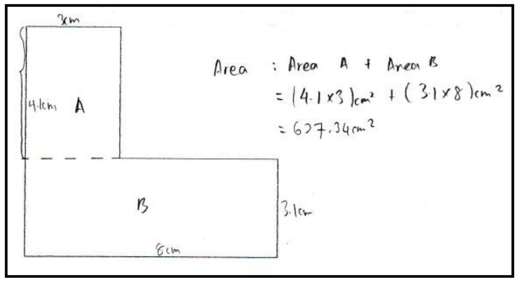


Figure N39. Liana measures the length and the width of each rectangle by ruler and then calculates its area.

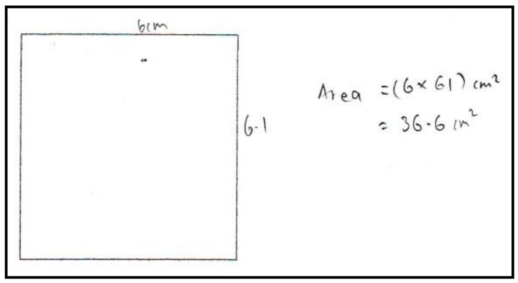


Figure N40. Liana measures the length of two adjacent sides of the square by ruler and then calculates its area.

In Excerpt L13, Liana partitioned L-shape into two rectangles, labelled as A and B respectively. She measured the length and the width of each rectangle by ruler and then calculated its area using rectangle area formulae as 627.34 cm^2 (Based on her measurement, it should be 37.1 cm^2, not 627.34 cm^2), as shown in Figure N39. Liana also measured the length of the two adjacent sides of the square by ruler and then calculated its area using square area formula as 36.6 cm^2, as shown in Figure N40. She concluded that they did not have the same area.

When probed for alternative method of comparing the area, Liana repartitioned L-shape into two rectangles for which area measurement formulae were known. Excerpt L14 depicts the formal method of measuring the side and applying the area formula that she used to compare the area of the given pair of shapes (Liana/L320-336).

Excerpt L14

R: Could you think of other way of finding out whether they had the same area?

S: For the shape of "L", I mean the method is the same. Just the way we separate is different. (Repartitions the L-shape into two rectangles, as shown in Figure N41. Measures the length and the width of each rectangle using ruler and then calculates its area using formula of rectangle). So, for this one (points to the given square) we get same as the previous method. So, the L-shape is divided into area A and area B. But this time, the separation line will be here. So, using the same formula (area of rectangle), and then the answer is 37.1.

R: Could you think of other way of finding out whether they had the same area?

S: No.

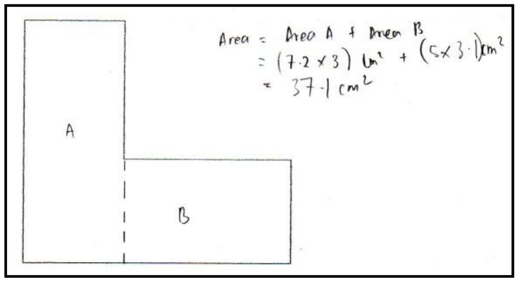


Figure N41. Liana repartitions the L-shape into two rectangles and then calculates its area.

In Excerpt L14, Liana repartitioned L-shape into two rectangles, labelled as A and B respectively. She measured the length and the width of each rectangle by ruler and then calculated its area using rectangle area formulae as 37.1 cm^2, as shown in Figure N41. For the square, Liana suggested to use the previous method, as shown in Figure N40.

Summary

In summary, Liana produced two similar formal methods of determining whether the given pair of shapes had the same area, namely measuring the length of side by ruler and applying area formulae. In the first method, Liana partitioned L-shape into two rectangles, as shown in Figure N39. In the second method, she repartitioned L-shape into two rectangles, as shown in Figure N41.

Comparing Perimeter (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Liana explained that she was unable to determine which shape has the longer perimeter. Excerpt L15 shows the justification that she made (Liana/L349-363).

Excerpt L15

R: (Puts the following table in front of Liana). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 1	25 paper clips	12 sticks

S: I couldn't tell.
R: Why?
S: Because paper clips can be various because some got a very big shape and then some have a very small shape. So, I can not determine exact length of all over, total of a shape, the perimeter of the shape. So, it goes for the, same goes for the sticks. So, I can not tell which one have the longer perimeter.
R: But in this case, 25 is larger than 12.
S: Yeah but it doesn't depends on the amount of the paper clips. So as the sticks. What perimeter is the measurement of the line of each object.

In Excerpt L15, Liana explained that she was unable to determine which shape has the longer perimeter as there were various sizes of paper clip and stick. It indicated that Liana focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. Liana knew that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters.

She explained that even though 25 is larger than 12 but it does not depend on the number of paper clips and sticks. Liana elaborated that the measurement (the length) of each paper clip and stick that matter.

In Set 2, Liana explained that two conclusions can be made in this case. Excerpt L16 depicts the justification that she made (Liana/L391-400).

Excerpt L16

R: (Puts the following table in front of Liana). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 2	10 paper clips	15 paper clips

S: Actually I got two conclusions here. If the shape of the paper clips for both shape A and shape B as the same, so for shape B will take the longer perimeter. But if the shape of the paper clips is varied, I mean, one with small and one with large shape, so it can not be determined unless you measured it by yourself.

In Excerpt L16, Liana explained that if the paper clips for both shapes A and B were of the same size, then shape B has the longer perimeter. Liana also explained that if the paper clips for shapes A and B were of the varied size, then she was unable to determine which shape has the longer perimeter. It indicated that she focused on the unit of measure when comparing perimeters in Set 2 with common nonstandard units. Liana knew that common nonstandard units (such as paper clips) are not reliable for comparing perimeters.

In another situation when shapes A and B had the same perimeter, Liana explained that the paper clips in shape A is longer than the paper clips in shape B. Excerpt L17 demonstrates her justification about their units of measurement (Liana/L401-412).

Excerpt L17

R: If shapes A and B had the same perimeter, what would you tell about their units of measure?
S: The paper clips might contain varied of shape.
R: Could you tell me more about it?
S: May be shape A will have the larger paper clips. Whereby the shape, for shape B, the paper clips will be a little bit small so that therefore (sic) they have the same perimeter.
R: Why?
S: Because the shape for B, it required 15 paper clips. So, the number of the paper clips is larger than the number paper clips required for shape A. So, I think the shape for B used small size of paper clips and then therefore they will have the same perimeter like I said just now.

In Excerpt L17, Liana explained that the paper clips in shape A is longer than the paper clips in shape B so that they had the same perimeter. She elaborated that the paper clip in shape B is shorter and thus it needed more paper clips (15) than shape A (10 paper clips) to get the same perimeter as shape A. It indicated that Liana understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length.

In Set 3, Liana stated that shape A has the longer perimeter. Excerpt L18 reveals her choice of shape that has the longer perimeter and the justification that she made (LianaL433-445).

Excerpt L18

R: (Puts the following table in front of Liana). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 3	16 cm	13 cm

S: Shape A. Yeah, shape A will have the longer perimeter.
R: Why?
S: Because 16 cm is longer than 13 cm. So, shape A will have the longer perimeter than shape B.
R: Why did you choose shape A?
S: Shape A because it is 16 cm. As 16 cm minus 13 cm, it will give the different is 3. So, it means that shape A is longer than shape B. If we take 13 cm minus 16 cm, we will have negative three.

In Excerpt L18, Liana explained that shape A has the longer perimeter because 16 cm is longer than 13 cm. She elaborated that 16 minus 13 equal to 3 whereas 13 minus 16 equal to negative 3. Thus, Liana concluded that shape A has the longer perimeter. It indicated that she focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Liana knew that common standard unit (such as cm) is reliable for comparing perimeters.

Summary

In summary, Liana focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. She knew that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. Liana focused on the unit of measure when comparing perimeters in Set 2 with common nonstandard units. She knew that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. Liana understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length. She focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Liana knew that common standard unit (such as cm) is reliable for comparing perimeters.

Comparing Area (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Liana explained that she was unable to determine which shape has the larger area. Excerpt L19 shows the justification that she made (Liana/L475-487).

Excerpt L19

R: (Puts the following table in front of Liana). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 1	25 triangles	12 squares

S: I can not tell because I don't know the length for the shape. So, I can not calculate the exact area for the shape. So, I can not determine which one is the larger.
R: In this case, 25 is larger than 12.

S: Yes, 25 is larger than 12 but in the square we have, there are two triangles there. It might be more than two triangles in a square. So, then the squares also might be a very big square. So, I can not simply say that shape A will have a larger area compared to shape B.

In Excerpt L19, Liana explained that she was unable to determine which shape has the larger area because she did not know the length of the shapes (triangle and square) and thus unable to calculate the exact area for the shapes (triangle and square). Liana elaborated that even though 25 is larger than 12 but the square might be 'a very big square'. It indicated that Liana focused on the unit of measure when comparing area in Set 1 with nonstandard units. She knew that nonstandard units (such as triangle and square) are not reliable for comparing areas.

In Set 2, Liana explained that she was unable to determine which shape has the larger area. Excerpt L20 depicts the justification that she made (Liana/L517-530).

Excerpt L20

R: (Puts the following table in front of Liana). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 2	10 squares	15 squares

S: Eh I can not simply say that shape B have the bigger area compared to shape A. It is because I must see the shape of the squares first to determine whether shape B is the larger or shape A is the larger because the shape of the square might be different. May be the shape, the square shape for A might be larger than the shape of the square for the B. So, if the square of shape A is larger than shape B, it might be the larger area, it might be the larger area compared to shape A and shape B. So, actually it depends on how the shape of the object looks like and how big it is.

In Excerpt L20, Liana explained that she was unable to determine which shape has the larger area because the squares in shapes A and B might be different (of area). She elaborated that a square from shape A might be larger than a square from shape B and vice versa. It indicated that she focused on the unit of measure when comparing areas in Set 2 with common nonstandard units. Liana knew that common nonstandard units (such as squares) are not reliable for comparing areas.

In another situation when shapes A and B had the same area, Liana explained that shape A has the larger squares. Excerpt L21 demonstrates her justification about their units of measurement (Liana/L542-549).

Excerpt L21

R: If shapes A and B had the same area, what would you tell about their units of measure?
S: Shape A will have the larger squares. Shape B will have smaller squares so that they have the same area.
R: Why?
S: Because shape B required 15 squares. The number of squares in shape B is larger than the number of squares in shape A. So, shape B used smaller size of squares so that they have the same area.

In Excerpt L21, Liana explained that shape A has the larger squares and shape B has the smaller squares so that they have the same area. She elaborated that the number of squares in shape B is larger than the number of squares in shape A and thus shape B used smaller size of squares so that they have the same area. It indicated that Liana understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area.

In Set 3, Liana stated that shape A has the larger area. Excerpt L22 reveals her choice of shape that has the larger area and the justification that she made (Liana/L559-571).

Excerpt L22

R: (Puts the following table in front of Liana). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 3	16 cm^2	13 cm^2

S: Shape A will have the larger area compared to B.
R: Why?
S: Because we know the measurement that for each unit is the same. So, 16 is larger than 13. So, shape A will have the larger area.
R: What do you mean by "the unit is the same"?
S: It is cm^2 (misreads cm^2 as cm square). So, the unit is the same. It means that they have do the measurement. So, 16 cm^2 is larger than 13 cm^2. So, of course shape A will have a larger area.

In Excerpt L22, Liana explained that shape A has the larger area because they used the same unit, namely cm^2 and 16 is larger than 13. It indicated that Liana focused on the number of unit when comparing areas in Set 3 with common standard unit. Liana knew that common standard unit (such as cm^2) is reliable for comparing areas.

Summary

In summary, Liana focused on the unit of measure when comparing areas in Set 1 with nonstandard units. She knew that nonstandard units (such as triangle and square) are not reliable for comparing areas. Liana focused on the unit of measure when comparing areas in Set 2 with common nonstandard units. She knew that common nonstandard units (such as squares) are not reliable for comparing areas. Liana understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area. She focused on the number of unit when comparing areas in Set 3 with common standard unit. Liana knew that common standard unit (such as cm^2) is reliable for comparing areas.

Linguistic Knowledge

Liana read 16 cm^2 and 13 cm^2 literally as '16 centimetre square' and '13 centimetre square' respectively, as shown in Excerpt L22. In another situation, Excerpt L23 exhibits how she wrote 16 cm^2 and 13 cm^2 in English words (Liana/L579-586).

Excerpt L23

R: (Puts a blank paper written the following measurements in front of Liana).
16 cm^2
13 cm^2
How would you write these measurements in English words?
S: (Writes the following).

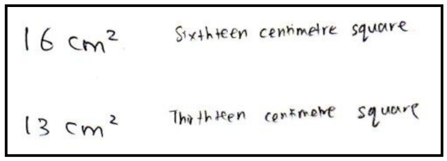


Figure N42. Liana writes 16 cm^2 and 13 cm^2 in English words.

In Excerpt L23, Liana wrote 16 cm^2 and 13 cm^2 literally as 'sixteen centimetre square' and 'thirthteen (sic) centimetre square', as shown in Figure N42. The correct answer should be 'sixteen square centimetres' and 'thirteen square centimetres'. It indicated that she did not know about the conventions pertaining to writing and reading of Standard International (SI) area measurement units.

Converting Standard Units of Area Measurement

Procedural Knowledge

Liana realized that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2. Excerpt L24 shows the algorithms that Liana used when she was converting 3 cm^2 to mm^2 (Liana/L593-628).

Excerpt L24

R: (Puts a card written the following scenario in front of Liana). Some Form One teachers noticed that several of their students seemed to multiply by 10, 100, and 1000, respectively when they were converting units of area from cm^2 to mm^2, m^2 to cm^2, and km^2 to m^2:

$3\ cm^2 = 3 \times 10\ mm^2 = 30\ mm^2$
$4.7\ m^2 = 4.7 \times 100\ cm^2 = 470\ cm^2$
$1.25\ km^2 = 1.25 \times 1000\ m^2 = 1250\ m^2$

What would you do if you were teaching Form One and you noticed that several of your students were doing this?

S: Well, I will explain to them. It is actually not the same as the just simply multiply 10, 100, and 1000.

R: Perhaps you may start with the first question?

S: Em (converts 3 cm^2 to mm^2, as shown in Figure N43). So, the answer will be 300 mm^2 (misreads 300 mm^2 as 300 mm square) for the first one.

R: Could you explain your solution?

S: (Draws a diagram to illustrate the conversion from cm^2 to mm^2, as shown in Figure N44). Em the first part is if we look at this picture here. This is "a", this is "b" right. So, a cm is change, b cm is change. So, if we change to the millimetre unit, both going to change to the millimetre unit. So, we will have a mm times b mm. So, it is actually twice. It is not just simply times 10. It is actually twice greater. So, we will get the power square here. It is twice, I mean power of two. So, this is what my teacher taught me when my previous school. He said that if you see here is two. So, when times something here, here also have to have the power of two and you will have the correct answer. Em so, the reason is because not only one side is changing. Its two side is changing.

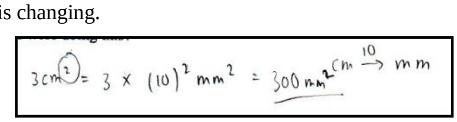


Figure N43. Liana converts 3 cm^2 to mm^2.

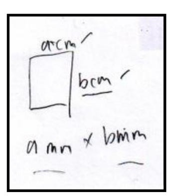


Figure N44. Liana draws a diagram to illustrate the conversion from cm^2 to mm^2.

In Excerpt L24, Liana has successfully converted 3 cm^2 to mm^2. She viewed 3 cm^2 as the product of 3 times 1 cm^2. Liana knew the relationship between the standard units of length measurement that 1 cm = 10 mm. She also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Liana times ten squared, $(10)^2$, when she converted 3 cm^2 to mm^2, as shown in Figure N43. Liana drew a diagram to illustrate the conversion from cm^2 to mm^2, as shown in Figure N44. She elaborated that when we convert cm^2 to mm^2, we have to convert cm to mm twice and we will get the 'power of two'. Liana recalled the hints that her teacher taught her in school. Her teacher reminded her to take the power of two, namely $(10)^2$, whenever she saw a '2' in the unit of cm^2 when converting cm^2 to mm^2 in order to get the correct answer. Liana emphasized that the conversion involved not only one side of the diagram but both sides, as shown in Figure N44.

Liana found that the students made a mistake when they were converting unit of area from 4.7 m^2 to cm^2. Excerpt L25 depicts the algorithms that Liana used when she was converting 4.7 m^2 to cm^2 (Liana/L629-641).

Excerpt L25

R: What about the second question?

S: (Converts 4.7 m^2 to cm^2, as shown in Figure N44). Em the method here actually the same. The reason is also the same because the changes have occurred on the both sides. So, then the power of two here. So, we times something, we have to make it the same, I mean, hundred power of two. Here is 4.7 m^2 (misreads 4.7 m^2 as 4.7 m square). If it is 4.7 m^3 (misreads 4.7 m^3 as 4.7 m cube), then we multiply by hundred to the power of three, it is not the power of two. So, the problem here is actually depends on the power of the unit here.

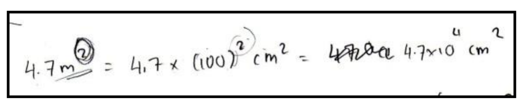


Figure N45. Liana converts 4.7 m^2 to cm^2.

In Excerpt L25, Liana has successfully converted 4.7 m^2 to cm^2. She wrote the answer in the standard form, namely 4.7 x 10^4 cm^2. Liana explained that the method of conversion and the reasoning behind it is the same as the previous question where she converted 3 cm^2 to mm^2. She elaborated that the conversion involved both sides, namely a and b, as shown in Figure N44. Liana viewed 4.7 m^2 as the product of 4.7 times 1 m^2. She knew the relationship between the standard units of length measurement that 1 m = 100 cm. Liana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Liana times one hundred squared, namely $(100)^2$, when she converted 4.7 m^2 to cm^2, as shown in Figure N45.

Liana also found that the students made a mistake when they were converting unit of area from 1.25 km^2 to m^2. Excerpt L26 demonstrates the algorithms that Liana used when she was converting 1.25 km^2 to m^2 (Liana/L642-652).

Excerpt L26

R: What about the third question?

S: (Converts 1.25 km^2 to m^2, as shown in Figure N46). Well, the answer wouldn't be much different. It is 1.25 km^2 (misreads 1.25 km^2 as 1.25 km square). So, we have to multiply by thousand power of square to get the correct answer when we changing unit here, km^2 to m^2.

R: What do you get?

S: 1.25 x 10^6 m^2.

$$1.25\ km^2 = 1.25 \times (1000)^2\ m^2 = 1.25 \times 10^6\ m^2$$

Figure N46. Liana converts 1.25 km^2 to m^2.

In Excerpt L26, Liana has successfully converted 1.25 km^2 to m^2. She wrote the answer in the standard form, namely 1.25 x 10^6 m^2. Liana explained that the solution would not much be different (from the other two questions where she converted 3 cm^2 to mm^2 and 4.7 m^2 to cm^2). Liana viewed 1.25 km^2 as the product of 1.25 times 1 km^2. She knew the relationship between the standard units of length measurement that 1 km = 1000 m. Liana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Liana times one thousand squared, namely $(1000)^2$, in order to get the correct answer when she converted 1.25 km^2 to m^2, as shown in Figure N46.

Summary

In summary, Liana realized that the students made mistakes when they were converting 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2 respectively. She knew the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. Liana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. She viewed 3 cm^2 as the product of 3 times 1 cm^2, 4.7 m^2 as the product of 4.7 times 1 m^2, and 1.25 km^2 as the product of 1.25 times 1 km^2. Thus, Liana times $(10)^2$, $(100)^2$, and $(1000)^2$ respectively when she converted 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2, as shown in Figures N43, N45, and N46.

Conceptual Knowledge

Liana knew the relationships between the standard units of length measurement such as 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. These can be seen in Figures N43, N45, and N46. She also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring, as shown in Figures N43, N45, and N46.

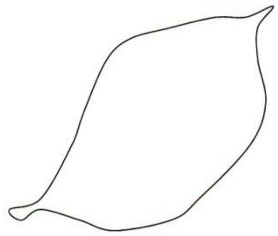
Relationship between Perimeter and Area
(Same Perimeter, Same Area?)

Conceptual Knowledge

Liana did not know that there is no direct relationship between perimeter and area. She did not know that two shapes with the same perimeter can have different areas. Thus, Liana was not sure whether the student's method of calculating the area of the leaf was correct or not. Excerpt L27 shows Liana's responses to the Form One student (Liana/L662-693).

Excerpt L27

R: (Puts a card written the following scenario in front of Liana). This is a picture of a leaf. A Form One student said that he had found a way to calculate the area of the leaf. The student placed a thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle.



How would you respond to this student?

S: Well, we are, myself does not sure whether those method will be apply to determine the area of the leaf. So, what I will answer, my answer for the student would be: I say, well, I will discuss first with some of my friends that expertise in science to find whether your, well, way of solution can be used to determine the area of the leaf.

R: Do you think the student's method works?

S: Well I can not simply say it is works or not before having my discussion with others because me and my self does not sure about this kind of method.

In Excerpt L27, Liana stated that she was not sure whether the method claimed by the student can be used to determine the area of the leaf. Thus, Liana expressed that she need to seek her friends' expertise in science to find out whether the method claimed by the student can be used to determine the area of the leaf. Liana explained that she could not simply say the student's method works or not as she was not sure about the correctness of the method.

Summary

In summary, Liana did not know that there is no direct relationship between perimeter and area. She did not know that two shapes with the same perimeter can have different areas. Thus, Liana was not sure whether the student's method of calculating the area of the leaf was correct or not.

Ethical Knowledge

In Task 5.1, Liana was not sure whether the student's method of calculating the area of the leaf was correct or not, as shown in Excerpt L27. Thus, Liana said that she needed to verify it. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter has the same area. Liana did not attempt to examine the possible pattern of the relationship between perimeter and area.

Liana expressed that she need to seek her friends' expertise in science to find out whether the method claimed by the student can be used to determine the area of the leaf. It indicated that Liana relied on authority, namely other people's view, to verify the correctness of the student's method of calculating the area of the leaf. She did not attempt to formulate generalization pertaining to the relationship between perimeter and area. Liana never tests the student's generalization that two shapes with the same perimeter have the same area.

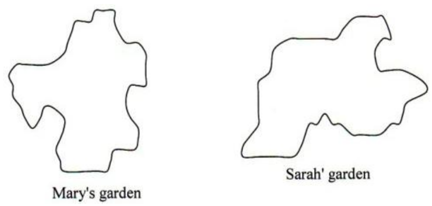
Relationship between Perimeter and Area
(Longer Perimeter, Larger Area?)

Conceptual Knowledge

Liana did not know that there is no direct relationship between perimeter and area. She did not know that the garden with the longer perimeter could have a smaller area. Thus, Liana was not sure whether Mary's claim that the garden with the longer perimeter has the larger area was correct or not. Excerpt L28 shows Liana's responses to the claim made by Mary that the garden with the longer perimeter has the larger area (Liana/L708-735, 754-756).

Excerpt L28

R: (Puts a card written the following scenario in front of Liana). Mary and Sarah are discussing whose garden has the larger area to plant flowers. Mary claims that all they have to do is walk around the two gardens to get the perimeter and the one with the longer perimeter has the larger area. How would you respond to these students?



S: Em…(silent for a while) well, to this student, I will say that you can not simply determine the area of your garden by simply measuring the perimeter of your garden.
R: Why?
S: Because the area and perimeter are two different things. So, perimeter is may be a part of measurement of the area but can not simply say that the longer is the perimeter, the larger will be the area. So, this is a very wrong concept eh determine the area. Eh may be we will use another method in calculating the area of your garden. Only by then we can determine which one is the larger.
.
.
.
R: Do you think the student's method works?
S: We can not simply say it is works or not. I have to do some research first.

In Excerpt L28, Liana stated that the area of a garden could not be determined by simply measuring the perimeter of the garden as area and perimeter were two different things (concepts). She emphasized that one cannot simply say that the longer the perimeter, the larger the area will be. Liana expressed that this was a wrong "concept" of determining the area (of the gardens). She suggested using other method to calculate the area of the gardens and only then can the larger area (of the garden) be determined. Nevertheless, Liana was not sure whether Mary's claim that the garden with the longer perimeter has the larger area was correct or not. She stated that one cannot simply say whether the method works or not. Liana expressed that she has to do some research (to verify it).

Summary

In summary, Liana did not know that there is no direct relationship between perimeter and area. She did not know that the garden with the longer perimeter could have a smaller area. Thus, Liana was not sure whether Mary's claim that the garden with the longer perimeter has the larger area was correct or not.

Ethical Knowledge

In Task 5.2, Liana was not sure whether Mary's claim that the garden with the longer perimeter has the larger area was correct or not. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. She stated that one cannot simply say whether the method works or not. Liana expressed that she has to do some research (to verify it). Liana did not attempt to examine the possible pattern of the relationship between perimeter and area. She did not formulate generalization pertaining to the relationship between perimeter and area. Liana did not test Mary's generalization that the garden with the longer perimeter has the larger area. She stated that one cannot simply say whether the method works or not. Liana expressed that she has to do some research (to verify it).

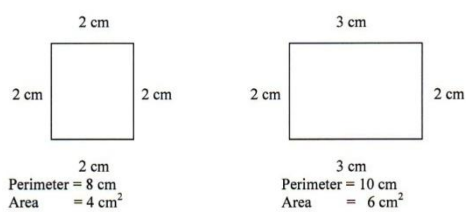
Relationship between Perimeter and Area
(Perimeter Increases, Area Increases?)

Conceptual Knowledge

Liana did not know that there is no direct relationship between perimeter and area. She did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Liana thought that the student's "theory" was correct. This is shown in Excerpt L29 (Liana/L790-828).

Excerpt L29

R: (Puts a card written the following scenario in front of Liana). Suppose that one of your Form One students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:



How would you respond to this student?

S: I will say to the student, my answer would be, usually is correct because she has prove here the figure, the measurement as well and by doing this calculation, actually clear that area A (refers to the square on the left hand side as shown above) and B (refers to the rectangle on the right hand side as shown above) have different area whereby the area B is higher than area A. And then her theory also correct. I will bet, as the perimeter increases, the area also increases. But it only apply for this kind of shape. …

In Excerpt L29, Liana thought that the student's "theory" was correct. She agreed with the student that as the perimeter of a closed figure increases, the area also increases. Liana explained that the "theory" was correct because the student has proven it with the picture together with the measurement of perimeter and area that clearly showed that as the perimeter of the figure increases (from 8 cm to 10 cm), the area also increases (from 4 cm^2 to 6 cm^2). She reiterated that the student's "theory was correct. Liana even bet that as the perimeter increases, the area also increases but it only applied for this kind of shape (rectangle).

When probed for other shapes, Liana stated that she was not sure whether the "theory" applied to a circle as well. Excerpt L30 is illustrative (Liana/L831-855).

Excerpt L30

R: Do it works for other shapes as well?

S: Well for circle, I have to check it first because I forgot the formula of the perimeter for the circle. If it is also contain the value of radius, so it also means the perimeter will take impact of the area also.

R: Do it works for other shapes?

S: Well, I think it also can be applied to other shapes.

R: Could you show me how it is?

S: Well, let's say for triangle (draws two triangles and then calculates its area respectively, as shown in Figure N47). Eh wait, wait, I just simply write roughly (the value of the base and height)….(silent for a while) well, if we will get, meaning here, this is short and this is long right. So, roughly we can say that this (points to the smaller triangle on the left hand side) will have smaller perimeter than this one (points to the larger triangle on the right hand side). The area would be …(calculating the respective area). So, whereby calculation, we can see the area for A (points to the smaller triangle) is smaller than the area for B here (points to the larger triangle). So, the perimeter for A also smaller than the perimeter for B. So, I see the theory just that the student said is correct.

R: So, what's your conclusion?

S: Em but I can not say it's all because since I'm not sure about the circle's formula for perimeter. So, she might be correct and might be not. May be after I do some research first and then I can give the answer.

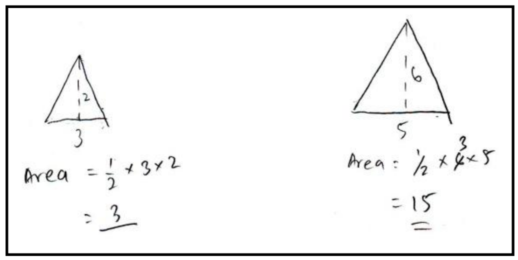


Figure N47. Liana draws two triangles and then calculates its area respectively.

In Excerpt L30, Liana stated that she was not sure whether the "theory" applied to a circle as well because she was unable to recall the formula for circumference (perimeter of a circle). When probed further for other shapes, Liana thought that the "theory" could also be applied to other shapes. She drew two triangles, roughly assigned values to the base and the height of the triangles, and then calculated its area as 3 and 15 respectively, as shown in Figure N47. Liana stated that it could be roughly said that the triangle on the left hand side of the Figure N47 has smaller (shorter) perimeter than the triangle on the right hand side. She explained that the triangle on the left hand side of the Figure N47 has smaller area than the triangle on the right hand side. Thus, Liana stated that the student's "theory" was correct. Nevertheless, Liana explained that she could not say that the "theory" could be applied to all (shapes) as she was not sure about the formula for circumference. Liana expressed that the student might be correct and might not be correct. Thus, Liana said that she needed to do some research to verify it.

The researcher wrote the formula for the circumference of the circle, $2\pi r$, and then showed it to Liana as she was unable to recall it. Excerpt L31 is illustrative (Liana/L862-872).

Excerpt L31

R: (Writes the formula for the circumference of the circle, $2\pi r$, and then shows it to Liana as she can't remember it).

S: $2\pi r$. Em (draws two circles and then writes the formula for the circumference and area respectively, as shown in Figure N48). Well, I think this is the formula. If I mentioned correct about the circle, I think the student's theory is correct.

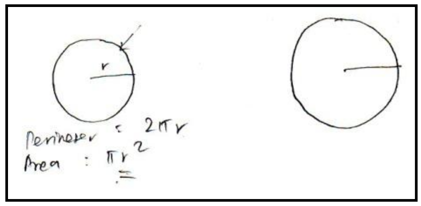


Figure N48. Liana draws two circles and then writes the formula for the circumference and area respectively.

In Excerpt L31, Liana drew two circles and then wrote the formulae for the circumference and area respectively, as shown in Figure N48. She concluded that the student's "theory" was correct. Liana did not know that the student's claim about the relationship between perimeter and area is not a theory. The claim is a conjecture. She also did not know that an example is not a proof and a theory cannot be proved by an example.

Summary

In summary, Liana did not know that there is no direct relationship between perimeter and area. She did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Liana thought that the student's "theory" was correct.

Ethical Knowledge

In Task 5.3, the student formulated a generalization that as the perimeter of a closed figure increases, the area also increases. Liana thought that the student's "theory" was correct. She generated two examples to test the student's generalization, as shown in Excerpts L30 and L31. The results of her examples concurred with the student's "theory" that as the perimeter of a closed figure increases, the area also increases. In reality, when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same.

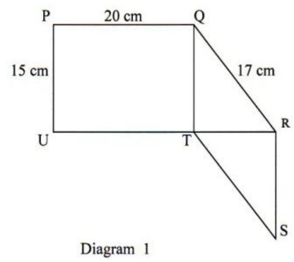
Calculating Perimeter and Area (Rectangle and Parallelogram/Triangle)

Procedural knowledge

After read through Task 6.1, Liana labelled the missing sides of Diagram 1 that required for calculating the perimeter and area of the diagram, as shown in Figure N49 in Excerpt L32 (Liana/L903-932).

Excerpt L32

R: (Puts a card written the following problem in front of Liana). Suppose that one of your Form One students asks you for help with the following problem:



In Diagram 1, PQTU is a rectangle and QRST is a parallelogram. UTR is a straight line.
Calculate
(c) the perimeter of the diagram,
(d) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 1, as shown in Figure N49).

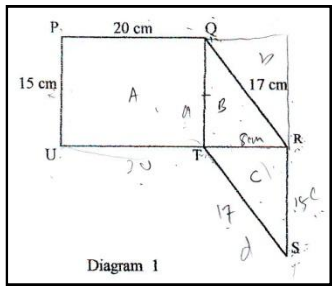
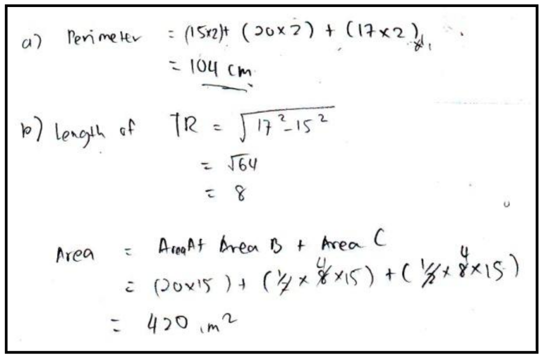


Figure N49. Liana labels the missing sides of Diagram 1.

In Excerpt L32, Liana labelled RS, ST, and TU as 15, 17, and 20 respectively on Diagram 1, as shown in Figure N49. After Liana labelled the missing sides of Diagram 1 that required for calculating the perimeter and area of the diagram, she started to calculate the perimeter and the area of Diagram 1. Excerpt L33 depicts how Liana has successfully calculated the perimeter and the area of Diagram 1 (Liana/L934-940, L981-988, L1001-1005).

Excerpt L33

S: (Calculates the perimeter and area of Diagram 1, as shown in Figure N50).
R: Could you explain your solution?
S: Ok for perimeter, we just plus lengths of PQ, QR, RS, ST, TU, and UP. That's will result the perimeter.
.
.
.
R: Could you explain your solution for the perimeter?
S: For the perimeter, just now I said just take the outer line of the diagram and then just plus all together and you'll get 104 cm.
R: Why didn't you include your QT and TR for the perimeter?
S: Perimeter is the length of the outer shape. So, QT and TR, the lines lies inside those shape. So, it doesn't count as a perimeter.
R: What do you get for the perimeter?
S: 104 cm.
.
.
.
R: Could you explain your solution for the area?
S: Solution for this area, I separate it two shapes and two parts, A, B, C. I'll calculate the area for each part and then plus all them together.
R: What do you get?
S: 420.



a) Perimeter $= (15\times2) + (20\times2) + (17\times2)$

$= 104$ cm

b) length of TR $= \sqrt{17^2 - 15^2}$

$= \sqrt{64}$

$= 8$

Area = Area A + Area B + Area C

$= (20\times15) + (\frac{1}{\cancel{2}} \times \overset{4}{\cancel{8}} \times 15) + (\frac{1}{\cancel{2}} \times \overset{4}{\cancel{8}} \times 15)$

$= 420$ cm^2

Figure N50. Liana calculates the perimeter and the area of Diagram 1.

In Excerpt L33, Liana used the doubling-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N50. She doubled the length of sides UP, PQ, and QR and then summed them up to get the perimeter of the diagram as 104 cm. Liana explained that the perimeter of Diagram 1 was just the sum of the lengths PQ, QR, RS, ST, TU, and UP. She defined perimeter of a shape as the length of the outer line of the shape. Thus, Liana explained that she did not include QT and TR as they were the lines lied inside Diagram 1.

Liana realized that she needed to find the length of TR before she could calculate the area of Diagram 1. Liana had successfully using the Pythagoras' theorem to find the length of TR as 8 cm, as shown in Figure N50. Liana used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N50. She partitioned Diagram 1 into a rectangle PQTU (labelled as A) and two triangles QRT (labelled as B) and RST (labelled as C). Liana calculated the area of A, B, and C using the area formulae of rectangle and triangles respectively and then summed them up to get the area of the diagram as 420 cm^2.

Summary

In summary, Liana has successfully calculated the perimeter of Diagram 1 by using the doubling-and-sum algorithm. She has also correctly calculated the area of Diagram 1 by using the partition-and-sum algorithm to calculate the area of the diagram.

Linguistic knowledge

Liana used the correct standard units of measurement for perimeter (cm) and area (cm^2) when she wrote the answers for these measurements, as shown in Figures N50 and N51.

Strategic Knowledge

When probed to check the answer for the perimeter, Liana suggested that she would use an alternative method to verify the answer. Excerpt L34 is illustrative (Liana/L989-992).

Excerpt L34

R: How would you check your answer for the perimeter?
S: Well, this calculation here, I put it like this: 15 times 2, 20 times 2, and 17 times 2. And the other round, I can just 15 plus 20 plus 17 plus 15 plus 17 plus 20. I have the perimeter.

In Excerpt L34, Liana suggested that she would check the answer for perimeter by using an alternative method, namely list all-and-sum strategy. Liana explained that she could just "15 plus 20 plus 17 plus 15 plus 17 plus 20" (Liana/L991-992) to check the answer for the perimeter.

When probed to check the answer for the area, Liana used an alternative procedure (alternative method) to generate an answer which could be used to verify her original answer. Excerpt L35 is illustrative (Liana/L1006-1026).

Excerpt L35

R: How would you check your answer for the area?
S: Actually there is a way where we calculating area in this shape (points to the parallelogram QRST). Em I can just simply translate (it should be transform, not translate) this area, this shape here (points to triangle TRS) to here. (Moves triangle RST under the translation T_{SR} to form a rectangle with the dimensions of 15 cm by 8 cm, as shown in Figure N49). It is the same.
R: Could you show me how it is?
S: (Draws the large transformed rectangle, labels its vertices and dimensions and then calculates its area, as shown in Figure N51). So, S here I transfer to here. The length is, here is 8 (points to the transformed QS, as shown in Figure N51) and this is 15, 20. So, actually it is just can be simplified which is A and B (partitions the large transformed rectangle into two smaller rectangles and labels them as A and B respectively as shown in Figure N51) that the total area would be area A plus area B.
R: What do you get?
S: 420.

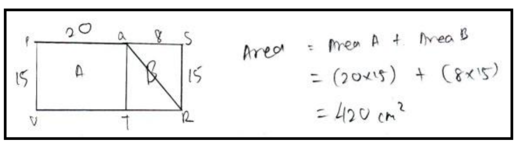


Figure N51. Liana uses alternative method to calculate the area of Diagram 1.

In Excerpt L35, Liana checked the answer for area by moving triangle RST under the translation T_{SR} to form a rectangle with the dimensions of 15 cm by 8 cm, as shown in Figures N49 and N51. She drew a large rectangle, as shown in Figure N51. Liana partitioned the large transformed rectangle into two smaller rectangles and labels them as A and B respectively, as shown in Figure N51. She calculated its area by using area formula of rectangle and summed up the area as 420 cm^2.

Ethical Knowledge

Liana has successfully calculated the perimeter and area of Diagram 1. Nevertheless, she did not check the correctness of the answers for perimeter as well as area. When probed to check answers, then only Liana suggested the strategies that she would use to check the answers for perimeter and area. Liana wrote the measurement units (without probed) for the answers of the perimeter and area that she has calculated, as shown in Figures N50 and N51.

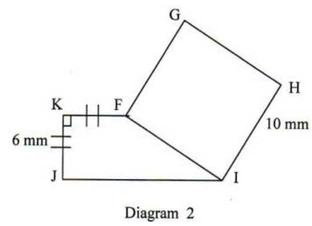
Calculating Perimeter and Area
(Square and Trapezium/Triangle)

Procedural Knowledge

After read through Task 6.2, Liana labelled the missing sides of Diagram 2 that required for calculating the perimeter and area of the diagram, as shown in Figure N52 in Excerpt L36 (Liana/L1039-1080).

Excerpt L36

R: (Puts a card written the following problem in front of Liana). Suppose that one of your Form One students asks you for help with the following problem:



In Diagram 2, FGHI is a square and FIJK is a trapezium.
Calculate
(c) the perimeter of the diagram,
(d) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 2, as shown in Figure N52). First, I will ask them what is the properties of a square. They might say that all those sides will have the equal length. So, I just put symbol like this ┼ (to indicate equal length) and then here, it is also said that this actually have this symbol ╫ means that it has the same length. So, this will be 6 mm here (points to KF)…(silent for a while) so, if we divide here (draws dotted line along FM), here will also get 6 mm (points to JM). The only problem here is, what is the length here (points to MI, also labels as "a"). So, I can, I just simply ask them: What is the formula for hypotenuse? And then here is 10 mm (points to FI), here 6 mm (points to FM). So, the formula for hypotenuse would be (calculates the length of MI, also labels as "a", as shown in Figure N53). "a", the length here is 8 mm.

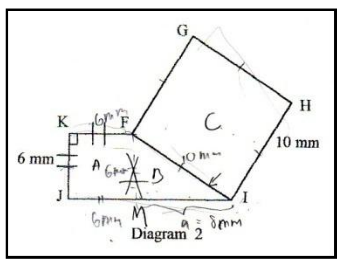


Figure N52. Liana labels the missing sides of Diagram 2.

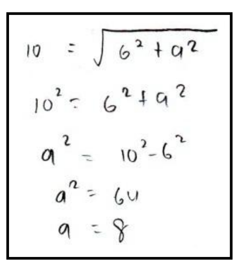


Figure N53. Liana calculates the length of MI, also labels as "a".

In Excerpt L36, Liana labelled KF, FM, JM, and FI as 6 mm, 6 mm, 6 mm, and 10 mm respectively on Diagram 2. Liana realized that she needed to find the length of MI (also labels as "a"). Liana partitioned trapezium FIJK into a square and a triangle, as shown in Figure N52. She has successfully calculated the length of MI as 8 mm using Pythagoras' theorem, as shown in Figure N53. Excerpt L37 depicts how Liana has successfully calculated the perimeter of Diagram 2 (Liana/L1080-1088).

Excerpt L37

S: I will ask them, to measure perimeter, so they might say plus all those length of sides but I'll ask them whether this side (points to FI) is included. Some may get the right answer, some may not and then I will just simply say that the perimeter is the, all total length of the side, outer of the shape. If the lines lies in inner side of shape, it is doesn't count. So, the perimeter would be (calculates the perimeter of Diagram 2, as shown in Figure N54).

a) perimeter

$= (6 \times 3) + 8 + (10 \times 3)$

$= 56$ mm

Figure N54. Liana calculates the perimeter of Diagram 2.

In Excerpt L37, Liana defined perimeter of a shape as the total length of outer sides of the shape. She explained that it excluded the length of the inner side (such as FI of Diagram 2) of the shape. Liana used the tripling-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N54. She tripled the length of sides JK and HI, and plus the length of MI. Liana summed them up to get the perimeter of the diagram as 56 mm.

Excerpt L38 demonstrates how Liana has successfully calculated the area of Diagram 2 (Liana/L1090-1102).

Excerpt L38

S: Em the area would (calculates the area of Diagram 2, as shown in Figure N55). This is one of the ways. There is another way whereby we can join this together become A here, A plus B. (Suggests alternative method to calculate the area of Diagram 2, as shown in Figure N56). That's also can be calculated to determine the area but I forgot the formula for trapezium.

b) Area $= \text{Area}(A + B + C)$

$= (6 \times 6) + (\cancel{\tfrac{1}{2}} \times \overset{4}{\cancel{8}} \times 6) + (10 \times 10)$

$= 160 \text{ mm}^2$

Figure N55. Liana calculates the area of Diagram 2.

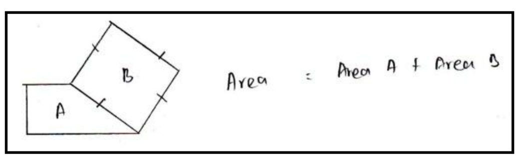


Figure N56. Liana suggests alternative method to calculate the area of Diagram 2.

In Excerpt L38, Liana used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N55. She partitioned Diagram 2 into square FMJK (labelled as A), triangle FIM (labelled as B), and square FGHI (labelled as C), as shown in Figure N52. Liana calculated the area of A, B, and C separately using the area formulae of square, triangle, and square respectively and then summed them up to get the area of the diagram as 160 mm^2, as shown in Figure N55. Liana also suggested an alternative method to calculate the area of Diagram 2, as shown in Figure N56. In the alternative method, she suggested to repartition Diagram 2 into trapezium FIJK and square FGHI. Nevertheless, Liana stated that she was unable to recall the formula for the area of a trapezium.

Summary

In summary, Liana has successfully calculated the perimeter of Diagram 2 using the tripling-and-sum algorithm. She has also correctly calculated the area of Diagram 2 using the partition-and-sum algorithm.

Linguistic Knowledge

Liana used the correct standard units of measurement for perimeter (mm) and area (mm^2) when she wrote the answer of these measurements, as shown in Figures N54 and N55.

Strategy Knowledge

When probed to check the answer for the perimeter, Liana suggested that she would use alternative method, namely list all-and-sum strategy, to verify the answer. Excerpt L39 is illustrative (Liana/L1108-1111).

Excerpt L39

R: How would you check the answer for the perimeter?
S: For here, I simply multiply 6 by 3, and 10 by 3 because it has 3 sides here. But if when you want to check, we can use another way by simply plus all them together.

In Excerpt L39, Liana suggested that she would check the answer for perimeter by using an alternative method, namely list all-and-sum strategy.

Liana suggested that she would use an alternative method to verify the answer for the area. Excerpt L40 is illustrative (Liana/L1116-1122).

Excerpt L40

R: How would you check the answer for the area?
S: I can use the area of trapezium, trapezium shape here plus the area here (points to the square FGHI). If they have the same answer, then my answer is correct.
R: Could you show me how it is?
S: Eh I doesn't remember this formula (points to the trapezium FIJK).

In Excerpt L40, Liana suggested that she would use the repartition-and-sum strategy to check the answer for the area of Diagram 2. Liana suggested that she would check the answer for the area by repartitioned Diagram 2 into trapezium FIJK and square FGHI, as shown in Figure N56. Nevertheless, Liana stated that she was unable to recall the formula for the area of a trapezium.

Ethical Knowledge

Liana has successfully calculated the perimeter and area of Diagram 2. Nevertheless, she did not check the correctness of the answer for perimeter. When probed to check answer, then only Liana suggested the strategy that she would use to check the answer for perimeter. Liana suggested the strategy that she would use to the answer for area without being probed. She wrote the measurement units (without probed) for the answers of perimeter and area, as shown in Figures N54 and N55.

Fencing Problem

Strategic Knowledge

Liana was unable to recall the method to solve the fencing problem. Excerpt L41 is illustrative (Liana/L1167-1181).

Excerpt L41

R: (Puts a card written the following problem in front of Liana). Suppose that one of your students asks you for help with the following problem:

A gardener has 84 m of fencing to enclose a garden along three sides, with the fourth side of the garden being formed by a wall. (Assume that the wall is perfectly straight). What are the dimensions of a rectangular garden that will yield the largest area being enclosed?

How would you solve this problem?

S: (Attempts to use differentiation method to solve this problem. Draws a diagram to represents the fencing of the rectangular garden, as shown in Figure N57). …(Silent for a while) em I forgot how to solve the problem.

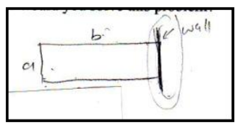


Figure N57. Liana draws a diagram to represents the fencing of the rectangular garden.

In Excerpt L41, Liana drew a diagram to represents the fencing of the rectangular garden, as shown in Figure N57. Nevertheless, Liana stated that she was unable to recall the method to solve the fencing problem. When probed further, Liana had attempted to use differentiation method to solve the fencing problem. Excerpt L42 is illustrative (Liana/L1189-1208).

Excerpt L42

R: Would you like to try to solve this problem?
S: But I don't remember the rate. The sum, the. I don't remember the method.
R: Can you try any method that you know?
S: (Writes an equation to represent the perimeter and area of the rectangular garden respectively, as shown in Figure N58). Now I change. I got an idea here. Now it will require to solve these two unknowns here.
R: Could you tell me more about it?
S: Well, the formula for the area is "a times b" and then perimeter is "a plus 2b". This is for the fence because the fence is required here only. So, I didn't add up the wall here. So, "a plus 2b" actually is same amount of 84. So, I required another equation to solve these two unknowns. The problem that I faced here is how to manipulate the equation of the area here. Ok as I recall that's something have to do with the two equations here. But I just can't remember what is it.
R: Would you like to try?
S: …(Silent for a while) no, I don't think so. I can't remember it.

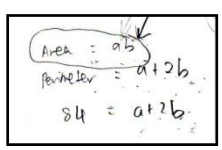


Figure N58. Lina writes an equation to represent the perimeter and area of the rectangular garden respectively.

In Excerpt L42, Liana had attempted to use differentiation method to solve the fencing problem. She wrote an equation to represent the perimeter and area of the rectangular garden respectively, as shown in Figure N58. Liana explained that the area for the rectangular garden is ‘a times b’ whereas the perimeter for the rectangular garden is ‘a plus 2b’ as the fourth side of the garden being formed by a wall. She elaborated that $a + 2b = 84$. Liana encountered the problem of how to manipulate the equation of the area, namely area = ab. Liana knew that she needed to manipulate the two equations, as shown in Figure N58. Nevertheless, Liana was unable to proceed because she was unable to recall the differentiation method to solve the fencing problem.

Summary

In summary, Liana had attempted to use differentiation method to solve the fencing problem. Nevertheless, Liana has unsuccessfully solved the fencing problem as she was unable to recall the method.

Ethical Knowledge

Liana had attempted to use differentiation method to solve the fencing problem. Nevertheless, Liana has unsuccessfully solved the fencing problem as she was unable to recall the method. Liana did not attempt to use other method to solve the fencing problem.

Developing Area Formulae

Procedural Knowledge

Liana could recall the formula for the area of a rectangle, namely '$a \times b$', as shown in Figure N59. Nevertheless, she was unable to develop it. Excerpt L43 is illustrative (Liana/L1244-1265, L1288-1295).

Excerpt L43

R: (Puts a card written the following scenario in front of Liana). Suppose that a Form One student comes to you and says that he does not know how to develop (derive) the formula for calculating the area of the following figures:
(a) Rectangle,
(b) Parallelogram,
(c) Triangle, and
(d) Trapezium.
How would you show him the way to develop (derive) the formula for calculating the area of these figures? Let's start with rectangle.

S: (Draws a rectangle, labels its vertices and dimensions. Writes its area formula, as shown in Figure N59). For rectangle, as we know that the length here and here (points to "a" and "b" respectively, as shown in Figure N59) is not the same. So, the area is "a times b". So, I simply just write that way.

R: How do you get the formula?

S: …(silent for a while) may be I have to do some research first because usually when we are in school, it didn't show any derivation. We just have to memorize.

.
.
.

R: Just now you mentioned that the area formula for a rectangle is "a times b". What do your "a" stands for?

S: "a" is, stands for the length of this side.

R: What do your "b" stands for?

S: "b" here, the length of this one.

R: What do you call it?

S: Well, if I put here, this A, B, C, D (refers to the rectangle, as shown in Figure N59). Then "b" is represents the distance, the length of AB.

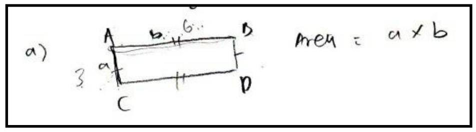


Figure N59. Liana draws a rectangle and then writes its area formula.

In Excerpt L43, Liana stated that the formula for the area of a rectangle is 'a times b'. Nevertheless, she was unable to develop it. Liana explained that she needed to do some research to find out how to develop the formula as it was not shown to her during her school days. Liana expressed that she just has to memorize the formula without knowing how to develop it. Liana just memorized the formula. She did not attempt to develop the formula. Liana explained that *a* represents 'the length of this side' and *b* represents 'the length of this one'. She also explained that *b* represents 'the length of AB'.

Liana neither could recall the formula for the area of a parallelogram nor able to develop it. Excerpt L44 is illustrative (Liana/L1271-1287, L1296-1303).

Excerpt L44

R: How would you show him the way to develop (derive) the formula for calculating the area of a parallelogram?

S: (Draws a parallelogram and then writes its area formula, as shown in Figure N60). It is two ways in calculating the parallelogram. First, formula for the parallelogram itself but I didn't remember and then if we can not memorize it, then you can just simply use the formula of triangle and also rectangular here. So, the area of "A plus B plus C" will be the same as the area of the parallelogram here. Area for "A" with the thing of, area of the triangle here. So, the formula is half times base times height. And formula for square, eh rectangular em multiplication of both sides, "a times b". So, area "C", area "A" equals to area "C". So, we just simplify area "C". Area "A" plus area "C" equals to "2 times area A".

.
.
.

R: (Decided to let Liana knows the area formula of a parallelogram as she was unable to recall it). The area formula of a parallelogram is "base times height".

S: (Writes down the formula as follow).

Parallelogram = Base x height

R: Could you show me how would you develop the formula?

S: …(silent for a while) no, I don't know how to derive (develop) the formula.

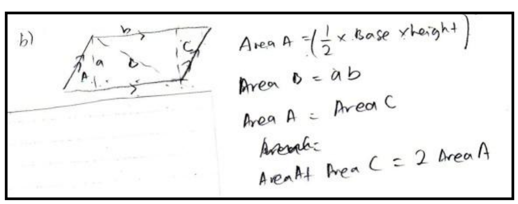


Figure N60. Liana draws a parallelogram and writes its area formula.

In Excerpt L44, Liana admitted that she could not recall the formula for the area of a parallelogram. Liana explained that a parallelogram can be partitioned into triangle "A", rectangle "B", and triangle "C" and thus its area can be calculated as the sum of '2 area A and area B' because area A = area C, as shown in Figure N60. The researcher decided to let Liana knew the area formula of a parallelogram as she was unable to recall it. Nevertheless, Liana also admitted that she did not how to develop the formula for the area of a parallelogram.

Liana could recall the formula for the area of a triangle, namely '$\frac{1}{2} \times height \times base$', as shown in Figure N61. She also knew how to develop the formula for the area of a triangle. Excerpt L45 is illustrative (Liana/L1307-1312, L1317-1329).

Excerpt L45

R: What is the formula for the area of a triangle?
S: (Writes the area formula of a triangle, as shown in Figure N61).
.
.
.
R: How would you show your student the way to develop (derive) the formula for calculating the area of a triangle?
S: (Develops the formula for the area of a triangle, as shown in Figure N62). Let's say this is base and this is height. If we have a square here and we divide here and it becomes two triangles here. So, that's how we get the half. Then, if the formula for the square, the height will be "a" and the base will be "b". So, we simply times "a and b" right. This is the formula for the square. But in the square, that's lies two triangles. That's why it’s a half here.

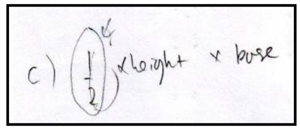


Figure N61. Liana writes the area formula of a triangle.

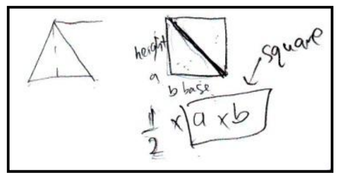


Figure N62. Liana develops the formula for the area of a triangle.

In Excerpt L45, Liana used the partition strategy to develop the formula for the area of a triangle. Liana developed the formula for the area of a triangle based on the formula for the area of a square. She explained that a square can be partitioned into two triangles and thus there is a half in the formula for the area of a triangle. Liana stated that the formula for the area of a square is ‘$a \times b$’, where a and b represents the height and the base of the square. She then wrote the formula for the area of a triangle as ‘$\frac{1}{2} \times a \times b$’, as shown in Figure N62.

Liana neither could recall the formula for the area of a trapezium nor able to develop it. Excerpt L46 is illustrative (Liana/ L1330-1353).

Excerpt L46

R: How would you show your student the way to develop (derive) the formula for calculating the area of a trapezium?
S: …(silent for a while) I have no idea.
R: Can you recall the formula?
S: Cannot.
R: (Decided to let Liana knows the area formula of a trapezium as she was unable to recall it). The area formula of a trapezium is "$\frac{1}{2}$ (a + b) h ".
S: (Writes down the formula as follow).

Area = $\frac{1}{2}$(a + b) h

(Draws a trapezium, as shown in Figure N63). For trapezium, if we complete the shape here, it is actually a rectangular shape but we have taken this out (a triangle). The shape is like this. So, how to derive this. When we cut this out, then…(silent for a while) it can see its with, I mean, a line here the shape we taken out is a triangle. I think because the shape here is like this and that's why where the half come and then "a plus b" eh I don't know how to explain.
R: Could you show me how would you derive the formula?
S: …(Silent for a while) I don't know how to derive.

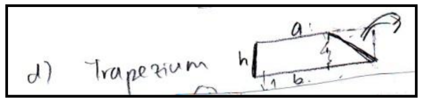


Figure N63. Liana draws a trapezium.

In Excerpt L46, Liana admitted that she neither could recall the formula for the area of a trapezium nor able to develop it. The researcher decided to let Liana knew the area formula of a trapezium as she was unable to recall it. Based on the formula provided by the researcher, Liana draws a trapezium, as shown in Figure N63. Liana explained that if a triangle was cut out from a rectangle, it would form a trapezium and she thought that that was why ‘a half’ came into the area formula of a trapezium. Liana expressed that she did not know how the ‘$a + b$’ came into the area formula of a trapezium. Liana reiterated that she did not know how to develop the formula.

Summary

In summary, Liana could recall the formula for the area of a rectangle and triangle. Nevertheless, she was only able to develop the formulae for the area of a triangle using the partition strategy. Liana did not attempt to develop the formula for the area of a rectangle.

Conceptual Knowledge

Liana could recall the formula for the area of a rectangle. Nevertheless, she was unable to develop the formula. It was apparent that Liana lack of conceptual knowledge underpinning the formula for the area of a rectangle.

Liana could not recall the formula for the area of a parallelogram. She was unable to develop the formula. It was apparent that she did not know the relationship between the area of a parallelogram and the area of a rectangle. Had Liana been known of this relationship, she would know how to develop the formula for the area of a parallelogram.

Liana could recall the formula for the area of a triangle. She was able to develop the formula. Liana developed the formula for the area of a triangle based on the formula for the area of a square. A square is a special case of a rectangle. It indicated that she knew the relationship between the formulae for the area of a triangle and rectangle that encloses it. Liana understands the relationship that the area of a triangle is half of the area of the rectangle that encloses it.

Liana could not recall the formula for the area of a trapezium. She was unable to develop the formula. It was quite clear that she did not know the relationship between the area formulae of a rectangle, parallelogram, triangle, and trapezium. Had Liana been known of this relationship, she would know how to develop the formula for the area of a trapezium.

Linguistic Knowledge

Liana used appropriate mathematical symbols to write the formula for the area of a rectangle, namely '$a \times b$', as shown in Figure N59. Nevertheless, she used inappropriate mathematical terms 'the length of this side' and 'the length of this one' to explain the meaning of the mathematical symbols a and b that she employed. Liana explained that a represents "…the length of this side." (Liana/L1290) and b represents "…the length of this one." (Liana/L1292). Actually, a and b in her formula represents the width and the length of the rectangle. Conventionally, the formula for the area of a rectangle is written as 'l x w', where l and w represents the length and the width of the rectangle.

Liana used appropriate mathematical symbols to write the formula for the area of a triangle, namely '$\frac{1}{2} \times a \times b$', as shown in Figure N62. She also used appropriate mathematical terms 'height' and 'base' to explain the meaning of the symbols that she employed. Liana explained that "…the height will be a and the base will be b. …" (Liana/L1326-1327).

Strategic Knowledge

Liana used the partition strategy to develop the formula for the area of a triangle. She developed the formula for the area of a triangle based on the formula for the area of a square, as shown in Excerpt L45. Liana explained that a square can be partitioned into two triangles and thus there is a half in the formula for the area of a triangle. Liana stated that the formula for the area of a square is '$a \times b$', where a and b represents the height and the base of the square. She then wrote the formula for the area of a triangle as '$\frac{1}{2} \times a \times b$', as shown in Figure N62.

Ethical Knowledge

Liana could recall the formula for the area of a rectangle but she did not attempt to develop the formula, as shown in Excerpt L43. Liana could not recall the formula for the area of a parallelogram and she did not attempt to develop the formula, as shown in Excerpt L44. Liana had succeeded in developing the formula for the area of a triangle, as shown in Excerpt L45. Liana could not recall the formula for the area of a trapezium and she did not attempt to develop the formula, as shown in Excerpt L46.

Level of Subject Matter Knowledge

In this section, Liana' levels (low, medium, high) of subject matter knowledge of perimeter and area was analyzed in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

Liana secured a high level of conceptual knowledge of perimeter and area when she obtained 72.0% of appropriate mathematical elements of conceptual knowledge of perimeter and area during the clinical interview. Liana secured a high level of procedural knowledge of perimeter and area when she obtained 72.7% of appropriate mathematical elements of procedural knowledge of perimeter and area. Liana gained a low level of linguistic knowledge of perimeter and area when she obtained 25.6%% of appropriate mathematical elements of linguistic knowledge of perimeter and area. Liana achieved a medium level of strategic knowledge of perimeter and area when she obtained 57.1% of appropriate mathematical elements of strategic knowledge of perimeter and area. Liana gained a low level of ethical knowledge of perimeter and area when she obtained 20.4% of appropriate mathematical elements of ethical knowledge of perimeter and area. Liana gained an overall low level of subject matter knowledge of perimeter and area when she obtained 38.7% of appropriate mathematical elements of subject matter knowledge of perimeter and area.

Mazlan

Mazlan lives in Tumpat, Kelantan. Mazlan is 21 years 8 months old when he was interviewed. Currently, he is pursuing a 4-year Bachelor of Science with Education (B.Sc.Ed.) program at a public university. He majored and minored in mathematics and chemistry respectively. He obtained grade 1A in Mathematics and Additional Mathematics in his 2003 SPM examination (equivalent to O level examination). He also scored A in Mathematics in the 2004 Matriculation examination (equivalent to A level examination). Mazlan performed satisfactory in his mathematics content courses at the university level when he secured two B, two C+, one C and one D+ in six mathematics content courses he had completed during the first and second year of his studies. The detail of his performance is shown in Table N4.

Table N4

Mathematics Performance of Mazlan

Courses	Grades
1. Calculus for Science Students I	C
2. Algebra for Science Students	B
3. Statistics for Science Students	D+
4. Calculus for Science Students II	C+
5. Differential Equation I	B
6. Programming for Scientific Applications	C+

At the time of data collection, Mazlan was in his second semester of third year studies. He attained 2.70 in the Cumulative Grade Point Average (CGPA) for his first two years of studies at the public university. He does not have any teaching experience prior to this interview.

Notion of Perimeter

Conceptual Knowledge

Mazlan has selected shapes “A”, “C”, and “H” as having a perimeter. Excerpt M1 shows his choice of shapes that have a perimeter (Mazlan/L133-135).

Excerpt M1

R: (Puts a handout comprises 12 shapes in front of Mazlan). Tick the shapes that have a perimeter.
S: (Ticks shapes "A", "C", and "H", as shown in Figure N64).

In Excerpt M1, Mazlan has selected three simple closed curves (A, C, H) that have a perimeter. Nevertheless, he did not select another simple closed curve (K) and the two closed but not simple curves (D, I) that have a perimeter. Mazlan also did not select the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that his notion of perimeter was limited to common simple closed curves (triangle, circle, and trapezium). He did not select the two simple but not closed curves (B, G) as well as the two 1-dimensional shapes (E, L) that do not have a perimeter. In other words, Mazlan did not select an open shape (including the lines) as having a perimeter.

When asked to justify his selection, Mazlan explained that he selected shapes "A", "C", and "H" because these shapes are closed. Excerpt M2 depicts his justification of selecting each of these shapes (Mazlan/L144-152).

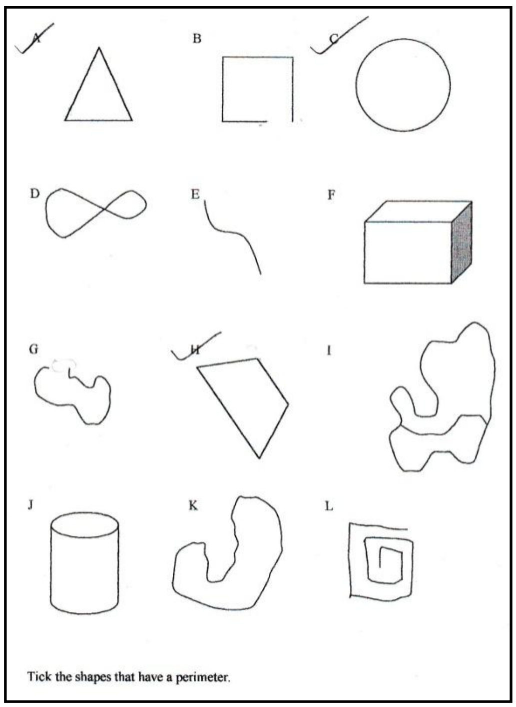


Figure N64. Mazlan's selection of shapes that have a perimeter.

Excerpt M2

R: Why did you select shape "A"?
S: *Sebab* "A" *adalah satu bentuk yang* closed, *yang tertutup. Maknanya ia bermula dari situ dan* end off *di sini.* [Because "A" is a closed shape. It starts from here and end off here.]
R: Why did you select shape "C"?
S: "C" *sama juga sebab bendanya tertutup, tak ada putus-putus, tak ada* curve, *tak ada ini* (points to the gap on shape "B"), smooth. ["C" also same because it is closed, continuous, no curve, no gap, smooth.]
R: Why did you select shape "H"?
S: "H" same with the "A", closed. We can calculate the perimeter when we have a value, the outer length.

Mazlan explained that he did not select shapes “D”, “I”, and “K” because they have curves. He elaborated that a curve does not have a perimeter. Excerpt M3 demonstrates his justification for not selecting shapes “D”, “I”, and “K” as having a perimeter (Mazlan/L156-158, L168-170, L174-176).

Excerpt M3

R: Why didn't you select shape "D"?
S: "D" because curve. It has curve that we can not calculate the curve *di sini* [here].
.
.
.
R: Why didn't you select shape "I"?
S: "I" is same because the length that has a curve. That's why it doesn't has a perimeter. Curved, curved.
.
.
.
R: Why didn't you select shape "K"?
S: "K" is the same with the "I" because it has a curve and it doesn't has a perimeter.

Mazlan explained that he did not select shapes “F” and “J” because they are 3-dimensional objects and 3-dimensional object does not have a perimeter. He elaborated that perimeter is limited to 2-dimensional shapes. Excerpt M4 reveals his justification for not selecting shapes “F” and “J” as having a perimeter (Mazlan/L162-164, L171-173).

Excerpt M4

R: Why didn't you select shape "F"?
S: Em because 3D object that perimeter has a 2D only. That's why 3D is not has a perimeter.
.
.
.
R: Why didn't you select shape "J"?
S: "J" is the same with "F", got the 3D object. So, that's why it doesn't has a perimeter.

Mazlan explained that he did not select shape “B” because it is not closed. Mazlan also explained that he did not select shape “G” because it was open. Excerpt M5 exhibits his justification for not selecting shapes “B” and “G” as having a perimeter (Mazlan/L153-155, L165-167).

Excerpt M5

R: Why didn't you select shape "B"?
S: We can see here "B" is not closed shape. That's why doesn't has a perimeter.
.
.
.
R: Why didn't you select shape "G"?
S: It's an open that does not has a closed length. That's why it doesn't has a perimeter.

Mazlan explained that he did not select shape “E” because it is just a line and thus it does not have a perimeter. Mazlan also explained that he did not select shape “L” because it was open and thus it does not have a perimeter. Excerpt M6 illustrates his justification for not selecting shapes “E” and “L” as having a perimeter (Mazlan/L159-161, L177-179).

Excerpt M6

R: Why didn't you select shape "E"?
S: It is just a line. So, does not has a perimeter that perimeter has closed system, closed shape.
.
.
.
R: Why didn't you select shape "L"?
S: "L" is because it is open that does not has a closed system. That's why it doesn't has a perimeter.

Summary

In summary, Mazlan has selected three simple closed curves (A, C, H) that have a perimeter. It indicated that his notion of perimeter was limited to common simple closed curves (triangle, circle, and trapezium). He justified his selection by explaining that these shapes are closed.

Linguistic Knowledge

Mazlan used appropriate mathematical term ‘closed’ to justify his selection of shapes that have a perimeter. Mazlan explained that he selected shapes “A”, “C”, and “H” because these shapes are closed, as shown in Excerpt M2. ‘Curve’ is an appropriate mathematical term, as shown in Excerpt M3. Nevertheless, it was not the appropriate justification for not selecting shapes “D”, “I”, and “K” that have a perimeter as we still can find perimeter for closed curves.

Mazlan used appropriate mathematical symbol ‘3D’ to represents 3-dimensional objects, “F” and “J”, as shown in Excerpt M4. Nevertheless, ‘3-dimensional objects’ was not the appropriate justification for not selecting shapes “F” and “J” that have a perimeter as we still can find perimeter for the faces of solids.

Mazlan used appropriate negation ‘not closed’ as his justification for not selecting shape “B” as having a perimeter. Mazlan explained that he did not select shape “B” because it is not closed, as shown in Excerpt M5. Mazlan used appropriate mathematical term ‘open’ as his justification for not selecting shape “G” as having a perimeter. Mazlan explained that he did not select shape “G” because it was open. Mazlan used appropriate mathematical terms ‘line’ and ‘open’ as his justification for not selecting shapes “E” and “L” as having a perimeter. Mazlan explained that he did not select shape “E” because it is just a line and thus it does not have a perimeter. Mazlan also explained that he did not select shape “L” because it was open and thus it does not have a perimeter.

Mazlan used appropriate mathematical symbol ‘2D’ to represents 2-dimensional shapes, as shown in Excerpts M4.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Mazlan had taken the effort to justify the selection of shapes that have a perimeter, as shown in Excerpt M2. He provided appropriate justification for selecting shapes “A”, “C”, and “H” that have a perimeter. Mazlan provided justification for not selecting shapes “D”, “I”, and “K” that have a perimeter, as shown in Excerpt M3. ‘Curve’ is an appropriate mathematical term, as shown in Excerpt M3. Nevertheless, it was not the appropriate justification for not selecting shapes “D”, “I”, and “K” that have a perimeter as we still can find perimeter for closed curves.

Mazlan also provided justification for not selecting shapes “F” and “J” that have a perimeter, as shown in Excerpt M4. Nevertheless, ‘3-dimensional shapes’ was not the appropriate justification for not selecting shapes “F” and “J” that have a perimeter as we still can find perimeter for the faces of solids.

Mazlan also had taken the effort to provide justification for not selecting other shapes that do not have a perimeter. He provided appropriate justification for not selecting shapes “B” and “G” as having a perimeter, as shown in Excerpt M5. Mazlan also provided appropriate justification for not selecting shapes “E” and “L” as having a perimeter, as shown in Excerpt M6.

Notion of Area

Conceptual Knowledge

Mazlan has successfully selected shapes “A”, “C”, “D”, “F”, “H”, “I”, “J”, and “K” that have an area. Excerpt M7 shows his choice of shapes that have an area (Mazlan/L188-191).

Excerpt M7

R: (Puts a handout comprises 12 shapes in front of Mazlan). Tick the shapes that have an area.
S: (Ticks shapes "A", "C", "D", “F”, "H", "I", “J”, and "K", as shown in Figure N65).

In Excerpt M7, Mazlan has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. He also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that Mazlan had a static perspective of the notion of area. Based on this perspective, area can be viewed as the amount of surface enclosed within a boundary. It also indicated that his notion of area was not only limited to 2-dimensional shapes (closed plane shapes), but also inclusive of 3-dimensional shapes. Mazlan did not select the two open shapes (B, G) as well as the two 1-dimensional shapes (E, L) that do not have an area. In other words, Mazlan did not select an open shape (including the lines) as having an area. It can be inferred that he did not has a dynamic perspective of area or, at least, this knowledge was not accessible to him during the clinical interview.

When asked to justify his selection, Mazlan explained that he selected shapes "A", "C", "D", "H", "I", and "K" because they are closed shapes. Mazlan also explained that he selected shapes “F” and “J” because they are 3D objects and their area (surface area) can be calculated. Excerpt M8 depicts his justification of selecting each of these shapes (Mazlan/L197-217).

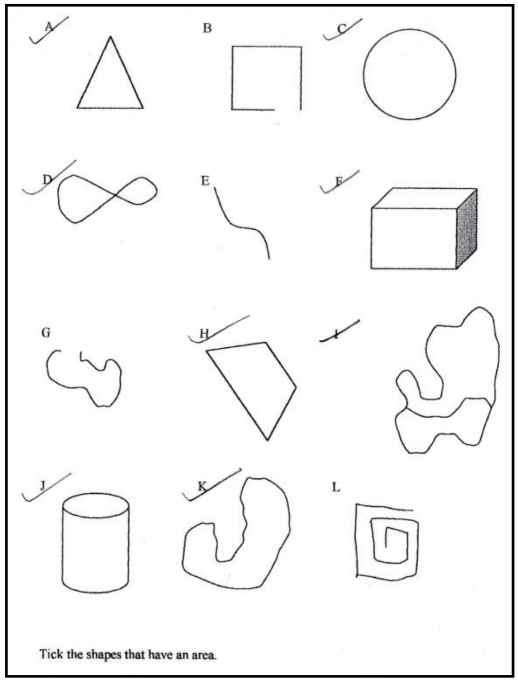


Figure N65. Mazlan's selection of shapes that have an area.

Excerpt M8

R: Why did you select shape "A"?
S: "A" because area, it must has a closed shape object. That's why I select "A".
R: Why did you select shape "C"?
S: "C" is same with the "A" because the closed. That's why it has the area.
R: Why did you select shape "D"?
S: "D" same with "A" and "C", has a closed. That's why we can calculate the area of "D". But it is quite a little bit of difficulty.
R: Why did you select shape "F"?
S: "F" 3D object that we can have the area of the object, 3D.
R: Why did you select shape "H"?
S: "H" is same with "A", "C", and "D" that have an area of the shape.
R: Why did you select shape "I"?
S: "I" because same with "D". We can calculate the area of "I" because it has a closed shape that it is quite difficult but it has an area.
R: Why did you select shape "J"?
S: "J" is same with "F" that 3D object that we can calculate the area. The area of the "J" that has a closed shape.
R: Why did you select shape "K"?
S: "K" is same with "I" because it is closed that we can calculate the area.

Mazlan explained that he did not select shapes "B" and "G" because they are open. Excerpt M9 demonstrates his justification for not selecting shapes "B" and "G" as having an area (Mazlan/L218-220, L224-226).

Excerpt M9

R: Why didn't you select shape "B"?
S: Because "B" is open that does not close. It is *ada putusnya* [discontinuous]. That area can not calculate.
.
.
.
R: Why didn't you select shape "G"?
S: "G" is same with "B", doesn't has a closed shape object that we can not calculate the area because it is open.

Mazlan explained that he did not select shape "E" because it is just a line and line does not have area or volume. Mazlan also explained that he did not select shape "L" because it is open and thus its area cannot be calculated. Excerpt M10 reveals his justification for not selecting shapes "E" and "L" as having an area (Mazlan/L221-223, L227-229).

Excerpt M10

R: Why didn't you select shape "E"?
S: "E" just a line. Line doesn't has any area or volume. So, that's why we can not calculate the area.
.
.
.
R: Why didn't you select shape "L"?
S: We can not calculate the "L" because "L" is open that we can't calculate the area.

Summary

In summary, Mazlan has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. He also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that Mazlan had a static perspective of the notion of area. His notion of area was not only limited to 2-dimensional shapes (closed plane shapes), but also inclusive of 3-dimensional shapes. Mazlan justified his selection by explaining that he selected shapes "A", "C", "D", "H", "I", and "K" because they are closed shapes. Mazlan also explained that he selected shapes “F” and “J” because they are 3D objects and their area (surface area) can be calculated.

Linguistic Knowledge

Mazlan used appropriate mathematical term ‘closed’ to justify his selection of shapes that have an area. Mazlan explained that he selected shapes “A”, “C”, “D”, “H”, “I”, and “K” because they are closed shapes, as shown in Excerpt M8. Mazlan used appropriate mathematical symbol ‘3D’ to justify his selection of shapes that have an area. Mazlan explained that he selected shapes “F” and “J” because they are 3D objects and their area (surface area) can be calculated.

Mazlan used appropriate mathematical term ‘open’ as his justification for not selecting shapes “B”, “G”, and “L” as having an area. Mazlan explained that he did not select shapes “B”, “G”, and “L” because they are open, as shown in Excerpts M9 and M10. Mazlan also used appropriate mathematical term ‘line’ as his justification for not selecting shape “E” as having an area. Mazlan explained that he did not select shape “E” because it is just a line and line does not have area or volume, as shown in Excerpt M10.

Ethical Knowledge

Mazlan had taken the effort to justify the selection of shapes that have an area, as shown in Excerpt M8. He provided appropriate justification for selecting shapes “A”, “C”, “D”, ”F”, “H”, “I”, “J”, and “K” that have an area.

Mazlan also had taken the effort to provide justification for not selecting other shapes that do not have an area. He provided appropriate justification for not selecting shapes “B” and “G” as having an area, as shown in Excerpt M9. Mazlan also provided appropriate justification for not selecting shapes “E” and “L” as having an area, as shown in Excerpt M10.

Notion of the Units of Area

Conceptual Knowledge

Initially, Mazlan stated that a square, rectangle, and triangle can be used as the unit of area. Excerpt M11 is illustrative (Mazlan/L244-252, L265-266).

Excerpt M11

R: (Puts a card written the following scenario in front of Mazlan). Ali, Chong, and David are discussing about the units of area. Ali says that we can use a square as the unit of area. Chong says that we can use a rectangle as the unit of area. David says that we can use a triangle as the unit of area. How would you respond to these students?

S: (Draws a square, rectangle, and triangle and writes its respective area formula and unit, as shown in Figure N66). …So, *ketiga-tiga ini boleh menjadi* [all the three can be] unit of area, depends on the shape.

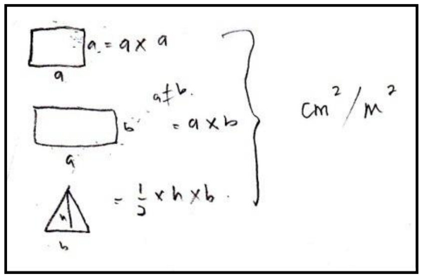


Figure N66. Mazlan draws a square, rectangle, and triangle and writes its respective area formula and unit.

In Excerpt M11, Mazlan drew a square, rectangle, and triangle and wrote its respective area formula and unit, as shown in Figure N66. Initially, He stated that a square, rectangle, and triangle can be used as the unit of area, subjected to the shape being measured.

Subsequently, Mazlan stated that a square can be used to measure the area of an A4-sized rectangular paper. Nevertheless, he thought that a rectangle cannot be used to measure the area of the paper. Excerpt M12 is illustrative (Mazlan/L267-276).

Excerpt M12

R: (Puts a blank A4-sized paper in front of Mazlan). Can we use a square to measure the area of this paper?
S: *Boleh* [can].
R: Can we use a rectangle to measure the area of this paper?
S: *Eh tak boleh* [cannot].
R: Why?
S: Rectangle is two, *kita* [we] "a times b". *Maknanya* [It means], "b" is not the same length of the "a". That's why we can not use a rectangle to calculate the area. So, easily we use square because square is the same length. So, it's easier to use that (refers to the square) we calculate the area.

In Excerpt M12, Mazlan explained that a rectangle cannot be used to measure the area of the paper as the sides of the rectangle are not of the same length. He also explained that a square can easily be used to measure the area of the paper as the sides of a square have the same length.

Initially, Mazlan thought that a triangle cannot be used to measure the area of the paper. Excerpt M13 is illustrative (Mazlan/L281-292).

Excerpt M13

R: Can we use a triangle to measure the area of this paper?
S: *Segitiga em tak boleh* [Triangle em cannot].
R: Why?

S: *Em sebabnya segitiga* [because triangle]...(silent for a while and then draws a square. Partitions it into two triangles, as shown in Figure N67). *Eh boleh, boleh* (laughs) sebabnya *segitiga itu daripada segiempat sama, makna segiempat sama dibahagi dengan dua.* So, *samalah tapi kita kena kerja dua kalilah dengan ini* (refers to the triangle) *sebab ia separuh daripada segiempat sama.* [Eh can, can (laughs) because that triangle comes from square, square divided into two. So, same but we have to double work with this (refers to the triangle) because it is half of a square].

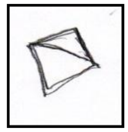


Figure N67. Mazlan draws a square and then partitions it into two triangles.

In Excerpt M13, Mazlan drew a square and then partitioned it into two triangles, as shown in Figure N67. He changed his mind and stated that a triangle can be used to measure the area of the paper. Mazlan explained that a triangle came from a square that had been partitioned into two triangles. He elaborated that we needed to double work if we used a triangle to measure the area of the paper as a triangle is half of a square.

Summary

In summary, initially, Mazlan stated that a square, rectangle, and triangle can be used as the unit of area. Subsequently, he thought that a rectangle cannot be used to measure the area of the A4-sized rectangular paper. Mazlan stated that a square can be used to measure the area of the paper. Mazlan also stated that a triangle can be used to measure the area of the paper. It indicated that his notion of the unit of area was limited to square and triangle.

Mazlan explained that a square can easily be used to measure the area of the A4-sized rectangular paper as the sides of a square have the same length. He explained that a rectangle cannot be used to measure the area of the paper as the sides of the rectangle are not of the same length. Mazlan also explained that a triangle can be used to measure the area of the paper as a triangle came from a square that had been partitioned into two triangles. It indicated that he was unable to provide the appropriate justification that any shape that tessellates a plane can be used as a unit of area measurement.

Linguistic knowledge

Mazlan used inappropriate mathematical term ‘same length’ to justify that a square can be used as the unit of area. He explained that a square can easily be used to measure the area of the A4-sized rectangular paper as the sides of a square have the same length, as shown in Excerpt M12.

Mazlan used inappropriate mathematical negation ‘not same length’ to justify that a rectangle cannot be used as the unit of area. He explained that a rectangle cannot be used to measure the area of the paper as the sides of the rectangle are not of the same length, as shown in Excerpt M12.

Mazlan used inappropriate mathematical term ‘a triangle came from a square’ to justify that a triangle can be used as the unit of area. He explained that a triangle can be used to measure the area of the paper as a triangle came from a square that had been partitioned into two triangles.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Mazlan had taken the effort to justify the shapes that can be used as a unit of area measurement. Nevertheless, he was unable to provide an appropriate justification for the shapes that can be used as a unit of area measure. This can be seen in Excerpts M12 and M13. In reality, any shape that tessellates a plane can be used as a unit of area measurement.

Mazlan also had taken the effort to justify the shape that he thought cannot be used as a unit of area measurement. Nevertheless, Mazlan was unable to provide an appropriate justification for the shapes that can be used as a unit of area measure. This can be seen in Excerpt M12.

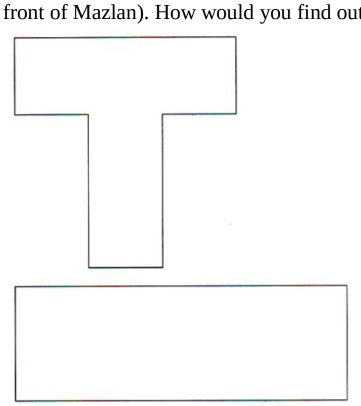
Comparing Perimeter (No Dimension Given)

Strategic Knowledge

Mazlan used the formal method of measuring the side and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter. Excerpt M14 shows the formal method that he used to compare the perimeter of the given pair of shapes (Mazlan/L345-361).

Excerpt M14

R: (Puts the following pair of shape in front of Mazlan). How would you find out whether they had the same perimeter?



S: *Em kita* [we] calculate.
R: Could you tell me more about it?
S: *Em kalau kita dapat* length*, kita akan dapat tahulah perimeternya.* [if we knew the length, we will know its perimeter] (Measures the length of each side by ruler and then calculates its perimeter respectively, as shown in Figure N68). Ha same, both 24.
R: Could you tell me more about it?
S: *Cara* we *guna* ruler *ini* [The way we use this ruler]. So, *letak dekat sini* [puts it here], the edge of *ini* [this]. So, *its measure is 6 cm.* So, with this length, 2 cm. So, this ok 2 cm. This 4 cm ok. This 2 cm. So, just one side. That's why same with the other side: 2, 2, 2, 2, 4, 4, 2, as shown in Figure N68, for the T-shape). Same with this rectangle. 9 just one side with 3. So, 9, 9, 3, 3. 24.

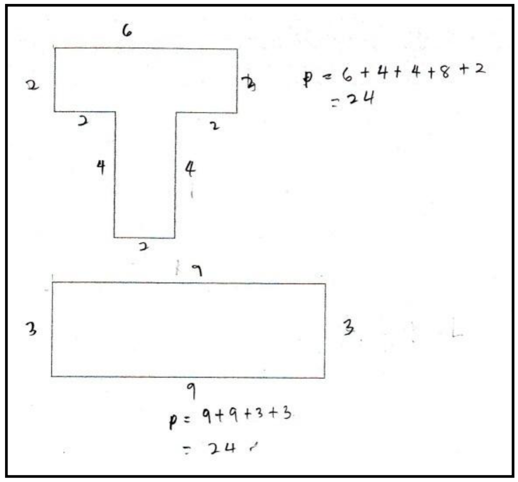


Figure N68. Mazlan measures the length of each side by ruler and then calculates its perimeter respectively.

In Excerpt M14, Mazlan measured the length of each side of the given T-shape by ruler and then calculated its perimeter correctly as 24 (it should be 24 cm). He also measured the length of each side of the given rectangle by ruler and then calculated its perimeter correctly as 24 (it should be 24 cm), as shown in Figure N68. Mazlan concluded that the given pair of shape had the same perimeter. When probed for alternative method of comparing the perimeter, Mazlan was unable to provide other method of comparing the perimeter. Excerpt M15 is illustrative (Mazlan/L383-385).

Excerpt M15

R: Could you think of other way of finding out whether they had the same perimeter?
S: *Em…*(silent for a while) tak ada [no].

Summary

In summary, Mazlan produced one method of determining whether the given pair of shape had the same perimeter. In this formal method, he measured the length of sides by ruler and applied the definition of perimeter.

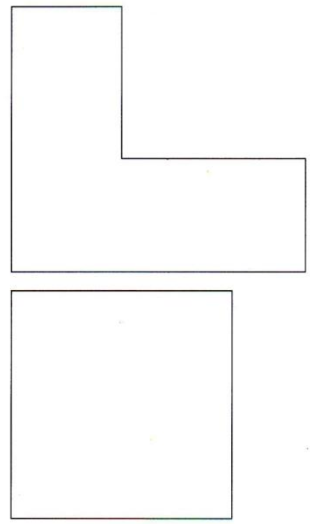
Comparing Area (No Dimension Given)

Strategic Knowledge

Mazlan partitioned L-shape into two rectangles for which area measurement formulae were known. Excerpt M16 shows the formal method of measuring the side and applying the area formula that he used to compare the area of the given pair of shape (Mazlan/L413-427).

Excerpt M16

R: (Puts the following pair of shape in front of Mazlan). How would you find out whether they had the same area?



S: *Em cara pertama mengguna* [first method use] measure, measuring by ruler. We measure by ruler. Then after we measured it, we use, apply with the formula, formula of the area. "L" we can divide by two (partitions L-shape into two rectangles, as shown in Figure N69. Measures its lengths and widths with ruler respectively. Then calculates its area and total area, as shown in Figure N69). *Ini sama juga* [This same also] area of the square, *panjang darab lebar* [length times width] (Measure the length of two adjacent sides of the square with ruler and then calculate its area, as shown in Figure N70).

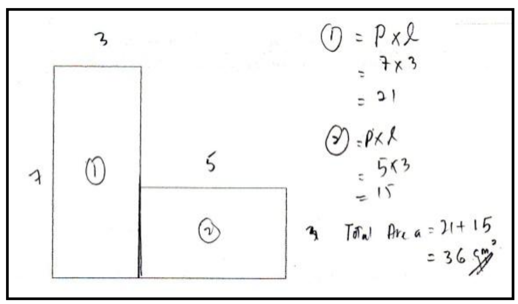


Figure N69. Mazlan measures the length of each side by ruler and then calculates its area.

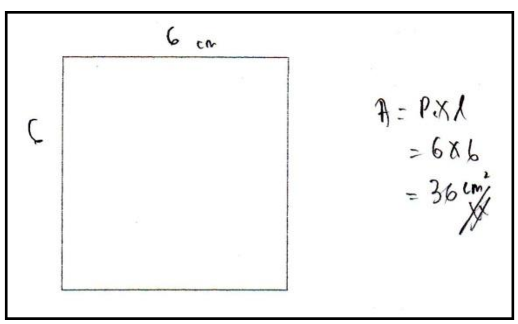


Figure N70. Mazlan measures the length of two adjacent sides by ruler and then calculates its area.

In Excerpt M16, Mazlan partitioned L-shape into two rectangles, labelled as ① and ② respectively. He measured its lengths and widths by ruler respectively and then calculated its area using rectangle area formulae, as shown in Figure N69. Mazlan also measured the length of two adjacent sides of the square by ruler and then calculated its area using rectangle area formula, as shown in Figure N70. When probed for alternative method of comparing the area, Mazlan was unable to provide other method of comparing the area. Excerpt M17 is illustrative (Mazlan/L430-432).

Excerpt M17

R: Could you think of other way of finding out whether they had the same area?
S: *Cara lain* [other method] *em*…(silent for a while) *tak ada* [no]. *Tak ada* [No].

Summary

In summary, Mazlan produced one method of determining whether the given pair of shape had the same area. In this formal method, he measured the length of sides by ruler and applied the area formulae.

Comparing Perimeter (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Mazlan stated that shape B has the longer perimeter. Excerpt M18 shows the justification that he made (Mazlan/L475-483, L504-512).

Excerpt M18

R: (Puts the following table in front of Mazlan). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 1	25 paper clips	12 sticks

S: Shape B.
R: Why?
S: Because shape B has 12 sticks. So, stick is longer than paper clip. So, even it has 12 only that it is longer than shape A.
R: In this case, 25 is larger than 12. So, why can't shape A has the longer perimeter?
S: Em *ini sebabnya* [this is why] we can not judge the measure by just look at the value of the paper clip, stick how much. So, we must see what the things that used to calculate the perimeter of the shape A. For example, shape A used paper clips. As we know, paper clip just, example just 2 cm. Even shape B used 12 sticks, that sticks as we know, stick longer than paper clip. So, the stick, shape B is longer perimeter compared with the shape A.

In Excerpt M18, Mazlan explained that shape B has the longer perimeter because he thought that a stick is longer than a paper clip. It indicated that Mazlan focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. Nevertheless, he did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters.

He stated that measurement cannot be judged just by looking at the number of unit of paper clip (25) and stick (12). Mazlan pointed out that we must look at the unit of measure used to calculate the perimeter of shape A (paper clip). He gave an example that a paper clip might just 2 cm long. He elaborated that even though shape B used 12 sticks, shape B has the longer perimeter compared to shape A as a stick is longer than a paper clip.

In Set 2, Mazlan stated that shape B has the longer perimeter. Excerpt M19 depicts the justification that he made (Mazlan/L546-555).

Excerpt M19

R: (Puts the following table in front of Mazlan). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 2	10 paper clips	15 paper clips

S: For me shape B is the larger, has longer perimeter than shape A.
R: Why?
S: It is because if we look at here, shape A just 10 paper clips compared with shape B has 15. So, 15 is the larger than 10 paper clips. So, that's why shape B is longer perimeter than shape A.

In Excerpt M19, Mazlan explained that shape B has the longer perimeter because shape B has 15 paper clips compared to shape A with 10 paper clips. He elaborated that 15 is larger than 10. It indicated that Mazlan focused on the number of unit when comparing perimeters in Set 2 with common nonstandard units. He did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters.

In another situation when shapes A and B had the same perimeter, Mazlan explained that the paper clips in shape A is longer than the paper clips in shape B. Excerpt M20 demonstrates his justification about their units of measurement (Mazlan/L556-571).

Excerpt M20

R: If shapes A and B had the same perimeter, what can you say about their units of measure?
S: *Kalau sama* [If same] (refers to perimeter), may be *yang ini lebih panjang sikit daripada ini* [this one longer than this].
R: Which one is longer?
S: Shape A.
R: Why?
S: *Sebab 15 pendek sikit* [Because 15, shorter]. …Shape A *lebih panjang lagi* [longer]. So, *barulah perimeter yang ini* [then only this perimeter] (points to shape B) *sama dengan ini* [equal to this] (points to shape A).

In Excerpt M20, Mazlan explained that the paper clips in shape A is longer than the paper clips in shape B. He elaborated that the paper clip in shape B is shorter whereas paper clip in shape A is longer. Then only they had the same perimeter. It indicated that Mazlan understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length.

In Set 3, Mazlan stated that shape A has the longer perimeter. Excerpt M21 reveals his choice of shape that has the longer perimeter and the justification that he made (Mazlan/L615-627).

Excerpt M21

R: (Puts the following table in front of Mazlan). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 3	16 cm	13 cm

S: Shape A.
R: Why?
S: We can see here, shape A is 16 cm, Shape B is 13 cm. So, shape A is longer than shape B.
R: Could you tell me more about it?
S: Because 16 cm is larger than 13 cm. Even any shape that we use, if we compared the value of the perimeter, 16 is larger than 13. So, shape A has longer perimeter than shape B.

In Excerpt M21, Mazlan explained that shape A has the longer perimeter because 16 cm is larger than 13 cm. He elaborated that 16 is larger than 13 if we compared the value of the perimeters. Thus, Mazlan concluded that shape A has the longer perimeter than shape B. It indicated that he focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Mazlan knew that common standard unit (such as cm) is reliable for comparing perimeters.

Summary

In summary, Mazlan focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. Nevertheless, he did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. Mazlan focused on the number of unit when comparing perimeters in Set 2 with common nonstandard units. He did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. Mazlan understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length. He focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Mazlan knew that common standard unit (such as cm) is reliable for comparing perimeters.

.

Comparing Area (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Mazlan stated that shape A has the larger area. Excerpt M22 shows the justification that he made (Mazlan/L657-669).

Excerpt M22

R: (Puts the following table in front of Mazlan). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 1	25 triangles	12 squares

S: Shape A.
R: Why?
S: Because we use a triangle. The value of triangle is more than square.
R: What do you mean by "the value of triangle is more than square"?
S: As we know, triangle is the half of the square. Like the cutting of the square, we can get the triangle. You see here 25. So, 25 divide by 2, you can have 12.5. So, larger than 12. So, shape A has the larger area than shape B.

In Excerpt M22, Mazlan explained that shape A has the larger area because it has more squares, namely 12.5 squares, compared to shape B with 12 squares. He elaborated that a triangle is a half of a square and thus 25 divided by 2 equals to 12.5. It indicated that Mazlan focused on the number of unit when comparing areas in Set 1 with nonstandard units. He did not know that nonstandard units (such as triangle and square) are not reliable for comparing areas.

In Set 2, Mazlan stated that shape B has the larger area. Excerpt M23 depicts the justification that he made (Mazlan/L699-707).

Excerpt M23

R: (Puts the following table in front of Mazlan). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 2	10 squares	15 squares

S: Shape B is the larger than shape A.
R: Why?
S: It is because 15 squares is larger than 10 squares. So, that's why shape B is larger than shape A.

In Excerpt M22, Mazlan explained that shape B has the larger area because shape B has 15 squares compared to shape A with 10 squares. He elaborated that 15 squares is larger than 10 squares. It indicated that Mazlan focused on the number of unit when comparing areas in Set 2 with common nonstandard units. He did not know that common nonstandard units (such as squares) are not reliable for comparing areas.

In another situation when shapes A and B had the same area, Mazlan explained that the squares in shape A is larger than the squares in shape B. Excerpt M24 demonstrates his justification about their units of measurement (Mazlan/L708-715).

Excerpt M24

R: If shapes A and B had the same area, what can you say about their units of measure?
S: *Yang ini lebih besar daripada ini* [This one is larger than this].
R: Which one is larger?
S: Shape A.
R: Why?
S: *Sebab 15 punya kecil sikit* [Because the 15 one smaller]. Shape A *lebih besar lagi* [larger]. So, *barulah* area *ini* [then only this area] (points to shape A) *sama dengan* area *ini* [equals to this area] (points to shape B).

In Excerpt M24, Mazlan explained that the squares in shape A is larger than the squares in shape B. He elaborated that the square in shape B is smaller whereas square in shape A is larger. Then only they had the same area. It indicated that Mazlan understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area.

In Set 3, Mazlan stated that shape A has the larger area. Excerpt M25 reveals his choice of shape that has the larger area and the justification that he made (MazlanL741-753).

Excerpt M25

R: (Puts the following table in front of Mazlan). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 3	16 cm^2	13 cm^2

S: Shape A has larger area than shape B.
R: Why?
S: Because 16 centimetre square (misreads cm^2 as centimetre square) is larger than 13 centimetre square (misreads cm^2 as centimetre square).
R: Could you tell me more about it?
S: The reason is we for this case, we have use centimetre square, same unit. So, we must see the value of the unit, 16 and 13. So, 16 is the larger than 13. That's why shape A has larger area than shape B.

In Excerpt M25, Mazlan explained that shape A has the larger area because 16 cm^2 is larger than 13 cm^2. He stated that we used the same unit, namely square centimetre, to compare areas. Mazlan elaborated that 16 is larger than 13 if we compared the value of the areas. Thus, he concluded that shape A has the larger area than shape B. It indicated that Mazlan focused on the number of unit when comparing areas in Set 3 with common standard unit. He knew that common standard unit (such as cm^2) is reliable for comparing areas.

Summary

In summary, Mazlan focused on the number of unit when comparing areas in Set 1 with nonstandard units. He did not know that nonstandard units (such as triangle and square) are not reliable for comparing areas. Mazlan focused on the number of unit when comparing areas in Set 2 with common nonstandard units. He did not know that common nonstandard units (such as squares) are not reliable for comparing areas. He understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area. Mazlan focused on the number of unit when comparing areas in Set 3 with common standard unit. He knew that common standard unit (such as cm^2) is reliable for comparing areas.

Linguistic Knowledge

Mazlan read 16 cm^2 and 13 cm^2 literally as '16 centimetre square' and '13 centimetre square' respectively, as shown in Excerpt M25. In another situation, Excerpt M26 exhibits how Mazlan wrote 16 cm^2 and 13 cm^2 in English words (Mazlan/L754-761).

Excerpt M26

R: (Puts a blank paper written the following measurements in front of Mazlan).
16 cm^2
13 cm^2
How would you write these measurements in English words?
S: (Writes the following).

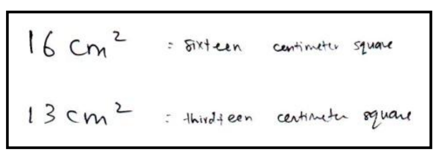


Figure N71. Mazlan writes 16 cm^2 and 13 cm^2 in English words.

In Excerpt M26, Mazlan wrote 16 cm^2 and 13 cm^2 literally as 'sixteen centimetre square' and 'thirdteen (sic) centimetre square', as shown in Figure N71. The correct answer should be 'sixteen square centimetres' and 'thirteen square centimetres'. It indicated that he did not know about the conventions pertaining to writing and reading of Standard International (SI) area measurement units.

Converting Standard Units of Area Measurement

Procedural Knowledge

Mazlan has incorrectly converted 3 cm^2 to 3 x 10^{-4} mm^2. Excerpt M27 shows the algorithms that Mazlan used when he was converting 3 cm^2 to mm^2 (Mazlan/L776-805).

Excerpt M27

R: (Puts a card written the following scenario in front of Mazlan). Some Form One teachers noticed that several of their students seemed to multiply by 10, 100, and 1000, respectively when they were converting units of area from cm^2 to mm^2, m^2 to cm^2, and km^2 to m^2:

$3\ cm^2 = 3 \times 10\ mm^2 = 30\ mm^2$
$4.7\ m^2 = 4.7 \times 100\ cm^2 = 470\ cm^2$
$1.25\ km^2 = 1.25 \times 1000\ m^2 = 1250\ m^2$

What would you do if you were teaching Form One and you noticed that several of your students were doing this?

S: (Converts 3 cm^2 to mm^2, as shown in Figure N72).

R: Could you explain your solution?

S: *Tiga sentimeter padu. Maknanya kita* convert *kepada meter padu dahulu. Ok darab sepuluh negatif satu meter padu*. So, *barulah kita* convert *kepada yang kita nak, milimeter.* So, *tiga darab sepuluh negatif satu darab sepuluh negatif tiga milimeter*. *Macam inilah.* [Three centimeter cubic (sic). It means we convert to meter cubic (sic) first. Ok times ten negative one meter cubic (sic). So, then only we convert to what we want, millimeter. So, three times ten negative one times ten negative three (sic) millimeter. This is the way.]

R: So, what do you get?

S: *Tiga darab sepuluh negatif empat milimeter padu* [Three times ten negative four millimeter cubic (sic).]

R: Please write your answer.

S: (Writes the following). $3 \times 10^{-4}\ mm^2$.

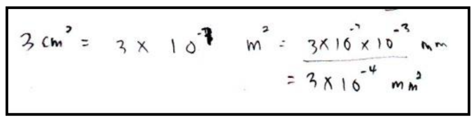


Figure N72. Mazlan converts 3 cm^2 to mm^2.

In Excerpt M27, Mazlan converted 3 cm^2 to m^2 first and then from m^2, he converted it to mm^2. Mazlan thought that 1 m^2 = 10 cm^2. Thus, Mazlan multiplied 10^{-1} (ten to the power of negative one) when he converted 3 cm^2 to m^2. Mazlan also thought that 1 m^2 = 10^{-3} mm^2. Therefore, Mazlan multiplied 10^{-3} (ten to the power of negative three) when he converted m^2 to mm^2, as shown in Figure N72. It indicated that Mazlan did not know the relationship between the standard units of length measurement that 1 cm = 10 mm and 1 m = 100 cm. It also indicated that he did not know the relationship between the standard units of area measurement that 1 cm^2 = 100 mm^2 and 1 m^2 = 10 000 cm^2.

Mazlan has incorrectly converted 4.7 m^2 to 470 cm^2. Excerpt M28 depicts the algorithms that Mazlan used when he was converting 4.7 m^2 to cm^2 (Mazlan/L806-813).

Excerpt M28

R: What about the second one (points to 4.7 m^2)?

S: 4.7 *meter padu sama dengan*, convert *dalam sentimeter padu* [4.7 meter cubic equal to, convert in centimeter cubic (sic)]. (Converts 4.7 m^2 to cm^2, as shown in Figure N73).

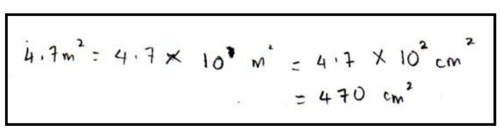


Figure N73. Mazlan converts 4.7 m^2 to cm^2.

In Excerpt M28, Mazlan thought that 1 m = 10 cm. Thus, Mazlan multiplied $(10^1\ cm)^2$ or $10^2\ cm^2$ when he converted 4.7 m^2 to cm^2, as shown in Figure N73. It indicated that Mazlan did not know the relationship between the standard units of length measurement that 1 m = 100 cm. It also indicated that he did not know the relationship between the standard units of area measurement that 1 m^2 = 10 000 cm^2. Nevertheless, Mazlan knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. This can be seen when he squared (10^1 cm) to get 10^2 cm^2, as shown in Figure N73.

Mazlan has incorrectly converted 1.25 km^2 to 1250 x 10^6 m^2. Excerpt M29 demonstrates the algorithms that Mazlan used when he was converting 1.25 km^2 to m^2 (Mazlan/L814-818).

Excerpt M29

R: What about the third one (points to 1.25 km^2)?
S: *Em*…(silent for a while) *kilometer* [kilometre]. (Converts 1.25 km^2 to m^2, as shown in Figure N74).

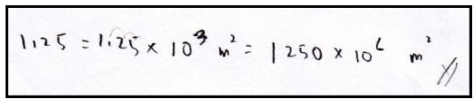


Figure N74. Mazlan converts 1.25 km^2 to m^2.

In Excerpt M29, Mazlan knew the relationship between the standard units of length measurement that 1 km = 1000 m. He also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Mazlan multiplied $(10^3\ m)^2$ or 10^6 m^2 when he converted 1.25 km^2 to m^2, as shown in Figure N74.

Summary

In summary, Mazlan has incorrectly converted 3 cm^2 to 3 x 10^{-4} mm^2, 4.7 m^2 to 470 cm^2, and 1.25 km^2 to 1250 x 10^6 m^2. He thought that 1 m = 10 cm and 1 m^2 = 10 cm^2, as shown in Excerpts M28 and M27 respectively. It indicated that Mazlan did not know the relationship between the standard units of length measurement that 1 cm = 10 mm and 1 m = 100 cm. It also indicated that he did not know the relationship between the standard units of area measurement that 1 cm^2 = 100 mm^2 and 1 m^2 = 10 000 cm^2. Nevertheless, Mazlan knew that 1 km = 1000 m. He also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Mazlan multiplied $(10^3\ m)^2$ or 10^6 m^2 when he converted 1.25 km^2 to m^2, as shown in Figure N74.

Conceptual Knowledge

The analysis of the previous section, namely procedural knowledge, indicated that Mazlan did not know the relationship between the standard units of length measurement that 1 cm = 10 mm and 1 m = 100 cm. It also indicated that he did not know the relationship between the standard units of area measurement that 1 cm^2 = 100 mm^2 and 1 m^2 = 10 000 cm^2. Mazlan thought that 1 m = 10 cm and 1 m^2 = 10 cm^2, as shown in Excerpts M28 and M27 respectively. Nevertheless, he knew that 1 km = 1000 m. Mazlan also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Mazlan multiplied $(10^3\ m)^2$ or 10^6 m^2 when he converted 1.25 km^2 to m^2, as shown in Figure N74.

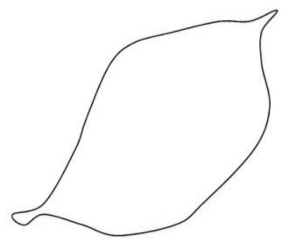
Relationship between Perimeter and Area
(Same Perimeter, Same Area?)

Conceptual Knowledge

Mazlan did not know that there is no direct relationship between perimeter and area. He did not know that two shapes with the same perimeter can have different areas. Thus, Mazlan thought that the student's method of calculating the area of the leaf was correct. Excerpt M30 shows Mazlan's responses to the Form One student (Mazlan/L851-887).

Excerpt M30

R: (Puts a card written the following scenario in front of Mazlan). This is a picture of a leaf. A Form One student said that he had found a way to calculate the area of the leaf. The student placed a piece of thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle.



How would you respond to this student?

S: Ok this is one method that how to fine the area of the leaf. It is a good eh we put the thread here. Join this point and then we can, anything, even we can use triangle or square or circle (draws the following diagrams, as shown in Figure N75).

R: Could you tell me more about it?

S: *Apabila kita dapat benang* [When we get the thread], this thread, we can form rectangle for that student. We get the rectangle even though we can make the triangle, we can get the circle, we can get the square may be. So, when we calculate the area of this (points to the triangle, as shown in Figure N75), this (points to the circle, as shown in Figure N75), this (points to the square, as shown in Figure N75), ok it is the same with the area of leaf.

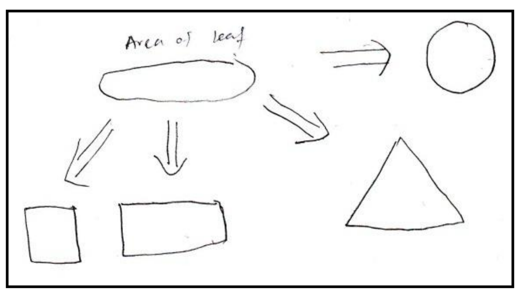


Figure N75. Mazlan draws diagrams to show that the thread can be rearranged to form other shapes such as triangle, square, or circle besides rectangle.

In Excerpt M30, Mazlan stated that this was one method to find the area of the leaf. He agreed with the student that the area of the leaf same as the area of the rectangle formed. Mazlan explained that the thread can also be used to form other shapes such as triangle, square, or circle besides rectangle, as shown in Figure N75. He expressed that the area of the leaf same as the area of the triangle, square, or circle formed.

When probed further, Mazlan thought that the student's method of calculating the area of the leaf was correct. Excerpt M31 depicts his explanation for the correctness of the method proposed by the student (Mazlan/L888-893).

Excerpt M31

R: Would the student's method correct?

S: Yes, *betul* [correct]. *Kaedah budak ini betul* [This student's method correct].

R: Why?

S: Because *dia ambil perimeter daun ini dan dia buat bentuk* [he took the perimeter of this leaf and he forms shape], shape, anything shape that *dia buat* [he formed] rectangle. So, *dia* [he] calculate area of that rectangle. So, *dia dapat* [he got the] area. So, *sama dengan luas daun ini* [equal to the area of this leaf].

In Excerpt M31, Mazlan explain that the student took the perimeter of the leaf and rearranged it to form a rectangle. He elaborated that the student calculated the area of the rectangle and the area of the leaf same as the area of the rectangle formed.

Summary

In summary, Mazlan did not know that there is no direct relationship between perimeter and area. He did not know that two shapes with the same perimeter can have different areas. Thus, Mazlan thought that the student's method of calculating the area of the leaf was correct.

Ethical Knowledge

In Task 5.1, Mazlan thought that the student's method of calculating the area of the leaf was correct. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter have the same area. Mazlan did not attempt to examine the possible pattern of the relationship between perimeter and area.

He did not attempt to formulate generalization pertaining to the relationship between perimeter and area. Mazlan never tests the student's generalization that two shapes with the same perimeter have the same area.

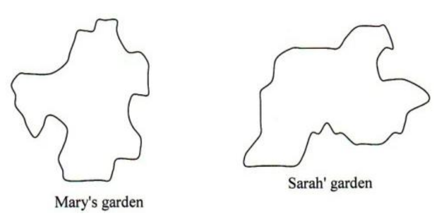
Relationship between Perimeter and Area
(Longer Perimeter, Larger Area?)

Conceptual Knowledge

Mazlan did not know that there is no direct relationship between perimeter and area. He did not know that the garden with the longer perimeter could have a smaller area. Thus, Mazlan thought that Mary's claim was correct. Excerpt M32 shows Mazlan's responses to the claim made by Mary that the garden with the longer perimeter has the larger area (Mazlan/L928-951).

Excerpt M32

R: (Puts a card written the following scenario in front of Mazlan). Mary and Sarah are discussing whose garden has the larger area to plant flowers. Mary claims that all they have to do is walk around the two gardens to get the perimeter and the one with the longer perimeter has the larger area. How would you respond to these students?



S: Ok *betul di mana* [correct where] longer perimeter *adalah mempunyai* [have] larger area.

R: Why?

S: *Daripada perimeter yang kita dapat itu, kita akan dapat mempunyai* area garden *itu.* So, *di mana perimeter yang besar mempunyai* area *yang besarlah.* [From the perimeter that we got, we will get the area of that garden. So, longer perimeter has the larger area.]

In Excerpt M32, Mazlan thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Mazlan explained that from the perimeters of the garden, the areas of the garden could be obtained. Thus, he elaborated that the garden with the longer perimeter has the larger area.

When probed further, Mazlan reiterated that Mary's method was correct. Excerpt M33 depicts his explanation for the correctness of the method claimed by Mary (Mazlan/L952-956).

Excerpt M33

R: Would the student's method correct?
S: *Betul* [Correct].
R: Why?
S: *Mula-mula dapatkan* [First of all, get the] perimeter first. *Lepas dapat perimeter baru dapat* [After got the perimeter then only get the] area. When the perimeter has longer, then larger area.

In Excerpt M33, Mazlan stated that Mary's method was correct. He reiterated that from the perimeters of the garden, the areas of the garden could be obtained. Thus, he concluded that the garden with the longer perimeter has the larger area

Summary

In summary, Mazlan did not know that there is no direct relationship between perimeter and area. He did not know that the garden with the longer perimeter could have a smaller area. Thus, Mazlan thought that Mary's claim was correct.

Ethical Knowledge

In Task 5.2, Mazlan thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Mazlan did not attempt to examine the possible pattern of the relationship between perimeter and area. He did not attempt to formulate generalization pertaining to the relationship between perimeter and area. Mazlan never tests Mary's generalization that that the garden with the longer perimeter has the larger area.

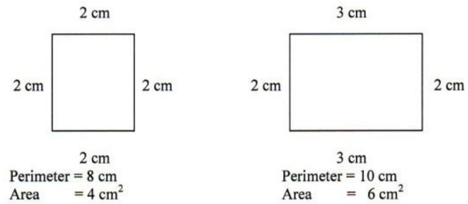
Relationship between Perimeter and Area
(Perimeter Increases, Area Increases?)

Conceptual Knowledge

Mazlan did not know that there is no direct relationship between perimeter and area. He did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Mazlan thought that the student's "theory" was correct. This is shown in Excerpt M34 (Mazlan/L974-1000).

Excerpt M34

R: (Puts a card written the following scenario in front of Mazlan). Suppose that one of your Form One students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:



How would you respond to this student?

S: Ok *adalah satu penemuan yang baik kerana betul apabila perimeter adalah besar,* area *pun juga akan besar* [it is a good discovery because correct. When the perimeter large, the area also will large]. For example, this is a rectangle, *perimeternya* [its perimeter] 10 dan [and] square *adalah* [is] 8. *Bila kita* [When we] calculate the area, we can get 4 centimeter square (misreads 4 cm^2 as 4 centimeter square). Then rectangle 6 centimeter square (misreads 6 cm^2 as 6 centimeter square). So, it is shown that when the perimeter increases, the area also increases.

In Excerpt M34, Mazlan praised the student for the good "discovery' because he thought that as the perimeter of a closed figure increases, the area also increases. Mazlan referred to the example generated by the student that indicated that as the perimeter of a closed figure increases from 8 cm to 10 cm, the area also increases from 4 cm^2 to 6 cm^2. Thus, he concluded that when the perimeter increases, the area also increases.

Mazlan did not know that the student's claim about the relationship between perimeter and area is not a theory. The claim is a conjecture. He also did not know that an example is not a proof and a theory cannot be proved by an example.

Summary

In summary, Mazlan did not know that there is no direct relationship between perimeter and area. He did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Mazlan thought that the student's "theory" was correct.

Ethical Knowledge

In Task 5.3, the student formulated a generalization that as the perimeter of a closed figure increases, the area also increases. Mazlan referred to the example generated by the student that indicated that as the perimeter of a closed figure increases from 8 cm to 10 cm, the area also increases from 4 cm^2 to 6 cm^2. Thus, he concluded that when the perimeter increases, the area also increases. Mazlan thought that the student's "theory" was correct. He did not attempt to test the student's generalization, as shown in Excerpt M34. He accepted the student's generalization without attempting to generate an example or counterexample to test it. In reality, when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same.

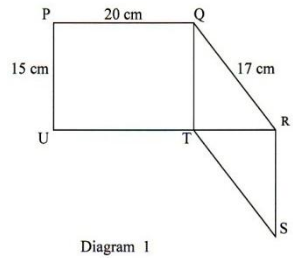
Calculating Perimeter and Area
(Rectangle and Parallelogram/Triangle)

Procedural knowledge

After read through Task 6.1, Mazlan labelled the missing sides of Diagram 1 that required for calculating the perimeter and area of the diagram, as shown in Figure N76 in Excerpt M35 (Mazlan/L1053-1081).

Excerpt M35

R: (Puts a card written the following problem in front of Mazlan). Suppose that one of your Form One students asks you for help with the following problem:



In Diagram 1, PQTU is a rectangle and QRST is a parallelogram. UTR is a straight line.
Calculate
(e) the perimeter of the diagram,
(f) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 1, as shown in Figure N76).

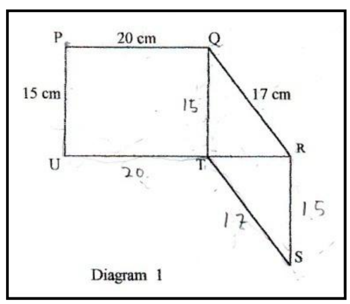


Figure N76. Mazlan labels the missing sides of Diagram 1.

In Excerpt M35, Mazlan labelled QT, UT, RS, and TS as 15, 20, 15, and 17 respectively on Diagram 1, as shown in Figure N76. Excerpt M36 depicts how Mazlan has calculated the perimeter and the area of Diagram 1 (Mazlan/L1083-1101).

Excerpt M36

S: Ok first one, we add all of the length that consists of the diagram is perimeter (calculates the perimeter and the area of Diagram 1, as shown in Figure N77).

R: Could you explain your solution?

S: Ok as we know perimeter is the length of all outside the this (points to Diagram 1, as shown in Figure N76), not include the *yang dalam punya, tak termasuk yang dalam* [the inner, not include the inner]. So, *kita* [we] start *daripada* [from] P. So, PQ plus QR plus RS plus ST plus TU plus UP. So, we can get the perimeter of the diagram.

R: So, what do you get?

S: I get 104 cm.

R: Then, what about the area?

S: We can divide the area. The first one, the first part is the rectangle, *panjang darab lebar* [length times width], *20 darab* [times] *15*. Then area of the parallelogram, ok 15 times 17. So, we can get 555 cm square (misreads cm^2 as cm square).

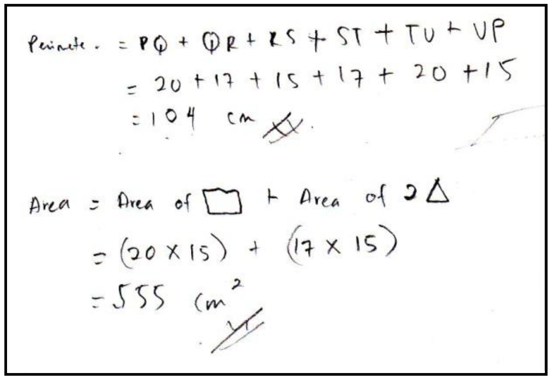


Figure N77. Mazlan calculates the perimeter and the area of Diagram 1.

In Excerpt M36, Mazlan has successfully calculated the perimeter of Diagram 1. He explained that the perimeter of Diagram 1 is the total length of sides that surrounded the diagram. Mazlan used the list all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N77. He listed all the length of sides that surrounded the diagram and then sum them up to get the perimeter of the diagram as 104 cm.

Nevertheless, Mazlan has incorrectly calculated the area of Diagram 1. He used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N77. Mazlan partitioned Diagram 1 into a rectangle PQTU and two triangles, namely QRT and RST. He correctly calculated the area of the rectangle as 300 cm^2. Mazlan viewed the two triangles as parallelogram QRST. Nevertheless, Mazlan confused with the slanted side and the height of the parallelogram that he used the slanted side QR as the height (TR = 8 cm) of the parallelogram. Thus, Mazlan incorrectly calculated the area of the parallelogram as '17 x 15 = 255 cm^2' (The area of the parallelogram should be '15 x 8 = 120 cm^2'). Consequently, he got the area of the diagram as 555 cm^2 (The correct answer should be 420 cm^2, not 555 cm^2).

Summary

In summary, Mazlan has successfully calculated the perimeter of Diagram 1 by using the list all-and-sum algorithm. He incorrectly calculated the area of Diagram 1. Mazlan confused with the slanted side and the height of the parallelogram that he used the slanted side QR as the height (TR) of the parallelogram to calculate the area of Diagram 1.

Linguistic knowledge

Mazlan used the correct standard units of measurement for perimeter (cm) and area (cm^2) when he wrote the answers for these measurements, as shown in Figure N77.

Strategic Knowledge

When probed to check the answer for the perimeter, Mazlan suggested that he would use the recalculating strategy to verify the answer. Excerpt M37 is illustrative (Mazlan/L1102-1111).

Excerpt M37

R: How would you check your answer for the perimeter?

S: Em I am very very (sic) sure that my answer is correct because rectangle we know the properties of rectangle which is 15 (points to the length of QT) same with the…(silent for a while) *berlawanan* [opposite] side *di mana sama* [where it equals to] 15 (points to the length of PU). 20 *dengan* [with] 20. So, that's why 20 *di sini* [here] (point to the length of UT). *Kerana kita tahu juga* [Because we also know] properties of parallelogram *di mana* [where] parallel side another 15 (point to the length of QT), 15 (point to the length of RS). 17 (point to the length of QR), 17 (point to the length of TS). So, we can get the value of ST and TU by using the properties of rectangle and parallelogram. So, we add up.

In Excerpt M37, Mazlan suggested that he would check the answer for the perimeter by the recalculating strategy that using the same method and calculate again. Mazlan was very optimistic that his answer was correct as he used the properties of rectangle and parallelogram to label the missing sides of Diagram 1, as shown in Figure N76. It indicated that Mazlan would check the length of the missing sides of Diagram 1 and recalculate the perimeter to verify his answer.

When probed to check the answer for the area, Mazlan suggested that he would use the recalculating strategy to verify the answer. Excerpt M38 is illustrative (Mazlan/L1112-1118).

Excerpt M38

R: How would you check your answer for the area?

S: Ok we get the value of the UT by using the properties of rectangle. We use the formula of shape diagram rectangle which is *panjang darab lebar* [length times width], 20 times 15. So, you can get area of the rectangle. Then we calculate the area of the parallelogram ok which we know the side of length of the ST by 17 times 15. So, we can get the area of the parallelogram.

In Excerpt M38, Mazlan suggested that he would check the answer for the area by the recalculating strategy that using the same method and calculate again. It indicated that Mazlan would recalculate the areas of rectangle PQTU and parallelogram QRST, as shown in Figure N77, to verify his answer.

Ethical Knowledge

Mazlan has successfully calculated the perimeter of Diagram 1. Nevertheless, he incorrectly calculated the area of Diagram 1. Mazlan did not check the correctness of the answers for the perimeter as well as the area. When probed to check answers, then only Mazlan suggested the strategies that he would use to check the answers for the perimeter and the area. Mazlan wrote the measurement units (without probed) for the answers of the perimeter and area that he has calculated, as shown in Figure N77.

Calculating Perimeter and Area
(Square and Trapezium/Triangle)

Procedural Knowledge

After read through Task 6.2, Mazlan labelled some of the missing sides of Diagram 2 that required for calculating the perimeter and area of the diagram, as shown in Figure N78 in Excerpt M39 (Mazlan/L1157-1182).

Excerpt M39

R: (Puts a card written the following problem in front of Mazlan). Suppose that one of your Form One students asks you for help with the following problem:

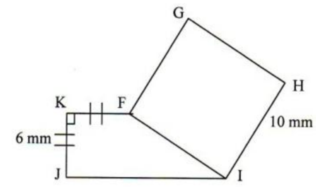


Diagram 2

In Diagram 2, FGHI is a square and FIJK is a trapezium.
Calculate
(e) the perimeter of the diagram,
(f) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 2, as shown in Figure N78).

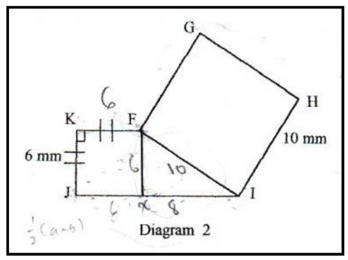


Figure N78. Mazlan labels some of the missing sides of Diagram 2.

In Excerpt M39, Mazlan labelled KF, JX, FI, and FX as 6, 6, 10, and 6 respectively on diagram 2, as shown in Figure N78. Excerpt M40 depicts how Mazlan has successfully calculated the perimeter and the area of Diagram 2 (Mazlan/L1185-1199).

Excerpt M40

S: (Calculates the perimeter and area of Diagram 2, as shown in Figure N79).

R: Could you explain your solution?

S: Ok as we know the perimeter is the length of all the diagram outside length which is from here JK plus KF plus length of the FG, length of GH plus HI plus IJ. So, we can get the value. We can get the 56 cm (wrong unit. It should be mm).

R: What about the area?

S: Area. We use divide by two parts. The first part is the trapezium and then the second part is the area of the square. We use the formula of the area of the trapezium and then plus the area of the square which is half times "a plus b" times height plus *panjang darab lebar* [length times width]. So we get the area of the diagram.

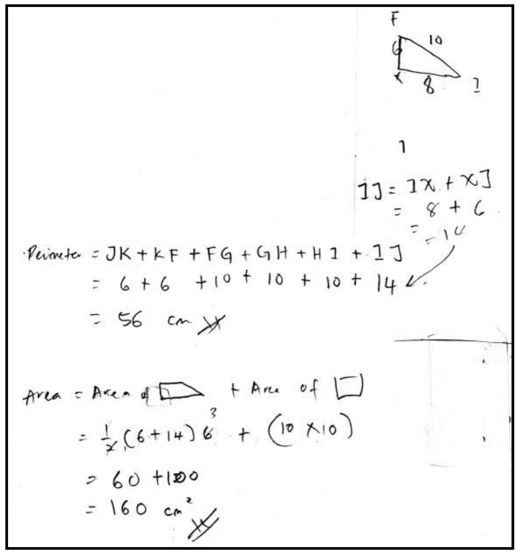


Figure N79. Mazlan calculates the perimeter and area of Diagram 2.

In Excerpt M40, Mazlan realized that he needed to find the length of XI. Mazlan partitioned trapezium FIJK into a square and a triangle, as shown in Figure N78. He has successfully finding the length of XI as 8 (cm) by using Pythagoras' triad (6, 8, 10), as shown in Figure N79. Mazlan labelled XI as 8, as shown in Figure N78. Mazlan used the list all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N79. He listed all the length of sides that surrounded the diagram and then summed them up to get the perimeter of the diagram as 56 cm (wrong unit. It should be mm). Mazlan explained that the perimeter of Diagram 2 is the total length of outer sides that surrounded the diagram.

In Excerpt M40, Mazlan used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N79. He partitioned Diagram 2 into trapezium FIJK and square FGHI. Mazlan calculated the area of the trapezium and square separately using the area formulae of trapezium and square respectively and then summed them up to get the area of the diagram as 160 cm^2 (wrong unit. It should be mm^2). He explained that the formula for the area of the trapezium and square is "half times "a plus b" times height" and "*panjang darab lebar* [length times width]" respectively (Mazlan/L1119).

Summary

In summary, Mazlan has successfully calculated the perimeter of Diagram 2 using the list all-and-sum algorithm. He has also correctly calculated the area of Diagram 2 using the partition-and-sum algorithm.

Linguistic Knowledge

Mazlan mistakenly used the incorrect units of measurement for perimeter (cm) and area (cm^2) when he wrote the answer of these measurements, as shown in Figure N79. The correct units of measurement for perimeter and area should be mm and mm^2.

Strategic Knowledge

When probed to check the answer for the perimeter, Mazlan indicated that he would use the recalculating strategy to verify the answer. Excerpt M41 is illustrative (Mazlan/L1200-1215).

Excerpt M41

R: How would you check your answer for the perimeter?

S: Ok first may be student can't get the value of IJ. Because the value of the IJ is not given. But for this case, we can divide JI by two which is 6 over here (points to FX). We know KF. We call X, X something. Ok KFXJ is square. So, JX is 6. Ok IJ equal to IX plus XJ actually. (Writes the following).
IJ = IX + XJ
= 8 + 6
= 14
Just now we get XJ, 6. FI we already know 10 because it is a square (refers to FGHI). FX we already know 6 because KFXJ is a square. We don't know the value of IX. IX consisting of the length of triangle FXI. We put the outside F, X, I (labels the triangle as F, X, I). Ok we know already 10 (refers to FI) and 6 (refers to FX). XI we can calculate by the formula or we just use the special triangle which has 6, 8, 10. So, we used this. We already know about XI. XI is surely 8.

In Excerpt M41, Mazlan indicated that he would check the answer for the perimeter by the recalculating strategy that using the same method and calculate again. Mazlan would check the length of sides that he used to calculate the perimeter, as listed in Figure N79. He was concerned that the student might encounter problem to find the length of IJ. Mazlan explained that KFXJ is a square and thus, FX and JX is 6 (mm) respectively. He elaborated that FI is 10 (mm) because FGHI is a square. Mazlan pointed out that the length of IX can be calculated by the "formula" (Pythagoras' theorem) or using the "special triangle which has 6, 8, 10" (Pythagoras' triad: 6, 8, 10). He used the "special triangle" to find the length of IX as 8 (mm), as shown in Figure N79.

When probed to check the answer for the area, Mazlan used alternative method, namely repartition-and-sum strategy, to verify the answer. Excerpt M42 is illustrative (Mazlan/L1228-1250).

Excerpt M42

R: How would you check your answer for the area?

S: Area is same. For me, we used the formula of the trapezium and then area of the square. Student must know the formula of the area, how to calculate the area of the trapezium which is half times "a plus b" times h, height and then area of the square, *panjang darab lebar* [length times width]. *Setelah kita dapat tahu idea tadi, kita dapat kira* [After we know the idea, we can calculate] half times "6 plus 14" times height, 6. Actually, for the student, if he forgot how to calculate the area of trapezium, trapezium can divide by two parts which is first part is the square and the second part is the triangle. So, we calculate square first and then we calculate the area of the triangle.

R: Can you show me how it is?

S: (Uses alternative method to calculate the area of trapezium FIJK, as shown in Figure N80).

R: How do you get 24 here?

S: Area of the triangle. *Satu perdua darab panjang darab lebar darab tinggi. Lupa. Lebar darab panjanglah* [One over two times length times width times height. Forget. Length times width]. So, we can get the value *ini* [this value] (points to 60). So, same *lah.*

R: How do you get "half times 8 times 6 times 10" here?

S: I'm forgot already.

R: What do you forget?

S: The area of the triangle. *Lupa* [Forget].

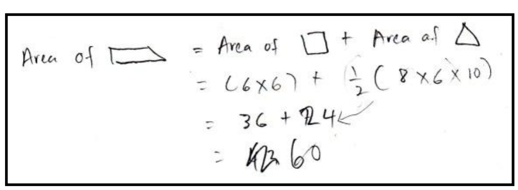


Figure N80. Mazlan uses alternative method to calculate the area of trapezium FIJK.

In Excerpt M42, Mazlan used the repartition-and-sum strategy to check the answer for the area of Diagram 2. Mazlan reiterated that the formula for the area of the trapezium and square is "half times "a plus b" times height" and "*panjang darab lebar* [width times length]" respectively (Mazlan/L1231-1232). He was concerned that the student might forget how to calculate the area of the trapezium. Thus, Mazlan pointed out that the trapezium can be partitioned into a square and a triangle. He used this alternative method to calculate the area of trapezium FIJK, as shown in Figure N80. When probed for the '24' in Figure N80, Mazlan initially thought that the formula for the area of a triangle is 'One over two times length times width times height'. Subsequently, he thought that the formula is 'width times length'. When probed further for the 'half times 8 times 6 times 10' in Figure N80, Mazlan stated that he was unable to recall the formula for the area of a triangle.

Ethical Knowledge

Mazlan has successfully calculated the perimeter and area of Diagram 2. Nevertheless, he did not check the correctness of the answers for perimeter and area. When probed to check answers, then only Mazlan suggested the strategy that he would use to check the answers for perimeter and area.

Mazlan incorrectly wrote the measurement units (without probed) for the answers of perimeter and area as cm and cm^2 respectively, as shown in Figure N79. The correct measurement units for the answers of perimeter and area should be mm and mm^2 respectively. Nevertheless, he did not write the measurement unit for the answer of the area of trapezium FIJK, as shown in Figure N80.

Fencing Problem

Strategic Knowledge

Mazlan used trial and error strategy to solve the fencing problem. Excerpt M43 is illustrative (Mazlan/L1280-1312).

Excerpt M43

R: (Puts a card written the following problem in front of Mazlan). Suppose that one of your students asks you for help with the following problem:

A gardener has 84 m of fencing to enclose a garden along three sides, with the fourth side of the garden being formed by a wall. (Assume that the wall is perfectly straight). What are the dimensions of a rectangular garden that will yield the largest area being enclosed?

How would you solve this problem?

S: (Uses trial and error strategy to solve the fencing problem, as shown in Figure N81). Ok the garden something like this which is the length of garden is 28 and the *lebar adalah* [length is] 14. So, this is the dimension of the garden.

R: How did you get that?

S: Em by divide by the, 84 *bahagi* [divide by] 6 *sama dengan* [equal to] 14.

R: Why did you divide by 6?

S: *Sebab saya bahagikan buat macam ini* [Because I divide like this]: 1, 2, 3, 4, 5, 6. So, side *ini sama dengan* side *ini* [this side equal to this side]. Side *ini sama dengan* side *ini* [this side equal to this side]. *Makna* 6 *panjang* [It means 6 lengths], 6x *lah*, x *adalah* [is] length. 6 *panjang ini* [this length], So *kita dapat semula ini* [we get back this] (refers to 84). *Macam mana kita nak tahu satu ini* [How do we know this one]. X *sama dengan* [equal to] 84 *bahagi* [divide by] 6, 14.

R: What is the largest area?

S: Area = 28 x 14 = 392 m^2.

R: How do you know 392 m^2 is the largest area?

S: *Em* (recalculates the area). *Em luas ini yang paling besar* [this is the largest area].

R: What are the dimensions that will yield the largest area?

S: *Ini* [This] (points to the rectangle with the dimensions of 28 by 14). 28 times 14.

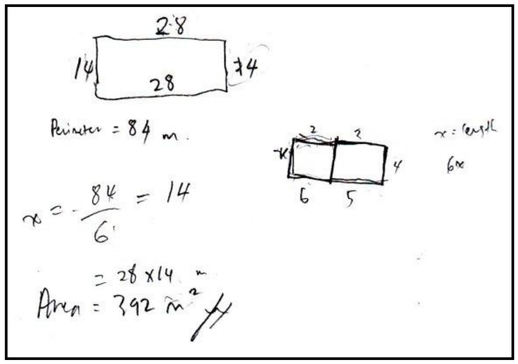


Figure N81. Mazlan uses trial and error strategy to solve the fencing problem.

In Excerpt M43, Mazlan divided the 84 m of fencing into six equal parts, labelled as 6x, where each part was 14 m, labelled as x. He thought that the length and the width of the rectangular garden was 28 m, labelled as 2x, and 14 m, labelled as x, respectively. Mazlan calculated the area of the rectangular garden as 28 x 14 = 392 m^2. He thought that the largest area being enclosed was 392 m^2 and 28 m by 14 m was the dimension of the rectangular garden that will yield the largest area being enclosed. When probed to explain why 392 m^2 was the largest area being enclosed, Mazlan was unable to justify it. He just recalculated the area of the rectangular garden and reiterated that "*Em luas ini yang paling besar* [this is the largest area]." (Mazlan/L1309).

Mazlan did not aware that 392 m^2 was not the largest area being enclosed and 28 m by 14 m was not the dimension of the rectangular garden that will yield the largest area being enclosed. In fact, 882 m^2 is the largest area being enclosed and 42 m by 21 m is the dimension of the rectangular garden that will yield the largest area being enclosed.

When probed to check the answer for the fencing problem, Mazlan used the same strategy, namely trial and error strategy, to verify the answer. Excerpt M44 is illustrative (Mazlan/L1313-1321).

Excerpt M44

R: How would you check your answer?

S: *Semak* [Check]. Ok *kita cuba bina satu lagi* [we try to construct another] rectangle (draws another rectangle and then calculates its area, as shown in Figure N82). *Contohnya* [For example], *84, 4 ada lebih* [extra 4]. *80 bahagi 2, maknyanya 40* [80 divided by 2, it means 40]. *Lagi 4* [4 more]*: 2, 2.* So, area *sini* [this area] 40 times 2, 80. So, *ini menunjukkan inilah* (points to 392 m^2) *yang paling besar* [it shows that this is (points to 392 m^2) the largest area].

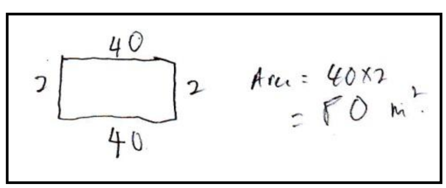


Figure N82. Mazlan draws another rectangle and then calculates its area.

In Excerpt M44, Mazlan drew another rectangle and then calculated its area as 80 m^2, as shown in Figure N82. He explained that 84 minus 4 equal to 80 and 80 divided by 2 equal to 40. The remaining 4 divided by 2 equal to 2. Mazlan took 40 and 2 as the length and the width of the rectangular garden respectively. He found that the area was 80 m^2 and reiterated that 392 m^2 was the largest area being enclosed.

Summary

In summary, Mazlan used trial and error strategy to solve the fencing problem. Nevertheless, he did not find the dimension of the rectangular garden that will yield the largest area being enclosed. Mazlan used the same strategy to check his answer.

Ethical Knowledge

When probed to check the answer for the fencing problem, Mazlan used the same strategy, namely trial and error strategy, to verify the answer. This can be seen in Excerpt M45. Mazlan did not write measurement units for the dimension (28 by 14) that he thought would yield the largest area being enclosed. Nevertheless, Mazlan wrote measurement units for its area (392 m^2) that he thought was the largest area being enclosed, as shown in Figure N81.

Developing Area Formulae

Procedural Knowledge

Mazlan could recall the formula for the area of a rectangle. Nevertheless, he was unable to develop it. Excerpt M45 is illustrative (Mazlan/L1344-1377).

Excerpt M45

R: (Puts a card written the following scenario in front of Mazlan). Suppose that a Form One student comes to you and says that he does not know how to develop (derive) the formula for calculating the area of the following figures:
(e) Rectangle,
(f) Parallelogram,
(g) Triangle, and
(h) Trapezium.
How would you show him the way to develop (derive) the formula for calculating the area of these figures? Let's start with rectangle.

S: (Draws a rectangle and then writes its area formula, as shown in Figure N83). Rectangle, area of rectangle *makna* [means] "a times b" which is *panjang* [length], "a times b", ab.

R: How do you get this formula?

S: *Yang kita tahu luas* [We know that the area of] rectangle *panjang darab lebar* [is length times width]. So, that's why *lah panjang adalah* [length is] *"a", lebar adalah* [width is] *"b", "*a times b*"*.

R: Could you tell me more about it?

S: *Panjag darab lebar maksudnya berapa banyak panjang itu darab dengan berapa banyak lebar itu. Itu adalah* area of the *sesuatu bahan. Macam itulah. Dalam bilik ini, panjag darab lebar. Itulah merangkumi,* consists *sector itu. Berapa kali panjang, berapa kali lebar.* So, *apabila darab dengan dua ini* (refers to the length and width of the rectangle), *akan menghasilkan* area. Product *nya* area. [Length times width means how much length times how much width. That is the area of a material. It just like that. In this room, length times width. So, when multiplied these two (refers to the length and width of the rectangle), will produce area. Its product is area].

R: If your student asks you, "why a times b", how would you respond to this student?

S: (Laughs) *macam mana nak jawab* [how to answer]? *Memang* [Of course] established *macam ini* [like this] (laughs). *Memang macam ini* [Of course like this], established *lah.*

R: What do you mean by "established"?

S: Established *aduh* [exclamation of pain, wonder]*! Memang sudah* [Of course already] fixed. Area *sejak dari dulu-dulu lagi* [since ancient times] (laughs), area *memang panjang darab lebar untuk* [of course length times width for] rectangle.

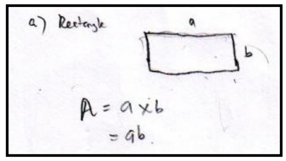


Figure N83 Mazlan draws a rectangle and writes its area formula.

In Excerpt M45, Mazlan stated that the formula for the area of a rectangle is ‘a times b’, where ‘a’ and ‘b’ is the length and the width of the rectangle, as shown in Figure N83. He expressed that the area of the rectangular mathematics teaching room (the room where the clinical interview was carried out) can be calculated using the formula ‘length times width’. Mazlan explained that the product of the length and the width of the rectangle is the area of the rectangle. He could recall the formula for the area of a rectangle. Nevertheless, Mazlan was unable to develop it. When probed for “why a times b”, he explained that the formula is indeed like that. Mazlan stated that it already established and fixed since ancient times. He just memorized the formula. Mazlan did not attempt to develop the formula.

Mazlan could recall the formula for the area of a parallelogram as ‘a times b’, as shown in Figure N84. He knew how to develop the formula for the area of a parallelogram. Excerpt M46 is illustrative (Mazlan/L1378-1391).

Excerpt M46

R: How would you develop (derive) the formula for calculating the area of a parallelogram?
S: (Draws a parallelogram and then writes its area formula, as shown in Figure N84). Area, "a *darab* [times] b", height. Actually parallelogram *adalah daripada* rectangle *sebenarnya* [is indeed from rectangle]. So, we can divide or we will cutting here. Put this over here (cut and paste). So, this times this.
R: What do you get?
S: "a times b"
R: How do you get that?
S: First thing, we cutting here and then put here. Ok same with the rectangle.

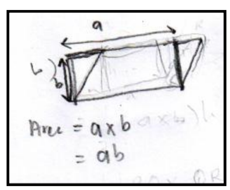


Figure N84. Mazlan draws a parallelogram and writes its area formula.

In Excerpt M46, Mazlan mentally cut out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle, as shown in Figure N84. Thus, the area of the parallelogram equals to the area of the rectangle formed and its area formula is ‘a times b’ or ‘base times height’.

Mazlan could recall the formula for the area of a triangle, namely ‘$\frac{1}{2} \times b \times h$’, as shown in Figure N85. Nevertheless, he was unable to develop it. Excerpt M47 is illustrative (Mazlan/L1392-1398).

Excerpt M47

R: How would you develop (derive) the formula for calculating the area of a triangle?
S: (Draws two triangles and then writes its area formula, as shown in Figure N85). Triangle, ok half times base times height.

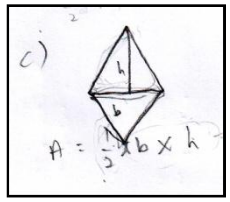


Figure N85 Mazlan draws two triangles and writes its area formula.

In Excerpt M47, Mazlan stated that the formula for the area of a triangle is ‘half times base times height’. Nevertheless, he was unable to develop it. Mazlan just memorized the formula. He did not attempt to develop the formula.

Mazlan could recall the formula for the area of a trapezium, namely ‘$\frac{1}{2}\,(a+b)h$’, as shown in Figure N86. Excerpt M48 is illustrative (Mazlan/L1410-1416).

Excerpt M48

R: How would you develop (derive) the formula for calculating the area of a trapezium?
S: (Draws a trapezium and then writes its area formula, as shown in Figure N86). Formula trapezium half of “a plus b” height.

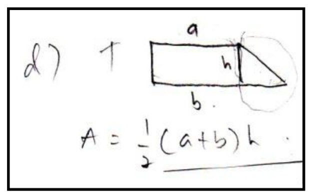


Figure N86. Mazlan draws a trapezium and writes its area formula.

In Excerpt M48, Mazlan stated that the formula for calculating the area of a trapezium is ‘half times (a + b) times height’. Mazlan attempted to develop the formula for the area of a trapezium using algebraic method but unsuccessful. Excerpt M49 is illustrative (Mazlan/L1417-1426).

Excerpt M49

R: How do you get this formula?
S: *Formula ini* [This formula] *aduh* [exclamation of pain, wonder]*! Satu perdua daripada* [One over two from] "a plus b" *darab* [times] height. Actually, can divide by two. (Tries to develop the formula for the area of a trapezium, as shown in Figure N87). Trapezium: Triangle and rectangle ok *di mana nilainya satu perdua darab dengan* [where its formula one over two times] base *darab* [times] height *tambah* [plus] length "ab". So, *kita tambahnya* [we plus it]*. "ha", "hb", "b", triangle simbolnya* [its symbol] but confusing of symbol. We can get *macam inilah* [like this].

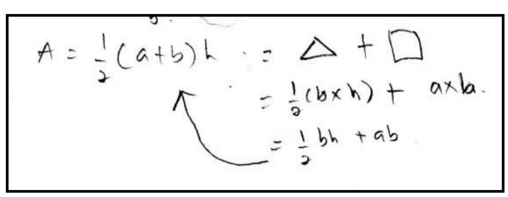


Figure N87. Mazlan tries to develop the formula for the area of a trapezium.

In Excerpt M49, Mazlan attempted to develop the formula for the area of a trapezium using algebraic method, as shown in Figure N87. He reiterated that that the formula for calculating the area of a trapezium is ‘half times (a + b) times height’. Mazlan partitioned the trapezium into a triangle and a rectangle, and circled the triangle, as shown in Figure N86. He incorrectly wrote the formula for the area of a triangle as '$\frac{1}{2}(b \times h)'$ (it should be '$\frac{1}{2}(b - a)h'$). Mazlan also incorrectly wrote the formula for the area of a rectangle as '$a \times b'$ (it should be '$a \times h$'), as shown in Figure N87. Consequently, he simplified them algebraically to become '$\frac{1}{2}bh + ab$' which was not equal to the formula for the area of a trapezium, namely '$\frac{1}{2}(a + b)h$', as shown in Figure N87.

Summary

In summary, Mazlan could recall the formula for the area of a rectangle, parallelogram, triangle, and trapezium. Nevertheless, he was only able to develop the formula for the area of a parallelogram. Mazlan did not attempt to develop the formulae for the area of a rectangle and triangle. Mazlan attempted to develop the formula for the area of a trapezium using algebraic method but unsuccessful.

Conceptual Knowledge

Mazlan could recall the formula for the area of a rectangle. Nevertheless, he was unable to develop the formula. It was apparent that Mazlan lack of conceptual knowledge underpinning the formula for the area of a rectangle.

Mazlan could recall the formula for the area of a parallelogram. He was able to develop the formula. Mazlan mentally transformed the parallelogram to a rectangle by cutting out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle. It indicated that he understands the relationship between the formula for the area of a parallelogram and rectangle. A parallelogram can always be transformed into a rectangle with the same base, same height, and the same area. Thus, the formula for the area of a parallelogram is exactly the same as the formula for the area of a rectangle, namely 'base times height'.

Mazlan could recall the formula for the area of a triangle. Nevertheless, he was unable to develop the formula. Mazlan did not know the relationship between the area of a triangle and the area of the rectangle that encloses it. Had he been known of this relationship, Mazlan would know how to develop the formula for the area of a triangle.

Mazlan could recall the formula for the area of a trapezium. Nevertheless, he was unable to develop the formula. It was quite clear that Mazlan did not know the relationship between the area formulae of a rectangle, parallelogram, triangle, and trapezium. Had he been known of this relationship, Mazlan would know how to develop the formula for the area of a trapezium.

Linguistic Knowledge

Mazlan used appropriate mathematical symbols to write the formula for the area of a rectangle, namely 'a x b', as shown in Figure N83. He also used appropriate mathematical terms 'length', 'times', and 'width' to state the formula for the area of a rectangle. Mazlan stated that "*Yang kita tahu luas* [We know that the area of] rectangle *panjang darab lebar* [is length times width]." (Mazlan/L1361).

Mazlan used appropriate mathematical symbols to write the formula for the area of a parallelogram, namely ‘a x b’, as shown in Figure N84. Mazlan used appropriate mathematical term ‘height’ to explain the meaning of the mathematical symbol *b* that he employed. Mazlan explained that “Area, "a *darab* [times] b", height. …” (Mazlan/L1384). Nevertheless, Mazlan did not explain the meaning of the mathematical symbol *a* that he employed. Actually, *a* represents the base of the parallelogram.

Mazlan used appropriate mathematical symbols to write the formula for the area of a triangle, namely ‘$\frac{1}{2} \times b \times h$’, as shown in Figure N85. He also used appropriate mathematical terms ‘half’, ‘base’ and ‘height’ to state the formula for the area of a triangle. Mazlan stated that “Triangle, ok half times base times height.” (Mazlan/L1398).

Mazlan used appropriate mathematical symbols to write the formula for the area of a trapezium, namely ‘$\frac{1}{2}\ (a+b)h$’, as shown in Figure N86. He used appropriate mathematical terms ‘half’ and ‘height’ to state the formula for the area of a trapezium. Nevertheless, Mazlan did not explain the meaning of the mathematical symbols a and b that he employed. Mazlan stated that “Formula trapezium half of “a plus b” height.” (Mazlan/L1416).

Strategic Knowledge

Mazlan used the cut and paste strategy to develop the formula for the area of a parallelogram. He mentally cut out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle, as shown in Figure N84.

Mazlan attempted to develop the formula for calculating the area of a trapezium using algebraic method but unsuccessful, as shown in Figure N87. Mazlan partitioned the trapezium into a triangle and a rectangle, and circled the triangle, as shown in Figure N86. He incorrectly wrote the formula for the area of a triangle as ‘$\frac{1}{2}(b \times h)'$ (it should be ‘$\frac{1}{2}(b-a)h'$). Mazlan also incorrectly wrote the formula for the area of a rectangle as ‘$a \times b'$ (it should be ‘$a \times h$’), as shown in Figure N87. Consequently, he simplified them algebraically to become ‘$\frac{1}{2}bh + ab$’ which was not equal to the formula for the area of a trapezium, namely ‘$\frac{1}{2}(a+b)h$’, as shown in Figure N87.

Ethical Knowledge

Mazlan could recall the formula for the area of a rectangle but he did not attempt to develop the formula, as shown in Excerpt M45. He had succeeded in developing the formula for the area of a parallelogram, as shown in Excerpt M46. Mazlan could recall the formula for the area of a triangle but he did not attempt to develop the formula, as shown in Excerpt M48. Mazlan could recall the formula for the area of a trapezium. Mazlan attempted to develop the formula for the area of a trapezium using algebraic method but unsuccessful, as shown in Figure N87.

Level of Subject Matter Knowledge

In this section, Mazlan' levels (low, medium, high) of subject matter knowledge of perimeter and area was analyzed in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

Mazlan gained a low level of conceptual knowledge of perimeter and area when he obtained 36.0% of appropriate mathematical elements of conceptual knowledge of perimeter and area during the clinical interview. Mazlan gained a low level of procedural knowledge of perimeter and area when he obtained 36.4% of appropriate mathematical elements of procedural knowledge of perimeter and area. Mazlan achieved a medium level of linguistic knowledge of perimeter and area when he obtained 62.8%% of appropriate mathematical elements of linguistic knowledge of perimeter and area. Mazlan achieved a medium level of strategic knowledge of perimeter and area when he obtained 57.1% of appropriate mathematical elements of strategic knowledge of perimeter and area. Mazlan achieved a medium level of ethical knowledge of perimeter and area when he obtained 53.1% of appropriate mathematical elements of ethical knowledge of perimeter and area. Mazlan achieved an overall medium level of subject matter knowledge of perimeter and area when he obtained 52.1% of appropriate mathematical elements of subject matter knowledge of perimeter and area.

Patrick

Patrick lives in Bau, Sarawak. Patrick is 21 years 7 months old when he was interviewed. Currently, he is pursuing a 4-year Bachelor of Science with Education (B.Sc.Ed.) program at a public university. He majored and minored in mathematics and chemistry respectively. He obtained grade 1A in Mathematics and 4B in Additional Mathematics in his 2003 SPM examination (equivalent to O level examination). He scored B+ in Mathematics in the 2004 Matriculation examination (equivalent to A level examination). Patrick performed moderately in his mathematics content courses at the university level when he secured three B, one C+ and one D in five mathematics content courses he had completed during the first and second year of his studies. The detail of his performance is shown in Table N5.

Table N5

Mathematics Performance of Patrick

Courses	Grades
1. Calculus for Science Students I	C+
2. Algebra for Science Students	B
3. Statistics for Science Students	B
4. Calculus for Science Students II	D
5. Differential Equation I	B

At the time of data collection, Patrick was in his second semester of third year studies. He attained 3.04 in the Cumulative Grade Point Average (CGPA) for his first two years of studies at the public university. He does not have any teaching experience prior to this interview.

Notion of Perimeter

Conceptual Knowledge

Patrick has selected shapes “A”, “C”, “D”, “H”, “I”, and “K” as having a perimeter. Excerpt P1 shows his choice of shapes that have a perimeter (Patrick/L200-202).

Excerpt P1

R: (Puts a handout comprises 12 shapes in front of Patrick). Tick the shapes that have a perimeter.
S: (Ticks shapes "A", "C", "D", "H", "I", and "K", as shown in Figure N88).

In Excerpt P1, Patrick has selected all simple closed curves (A, C, H, K) as well as all closed but not simple curves (D, I) that have a perimeter. Nevertheless, he did not select the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that his notion of perimeter was limited to simple closed curves, and closed but not simple curves, exclusive of 3-dimensional shapes. Patrick also did not select the two simple but not closed curves (B, G) as well as the two 1-dimensional shapes (E, L) that do not have a perimeter. In other words, Patrick did not select an open shape (including the lines) as having a perimeter.

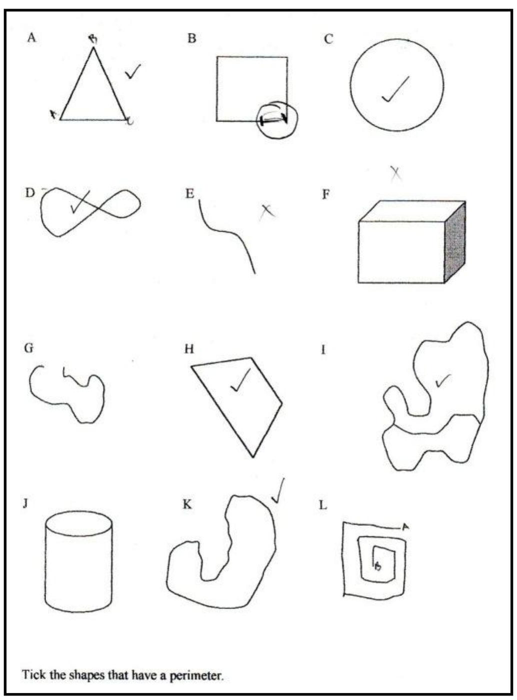


Figure N88. Patrick’s selection of shapes that have a perimeter.

When asked to justify his selection, Patrick explained that he selected shapes “A”, “C”, “D”, “H”, “I”, and “K” because all these shapes are covered. Patrick pointed out that the region of shape “D” is being covered even though it has two intersecting regions. He elaborated that shapes "H", "I", and "K" have a perimeter because it has a covered region even though it cannot be calculated using formula. Excerpt P2 depicts his justification for selecting each of these shapes (Patrick/L210-225, L232-237).

Excerpt P2

R: Why did you select shape "A"?
S: *Kerana ia melitupi daripada kawasan* [Because it covered the region of] ABC (labels A, B, and C on shape "A", as shown in Figure N88). *Ia akan balik kepada* [It will come back to] A. …
R: Why did you select shape "C"?
S: *Sebab ia melitupi, ia tak ada pun jarak* [Because it covered, it is no gap]. …
R: Why did you select shape "H"?
S: *Ia mempunyai penerangan yang sama bagi* "H", "I", and "K" *ini. sebab ia mempunyai kawasan yang mengelilingi.* So, *ia tak boleh kira menggunakan formula tapi ia masih ada perimeter, iaitu jarak keliling.* [It has the same explanation for "H", "I", and "K" because it has a covered region. So, it cannot be calculated using formula but it still has perimeter, namely the surround distance.]
.
.
.
R: Why didn't you select shape "D"?
S: (Changes his mind and ticks shape "D", as shown in Figure N88).
R: Why did you select "D" now?
S: *Kerana ia mempunyai kawasan itu dilitupi biarpun ia mempunyai dua kawasan yang bersilang ini. Tapi* yet *ia mempunyai kawasan yang boleh dikelilingi.* [Because its region is being covered even though it has two intersecting regions. But yet it has a covered region.]

Patrick explained that he did not select shapes “F” and “J” because they are 3-dimensional objects. Excerpt P3 demonstrates his justification for not selecting shapes “F” and “J” as having a perimeter (Patrick/L241-242, L248-249).

Excerpt P3

R: Why didn't you select shape "F"?
S: Em…(silent for a while) *kerana ia merupakan objek 3D* (laughs). [Because it is a 3D object.]
.
.
.
R: Why didn't you select shape "J"?
S: *Kerana kerana ia merupakan objek* 3D. [Because it is a 3D object.]

Patrick explained that he did not select shapes “B” and “G” because they are not joined. Excerpt P4 reveals his justification for not selecting shapes “B” and “G” as having a perimeter (Patrick/L226-231, L243-244).

Excerpt P4

R: Why didn't you select shape "B"?
S: *Sebab ia mempunyai jarak* d*ekat sini* [It has a gap here] (points to the gap on shape "B", as shown in Figure N88). *Ini yang menyebabkan perbezaan* [This caused the different].
R: Could you tell me more about it?
S: Ruangn yang tidak bercantum di antara ini [The space that not joined between this] (points to the gap on shape "B", as shown in Figure N88) *em menyebabkan ia bukan merupakan perimeter* [caused it not a perimeter].
.
.
.
R: Why didn't you select shape "G"?
S: *Ia macam "B" juga. Ia tidak bercantum.* [It’s like “B”. It is not joined.]

Patrick explained that he did not select shape “E” because it is not covered. He elaborated that shape “E” looks like a straight object that does not have perimeter because it is not covered. Patrick explained that he did not select shape “L” because it is not joined. Patrick pointed out that A and B on shape “L”, as shown in Figure N88, is not joined and thus it does not have region that being covered. Excerpt P5 exhibits his justification for not selecting shapes “E” and “L” as having a perimeter (Patrick/L238-240, L245-247).

Excerpt P5

R: Why didn't you select shape "E"?
S: *Sebab ia mempunyai* straight. *Macam benda yang* straight *memang tak ada perimeter sebab ia tidak mengelilingi.* [Because it is straight. Like straight object indeed doesn’t have perimeter because it is not covered.]
.
.
.
R: Why didn't you select shape "L"?
S: *Kerana* line *A dan B* [Because lines A and B] (points to A and B on shape "L", as shown in Figure N88) *tidak bercantum* [not joined]. So, *ia tidak mempunyai kawasan yang dilitupi* [it doesn’t have region that being covered].

Summary

In summary, Patrick has selected all simple closed curves (A, C, H, K) and all closed but not simple curves (D, I) that have a perimeter. It indicated that his notion of perimeter was limited to simple closed curves, and closed but not simple curves, exclusive of 3-dimensional shapes. He justified his selection by explaining that all these shapes are covered.

Linguistic Knowledge

Patrick used appropriate mathematical term ‘covered’ to justify his selection of shapes that have a perimeter. Patrick explained that he selected shapes “A”, “C”, “D”, “H”, “I”, and “K” because all these shapes are covered, as shown in Excerpt P2.

Patrick used appropriate mathematical symbol ‘3D’ to represents 3-dimensional objects, “F” and “J”, as shown in Excerpt P3. Nevertheless, ‘3-dimensional objects’ was not the appropriate justification for not selecting shapes “F” and “J” that have a perimeter as we still can find perimeter for the faces of solids.

Patrick used inappropriate negation ‘not joined’ as his justification for not selecting shapes “B” and “G” as having a perimeter. Patrick explained that he did not select shapes “B” and “G” because they are not joined, as shown in Excerpt P4. Patrick used appropriate negation ‘not covered’ as his justification for not selecting shape “E” as having a perimeter. Patrick explained that he did not select shape “E” because it is not covered, as shown in Excerpt P5. Patrick also used inappropriate negation ‘not joined’ as his justification for not selecting shape “L” as having a perimeter. Patrick explained that he did not select shape “L” because it is not joined, as shown in Excerpt P5.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Patrick had taken the effort to justify the selection of shapes that have a perimeter, as shown in Excerpt P2. He provided appropriate justification for selecting shapes “A”, “C”, “D”, “H”, “I”, and “K” that have a perimeter. Patrick also provided justification for not selecting shapes “F” and “J” that have a perimeter, as shown in Excerpt P3. Nevertheless, ‘3-dimensional objects’ was not the appropriate justification for not selecting shapes “F” and “J” that have a perimeter as we still can find perimeter for the faces of solids.

Patrick also had taken the effort to provide justification for not selecting other shapes that do not have a perimeter. He provided inappropriate justification for not selecting shapes “B” and “G” as having a perimeter, as shown in Excerpt P4. Nevertheless, Patrick provided appropriate justification for not selecting shapes “E” and “L” as having a perimeter, as shown in Excerpt P5.

Notion of Area

Conceptual Knowledge

Patrick has selected shapes “A”, “C”, “F”, “H”, and “J” as having an area. Excerpt P6 shows his choice of shapes that have an area (Patrick/L254-256).

Excerpt P6

R: (Puts a handout comprises 12 shapes in front of Patrick). Tick the shapes that have an area.
S: (Ticks shapes "A", "C", "D", “F”, "H", "I", “J”, and "K", as shown in Figure N89).

In Excerpt P6, Patrick has selected three of the 2-dimensional shapes (A, C, H) that have an area. He also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that his notion of area was limited to regular 2-dimensional shapes (such as triangle, circle, and trapezium) and 3-dimensional shapes (such as cuboid and cylinder), where its area or surface area can be calculated using formula. Patrick did not select the two open shapes (B, G) as well as the two 1-dimensional shapes (E, L) that do not have an area. In other words, Patrick did not select an open shape (including the lines) as having an area. It can be inferred that he did not has a dynamic perspective of area or, at least, this knowledge was not accessible to him during the clinical interview.

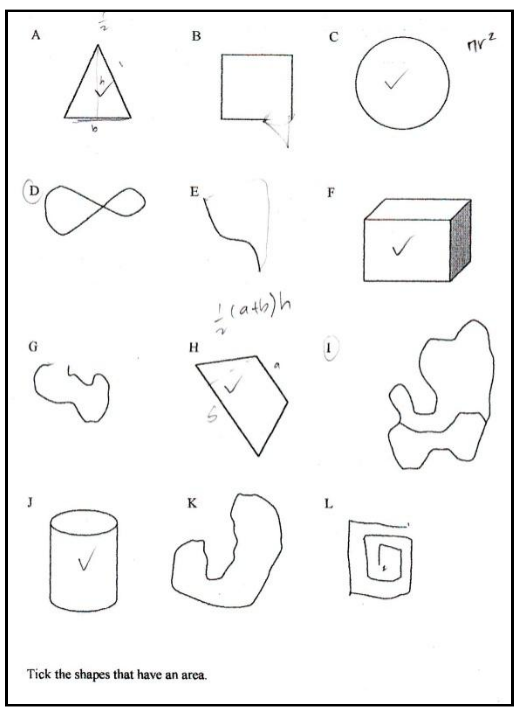


Figure N89. Patrick’s selection of shapes that have an area.

When asked to justify his selection, Patrick explained that he selected shapes "A", "C", and "H" because their area can be calculated using formula. It indicated that Patrick appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Patrick stated that there is a formula to calculate the area of a triangle but he was unable to recall it. Patrick just wrote half above the vertex of the triangle and labelled the base and the height of the triangle as b and h respectively, as shown in Figure N89. Patrick stated that the formula for calculating the area of shapes "C" (circle) and "H" (trapezium) is πr^2 and $\frac{1}{2}(a + b)\, h$ respectively. He wrote these formulae near by shapes "C" and "H" respectively, as shown in Figure N89. Patrick also explained that he selected shapes "F" and "J" because their area (surface area) can be calculated using formula. It also indicated that Patrick appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Patrick stated that there is a formula to calculate the area (surface area) of shapes "F" (cuboid) and "J" (cylinder) but he was unable to recall them. Excerpt P7 depicts his justification of selecting each of these shapes (Patrick/L263-281).

Excerpt P7

R: Why did you select shape "A"?
S: *Kerana ia mempunyai formula untuk mengira luas bagi segitiga.* [Because there is a formula to calculate the area of a triangle.]
R: Could you tell me more about it?
S: *Satu perdua, setengah* [One over two, half] ...(silent for a while) *macam mana* [how]*? Yang saya pasti ia ada formula tapi saya lupa dah.* [I am sure there is a formula but I can't recall it.]
R: Why did you select shape "C"?
S: *Pun mempunyai formula untuk mengira.* [There is also a formula to calculate (it).]
R: Could you tell me more about it?
S: πr^2.
R: Why did you select shape "F"?
S: *Mempunyai formula juga.* [There is a formula too.]

R: Could you tell me more about it?
S: *Pun saya lupa* [I also forget] (laughs).
R: Why did you select shape "H"?
S: "H" *pun ada formula* [also has a formula], $\frac{1}{2}$(a + b) h.
R: Why did you select shape "J"?
S: Em sama juga, ia ada formula. [Also the same, there is a formula.]
R: Could you tell me more about it?
S: …(Silent for while) *lupa dah* [already forgot].

Patrick explained that he did not select shapes "D", "I", and "K" because there is no specific formula that can be used to calculate their area. It indicated that Patrick appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Patrick pointed out that there is no specific formula that can be used to calculate the area of shape "D" and thus indicated that it has no area. The same goes for shapes "I" and "K" as well. Patrick elaborated that "I" has a complex shape and it is difficult to calculate its area. Patrick thus concluded that shape "I" has no area. Patrick expressed that "K" too has a complex shape and there is no specific formula that can be used to calculate its area. Excerpt P8 demonstrates his justification for not selecting shapes "D", "I", and "K" as having an area (Patrick/L288-292, L304-315).

Excerpt P8

R: Why didn't you select shape "D"?
S: …*tak ada satu formula yang khas untuk mengira* […no specific formula to calculate (its area)]. …
R: So, does shape "D" has an area?
S: …(Silent for a while and then moves his head to indicate no).
.
.

.
R: Why didn't you select shape "I"?
S: *Mempunyai bentuk yang kompleks dan juga sukar untuk dibuat pengiraan.* [(It) has a complex shape and also difficult to calculate (its area).]
R: Could you tell me more about it?
S: *Ia lebih kurang sama dengan konteks* "D"*, …memang sukar untuk dikira.* So, *kalau dalam konteks matematik, saya kata tak ada luas.* [It almost similar to context "D", …it's difficult to calculate (its area). So, within the context of mathematics, I would say no area].
R: Why didn't you select shape "K"?
S: *Em ia mempunyai* reason *yang sama dengan* "I" [it has the same reason with "I"].
R: Could you tell me more about it?
S: *Iaitu mempunyai bentuk yang kompleks dan juga tiada formula yang khusus untuk mengira bentuk yang macam ini* [It has a complex shape and also no specific formula to calculate (the area) of such shape].

Patrick explained that he did not select shape "B" because it is not covered and thus no area. Patrick explained that he did not select shape "G" because its line not joined. Excerpt P9 reveals his justification for not selecting shapes "B" and "G" as having an area (Patrick/L282-283, L302-303).

Excerpt P9

R: Why didn't you select shape "B"?
S: *Em kerana kawasannya tak dilitupi*. So, *tak ada.* [Em because it is not covered. So, no (area).]
.
.
R: Why didn't you select shape "G"?
S: *Sebab garisannya pun tak bersambung dan juga bentuknya kompleks.* [Because its line not joined and its shape also complex.]

Patrick explained that he did not select shape "E" because it is just a line and not joined. Patrick thus concluded that it has no area. Patrick also explained that he did not select shape "L" because the lines are not joined. Excerpt P10 exhibits his justification for not selecting shapes "E" and "L" as having an area (Patrick/L299-301, L316-319).

Excerpt P10

R: Why didn't you select shape "E"?
S: "E" *macam ia* just *satu* line, *macam tak ada bersambung. Ia tak mempunyai luas yang spesifik.* So, *ia memang tak ada luas.* ["E" like just a line, like not joined. It does not have a specific area. So, it indeeds no area.]
.
.
.
R: Why didn't you select shape "L"?
S: *Kerana garisan 1 dan 2 ini tidak bersambung dan juga ia tidak memberi apa-apa bentuk.* [Because these lines 1 and 2 (points to the labels 1 and 2 on shape "I", as shown in Figure N89) not joined and also it does not show any shape.]

Summary

In summary, Patrick has selected three of the 2-dimensional shapes (A, C, H) that have an area. He also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that his notion of area was limited to regular 2-dimensional shapes (such as triangle, circle, and trapezium) and 3-dimensional shapes (such as cuboid and cylinder), where its area or surface area can be calculated using formula. Patrick justified his selection by explaining that he selected shapes "A", "C", and "H" because their area can be calculated using formula. It indicated that Patrick appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Patrick also explained that he selected shapes "F" and "J" because their area (surface area) can be calculated using formula. It also indicated that Patrick appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured).

Linguistic Knowledge

Patrick used appropriate mathematical term ‘calculate’ to justify his selection of shapes “A”, “C”, and “H” that have an area. Patrick explained that he selected shapes "A", "C", and "H" because their area can be calculated using formula, as shown in Excerpt P7. Patrick also used appropriate mathematical term ‘calculate’ to justify his selection of shapes “F” and “J” that have an area. Patrick also explained that he selected shapes “F” and “J” because their area (surface area) can be calculated using formula, as shown in Excerpt P7.

Although ‘no specific formula that can be used to calculate their area’ is an appropriate negation but it was not an appropriate justification for not selecting shapes “D”, “I”, and “K” that have an area as their area still exist even though there is no specific formula that can be used to calculate their area, as shown in Excerpt P8.

Patrick used appropriate negation ‘not covered’ as his justification for not selecting shape “B” as having an area. Patrick explained that he did not select shape “B” because it is not covered and thus no area. Patrick used inappropriate negation ‘not joined’ as his justification for not selecting shape “G” as having an area. Patrick explained that he did not select shape “G” because its line not joined, as shown in Excerpt P9. Patrick also used inappropriate negation ‘not joined’ as his justification for not selecting shapes “E” and “L” as having an area. Patrick explained that he did not select shape “E” because it is just a line and not joined. Patrick thus concluded that it has no area. Patrick also explained that he did not select shape “L” because the lines are not joined, as shown in Excerpt P10.

Ethical Knowledge

Patrick had taken the effort to justify the selection of shapes that have an area, as shown in Excerpt P7. He provided appropriate justification for selecting shapes "A", "C", “F”, “H”, and "J" that have an area.

Patrick also had taken the effort to provide justification for not selecting other shapes that do not have an area. He provided inappropriate justification for not selecting shapes “D”, “I”, and “K” as having an area, as shown in Excerpt P8. Patrick provided appropriate justification for not selecting shape “B” as having an area, as shown in Excerpt P9. He provided inappropriate justification for not selecting shapes “E”, “G”, and “L” as having an area, as shown in Excerpts P9 and P10.

Notion of the Units of Area

Conceptual Knowledge

Patrick thought that square can only be used as the unit of area measurement to measure the area of certain shape such as rectangle and square. He elaborated that square cannot be used as the unit of area measurement to measure the (surface) area of a 3-dimensional shape such as a cone. Patrick expressed that we would not be able to figure out how to measure the (surface) area of a cone using a square as the unit of area measure. Excerpt P11 shows his behavior in explaining why square could only be used as the unit of area measurement to measure the area of certain shape (Patrick/L352-364).

Excerpt P11

R: (Puts a card written the following scenario in front of Patrick). Ali, Chong, and David are discussing about the units of area. Ali says that we can use a square as the unit of area. Chong says that we can use a rectangle as the unit of area. David says that we can use a triangle as the unit of area. How would you respond to these students?

S: *Pandangan Ali betul untuk* certain case *saja. Kalau kes segiempat tepat, segiempat sama, mungkin kita boleh buat pengiraan menggunakan* square as a unit of area. *Tapi* let's say *benda itu 3D, maksudnya apakah luas kon? Kita sudah tentu tak boleh gunakan* square as a unit of area *sebab kita tak akan dapat* figure *macam mana nak kira menggunakan* square *ini tadi.* [Ali's opinion is true for certain case only. If the case of rectangle, square, may be we can use a square as a unit of area. But let's say the shape is 3D, what is the (surface) area of a cone? We certainly cannot use square as a unit of area because we are unable to figure out how to calculate using this square.]

Patrick also thought that rectangle can only be used as the unit of area measurement to measure the area of certain shape such as rectangle. He explained that we can count the numbers of square units it takes to cover a region. Patrick also explained that we can count the numbers of rectangular units it takes to cover a region. Excerpt P12 is illustrative (Patrick/L365-390).

Excerpt P12

R: Chong says that we can use a rectangle as the unit of area. How would you respond to Chong?

S: *Pun sama juga. Mungkin kita boleh bagi dia* certain case (draws a large rectangle and then calculates its area, as shown in Figure N90). *Kita bagi dia satu segiempat tepat yang besar di mana kita boleh kira menggunakan formula dulu.* Let's say *ini 6, 2.* So, area ABCD *ini merupakan 6 kali 2, 12 m^2. Maka, di sini* just add up. *Mungkin akan semua tiga-tiga ini* (refers to square, rectangle, and triangle as the unit of area) *akan memberikan jawapan yang berbezalah. Tapi sebab di sini kita mempunyai formula,* so *kita mempunyai unit* meter square (misreads m^2 as meter square). So, *mereka menggunakan* square, triangle, and rectangle.

[Also the same. May be we can give him (Chong) certain case (draws a large rectangle and then calculates its area, as shown in Figure N90). We give him a large rectangle where we can calculate using formula first. Let's say this 6, 2. So, the area of ABCD is 6 times 2, 12 m^2. Then, just add up here. Perhaps all these three (refers to square, rectangle, and triangle as the unit of area) will give different answers. But because here we used formula, so we have unit "meter square" (it should be square metre). So, they used square, triangle, and rectangle.]

R: What do you mean by "certain case"?

S: Let's say *kalau kita diberi untuk mengirakan luas segiempat tepat, memanglah senang. Kalau segiempat tepat itu besar, lepas itu kita nak kira menggunakan segiempat sama yang kecil.* So, *kita* just *kira satu, dua, kita akan kira sampai memenuhi ruang ini* (points to the large rectangle that he has drawn, as shown in Figure N90). So, *kita akan dapat, mungkin 24* square *dan ukuran* square *itu baguslah itu. Untuk kes Chong pula,* rectangle...(silent for a while) *pun boleh. Em ia akan sama dengan seperti kes Ali ini tadi. Ia akan* just fit in, fit in, fit in, *dan mengira berapa kali* rectangle *ini digunakan untuk memenuhi ruang sesuatu kawasan itu, iaitu kawasan 2D sajalah.*

[Let's say we were asked to calculate the area of a rectangle, it indeed easy. If that rectangle is large, then we want to calculate using small squares. So, we just count one, two, we will count until it covers this space (points to the large rectangle that he has drawn, as shown in Figure N90). So, we will get, perhaps 24 square and that square measurement is fine. For the case of Chong, rectangle...(silent for a while) can also. Em it will same as Ali's case just now. It just fit in, fit in, fit in (sic), and count the numbers of rectangle used to cover the region, namely 2D region only.]

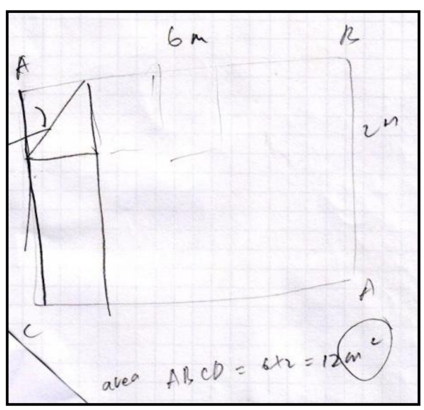


Figure N90. Patrick draws a large rectangle and then calculates its area.

In Excerpt P12, Patrick drew a large rectangle and then used the formula for the area of a rectangle to calculate its area as 12 m^2, as shown in Figure N90. He expressed that if a square, rectangle, and triangle were used as the unit of area measure then it would produce different answers. Nevertheless, Patrick explained that its unit of measurement would be the same, namely unit “meter square” (it should be square metre). He elaborated that it indeed easy to use a square as the unit of area measure to measure the area of a large rectangle as we just have to count the numbers of square units it takes to cover the rectangle, namely “perhaps 24 square units”. Patrick stated that a rectangle can be used as the unit of area measure to measure the area of that large rectangle. He pointed out that the case of Chong would similar to the case of Ali. It just count the numbers of rectangle units it takes to cover the large rectangle, “*iaitu kawasan 2D sajalah* [namely 2D region only].” (Patrick/L390).

Patrick was not sure whether a triangle can used as the unit of area measure because there are many types of triangles such as isosceles triangle and equilateral triangle and thus he stated that “…*ia mungkin boleh.* [it perhaps can.]” (Patrick/L393). Excerpt P13 is illustrative (Patrick/L391-395).

Excerpt P13

R: David says that we can use a triangle as the unit of area. How would you respond to David?

S: …(Silent for a while) *ia mungkin boleh. Yang* triangle *ini mungkin agak kompleks sebab* triangle *ini ia akan mempunyai bentuk yang banyak. Contoh, sama kaki, sama sisi.*
[…(Silent for a while) it perhaps can. This triangle perhaps is quite complex because it has many shapes. For example, isosceles triangle, equilateral triangle.]

Summary

In summary, Patrick thought that a square can only be used as the unit of area measurement to measure the area of certain shape such as rectangle and square. He also thought that a rectangle can only be used as the unit of area measurement to measure the area of certain shape such as rectangle. Patrick was not sure whether a triangle can used as the unit of area measure because there are many types of triangles such as isosceles triangle and equilateral triangle and thus he stated that “…*ia mungkin boleh.* [it perhaps can.]” (Patrick/L393). It indicated that his notion of the unit of area was limited to square and rectangle.

Patrick explained that we can count the numbers of square units it takes to cover a region. He also explained that we can count the numbers of rectangular units it takes to cover a region. It indicated that Patrick knew that a square and rectangle tessellate a plane and thus can be used as the units of area measurement. Nevertheless, he did not realize that a triangle also tessellates a plane and thus can be used as the unit of area measurement.

Linguistic knowledge

Mazlan used appropriate mathematical term ‘‘*memenuhi* [cover]’’ to justify that a square and rectangle can be used as the unit of area. He explained that we can count the numbers of square units it takes to cover a region. Patrick also explained that we can count the numbers of rectangular units it takes to cover a region. He elaborated that:

> “…So, *kita* just *kira satu, dua, kita akan kira sampai memenuhi ruang ini* (points to the large rectangle that he has drawn, as shown in Figure N90). …[So, we just count one, two, we will count until it covers this space (points to the large rectangle that he has drawn, as shown in Figure N90). …]” (Patrick/L383-385)

Patrick also expressed that:

> “…*ia akan sama dengan seperti kes Ali ini tadi. Ia akan* just fit in, fit in, fit in (sic), *dan mengira berapa kali* rectangle *ini digunakan untuk memenuhi ruang sesuatu kawasan itu…* […it will same as Ali’s case just now. It just fit in, fit in, fit in (sic), and count the numbers of rectangle used to cover the region…]” (Patrick/L387-390)

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Patrick had taken the effort to justify the shapes that can be used as a unit of area measurement. He provided appropriate justification for the shapes that can be used as a unit of area measure. This can be seen in Excerpt P12. Patrick knew that a square and rectangle tessellate a plane and thus can be used as the units of area measurement.

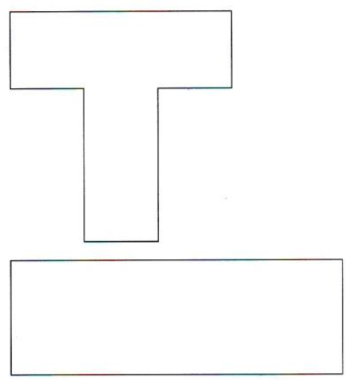
Comparing Perimeter (No Dimension Given)

Strategic Knowledge

Patrick used the formal method of measuring the side and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter. Excerpt P14 shows the formal method that he used to compare the perimeter of the given pair of shapes (Patrick/L460-474)

Excerpt P14

R: (Puts the following pair of shape in front of Patrick). How would you find out whether they had the same perimeter?



S: *Menggunakan pembaris* [Using ruler].
R: Could you show me how it is?
S: (Measures each side with a ruler and then uses a calculator to total up the length of each side, as shown in Figure N91). Ok *di sini kita sudah dapat jawapanlah* [here we already got the answer].
R: Could you tell me more about it?
S: *Em sini kalau kita mengira menggunakan pembaris, kedua-dua ini mempunyai jarak yang hampir sama di mana perimeter A kita mempunyai perimeter 24.4 cm. B ini 24 cm. Di sini mungkin A dan B ini mempunyai sedikit* error *dalam penggunaan pembaris.* [Em here we calculate using ruler, both of these almost have the same distance where perimeter A is 24.4 cm. This B 24 cm. Here probably these A and B have a bit of error of measurement using the ruler.]

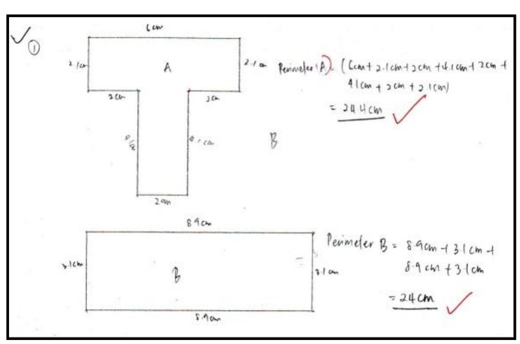


Figure N91. Patrick measures the length of each side of the T-shape and rectangle by ruler and then calculates its perimeter respectively.

In Excerpt P14, Patrick measured the length of each side of the given T-shape by ruler. She then labelled the lengths and calculated its perimeter as 24.4 cm, as shown in Figure N91. Patrick also measured the length of each side of the given rectangle by ruler. She then labelled the lengths and calculated its perimeter as 24 cm, as shown in Figure N91. Patrick explained that the given pair of shapes had approximately the same perimeter. He attributed the differences of the perimeter to the error of measurement using ruler.

When probed for alternative method of comparing the perimeter, Patrick used a formal method of measuring the side by thread and ruler. Excerpt P15 depicts how he used thread and ruler to determine each perimeter and then compare their measurements (Patrick/L475-494).

Excerpt P15

R: Could you think of other way of finding out whether they had the same perimeter?
S: *Menggunakan benang* [Using thread].
R: Could you show me how it is?
S: (Measures the length of each side of the T-shape using the thread. Marks the length of each side with red pen. Cuts the thread after measures the last side of the T-shape and then puts it on the ruler to determine its total length). So, *lepas kita telah ukur lilit menggunakan benang, kita kira panjangnya menggunakan pembaris* [after we measured the surround using the thread, we calculate its length using ruler]. (Writes the following).
Perimeter A - *menggunakan benang* [using the thread] = 24.3 cm
So, A (refers to the T-shape) *akan memberikan nilai* [will give the value of] 24.3 cm. (Measures the length of each side of the rectangle using another piece of thread. Marks the length of each side with red pen. Cuts the thread after measures the last side of the rectangle and then puts it on the ruler to determine its total length). (Writes the following).
Perimeter B - *menggunakan benang* [using the thread] = 23.9 cm
So, B (refers to the rectangle) 23.9 cm.
R: Do they have the same perimeter?
S: *Lebih kurang sama tapi nilainya memberikan perpuluhan* [Almost the same but its value in decimals].

In Excerpt P15, Patrick measured the length of each side of the T-shape by thread. He marked the length of each side with red pen. Patrick cut the thread after measured the last side of the T-shape and then put it on the ruler to determine its total length (perimeter). He got the perimeter of the T-shape as 24.3 cm. Patrick also measured the length of each side of the rectangle using another piece of thread. He marked the length of each side with red pen. Patrick cut the thread after measured the last side of the rectangle and then put it on the ruler to determine its total length (perimeter). He got the perimeter of the T-shape as 23.9 cm. Patrick concluded that they almost had the same perimeter.

When probed for other method of comparing the perimeter, Patrick used a formal method of measuring the side by a compass and ruler. Excerpt P16 demonstrates how he used a compass and ruler to determine each perimeter and then compare their measurements (Patrick/L495-513).

Excerpt P16

R: Could you think of other way of finding out whether they had the same perimeter?
S: Use *jangka lukis* [compasses] and ruler.
R: Could you show me how it is?
S: (Measures the length of each side of the T-shape using the compasses and then puts it on the ruler to determine its length. Jots down each measurement immediately and calculates its perimeter, as shown in Figure N92). *Jawapannya masih sama dengan jawapan A* [Its answer same as the answer of A] (refers to the perimeter of the T-shape using the first method). (Measures the length of each side of the rectangle using the compasses and then puts it on the ruler to determine its length. Jots down each measurement immediately and calculates its perimeter, as shown in Figure N93). We will get a different answer, 24.2 cm.

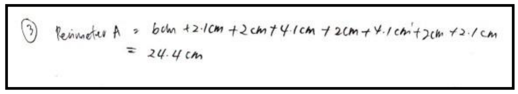


Figure N92. Patrick calculates the perimeter of the T-shape.

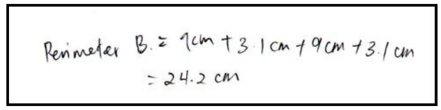


Figure N93. Patrick calculates the perimeter of the rectangle.

In Excerpt P16, Patrick measured the length of each side of the T-shape by a compass and then put it on the ruler to determine its length. He jotted down each measurement immediately and calculated its perimeter, as shown in Figure N92. Patrick also measured the length of each side of the rectangle by a compass and then puts it on the ruler to determine its length. He jotted down each measurement immediately and calculated its perimeter, as shown in Figure N93. Patrick concluded that they had different perimeter.

When probed further for other method of comparing the perimeter, Patrick used a formal method of measuring the side by a paper and ruler. Excerpt P17 reveals how he used a paper and ruler to determine each perimeter and then compare their measurements (Patrick/L514-536).

Excerpt P17

R: Could you think of other way of finding out whether they had the same perimeter?
S: Ok using paper.
R: Could you show me how it is?
S: (Takes a piece of A4-sized white blank paper. Marks the length of each side of the T-shape individually with a pencil on the longer side of the blank paper and then puts it on the ruler to determine its total length). (Writes the following).
Perimeter A = 24.4 cm
(Marks the length of each side of the rectangle individually with a pencil on the opposite side of the longer side of the blank paper and then puts it on the ruler to determine its total length). (Writes the following).
Perimeter B = 24.0 cm
So, using paper (refers to fourth method) and ruler (refers to first method) is the same answer.
R: Could you tell me more about it?
S: Em using straight line, something that is straight, give more precise answer compared to by this *jangka lukis* [compasses] and *benang* [thread].
R: Could you think of other way of finding out whether they had the same perimeter?
S: (Moves his head to indicate no).

In Excerpt P17, Patrick took a piece of A4-sized white blank paper. He marked the length of each side of the T-shape individually with a pencil on the longer side of the blank paper and then put it on the ruler to determine its total length (perimeter). Patrick got the perimeter of the T-shape as 24.4 cm. He also marked the length of each side of the rectangle individually with a pencil on the opposite side of the longer side of the blank paper and then put it on the ruler to determine its total length (perimeter). Patrick got the perimeter of the rectangle as 24.0 cm. He expressed that the first and fourth methods gave the same answer. Patrick elaborated that using "straight line" (such as ruler, paper and ruler) to measure perimeter would gave "more precise answer" compared to using compasses (the third method) and thread (the second method).

Summary

In summary, Patrick produced four formal methods of determining whether the given pair of shape had the same perimeter. In the first method, he used the formal method of measuring the side by ruler and applying the definition of perimeter. In the second method, Patrick used a formal method of measuring the side by thread and ruler. In the third method, he used a formal method of measuring the side by a compass and ruler. In the fourth method, Patrick used a formal method of measuring the side by a paper and ruler.

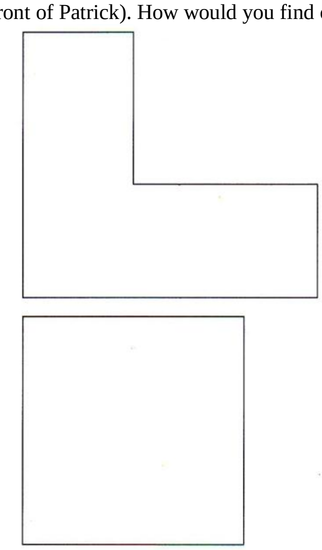
Comparing Area (No Dimension Given)

Strategic Knowledge

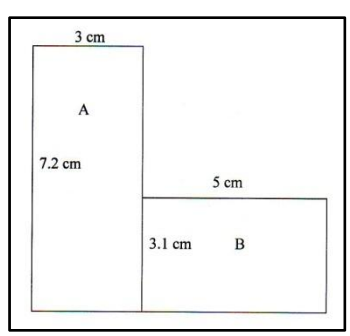
Patrick partitioned L-shape into two rectangles for which area measurement formulae were known. Excerpt P18 shows the formal method of measuring the side and applying the area formula that he used to compare the area of the given pair of shapes (Patrick/L589-612).

Excerpt P18

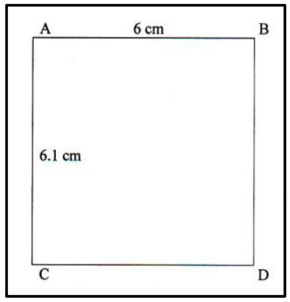
R: (Puts the following pair of shape in front of Patrick). How would you find out whether they had the same perimeter?



S: *Em gunakan pengukuran pembaris* [Em used the measurement of ruler].
R: Could you show me how it is?
S: (Partitions L-shape into two rectangles, labels as A and B respectively, in the following way).



(Measures the length and width of each rectangle as shown above and then applies formula to calculate its area respectively, as shown in Figure N94).



(Measures the length of the two adjacent length of the square as shown above and then applies formula to calculate its area, as shown in Figure N95). *Em tak sama* [Em unequal]. P (refers to the L-shape) *mempunyai luas yang lebih besar daripada* [has the larger area than] Q (refers to the square).

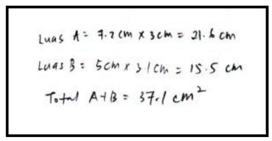


Figure N94. Patrick measures the length and the width of each rectangle by ruler and then calculates its area.

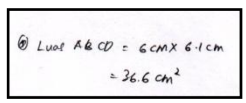


Figure N95. Patrick measures the length of the two adjacent sides of the square by ruler and then calculates its area.

In Excerpt P18, Patrick partitioned L-shape into two rectangles, labelled as A and B respectively. He measured the length and the width of each rectangle by ruler and then calculated its area using rectangle area formulae as 37.1 cm^2. Patrick also measured the length of the two adjacent sides of the square by ruler and then calculated its area using square area formula as 36.6 cm^2. He concluded that the L-shape has the larger area than the square.

When probed for alternative method of comparing the area, Patrick used a semi-formal method of tracing both shapes on a 1-cm grid paper and then counting the number of 1-cm grid covered by the shapes. Excerpt P19 depicts how he used this semi-formal method to determine each area and then compare their measurements (Patrick/L613-622).

Excerpt P19

R: Could you think of other way of finding out whether they had the same area?
S: *Gunakan gunting* [Used scissors].
R: Could you show me how it is?
S: (Takes a 1-cm grid paper. Cuts the L-shape and then traces it on the 1-cm grid paper. Counts the number of 1-cm grids covered by the L-shape). 36 *petak* [grids]. (Cuts the square and then traces it on the 1-cm grid paper. Counts the number of 1-cm grids covered by thesquare). 36 *petak* [grids]. *Dari sini, menggunakan grid memberikan luas yang sama* [From here, using the grid, give the same area].

In Excerpt P19, Patrick cut the L-shape and then traced it on a 1-cm grid paper. He counted the number of 1-cm grids covered by the L-shape, namely 36 grids. Patrick also cut the square and then traced it on a 1-cm grid paper. He counted the number of 1-cm grids covered by the square, namely 36 grids. Patrick concluded that they had the same area.

When probed for other method of comparing the area, Patrick used a semi-formal method of covering both shapes with a common grid and then counting the number of the grid required to cover the shapes. Excerpt P20 demonstrates how he used this semi-formal method to determine each area and then compare their measurements (Patrick/L623-651).

Excerpt P20

R: Could you think of other way of finding out whether they had the same area?

S: (Takes a 1-cm grid paper. Cuts a grid with the dimensions of 2 cm by 2 cm. Counts the number of grids required to cover the L-shape, as shown in Figure N96). (Writes the following). 36 cm (It should be 36 cm^2).

R: Could you tell me more about it?

S: This one we cut 2 cm, 2 cm. *Petak ini mempunyai keluasan 4 sentimeter persegi.* So, *di sini apabila kita letak di sini, ia akan memberikan 9 petak yang sama.* So, *9 darab 4 cm^2, ia akan memberikan luas 36* centimeter square (misreads cm^2 as centimeter square) menggunakan kertas. [This grid has the area of 4 square centimeters. So, here when we put here, it will give 9 similar grids. So, 9 times 4 cm^2, it will give the area of 36 square centimeters using the (grid) paper.] (Using the same grid with the dimensions of 2 cm by 2 cm, counts the number of grids required to cover the square, as shown in Figure N97). (Writes the following). 36 cm^2. Will give the same answer (refers to area of the square compared to the area of the L-shape).

R: So, what's the answer?

S: 36 because this one will give 9 *petak* [grids]. So, by using this one, the answer is the same.

R: Could you think of other way of finding out whether they had the same area?

S: …(Silent for a while) no.

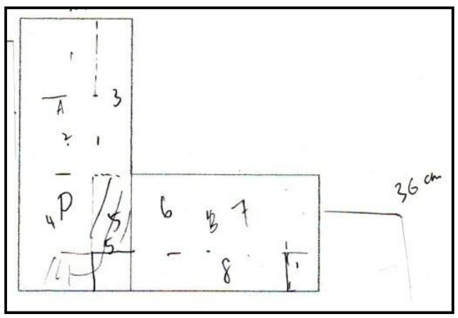


Figure N96. Patrick counts the number of the 2 cm by 2 cm grids required to cover the L-shape.

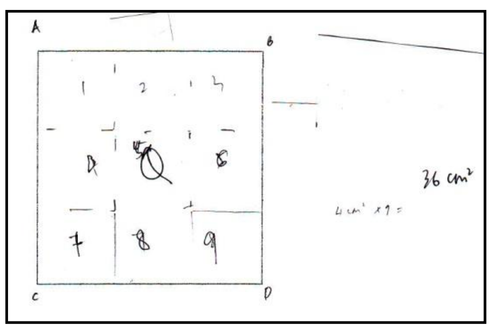


Figure N97. Patrick counts the number of the 2 cm by 2 cm grids required to cover the square.

In Excerpt P20, Patrick used the scissors to cut a grid with the dimensions of 2 cm by 2 cm. He counted the number of the grids required to cover the L-shape, as shown in Figure N96, namely 9 grids. Patrick wrote the area of the L-shape as 36 cm (It should be 36 cm^2). He explained that 9 times 4 cm^2 equal to 36 centimeter square (it should be 36 square centimeters). Patrick also counted the number of the grids required to cover the square, as shown in Figure N97, namely 9 grids. Patrick wrote the area of the square as 36 cm^2. He concluded that they had the same area.

Summary

In summary, Patrick produced one formal method and two semi-formal methods of determining whether the given pair of shapes had the same area. In the first method, he partitioned L-shape into two rectangles, as shown in Excerpt P18. Patrick measured the length and the width of each rectangle by ruler and applied area formulae. He also measured the length of the two adjacent sides of the square by ruler and applied area formulae. In the second method, he used a semi-formal method of tracing both shapes on a 1-cm grid paper and then counting the number of 1-cm grids covered by each shape, as shown in Excerpt P19. In the third method, Patrick used a semi-formal method of covering both shapes with a common grid and then counting the number of the grid required to cover the shapes.

Comparing Perimeter (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Patrick explained that he was unable to determine which shape has the longer perimeter. Excerpt P21 shows the justification that he made (Patrick/L660-670).

Excerpt P21

R: (Puts the following table in front of Patrick). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 1	25 paper clips	12 sticks

S: First, *saya kena tahu macam manakah bentuk klipnya. Kemudian,* stick *ini.* So, *apabila kita lihat panjang mana* stick *itu, kita boleh* compare.[First, I need to know the shape of the clip. Then, this stick. So, when we know the length of the stick, we can compare.]

R: Which shape has the longer perimeter?

S: *Em jika klip bagi* paper clip *ini sama panjang dengan satu* stick, maka shape A *akan lebih besar. Jika* paper clip *ini kecil daripada* stick *ini, mungkin* shape B mempunyai perimeter yang lebih besar. [Em if a paper clip and a stick are same length, then shape A will longer. If this paper clip is smaller (shorter) than this stick, may be shape B has the longer perimeter.]

In Excerpt P21, Patrick explained that he was unable to determine which shape has the longer perimeter as the length of each paper clip and stick were not known. Patrick elaborated that if a paper clip and a stick are same length, then shape A has the longer perimeter. He also elaborated that if a paper clip is smaller (shorter) than a stick, then may be shape B has the longer perimeter. It indicated that he focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. Patrick knew that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters.

In Set 2, Patrick stated that shape B has the longer perimeter. Excerpt P22 depicts the justification that he made (Patrick/L702-713).

Excerpt P22

R: (Puts the following table in front of Patrick). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 2	10 paper clips	15 paper clips

S: Shape B, shape B.

R: Why?

S: *Kerana kedua-dua ini menggunakan* paper clips *sebagai* measurement, *alat pengukur*. *Jadi, di sini disebabkan* shape B *mempunyai* 15 paper clips *dan* shape A *mempunyai* 10 paper clips. So, *di sini kita boleh lihat secara* direct *menyatakan bahawa* shape B *mempunyai perimeter yang lebih besar daripada* shape A. [Because both shapes used paper clip as the (unit of) measurement, measurement tool. So, here because shape B has 15 paper clips and shape A has 10 paper clips. So, shape B has the longer perimeter than shape A.]

In Excerpt P22, Patrick explained that shape B has the longer perimeter because both shapes A and B used paper clip as the (unit of) measurement and shape B has 15 paper clips compared to 10 paper clips in shape A. It indicated that he focused on the number of unit when comparing perimeters in Set 2 with common nonstandard units. Patrick did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters.

In another situation when shapes A and B had the same perimeter, Patrick explained that the paper clips in shape A is longer than the paper clips in shape B. Excerpt P23 demonstrates his justification about their units of measurement (Patrick/L714-723).

Excerpt P23

R: If shapes A and B had the same perimeter, what would you tell about their units of measure?

S: *Eh mereka menggunakan* paper clips *yang berbeza dari segi panjang.* [Eh they used paper clips of different length.]

R: Could you tell me more about it?

S: Paper clips *yang digunakan untuk mengukur* shape A *adalah lebih panjang berbanding dengan p*aper clips *yang digunakan untuk mengukur* shape B kerana 10 paper clips mempunyai sama ukuran dengan 15 paper clips. So, *di sini kita boleh kata* paper clips *untuk mengukur bentuk A adalah lebih besar atau lebih panjang daripada* paper clips *yang digunakan untuk mengukur* shape B. [Paper clips used in shape A is longer than paper clips used in shape B because 10 paper clips (of shape A) is same length as 15 paper clips (of shape B). So, here we can say that paper clips used to measure shape A is larger or longer than paper clips used to measure shape B.]

In Excerpt P23, Patrick stated that they used paper clips of different length. He explained that the paper clips in shape A is longer than the paper clips in shape B because 10 paper clips of shape A is same length as 15 paper clips of shape B. Patrick reiterated that paper clips used to measure shape A is larger or longer than paper clips used to measure shape B. It indicated that Patrick understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length.

In Set 3, Patrick stated that shape A has the longer perimeter. Excerpt P24 reveals his choice of shape that has the longer perimeter and the justification that he made (Patrick/L744-761).

Excerpt P24

R: (Puts the following table in front of Patrick). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 3	16 cm	13 cm

S: Shape A has the longer perimeter.

R: Why?

S: *Bentuk A mempunyai perimeter yang lebih besar kerana ia mempunyai 16 cm berbanding dengan bentuk B yang hanya memberikan nilai sebanyak 13 cm. Di sini kita boleh* direct *menyatakan* shape A *mempunyai perimeter yang lebih besar kerana mereka menggunakan unit yang sama.* [Shape A has the larger perimeter because it has 16 cm compared to shape B that only has 13 cm. Here we can directly stated that shape A has the larger perimeter because they used the same unit.]

R: What do you mean by "*unit yang sama* [same unit]"?

S: *Di sini mereka menggunakan unit sentimeter. Bentuk A menggunakan unit sentimeter. Begitu juga dengan bentuk B, menggunakan unit yang sama iaitu 13 cm.* So, *di sini kita akan melihat lebih besar nombor yang digunakan, menggunakan unit yang sama, maka lebih besarlah perimeter sesuatu benda itu.* [Here they used the unit of centimetre. Shape A used the unit of centimetre. So as shape B, used the same unit, namely 13 cm. So, here we will look at the larger number used, using the same unit, thus the larger the perimeter of that thing.]

In Excerpt P24, Patrick explained that shape A has the longer perimeter because they used the same unit, namely centimetre, and shape A has 16 cm compared to shape B that only has 13 cm. He reiterated that they used the same unit, namely centimetre, and thus the shape with the larger number (of unit) has the larger (longer) perimeter. It indicated that he focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Patrick knew that common standard unit (such as cm) is reliable for comparing perimeters.

Summary

In summary, Patrick focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. He knew that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. Patrick focused on the number of unit when comparing perimeters in Set 2 with common nonstandard units. He did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. Patrick understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length. He focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Patrick knew that common standard unit (such as cm) is reliable for comparing perimeters.

Comparing Area (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Patrick explained that he was unable to determine which shape has the larger area. Excerpt P25 shows the justification that he made (Patrick/L786-797).

Excerpt P25

R: (Puts the following table in front of Patrick). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 1	25 triangles	12 squares

S: *Em Set 1 bergantung kepada keluasan segiempat (sama) dan juga segitiga. Jika segiempat (sama) yang digunakan adalah lebih besar, maka bentuk B mempunyai luas yang lebih besar daripada bentuk A.* [Em Set 1 depends on the area of the square and triangle. If it used the larger square, then shape B has the larger area than shape A.]

R: In this case, 25 is larger than 12. So, why can't shape A has a larger area?

S: *Bergantung kepada bentuk segitiga yang digunakan untuk mengukur bentuk A tadi.* [It depends on the type of triangle used to measure (the area of) shape A.]

In Excerpt P25, Patrick explained that he was unable to determine which shape has the larger area as it depends on the area of the square and triangle. Patrick elaborated that if it used the larger square, then shape B has the larger area than shape A. He expressed that even though 25 is larger than 12, it depends on the type of triangle used to measure (the area of) shape A. It indicated that Patrick focused on the unit of measure when comparing area in Set 1 with nonstandard units. He knew that nonstandard units (such as triangle and square) are not reliable for comparing areas.

In Set 2, Patrick stated that shape B has the larger area. Excerpt P26 depicts his choice of shape that has the larger area and the justification that he made (Patrick/L828-839).

Excerpt P26

R: (Puts the following table in front of Patrick). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 2	10 squares	15 squares

S: *Bentuk B mempunyai keluasan yang lebih besar berbanding dengan bentuk A.* [Shape B has the larger area compared to shape A.]
R: Why?
S: *Kerana di sini mereka menggunakan unit yang sama.* So, *15 adalah lebih besar daripada 10.* So, *di sini oleh kerana mereka sama-sama menggunakan unit yang sama, maka bentuk B memberikan keluasan yang lebih luas daripada bentuk A.* [Because here they used the same unit. So, 15 is larger than 10. So, here because they used the same unit together, thus shape B has the larger area than shape A.]

In Excerpt P26, Patrick explained that shape B has the larger area because they used the same unit, namely squares, and 15 is larger than 10. It indicated that he focused on the number of unit when comparing areas in Set 2 with common nonstandard units. Patrick did not know that common nonstandard units (such as squares) are not reliable for comparing areas.

In another situation when shape A and B had the same area, Patrick explained that the unit (of measure) used to measure shape A is larger. Excerpt P27 demonstrates his justification about their units of measurement (Patrick/L840-850).

Excerpt P27

R: If shapes A and B had the same area, what can you say about their units of measure?
S: *Kita lihat di sini bahawa unit yang digunakan untuk mengukur bentuk A adalah lebih besar. Maksudnya,* square *yang digunakan adalah lebih besar berbanding dengan bentuk B di mana mereka menggunakan segiempat (sama) yang lebih kecil berbanding dengan yang digunakan untuk mengukur bentuk A.* [We find that the unit (of measure) used to measure shape A is larger. It means that the square used to measure shape A is larger than the square used to measure shape B.]
R: Why?
S: *Kerana ia memberikan yang kecil tetapi memberikan keluasan yang sama berbanding ia menggunakan* square *yang banyak tetapi mepunyai keluasan yang sama juga dengan A.* [Because it used less (squares) but it gives the same area compared to it used more squares but gives the same area as A.]

In Excerpt P27, Patrick explained that the unit used to measure shape A is larger. He elaborated that the square used to measure shape A is larger than the square used to measure shape B. Patrick expressed that it was due to shape A used less squares and shape B used more squares to give the same area. It indicated that Patrick understand the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area.

In Set 3, Patrick stated that shape A has the larger area. Excerpt P28 reveals his choice of shape that has the larger area and the justification that he made (Patrick/L870-883).

Excerpt P28

R: (Puts the following table in front of Patrick). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 3	16 cm^2	13 cm^2

S: Shape A.
R: Why?

S: *Kerana ia memberikan nilai yang lebih besar daripada bentuk B. di sini juga mereka menggunakan unit yang sama, iaitu sentimeter persegi.* So, *16 cm^2 adalah lebih besar daripada 13 cm^2.* ...16 dengan 13, maka kita akan kata nombor yang lebih besar adalah lebih besar keluasannya. [Because it gives the value larger than shape B. Here also they used the same unit, namely square centimetre. So, 16 cm^2 is larger than 13 cm^2. …16 with 13, then we will say the larger number is the larger area.]

In Excerpt P28, Patrick explained that shape A has the larger area because they used the same unit, namely cm^2, and 16 is larger than 13. It indicated that Patrick focused on the number of unit when comparing areas in Set 3 with common standard unit. He knew that common standard unit (such as cm^2) is reliable for comparing areas.

Summary

In summary, Patrick focused on the unit of measure when comparing areas in Set 1 with nonstandard units. He knew that nonstandard units (such as triangle and square) are not reliable for comparing areas. Patrick focused on the number of unit when comparing areas in Set 2 with common nonstandard units. He did not know that common nonstandard units are not reliable for comparing areas. Patrick understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area. He focused on the number of unit when comparing areas in Set 3 with common standard unit. Patrick knew that common standard unit (such as cm^2) is reliable for comparing areas.

Linguistic Knowledge

In another situation in Set 3, Excerpt P29 exhibits how Patrick wrote 16 cm^2 and 13 cm^2 in English words (Patrick/L884-891).

Excerpt P29

R: (Puts a blank paper written the following measurements in front of Patrick).
16 cm^2
13 cm^2
How would you write these measurements in English words?
S: (Writes the following).

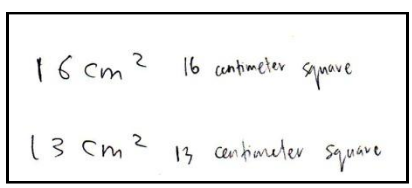


Figure N98. Patrick writes 16 cm^2 and 13 cm^2 in English words.

In Excerpt P29, Patrick wrote 16 cm^2 and 13 cm^2 literally as ‘16 centimeter square’ and ‘13 centimeter square’, as shown in Figure N98. The correct answer should be ‘sixteen square centimetres’ and ‘thirteen square centimetres’. It indicated that he did not know about the conventions pertaining to writing of Standard International (SI) area measurement units.

Converting Standard Units of Area Measurement

Procedural Knowledge

Patrick realized that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2. Excerpt P30 shows the algorithms that Patrick used when he was converting 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2 (Patrick/L903-939).

Excerpt P30

R: (Puts a card written the following scenario in front of Patrick). Some Form One teachers noticed that several of their students seemed to multiply by 10, 100, and 1000, respectively when they were converting units of area from cm^2 to mm^2, m^2 to cm^2, and km^2 to m^2:

3 cm^2 = 3 x 10 mm^2 = 30 mm^2
4.7 m^2 = 4.7 x 100 cm^2 = 470 cm^2
1.25 km^2 = 1.25 x 1000 m^2 = 1250 m^2

What would you do if you were teaching Form One and you noticed that several of your students were doing this?

S: (Writes the following).
1 cm = 10 mm
1 m = 100 cm
1 km = 1000 m
Em saya akan bagi dengan macam [Em I will give like] 1 cm = 10 mm, 1 m = 100 cm,1 km = 1000 m. So, *di sini apabila kita nak tukar* [here when we want to convert] 3 cm^2…(silent for a while and then converts 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2, as shown in Figure N99).

R: Could you explain your solution?

S: Ok 3 *sentimeter persegi. Di sini* 3 *darab* 10 mm *darab* 10 mm. 3 *di sini mewakili inilah* (points to the 3 in 3 cm^2). 1 cm = 10 mm. So, *sini* 10 mm *mewakili untuk unit* mm. *Ini juga* 10 mm. *Apabila kita darab, ia akan dapat* mm square (misreads mm^2 as mm square). So, *kita darab kita kena pasti untuk millimeter persegi. Sama juga dengan ini* (points to 4.7 m^2), 1 m = 100 cm. 4.7 m^2 = 4.7 x 100 cm x 100 cm. So, *unit* cm *darab unit* cm, *ia kena menjadi* cm square (misreads cm^2 as cm square). *Sama juga dengan* km (points to 1.25 km^2), 1 km = 1000 m. 1.25 k m^2= 1.25 x 1000 m x 1000 m = 1 250 000 m^2. [Ok 3 square centimetres. Here 3 times 10 mm times 10 mm. The 3 here represents this (points to the 3 in 3 cm^2). 1 cm = 10 mm. So, here 10 mm represents the unit mm. Here also 10 mm. When we multiplied, it will get mm square (misreads mm^2 as mm square). So, (when) we multiplied, we have to make sure (that the unit is) square millimetre. Same as this one (points to 4.7 m^2), 1 m = 100 cm. 4.7 m^2 = 4.7 x 100 cm x 100 cm. So, unit cm times unit cm, it has to become cm square (misreads cm^2 as cm square). Same as the km (points to 1.25 km^2), 1 km = 1000 m. 1.25 k m^2= 1.25 x 1000 m x 1000 m = 1 250 000 m^2.]

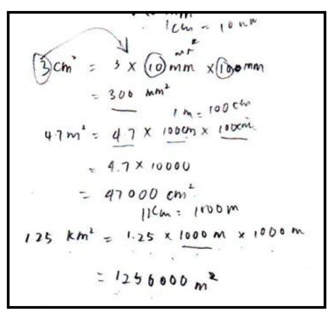


Figure N99. Patrick converts 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2.

In Excerpt P30, Patrick has successfully converted 3 cm^2 to mm^2. He viewed 3 cm^2 as the product of 3 times 1 cm times 1 cm. Patrick knew the relationship between the standard units of length measurement that 1 cm = 10 mm. He also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Patrick times 10 when he converted 1 cm to mm twice, as shown in Figure N99.

In Excerpt P30, Patrick has successfully converted 4.7 m^2 to cm^2. He viewed 4.7 m^2 as the product of 4.7 times 1 m times 1 m. Patrick knew the relationship between the standard units of length measurement that 1 m = 100 cm. He also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Patrick times 100 when he converted 1 m to cm twice, as shown in Figure N99.

In Excerpt P30, Patrick has also successfully converted 1.25 km^2 to m^2. He viewed 1.25 km^2 as the product of 1.25 times 1 km times 1 km. Patrick knew the relationship between the standard units of length measurement that 1 km = 1000 m. He also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Patrick times 1000 when he converted 1 km to m twice, as shown in Figure N99.

Summary

In summary, Patrick realized that the students made mistakes when they were converting 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2 respectively. He knew the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. Patrick also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. He viewed 3 cm^2 as the product of 3 times 1 cm times 1 cm, 4.7 m^2 as the product of 4.7 times 1 m times 1 m, and 1.25 km^2 as the product of 1.25 times 1 km times 1 km. Thus, Patrick times 10, 100, and 1000 respectively when he converted 1 cm to mm, 1 m to cm, and 1 km to m twice, as shown in Figure N99.

Conceptual Knowledge

Patrick knew the relationships between the standard units of length measurement such as 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. These can be seen in Figure N99. He also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring, as shown in Figure N99.

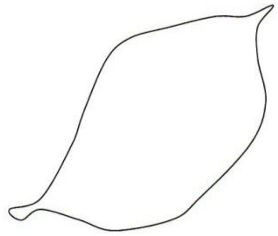
Relationship between Perimeter and Area
(Same Perimeter, Same Area?)

Conceptual Knowledge

Patrick did not know that there is no direct relationship between perimeter and area. He did not know that two shapes with the same perimeter can have different areas. Thus, Patrick thought that the student's method of calculating the area of the leaf was acceptable. Excerpt P31 shows Patrick's responses to the Form One student (Patrick/L959-1002).

Excerpt P31

R: (Puts a card written the following scenario in front of Patrick). This is a picture of a leaf. A Form One student said that he had found a way to calculate the area of the leaf. The student placed a thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle.



How would you respond to this student?

S: Can be acceptable (laughs).

R: Why?

S: Because also we are placed a thread around the boundary that we can form one rectangle. Then from the rectangle, we can come out with formula and then from formula we can come out with the area.

R: Could you tell me more about it?

S: We cannot, *kita tidak boleh mengenal apa, macam mana nak cari luas tertentu bagi bentuk ini. Tapi apabila kita meletak benang mengelilingi dan kita membentuk satu segitiga atau bentuk segiempat (tepat), mungkin kita akan boleh mencari luas kerana segitiga atau segiempat (tepat) mempunyai formula tersendiri untuk mencari luas.* So, *kita boleh* confirm-*kan luas ini apabila dibentuk dengan bentuk yang mempunyai formula untuk mencari luas, maka kita akan mendapat luas bentuk ini.*
[We cannot, we cannot recognize how to find the specific area for this shape. But when we place (a piece of) thread around and we form a triangle or rectangle, may be we can find the area because a triangle or rectangle has its own formula to find area. So, we can confirm this area when it formed with the shape that has a formula to find area, then we can find the area of this shape.]

R: Would the student's method works?

S: …(Silent for a while) *mungkin boleh diterima tetapi kita boleh menyukat keluasan daun ini menggunakan* [can be acceptable but we can measure the area of this leaf using] grid paper.

R: Would the student's method correct?

S: …(Silent for a while) Can be acceptable.

R: Do you agree with the student's method?

S: Yeah …(silent for a while) may be I will discuss with my colleagues.

In Excerpt P31, Patrick thought that the student’s method of calculating the area of the leaf was acceptable because the student placed a piece of thread around the boundary of the leaf and then rearranged the thread to form another shape, namely rectangle (with specific area formula), and got the area of the leaf as the area of a rectangle. He was wondering how to find the specific area of the leaf.

Patrick agreed with the student’s method of calculating the area of the leaf by placing a piece of thread around the boundary of the leaf and then rearranged the thread to form another shape, a triangle or rectangle that has its own formula to find area. He explained that the area of the leaf then can be determined as the area of a shape that has its own formula to find area. Patrick reiterated that the student’s method of calculating the area of the leaf was acceptable. He also suggested that the area of the leaf can be measured using grid paper. Even though Patrick agreed with the student’s method of calculating the area of the leaf, he would seek his colleagues’ view to verify it.

Summary

In summary, Patrick did not know that there is no direct relationship between perimeter and area. He did not know that two shapes with the same perimeter can have different areas. Thus, Patrick thought that the student’s method of calculating the area of the leaf was acceptable.

Ethical Knowledge

In Task 5.1, Patrick thought that the student's method of calculating the area of the leaf was acceptable, as shown in Excerpt P31. Even though Patrick agreed with the student's method of calculating the area of the leaf, he would seek his colleagues' view to verify it. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter has the same area. Patrick did not attempt to examine the possible pattern of the relationship between perimeter and area.

Patrick stated that he would seek his colleagues' view to verify the student's method of calculating the area of the leaf. It indicated that Patrick relied on authority, namely other people's view, to verify the correctness of the student's method of calculating the area of the leaf. He did not attempt to formulate generalization pertaining to the relationship between perimeter and area. Patrick never tests the student's generalization that two shapes with the same perimeter have the same area.

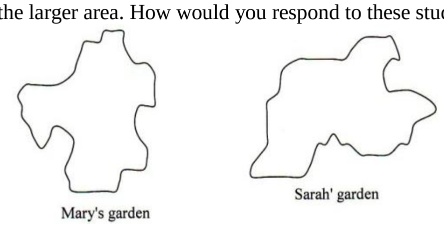
Relationship between Perimeter and Area
(Longer Perimeter, Larger Area?)

Conceptual Knowledge

Patrick did not know that there is no direct relationship between perimeter and area. He did not know that the garden with the longer perimeter could have a smaller area. Thus, Patrick thought that Mary's claim was correct. Excerpt P32 shows Patrick's responses to the claim made by Mary that the garden with the longer perimeter has the larger area (Patrick/L1051-1083).

Excerpt P32

R: (Puts a card written the following scenario in front of Patrick). Mary and Sarah are discussing whose garden has the larger area to plant flowers. Mary claims that all they have to do is walk around the two gardens to get the perimeter and the one with the longer perimeter has the larger area. How would you respond to these students?



S: I'll give an example. Let's say there are two rectangles here (draws two rectangles, labels its dimensions, and then calculates its perimeter and area, as shown in Figure N100).

R: Could you explain your solution?

S: Here we give them example. *Kita memberi contoh, A dan B. A mempunyai perimeter yang lebih besar daripada B. Selepas itu, kita membuat pengiraan. Jika* garden *itu adalah berbentuk segiempat (tepat), AB adalah 6 cm dan juga BC adalah 5 cm.* So, *luasnya adalah 30 cm^2*. So, *B pula memberikan perimeter yang kecil iaitu 5 tambah 4 tambah 5 tambah 4, jadi 18 cm. Begitu juga dengan luasnya, ia akan memberikan nilai yang kecil juga apabila perimeternya kecil.*
[Here we give them example. We give example, A and B. A has the longer perimeter than B. After that, we do calculation. If the garden is a rectangle, AB is 6 cm and also BC is 5 cm. So, its area is 30 cm^2. So, B has the shorter perimeter, namely 5 plus 4 plus 5 plus 4, thus 18 cm. As with its area, it will also give small value when its perimeter is small.]

R: Would Mary's method works?

S: Em (nods his head to indicate yes) *betul* [correct].

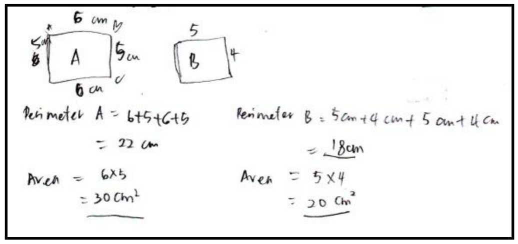


Figure N100. Patrick draws two rectangles, labels its dimensions, and then calculates its perimeter and area.

In Excerpt P32, Patrick gave an example where he drew two rectangles, labelled its dimensions, and then calculated its perimeter and area, as shown in Figure N100. Patrick found that rectangle A with the longer perimeter (22 cm) has the larger area (30 cm^2) compared to rectangle B with the perimeter of 18 cm and the area of 20 cm^2. Thus, he thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area.

Summary

In summary, Patrick did not know that there is no direct relationship between perimeter and area. He did not know that the garden with the longer perimeter could have a smaller area. Thus, Patrick thought that Mary's claim was correct.

Ethical Knowledge

In Task 5.2, Patrick thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Patrick attempted to examine the possible pattern of the relationship between perimeter and area. He generated an example where he drew two rectangles, labelled its dimensions, and then calculated its perimeter and area, as shown in Figure N100. Patrick found that rectangle A with the longer perimeter (22 cm) has the larger area (30 cm^2) compared to rectangle B with the perimeter of 18 cm and the area of 20 cm^2.

He formulated a generalization pertaining to the relationship between perimeter and area that the garden with the longer perimeter has the larger area that concurred with Mary's generalization. Patrick tested Mary's generalization that the garden with the longer perimeter has the larger area with the example that he generated, as shown in Figure N100. Nevertheless, he did not know that an example could not be used to determine the truth of a generalization. A counterexample can be used to refute the truth of a generalization.

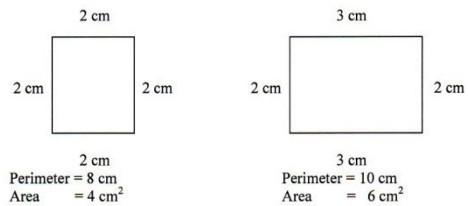
Relationship between Perimeter and Area
(Perimeter Increases, Area Increases?)

Conceptual Knowledge

Patrick did not know that there is no direct relationship between perimeter and area. He did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Patrick thought that the student's "theory" was correct. This is shown in Excerpt P33 (Patrick/L1130-1178).

Excerpt P33

R: (Puts a card written the following scenario in front of Patrick). Suppose that one of your Form One students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:



How would you respond to this student?

S: (Draws two equilateral triangles with the length of side 2 cm and 3 cm respectively and then calculates its perimeters and areas, as shown in Figure N101). (Patrick made some mistakes. The height for the first triangle is 1.7 cm, not 1.2 cm. Thus, the area for the first triangle is 1.7 cm^2, not 1.2 cm^2. The height for the second triangle is 2.6 cm, not 2.1 cm. Thus, the area for the second triangle is 3.9 cm^2, not 3.15 cm^2. He did not realize his mistakes.) Can be accepted.

R: Would this student's claim correct?

S: Em (nods his head to indicate yes).

R: Why?

S: Because I tried by using *segitiga di mana apabila segitiga sama, bila kita letak* 2 cm*, semua sisi mempunyai panjang yang sama;* 2 cm, 2 cm, 2 cm. *Perimeternya akan memberikan* 6 cm. So, *apabila kita cari* area, *setengah darab tapak darab tinggi.* So, *kita boleh dapat* 1.2 cm^2 (makes a mistake. It should be 1.7 cm^2). So, *kita menggunakan sisi yang sama panjang,* 3 cm *pula memberikan perimeter* 9 cm. Area*nya adalah setengah darab* 3 *darab tinggi* 2.1 (makes a mistake. It should be 2.6 cm, not 2.1 cm) *dengan menghasilkan* 3.15 cm^2 (makes a mistake. It should be 3.9 cm^2). So, *di sini kita lihat peningkatan perimeter akan memberikan peningkatan kepada luas.*

[Because I tried by using triangle, equilateral triangle. When we put 2 cm, all the sides have the same length: 2 cm, 2 cm, 2 cm. Its perimeter is 6 cm. So, when we find area, half times the base times the height. So, we can get 1.2 cm^2 (makes a mistake. It should be 1.7 cm^2). So, when we used the side with the same length, 3 cm, the perimeter is 9 cm. Its area is half times 3 times the height 2.1 (makes a mistake. It should be 2.6 cm, not 2.1 cm) equal to 3.15 cm^2 (makes a mistake. It should be 3.9 cm^2). So, here we see that as the perimeter increases, the area also increases.]

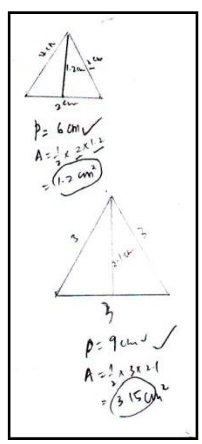


Figure N101. Patrick draws two equilateral triangles and then calculates its perimeters and areas.

In Excerpt P33, Patrick drew two equilateral triangles with the length of side 2 cm and 3 cm respectively and then calculated its perimeters and areas, as shown in Figure N101. He made some mistakes. The height for the first triangle is 1.7 cm, not 1.2 cm. Thus, the area for the first triangle is 1.7 cm^2, not 1.2 cm^2. The height for the second triangle is 2.6 cm, not 2.1 cm. Thus, the area for the second triangle is 3.9 cm^2, not 3.15 cm^2. Patrick did not realize his mistakes.

He stated that the student's "theory" can be accepted. When probed further, Patrick indicated that the student's claim was correct because his example showed that as the perimeter of the triangle increases from 6 cm to 9 cm, its area also increases from 1.7 cm^2 to 3.9 cm^2. The example generated by him concurred with the student's "theory that as the perimeter of a closed figure increases, the area also increases. Patrick did not know that the student's claim about the relationship between perimeter and area is not a theory. The claim is a conjecture. He also did not know that an example is not a proof and a theory cannot be proved by an example.

Summary

In summary, Patrick did not know that there is no direct relationship between perimeter and area. He did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Patrick thought that the student's "theory" was correct.

Ethical Knowledge

In Task 5.3, the student formulated a generalization that as the perimeter of a closed figure increases, the area also increases. Patrick thought that the student's "theory" was correct. He generated an example to test the student's generalization, as shown in Excerpt P33. The example generated by him concurred with the student's "theory that as the perimeter of a closed figure increases, the area also increases. In reality, when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same.

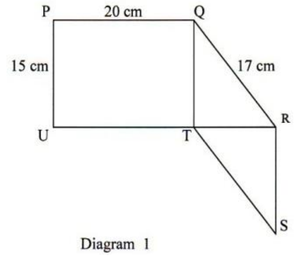
Calculating Perimeter and Area
(Rectangle and Parallelogram/Triangle)

Procedural knowledge

After read through Task 6.1, Patrick labelled the missing sides of Diagram 1 that required for calculating the perimeter and area of the diagram, as shown in Figure N102 in Excerpt P34 (Patrick/L1187-1240).

Excerpt P34

R: (Puts a card written the following problem in front of Patrick). Suppose that one of your Form One students asks you for help with the following problem:



In Diagram 1, PQTU is a rectangle and QRST is a parallelogram. UTR is a straight line.
Calculate
(g) the perimeter of the diagram,
(h) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 1, as shown in Figure N102). (Uses Pythagoras' theorem to calculate the length of TR, as shown in Figure N103).

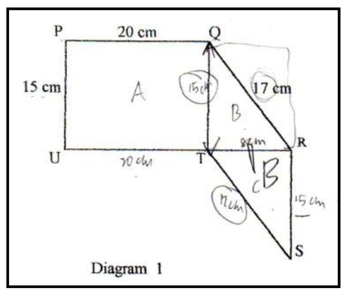


Figure N102. Patrick labels the missing sides of Diagram 1.

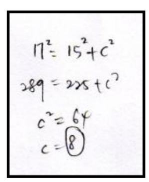


Figure N103. Patrick calculates the length of TR.

In Excerpt P34, Patrick labelled TU, QT, RS, and ST as 20 cm, 15 cm, 15 cm, and 17 cm respectively on Diagram 1, as shown in Figure N102. Patrick realized that he needed to find the length of TR in order to calculate the area of Diagram 1. Patrick has successfully calculated the length of TR using Pythagoras' theorem, as shown in Figure N103. Excerpt P35 depicts how Patrick has successfully calculated the perimeter of Diagram 1 (Patrick/L1242-1246, L1253-1256, L1263-1264).

Excerpt P35

S: (Calculates the perimeter of Diagram 1, as shown in Figure N104).

R: Could you explain your solution?

S: Ok for perimeter, *untuk perimeter, kita akan mengira lingkungan yang membentuk satu* figure *ini, iaitu daripada sini, dari PQ, QR, RS, ST, TU, dan UP. Kita tak akan kira untuk jarak QT dan juga RT. …*So, *kita akan hanya menambah jarak yang di luar-luar ini.* So, *kita akan dapat 104 cm.*
[Ok for perimeter, for perimeter, we will calculate the surround that form this figure, namely from here, from PQ, QR, RS, ST, TU, and UP. We will not include the length of QT and also RT. …So, we will only add these outer lengths. So, we will get 104 cm.]

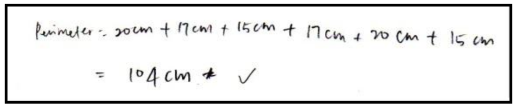


Figure N104. Patrick calculates the perimeter of Diagram 1.

In Excerpt P35, Patrick used the list all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N104. He listed all the length of sides that surrounded the diagram, namely PQ, QR, RS, ST, TU, and UP, and then summed them up to get the perimeter of the diagram as 104 cm. Patrick explained that QT and RT were not included in the calculation of the perimeter as they were not the "outer lengths" that surrounded Diagram 1.

Excerpt P36 demonstrates how Patrick has successfully calculated the area of Diagram 1 (Patrick/L1248-1251, L1265-1275).

Excerpt P36

S: (Calculates the area of Diagram 1, as shown in Figure N105).

R: Could you explain your solution?

S: *Bagi* area, *untuk* area *kita akan bahagi kepada tigalah, A, B, C. Untuk A, kita akan menggunakan formula segiempat tepat iaitu sisi PQ darab sisi PU. PQ sama dengan 20 cm dan PU sama dengan 15.* So, *kita akan dapat 300 cm^2. Untuk B pula, kita terpaksa menggunakan rumus trigonometri iaitu hipotenus sama dengan sisi bertentangan dan juga sisi bersebelahan kuasa dualah. Kita akan dapat sisi TR. RT* (sic) *ini adalah sama dengan 8 cm.* So, *kita darab satu perdua darab tapak darab tinggi, 15 cm. Dari sini dapat 60. S*o, *C pula, kita akan menggunakan formula yang sama, satu perdua darab tinggi darab tapak. Kita dapat 60.* So, *kita tambah ketiga-tiga A, B, C, kita akan dapat 420 cm^2.*
[For area, for area, we will partition it into three, A, B, C. For A, we will use the formula for a rectangle, namely the side of PQ times the side of PU. PQ equal to 20 cm and PU equal to 15. So, we will get 300 cm^2. For B, we have to use the formula of trigonometry, namely hypotenuse equal to the square of the opposite side and the square of the adjacent side. We will get the side of TR. This RT (sic) is equal to 8 cm. So, we multiply, one over two times the base times the height, 15 cm. From here, get 60. So, for C, we will use the same formula, one over two times the height times the base. We get 60. So, we add all the three A, B, C, we will get 420 cm^2.]

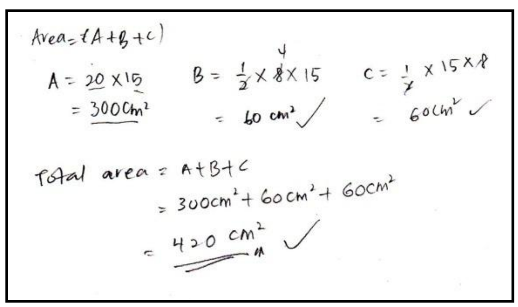


Figure N105. Patrick calculates the area of Diagram 1.

In Excerpt P36, Patrick used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N105. He partitioned Diagram 1 into a rectangle PQTU (labelled as A) and two triangles QRT (labelled as B) and RST (labelled as C). Patrick calculated the area of A, B, and C using the area formulae of rectangle and triangles respectively and then summed them up to get the area of the diagram as 420 cm^2.

Summary

In summary, Patrick has successfully calculated the perimeter of Diagram 1 using the list all-and-sum algorithm. He has also correctly calculated the area of Diagram 1 using the partition-and-sum algorithm.

Linguistic knowledge

Patrick used the correct standard units of measurement for perimeter (cm) and area (cm^2) when he wrote the answers for these measurements, as shown in Figures N106 and N107.

Strategic Knowledge

When probed to check the answer for the perimeter, Patrick suggested that he would use the recalculating strategy to verify the answer. Excerpt P37 is illustrative (Patrick/L1276-1282).

Excerpt P37

R: How would you check your answer for the perimeter?

S: *Untuk perimeter, kita akan lihat definisinyalah. Ia merupakan jarak yang melitupi sesuatu kawasan yang dilitupi itu. Tapi kita tak akan ambil jarak yang di dalam.* So, *kita* just *mengira ukuran yang di luar sajalah.*
[For perimeter, we will look at its definition. It is the length that surrounds an enclosed region. But we will not include the inner length. So, we just calculate the outer lengths only.]

In Excerpt P37, Patrick suggested that he would check the answer for perimeter by recalculating strategy that using the same method and calculate again. Patrick defined perimeter as the length that surrounded an enclosed region. He explained that inner lengths (QT and RT) would not be included in the calculation of the perimeter.

When probed to check the answer for the area, Patrick used an alternative procedure (alternative method) to generate an answer which could be used to verify his original answer. Excerpt P38 is illustrative (Patrick/L1283-1298).

Excerpt P38

R: How would you check your answer for the area?

S: Ok *di sini kalau kita tak menggunakan formula ini* (point to the area formula of a triangle, half times base times height that he has used to calculate the area of triangle TQR), *kita akan guna formula yang kedua untuk A dan ini* (points to triangles TQR and TRS) *kita akan gabung jadi B* (refers to the rectangle with the length QT and the height TR). *Ini* (points to triangle TRS) *akan pergi atas sini.* (Uses alternative method to calculate the area of Diagram 1, as shown in Figure N106). *So, kita akan menggunakan dua kaedahlah. So, kedua-dua kaedah akan memberikan jawapan yang sama.* So, *kita boleh membuktikan bahawa* area *ini adalah 420 sebab menggunakan dua kaedah yang berbeza tapi kita mendapat jawapan yang sama.*
[Ok here if we don't want to use this formula (point to the area formula of a triangle, half times base times height that he has used to calculate the area of triangle TQR), we can use a second formula for "A" and this (points to triangles TQR and TRS) we will combine them as "B" (refers to the rectangle with the length QT and the height TR). This (points to triangle TRS) will go up here. (Uses alternative method to calculate the area of Diagram 1, as shown in Figure N106). S0, both methods gave the same answer. So, we can prove that this area is 420 because two different methods gave the same answer.]

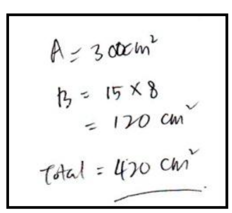


Figure N106. Patrick uses alternative method to calculate the area of Diagram 1.

In Excerpt P38, Patrick checked the answer for area by moving triangle RST under the translation T_{SR} to form a rectangle (labelled as new "B") with the dimensions of 15 cm by 8 cm, as shown in Figure N102. He calculated area of "B" by using area formula of a rectangle, namely 15 x 8 = 120 cm^2. Patrick explained that both methods gave the same answer, namely 420 cm^2. Thus, it confirms that the answer was correct.

Ethical Knowledge

Patrick has successfully calculated the perimeter and area of Diagram 1. Nevertheless, he did not check the correctness of the answers for perimeter as well as area. When probed to check answers, then only Patrick suggested the strategies that he would use to check the answers for perimeter and area. Patrick wrote the measurement units (without probed) for the answers of the perimeter and area that he has calculated, as shown in Figures N104, N105, and N106.

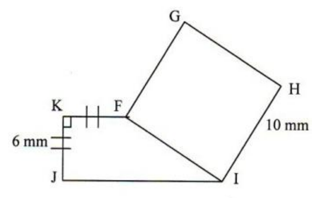
Calculating Perimeter and Area
(Square and Trapezium/Triangle)

Procedural Knowledge

After read through Task 6.2, Patrick labelled the missing sides of Diagram 2 that required for calculating the perimeter and area of the diagram, as shown in Figure N107 in Excerpt P39 (Patrick/L1317-1343, L1352-1355).

Excerpt P39

R: (Puts a card written the following problem in front of Patrick). Suppose that one of your Form One students asks you for help with the following problem:



In Diagram 2, FGHI is a square and FIJK is a trapezium.
Calculate
(g) the perimeter of the diagram,
(h) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 2, as shown in Figure N107). (Calculates the value of "A" using Pythagoras' theorem, as shown in Figure N108).

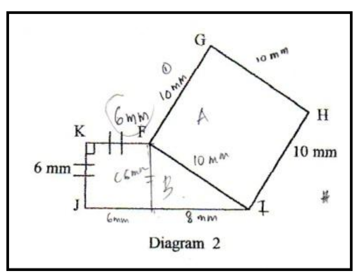


Figure N107. Patrick labels the missing sides of Diagram 2.

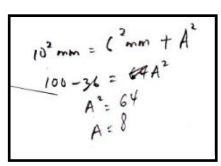


Figure N108. Patrick calculates the value of "A" using Pythagoras' theorem.

In Excerpt P39, Patrick labelled FG, GH, FI, KF, the opposite side of KF, and the opposite side of KJ as 10 mm, 10 mm, 10 mm, 6 mm, 6 mm, and 6 mm respectively on Diagram 2, as shown in Figure N107. Patrick realized that he needed to find the value of “A”, as shown in Figure N108. Patrick partitioned trapezium FIJK into a square and a triangle, as shown in Figure N107. He has successfully calculated the value of “A” as 8 (mm) using Pythagoras’ theorem, as shown in Figure N108.

Excerpt P40 depicts how Patrick has successfully calculated the perimeter of Diagram 2 (Patrick/L1345-1350, L1362-1364).

Excerpt P40

S: (Calculates the perimeter of Diagram 2, as shown in Figure N109).

.
.
.

R: Could you explain your solution?

S: Em for the perimeter, we just calculate the outside measurement. So, we get 56 cm (wrong unit. It should be mm).

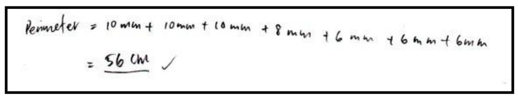


Figure N109. Patrick calculates the perimeter of Diagram 2.

In Excerpt P40, Patrick used the list all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N109. He listed all the length of sides that surrounded the diagram and then summed them up to get the perimeter of the diagram as 56 cm (wrong unit. It should be mm). Patrick explained that we just calculate the “outside measurement” for the perimeter.

Excerpt P41 demonstrates how Patrick has successfully calculated the area of Diagram 2 (Patrick/L1357-1360, L1364-1378).

Excerpt P41

S: (Calculates the area of Diagram 2, as shown in Figure N110).

.
.
.

R: Could you explain your solution?

S: …So, for the area, we used two methods, that is Method 1, the area of square plus the area of the trapezium. So, here we can get the area of the square equal to 10 mm times 10 mm equal to 100 mm^2. So, for area B, that is the trapezium, we use the formula one over two times the base plus the opposite side times the height. We get 60 mm square (misreads mm^2 as mm square). So, the total is 160 for the area. …

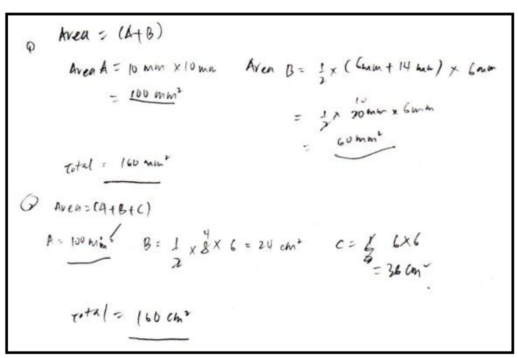


Figure N110. Patrick calculates the area of Diagram 2.

In Excerpt P41, Patrick explained that there were two methods to calculate the area of Diagram 2. In the first method, he used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N110. Patrick partitioned Diagram 2 into square FGHI and trapezium FIJK. He calculated the area of the square and trapezium separately using the area formulae of square and trapezium respectively and then summed them up to get the area of the diagram as 160 mm^2. Patrick stated that the second method is meant to check the correctness of the answer for the area of Diagram 2. Thus, the second method will be discussed in the later section, namely strategic knowledge.

Summary

In summary, Patrick has successfully calculated the perimeter of Diagram 2 using the list all-and-sum algorithm. He has also correctly calculated the area of Diagram 2 using the partition-and-sum algorithm.

Linguistic Knowledge

Patrick mistakenly used the incorrect units of measurement for perimeter (cm) when he wrote the answer of this measurement, as shown in Figure N109. The correct unit of measurement for perimeter should be mm. Patrick also mistakenly used the incorrect units of measurement for area (cm^2) in the second method when she wrote the answer of this measurement, as shown in Figure N110. The correct units of measurement for area should be mm^2.

Strategy Knowledge

When probed to check the answer for the perimeter, Patrick suggested that he would use the recalculating strategy to verify the answer. Excerpt P42 is illustrative (Patrick/L1379-1382).

Excerpt P42

R: How would you check your answer for the perimeter?

S: Perimeter we can just calculate again, but then without calculate the line FI. So, if we get the same answer (laughs), that means we get the same solution also. The solution is correct.

In Excerpt P42, Patrick suggested that he would check the answer for the perimeter by the recalculating strategy that using the same method and calculate again. Patrick stated that we can just calculate again but exclusive of the length of FI. He explained that if the answer was the same, then the answer was correct.

Patrick used alternative method to verify the answer for the area. Excerpt P43 is illustrative (Patrick/L1370-1378).

Excerpt P43

S: … So, we want to determine whether our method for finding the area of this diagram correct or not, we use the second method that is A plus B plus C. A is the square, B is the triangle and then C is the square also. So, for the A, for the big square, equal to 100 mm square (misreads mm^2 as mm square) and then B, the triangle, we use the formula one over two times the base times the height equal to 24 cm square (wrong unit. It should be mm^2. Misreads cm^2 as cm square). And then C, the square, 6 times 6 equal to 36 and then the total is also that 160.

In Excerpt P43, Patrick used the repartition-and-sum strategy to check the answer for the area of Diagram 2, as shown in Figure N110. He repartitioned Diagram 2 into a large square (FGHI, labelled as A), a triangle (labelled as B), and a small square (labelled as C), as shown in Figure N107. Patrick calculated the area of A, B, and C separately using the area formulae of a square, triangle, and square respectively and then summed them up to get the area of the diagram as 160 cm^2 (wrong unit. It should be mm^2), as shown in Figure N110.

Ethical Knowledge

Patrick has successfully calculated the perimeter and area of Diagram 2. Nevertheless, he did not check the correctness of the answer for perimeter. When probed to check answer, then only Patrick suggested the strategy that he would use to check the answer for perimeter. Patrick checked the correctness of the answer for area without being probed.

Patrick incorrectly wrote the measurement unit for the answer of perimeter as cm, as shown in Figure N109. The correct measurement unit for the answer of perimeter should be mm. He also incorrectly wrote the measurement unit for the answer of area in the second method as cm^2, as shown in Figure N110. The correct measurement unit for the answer of area should be mm^2.

Fencing Problem

Strategic Knowledge

Patrick used looking for a pattern strategy to solve the fencing problem. Excerpt P44 is illustrative (Patrick/L1409-1430).

Excerpt P44

R: (Puts a card written the following problem in front of Patrick). Suppose that one of your students asks you for help with the following problem:

A gardener has 84 m of fencing to enclose a garden along three sides, with the fourth side of the garden being formed by a wall. (Assume that the wall is perfectly straight). What are the dimensions of a rectangular garden that will yield the largest area being enclosed?

How would you solve this problem?

S: (Uses looking for a pattern strategy to solve this problem. Draws the possible rectangular gardens and calculates their respective areas, as shown in Figures N111 and N112).

In Excerpt P44, Patrick started off with the width and the length of the rectangular garden as 1 m and 82 m respectively and this yielded the smallest area being enclosed, namely 82 m^2. He then increased the width of the rectangular garden, one metre at a time, to 10 m and reduced the length of the rectangular garden accordingly to 64 m. Consequently, the area increased to 640 m^2, as shown in Figure N111. Patrick saw a pattern that area increases as he increases the width of the rectangular garden while reduces its length accordingly. He increased the width of the rectangular garden to 12 m instead of 11 m and reduced its length to 60 m. The area increased to 720 m^2, as shown in Figure N112.

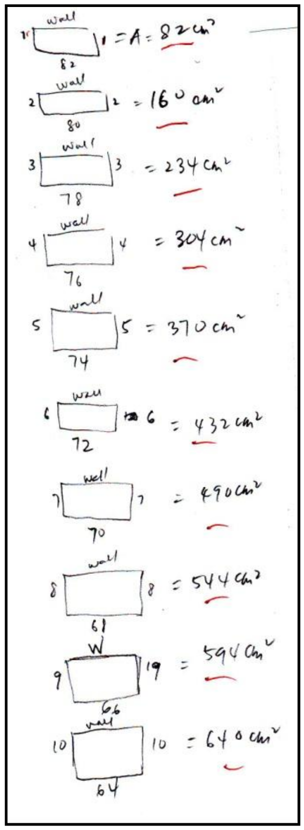


Figure N111. Patrick draws the possible rectangular gardens and calculates their areas.

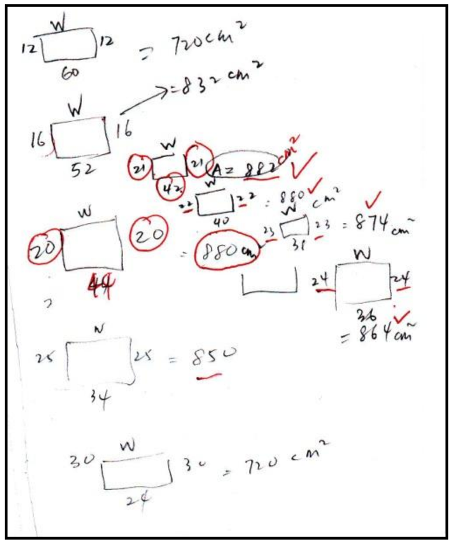


Figure N112. Patrick continues to draw the possible rectangular gardens and calculates their areas.

Subsequently, Patrick increased the width of the rectangular garden, four metres at a time, to 20 m and reduced the length of the rectangular garden accordingly to 44 m. Consequently, the area increased to 880 m^2. He then increased the width of the rectangular garden, five metres at a time, to 30 m and reduced the length of the rectangular garden accordingly to 24 m. Consequently, the area decreased to 720 m^2. Patrick realized that the area of the rectangular garden decreasing when he increased the width of the rectangular garden, five metres at a time, from 20 m to 25 m and reduced the length of the rectangular garden accordingly from 44 m to 34 m. Consequently, the area decreased from 880 m^2 to 850 m^2.

Patrick became more cautious and he decided to increase the width of the rectangular garden, one metre at a time, from 20 m to 21 m and reduced the length of the rectangular garden accordingly from 44 m to 42 m. Consequently, the area increased from 880 m^2 to 882 m^2. Patrick continued to increase the width of the rectangular garden, one metre at a time, to 24 m and reduced the length of the rectangular garden accordingly to 36 m. Consequently, the area decreased to 864 m^2. Patrick concluded that 882 m^2 was the largest area being enclosed. He circled the answer and then put a "longer" tick (√) behind it, as shown in Figure N112. Table N6 summarizes the dimensions of the rectangular garden and its area that Patrick has figured out.

Table N6

Dimensions of Rectangular Garden and its Area That Patrick has Figured out

Length (cm)	Width (cm)	Width (cm)	Area (cm^2)
82	1	1	82
80	2	2	160
78	3	3	234
76	4	4	304
74	5	5	370
72	6	6	432
70	7	7	490
68	8	8	544
66	9	9	594
64	10	10	640
60	12	12	720
52	16	16	832
44	20	20	880
34	25	25	850
24	30	30	720
42	21	21	882
40	22	22	880
38	23	23	874
36	24	24	864

When probed further, Patrick elaborated how he used looking for a pattern strategy to solve the fencing problem. Excerpt P45 is illustrative (Patrick/L1432-1448).

Excerpt P45

R: Could you explain your solution?

S: Ok here we can find out by *mencari pola*, pattern. So, here if we put 1, 1, 82, the area keep on increase. But then until, *sampai kepada satu tahap yang mana sisi yang bertentangan adalah* 20 *dan sisi yang tinggal* 44. This *adalah* 880 m^2 (reads m^2 as *meter persegi* [square metre]) *dan di sini kita nak buat dia* step by step: 21, 22, 23. 21, 882. *Apabila tepi ini* (point to the width of the rectangle) *jadi* 22, 880. 23, 23, 874. So, 24, *ia akan* 864. 25, *ia akan semakin menurun pula.* So, climax-*nya adalah apabila tepi ini* 21 (point to the width of the rectangle), *sini* 42 (point to the length of the rectangle).
[Ok here we can find out by looking for a pattern. So, here if we put 1, 1, 82, the area keep on increase. But then until, it reaches a level where the opposite sides is 20 and the remaining side is 44. This is 880 m^2 (reads m^2 as *meter persegi* [square metre]) and here we want to make it step by step: 21, 22, 23. 21, 882. When this side (point to the width of the rectangle) becomes 22, 880. 23, 23, 874. So, 24, it will (be) 864. 25, it will increasingly decrease. So, its climax is when this side 21 (point to the width of the rectangle), here 42 (point to the length of the rectangle).]

R: So, what are the dimensions that will give you the largest area?

S: Eh 21, 42, 21.

R: What is your maximum area?

S: 882.

In Excerpt P45, Patrick elaborated how he used looking for a pattern strategy to solve the fencing problem. Patrick explained that he started off with the width and the length of the rectangular garden as 1 m and 82 m respectively and this yielded the area 82 m^2. Patrick expressed that the area kept on increasing as he increased the width of the rectangular garden and reduced the length of the rectangular garden accordingly. Patrick noticed that when he increased the width of the rectangular garden to 20 m and reduced its length to 44 m, the area increased to 880 m^2. Patrick found that when he further increased the width of the rectangular garden, one metre at a time, from 20 m to 25 m and reduced the length of the rectangular garden accordingly from 44 m to 34 m, he noticed that the area increased from 880 m^2 to 882 m^2, and then kept on decreasing from 882 m^2 to 850 m^2. Thus, Patrick pointed out that it reached its "climax" (the largest area being enclosed) when the width and the length of the rectangular garden is 21 m and 42 m respectively. Patrick reiterated that 882 m^2 was the largest area being enclosed and 42 m by 21 m is the dimension of the rectangular garden that will yield the largest area being enclosed.

Summary

In summary, Patrick has successfully solved the fencing problem using the looking for a pattern strategy. Patrick used the same strategy, namely the looking for a pattern strategy, to check the answer for the fencing problem without being probed.

Ethical Knowledge

Patrick used the same strategy, namely the looking for a pattern strategy, to check the answer for the fencing problem without being probed. This can be seen in Excerpts P44 and P45. Patrick wrote the area measurement units throughout Task 7. This can be seen in Figures N111 and N112.

Developing Area Formulae

Procedural Knowledge

Patrick drew a rectangle, parallelogram, triangle, and trapezium. He then wrote their respective area formulae. Excerpt P46 is illustrative (Patrick/L1494-1510).

Excerpt P46

R: (Puts a card written the following scenario in front of Patrick). Suppose that a Form One student comes to you and says that he does not know how to develop (derive) the formula for calculating the area of the following figures:

(i) Rectangle,
(j) Parallelogram,
(k) Triangle, and
(l) Trapezium.

How would you show him the way to develop (derive) the formula for calculating the area of these figures? Let's start with rectangle.

S: (Draws a rectangle, parallelogram, triangle, and trapezium, and then writes their respective area formulae, as shown in Figure N113).

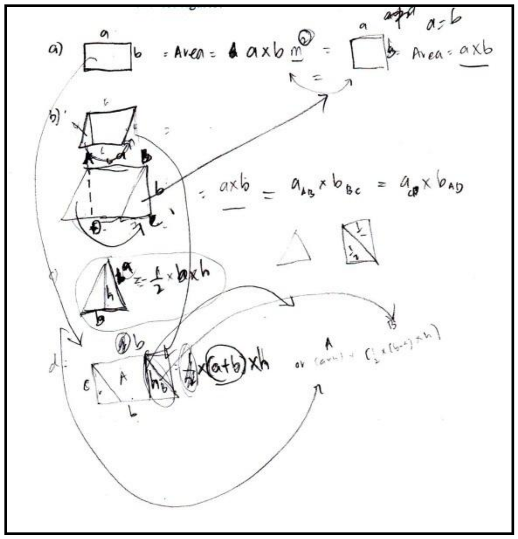


Figure N113. Patrick draws a rectangle, parallelogram, triangle, and trapezium, and then writes their respective area formulae.

Patrick could recall the formula for the area of a rectangle. Nevertheless, he was unable to develop it. Excerpt P47 is illustrative (Patrick/L1513-1525).

Excerpt P47

R: How would you develop (derive) the formula for calculating the area of a rectangle?

S: So rectangle, *apa yang kita perlu buat adalah lukiskan bentuk itu. Selepas itu, kita mewakili setiap sisi ini* (points to the length and width of the rectangle) *dengan a, b.* So, *di sini* area *untuk segiempat tepat ini adalah a darab b.* So, *untuk lebih memudahkan dia orang ingat, kita bagi* comparison-*lah. Macam mana untuk cari* area *untuk segiempat sama pula. Sama juga dengan itu, a* times *a. Maksudnya, nilai a adalah sama nilai dengan b. Nilai a sama dengan b. Maksudnya a* times *b lah.* So, *cara formula untuk mencari segiempat tepat dan segiempat sama adalah sama, sisi yang berlainan akan darab.*
[So rectangle, what we need to do is to draw the shape. After that, we represent each side (points to the length and width of the rectangle) with *a, b*. So, here area for this rectangle is *a* times *b*. So, to facilitate him to memorize, we make comparison, how to find the area of a square. Similar to that, *a* times *a*. It means the value of *a* same as the value of *b*. Value of *a* equal to *b*. It means *a* times *b*. So, the formula method of finding (the area of) a rectangle and square is the same, different side will be multiplied.]

R: How do you get the formula "a times b"?

S: …(Silent for a while and then moves his head to indicate no idea).

In Excerpt P47, Patrick suggested to draw a rectangle and then to represent its length and width as *a* and *b*. He could recall the formula for the area of a rectangle. Patrick stated the formula for the area of a rectangle is '$a \times b$', as shown in Figure N113. He suggested to make comparison with the formula for the area of a square, namely '$a \times a$', in order to facilitate the student to memorize the formula for the area of a rectangle. Nevertheless, Patrick was unable to develop it. He just memorized the formula. Patrick did not attempt to develop the formula.

Patrick could recall the formula for the area of a parallelogram as '$a \times b$', as shown in Figure N113. He also knew how to develop the formula for the area of a parallelogram. Excerpt P48 is illustrative (Patrick/L1526-1543).

Excerpt P48

R: How would you develop (derive) the formula for calculating the area of a parallelogram?

S: Parallelogram. *Kita akan lukis juga bentuk* parallelogram. Ok *untuk menyenangkan pengiraan kita akan bahagi kepada dualah Ini* (points to the right-angled triangle from one end of the parallelogram) *kita akan pindah ke sini* (points to the other end of the parallelogram). So, last *ia akan dapat segiempat tepat.* So, *kita akan gunakan formula*.
[Parallelogram. We will draw a parallelogram. Ok to ease the calculation, we will partition it into two. This (points to the right-angled triangle from one end of the parallelogram) we will move to here (points to the other end of the parallelogram). So, (at) last, it will get a rectangle. So, we will use its formula.]

R: So, what's the formula for the area of a parallelogram?

S: *a* times *b lah.*

R: What does the "a" stand for?

S: "a" is here (points to the base of the parallelogram), "b" is here (points to the height of the parallelogram).

R: Could you tell me more about it?

S: "a" *mewakili jarak, eh ukuran daripada AB dan "b" mewakili ukuran daripada BC.* So, *kita tahu yang jarak AB akan sama dengan CD dan juga BC adalah sama dengan AD.* So, *kita boleh* preferable *lah ini, sama ada* "a" *mewakili AB* times "b" *mewakili BC* (points to the formula $a_{AB} \times b_{BC}$, as shown in Figure N113) *atau* "a" *mewakili CD* times "b" *mewakili AD* (points to the formula $a_{CD} \times b_{AD}$, as shown in Figure N113).
["a" represents the distance, the measurement of *AB* and *"b"* represents the measurement of *BC*. So, we know that the distance of *AB* equals to the distance of *CD* and also *BC* equals to *AD*. So, we can preferable this, either "a" represents *AB* times "b" represents *BC* (points to the formula $a_{AB} \times b_{BC}$, as shown in Figure N113) or "a" represents *CD* times "b" represents *AD* (points to the formula $a_{CD} \times b_{AD}$, as shown in Figure N113).

In Excerpt P48, Patrick suggested to draw a parallelogram. He could recall the formula for the area of a parallelogram. Patrick stated that the formula for the area of a parallelogram is '$a \times b$', as shown in Figure N113. He also knew how to develop the formula for the area of a parallelogram. Patrick mentally cut out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle, as shown in Figure N113. Thus, the area of the parallelogram equals to the area of the rectangle formed and its area formula is 'a times b' or 'base times height'. He explained that the formula for the area of a parallelogram could also be written as $a_{AB} \times b_{BC}$ or $a_{CD} \times b_{AD}$, as shown in Figure P28, because $AB = CD$ and $BC = AD$.

Patrick could recall the formula for the area of a triangle, namely '$\frac{1}{2} \times b \times h$', as shown in Figure N113. Nevertheless, he was unable to develop it. Excerpt P49 is illustrative (Patrick/L1544-1548).

Excerpt P49

R: How would you derive the formula for calculating the area of a triangle?

S: *Eh juga sama. Kita akan lukis* diagram *itu sendiri.* So, *kita akan letak symbol a, b. "a" mewakili sendeng, "b" mewakili tapak dan "h" mewakili ketinggian.* So, *di sini kita* just *letak satu perdua darab tapak,* base times height (points to the formula $\frac{1}{2} \times b \times h$).
[Eh also the same. We will draw its diagram. So, we will assign symbols *a, b. "a"* represents the slant, *"b"* represents the base and "h" represents the height . So, here we just put one over two times base times height (points to the formula $\frac{1}{2} \times b \times h$).]

In Excerpt P49, Patrick suggested to draw a triangle. He assigned the symbols a, b, and h to represent the slant, base, and height of the triangle, as shown in Figure N113. Patrick could recall the formula for the area of a triangle. He stated that the formula for the area of a triangle is 'one over two times base times height'. Nevertheless, Patrick did not know how to develop the formula for the area of a triangle. Patrick just memorized the formula. He did not attempt to develop the formula.

Patrick could recall the formula for the area of a trapezium, namely '$\frac{1}{2} \times (a + b) \times h$', as shown in Figure N113. Nevertheless, he was unable to develop it. Patrick attempted to develop the formula using algebraic method but unsuccessful. Excerpt P50 is illustrative (Patrick/L1552-1555, L1558-1572).

Excerpt P50

R: How would you derive the formula for calculating the area of trapezium?

S: Em draw the diagram and then come out one over two times "a", base plus opposite one times "h" (points to the formula $\frac{1}{2} \times (a + b) \times h$).

.
.
.

R: How would you derive the formula for calculating the area of a trapezium?

S: One over two…(silent for a while) I also don't kow how to derive.

R: Would you like to try?

S: And then times "*a* plus *b*". "*a*" *mewakili sisi yang bertentangan dengan tapak*. *"a tambah b"*. *Lepas itu kalikan dengan ketinggian…*(silent for a while) *sebenarnya, kita boleh dapat dengan dua formulalah. Satu, kita menggunakan rumus trapezium itu sendiri. Satu, kita guna rumus satu iaitu* rectangle *ini dan juga dengan* triangle *ini*. So, *kita akan dapat ini* (point to ($a \times b$)), *ini* (point to $\frac{1}{2} \times (b - a) \times h$), *ini* (points to $(a \times b) + \frac{1}{2} \times (b - a) \times h$ and then moves his hand to indicate it equals to $\frac{1}{2} \times (a + b) \times h$).

[And then times "*a* plus *b*". "*a*" represents the side that opposite the base. "*a* plus *b*". After that, times the height…(silent for a while) actually, we can get two formulae. One, we used the formula of the trapezium itself. (Another) one, we used the formula of a rectangle and also a triangle. So, we will get this (point to ($a \times b$)), this (point to $\frac{1}{2} \times (b - a) \times h$), this (points to $(a \times b) + \frac{1}{2} \times (b - a) \times h$ and then moves his hand to indicate it equals to $\frac{1}{2} \times (a + b) \times h$).]

R: Do you have any idea how to derive the formula for calculating the area of trapezium?

S: …(Silent for a while and then moves his head to indicate no idea).

In Excerpt P50, Patrick suggested to draw a trapezium. He could recall the formula for the area of a trapezium. Patrick stated that the formula for calculating the area of a trapezium is ‘one over two times "*a*", base plus opposite one times "*h*" (points to the formula $\frac{1}{2} \times (a + b) \times h$)’, as shown in Figure N113. Patrick admitted that he did not know how to develop it. Nevertheless, Patrick attempted to develop the formula using algebraic method but unsuccessful. Patrick explained that the area of a trapezium can be calculated using the formula for the area of a trapezium itself or using the combination of the formula for the area of a rectangle, namely $a \times b$ (wrong formula. It should be $a \times h$), and a triangle, namely $\frac{1}{2} \times (b - a) \times h$). He moved his hand to indicate that $a \times b$ (wrong formula. It should be $a \times h$) + $\frac{1}{2} \times (b - a) \times h$) equals to $\frac{1}{2} \times (a + b) \times h$). Patrick was unable to show how $a \times b$ (wrong formula. It should be $a \times h$) + $\frac{1}{2} \times (b - a) \times h$) could be simplified as $\frac{1}{2} \times (a + b) \times h$). He moved his head to indicate that he has no idea how to develop the formula.

Summary

In summary, Patrick could recall the formula for the area of a rectangle, parallelogram, triangle, and trapezium. Nevertheless, he was only able to develop the formula for the area of a parallelogram. Patrick did not attempt to develop the formulae for the area of a rectangle and triangle. He attempted to develop the formula for the area of a trapezium using algebraic method but unsuccessful.

Conceptual Knowledge

Patrick could recall the formula for the area of a rectangle. Nevertheless, he was unable to develop the formula. It was apparent that Patrick lack of conceptual knowledge underpinning the formula for the area of a rectangle.

Patrick could recall the formula for the area of a parallelogram. He was able to develop the formula. Patrick mentally transformed the parallelogram to a rectangle by cutting out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle. It indicated that he understands the relationship between the formula for the area of a parallelogram and rectangle. A parallelogram can always be transformed into a rectangle with the same base, same height, and the same area. Thus, the formula for the area of a parallelogram is exactly the same as the formula for the area of a rectangle, namely ‘base times height’.

Patrick could recall the formula for the area of a triangle. Nevertheless, he was unable to develop the formula. Patrick did not know the relationship between the area of a triangle and the area of the rectangle that encloses it. Had he been known of this relationship, Patrick would know how to develop the formula for the area of a triangle.

Patrick could recall the formula for the area of a trapezium. Nevertheless, he was unable to develop the formula. It was quite clear that Patrick did not know the relationship between the area formulae of a rectangle, parallelogram, triangle, and trapezium. Had he been known of this relationship, Patrick would know how to develop the formula for the area of a trapezium.

Linguistic Knowledge

Patrick used appropriate mathematical symbols to write the formula for the area of a rectangle, namely '$a \times b$', as shown in Figure N113. Patrick used inappropriate mathematical term '*sisi yang berlainan* [different side]' to explain the meaning of the symbols *a* and *b* that he employed. He explained that "…*sisi yang berlainan akan darab.* […different side will be multiplied.]" (Patrick/L1523). Actually, *a* and *b* represents the length and the width of the rectangle.

Patrick used appropriate mathematical symbols to write the formula for the area of a parallelogram, namely '$a \times b$', as shown in Figure N113. Nevertheless, Patrick used inappropriate mathematical terms 'distance' and 'measurement' to explain the meaning of the mathematical symbols *a* and *b* that he employed. Patrick explained that "*a mewakili jarak, eh ukuran daripada AB dan b mewakili ukuran daripada BC.* …[*a* represents the distance, the measurement of *AB* and *b* represents the measurement of *BC*. …]" (Patrick/L1538-1539). Actually, *a* and *b* represents the base and the height of the parallelogram.

Patrick used appropriate mathematical symbols to write the formula for the area of a triangle, namely '$\frac{1}{2} \times b \times h$', as shown in Figure N113. He also used appropriate mathematical terms 'one over two', 'base' and 'height' to state the formula for the area of a triangle. Patrick stated that ""…*satu perdua darab tapak,* base times height (points to the formula $\frac{1}{2} \times b \times h$). […one over two times base times height (points to the formula $\frac{1}{2} \times b \times h$).]" (Patrick/L1547-1548).

Patrick used appropriate mathematical symbols to write the formula for the area of a trapezium, namely '$\frac{1}{2} \times (a + b) \times h$', as shown in Figure N113. He also used appropriate mathematical terms 'one over two' and 'height' but inappropriate mathematical term 'base' to state the formula for the area of a trapezium. Patrick stated that "…one over two times "a", base plus opposite one times "h" (points to the formula $\frac{1}{2} \times (a + b) \times h$)." (Patrick/L1554-1555).

Strategic Knowledge

Patrick used the cut and paste strategy to develop the formula for the area of a parallelogram. He mentally cut out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle, as shown in Figure N113. Patrick attempted to develop the formula for calculating the area of a trapezium using algebraic method but unsuccessful, as shown in Excerpt P50.

Patrick explained that the area of a trapezium can be calculated using the formula for the area of a trapezium itself or using the combination of the formula for the area of a rectangle, namely $a \times b$ (wrong formula. It should be $a \times h$), and a triangle, namely $\frac{1}{2} \times (b - a) \times h$). He moved his hand to indicate that $a \times b$ (wrong formula. It should be $a \times h$) + $\frac{1}{2} \times (b - a) \times h$) equals to $\frac{1}{2} \times (a + b) \times h$). Patrick was unable to show how $a \times b$ (wrong formula. It should be $a \times h$) + $\frac{1}{2} \times (b - a) \times h$) could be simplified as $\frac{1}{2} \times (a + b) \times h$). He moved his head to indicate that he has no idea how to develop the formula.

Ethical Knowledge

Patrick could recall the formula for the area of a rectangle but he did not attempt to develop the formula, as shown in Excerpt P47. He had succeeded in developing the formula for the area of a parallelogram, as shown in Excerpt P48. Patrick could recall the formula for the area of a triangle but he did not attempt to develop the formula, as shown in Excerpt P49. Patrick could recall the formula for the area of a trapezium. He attempted to develop the formula for the area of a trapezium using algebraic method but unsuccessful, as shown in Excerpt P50.

Level of Subject Matter Knowledge

In this section, Patrick' levels (low, medium, high) of subject matter knowledge of perimeter and area was analyzed in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

Patrick achieved a medium level of conceptual knowledge of perimeter and area when he obtained 56.0% of appropriate mathematical elements of conceptual knowledge of perimeter and area during the clinical interview. Patrick secured a high level of procedural knowledge of perimeter and area when he obtained 72.7% of appropriate mathematical elements of procedural knowledge of perimeter and area. Patrick achieved a medium level of linguistic knowledge of perimeter and area when he obtained 51.2% of appropriate mathematical elements of linguistic knowledge of perimeter and area. Patrick secured a high level of strategic knowledge of perimeter and area when he obtained 78.6% of appropriate mathematical elements of strategic knowledge of perimeter and area. Patrick achieved a medium level of ethical knowledge of perimeter and area when he obtained 46.9% of appropriate mathematical elements of ethical knowledge of perimeter and area. Patrick achieved an overall medium level of subject matter knowledge of perimeter and area when he obtained 54.9% of appropriate mathematical elements of subject matter knowledge of perimeter and area.

Roslina

Roslina lives in Sungai Buloh, Selangor. Roslina is 21 years 8 months old when she was interviewed. Currently, she is pursuing a 4-year Bachelor of Science with Education (B.Sc.Ed.) program at a public university. She majored and minored in biology and mathematics respectively. She obtained grade 1A in Mathematics and 2A in Additional Mathematics in her 2003 SPM examination (equivalent to O level examination). She scored A− in Mathematics in the 2005 Matriculation examination (equivalent to A level examination). Roslina performed moderately in her mathematics content courses at the university level when she secured one B+, one B, and two C+ in four mathematics content courses she had completed during the first and second year of her studies. The detail of her performance is shown in Table N7.

Table N7

Mathematics Performance of Roslina

Courses	Grades
1. Calculus for Science Students I	C+
2. Statistics for Science Students	B+
3. Calculus for Science Students II	C+
4. Differential Equations I	B

At the time of data collection, Roslina was in her second semester of third year studies. She attained 3.15 in the Cumulative Grade Point Average (CGPA) for her first two years of studies at the public university. She does not have any teaching experience prior to this interview.

Notion of Perimeter

Conceptual Knowledge

Roslina has successfully selected all the shapes that have a perimeter, namely "A", "C", "D",”F”, "H", "I", “J”, and "K". Excerpt R1 shows her choice of shapes that have a perimeter (Roslina/L94-97).

Excerpt R1

R: (Puts a handout comprises 12 shapes in front of Roslina). Tick the shapes that have a perimeter.
S: Oh just tick. (Ticks shapes "A", "C", "D",”F”, "H", "I", “J”, and "K", as shown in Figure N114). I think that’s all.

In Excerpt R1, Roslina has selected all simple closed curves (A, C, H, K) as well as all closed but not simple curves (D, I) that have a perimeter. She also selected the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that her notion of perimeter was not only limited to simple closed curves, and closed but not simple curves, but also inclusive of 3-dimensional shapes. Roslina did not select the two simple but not closed curves (B, G) as well as the two 1-dimensional shapes (E, L) that do not have a perimeter. In other words, Roslina did not select an open shape (including the lines) as having a perimeter.

When asked to justify her selection, Roslina explained that she selected shapes "A", "C", "D",”F”, "H", "I", “J”, and "K" because they are closed shapes. She emphasized that the calculation of the perimeter for shape “I” did not include the inner line.

Roslina stated that “J” is a cylinder and elaborated that its perimeter and area can be calculated. Excerpt R2 depicts her justification of selecting each of these shapes (Roslina/L104-119).

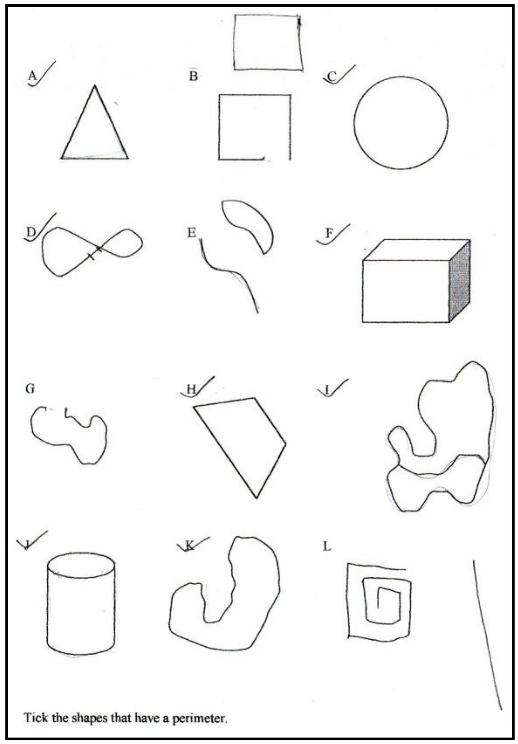


Figure N114. Roslina’s selection of shapes that have a perimeter.

Excerpt R2

R: Why did you select shape "A"?
S: Because the shape *tertutup* [closed]. It’s not open right. So, we can calculate the *ukurlilit* [circumference] of the shape. And so as "C", it’s not open and "D".
R: Why did you select shape "C"?
S: "C" is round shape. So, we can calculate the entire *ukurlilit* [circumference]. I don't know *ukurlilit* [circumference] in English. So, it’s easy for us as it is a closed shape. And then so as "D". It’s a closed shape, I mean that's no, I think that's no weird as picture "B". Ok and then picture "F" same as "A", "C", and "D", we can calculate. "H" yes, same. This is a trapezium. We can calculate the perimeter. "I" yes, we just (calculate) perimeter the whole, I mean the outside line. We don't have to calculate the inside ok. So as "J" and "K".
R: Why did you select shape "J"?
S: "J" is cylinder. We can calculate perimeter and area. However, we want to create a method of "J". We just can calculate the *ukurlilit* [circumference], not the inside. So as "K", "K" yeah.

Roslina explained that she did not select shapes “B” and “G” because they are open and thus their perimeter cannot be calculated. Roslina pointed out that the perimeter of shape “B” can be calculated if the shape is closed. She drew a complete square above shape "B", as shown in Figure N114. Excerpt R3 reveals her justification for not selecting shapes “B” and “G” as having a perimeter (Roslina/L120-127, L137-139).

Excerpt R3

R: Why didn't you select shape "B"?
S: "B" because the shape is, I mean they don't touch. That's an open space here. To calculate the perimeter, I think that the properties we have to make sure that the shape is one point touch to other point. That's no space, no hole here, like "A". "B" can be calculated if the shape is closed. (Draws a complete square above shape "B", as shown in Figure N114). So, yes, we can calculate the perimeter and can calculate the area as well.

.
.

.
R: Why didn't you select shape "G"?
S: "G" is same as "B". As this, there is a space here. So, we can not calculate the perimeter.

Roslina explained that she did not select shapes "E" and "L" because they are just a line and thus there is no perimeter. She drew a closed shape near shape "E", as shown in Figure N114, and stated that its perimeter can be calculated. Roslina also explained that shape "L" can be elaborated (expanded) to be just a straight line. She drew a straight line next to shape "L", as shown in Figure N114. Excerpt U4 exhibits her justification for not selecting shapes "E" and "L" as having a perimeter (Roslina/L165-168, L171-172).

Excerpt R4

R: Why didn't you select shape "E"?
S: Because "E" is not a shape that we can, I mean, we can not calculate the *ukurlilit* [circumference].
R: Why?
S: It is just a line. There is no, I mean for perimeter, line have to be like this (draws a closed shape near shape "E", as shown in Figure N114). Yes, it can be (refers to the closed shape that she has just drawn). I mean that this is something that we can calculate. May be this is an area. So, we can calculate the *ukurlilit* [circumference]. So, this is not (points to shape "E").
.
.
.
R: Why didn't you select shape "L"?
S: "L" is a shape but if we elaborate it, it's just a line, a straight line. (Draws a straight line next to shape "L", as shown in Figure N114). So, that's nothing to calculate there. May be the length yes. We can calculate the length.

Summary

In summary, Roslina has selected all simple closed curves (A, C, H, K) and all closed but not simple curves (D, I) that have a perimeter. She also selected the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that her notion of perimeter was not only limited to simple closed curves, and closed but not simple curves, but also inclusive of 3-dimensional shapes. Roslina justified her selection by explaining that all these shapes are closed.

Linguistic Knowledge

Roslina used appropriate mathematical term 'closed' to justify her selection of shapes that have a perimeter. Roslina explained that she selected shapes "A", "C", "D", "F", "H", "I", "J", and "K" because they are closed shapes, as shown in Excerpt R2.

Roslina used appropriate mathematical term 'open' as her justification for not selecting shapes "B" and "G" as having a perimeter. Roslina explained that she did not select shapes "B" and "G" because they are open and thus their perimeter cannot be calculated, as shown in Excerpt R3. Roslina also used appropriate mathematical term 'line' as her justification for not selecting shape "E", and "L" as having a perimeter. Roslina explained that she did not select shapes "E" and "L" because they are just a line and thus there is no perimeter, as shown in Excerpt R4.

The perimeter of a circle is given a specific name known as circumference. Roslina inappropriately used the mathematical term '*ukurlilit* [circumference]' to include perimeters of noncircular shapes such as shapes "A" and "E", as shown in Excerpts R2 and R4.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Roslina had taken the effort to justify the selection of shapes that have a perimeter, as shown in Excerpt R2. She provided appropriate justification for selecting shapes “A”, “C”, “D”, ”F”, “H”, “I”, “J”, and “K” that have a perimeter.

Roslina also had taken the effort to provide justification for not selecting other shapes that do not have a perimeter. She provided appropriate justification for not selecting shapes “B” and “G” as having a perimeter, as shown in Excerpt R3. Roslina also provided appropriate justification for not selecting shapes “E” and “L” as having a perimeter, as shown in Excerpt R4.

Notion of Area

Conceptual Knowledge

Roslina has selected shapes “A”, “C”, “F”, “H”, and “J” as having an area. Excerpt R5 shows her choice of shapes that have an area (Roslina/L147-149).

Excerpt R5

R: (Puts a handout comprises 12 shapes in front of Roslina). Tick the shapes that have an area.
S: (Ticks shapes "A", "C", "D", “F”, "H", "I", “J”, and "K", as shown in Figure N115).

In Excerpt R5, Roslina has selected three of the 2-dimensional shapes (A, C, H) that have an area. She also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that her notion of area was limited to regular 2-dimensional shapes (such as triangle, circle, and trapezium) and 3-dimensional shapes (such as cuboid and cylinder), where its area or surface area can be calculated using formula. Roslina did not select the two open shapes (B, G) as well as the two 1-dimensional shapes (E, L) that do not have an area. In other words, Roslina did not select an open shape (including the lines) as having an area. It can be inferred that she did not has a dynamic perspective of area or, at least, this knowledge was not accessible to her during the clinical interview.

When asked to justify her selection, Roslina explained that she selected shapes "A", "C", and "H" because their area can be calculated using formula. It indicated that Roslina appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). She stated that the formula for the area of a triangle is $\frac{1}{2}(h \times b)$. Roslina wrote the formula above shape “A”, as shown in Figure N115. She named shape “C” as ‘a round shape’ (it should be ‘a circle’). Roslina stated that the formula for the area of shape “C” is $2\pi j$. She wrote the formula above shape “C”, as shown in Figure N115. The right formula for the area of a circle should be πj^2 or πr^2, where j or r is the radius of the circle, as $2\pi j$ is the formula for the circumference of a circle. Roslina stated that “H” is a trapezium. There is a formula for the area of a trapezium but she was unable to recall it.

Roslina also explained that he selected shapes “F” and “J” because their area (surface area) can be calculated using formula. It also indicated that Roslina appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). She stated that “F” is a cuboid. Roslina labelled a, b, and c on shape “F”, as shown in Figure N115, and stated that its area is ‘a times b times c’. Nevertheless, ‘a times b times c’ is the formula for the volume of a cuboid, not the area formula as she said. She stated that “J” is a cylinder and its area (surface area) can be calculated (using formula). Excerpt R6 depicts her justification of selecting each of these shapes (Roslina/L157-173).

Excerpt R6

R: Why did you select shape "A"?

S: Because this is a triangle and the triangle have the formula to calculate the area. So, it’s the half the height times base (writes the formula $\frac{1}{2}(h \times b)$ above shape "A", as shown in Figure N115). So, we can calculate the area.

"C" is a round shape here. So, sure has a formula right. Em $2\pi j$, j is the *jejari* [radius] (writes $2\pi j$ above shape "C", as shown in Figure N115. The right formula for the area of a circle should be πj^2 or πr^2 as $2\pi j$ is the formula for the circumference of a circle).

And then "F" is cuboid. Cuboid for area is the, I just symbolized it "a, b, and c" (labels "a, b, and c" on shape "F", as shown in Figure N115). So, the area is "a times b times c" (writes the volume formula "a times b times c" for the cuboid, not the area formula as she said).

For "H" is trapezium. Trapezium also has a formula. But I am not very (sure), I can not recall it but there is a formula for trapezium.

And this is cylinder, "J". Cylinder, we also can calculate the area for it, yes.

Roslina explained that she did not select shapes “D”, “I”, and “K” because there is no specific formula that can be used to calculate their area. It indicated that Roslina appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Roslina pointed out that there is no single formula that can be used to calculate the area of shape “D”. She emphasized that “…Area will be strictly with the formula.…” (Roslina/L182) and thus indicated that it has no area. The same goes for shapes “I” and “K” as well. Excerpt R7 demonstrates her justification for not selecting shapes “D”, “I”, and “K” as having an area (Roslina/L181-183, L193-199).

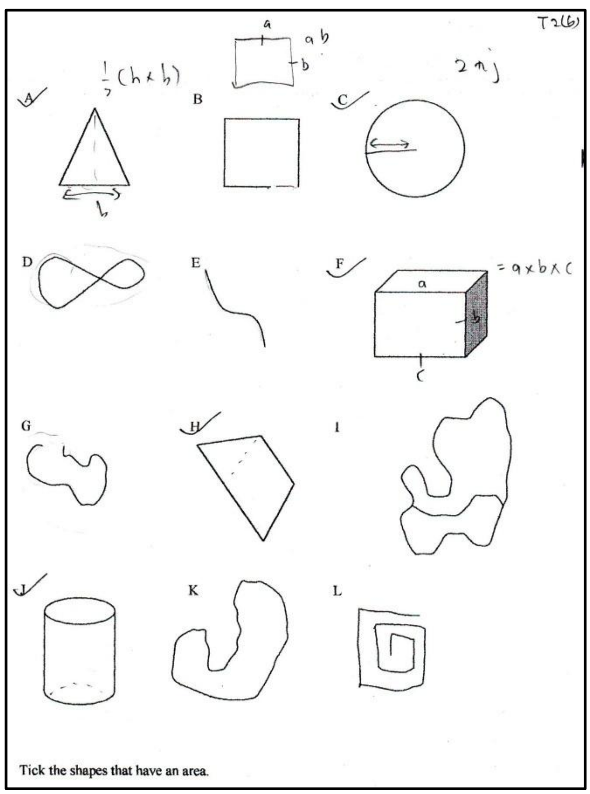


Figure N115. Roslina’s selection of shapes that have an area.

Excerpt R7

S: So, for "D", there is no single formula to calculate this kind of shape in area. Area will be strictly with the formula. So, no, you can not calculate the area for picture "D".

.
.
.

R: Why didn't you select shape "I"?

S: "I" because "I" is a, the point is one point touch to other point but there is no simply a formula to calculate the area of this shape, this kind of shape.

R: Why didn't you select shape "K"?

S: "K", yes the line touch to each others but there is no simply a formula to calculate (the area of) this kind of shape.

Roslina explained that she did not select shape “B” because it is not a closed shape. Roslina elaborated that if it is a closed shape, it can be called s *kubus* [cube] (it should be a rectangle, not a *kubus* [cube] as she mentioned). She drew a rectangle above shape "B", labelled its dimensions as "a" and "b" and then wrote its area formula as "ab", as shown in Figure N115). Roslina expressed that if it is a “*kubus* [cube]”, its area can be calculated as "a times b" (Roslina/L180). Roslina explained that she did not select shape “G” because it is just a line and thus we cannot calculate the area. Excerpt R8 reveals her justification for not selecting shapes “B” and “G” as having an area (Roslina/L174-180, L189-192).

Excerpt R8

R: Why didn't you select shape "B"?

S: "B" because as it same as perimeter (refers to the previous task, Task 1.1), it is not a closed shape. It’s not a closed shape. If it is a closed shape, it can called as *kubus* [cube] (it should be a rectangle, not a *kubus* [cube]). (Draws a rectangle near by shape "B", labels its dimensions as "a" and "b" and then writes its area formula as "ab" as shown above). So, if it is a *kubus*, then we can calculate the area as "a times b".

.
.
.

R: Why didn't you select shape "G"?

S: "G" because "G" as you can see, there's no straight line to form a shape. So, its just simply draw the line and then there's no point touch one to one point. So, we can not calculate the area.

Roslina explained that she did not select shapes “E” and “L” because they are just a line and thus we cannot calculate their area. Excerpt R9 exhibits her justification for not selecting shapes “E” and “L” as having an area (Roslina/L186-188, L200-203).

Excerpt R9

R: Why didn't you select shape "E"?

S: "E" because it’s just a line and then that's no shape. So, we can not calculate the area for this shape "E".

.
.
.

R: Why didn't you select shape "L"?

S: "L" as same as the perimeter (refers to the previous task, Task1.1). It’s just a line. We can not calculate the area for this because it’s just a line.

Summary

In summary, Roslina has selected three of the 2-dimensional shapes (A, C, H) that have an area. She also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that her notion of area was limited to regular 2-dimensional shapes (such as triangle, circle, and trapezium) and 3-dimensional shapes (such as cuboid and cylinder), where its area or surface area can be calculated using formula. Roslina justified her selection by explaining that she selected shapes "A", "C", and "H" because their area can be calculated using formula. It indicated that Roslina appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Roslina also explained that she selected shapes “F” and “J” because their area (surface area) can be calculated using formula. It also indicated that Roslina appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured).

Linguistic Knowledge

Roslina used appropriate mathematical term ‘calculate’ to justify her selection of shapes “A”, “C”, and “H” that have an area. Roslina explained that she selected shapes "A", "C", and "H" because their area can be calculated using formula, as shown in Excerpt R6. Roslina also used appropriate mathematical term ‘calculate’ to justify her selection of shapes “F” and “J” that have an area. Roslina also explained that she selected shapes “F” and “J” because their area (surface area) can be calculated using formula, as shown in Excerpt R6.

Although ‘no specific formula that can be used to calculate their area’ is an appropriate negation but it was not an appropriate justification for not selecting shapes “D”, “I”, and “K” that have an area as their area still exist even though there is no specific formula that can be used to calculate their area, as shown in Excerpt R7.

Roslina used appropriate negation ‘not closed’ as her justification for not selecting shape “B” as having an area. Roslina explained that she did not select shape “B” because it is not a closed shape. Roslina used appropriate mathematical term ‘line’ as her justification for not selecting shape “G” as having an area. Roslina explained that she did not select shape “G” because it is just a line and thus we cannot calculate the area, as shown in Excerpt R8. Roslina also used appropriate mathematical term ‘line’ as her justification for not selecting shapes “E” and “L” as having an area. Roslina explained that she did not select shapes “E” and “L” because they are just a line and thus we cannot calculate their area, as shown in Excerpt R9.

Roslina used inappropriate mathematical term ‘round shape’ instead of ‘circle’ to name shape “C”. She named shape “C” as ‘a round shape’ (it should be ‘a circle’), as shown in Excerpt R6. Roslina also used inappropriate mathematical term ‘*kubus* [cube]’ instead of ‘rectangle’ to name shape “B” when it is closed. Roslina elaborated that if it is a closed shape, it can be called s *kubus* [cube], as shown in Excerpt R8.

Ethical Knowledge

Roslina had taken the effort to justify the selection of shapes that have an area, as shown in Excerpt R6. She provided appropriate justification for selecting shapes "A", "C", “F”, “H”, and "J" that have an area.

Roslina also had taken the effort to provide justification for not selecting other shapes that do not have an area. She provided inappropriate justification for not selecting shapes “D”, “I”, and “K” as having an area, as shown in Excerpt R7. Roslina provided appropriate justification for not selecting shapes “B” and “G” as having an area, as shown in Excerpt R8. She also provided appropriate justification for not selecting shapes “E” and “L” as having an area, as shown in Excerpt R9.

Notion of the Units of Area

Conceptual Knowledge

Roslina stated that a square can be used as the unit of area measurement. She thought that a rectangle and triangle cannot be used as the unit of area measurement. It indicated that her notion of the unit of area was limited to square. Excerpt R10 shows her behavior in explaining why a square can be used as the unit of area measurement but a rectangle and triangle cannot be used as the unit of area measurement (Roslina/L224-262).

Excerpt R10

R: (Puts a card written the following scenario in front of Roslina). Ali, Chong, and David are discussing about the units of area. Ali says that we can use a square as the unit of area. Chong says that we can use a rectangle as the unit of area. David says that we can use a triangle as the unit of area. How would you respond to these students?

S: Ok I take this one (takes a 1-cm grid paper). So, for example, I will respond to the idea that Ali used. Ali used the square as a unit of area because for example, may be they want to calculate the area of a particular place that we shape. (Draws a large square on the grid paper, as shown in Figure N116). Ali chose that, why didn't we use a square as a unit of area. (Draws a 1-cm square on the grid paper, as shown in Figure N117). This is a square (points to the square that she has just drawn, as shown in Figure N117). The length of that (square) are (sic) the same. So, if we want to calculate this right, we just put, simply may be there is a model of square and then we just put there and then they can calculate. That is about one, two, three, four, five, six, seven, eight, nine, ten, eleven. So, this is eleven squares. And that one, two, three, four, five, six, seven, eight, nine, ten, eleven. This is eleven squares too. So, to calculate the area, it's just simple. They just calculate "11 times 11". (Writes the following).

$11 \times 11 = 121 \text{ cm}^2$

So, they get the answer right, 121 cm^2 (misreads 121 cm^2 as 121 cm square). Chong decided that they just use rectangle. It's not possible as we know rectangle is not the same the length. So, it's impossible and David said they used a triangle. Triangle, there's a three sides. So, if they want to calculate the area using the triangle, it is impossible, very impossible. It is not possible to do that.

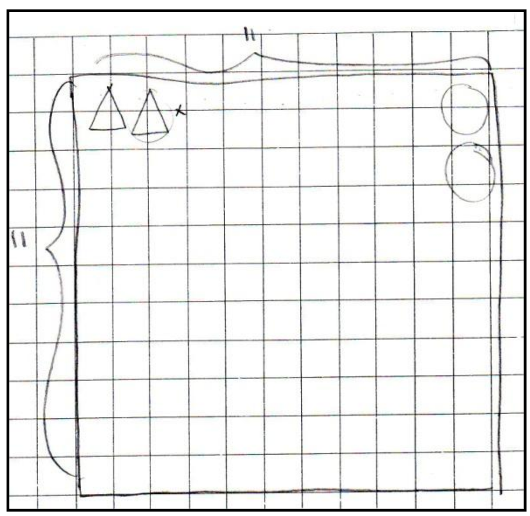


Figure N116. Roslina draws a large square on a 1-cm grid paper.

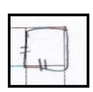


Figure N117. Roslina draws a 1-cm square on the grid paper.

In Excerpt R10, Roslina drew a large square with the dimension of 11 cm by 11 cm on a 1-cm grid paper and she also drew a 1-cm square on the grid paper, as shown in Figures N116 and N117. She explained that the sides of a square have the same length. Roslina mentally covered the large square with the 1-cm square and counted the number of the 1-cm square it takes to cover the 1-cm square on the first row of the large square, namely about 11 units. She then counted the number of the 1-cm square it takes to cover the 1-cm square on the first column of the large square, namely 11 units. Roslina calculated the area of the large square by multiplying the two adjacent length of the square, namely 11 x 11 = 121 cm^2. She explained that the sides of a rectangle are not of the same length and thus it was impossible to use it as the unit of area measurement. Roslina also explained that a triangle has three sides and thus it was impossible to use it as the unit of area measurement.

Summary

In summary, Roslina stated that a square can be used as the unit of area measurement. She thought that a rectangle and triangle cannot be used as the unit of area measurement. It indicated that her notion of the unit of area was limited to square. Roslina indicated that a square can be used as the unit of area measurement because the sides of a square have the same length. She explained that the sides of a rectangle are not of the same length and thus it was impossible to use it as the unit of area measurement. Roslina also explained that a triangle has three sides and thus it was impossible to use it as the unit of area measurement. It indicated that she was unable to provide the appropriate justification that any shape that tessellates a plane can be used as a unit of area measurement.

Linguistic knowledge

Roslina used inappropriate mathematical term ‘same length’ to justify that a square can be used as the unit of area. She indicated that a square can be used as the unit of area measurement because the sides of a square have the same length.

Roslina used inappropriate mathematical negation ‘not same length’ to justify that a rectangle cannot be used as the unit of area. She explained that the sides of a rectangle are not of the same length and thus it was impossible to use it as the unit of area measurement.

Roslina used inappropriate mathematical term ‘a triangle has three sides’ to justify that a triangle cannot be used as the unit of area. She explained that a triangle has three sides and thus it was impossible to use it as the unit of area measurement.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Roslina had taken the effort to justify the shape that can be used as a unit of area measurement. Nevertheless, she was unable to provide an appropriate justification for the shape that can be used as a unit of area measure. This can be seen in Excerpt R10. In reality, any shape that tessellates a plane can be used as a unit of area measurement. Roslina also had taken the effort to justify the shapes that cannot be used as a unit of area measurement. Nevertheless, she was unable to provide an appropriate justification for the shapes that cannot be used as a unit of area measure. This can be seen in Excerpt R10.

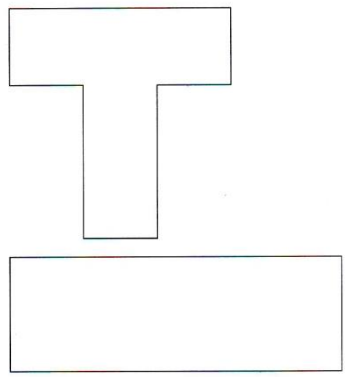
Comparing Perimeter (No Dimension Given)

Strategic Knowledge

Roslina used the formal method of measuring the side and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter. Excerpt R11 shows the formal method that she used to compare the perimeter of the given pair of shapes (Roslina/L358-379)

Excerpt R11

R: (Puts the following pair of shape in front of Roslina). How would you find out whether they had the same perimeter?



S: First, we can use the ruler.
R: Could you show me how it is?
S: (Measures the length of each side of the T-shape by ruler and then calculates its perimeter, as shown in Figure N118). (Measures the length of each side of the rectangle by ruler and then calculates its perimeter, as shown in Figure N119).

R: Could you explain your solution?
S: Ok just calculate. This is 6 cm and then the other side is 2 cm. This is 2 cm and then this is 2 cm. 2 cm and then this is 4, 4, and this is 2. So, we just sum it together. So, 6 plus 2 plus 2 plus 2 plus 2 plus 4 plus 4 plus 2. So, that is 24 cm.
R: What about the perimeter of the rectangle?
S: We calculate it ok. This is 9 cm. This is 9 cm. This is 3 cm. This is 3 cm. So, 9 plus 9 plus 3 plus 3 is 24. So, it's the same. 24 cm is the same perimeter.

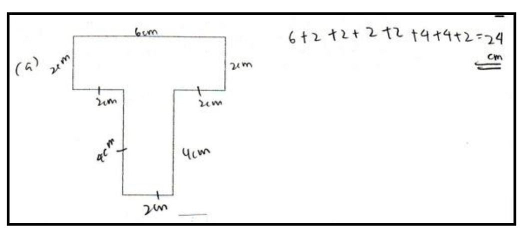


Figure N118. Roslina measures the length of each side of the T-shape by ruler and then calculates its perimeter.

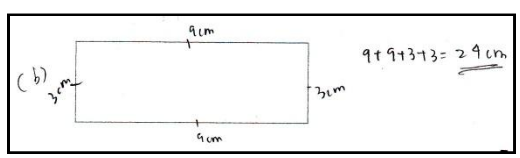


Figure N119. Roslina measures the length of each side of the rectangle by ruler and then calculates its perimeter.

In Excerpt R11, Roslina measured the length of each side of the given T-shape by ruler and then calculated its perimeter correctly as 24 cm, as shown in Figure N118. She measured the length of each side of the given rectangle by ruler and then calculated its perimeter correctly as 24 cm, as shown in Figure N119. Roslina concluded that the given pair of shapes had the same perimeter.

When probed for alternative method of comparing the perimeter, Roslina used another formal method of measuring the side by thread and ruler. Excerpt R12 demonstrates how she used thread and ruler to determine each perimeter and then compare their measurement (Roslina/L380-409).

Excerpt R12

R: Could you think of other way of finding out whether they had the same perimeter?
S: May be the teacher just give students to use, may be we use the, what do you call this, *benang* [thread].
R: We call it thread in English.
S: Yeah thread. Ok may be the teacher give the student to use this (holds the thread). May be they can use the thread. (Measures the length of each side of the T-shape using thread. Stops at the end of each side. Then puts the total length on the ruler to get its measurement). They just calculate like this and then they stop at this point. This is continue to other side. May be it's not as accurate as the ruler and then they continue to this point Ok this point and then use the ruler to get the reading. Yeah, 24. So, they get 24 and so as this shape (points to the rectangle). (Measures the length of each side of the rectangle using thread. Stops at the end of each side. Then puts the total length on the ruler to get its measurement). Just simply use the thread to calculate it. …
R: What do you get?
S: I got 24. Same as this (points to the T-shape), same for here. It's just the same.
R: Could you think of other way of finding out whether they had the same perimeter?
S: Perimeter. *Yang ini* [This one] ruler (refers to the first method), *benang* [thread] (refers to the second method). Em…(silent for a while) eh I don't know yet because perimeter is the *ukurlilit* [circumference]. So, I just think about using the ruler and the *benang* [thread] only. I don't know, I don't know other method.

In Excerpt R12, Roslina measured the length of each side of the T-shape by thread and then put it on a ruler to determine its total length (perimeter). She also measured the length of each side of the rectangle by thread and then put it on a ruler to determine its total length (perimeter). Roslina concluded that the given pair of shapes had the same perimeter, namely 24 (it should be 24 cm), as in Excerpt R12.

Summary

In summary, Roslina produced two formal methods of determining whether the given pair of shape had the same perimeter. In the first method, Roslina measured the length of sides by ruler and applied the definition of perimeter. In the second method, she measured the length of sides by thread and then put it on a ruler to determine its total length.

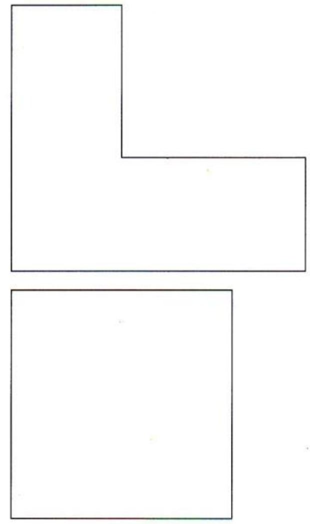
Comparing Area (No Dimension Given)

Strategic Knowledge

Roslina partitioned L-shape into two rectangles for which area measurement formulae were known. Excerpt R13 shows the formal method of measuring the side and applying the area formula that she used to compare the area of the given pair of shapes (Roslina/L448-477).

Excerpt R13

R: (Puts the following pair of shape in front of Roslina). How would you find out whether they had the same area?



S: Using the formula.

R: Could you show me how it is?

S: (Partitions the L-shape into two rectangles, labels as "a" and "b" respectively, as shown in Figure N120. Measures the length and width of each rectangle using ruler and then calculates its total area using area formula of rectangle). Ok for this shape (points to the L-shape), we divide it into two, I mean two shapes because it’s easy. So, I calculate it. This is "a" and this "b". I calculate the "a" first. The first side is 3 cm and the second side is 4 cm. For "a", the area would be "3 times 4". So, it will be 12 cm (wrong unit. It should be 12 cm^2). And then for "b", the first side would be 8 cm and then the second side would be 3 cm. It will be, for "b", its "8 times 3", 24 cm (wrong unit. It should be 24 cm^2). So, the total, we have to sum "a" and "b" as we divide into two right? So, we have to total it. So, it would be 36 cm^2 (misreads 36 cm^2 as 36 cm square), is the area. And then for the second picture (refers to the square), it’s easy. We just measure the side. (Measures the length of the two adjacent sides of the square using ruler and then calculates its area using area formula of square, as shown in Figure N121). This is 6 cm and this is 6 cm. So, "6 times 6 is 36 cm^2 (misreads 36 cm^2 as 36 cm square). So, they are the same length (sic), they had the same area. So, I think this is just the method I used.

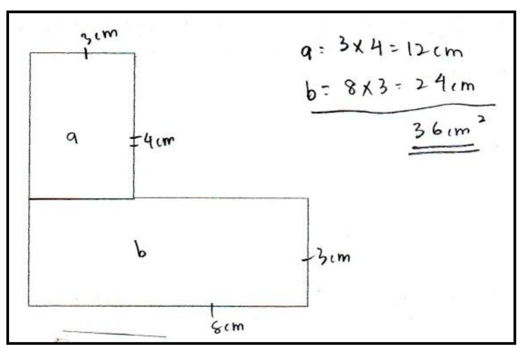


Figure N120. Roslina measures the length and the width of each rectangle by ruler and then calculates its area.

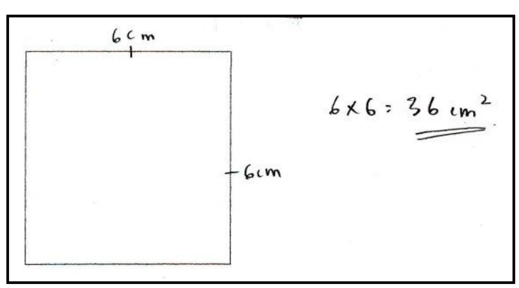


Figure N121. Roslina measures the length of the two adjacent sides of the square by ruler and then calculates its area.

In Excerpt R13, Roslina partitioned L-shape into two rectangles, labelled as "a" and "b" respectively, as shown in Figure N120. She measured the length and the width of each rectangle by ruler and then calculated its area using rectangle area formulae as 36 cm^2. Roslina also measured the length of the two adjacent sides of the square by ruler and then calculated its area using square area formula as 36 cm^2, as shown in Figure N121. She concluded that they had the same area.

When probed for alternative method of comparing the area, Roslina used a semi-formal method of tracing both shapes on a 1-cm grid paper and then counts the number of 1-cm square enclosed by each shape, as shown in Figures N120 and N121. Excerpt R14 depicts how she used this semi-formal method to determine each area and then compare their measurements (Roslina/L478-513).

Excerpt R14

R: Could you think of other way of finding out whether they had the same area?

S: Em…(silent for a while) other method. I think we can use the square grid that like Ali, David and Chong (refers to Task 3.1).

R: Could you show me how it is?

S: Yeah ok I'll take the 1 cm one (takes a 1-cm grid paper). (Traces the L-shape on the 1-cm grid paper, as shown in Figure N122. Counts the numbers of 1-cm grid enclosed by the outline of the L-shape and then writes its area, as shown in Figure N122). Ok for example, I just draw the shape here (the L-shape). So, this is 3 cm. This is 7 cm. This is 7. So, 7. This is 1 cm. So, we just calculate 1, 2, 3, 4, 5, 6, 7. Ok it is 8: 1, 2, 3, 4, 5, 6, 7, 8. 1, 2, 3. 1, 2, 3. Five, 1, 2, 3, 4, 5. Ok for this shape (the L-shape), as we know the grid is 1-cm grid. So, I just draw it. If it is 3 cm, so I just calculate 3, 3 grids, 3 box, I called it box. So, for the area, I just calculate how many squares grids is inside the picture (refers to the L-shape). So, I just calculate that is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36. This is 36 box. So, each box is consists of 1-cm grid. So, there 36 cm^2 (misreads 36 cm^2 as 36 cm square). This is the area. (Traces the given square on the 1-cm grid paper, as shown in Figure N123. Counts the numbers of 1-cm grid enclosed by the outline of the shape and then writes its area, as shown in Figure N123). And for the second picture (refers to the given square), we calculate the, how many square here. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36. So, there also 36 also. So, this is the second method.

R: Could you think of other way of finding out whether they had the same area?

S: Em no. I think that's it.

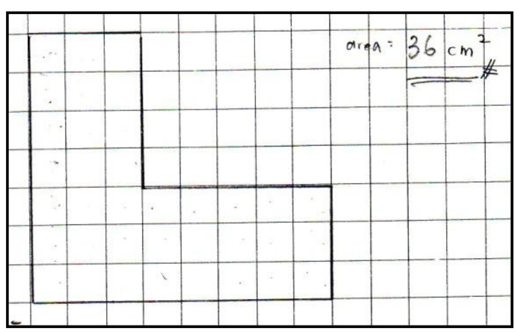


Figure N122. Roslina traces the L-shape on the 1-cm grid paper and counts the number of 1-cm square enclosed by the shape.

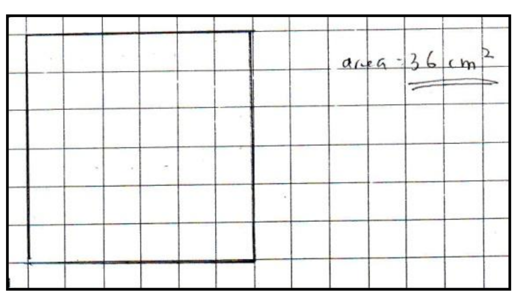


Figure N123. Roslina traces the square on the 1-cm grid paper and counts the number of 1-cm square enclosed by the shape.

In Excerpt R14, Roslina traced the L-shape on the 1-cm grid paper. She counted the number of 1-cm square enclosed by the shape, as shown in Figure N122. Roslina also traced the square on the 1-cm grid paper. She counted the number of 1-cm square enclosed by the shape, as shown in Figure N123. She implicitly concluded that they had the same area.

Summary

In summary, Roslina produced one formal method and one semi-formal method of determining whether the given pair of shapes had the same area.

In the first method, she partitioned L-shape into two rectangles, as shown in Figure N118. Roslina measured the length of side by ruler and applied area formulae. In the second method, she used a semi-formal method of tracing both shapes on a 1-cm grid paper and then counts the number of 1-cm square enclosed by each shape.

Comparing Perimeter (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Roslina stated that shape B has the longer perimeter. Excerpt R15 shows the justification that she made (Roslina/ L537-551).

Excerpt R15

R: (Puts the following table in front of Roslina). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 1	25 paper clips	12 sticks

S: May be shape B because stick I thought may be stick is longer than the paper clips. Paper clip is just small and they use 12 sticks to get the perimeter. So, paper clip is small. So, I think shape B has the larger number, larger amount of perimeter.

R: But 25 is larger than 12.

S: Yes, it's the number right? But I mean the paper clips, the size is small. So, may be their length is about 2 cm. May be the stick is 5 cm. So, if one paper clip equals to 2 cm, then they got 50 right. But for stick, they have 12 sticks. May be their length is 5. So, they get 60. So, may be shape B is longer than shape A.

In Excerpt R15, Roslina explained that shape B has the longer perimeter because she thought that a stick is longer than a paper clip. It indicated that she focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. Nevertheless, Roslina did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. She gave an example that the length of a paper clip and a stick might just 2 cm and 5 cm respectively. Thus, the length of 25 paper clips and 12 sticks is 50 (cm) and 60 (cm) respectively. Roslina concluded that shape B has the longer perimeter than shape A.

In Set 2, Roslina stated that shape B has the longer perimeter. Excerpt R16 depicts the justification that she made (Roslina/L579-589).

Excerpt R16

R: (Puts the following table in front of Roslina). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 2	10 paper clips	15 paper clips

S: Shape B.
R: Why?
S: Because they use the same thing, paper clips. But the number of paper clips is different. Shape A, they used 10. But shape B, they used extra 5 paper clips, 15. So, it shows us really that shape B has the bigger perimeter compared to shape A.

In Excerpt R16, Roslina explained that shape B has the longer perimeter because she thought that they used the same paper clips for both shapes A and B, and shape B has 15 paper clips compared to shape A with 10 paper clips. She elaborated that 15 is larger than 10. It indicated that Roslina focused on the number of unit when comparing perimeters in Set 2 with common nonstandard units. Nevertheless, she did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters.

In another situation when shapes A and B had the same perimeter, Roslina explained that the paper clips in shape A is longer than the paper clips in shape B. Excerpt R17 demonstrates her justification about their units of measurement (Roslina/L590-603).

Excerpt R17

R: If shapes A and B had the same perimeter, what would you tell about their units of measure?
S: The size of the paper clips. May be 10 paper clips, the size is slightly bigger. I mean the size is slightly big compared to paper clips B. May be for shape A, they used 10 paper clips but the size of the paper clips may be long compared to paper clips B. (Draws the following diagrams, as shown in Figure N124).

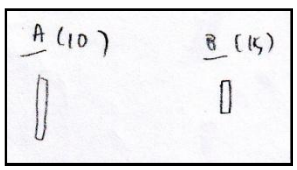


Figure N124. Roslina draws the length of a paper clip in shapes A and B.

May be half of it or I mean this is big compared to the B.
R: Which paper clip is longer?
S: May be shape A, yeah.

In Excerpt R17, Roslina explained that the paper clips in shape A is longer than the paper clips in shape B. She drew the length of a paper clip in shapes A and B, as shown in Figure N124, to show that the paper clip in shape A is longer. It indicated that Roslina understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length.

In Set 3, Roslina stated that shape A has the longer perimeter. Excerpt R18 reveals her choice of shape that has the longer perimeter and the justification that she made (Roslina/L613-623, L630-634).

Excerpt R18

R: (Puts the following table in front of Roslina). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 3	16 cm	13 cm

S: Shape A.
R: Why?
S: Because obviously shape A has 16 cm and shape B is 13 cm. So, it shows that shape A has the bigger perimeter compared to shape B. Because the unit is the same but the number 16 is greater than 13. So, shape A.
.
.
.
R: Just now you said "the unit is the same". Could you tell me more about it?
S: Unit, ok cm, cm is the same unit. So, what we have to do is we look at the number, I mean the length. So, 16 in mathematics is bigger compared to 13 right. …

In Excerpt R18, Roslina explained that shape A has the longer perimeter because they used the same unit, namely centimetre, and 16 is larger than 13. She elaborated that since they used the same unit of measurement, namely centimetre, so we just have to focus on the number (number of unit). Roslina reiterated that 16 is larger than 13 and thus shape A has the longer perimeter. It indicated that she focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Roslina knew that common standard unit (such as cm) is reliable for comparing perimeters.

Summary

In summary, Roslina focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. Nevertheless, she did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. Roslina focused on the number of unit when comparing perimeters in Set 2 with common nonstandard units. Nevertheless, she did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. Roslina understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length. She focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Roslina knew that common standard unit (such as cm) is reliable for comparing perimeters.

Comparing Area (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Roslina stated that shape B has the larger area compared to shape A. Excerpt R19 shows her choice of shape that has the larger area and the justification that she made (Roslina/L655-670).

Excerpt R19

R: (Puts the following table in front of Roslina). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 1	25 triangles	12 squares

S: Which shape? I think, I think would be B.
R: Why?
S: Because they used square. Squares although they used only 12 squares. I think may be the area of one square is larger compared to one triangle. It's because the formula in triangle half of the height and the base but for square they just simply times length "a" and length "b". So, shape B has the bigger area compared to A.
R: But in this case, 25 is larger than 12.
S: Yes the number, I mean 25 but what make sense is the shape used. So, I think square would be large area compared to triangle although it is 25 and it is 12. But I think it is square will be larger area.

In Excerpt R19, Roslina explained that shape B has the larger area because the area of a square is larger compared to the area of a triangle. She elaborated that the formula for the area of a triangle is 'half of the height and the base' but for a square, it is just simply times length "a" and length "b". Thus, Roslina concluded that shape B has the larger area compared to shape A. She expressed that even though 25 is larger than 12 but she reiterated that the area of a square is larger compared to the area of a triangle. It indicated that Roslina focused on the unit of measure when comparing areas in Set 1 with nonstandard units. Nevertheless, she did not know that nonstandard units (such as triangle and squares) are not reliable for comparing areas.

In Set 2, Roslina stated that shape B has the larger area compared to shape A. Excerpt R20 depicts her choice of shape that has the larger area and the justification that she made (Roslina/L697-707).

Excerpt R20

R: (Puts the following table in front of Roslina). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 2	10 squares	15 squares

S: Shape B.
R: Why?
S: Because they used the same shape, squares and then the different between them are only the numbers. Shape A used 10 squares and shape B used 15 squares. So, it is obviously shows that shape B has the bigger area compared to A. They used 15 squares.

In Excerpt R20, Roslina explained that shape B has the larger area because she thought that they used the same shape, namely squares (as the unit of measure), and shape B has 15 squares compared to shape A with 10 squares. It indicated that she focused on the number of unit when comparing areas in Set 2 with common nonstandard units. Roslina did not know that common nonstandard units (such as squares) are not reliable for comparing areas.

In another situation when shape A and B had the same area, Roslina explained that the size of the squares from shapes A and B are different. Excerpt R21 demonstrates her justification about their units of measurement (Roslina/L708-716).

Excerpt R21

R: If shapes A and B had the same area, what can you say about their units of measure?
S: May be the size of the squares. Their area are the same. May be the size of the squares make sense because this, they used 10 squares. May be the squares are big.
R: Which one "big"?
S: Shape A. Shape B, they used 15 squares, many many squares but their squares are small. So, 15 small squares and 10 big squares will make their area the same.

In Excerpt R21, Roslina explained that the square from shape A is big and the square from shape B is small and thus 15 small squares and 10 big squares will make their area the same. It indicated that Roslina understand the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area.

In Set 3, Roslina stated that shape A has the larger area. Excerpt R22 reveals her choice of shape that has the larger area and the justification that she made (Roslina/L739-754).

Excerpt R22

R: (Puts the following table in front of Roslina). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 3	16 cm^2	13 cm^2

S: Shape A.
R: Why?
S: Because shape A and shape B, the unit are the same, cm^2 (misreads cm^2 as cm square).
R: Could you tell me more about it?
S: Because the unit are the same but the numbers, shape A has 16 cm^2 and shape B is 13 cm^2. So, obviously shape A has the larger area compared to shape B.
R: What do you mean by "the unit are the same"?
S: Same unit. Yes, the unit, cm^2 (misreads cm^2 as cm square), the same unit. …

In Excerpt R22, Roslina explained that shape A has the larger area because they used the same unit, namely cm^2, and shape A has 16 cm^2 compared to shape B with 13 cm^2. It indicated that Roslina focused on the number of unit when comparing areas in Set 3 with common standard units. She knew that common standard unit (such as cm^2) is reliable for comparing areas.

Summary

In summary, Roslina focused on the unit of measure when comparing areas in Set 1 with nonstandard units. Nevertheless, she did not know that nonstandard units (such as triangle and squares) are not reliable for comparing areas. Roslina focused on the number of unit when comparing areas in Set 2 with common nonstandard units. She did not know that common nonstandard units are not reliable for comparing areas. Roslina understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area. She focused on the number of unit when comparing areas in Set 3 with common standard units. Roslina knew that common standard unit (such as cm^2) is reliable for comparing areas.

Linguistic Knowledge

Roslina read 16 cm^2 and 13 cm^2 literally as '16 centimeter square' and '13 centimeter square' respectively, as shown in Excerpt R22. In another situation, Excerpt R23 exhibits how Roslina wrote 16 cm^2 and 13 cm^2 in English words (Roslina/L758-765).

Excerpt R23

R: (Puts a blank paper written the following measurements in front of Roslina).
16 cm^2
13 cm^2
How would you write these measurements in English words?
S: (Writes the following).

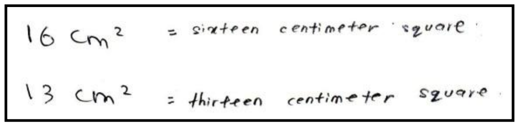


Figure N125. Roslina writes 16 cm^2 and 13 cm^2 in English words.

In Excerpt R23, Roslina wrote 16 cm^2 and 13 cm^2 literally as 'sixteen centimeter square' and 'thirteen centimeter square', as shown in Figure N125. The correct answer should be 'sixteen square centimetres' and 'thirteen square centimetres'. It indicated that she did not know about the conventions pertaining to writing and reading of Standard International (SI) area measurement units.

Converting Standard Units of Area Measurement

Procedural Knowledge

Roslina did not realize that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2. Excerpt R24 shows the algorithms that Roslina used when she was converting 3 cm^2 to mm^2 (Roslina/L775-799).

Excerpt R24

R: (Puts a card written the following scenario in front of Roslina). Some Form One teachers noticed that several of their students seemed to multiply by 10, 100, and 1000, respectively when they were converting units of area from cm^2 to mm^2, m^2 to cm^2, and km^2 to m^2:

$3\ cm^2 = 3 \times 10\ mm^2 = 30\ mm^2$
$4.7\ m^2 = 4.7 \times 100\ cm^2 = 470\ cm^2$
$1.25\ km^2 = 1.25 \times 1000\ m^2 = 1250\ m^2$

What would you do if you were teaching Form One and you noticed that several of your students were doing this?
S: This issue that this is 1 cm equals to 10 mm (writes the following).
1 cm = 10 mm
So, what the student do is the simplest, I mean it's the easiest way to calculate without using calculator. They have the prior knowledge. The prior knowledge is 1 cm equals to 10 mm. (Converts 3 cm^2 to mm^2, as shown in Figure N126). So, 3 cm^2 (misreads 3 cm^2 as 3 cm square) to convert it to mm^2 (misreads mm^2 as mm square). So, they just simply 3 times 10 mm. They get 30 mm (the unit should be mm^2, not mm). So, 30 mm^2 is equals to 3 cm^2. So, what they do, may be I do the same thing, yeah.
R: Do you think the students' first solution correct?
S: Yes, solution are correct. Yes, 30 mm^2.

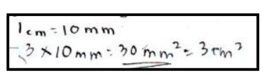


Figure N126. Roslina converts 3 cm^2 to mm^2.

In Excerpt R24, Roslina thought that the students' method of converting 3 cm^2 to mm^2 was the simplest or easiest way without using calculator. She knew that 1 cm = 10 mm. Nevertheless, Roslina thought that 1 cm^2 = 10 mm^2. Thus, Roslina times 10 when she converted 3 cm^2 to mm^2, as shown in Figure N126. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2. She also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Consequently, Roslina did not realize that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2. The students thought that 1 cm^2 = 10 mm^2. Thus, Roslina concluded that the students had correctly converted the unit of area for the first question, namely 3 cm^2 to 30 mm^2, because she thought that 1 cm^2 = 10 mm^2. Roslina stated that she did the same thing as the students did in converting unit of area from 3 cm^2 to mm^2.

Roslina did not realize that the students made a mistake when they were converting unit of area from 4.7 m^2 to cm^2. Excerpt R25 depicts the algorithms that Roslina used when she was converting 4.7 m^2 to cm^2 (Roslina/L800-809).

Excerpt R25

R: What about the second question (points to 4.7 m^2)?

S: We know, second, 1 m equals to 100 cm (writes the following).

1 m = 100 cm

So, what they do is 4.7 times 100. They just simply change the place of the dot. The decimal place. So, this is two, two zero, two times. So, they got 470 cm^2. (Converts 4.7 m^2 to cm^2, as shown in Figure N127). So, yes, the answer is correct.

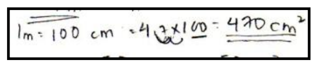


Figure N127. Roslina converts 4.7 m^2 to cm^2.

In Excerpt R25, Roslina knew that 1 m = 100 cm. Nevertheless, Roslina thought that 1 m^2 = 100 cm^2. Thus, Roslina times 100 when she converted 4.7 m^2 to cm^2, as shown in Figure N127. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 m^2 = 10 000 cm^2. She also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Consequently, Roslina did not realize that the students made a mistake when they were converting unit of area from 4.7 m^2 to cm^2. The students thought that 1 m^2 = 100 cm^2. Thus, Roslina concluded that the students had correctly converted the unit of area for the second question, namely 4.7 m^2 to 470 cm^2, because she thought that 1 m^2 = 100 cm^2. Roslina did the same thing as the students did in converting unit of area from 4.7 m^2 to cm^2.

Roslina did not realize that the students made a mistake when they were converting unit of area from 1.25 km^2 to m^2. Excerpt R26 demonstrates the algorithms that Roslina used when she was converting 1.25 km^2 to m^2 (Roslina/L810-817).

Excerpt R26

R: What about the third question (points to 1.25 km^2)?

S: So, they times it with one thousand. They just, they simply (counts the number of place that she needs to move to the right): One, two, three. 1.25 times 1000: one, two, three (counts the number of place that she needs to move to the right). Yes, they got the correct answer. (Converts 1.25 km^2 to m^2, as shown in Figure N128).

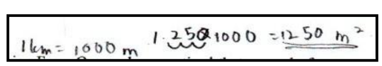


Figure N128. Roslina converts 1.25 km^2 to m^2.

In Excerpt R26, Roslina knew that 1 km = 1000 m. Nevertheless, Roslina thought that 1 km^2 = 1000 m^2. Thus, Roslina times 1000 when she converted 1.25 km^2 to m^2, as shown in Figure N128. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 km^2 = 1000 000 m^2. She also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Consequently, Roslina did not realize that the students made a mistake when they were converting unit of area from 1.25 km^2 to m^2. The students thought that 1 km^2 = 1000 m^2. Thus, Roslina concluded that the students had correctly converted the unit of area for the third question, namely 1.25 km^2 to 1250 m^2, because she thought that 1 km^2 = 1000 m^2. Roslina did the same thing as the students did in converting unit of area from 1.25 km^2 to m^2.

Summary

In summary, Roslina did not realize that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2, as shown in Figures N126, N127, and N128. She thought that 1 cm^2 = 10 mm^2, 1 m^2 = 100 cm^2, and 1 km^2 = 1000 m^2, as shown in Figures N126, N127, and N128. It indicated that Roslina did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2, 1 m^2 = 10 000 cm^2, and 1 km^2 = 1000 000 m^2. She also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Nevertheless, Roslina knew the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m, as shown in Figures N126, N127, and N128.

Conceptual Knowledge

The analysis of the previous section, namely procedural knowledge, indicated that Roslina did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2, 1 m^2 = 10 000 cm^2, and 1 km^2 = 1 000 000 m^2. She thought that 1 cm^2 = 10 mm^2, 1 m^2 = 100 cm^2, and 1 km^2 = 1000 m^2, as shown in Figures N126, N127, and N128. Roslina also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Nevertheless, Roslina knew the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m.

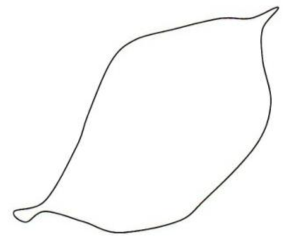
Relationship between Perimeter and Area
(Same Perimeter, Same Area?)

Conceptual Knowledge

Roslina did not know that there is no direct relationship between perimeter and area. She did not know that two shapes with the same perimeter can have different areas. Thus, Roslina thought that the student’s method of calculating the area of the leaf was correct. Excerpt R27 shows Roslina’s responses to the Form One student (Roslina/L859-898).

Excerpt R27

R: (Puts a card written the following scenario in front of Roslina). This is a picture of a leaf. A Form One student said that he had found a way to calculate the area of the leaf. The student placed a thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle.



How would you respond to this student?

S: The method that the student did I think is correct. Because they used the thread. They just calculate the outline of the leaf. So, they calculate it. (Measures the perimeter of the leaf using the thread and finds that the perimeter is 24 cm). They get the length and then the student draws the shape of a rectangle. (Draws a rectangle. Labels its dimensions as 8 cm by 4 cm based on the perimeter of the leaf, 24 cm. Calculates the area of the rectangle, as shown in Figure N129). The perimeter is 24 cm. So, the area would be 4 times 8, 32 cm^2 (misreads 32 cm^2 as 32 cm square).

R: Do you think the student's method works?

S: Yes, works because they just use the thread. They got the length and then they calculate the length. Then they draw the rectangle which their perimeter is the same as the length and then they simply just times to get the area.

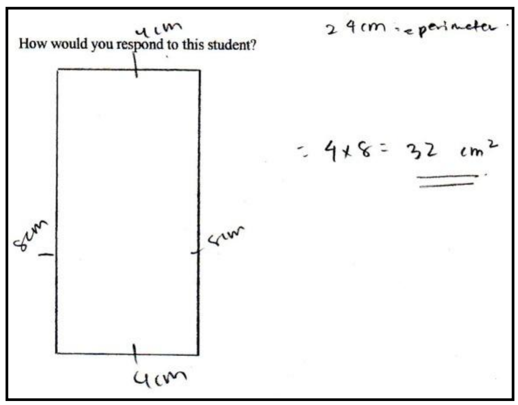


Figure N129. Roslina draws a rectangle, labels its dimensions and calculates its area.

In Excerpt R27, Roslina thought that the student's method of calculating the area of the leaf was correct. She agreed with the student that the area of the leaf same as the area of the rectangle formed. Roslina explained that the student used the perimeter of the leaf to form other shape, namely rectangle, and thus the area of the leaf same as the area of the rectangle. She tried out the student's method by placing a piece of thread around the boundary of the leaf and got the perimeter of the leaf as 24 cm. Roslina drew a rectangle, labelled its dimensions as 8 cm by 4 cm based on the perimeter of the leaf, namely 24 cm, and then calculated its area as 32 cm^2, as shown in Figure N129. She reiterated that the student's method works.

Summary

In summary, Roslina did not know that there is no direct relationship between perimeter and area. She did not know that two shapes with the same perimeter can have different areas. Roslina thought that the student's method of calculating the area of the leaf was correct.

Ethical Knowledge

In Task 5.1, Roslina thought that the student's method of calculating the area of the leaf was correct. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter have the same area. Roslina had attempted to examine the possible pattern of the relationship between perimeter and area by placing a piece of thread around the boundary of the leaf and got the perimeter of the leaf as 24 cm. Roslina drew a rectangle, labelled its dimensions as 8 cm by 4 cm based on the perimeter of the leaf, namely 24 cm, and then calculated its area as 32 cm^2, as shown in Figure N129. Based on this example, she thought that the student's generalization that two shapes with the same perimeter have the same area was correct.

Roslina formulated a generalization pertaining to the relationship between perimeter and area that two shapes with the same perimeter have the same area. Roslina tested the student's generalization that two shapes with the same perimeter have the same area with an example. She did not know that an example cannot be used to determine the truth of a generalization. A counterexample is sufficient to refute the truth of a generalization.

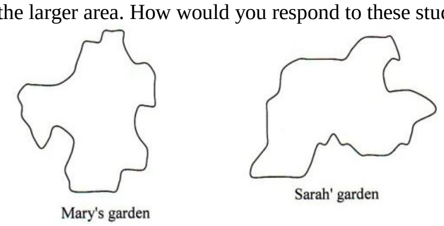
Relationship between Perimeter and Area
(Longer Perimeter, Larger Area?)

Conceptual Knowledge

Roslina did not know that there is no direct relationship between perimeter and area. She did not know that the garden with the longer perimeter could have a smaller area. Thus, Roslina thought that Mary's claim was correct. Excerpt R28 shows Roslina's responses to the claim made by Mary that the garden with the longer perimeter has the larger area (Roslina/L915-947).

Excerpt R28

R: (Puts a card written the following scenario in front of Roslina). Mary and Sarah are discussing whose garden has the larger area to plant flowers. Mary claims that all they have to do is walk around the two gardens to get the perimeter and the one with the longer perimeter has the larger area. How would you respond to these students?



S: Em…(silent for a while) ok I'll say that the longer perimeter has larger area. We have to find the perimeter first. Yes, it's true that longer perimeter has the larger area. We have to calculate what is the perimeter. So, they calculate using the thread and then if they get the perimeter, same as the method before (refers to the method used in the previous task, Task 5.1), they draw a rectangle and then they calculate the area. If Mary's perimeter is long compared to Sarah's, so Mary's garden is bigger compared to, is bigger than Sarah's.

R: Do Mary's method works?

S: Ok Mary has suggested that we have to walk around to get the perimeter. Yes, that is one method. That's a method. Yes, the method that Mary used is right. Ok they just go around and then calculate the perimeter. If Mary's garden's perimeter is longer compared to Sarah's garden, so Mary's garden would be, the area would be bigger compared to Sarah's.

In Excerpt R28, Roslina thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Roslina stated that the area of the gardens can be measured using the thread method in the previous task, namely Task 5.1. She explained that if Mary's garden had the longer perimeter than Sarah's, then Mary's garden has the larger area than Sarah's. Roslina thought that Mary's method works and reiterated that Mary's claim was correct.

When probed further, Roslina generated an example that concurred with Mary's claim. Excerpt R29 is illustrative (Roslina/L956-968).

Excerpt R29

R: Just now you mentioned that "if the perimeter is longer, the area would be larger". Could you tell me more about it?

S: Ok I assumed that Mary's garden may be is perimeter 24 and this is 12 (Sarah's) and then they draw a rectangle. (Draws two rectangles. Labels its dimensions respectively and then calculate its area, as shown in Figure N130). So, this is 8, 8, 4, 4. So, 12 may be this is 4, 4, 2, 2. Ok this is 12 cm and 24 is perimeter and then to calculate the area, 8 times 4 and this is 4 times 2. So, its true this is 32 cm^2 (misreads 32 cm^2 as 32 cm square) and this is 8. So, longer perimeter would be larger area.

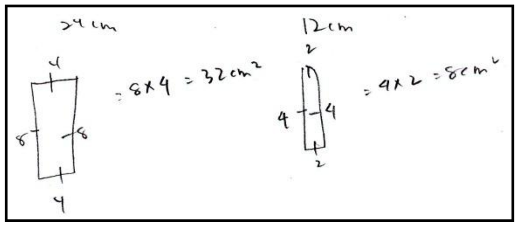


Figure N130. Roslina draws two rectangles and then calculate its area respectively.

In Excerpt R29, Roslina assumed that the perimeter of Mary's and Sarah's gardens were 24 cm and 12 cm respectively. She used the thread method in the previous task, Task 5.1, to transform the gardens into two rectangles. Roslina drew two rectangles to represent these gardens. She labelled its dimensions and then calculated its area as 32 cm^2 and 8 cm^2, as shown in Figure N130. Thus, Roslina concluded that (the garden with the) longer perimeter (24 cm) has the larger area (32 cm^2).

Summary

In summary, Roslina did not know that there is no direct relationship between perimeter and area. She did not know that the garden with the longer perimeter could have a smaller area. Thus, Roslina thought that Mary's claim was correct.

Ethical Knowledge

In Task 5.2, Roslina thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Roslina generated an example to examine the possible pattern of the relationship between perimeter and area, as shown in Figure N130. She assumed that the perimeter of Mary's and Sarah's gardens were 24 cm and 12 cm respectively. Roslina used the thread method in the previous task, Task 5.1, to transform the gardens into two rectangles. She drew two rectangles to represent these gardens. Roslina labelled its dimensions and then calculated its area as 32 cm^2 and 8 cm^2. Thus, she concluded that (the garden with the) longer perimeter (24 cm) has the larger area (32 cm^2). The example that Roslina generated concurred with Mary's claim that the garden with the longer perimeter has the larger area.

Roslina formulated a generalization pertaining to the relationship between perimeter and area that the garden with the longer perimeter has the larger area. Roslina tested Mary’s generalization that the garden with the longer perimeter has the larger area with the example that she generated. Her example concurred with Mary’s claim that the garden with the longer perimeter has the larger area. Nevertheless, Roslina did not know that an example could not be used to determine the truth of a generalization. A counterexample can be used to refute the truth of a generalization.

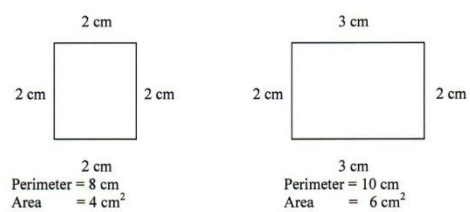
Relationship between Perimeter and Area
(Perimeter Increases, Area Increases?)

Conceptual Knowledge

Roslina did not know that there is no direct relationship between perimeter and area. She did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Roslina thought that the student’s “theory” was correct. This is shown in Excerpt R30 (Roslina/L996-1022).

Excerpt R30

R: (Puts a card written the following scenario in front of Roslina). Suppose that one of your Form One students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:



How would you respond to this student?

S: Ok I'll say that yes your theory is right. What you found is right and then it’s true that when the perimeter increases, the area would be increases. She proves it and it is true because as I said before, when the perimeter is large compared to other shape, the area would be also large. We can prove it by calculate the length, the perimeter and then we find the area. Yes, when the perimeter increases, the area would be increases.

In Excerpt R30, Roslina thought that the student’s “theory” was correct. She agreed with the student that as the perimeter of a closed figure increases, the area also increases. Roslina explained that when the perimeter (of a closed figure) increases, the area also increases. She expressed that the student has proven it and it was true because when the perimeter (of a shape) is longer compared to other shape, the area also larger. Roslina stated that it can be proven by calculating the perimeter and then the area. She concurred with the student that when the perimeter (of a closed figure) increases, the area also increases.

Roslina did not know that the student’s claim about the relationship between perimeter and area is not a theory. The claim is a conjecture. She also did not know that an example is not a proof and a theory cannot be proved by an example.

Summary

In summary, Roslina did not know that there is no direct relationship between perimeter and area. She did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Roslina thought that the student's "theory" was correct.

Ethical Knowledge

In Task 5.3, the student formulated a generalization that as the perimeter of a closed figure increases, the area also increases. Roslina explained that when the perimeter (of a closed figure) increases, the area also increases. She expressed that the student has proven it and it was true because when the perimeter (of a shape) is longer compared to other shape, the area also larger. Roslina thought that the student's "theory" was correct. She did not attempt to test the student's generalization, as shown in Excerpt R33. Roslina accepted the student's generalization without attempting to generate an example or counterexample to test it. In reality, when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same.

Calculating Perimeter and Area
(Rectangle And Parallelogram/Triangle)

Procedural knowledge

After read through Task 6.1, Roslina labelled the missing sides of Diagram 1 that required for calculating the perimeter and area of the diagram, as shown in Figure N131 in Excerpt R31 (Roslina/L1088-1117).

Excerpt R31

R: (Puts a card written the following problem in front of Roslina). Suppose that one of your Form One students asks you for help with the following problem:

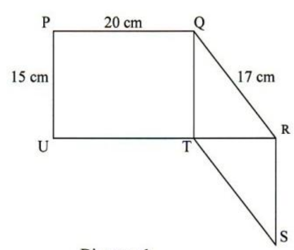


Diagram 1

In Diagram 1, PQTU is a rectangle and QRST is a parallelogram. UTR is a straight line.
Calculate
(i) the perimeter of the diagram,
(j) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 1, as shown in Figure N131).

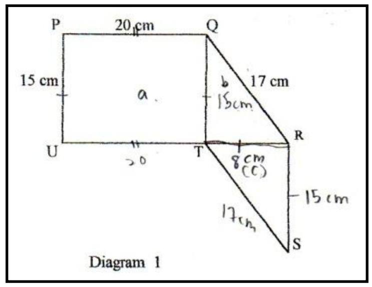


Figure N131. Roslina labels the missing sides of Diagram 1.

In Excerpt R31, Roslina labelled UT, QT, TS, and RS as 20, 15 cm, 17 cm, and 15 cm respectively on Diagram 1. Excerpt R32 depicts how Roslina has successfully calculated the perimeter of Diagram 1 (Roslina/L1119-1120, L1128-1135).

Excerpt R32

S: Ok first, when we want to calculate the perimeter of the diagram, we make sure we know all the length of each side. …So, we want to calculate the perimeter. We just calculate the outline of the shape. (Calculates the perimeter of Diagram 1, as shown in Figure N132). So, the perimeter would be 20 plus 15 plus 20 plus 17 plus 17 plus 15. So, they should be 104 cm.

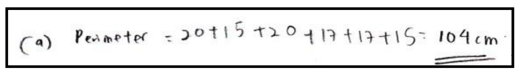


Figure N132. Roslina calculates the perimeter of Diagram 1.

In Excerpt R32, Roslina used the list all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N132. She listed all the length of sides that surrounded the diagram and then summed them up to get the perimeter of the diagram as 104 cm. Excerpt R33 demonstrates how Roslina has successfully calculated the area of Diagram 1 (Roslina/L1136-1158).

Excerpt R33

S: So, for area, we have to divide it into three shapes. I named it "a", "b", and "c". (Calculates the length of TR using Pythagoras' theorem, as shown in Figure N133). (Calculates the area of Diagram 1, as shown in Figure N134). So, for "a", the rectangle, the area would be 20 times 15. So, this should be (uses calculator to find the product) 300 cm^2 (misreads 300 cm^2 as 300 cm square). And for "b", it's a triangle. This 15 (points to QT), this is 17 (points to QR). So, we have to use, to get this length (points to TR). We have to use theorem Pythagoras. TR equals to 17^2 minus 15^2. So, we get (uses calculator to simplify it) 64, its square root. So, 64 square root (sic) equals to 8. So, this is 8 cm (points to TR). So, they have to know how to use theorem Pythagoras. So, this will be half times 15 times 8 (uses calculator to get the product). They should get 60 cm^2 (misreads 60 cm^2 as 60 cm square). And this is the same. So, the "c" also would be 60 cm^2 (misreads 60 cm^2 as 60 cm square). So, the area would be 300 plus 60 plus 60. So, 420 cm^2 (misreads 420 cm^2 as 420 cm square).

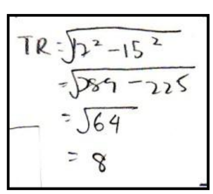


Figure N133. Roslina calculates the length of TR using Pythagoras' theorem.

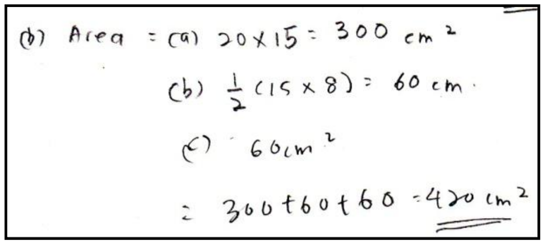


Figure N134. Roslina calculates the area of Diagram 1.

In Excerpt R33, Roslina used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure R21. She partitioned Diagram 1 into a rectangle PQTU (labelled as "a") and two triangles QRT (labelled as "b") and RST (labelled as "c"). Roslina knew that she needed to find the length of TR. She has successfully calculated the length of TR using Pythagoras' theorem as 8 cm, as shown in Figure N133. Roslina calculated the area of "a", "b", and "c" using the area formulae of rectangle and triangles respectively and then summed them up to get the area of the diagram as 420 cm^2, as shown in Figure N134.

Summary

In summary, Roslina has successfully calculated the perimeter of Diagram 1 by using the list all-and-sum algorithm. She has also correctly calculated the area of Diagram 1 by using the partition-and-sum algorithm.

Linguistic knowledge

Roslina used the correct standard units of measurement for perimeter (cm) and area (cm^2) when she wrote the answers for these measurements, as shown in Figures N132 and N134.

Strategic Knowledge

When probed to check the answer for the perimeter, Roslina suggested that she would use the recalculating strategy to verify the answer. Excerpt R34 is illustrative (Roslina/L1159-1163).

Excerpt R34

R: How would you check your answer for the perimeter?
S: Check. We have to calculate more than one time. So, just calculate it again. So, make sure that my answer is correct or I ask the student ok why don't you try out yourself. If our answer is the same, so the answer is correct.

In Excerpt R34, Roslina suggested that she would check the answer for perimeter by recalculating strategy that using the same method and calculate again. She explained that if the answer came out to be the same, then the original answer was correct.

When probed to check the answer for the area, Roslina suggested that she would also use the recalculating strategy to verify the answer. Excerpt R35 is illustrative (Roslina/L1164-1168).

Excerpt R35

R: How would you check your answer for the area?
S: Em we use the same method. Ok divide the shape into three different shapes that is easy to calculate where the shape has the formula. Then I'll calculate again and sum the shape together. If my answer still the same, so my answer is correct.

In Excerpt R35, Roslina suggested that she would check the answer for the area by recalculating strategy that using the same method and calculate again. She reiterated that if the answer came to be the same, then the original answer was correct.

Ethical Knowledge

Roslina has successfully calculated the perimeter and area of Diagram 1. Nevertheless, she did not check the correctness of the answers for perimeter as well as area. When probed to check answers, then only Roslina suggested the strategies that she would use to check the answers for perimeter and area. Roslina wrote the measurement units (without probed) for the answers of the perimeter and area that she has calculated, as shown in Figures N132 and N134.

Calculating Perimeter and Area
(Square and Trapezium/Triangle)

Procedural Knowledge

After read through Task 6.2, Roslina labelled the missing sides of Diagram 2 that required for calculating the perimeter and area of the diagram, as shown in Figure N135 in Excerpt R36 (Roslina/L1179-1222).

Excerpt R36

R: (Puts a card written the following problem in front of Roslina). Suppose that one of your Form One students asks you for help with the following problem:

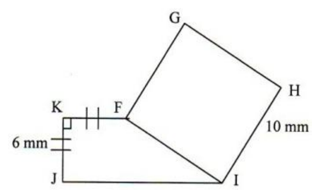


Diagram 2

In Diagram 2, FGHI is a square and FIJK is a trapezium.
Calculate
(i) the perimeter of the diagram,
(j) the area of the diagram.

How would you solve this problem?
S: I show the student.
R: Can you show me how it is?
S: (Labels Diagram 2, as shown in Figure N135). Ok first, to calculate the perimeter, we know this length (6 cm, the opposite length of KF) but we don't know what is this length (points to the length labels as "a" later as shown above). This is square. So, its 6 cm (the opposite length of KJ). We don't know what is this length (labels as "a" later, as shown in Figure N135). So, to know this length, we have to use the theorem Pythagoras. So, this side we want to find. I symbol it as "a". (Calculates the value of "a" using Pythagoras' theorem, as shown in Figure N136). "a" equals to 10^2 (reads as 10 squared) minus 6^2 (reads as 6 squared) square root and then this is 100 minus 36. We got 64 and then we got 8. So, this is 8 cm.

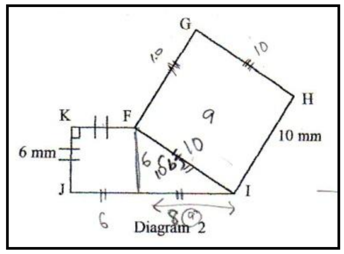


Figure N135. Roslina labels the missing sides of Diagram 2.

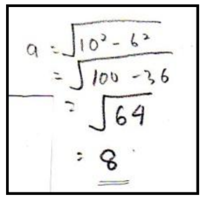


Figure N136. Roslina calculates the value of "a" using Pythagoras' theorem.

In Excerpt R36, Roslina labelled FG, GH, FI, the opposite side of KF, and the opposite side of KJ as 10, 10, 10, 6, and 6 respectively on Diagram 2. Roslina realized that she needed to find the value of “a”, as shown in Figure N135. Roslina partitioned trapezium FIJK into a square and a triangle. She has successfully calculated the value of “a” as 8 (mm) using Pythagoras’ theorem, as shown in Figure N136.

Excerpt R37 depicts how Roslina has successfully calculated the perimeter of Diagram 2 (Roslina/L1223-1229).

Excerpt R37

S: (Calculates the perimeter of Diagram 2, as shown in Figure N137). The perimeter is the outline of the shape. So, we just sum it: 10 plus 10 plus 10 plus 8 plus 6 plus 6 plus 6. So, we get (sums up it mentally) 56 mm.

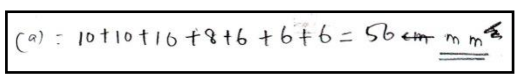


Figure N137. Roslina calculates the perimeter of Diagram 2.

In Excerpt R37, Roslina used the list all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N137. She listed all the length of sides that surrounded the diagram and then mentally summed them up to get the perimeter of the diagram as 56 mm. Roslina defined perimeter as the (length of the) outline of a shape. Initially, she wrongly wrote the unit of the perimeter of Diagram 2 as cm. Subsequently, Roslina realized her mistake, cancelled it and wrote the correct unit as mm, as shown in Figure N137.

Excerpt R38 demonstrates how Roslina has successfully calculated the area of Diagram 2 (Roslina/L1230-1242).

Excerpt R38

S: So, for the area, I will show them to calculate the area, easy we divide it into three shapes which is the square, the square and then this is the triangle. (Calculates the area of Diagram 2, as shown in Figure N138). So, the first "a", "a" would be 10 times 10. I got 100 mm^2 (misreads 100 mm^2 as 100 mm square). For "b", the triangle is half times 6 times 8. So, 48 divide by 2, 24. This 24 mm^2 (misreads 24 mm^2 as 24 mm square). And then "c", this is also the square, 6 times 6. So, we got 36 mm^2 (misreads 36mm^2 as 36 mm square). So, the area would be 100 plus 24 plus 36. We got 160 mm^2 (misreads 160 mm^2 as 160 mm square). So, this is the area.

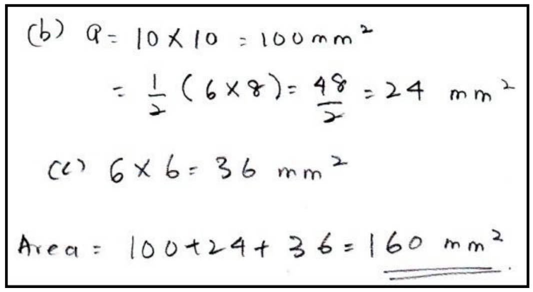


Figure N138. Roslina calculates the area of Diagram 2.

In Excerpt R38, Roslina used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N138. She partitioned Diagram 2 into a large square (FGHI, labelled as “a”), a triangle (labelled as “b”) and a small square (labelled as “c”). Roslina calculated the area of the large square, triangle, and small square separately using the area formulae of a square, triangle, and square respectively and then summed them up to get the area of the diagram as 160 mm^2.

Summary

In summary, Roslina has successfully calculated the perimeter of Diagram 2 using the list all-and-sum algorithm. She has also correctly calculated the area of Diagram 2 using the partition-and-sum algorithm.

Linguistic Knowledge

Roslina used the correct standard units of measurement for perimeter (mm) and area (mm^2) when she wrote the answer of these measurements, as shown in Figures N137 and N138.

Strategy Knowledge

When probed to check the answer for the perimeter, Roslina suggested that she would use the recalculating strategy to verify the answer. Excerpt R39 is illustrative (Roslina/L1243-1245).

Excerpt R39

R: How would you check your answer for the perimeter?
S: I calculate it again and then when the answer is the same, so my answer is correct.

In Excerpt R39, Roslina suggested that she would check the answer for the perimeter by the recalculating strategy that using the same method and calculate again. Roslina explained that she would recalculate the perimeter and if the answer was the same, then her answer was correct.

When probed to check the answer for the area, Roslina suggested that she would use the recalculating strategy to verify the answer. Excerpt R40 is illustrative (Roslina/L1246-1249).

Excerpt R40

R: How would you check your answer for the area?
S: Oh I will calculate that my theorem Pythagoras' answer is the correct one, my formula is correct and if my answer is right, so I think my answer will be right.

In Excerpt R40**,** Roslina suggested that she would check the answer for the area by the recalculating strategy. Roslina explained that she would check whether the value of "a" that she calculated, as shown in Figure N136, the formula that she used and her answer was correct.

Ethical Knowledge

Roslina has successfully calculated the perimeter and area of Diagram 2. Nevertheless, she did not check the correctness of the answers for perimeter and area. When probed to check answers, then only Roslina suggested the strategy that she would use to check the answers for perimeter and area. Roslina wrote the measurement units (without probed) for the answers of perimeter and area, as shown in Figures N137 and N138.

Fencing Problem

Strategic Knowledge

Roslina used trial and error strategy to solve the fencing problem. Excerpt R41 is illustrative (Roslina/L1266-1292).

Excerpt R41

R: (Puts a card written the following problem in front of Roslina). Suppose that one of your students asks you for help with the following problem:

A gardener has 84 m of fencing to enclose a garden along three sides, with the fourth side of the garden being formed by a wall. (Assume that the wall is perfectly straight). What are the dimensions of a rectangular garden that will yield the largest area being enclosed?

How would you solve this problem?
S: Ok first, I will draw the fence. (Draws a rectangular garden. Labels its dimensions and then calculates its area, as shown in Figure N139). So, gardener has 84 m fence to enclose a garden along three sides. So, one side, two side, three side and then it says that it will be rectangular garden. The fourth side, this one is a wall. So, this is a wall ok. Assumed it is a wall and then this is the fence. So, to get the largest area we enclosed, I assume that this is 80 m, this is 2m and this is 2m. So, this is 80, 82, 84 m fence. Then to get the area, 80 times 2, I get 160 m^2 (misreads 160 m^2 as 160 m square).
R: What do you get?
S: My answer would be 160.
R: What is the largest area being enclosed?
S: 160.

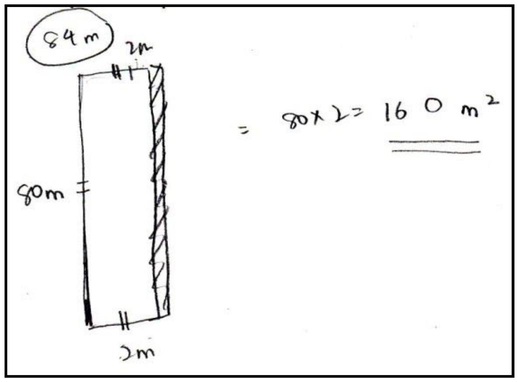


Figure N139. Roslina draws a rectangular garden and then calculates its area.

In Excerpt R41, Roslina drew a rectangular garden with the dimensions of 80 m by 2 m, labelled its dimensions and then calculated its area as 160 m^2, as shown in Figure N139. She assumed that the length and the width of the rectangular garden were 80 m and 2 m respectively. Roslina thought that the largest area being enclosed was 160 m^2 and 80 m by 2 m was the dimension of the rectangular garden that will yield the largest area being enclosed.

When probed to explain why 160 m^2 was the largest area being enclosed, Roslina was unable to justify it. Excerpt R42 is illustrative (Roslina/L1293-1308).

Excerpt R42

R: How do you know "160" will give you the largest area here?

S: Because I will make sure that the rectangular will have two sides which will be different. So, my first choice is 80 and 2. So, 84 and then my other choice would be for example 82 and 1. (Draws another rectangular garden. Labels its dimensions and then calculates its area, as shown in Figure N140). Like this and then this is the wall. If I want to get the area of the rectangle, this will be 82 times 1. So, this is 82 m^2 (misreads 82 m^2 as 82 m square). So, surely that this is the largest area being enclosed (refers to her first choice, as shown in Figure N139). So, I think I'll say that this is the largest area.

R: What is the dimension of your largest area?

S: 80 times 2.

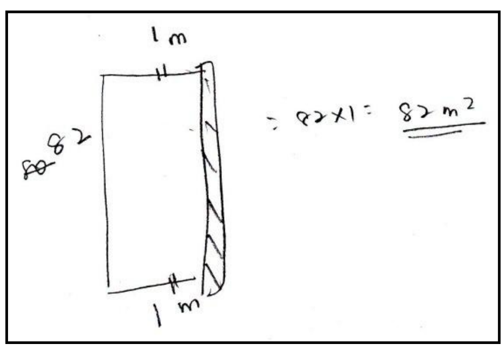


Figure N140. Roslina draws another rectangular garden and then calculates its area.

In Excerpt R42, Roslina was unable to justify why 160 m^2 was the largest area being enclosed. She merely explained that 80 m by 2 m was her first choice. Roslina expressed that her other choice would be 82 m by 1 m. Roslina drew another rectangular garden with the dimensions of 82 m by 1 m, labelled its dimensions and then calculated its area as 80 m^2, as shown in Figure N140. Roslina reiterated that the largest area being enclosed was 160 m^2 and 80 m by 2 m was the dimension of the rectangular garden that will yield the largest area being enclosed.

When probed to check the answer for the fencing problem, Roslina suggested that she would use the list all-and-compare strategy, to verify the answer. Excerpt R43 is illustrative (Roslina/L1309-1322).

Excerpt R43

R: How would you check your answer?
S: I will list all the possible answer and then I will try out. I will make sure that my one side will be long. I will make it as long as it can. I will measure that's many possible and then I will compare. When I compared these answers, I will make sure that the length have to be 84 m of fencing. And then when I get it, I found that when the dimension is 80 times 2, 80 m and 2 m, the area would be the largest.
R: Just now you mentioned that you "will list all the possible answer". Could you tell me more about it?
S: Ok this is one possible answer (points to 160 m^2) and this is the other (points to 82 m^2). Then I will find the length that can make it to 84 m fence and then when I list, I found that this is the largest (points to 160 m^2). So, I would say that dimension 80 m and 2 m will be the, yield the largest area being enclosed.

In Excerpt R43, Roslina suggested that she would use the list all-and-compare strategy, to verify the answer. Roslina explained that she would list all the possible answers and then compare them. Roslina emphasized that the total length has to be 84 m of fencing. Roslina concluded that she would find that the dimension of 80 m by 2 m will yield the largest area being enclosed.

Summary

In summary, Roslina used trial and error strategy to solve the fencing problem. Nevertheless, she did not find the dimension of the rectangular garden that will yield the largest area being enclosed. Roslina suggested that she would use the list all-and-compare strategy to verify the answer.

Ethical Knowledge

When probed to check the answer for the fencing problem, Roslina suggested that she would use the list all-and-compare strategy to verify the answer. This can be seen in Excerpt R43. Roslina wrote measurement units for the dimension (80 m by 2 m) that she thought would yield the largest area being enclosed. She also wrote measurement unit for its area (160 m^2) that she thought was the largest area being enclosed, as shown in Figure N139.

Developing Area Formulae

Procedural Knowledge

Roslina could recall the formula for the area of a rectangle, namely '$a \times b$', as shown in Figure N141. Nevertheless, she was unable to develop it. Excerpt R44 is illustrative (Roslina/L1365-1384, L1410-1413, L1420-1423).

Excerpt R44

R: (Puts a card written the following scenario in front of Roslina). Suppose that a Form One student comes to you and says that he does not know how to develop (derive) the formula for calculating the area of the following figures:
(m) Rectangle,
(n) Parallelogram,
(o) Triangle, and
(p) Trapezium.
How would you show him the way to develop (derive) the formula for calculating the area of these figures? Let's start with rectangle.

S: Ok first, for rectangle, I draw the shape of rectangle first. (Draws a rectangle. Labels its dimension and then writes its area formula, as shown in Figure N141). I tell that rectangle have two different sides. This side should be equals to this side (points to the lengths of the rectangle). This side equals to this side (points to the widths of the rectangle). So, the area of rectangle would be "a times b".

.
.
.

R: How would you develop (derive) the formula for calculating the area of a rectangle?

S: "a times b". I symbolized this length, this side is "a" and this "b" and then I said that the formula to find the area of rectangle is "a times b".

.
.
.

R: How would you show him the way to develop (derive) the formula for calculating the area of rectangle?

S: Oh how to derive formula. Em I don't know how to derive the formula. I just know how to use the formula.

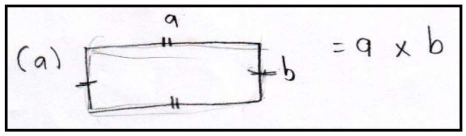


Figure N141. Roslina draws a rectangle and then writes its area formula.

In Excerpt R44, Roslina indicated that the opposite sides of a rectangle are congruent (same length). She stated that the formula for the area of a rectangle is '*a* times *b*'. She used *a* and *b* to represent the length and the width of the rectangle. Nevertheless, she was unable to develop it. Roslina expressed that she just knew how to use the formula. Roslina just memorized the formula. She did not attempt to develop the formula.

Roslina neither could recall the formula for the area of a parallelogram nor able to develop it. Excerpt R45 is illustrative (Roslina/L1384-1394, L1426-1427).

Excerpt R45

S: (Draws a parallelogram. Labels its base and height as "b" and "h" respectively and then writes its area formula, as shown in Figure N142). And then for parallelogram, the shape like this. This side is the same. So, the area would be half times the height times the side bottom it. So, I labeled it as "b". (The correct formula for the area of a parallelogram is "base times height or b x h", not $\frac{1}{2}$(h x b), as shown in Figure N142). … I don't know how to derive it.

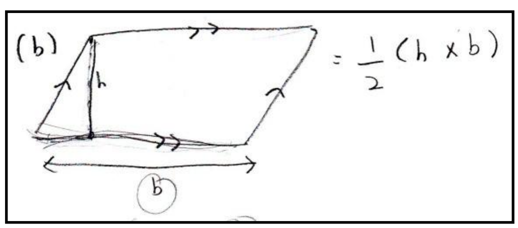


Figure N142 Roslina draws a parallelogram and then writes its area formula.

In Excerpt R45, Roslina indicated that the parallel sides of a parallelogram are congruent (same length). She could not recall the formula for the area of a parallelogram. Roslina thought that the formula for the area of a parallelogram is "half times the height times the side bottom it" or $\frac{1}{2}$(h x b), as shown in Figure N142. The correct formula for the area of a parallelogram is "base times height or b x h", not $\frac{1}{2}$(h x b), as shown in Figure N142. Roslina admitted that she did not know how to develop it.

Roslina could recall the formula for the area of a triangle, namely ‘$\frac{1}{2}\ (h\ \times b)$’, as shown in Figure N143. Nevertheless, she was unable to develop it. Excerpt R46 is illustrative (Roslina/L1395-1402).

Excerpt R46

S: And then (c) is triangle. (Draws a triangle. Labels its base and height as "b" and "h" respectively and then writes its area formula, as shown in Figure N143). Triangle shape is, there are three sided, three sides. So, this is the height and then the formula is half times the height with the base.

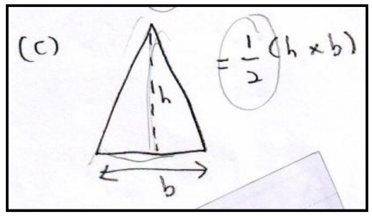


Figure N143. Roslina draws a triangle and then writes its area formula.

In Excerpt R46, Roslina stated that the formula for the area of a triangle is ‘half times the height times the base’ or ‘$\frac{1}{2}\ (h\ \times b)$’, as shown in Figure N143. Roslina was unable to develop the formula. Roslina just memorized the formula. She did not attempt to develop the formula.

Roslina neither could recall the formula for the area of a trapezium nor able to develop it. Excerpt R47 is illustrative (Roslina/ L1403-1409).

Excerpt R47

S: And (d) is the trapezium. (Draws a prism instead of a trapezium, as shown in Figure N144). Trapezium is a shape…(silent for a while) I forgot, I don't know, I can not recall the formula of trapezium.

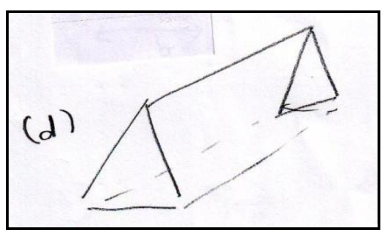


Figure N144. Roslina draws a prism instead of a trapezium.

In Excerpt R47, Roslina drew a prism instead of a trapezium, as shown in Figure N144. Roslina admitted that she was unable to recall the formula for the area of a trapezium. It also indicated that Roslina was unable to develop the formula.

Summary

In summary, Roslina could recall the formula for the area of a rectangle and triangle. Nevertheless, she was unable to develop these formulae. Roslina did not attempt to develop the formula for the area of a rectangle and triangle.

Conceptual Knowledge

Roslina could recall the formula for the area of a rectangle. Nevertheless, she was unable to develop the formula. It was apparent that Roslina lack of conceptual knowledge underpinning the formula for the area of a rectangle.

Roslina could not recall the formula for the area of a parallelogram. She was unable to develop the formula. It was apparent that she did not know the relationship between the area of a parallelogram and the area of a rectangle. Had Roslina been known of this relationship, she would know how to develop the formula for the area of a parallelogram.

Roslina could recall the formula for the area of a triangle. Nevertheless, she was unable to develop the formula. Roslina did not know the relationship between the area of a triangle and the area of the rectangle that encloses it. Had she been known of this relationship, Roslina would know how to develop the formula for the area of a triangle.

Roslina could not recall the formula for the area of a trapezium. She was unable to develop the formula. It was quite clear that she did not know the relationship between the area formulae of a rectangle, parallelogram, triangle, and trapezium. Had Roslina been known of this relationship, she would know how to develop the formula for the area of a trapezium.

Linguistic Knowledge

Roslina used appropriate mathematical symbols to write the formula for the area of a rectangle, namely 'a x b', as shown in Figure N141. Nevertheless, Roslina used inappropriate mathematical terms 'this length' and 'this side' to explain the meaning of the symbols *a* and *b* that she employed. Roslina explained that "…this length, this side is *a* and this *b*." (Roslina/L1412). Actually, *a* and *b* represents the length and the width of the rectangle.

Roslina used appropriate mathematical symbols to write the formula for the area of a triangle, namely '$\frac{1}{2}\ (h\ \times b)$', as shown in Figure N143. Roslina used appropriate mathematical terms 'half', 'times', 'height', and 'base' to state the formula for the area of a triangle. Roslina stated that "…the formula is half times the height with the base." (Roslina/L1402).

Strategic Knowledge

Roslina could recall the formula for the area of a rectangle and triangle. Nevertheless, she was unable to develop these formulae. Roslina did not attempt to develop these formulae.

Ethical Knowledge

Roslina could recall the formula for the area of a rectangle but she did not attempt to develop the formula, as shown in Excerpt R44. Roslina could not recall the formula for the area of a parallelogram and she did not attempt to develop the formula, as shown in Excerpt R45. Roslina could recall the formula for the area of a triangle but she did not attempt to develop the formula, as shown in Excerpt R46. Roslina could not recall the formula for the area of a trapezium and she did not attempt to develop the formula, as shown in Excerpt R47.

Level of Subject Matter Knowledge

In this section, Roslina' levels (low, medium, high) of subject matter knowledge of perimeter and area was analyzed in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

Roslina gained a low level of conceptual knowledge of perimeter and area when she obtained 32.0% of appropriate mathematical elements of conceptual knowledge of perimeter and area during the clinical interview. Roslina gained a low level of procedural knowledge of perimeter and area when she obtained 36.4% of appropriate mathematical elements of procedural knowledge of perimeter and area. Roslina achieved a medium level of linguistic knowledge of perimeter and area when she obtained 65.1% of appropriate mathematical elements of linguistic knowledge of perimeter and area. Roslina achieved a medium level of strategic knowledge of perimeter and area when she obtained 64.3% of appropriate mathematical elements of strategic knowledge of perimeter and area. Roslina obtained a medium level of ethical knowledge of perimeter and area when she obtained 55.1% of appropriate mathematical elements of ethical knowledge of perimeter and area. Roslina achieved an overall medium level of subject matter knowledge of perimeter and area when she obtained 53.5% of appropriate mathematical elements of subject matter knowledge of perimeter and area.

Suhana

Suhana lives in Kuantan, Pahang. Suhana is 20 years 10 months old when she was interviewed. Currently, she is pursuing a 4-year Bachelor of Science with Education (B.Sc.Ed.) program at a public university. She majored and minored in physics and mathematics respectively. She obtained grade 1A in Mathematics and Additional Mathematics in her 2003 SPM examination (equivalent to O level examination). She scored A in Mathematics in the 2005 Matriculation examination (equivalent to A level examination). Suhana performed satisfactory in her mathematics content courses at the university level when she secured one B, one C+, and one D− in three mathematics content courses she had completed during the first and second year of her studies. The detail of her performance is shown in Table N8.

Table N8

Mathematics Performance of Suhana

Courses	Grades
1. Calculus for Science Students I	C+
2. Statistics for Science Students	D−
3. Calculus for Science Students II	B

At the time of data collection, Suhana was in her second semester of third year studies. She attained 2.52 in the Cumulative Grade Point Average (CGPA) for her first two years of studies at the public university. She does not have any teaching experience prior to this interview.

Notion of Perimeter

Conceptual Knowledge

Suhana has selected shapes “A”, “C”, “D”, “H”, “I”, and “K” as having a perimeter. Excerpt S1 shows her choice of shapes that have a perimeter (Suhana/L115-117).

Excerpt S1

R: (Puts a handout comprises 12 shapes in front of Suhana). Tick the shapes that have a perimeter.
S: (Ticks shapes "A", "C", "D", "H", "I", and "K", as shown in Figure N145).

In Excerpt S1, Suhana has selected all simple closed curves (A, C, H, K) as well as all closed but not simple curves (D, I) that have a perimeter. Nevertheless, she did not select the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that her notion of perimeter was limited to simple closed curves, and closed but not simple curves, exclusive of 3-dimensional shapes. Suhana also did not select the two simple but not closed curves (B, G) as well as the two 1-dimensional shapes (E, L) that do not have a perimeter. In other words, Suhana did not select an open shape (including the lines) as having a perimeter.

When asked to justify her selection, Suhana explained that she selected shapes “A”, “C”, “D”, “H”, “I”, and “K” because they have closed boundary. She emphasized that shape “D” has two boundaries. Suhana pointed out that initially she was not sure whether shape “I” has a perimeter as it has two regions. Subsequently, Suhana selected shape “I” because its surrounding is closed. Excerpt S2 depicts her justification for selecting each of these shapes (Suhana/L123-137).

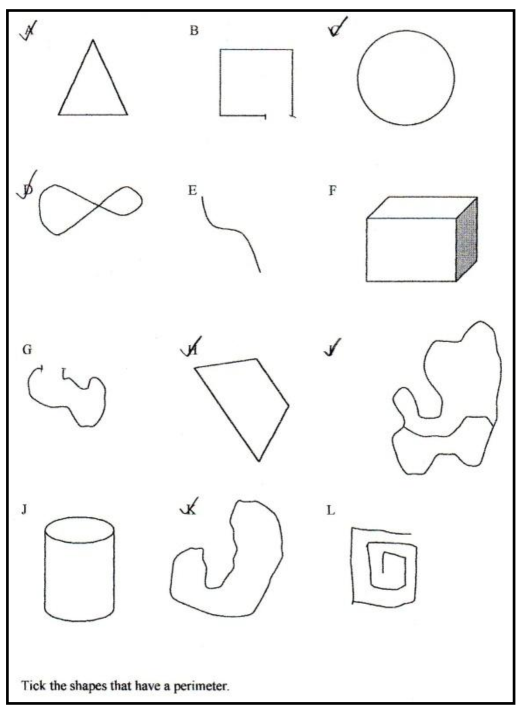


Figure N145. Suhana’s selection of shapes that have a perimeter.

Excerpt S2

R: Why did you select shape "A"?
S: "A" because it has closed boundary and can demonstrate.

R: Why did you select shape "C"?
S: Em same as "A".
R: Could you tell me more about it?
S: It has closed boundary and then the shape looks familiar, em the circle.
R: Why did you select shape "D"?
S: "D" because "D" has two closed boundary and then em that's all.
R: Why did you select shape "H"?
S: "H" because it is closed boundary.
R: Why did you select shape "I"?
S: "I" ok at first I don't know if I have to select "I" or not because it has two area. Because *dekat sekeliling ini tertutup* [this surrounding is closed].
R: Why did you select shape "K"?
S: "K" because of the closed boundary and empty, closed boundary.

Suhana explained that she did not select shapes “F” and “J” because they are 3-dimensional shapes. Suhana thought that 3-dimensional shapes do not have perimeter. She elaborated that perimeter is limited to 2-dimensional shapes. Excerpt S3 demonstrates her justification for not selecting shapes “F” and “J” as having a perimeter (Suhana/L143-145, L149-151).

Excerpt S3

R: Why didn't you select shape "F"?
S: "F" because it's 3D. It's 3D picture. So, I think it doesn't have perimeter. Perimeter only apply to 2D.
.
.
.
R: Why didn't you select shape "J"?
S: "J" because it is 3D. So, it's impossible. It is confusing on how to calculate the perimeter.

Suhana explained that she did not select shape “B” because it is not closed. Suhana also explained that she did not select shape “G” because the line is not connected to each other. Excerpt S4 reveals her justification for not selecting shapes “B” and “G” as having a perimeter (Suhana/L138-140, L146-148).

Excerpt S4

R: Why didn't you select shape "B"?
S: Not, not closed boundary. It is not…(silent for a while) em it is not closed. The shape not closed.
.
.
.
R: Why didn't you select shape "G"?
S: "G" because it is not, the line is not connected to each other. It has space between it.

Suhana explained that she did not select shapes “E” and “L” because they are just a line. Suhana elaborated that a line does not have perimeter but it has length. Excerpt S5 exhibits her justification for not selecting shapes “E” and “L” as having a perimeter (Suhana/L141-142, L152-154).

Excerpt S5

R: Why didn't you select shape "E"?
S: "E", it's a line, not a straight line. But only a line.
.
.
.
R: Why didn't you select shape "L"?
S: Because "L" is a line. So, I think it doesn't have perimeter. But it has the length.

Summary

In summary, Suhana has selected all simple closed curves (A, C, H, K) and all closed but not simple curves (D, I) that have a perimeter. It indicated that her notion of perimeter was limited to simple closed curves, and closed but not simple curves, exclusive of 3-dimensional shapes. She justified her selection by explaining that they have closed boundary.

Linguistic Knowledge

Suhana used appropriate mathematical term ‘closed’ to justify her selection of shapes that have a perimeter. Suhana explained that she selected shapes “A”, “C”, “D”, “H”, “I”, and “K” because they have closed boundary, as shown in Excerpt S2.

Suhana used appropriate mathematical symbol ‘3D’ to represents 3-dimensional shapes, “F” and “J”, as shown in Excerpt S3. Nevertheless, ‘3-dimensional shapes’ was not the appropriate justification for not selecting shapes “F” and “J” that have a perimeter as we still can find perimeter for the faces of solids.

Suhana used appropriate negation ‘not closed’ as her justification for not selecting shape “B” as having a perimeter. Suhana explained that she did not select shape “B” because it is not closed. Suhana used inappropriate negation ‘not connected’ as her justification for not selecting shape “G” as having a perimeter. Suhana explained that she did not select shape “G” because the line is not connected to each other, as shown in Excerpt S4. Suhana used appropriate mathematical term ‘line’ as her justification for not selecting shapes “E” and “L” as having a perimeter. Suhana explained that she did not select shapes “E” and “L” because they are just a line, as shown in Excerpt S5. Suhana used appropriate mathematical symbol ‘2D’ to represents 2-dimensional shapes, as shown in Excerpt S3.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Suhana had taken the effort to justify the selection of shapes that have a perimeter, as shown in Excerpt S2. She provided appropriate justification for selecting shapes “A”, “C”, “D”, “H”, “I”, and “K” that have a perimeter. Suhana also provided justification for not selecting shapes “F” and “J” that have a perimeter, as shown in Excerpt S3. Nevertheless, ‘3-dimensional shapes’ was not the appropriate justification for not selecting shapes “F” and “J” that have a perimeter as we still can find perimeter for the faces of solids.

Suhana also had taken the effort to provide justification for not selecting other shapes that do not have a perimeter. She provided appropriate justification for not selecting shape “B” as having a perimeter. Nevertheless, Suhana provided inappropriate justification for not selecting shape “G” as having a perimeter, as shown in Excerpt S4. She provided appropriate justification for not selecting shapes “E” and “L” as having a perimeter, as shown in Excerpt S5.

Notion of Area

Conceptual Knowledge

Suhana has successfully selected all the shapes that have an area, namely "A", "C", "D",”F”, "H", "I", “J”, and "K". Excerpt S6 shows her choice of shapes that have an area (Suhana/L170-173).

Excerpt S6

R: (Puts a handout comprises 12 shapes in front of Suhana). Tick the shapes that have an area.
S: (Ticks shapes "A", "C", "D",”F”, "H", "I", “J”, and "K", as shown in Figure N146).

In Excerpt S6, Suhana has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. She also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that Suhana had a static perspective of the notion of area. Based on this perspective, area can be viewed as the amount of surface enclosed within a boundary. It also indicated that her notion of area was not only limited to 2-dimensional shapes (closed plane shapes), but also inclusive of 3-dimensional shapes. Suhana also did not select the two open shapes (B, G) as well as the two 1-dimensional shapes (E, L) that do not have an area. In other words, Suhana did not select an open shape (including the lines) as having an area. It can be inferred that she did not has a dynamic perspective of area or, at least, this knowledge was not accessible to her during the clinical interview.

When asked to justify her selection, Suhana explained that she selected shapes "A", "C", "D", "H", "I", and "K" because they are closed shapes. Excerpt S7 depicts her justification of selecting each of these shapes (Suhana/L179-184, L188-191, L194-195).

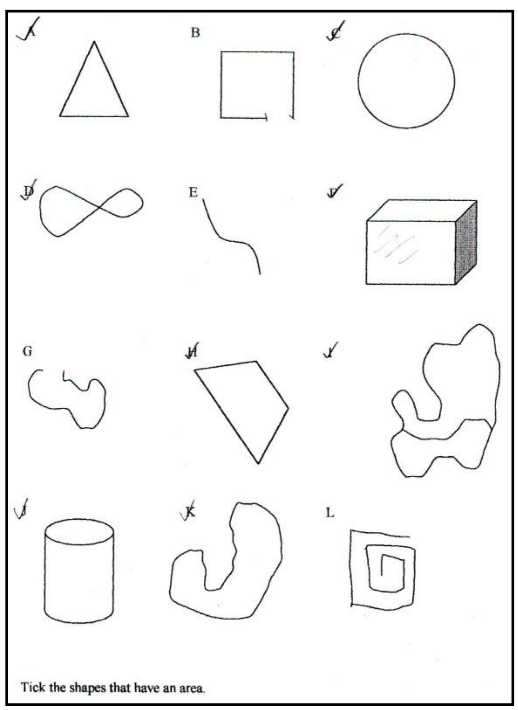


Figure N146. Suhana’s selection of shapes that have an area.

Excerpt S7

R: Why did you select shape "A"?
S: "A" because it is closed, closed shape and triangle.
R: Why did you select shape "C"?
S: "C" also closed, closed shape and em closed shape.
R: Why did you select shape "D"?
S: Yes closed shape, closed and have closed shape and it have two area.
.
.
.
R: Why did you select shape "H"?
S: "H" closed, closed shape, closed boundary.
R: Why did you select shape "I"?
S: The same as "D" because it has closed shape although it has two area.
.
.
.
R: Why did you select shape "K"?
S: "K" is closed.

Suhana explained that she selected shapes “F” and “J” because 3D has surface area. Excerpt S8 demonstrates her justification of selecting each of these shapes (Suhana/L185-187, L192-193).

Excerpt S8

R: Why did you select shape "F"?
S: "F", it's a cube, cuboid, a cube and then it's 3D. So, it has area too. Area inside, area of the *permukaan* [surface area].
.
.
.
R: Why did you select shape "J"?
S: "J" is a cylinder. So, it has three *permukaan* [surface]. So, it has an area.

Suhana explained that she did not select shape “B” because it is an open shape. Suhana also explained that she did not select shape “G” because it is not a closed shape. Excerpt S9 reveals her justification for not selecting shapes “B” and “G” as having an area (Suhana/L196-197, L200-201).

Excerpt S9

R: Why didn't you select shape "B"?
S: "B", it's open shape. So, it is not, it didn't have area.
.
.
.
R: Why didn't you select shape "G"?
S: "G" is not closed shape.

Suhana explained that she did not select shapes “E” and “L” because it is just a line. Suhana also pointed out that shape “L” is not closed. Excerpt S10 exhibits her justification for not selecting shapes “E” and “L” as having an area (Suhana/L198-199, L202-203).

Excerpt S10

R: Why didn't you select shape "E"?
S: "E" is just a line and it has no area at all.
.
.
.
R: Why didn't you select shape "L"?
S: "L" because "L" is not closed and then it's a line.

Summary

In summary, Suhana has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. She also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that Suhana had a static perspective of the notion of area. Her notion of area was not only limited to 2-dimensional shapes (closed plane shapes), but also inclusive of 3-dimensional shapes. Suhana justified her selection by explaining that she selected shapes "A", "C", "D", "H", "I", and "K" because they are closed shapes. Suhana explained that she selected shapes “F” and “J” because 3D has surface area.

Linguistic Knowledge

Suhana used appropriate mathematical term ‘closed’ to justify her selection of shapes that have an area. Suhana explained that she selected shapes "A", "C", "D", "H", "I", and "K" because they are closed shapes, as shown in Excerpt S7. Suhana used appropriate mathematical symbol ‘3D’ to justify her selection of shapes that have an area. Suhana explained that she selected shapes “F” and “J” because 3D has surface area, as shown in Excerpt S8.

Suhana used appropriate mathematical term ‘open’ as her justification for not selecting shape “B” as having an area. Suhana explained that she did not select shape “B” because it is an open shape. Suhana also used appropriate negation ‘not closed’ as her justification for not selecting shape “G” as having an area. Suhana explained that she did not select shape “G” because it is not a closed shape, as shown in Excerpt S9. Suhana also used appropriate mathematical term ‘line’ as her justification for not selecting shapes “E” and “L” as having an area. Suhana explained that she did not select shapes “E” and “L” because it is just a line, as shown in Excerpt S10. Suhana used appropriate mathematical symbol ‘3D’ to represents 3-dimensional shapes, “F” and “J”, as shown in Excerpt S8.

Ethical Knowledge

Suhana had taken the effort to justify the selection of shapes that have an area, as shown in Excerpts S7 and S8. She provided appropriate justification for selecting shapes “A”, “C”, “D”, ”F”, “H”, “I”, “J”, and “K” that have an area.

Suhana also had taken the effort to provide justification for not selecting other shapes that do not have an area. She provided appropriate justification for not selecting shapes “B” and “G” as having an area, as shown in Excerpt S9. Suhana also provided appropriate justification for not selecting shapes “E” and “L” as having an area, as shown in Excerpt S10.

Notions of the Units of Area

Conceptual Knowledge

Suhana thought that a rectangle and triangle cannot be used as the unit of area measurement to measure the area of other shape. Excerpt S11 is illustrative (Suhana/L225-237).

Excerpt S11

R: (Puts a card written the following scenario in front of Suhana). Ali, Chong, and David are discussing about the units of area. Ali says that we can use a square as the unit of area. Chong says that we can use a rectangle as the unit of area. David says that we can use a triangle as the unit of area. How would you respond to these students?

S: (Draws a square, rectangle, and triangle and then writes the area formula for the triangle, as shown in Figure N147). Ok first Ali, square. Chong, rectangle. David, triangle. Ok if you are using David's idea, it is impossible to apply to other shape. The same as Chong. He used rectangle, can not use on other shape.

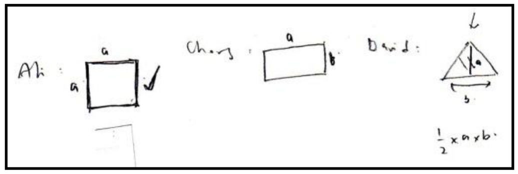


Figure N147. Suhana draws a square, rectangle, and triangle and then writes the area formula for the triangle.

In Excerpt S11, Suhana drew a square, rectangle, and triangle and then wrote the area formula for the triangle, as shown in Figure N147. She thought that a rectangle cannot be used as the unit of area measurement to measure the area of other shape. Suhana also thought that a triangle cannot be used as the unit of area measurement to measure the area of other shape.

When probed for the idea suggested by Ali, Suhana explained that a square can be used as the unit of area measurement because the sides of a square have the same length, labelled as ‘a’, as shown in Figure N147). Excerpt S12 is illustrative (Suhana/L250-255).

Excerpt S12

R: Can we use a square as the unit of area?
S: Ok *kalau* [if] a square *boleh* [can].
R: Why?
S: Because ok square have same length, "a", this side and this side (points to the square that she has drawn, as shown in Figure N147). So, it is easy to measure the area.

. When probed for the idea suggested by Chong, Suhana explained that it was difficult to use a rectangle as the unit of area measurement as the length and the width of the rectangle were of different length. Excerpt S13 is illustrative (Suhana/L256-258).

Excerpt S13

R: Can we use the rectangle as the unit of area?
S: It is a bit difficult because the side is not the same. So, it is difficult to calculate the area.

When probed for the idea suggested by David, Suhana stated that the formula for calculating the area of a triangle is ‘half times a times b’, as shown in Figure N147. Suhana explained that it is difficult to use a triangle as the unit of area measurement and she thus concluded that a triangle cannot be used as the unit of area measurement. Excerpt S14 is illustrative (Suhana/L259-263).

Excerpt S14

R: Can we use the triangle as the unit of area?
S: Triangle. If we want to calculate the area of the triangle, that will be "half times a times b". So, if the figure in, if we are taking the area like this one (points to the triangle that she has drawn, as shown in Figure N147) and then it is difficult to calculate it. So, cannot.

Summary

In summary, Suhana stated that a square can be used as the unit of area measurement. She thought that a rectangle and triangle cannot be used as the unit of area measurement to measure the area of other shape. It indicated that her notion of the unit of area was limited to a square. Suhana explained that a square can be used as the unit of area measurement because the sides of a square have the same length. It indicated that she was unable to provide the appropriate justification that any shape that tessellates a plane can be used as a unit of area measurement. Suhana explained that a rectangle and triangle cannot be used as the unit of area measurement because they were difficult to use them.

Linguistic knowledge

Suhana used inappropriate mathematical term ‘same length’ to justify that a square can be used as the unit of area. She explained that a square can be used as the unit of area measurement because the sides of a square have the same length.

Suhana used inappropriate words ‘difficult to use’ to justify that a rectangle cannot be used as the unit of area. Suhana explained that a rectangle cannot be used as the unit of area because it was difficult to use it.

Suhana used inappropriate words ‘difficult to use’ to justify that a triangle cannot be used as the unit of area. Suhana explained that a triangle cannot be used as the unit of area because it was difficult to use it.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Suhana had taken the effort to justify the shape that can be used as a unit of area measurement. Nevertheless, she was unable to provide an appropriate justification for the shape that can be used as a unit of area measure. This can be seen in Excerpt S12. In reality, any shape that tessellates a plane can be used as a unit of area measurement.

Suhana also had taken the effort to justify the shapes that cannot be used as a unit of area measurement. Nevertheless, she was unable to provide an appropriate justification for the shapes that she thought cannot be used as a unit of area measure. This can be seen in Excerpts S13 and S14.

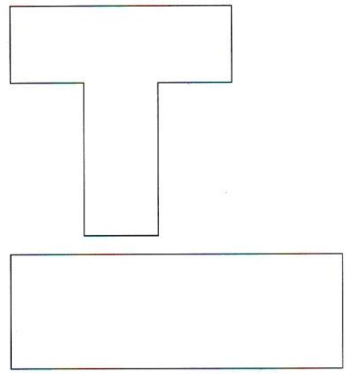
Comparing Perimeter (No Dimension Given)

Strategic Knowledge

Suhana used the formal method of measuring the side and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter. Excerpt S15 shows the formal method that she used to compare the perimeter of the given pair of shapes (Suhana/L319-337).

Excerpt S15

R: (Puts the following pair of shape in front of Suhana). How would you find out whether they had the same perimeter?



S: Em using ruler.
R: Could you show me how it is?
S: Measure the length and then add. (Measure the length of each side of the T-shape by ruler and then calculates its perimeter, as shown in Figure N148). (Measure the length of each side of the rectangle by ruler and then calculates its perimeter, as shown in Figure N149).
R: Could you explain your solution?
S: When I calculate, it is the same.
R: What did you get?
S: 24 cm.

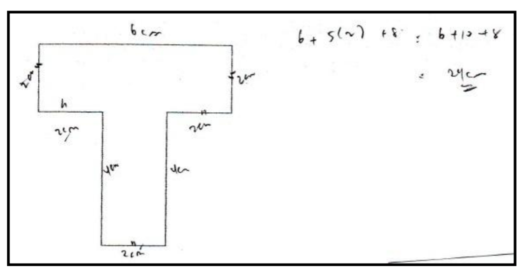


Figure N148. Suhana measures the length of each side of the T-shape by ruler and then calculates its perimeter.

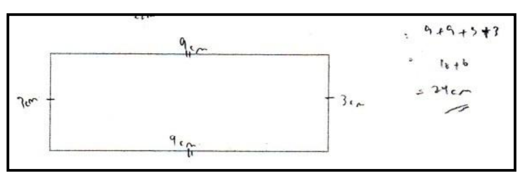


Figure N149. Suhana measures the length of each side of the rectangle by ruler and then calculates its perimeter.

In Excerpt S15, Suhana measured the length of each side of the given T-shape by ruler and then calculated its perimeter correctly as 24 cm, as shown in Figure N148. She measured the length of each side of the given rectangle by ruler and then calculated its perimeter correctly as 24 cm, as shown in Figure N149. Suhana concluded that the given pair of shapes had the same perimeter.

When probed for alternative method of comparing the perimeter, Suhana used a semi-formal method of measuring the side by thread. Excerpt S16 demonstrates how she used the thread to compare the perimeter (Suhana/L338-351).

Excerpt S16

R: Could you think of other way of finding out whether they had the same perimeter?
S: Using the *benang* [thread].
R: Could you show me how it is?
S: First I put here (Uses the thread to measure the length of each side of the T-shape. Marks with pencil, the length of each side, on the thread. Cut it and then uses the same portion of the thread to measure the total length of the rectangle). Then I put it on the rectangle (measures the total length of the rectangle using the same portion of the thread).
R: Could you explain your solution?
S: Ok first *gunakan benang ini* [use this thread], by using the *benang* [thread] ok, I measure the length of this T-shape first and then using the same *benang* [thread], I measure this one (points to the rectangle). So, it is the same. It has the same length, same perimeter.

In Excerpt S16, Suhana measured the length of each side of the T-shape by thread. She marked with pencil, the length of each side, on the thread. Suhana cut it and then used the same portion of the thread to measure the total length of the rectangle. She concluded that the given pair of shapes had the same perimeter, as shown in Excerpt S16.

When probed for other method of comparing the perimeter, Suhana used a semi-formal method of measuring the side by a piece of blank A4-sized paper. Excerpt S17 depicts how she used the paper to compare the perimeter (Suhana/L352-362).

Excerpt S17

R: Could you think of other way of finding out whether they had the same perimeter?
S: Em ok if I don't have the ruler, I will use the paper (takes a blank A4-sized paper) and then just measure like that.
R: Could you show me how it is?
S: (Marks the length of each side of the T-shape on the length of the blank paper. Repeats the same for the rectangle and sees whether it ends at the same point. Finds a little bit of different of the total length for the T-shape and the rectangle). Measure this one (refers to the T-shape). Then here (refers to the rectangle). Em it is not accurate *lah*. So, this is a bit different.

In Excerpt S17, Suhana marked the length of each side of the T-shape on the length of the blank paper. She repeated the same for the rectangle and sees whether it ended at the same point. Suhana found that there was a little bit of different of the total length for the T-shape and the rectangle. She thus concluded that this method of comparison was not accurate.

When probed for other method of comparing the perimeter, Suhana used a semi-formal method of putting a 1-cm grid paper on both the given shapes, as shown in Figures N150 and N151. Excerpt S18 reveals how she used this semi-formal method to determine each perimeter and then compare their measurements (Suhana/L363-388).

Excerpt S18

R: Could you think of other way of finding out whether they had the same perimeter?
S: Using this grid (1-cm grid paper).
R: Could you show me how it is?
S: (Puts the grid paper on the T-shape and then writes the length of each side on the grid paper, as shown in Figure N150). (Writes its perimeter as follow).

$$6 + 5\,(2) + 8 = 6 + 10 + 8$$
$$= 24 \text{ grid}$$

(Puts the grid paper on the rectangle and then writes the length of each side on the grid paper, as shown in Figure N151). (Writes its perimeter as follow).

$$2\,(9) + 2\,(3) = 18 + 6$$
$$= 24 \text{ grid}$$

R: What did you get?
S: 24 grids.
R: So, what's your conclusion?
S: It has the same shape. Eh, it is the same perimeter.

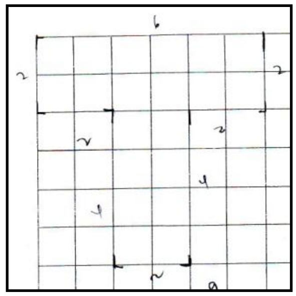


Figure N150. Suhana puts the grid paper on the T-shape and then writes the length of each side on the grid paper.

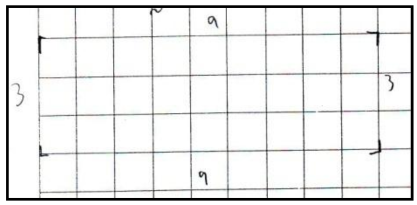


Figure N151. Suhana puts the grid paper on the rectangle and then writes the length of each side on the grid paper.

In Excerpt S18, Suhana put the grid paper on the T-shape and then wrote the length of each side on the grid paper, as shown in Figure N150. She calculated the perimeter of the T-shape as 24 grids (it should be 24 cm). Suhana also put the grid paper on the rectangle and then wrote the length of each side on the grid paper, as shown in Figure N151. She also calculated the perimeter of the rectangle as 24 grids (it should be 24 cm). Suhana concluded that the given pair of shapes had the same perimeter.

When probed further for other method of comparing the perimeter, Suhana used an informal method of cut-and-paste. Excerpt S19 exhibits how she used this informal method to compare the perimeter (Suhana/L389-398).

Excerpt S19

R: Could you think of other way of finding out whether they had the same perimeter?
S: First I cut this (T-shape). (Uses the scissors, cuts the T-shape along its outline and then superimposes on the rectangle). Then I put on the second picture (rectangle). Em it's not accurate because it has error here.
R: Could you think of other way of finding out whether they had the same perimeter?
S: Em *tak ada* [no], no.

In Excerpt S19, Suhana used the scissors to cut the T-shape along its outline and then superimposed it on the rectangle. She concluded that this method was not accurate as it has error.

Summary

In summary, Suhana produced one formal, three semi-formal and one informal methods of determining whether the given pair of shape had the same perimeter. In the first method, she used the formal method of measuring the length of side by ruler and applying the definition of perimeter. In the second method, Suhana used a semi-formal method of measuring the side by thread. In the third method, she used a semi-formal method of measuring the side by a piece of blank A4-sized paper. In the fourth method, Suhana used a semi-formal method of putting a 1-cm grid paper on both the given shapes. In the fifth method, she used an informal method of cut-and-paste.

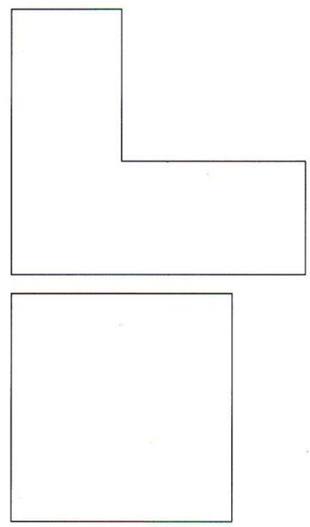
Comparing Area (No Dimension Given)

Strategic Knowledge

Suhana partitioned L-shape into two rectangles for which area measurement formulae were known. Excerpt S20 shows the formal method of measuring the side and applying the area formula that she used to compare the area of the given pair of shapes (Suhana/L424-449).

Excerpt S20

R: (Puts the following pair of shape in front of Suhana). How would you find out whether they had the same area?



S: Measure (by ruler).
R: Could you show me how it is?
S: (Measures the length of the two adjacent sides of the square by ruler and then calculates its area, as shown in Figure N152). (Partitions L-shape into two rectangles, labels as "A" and "B" respectively, as shown in Figure N153. Measures the length and the width of each rectangle by ruler and then alculates its total areas, as shown in Figure N153).
R: Could you explain your solution?
S: First I calculate this box (points to the square): 6 times 6 equals 36 and then this one (points to the L-shape), I divide into two. This (draws the dotted line to partition L-shape into two rectangles, as shown in Figure N153) and then we have "A" part and "B" part. "A" part has the length 7 cm and this one is 3. This one is 3 and this is 5. So, 7 times 3, 21. 5 times 3, 15. 36.

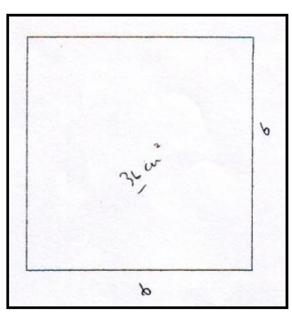


Figure N152. Suhana measures the length of the two adjacent sides of the square by ruler and then calculates its area.

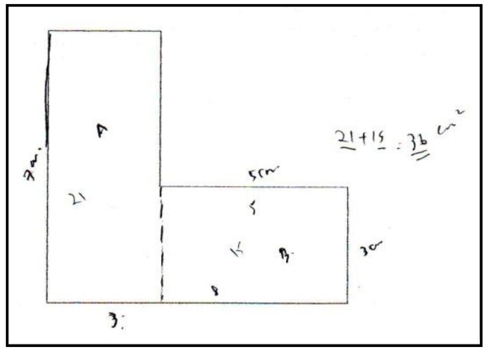


Figure N153. Suhana measures the length and the width of each rectangle by ruler and then calculates its area.

In Excerpt S20, Suhana measured the length of the two adjacent sides of the square by ruler and then calculated its area using square area formula as 36 cm^2, as shown in Figure N152. She partitioned L-shape into two rectangles, labelled as A and B respectively, as shown in Figure N153. Suhana measured the length and the width of each rectangle by ruler and then calculated its area using rectangle area formulae as 36 cm^2. She implicitly concluded that they had the same area.

When probed for alternative method of comparing the area, Suhana used a semi-formal method of tracing both shapes on a 1-cm grid paper and then determine its length and width by counting the number of 1-cm square on its length and width, as shown in Figures N154 and N155. Excerpt S21 depicts how she used this semi-formal method to determine each area and then compare their measurements (Suhana/L450-480).

Excerpt S21

R: Could you think of other way of finding out whether they had the same area?

S: Using this grid (takes a 1-cm grid paper). (Traces the square on the grid paper, as shown in Figure N154, by putting the grid paper on top of the square. Counts the number of 1-cm grid on the length of the two adjacent sides of the traced square and labels its lengths as 6 respectively. Multiplies the length of the two adjacent sides to get the area as 36 cm^2). (Traces the L-shape on the grid paper, as shown in Figure N155, by putting the grid paper on top of the L-shape. Partitions the traced L-shape into two rectangles, labels as "A" and "B" respectively, as shown in Figure N155. Counts the number of 1-cm grid on the length and the width of each rectangle. Labels its length and width as "7 and 3" and "5 and 3" respectively. Multiplies the length and the width respectively to get the area as 21 and 15. Writes its total area as 36 cm^2).

R: Could you show me how it is?

S: Ok first I will draw the picture (the square) on this grid and then I got 6 times 6, 36. This one (L-shape) I divide into two like before, "A" and "B". When I 3 times 7, 21 and this one 5 times 3, 15. So, add on equals to 36.

In Excerpt S21, Suhana traced the square on the 1-cm grid paper, as shown in Figure N154, by putting the grid paper on top of the square. She counted the number of 1-cm grid on the length of the two adjacent sides of the traced square and labelled its lengths as 6 respectively. Suhana multiplied the length of the two adjacent sides to get the area as 36 cm^2. She also traced the L-shape on the grid paper, as shown in Figure N155, by putting the grid paper on top of the L-shape. Suhana partitioned the traced L-shape into two rectangles, labels as "A" and "B" respectively, as shown in Figure N155. She counted the number of 1-cm grid on the length and the width of each rectangle and labelled its length and width as "7 and 3" and "5 and 3" respectively. Suhana multiplied the length and the width respectively to get the area as 21 and 15. She wrote its total area as 36 cm^2. Suhana implicitly concluded that they had the same area.

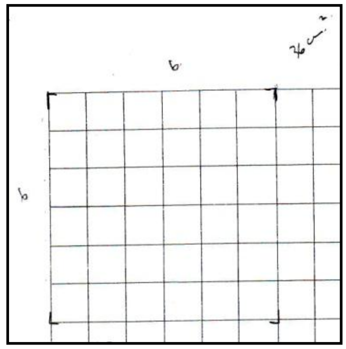


Figure N154. Suhana traces the square on the 1-cm grid paper and then calculates its area.

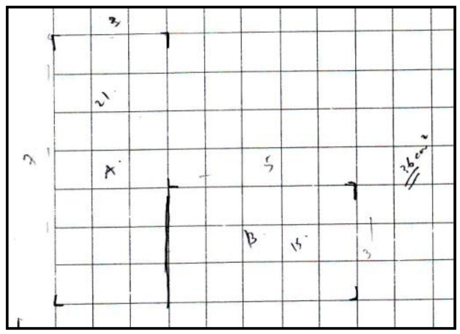


Figure N155. Suhana traces the L-shape on the 1-cm grid paper and then calculates its area.

When probed for other method of comparing the area, Suhana used an informal method of cut-and-paste. Excerpt S22 demonstrates how she used this informal method to compare the area (Suhana/L481-491).

Excerpt S22

R: Could you think of other way of finding out whether they had the same area?
S: …(Silent for a while) cut this one (points to the L-shape) and then put on the surface (of the square). (Cuts out L-shape by scissors and then superimpose it on the square).
R: Could you explain your solution?
S: Em first I cut the picture, the L-picture and then I put it into the square and it equals the square.
R: Could you think of other way of finding out whether they had the same area?
S: …(Silent for a while) no.

In Excerpt S22, Suhana used the scissors to cut the L-shape along its outline and then superimposed it on the square. She concluded that the area of the L-shape equal to area of the square.

Summary

In summary, Suhana produced one formal method, one semi-formal method, and one informal method of determining whether the given pair of shapes had the same area. In the first method, she partitioned L-shape into two rectangles, as shown in Figure N153. Suhana measured the length of side by ruler and applied area formulae. In the second method, she used a semi-formal method of tracing both shapes on a 1-cm grid paper and then determine its length and width by counting the number of 1-cm square on its length and width, as shown in Figures N154 and N155. In the third method, Suhana used an informal method of cut-and-paste.

Comparing Perimeter (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Suhana stated that shape B has the longer perimeter. Excerpt S23 shows the justification that she made (Suhana/ L521-540).

Excerpt S23

R: (Puts the following table in front of Suhana). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 1	25 paper clips	12 sticks

S: Shape B.
R: Why?
S: Because stick has length 15 cm and above while paper clip have (sic) only 2, 3, 2.5 cm. (Calculates the perimeters of shapes A and B respectively, as shown in Figure N156). So, 2.5 times 25, 62.5 cm and shape B, 12 times 15, 180. So, shape B.
R: How do you know that the length of the paper clip is 2.5 cm?
S: Because imagine, just by imagine.
R: How do you know that the length of the stick is 15 cm?
S: Because stick has the length like this one (points to a pencil on the table) and then I put next to the ruler and it's more than 15. So, I take it as 15.
R: But in this case, 25 is larger than 12.
S: Yeah but the thing that we used is not the same and in my opinion, stick is longer than paper clip and when we calculate the perimeter, shape B has the longer perimeter.

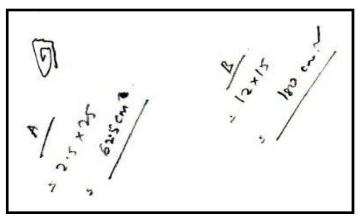


Figure N156. Suhana calculates the perimeters of shapes A and B respectively.

In Excerpt S23, Suhana explained that shape B has the longer perimeter because she thought that a stick is longer than a paper clip. It indicated that she focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. Nevertheless, Suhana did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. She imagined that the length of a paper clip and a stick is about 2.5 cm and 15 cm respectively. Thus, Suhana calculated the perimeters of shapes A and B as 62.5 cm^2 (the correct unit should be cm) and 180 cm^2 (the correct unit should be cm) respectively, as shown in Figure N156. She reiterated that shape B has the longer perimeter.

In Set 2, Suhana stated that shape B has the longer perimeter. Excerpt S24 depicts the justification that she made (Suhana/L579-588).

Excerpt S24

R: (Puts the following table in front of Suhana). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 2	10 paper clips	15 paper clips

S: Shape B.
R: Why?

S: Because it has, it use 15 paper clips while shape A use 10. So, 10 to 15, 15 is bigger than 10. So, shape B has the longer perimeter than shape A.

In Excerpt S24, Suhana explained that shape B has the longer perimeter because shape B has 15 paper clips compared to shape A with 10 paper clips and 15 is larger than 10. It indicated that Suhana focused on the number of unit rather than the unit of measure when comparing perimeters in Set 2 with common nonstandard units. Nevertheless, she did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters.

In another situation when shapes A and B had the same perimeter, Suhana explained that the paper clips in shape B is smaller (shorter) than the paper clips in shape A. Excerpt S25 demonstrates her justification about their units of measurement (Suhana/L589-605).

Excerpt S25

R: If shapes A and B had the same perimeter, what would you tell about their units of measure?
S: Shape B have (sic) the paper clips that is smaller than paper clips in shape A. So, 10 paper clips in shape A equal to 15 paper clips in shape B. (Writes the following).
10 A = 15 B
R: Which paper clip is smaller?
S: Shape B has the smaller paper clip than shape A.
R: Why?
S: Because 15 and 10, 15 has the larger number. So, if this one has a small and then times together. (Calculates the length of a paper clip in shape B, as shown in Figure N157). Ok let's say that shape A has paper clip that is 1 centimetre. So, becomes 10 cm and shape B equals to 15 times "we don't know", let's say "*t*". So, when we do the comparison, $10 = 15t$, $t = \frac{2}{3}$.

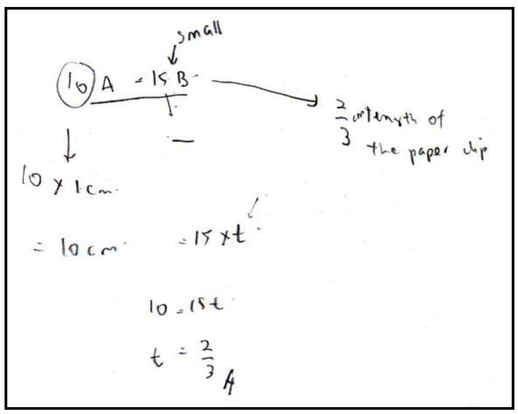


Figure N157. Suhana calculates the length of a paper clip in shape B.

In Excerpt S25, Suhana explained that the paper clips in shape B is smaller (shorter) than the paper clips in shape A. She elaborated that the length of 10 paper clips in shape A is equal to the length of 15 paper clips in shape B and wrote the relationship as 10 A = 15 B. Suhana calculated the length of a paper clip in shape B as $\frac{2}{3}$ (cm), as shown in Figure N157, when the length of a paper clip in shape A is 1 cm. It indicated that Suhana understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length.

In Set 3, Suhana stated that shape A has the longer perimeter. Excerpt S26 reveals her choice of shape that has the longer perimeter and the justification that she made (Suhana/L625-636).

Excerpt S26

R: (Puts the following table in front of Suhana). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 3	16 cm	13 cm

S: Shape A.
R: Why?
S: Because it is 16 cm while shape B is 13. So, 16 differ with 13 by 3. So, shape A has the longer perimeter, which is 3 over shape B.
R: In this case, how would you explain to your student that shape A has the longer perimeter?
S: I will compare the figure, 16 and 13. 16 is the bigger number.

In Excerpt S26, Suhana explained that shape A has the longer perimeter because 16 is larger than 13. It indicated that she focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Suhana knew that common standard unit (such as cm) is reliable for comparing perimeters.

Summary

In summary, Suhana focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. Nevertheless, she did not know that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters.

Suhana focused on the number of unit when comparing perimeters in Set 2 with common nonstandard units. Nevertheless, she did not know that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. Suhana understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length. She focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Suhana knew that common standard unit (such as cm) is reliable for comparing perimeters.

Comparing Area (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Suhana explained that she was unable to determine which shape has the larger area. Excerpt S27 shows the justification that she made (Suhana/L693-714).

Excerpt S27

R: (Puts the following table in front of Suhana). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 1	25 triangles	12 squares

S: Actually I can not determine which one has the larger area because shape A is using triangles whereas shape B is using the squares. And the triangle, I don't know the measurement, the length of the triangle and shape B too. So, if I say that shape A has the larger area, it may be true because of 25. But let's say that the square has the side that, has the side *yang lebih panjang* from this one. And then even it's 12, it will become, it will be the larger area. I don't know, just a guess.
R: Could you tell which one has the larger area?
S: No.
R: Why?
S: Because triangle and square is not a fixed unit. So, it may be differ by the side, the length of the side.

In Excerpt S27, Suhana explained that she was unable to determine which shape has the larger area as they were different shape, triangle and square, and she did not know the length of side of the triangle and square. Suhana elaborated that triangle and square is not a fixed unit and they may be differed in terms of the length of side. It indicated that Suhana focused on the unit of measure when comparing areas in Set 1 with nonstandard units. She knew that nonstandard units (such as triangle and square) are not reliable for comparing areas.

In Set 2, Suhana stated that shape B has the larger area. Excerpt S28 depicts her choice of shape that has the larger area and the justification that she made (Suhana/L746-753).

Excerpt S28

R: (Puts the following table in front of Suhana). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 2	10 squares	15 squares

S: …(Silent for a while) when comparing the figure, shape B.
R: Why?
S: Because 15 is larger than 10.

In Excerpt S28, Suhana explained that shape B has the larger area because 15 is larger than 10. It indicated that she focused on the number of unit rather than the unit of measure when comparing areas in Set 2 with common nonstandard units. Suhana did not know that common nonstandard units (such as squares) are not reliable for comparing areas.

In another situation when shape A and B had the same area, Suhana explained that the square in shape A has larger area than the square in shape B. Excerpt S29 demonstrates her justification about their units of measurement (Suhana/L754-773).

Excerpt S29

R: If shapes A and B had the same area, what can you say about their units of measure?
S: Ok shape A has the area with larger, the square in shape A has larger area than shape B.
R: Could you tell me more about it?
S: Ok square, then the length in shape A is bigger than the length in shape B.
R: Why?
S: (Writes the following).

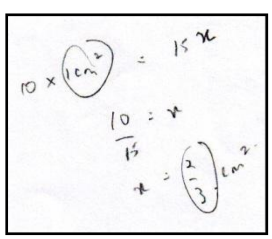


Figure N158. Suhana calculates the area of a square in shape B.

Because if we put this figure A inside the grid (points to the 1-cm grid paper). Let's say one grid equal to 1 cm^2 (misreads 1 cm^2 as 1 cm square). Times 10 and then B, 15 times "we don't know", x, become 10 over 15 equal to x. x equal to 2 over 3. So, when we putting the square, the shape B in our grid…(silent for a while) the area of one grid will become $\frac{2}{3}$ cm^2 (misreads $\frac{2}{3}$ cm^2 as $\frac{2}{3}$ cm square), smaller than A.

R: So, which one smaller?
S: B.

In Excerpt S29, Suhana explained that the square in shape A is larger than the square in shape B. She calculated the area of a square in shape B as $\frac{2}{3}$ cm^2, as shown in Figure N158, when the area of a square in shape A is 1 cm^2. It indicated that Suhana understand the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area.

In Set 3, Suhana stated that shape A has the larger area. Excerpt S30 reveals her choice of shape that has the larger area and the justification that she made (Suhana/L775-786).

Excerpt S30

R: (Puts the following table in front of Suhana). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 3	16 cm^2	13 cm^2

S: Shape A.
R: Why?
S: Because 16 and 13, 16 has the larger number than 13.
R: Could you tell me more about it?
S: Ok it used the fixed unit, cm^2 (misreads cm^2 as cm square), which we know that from the ruler, which is 1 cm equal to 1 cm too. So, we just compare the figure, 16 and 13, 16 bigger than 13.

In Excerpt S30, Suhana explained that shape A has the larger area because they used the fixed unit, namely cm^2, and 16 is larger than 13. It indicated that Suhana focused on the number of unit when comparing areas in Set 3 with common standard unit. She knew that common standard unit (such as cm^2) is reliable for comparing areas.

Summary

In summary, Suhana focused on the unit of measure when comparing areas in Set 1 with nonstandard units. She knew that nonstandard units (such as triangle and square) are not reliable for comparing areas. Suhana focused on the number of unit when comparing areas in Set 2 with common nonstandard units. She did not know that common nonstandard units are not reliable for comparing areas. Suhana understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area. She focused on the number of unit when comparing areas in Set 3 with common standard unit. Suhana knew that common standard unit (such as cm^2) is reliable for comparing areas.

Linguistic Knowledge

Suhana read 16 cm^2 and 13 cm^2 literally as '16 centimeter square' and '13 centimeter square' respectively, as shown in Excerpt S30. In another situation, Excerpt S31 exhibits how Suhana wrote 16 cm^2 and 13 cm^2 in English words (Suhana/L813-819).

Excerpt S31

R: (Puts a blank paper written the following measurements in front of Suhana).
16 cm^2
13 cm^2
How would you write these measurements in English words?
S: (Writes the following).

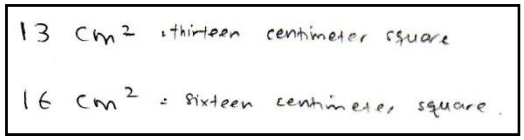


Figure N159. Suhana writes 16 cm^2 and 13 cm^2 in English words.

In Excerpt S31, Suhana wrote 16 cm^2 and 13 cm^2 literally as ‘sixteen centimeter square’ and ‘thirteen centimeter square’, as shown in Figure N159. The correct answer should be ‘sixteen square centimetres’ and ‘thirteen square centimetres’. It indicated that she did not know about the conventions pertaining to writing and reading of Standard International (SI) area measurement units.

Converting Standard Units of Area Measurement

Procedural Knowledge

Suhana realized that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2. Excerpt R32 shows the algorithms that Suhana used when she was converting 3 cm^2 to mm^2 (Suhana/L821-884).

Excerpt R32

R: (Puts a card written the following scenario in front of Suhana). Some Form One teachers noticed that several of their students seemed to multiply by 10, 100, and 1000, respectively when they were converting units of area from cm^2 to mm^2, m^2 to cm^2, and km^2 to m^2:

$3\ cm^2 = 3 \times 10\ mm^2 = 30\ mm^2$
$4.7\ m^2 = 4.7 \times 100\ cm^2 = 470\ cm^2$
$1.25\ km^2 = 1.25 \times 1000\ m^2 = 1250\ m^2$

What would you do if you were teaching Form One and you noticed that several of your students were doing this?
S: Ok what I will do. Ok first 3 cm I'll change to mm. (Converts 3 cm^2 to mm^2, as shown in Figure N160).
R: Could you explain your solution?
S: First cm into mm. Ok from cm to change it to mm, use 10. But in this case we are using square. So, we have to square this number. So, it becomes 300. Because first 3 cm change into mm, it will become 30 mm. But if we are using the 3 cm^2 to change, it becomes 3 times 10 square. Because it square, so square 10 mm and then because you want to change it to square too, so you square this one and become 300 mm^2 (misreads 300 mm^2 as 300 mm square).

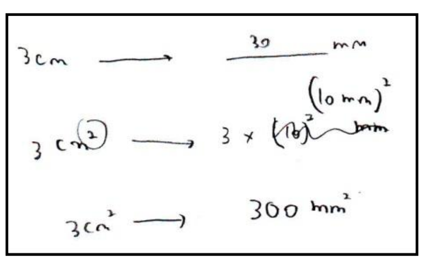


Figure N160, Suhana converts 3 cm^2 to mm^2.

In Excerpt R32, Suhana has successfully converted 3 cm^2 to mm^2. She viewed 3 cm as the product of 3 times 1 cm. Thus, Suhana times 10 when she converted 3 cm to mm because 1 cm = 10 mm. Similarly, Suhana viewed 3 cm^2 as the product of 3 times 1 cm^2. She knew the relationship between the standard units of length measurement that 1 cm = 10 mm. Suhana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Suhana times ten squared, $(10)^2$, when she converted 3 cm^2 to mm^2, as shown in Figure N160.

Suhana found that the students made a mistake when they were converting unit of area from 4.7 m^2 to cm^2. Excerpt R33 depicts the algorithms that Suhana used when she was converting 4.7 m^2 to cm^2 (Suhana/L845-854).

Excerpt R33

R: What about the second part (points to 4.7 m^2)?

S: (Converts 4.7 m^2 to cm^2, as shown in Figure N161). So the second, 4.7 m change into cm, 4.7 times 100 because 1 m equals to 100 cm. So, if 4.7 m^2 (misreads 4.7 m^2 as 4.7 m square) change to cm^2, 4.7 times $(100\text{ cm})^2$. So, become answer.

R: What do you get?

S: 470 000 cm^2. (The right answer should be 47 000 cm^2).

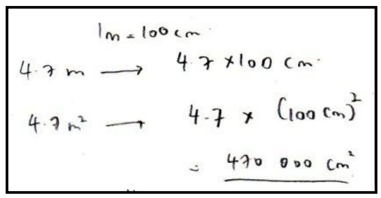


Figure N161. Suhana converts 4.7 m^2 to cm^2.

In Excerpt R33, Suhana has used appropriate algorithm in converting 4.7 m^2 to cm^2. She viewed 4.7 m as the product of 4.7 times 1 m. Thus, Suhana times 100 when she converted 4.7 m to cm because 1 m = 100 cm. Similarly, Suhana viewed 4.7 m^2 as the product of 4.7 times 1 m^2. She knew the relationship between the standard units of length measurement that 1 m = 100 cm. Suhana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Suhana times one hundred squared, namely $(100)^2$, when she converted 4.7 m^2 to cm^2, as shown in Figure N161. Nevertheless, Suhana made a mistake when she simplified the product of 4.7 times $(100\text{ cm})^2$ as 470 000 cm^2. The correct answer should be 47 000 cm^2.

Suhana also found that the students made a mistake when they were converting unit of area from 1.25 km^2 to m^2. Excerpt R34 demonstrates the algorithms that Suhana used when she was converting 1.25 km^2 to m^2 (Suhana/L855-863).

Excerpt R34

R: What about the third part (points to 1.25 km^2)?

S: (Converts 1.25 km^2 to m^2, as shown in Figure N162). The third one, 1.25 km change into m. 1 km is equal to 1000 m, become 1.25 times 1000 m. But if 1.25 km^2, 1.25 times $(1000\text{ m})^2$. So, we get the answer 125 000 000 m^2 (misreads 125 000 000 m^2 as 125 million m square). (The right answer should be 1 250 000 m^2).

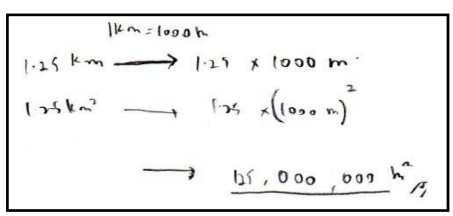


Figure N162. Suhana converts 1.25 km^2 to m^2.

In Excerpt R34, Suhana has also used appropriate algorithm in converting 1.25 km^2 to m^2. She viewed 1.25 km as the product of 1.25 times 1 km. Thus, Suhana times 1000 when she converted 1.25 km to m because 1 km = 1000 m. Similarly, Suhana viewed 1.25 km^2 as the product of 1.25 times 1 km^2. She knew the relationship between the standard units of length measurement that 1 km = 1000 m. Suhana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Suhana times one thousand squared, namely $(1000)^2$, when she converted 1.25 km^2 to m^2, as shown in Figure N162. Nevertheless, Suhana made a mistake when she simplified the product of 1.25 times $(1000\ m)^2$ as 125 000 000 m^2. The correct answer should be 1 250 000 m^2.

Summary

In summary, Suhana realized that the students made mistakes when they were converting 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2 respectively. She knew the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. Suhana also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. She viewed 3 cm^2 as the product of 3 times 1 cm^2, 4.7 m^2 as the product of 4.7 times 1 m^2, and 1.25 km^2 as the product of 1.25 times 1 km^2. Thus, Suhana times $(10)^2$, $(100)^2$, and $(1000)^2$ respectively when she converted 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2, as shown in Figures N160, N161, and N162.

Suhana has successfully converted 3 cm^2 to mm^2. She has used appropriate algorithm in converting 4.7 m^2 to cm^2. Nevertheless, Suhana made a mistake when she simplified the product of 4.7 times $(100\ cm)^2$ as 470 000 cm^2. The correct answer should be 47 000 cm^2. Suhana has also used appropriate algorithm in converting 1.25 km^2 to m^2. Nevertheless, Suhana made a mistake when she simplified the product of 1.25 times $(1000\ m)^2$ as 125 000 000 m^2. The correct answer should be 1 250 000 m^2.

Conceptual Knowledge

Suhana knew the relationships between the standard units of length measurement such as 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. These can be seen in Figures N160, N161, and N162. She also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring, as shown in Figures N160, N161, and N162.

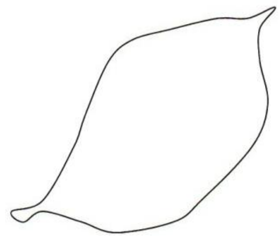
Relationship between Perimeter and Area
(Same Perimeter, Same Area?)

Conceptual Knowledge

Suhana knew that there is no direct relationship between perimeter and area. She knew that two shapes with the same perimeter can have different areas. Thus, Suhana knew that the student's method of calculating the area of the leaf was not correct. Excerpt S35 shows Suhana's responses to the Form One student (Suhana/L915-949).

Excerpt S35

R: (Puts a card written the following scenario in front of Suhana). This is a picture of a leaf. A Form One student said that he had found a way to calculate the area of the leaf. The student placed a thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle.



How would you respond to this student?

S: Ok first I will tell them about this one, perimeter and area is not same. Then I will transfer this picture (points to the leaf) into a grid paper and ask them to calculate the grid.

R: Could you show me how it is?

S: (Puts a 1-cm grid paper on the leaf and then traces its outline. Counts the number of 1-cm grid fully covered by the leaf. Combines parts of grids covered by the leaf to form a complete grid and then counts its number. The leaf was covered by 23 1-cm grids. Writes the total area as 23 cm^2. Uses thread to measure the perimeter of the traced leaf on 1-cm grid paper and writes its measurement as 21.5 cm and then rounds it off to 22 cm, as shown in Figure N163). Ok I will ask them to calculate this box (refers to 1-cm grids): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23. So, it will be 23 cm^2 and then I will ask them to use the *benang* [thread] here.

R: Could you show me how it is?

S: Ok (uses the thread to measure the perimeter of the traced leaf on the 1-cm grid paper).

R: What do you get?

S: For the area, 23 cm^2 and for the perimeter, 21.5 cm.

R: Do you think the student's method correct?

S: No.

R: Why?

S: Because this 21.5 cm, let's say 22 *lah*, we round it. (Draws a rectangle, labels its dimensions and then calculates its area as 30 cm^2, as shown in Figure N164). So, if we do it like this, 5 and 5 and then 6 and 6. So, if we calculate it, it will become 30 cm^2. 30 cm^2 is not the same as 23 cm^2. So, it is not correct.

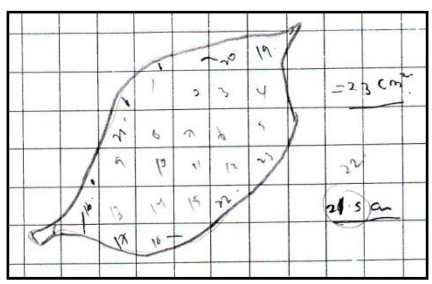


Figure N163. Suhana measures the perimeter and area of the traced leaf.

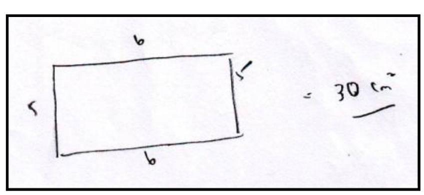


Figure N164. Suhana draws a rectangle, labels its dimensions and calculates its area.

In Excerpt S35, Suhana stated that perimeter and area were not the same. She put a 1-cm grid paper on the leaf and then traced its outline. Suhana counted the number of 1-cm grid fully covered by the leaf. She combined parts of grids covered by the leaf to form a complete grid and then counted its number. Suhana found that the leaf was covered by 23 1-cm grids. She wrote the total area of the leaf as 23 cm^2 on the grid paper. Suhana used a piece of thread to measure the perimeter of the traced leaf on 1-cm grid paper and wrote its measurement as 21.5 cm and then rounded it off to 22 cm, as shown in Figure N163.

Suhana drew a rectangle, labelled its dimensions as 6 by 5 and then calculated its area as 30 cm^2, as shown in Figure N164. She found that the area of the rectangle, namely 30 cm^2, is not the same as the area of the leaf which is 23 cm^2 even though they had the same perimeter as 22 cm. Thus, Suhana concluded that the student's method of calculating the area of the leaf was not correct.

Summary

In summary, Suhana knew that there is no direct relationship between perimeter and area. She knew that two shapes with the same perimeter can have different areas. Thus, Suhana knew that the student's method of calculating the area of the leaf was not correct.

Ethical Knowledge

In Task 5.1, Suhana knew that the student's method of calculating the area of the leaf was not correct. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter have the same area. Suhana had attempted to examine the possible pattern of the relationship between perimeter and area by comparing the area of the leaf and rectangle that had the same perimeter. She found that the area of the rectangle, namely 30 cm^2, is not the same as the area of the leaf which is 23 cm^2 even though they had the same perimeter as 22 cm. Thus, Suhana concluded that the student's method of calculating the area of the leaf was not correct.

Based on this counterexample, she knew that the student's generalization that two shapes with the same perimeter have the same area was not correct. Suhana formulated a generalization pertaining to the relationship between perimeter and area that two shapes with the same perimeter may have the different area. Suhana refuted the student's generalization that two shapes with the same perimeter have the same area with a counterexample. She knew that a counterexample is sufficient to refute the truth of a generalization.

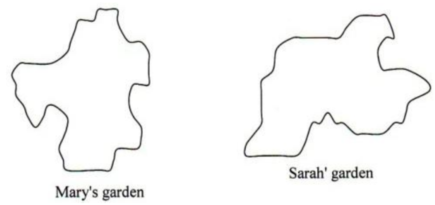
Relationship between Perimeter and Area
(Longer Perimeter, Larger Area?)

Conceptual Knowledge

Suhana knew that there is no direct relationship between perimeter and area. She knew that the garden with the longer perimeter could have a smaller area. Thus, Suhana knew that Mary's claim was not correct. Excerpt S36 shows Suhana's responses to the claim made by Mary that the garden with the longer perimeter has the larger area (Suhana/L973-1035).

Excerpt S36

R: (Puts a card written the following scenario in front of Suhana). Mary and Sarah are discussing whose garden has the larger area to plant flowers. Mary claims that all they have to do is walk around the two gardens to get the perimeter and the one with the longer perimeter has the larger area. How would you respond to these students?



S: Ok first I will ask them to draw picture on the grid paper.
R: Could you show me how it is?
S: Ok (Takes a $\frac{1}{2}$-cm grid paper. Puts the grid paper on Mary's garden and traces its outline. Counts the number of $\frac{1}{2}$-cm grid fully covered by the garden. Combines parts of grids covered by the garden to form a complete grid and then counts its number. The garden was covered by "34" $\frac{1}{2}$-cm grids. Writes the total area as 0.5 x 0.5 x 34 = 8.5 cm^2, as shown in Figure N165). (Puts the grid paper on Sarah's garden and traces its outline. Counts the number of $\frac{1}{2}$-cm grid fully covered by the garden. Combines parts of grids covered by the garden to form a complete grid and then counts its number. The garden was covered by "41" $\frac{1}{2}$-cm grids. Writes the total area as 0.5 x 0.5 x 41 = 10.25 cm^2, as shown in Figure N166).
R: What do you get?
S: This one (points to Mary's garden) 34.
R: Is it perimeter or area?
S: (Calculates the area as follow).
  0.5 x 0.5 x 34 = 8.5 cm^2
  8.5 cm^2 (misreads 8.5 cm^2 as 8.5 cm square).
R: How do you get that?
S: This is half centimeter. So, 0.5 times 0.5 times 34.
R: So, what do you get?
S: 8.5 cm^2. This one (points to Sarah's garden). (Traces Sarah's garden on the $\frac{1}{2}$-cm grid paper and then counts the number of grids covered by the garden). So, 41 box (grids) and then 0.5 (calculates its area as follow).
  0.5 x 0.5 x 41 = 10.25
  10.25.
R: What do you get?
S: 10.25 cm^2.
R: So, what's your conclusion?
S: Ok so Sarah has the bigger garden. They dispute about the perimeter right? So, I'll use the *benang* [thread]. (Uses different part of the thread to measure the perimeter of both gardens: Mary's 15.2 cm and Sarah's 16.7 cm).
R: What do you get?
S: From the perimeter, Mary has, got 15.2 cm and then Sarah, 16.7.
R: So, what's your conclusion?
S: Even though Sarah has the longer perimeter and larger area, but it's not true that the longer perimeter has the larger area because in this picture we has the non-uniform area. So, and also perimeter. …

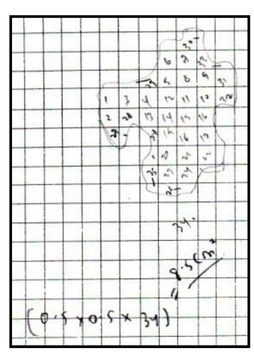


Figure N165. Suhana traces "Mary's garden" on a $\frac{1}{2}$-cm grid paper and calculates its area.

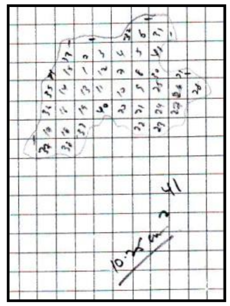


Figure N166. Suhana traces “Sarah’s garden” on a $\frac{1}{2}$-cm grid paper and calculates its area.

In Excerpt S36, Suhana traced “Mary's garden” on a $\frac{1}{2}$-cm grid paper. She counted the number of $\frac{1}{2}$-cm grids covered by the garden. Suhana found that the garden was covered by "34" $\frac{1}{2}$-cm grids. She then calculated the area of the garden as 0.5 x 0.5 x 34 = 8.5 cm^2, as shown in Figure N165. Suhana also traced “Sarah's garden” on the $\frac{1}{2}$-cm grid paper. She counted the number of $\frac{1}{2}$-cm grids covered by the garden. Suhana found that the garden was covered by "41" $\frac{1}{2}$-cm grids. She then calculated the area of the garden as 0.5 x 0.5 x 41 = 10.25 cm^2, as shown in Figure N166. Suhana concluded that Sarah has the larger garden.

She used a piece of thread to measure the perimeter of “Mary's garden” as 15.2 cm. Suhana used another piece of thread to measure the perimeter of “Sarah's garden” as 16.7 cm. She concluded that even though Sarah’s garden has the longer perimeter and larger area, but it was not true that the garden with longer perimeter has the larger area as the “picture” of the gardens involved the “non-uniform” (irregular shapes) perimeter and area. Suhana stated that she would compare with other “picture”.

Suhana tried to generate another example to refute Mary’s claim that the garden with the longer perimeter has the larger area. Excerpt S37 is illustrative (Suhana/L1035-1076).

Excerpt S37

S: …So. It's yet, I show, I will compare to the other picture.
R: Could you show me how it is?
S: So, I take the shape. This (Draws a rectangle. Labels its dimensions and then calculates its perimeter and area, as shown in Figure N167). This one is 8 times 2, 16 cm^2. Then I use the triangle. (Draws an equilateral triangle. Labels its dimensions and height and then calculates its perimeter and area, as shown in Figure N168). So, the perimeter, 18. Area, half times 6 times 5.2, 15.6 (uses calculator to get its product).
R: What about the perimeter of the rectangle?
S: 20.
R: Area?
S: 16.
R: So, what's your conclusion?
S: I can not use this example. (Draws a trapezium. Labels its dimensions and then calculates its perimeter and area, as shown in Figure N169).
R: Could you explain your solution?
S: Ok this one is 3, 5 and 3. So, this one is 3.6. The perimeter will be 14.6 (uses calculator to sum up the total length) while the area half times 3 times (3 plus 5), 12 (uses calculator to get its product).
R: What do you get?
S: 12.
R: How do you get "12"?
S: half times 3 times "3 plus 5" equal 12.
R: So, what's your conclusion?
S: …(Silent for a while) these are my example. Longer perimeter has larger area. The student's (Mary’s) opinion for the longer perimeter has larger area is true.
R: Why?
S: Because from my example, my question, when I compared, longer perimeter has larger area.

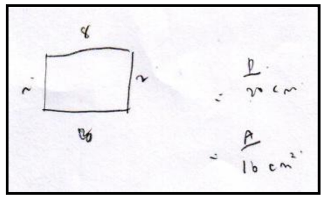


Figure N167. Suhana draws a rectangle and calculates its perimeter and area.

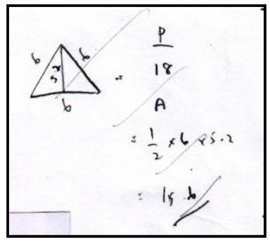


Figure N168. Suhana draws a triangle and calculates its perimeter and area.

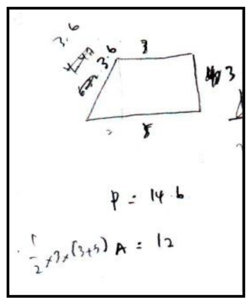


Figure N169. Suhana draws a trapezium and calculates its perimeter and area.

In Excerpt S37, Suhana drew a rectangle and calculated its perimeter and area as 20 cm and 16 cm^2 respectively, as shown in Figure N167. She also drew an equilateral triangle and calculated its perimeter and area as 18 cm and 15.6 cm^2 respectively, as shown in Figure N168. Suhana found that the rectangle has the longer perimeter (20 cm) and larger area (16 cm^2) while the triangle has the shorter perimeter (18 cm) and smaller area (15.6 cm^2).

She found that this example also concurred with Mary's claim that the garden with longer perimeter has the larger area. Thus Suhana stated that this example could not be used as she was trying to generate a counterexample that would refute Mary's claim. Suhana drew another diagram, namely a trapezium, and then calculated its perimeter and area as 14.6 cm and 12 cm^2 respectively, as shown in Figure N169. Suhana found that the rectangle has the longest perimeter and largest area compared to the triangle and trapezium that she has drawn. Suhana concluded that her examples showed that the shape with the longer perimeter has the larger area and thus she thought that Mary's claim was true.

When probed further, Suhana indicated that Mary's method of determining whose garden has the larger area was not correct. Excerpt S38 is illustrative (Suhana/L1077-1092).

Excerpt S38

R: Do you think the student's method correct?
S: (Moves her head to indicate no) no.

R: Why?
S: Because it is not apply to all of the perimeter and the area.
R: Could you tell me more about it?
S: I think the student got the conclusion just by this situation and it is just by luck and then my example is not, I can not find the suitable one. So, I can not say that their conclusion is not right. But at the same time I am not agree with them.
R: So, what's your conclusion?
S: Longer perimeter is not, *tak semestinya* [not necessarily] has larger area.
R: Why?
S: Because I have come across an example but I can not state here because I don't know that sometimes shorter perimeter has larger area too compared to longer perimeter.

In Excerpt S38, Suhana indicated that Mary's method was not correct because it did not apply to all shapes with their respective perimeter and area. Suhana stated that Mary came to the conclusion just based on this situation and was just by luck. Suhana expressed that since she was unable to find a suitable example (counterexample to refute Mary's claims), she could not say that Mary's conclusion (claim) was not true. Nevertheless, Suhana said that she did not agree with Mary. Suhana concluded that (the shape with the) longer perimeter does not necessarily has the larger area. She explained that sometimes (the shape with the) shorter perimeter has larger area too compared to (the shape with the) longer perimeter.

Summary

In summary, Suhana knew that there is no direct relationship between perimeter and area. She knew that the garden with the longer perimeter could have a smaller area. Thus, Suhana knew that Mary's claim was not correct.

Ethical Knowledge

In Task 5.2, Suhana knew that Mary's claim was not correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. She indicated that Mary's method was not correct because it did not apply to all shapes. Suhana stated that Mary came to the conclusion just based on this situation and was just by luck.

Suhana attempted to examine the possible pattern of the relationship between perimeter and area, as shown in Excerpts S36 and S37. She had generated two examples to examine the possible pattern of the relationship between perimeter and area. Nevertheless, Suhana found that both of her examples concurred with Mary's generalization that the garden with the longer perimeter has the larger area. Suhana concluded that (the shape with the) longer perimeter does not necessarily has the larger area. She explained that sometimes (the shape with the) shorter perimeter has larger area too compared to (the shape with the) longer perimeter. It indicated that she formulated a generalization that the garden with the longer perimeter could have a smaller area.

She had generated two examples to test Mary's generalization that the garden with the longer perimeter has the larger area. Nevertheless, Suhana found that both of her examples concurred with Mary's generalization that the garden with the longer perimeter has the larger area. She tried to generate a counterexample to refute Mary's claim but was unsuccessful.

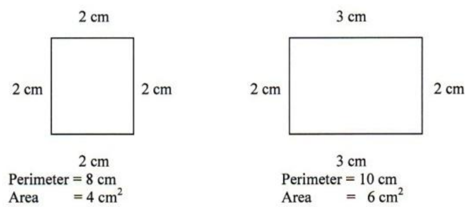
Relationship between Perimeter and Area
(Perimeter Increases, Area Increases?)

Conceptual Knowledge

Suhana did not know that there is no direct relationship between perimeter and area. She did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Suhana thought that the student's "theory" was correct. This is shown in Excerpt S39 (Suhana/L1155-1179).

Excerpt S39

R: (Puts a card written the following scenario in front of Suhana). Suppose that one of your Form One students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:



How would you respond to this student?

S: Em…(silent for a while) this is true.

R: Why?

S: Because the increasing of the perimeter, 8 to10 and then the area, 4 to 6. Em it's true. And for a triangle, for others, for other figure, it's also the same. It's true.

In Excerpt S39, Suhana thought that the student's "theory" was correct. Suhana agreed with the student that as the perimeter of a closed figure increases, the area also increases. She explained that when the perimeter increases from 8 (cm) to 10 (cm), the area also increases from 4 (cm^2) to 6 (cm^2). Thus, she concluded that the student's "theory" was correct. Suhana explained that the "theory" was also applied to other figure such as triangle.

Suhana did not know that the student's claim about the relationship between perimeter and area is not a theory. The claim is a conjecture. She also did not know that an example is not a proof and a theory cannot be proved by an example.

Summary

In summary, Suhana did not know that there is no direct relationship between perimeter and area. She did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Suhana thought that the student's "theory" was correct.

Ethical Knowledge

In Task 5.3, the student formulated a generalization that as the perimeter of a closed figure increases, the area also increases. She explained that when the perimeter increases from 8 (cm) to 10 (cm), the area also increases from 4 (cm^2) to 6 (cm^2). Suhana thought that the student's "theory" was correct. She did not attempt to test the student's generalization, as shown in

Excerpt S39. Suhana accepted the student's generalization without attempting to generate an example or counterexample to test it. In reality, when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same.

Calculating Perimeter and Area
(Rectangle and Parallelogram/Triangle)

Procedural knowledge

After read through Task 6.1, Suhana labelled the missing sides of Diagram 1 that required for calculating the perimeter and area of the diagram, as shown in Figure N170 in Excerpt S40 (Suhana/L1233-1268).

Excerpt S40

R: (Puts a card written the following problem in front of Suhana). Suppose that one of your Form One students asks you for help with the following problem:

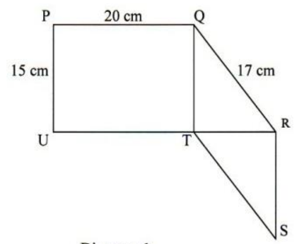


Diagram 1

In Diagram 1, PQTU is a rectangle and QRST is a parallelogram. UTR is a straight line.
Calculate
(k) the perimeter of the diagram,
(l) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 1, as shown in Figure N170).
Ok first I will ask them what is parallelogram. Then if the side is the same, ok put two lines (marks ╫ on QR and TS, as shown in Figure N170) and one line (marks ┼ on QT and RS, as shown in Figure N170). Then I will ask them to put the figure (such as QT = 15, RS = 15, ST = 17, TU = 20) around this picture (Diagram 1). …

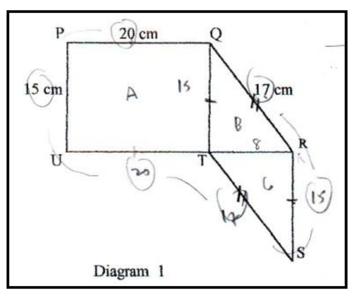


Figure N170 Suhana labels the missing sides of Diagram 1.

In Excerpt S40, Suhana put the symbol ╫ on QR and TS, and put the symbol ┼ on QT and RS, to indicate that they were of the same length respectively, as shown in Figure N170. She then labelled QT, RS, ST, and TU as 15, 15, 17, and 20 respectively on Diagram 1. Excerpt S41 depicts how Suhana has successfully calculated the perimeter of Diagram 1 (Suhana/L1268-1274).

Excerpt S41

S: …So, let them calculate (calculates the perimeter of Diagram 1, as shown in Figure N171. Uses calculator to sum up the total length). 2 ten plus 2 fifteen plus 2 seventeen, 104 cm.

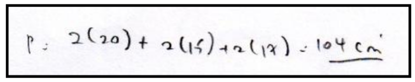


Figure N171. Suhana calculates the perimeter of Diagram 1.

In Excerpt S41, Suhana circled the length of sides that involved in the calculation of perimeter of Diagram 1, as shown in Figure N170. Suhana used the doubling-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N171. She doubled the length of sides UP, PQ, and QR and then summed them up to get the perimeter of the diagram as 104 cm. Excerpt S42 demonstrates how Suhana has successfully calculated the area of Diagram 1 (Suhana/L1275-1296).

Excerpt S42

R: What about the area?

S: (Partitions Diagram 1 into a rectangle PQTU (labelled as A) and two triangles QRT (labelled as B) and RST (labelled as C), as shown in Figure N170. Uses Pythagoras' theorem to find TR, as shown in Figure N172. Calculates the area of Diagram 1 as the total areas of A, B, and C, as shown in Figure N173). Area ok. I will ask them to *pecahkan* [partitions] this picture into three parts. Here is A, B, and C. And then same is, 15 (RS) and 17 (ST) is the same (as QT and QR respectively). So, I will ask them to calculate A, B, C: A equal to 20 times 15, 300. B equal to half, and then I will ask then to (find TR). This triangle (ΔQRT), ok using *Teorem Pithagoras* [Pythagoras' theorem], $17^2 - 15^2$ (reads as 17 square minus 15 square. Uses calculator to calculate $\sqrt{17^2 - 15^2} = 8$. Then labels 8 on TR). So, half times 8 times 15, 60. Then same also for C, half times 8 times 15, 60. So, area equal A plus B plus C equal 300 plus 60 plus 60, 420 cm^2.

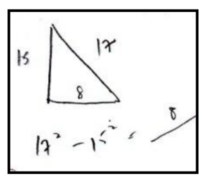


Figure N172. Suhana calculates the length of TR using Pythagoras' theorem.

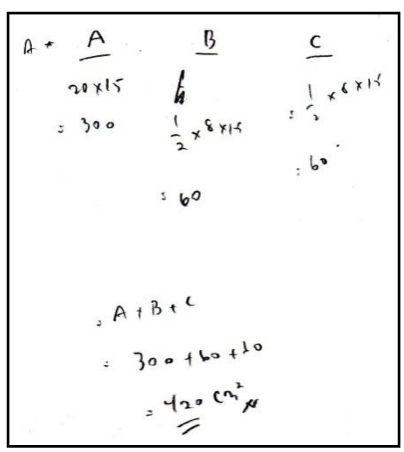


Figure N173. Suhana calculates the area of Diagram 1.

In Excerpt S42, Suhana used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N170. She partitioned Diagram 1 into a rectangle PQTU (labelled as A) and two triangles QRT (labelled as B) and RST (labelled as C). Suhana knew that she needed to find the length of TR. She has successfully calculated the length of TR using Pythagoras' theorem as 8, as shown in Figure N172. Suhana calculated the area of A, B, and C using the area formulae of rectangle and triangles respectively and then summed them up to get the area of the diagram as 420 cm^2, as shown in Figure N173.

Summary

In summary, Suhana has successfully calculated the perimeter of Diagram 1 by using the doubling-and-sum algorithm. She has also correctly calculated the area of Diagram 1 by using the partition-and-sum algorithm.

Linguistic knowledge

Suhana used the correct standard units of measurement for perimeter (cm) and area (cm^2) when she wrote the answers for these measurements, as shown in Figures N171 and N173.

Strategic Knowledge

When probed to check the answer for the perimeter, Suhana suggested that she would use the recalculating strategy to verify the answer. Excerpt S43 is illustrative (Suhana/L1326-1331).

Excerpt S43

R: If you were asked to check the answer for the perimeter, how would you check?
S: I calculate this one (points to the lengths of each side that surround Diagram 1). No other method. (Moves her head to indicate no idea). Just *saya tambah* figure figure *ini* [I just add these figures (numbers)] (points to the lengths of each side that surround Diagram 1, as shown in Figure N170), *itu saja* [that's all].

In Excerpt S43, Suhana suggested that she would check the answer for perimeter by recalculating strategy that using the same method and calculate again. Suhana explained that she would recalculate the total length that surround Diagram 1, as shown in Figure N170 and expressed that there is no other method (to check the answer for the perimeter).

When probed to check the answer for the area, Suhana suggested that she would also use the recalculating strategy to verify the answer. Excerpt S44 is illustrative (Suhana/L1336-1340).

Excerpt S44

R: How would you check your answer for the area?
S: I will calculate it again.
R: Could you tell me more about it?
S: The same as this one (points to her previous solution for the area, as shown in Figure N173).

In Excerpt S44, Suhana suggested that she would check the answer for the area by recalculating strategy that using the same method and calculate again. Suhana explained that she would recalculate the answer for the area, as shown in Figure N173.

Ethical Knowledge

Suhana has successfully calculated the perimeter and area of Diagram 1. Nevertheless, she did not check the correctness of the answers for perimeter as well as area. When probed to check answers, then only Suhana suggested the strategies that she would use to check the answers for perimeter and area. Suhana wrote the measurement units (without probed) for the answers of the perimeter and area that she has calculated, as shown in Figures N171 and N173.

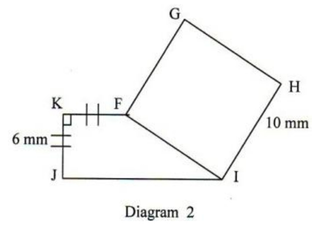
Calculating Perimeter and Area
(Square and Trapezium/Triangle)

Procedural Knowledge

After read through Task 6.2, Suhana labelled the missing sides of Diagram 2 that required for calculating the perimeter and area of the diagram, as shown in Figure N174 in Excerpt S45 (Suhana/L1373-1403, L1411-1419).

Excerpt S45

R: (Puts a card written the following problem in front of Suhana). Suppose that one of your Form One students asks you for help with the following problem:



In Diagram 2, FGHI is a square and FIJK is a trapezium.
Calculate
(k) the perimeter of the diagram,
(l) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 2, as shown in Figure N174). Ok first I will ask them what is square and then they, square has the same side, same length of side. So, this one is 10, 10, 10 (points to FG, GH, FI as shown in Figure N174). This one 6, 6, 6 (points to KF, FZ, ZJ as shown in Figure N174). This one is 8 (points to ZI, as shown in Figure N174).

.
.
.

R: How do you get 8 here (points to ZI, as shown in Figure N174).

S: Ok this is the *Teorem Pithagoras* [Pythagoras' theorem].

R: Could you show me how it is?

S: (Calculates ZI using Pythagoras' theorem, as shown in Figure N175). (Reads $FI^2 - FZ^2 = (ZI)^2$ as FI square minus FZ square equal to ZI square).

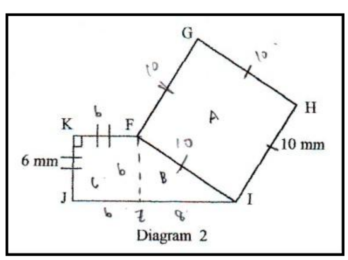


Figure N174. Suhana labels the missing sides of Diagram 2.

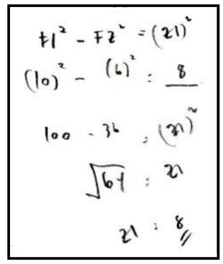


Figure N175. Suhana calculates the length of ZI using Pythagoras' theorem.

In Excerpt S45, Suhana labelled FG, GH, FI, KF, FZ, and ZJ as 10, 10, 10, 6, 6, and 6 respectively on Diagram 2, as shown in Figure N174. Suhana realized that she needed to find the length of ZI, as shown in Figure N174. Suhana partitioned trapezium FIJK into a square and a triangle. She has successfully calculated the length of ZI as 8 (mm) using Pythagoras' theorem, as shown in Figure N175.

Excerpt S46 depicts how Suhana has successfully calculated the perimeter of Diagram 2 (Suhana/L1404-1410).

Excerpt S46

S: (calculates the perimeter of Diagram 2, as shown in Figure N176).
R: How do you get 56 cm?
S: Eh mm (realizes the mistake and changes the unit to mm, as shown in Figure N176).

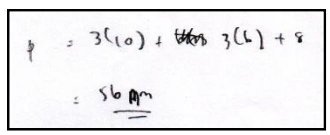


Figure N176. Suhana calculates the perimeter of Diagram 2.

In Excerpt S46, Suhana used the tripling-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N176. She tripled the length of sides JK and HI, and plus the length of ZI. Suhana mentally summed them up to get the perimeter of the diagram as 56 mm. Initially, she wrongly wrote the unit of the perimeter of Diagram 2 as cm. Subsequently, Suhana realized her mistake and wrote the correct unit as mm, as shown in Figure N176. Excerpt S47 demonstrates how she has successfully calculated the area of Diagram 2 (Suhana/L1420-1438).

Excerpt S47

R: What about the area?
S: Area. (Calculates the area of Diagram 2, as shown in Figure N177). I divide it into three parts: A, B, and C. So, A got 10 times 10 equal to 100 mm^2 (misreads 100 mm^2 as 100 mm square). B got half times 6 times 8 equal 24 mm^2 (misreads 24 mm^2 as 24 mm square). And C got 6 times 6 equal 36 mm^2 (misreads 36 mm^2 as 36 mm square). Overall, A plus B plus C equal 100 plus 24 plus 36 equal 160 mm^2 (misreads 160 mm^2 as 160 mm square).
R: What do you get?
S: 160 mm^2 (misreads 160 mm^2 as 160 mm square).
R: Could you explain your solution for the area?
S: Ok for the area, I divide it into three parts: A, B, and C. Then this one 10 times 10 is 100 mm^2 and then for B, half times 6 times 8, I got 12.
R: How do you get 12?
S: Eh 24. And then C, 6 times 6, 36. So, add all, I got 160 mm^2.

In Excerpt S50, Suhana used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N177. She partitioned Diagram 2 into square FGHI (labelled as A), triangle FIZ (labelled as B), and square KFZJ (labelled as C), as shown in Figure N174. Suhana calculated the area of A, B, and C separately using the area formulae of a square, triangle, and square respectively and then summed them up to get the area of the diagram as 160 mm^2.

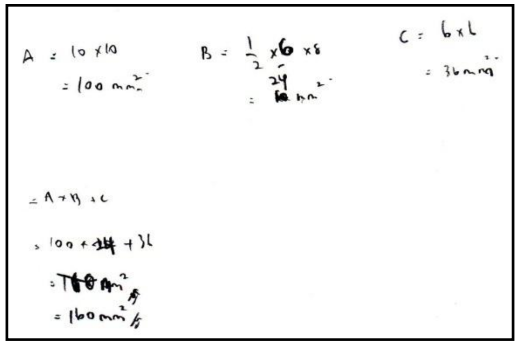


Figure N177. Suhana calculates the area of Diagram 2.

Summary

In summary, Suhana has successfully calculated the perimeter of Diagram 2 using the tripling-and-sum algorithm. She has also correctly calculated the area of Diagram 2 using the partition-and-sum algorithm.

Linguistic Knowledge

Suhana used the correct standard units of measurement for perimeter (mm) and area (mm^2) when she wrote the answer of these measurements, as shown in Figures N176 and N177.

Strategic Knowledge

When probed to check the answer for the perimeter, Suhana suggested that she would use the recalculating strategy to verify the answer. Excerpt S48 is illustrative (Suhana/L1439-1440).

Excerpt S48

R: How would you check your answer for the perimeter?
S: Perimeter. Just calculate again.

In Excerpt S48, Suhana suggested that she would check the answer for the perimeter by the recalculating strategy that using the same method and calculate again. Suhana stated that she would just calculate the perimeter again.

When probed to check the answer for the area, Suhana suggested that she would use the recalculating strategy to verify the answer. Excerpt S49 is illustrative (Suhana/L1441-1442).

Excerpt S49

R: How would you check your answer for the area?
S: The same, calculate again.

In Excerpt S49**,** Suhana suggested that she would check the answer for the area by the recalculating strategy. Suhana stated that she would just calculate the area again.

Ethical Knowledge

Suhana has successfully calculated the perimeter and area of Diagram 2. Nevertheless, she did not check the correctness of the answers for perimeter and area. When probed to check answers, then only Suhana suggested the strategy that she would use to check the answers for perimeter and area. Suhana wrote the measurement units (without probed) for the answers of perimeter and area, as shown in Figures N176 and N177.

Fencing Problem

Strategic Knowledge

Suhana used the trial and error method to solve the fencing problem. Excerpt S50 is illustrative (Suhana/L1452-1487).

Excerpt S50

R: (Puts a card written the following problem in front of Suhana). Suppose that one of your students asks you for help with the following problem:

A gardener has 84 m of fencing to enclose a garden along three sides, with the fourth side of the garden being formed by a wall. (Assume that the wall is perfectly straight). What are the dimensions of a rectangular garden that will yield the largest area being enclosed?

How would you solve this problem?

S: (Draws a diagram to list down the possible factors of 84. There is an error in the second row. The actual factors in the second row should be 2 and 42, not 41, as shown in Figure N178). (Based on the list of factors of 84, uses trial and error method to identify the factors that yield the largest area, as shown in Figure N179).

R: Could you explain your solution?

S: Ok first I ask them to draw the rectangle (points to the rectangle with the dimensions of $2x$ and $3x$, as shown in Figure N179).

R: What is your "x"?

S: "x" equal to 12. So, $2x \times 3x = 24 \times 36 = 864$ (uses calculator to get the product). And then second (points to the rectangle with the dimensions of x and $4x$, as shown in Figure N179), $x \times 4x = 14 \times 56 = 784$ (uses calculator to get the product. Third, (points to the rectangle with the dimensions of x and $2x$, as shown in Figure N179), $x \times 2x = 21 \times 42 = 882$ (uses calculator to get the product). And then fourth, (points to the rectangle with the dimensions of $3x$ and x, as shown in Figure N179), $3x \times x = 36 \times 12 = 432$ (uses calculator to get the product). (Draws an arrow pointed towards 882 and writes the word "biggest" to indicate "882" is the largest area being enclosed, as shown in Figure N179).

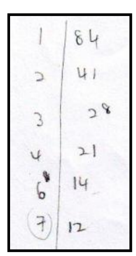


Figure N178. Suhana draws a diagram to list down the possible factors of 84.

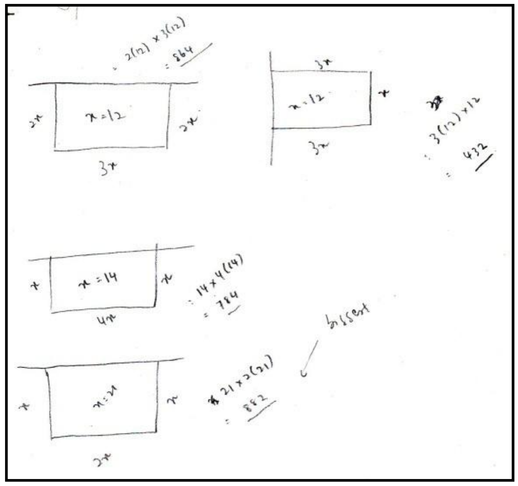


Figure N179. Suhana uses trial and error method to solve the fencing problem.

In Excerpt S50, Suhana drew a diagram to list down the possible factors of 84. There was an error in the second row. The actual factors in the second row should be 2 and 42, not 41, as shown in Figure N178. Based on the list of factors of 84, she used the trial and error method to solve the fencing problem by identifying the factors that yield the largest area, as shown in Figure N179. In the first trial, Suhana viewed 7 as the sum of 2, 3, and 2, and drew a rectangle with the dimension of $2x$ by $3x$. She calculated the area of the rectangle as $2x \times 3x = 24 \times 36 = 864$, where $x = 12$.

In the second trial, Suhana viewed 6 as the sum of 1, 4, and 1, and drew a rectangle with the dimension of x by $4x$. She calculated the area of the rectangle as $x \times 4x = 14 \times 56 = 784$, where $x = 14$. In the third trial, Suhana viewed 4 as the sum of 1, 2, and 1, and drew a rectangle with the dimension of x by $2x$. She calculated the area of the rectangle as $x \times 2x = 21 \times 42 = 882$, where $x = 21$. In the fourth trial, Suhana viewed 7 as the sum of 3, 1, and 3, and drew a rectangle with the dimension of $3x$ by x. She calculated the area of the rectangle as $3x \times x = 36 \times 12 = 432$, where $x = 12$.

Suhana drew an arrow pointed towards 882 and wrote the word "biggest" to indicate "882" is the largest area being enclosed, as shown in Figure N179. Excerpt S51 shows the justification that she made (Suhana/L1500-1504, L1523-1524).

Excerpt S51

R: How do you know "882" is the largest area?
S: By comparing these four figures (points to 864, 784, 882, and 432 in Figure N179).
R: Could you tell me more about it?
S: Ok I am configure with 864, 784, 882, and 432 and then by comparing these four figures, I suggesting that 882 has the largest area.
.
.
.
R: What are the dimensions that give you the largest area?
S: 21, 42, and 21.

In Excerpt S51, Suhana compared the areas of the rectangular garden that she had calculated. Suhana indicated that 882 was the largest area among the areas that she had calculated, namely 864, 784, 882, and 432, as shown in Figure N179. Thus, Suhana concluded that 882 (m^2) is the largest area being enclosed. She stated that 42 (m) by 21 (m) is the dimension of the rectangular garden that will yield the largest area being enclosed.

Summary

In summary, Suhana has successfully solved the fencing problem using the trial and error method. She used the compare strategy to verify the answer without being probed.

Ethical Knowledge

Suhana used the compare strategy to verify the answer without being probed. This can be seen in Excerpt S51. She did not write any measurement unit throughout Task 7. This can be seen in Excerpts S50 and S51, as well as Figures N178 and N179.

Developing Area Formulae

Procedural Knowledge

Suhana could recall the formula for the area of a rectangle, namely '$a \times b$', as shown in Figure N180. Nevertheless, she was unable to develop it. Excerpt S52 is illustrative (Suhana/L1537-1559).

Excerpt S52

R: (Puts a card written the following scenario in front of Suhana). Suppose that a Form One student comes to you and says that he does not know how to develop (derive) the formula for calculating the area of the following figures:
(a) Rectangle,
(b) Parallelogram,
(c) Triangle, and
(d) Trapezium.
How would you show him the way to develop (derive) the formula for calculating the area of these figures? Let's start with rectangle.
S: Rectangle. Ok this one (draws a rectangle and then writes its area formula, as shown in Figure N180). "*a* times *b*".
R: How do you get "$a \times b$"?
S: The longer side times the…(silent for a while) times two, two sides.
R: What do your "*a*" stands for?
S: "*a*" for the longer side and then "*b*" is the shorter side.
R: How do you get this formula?
S: Em…(silent for a while) I have learnt it by remembering the formula.

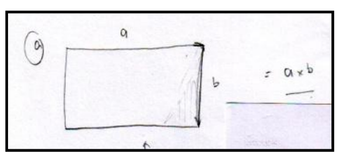


Figure N180. Suhana draws a rectangle and writes its area formula.

In Excerpt S52, Suhana stated that the formula for the area of a rectangle is '*a* times *b*'. She explained that *a* and *b* represent the longer and shorter sides of the rectangle. Nevertheless, Suhana was unable to develop the formula. Suhana expressed that she learnt the formula by memorizing it. Suhana just memorized the formula. She did not attempt to develop the formula.

Suhana could recall the formula for the area of a parallelogram as 'the *tapak* [base] times the *tinggi* [height]'. She also knew how to develop the formula for the area of a parallelogram. Excerpt S53 is illustrative (Suhana/L1560-1585).

Excerpt S53

R: How would you develop (derive) the formula for calculating the area of a parallelogram?
S: Parallelogram. (Develops the formula for the area of a parallelogram, as shown in Figure N181). This "*c*" and this "*d*". Because this one, em we take out this one (points to triangle "I") and then put here. It will become like the rectangle.
R: How do you get that?

S: Ok this parallelogram, …if we cut it into two and then put this one (points to triangle "I") here, it will become rectangle.
R: Could you show me how it is?
S: …This is "a". So, it becomes "a times b". This is "b". So, "a times b".
R: How do you get that?
S: From here we put this. Let's say this "k", this is "I". So, this is "k" and this is "I", em yeah.
R: What is the formula for the area of a parallelogram?
S: Parallelogram. (Draws a parallelogram and labels its *tapak* [base] and *tinggi* [height], as shown in Figure N182). This one, the *tapak* [base] times the *tinggi* [height].

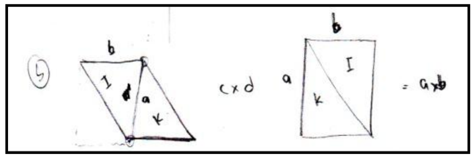


Figure N181. Suhana develops the formula for the area of a parallelogram.

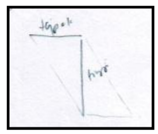


Figure N182. Suhana draws a parallelogram and labels its *tapak* [base] and *tinggi* [height].

In Excerpt S53, Suhana developed the formula for the area of a parallelogram, as shown in Figure N181. She drew a parallelogram where one of its diagonal perpendicular to the base (it does not look like perpendicular to the base, as shown in Figure N181, as Suhana used free hand to draw the parallelogram). Suhana initially labelled the base and the height of the parallelogram as *c* and *d* and wrote its area formula as $c \times d$. Subsequently, Suhana modified the label *c* to *b*, cancelled the label *d* and then rewrote it as *a* because she wanted to use the same symbols to represent the formula for the area of a rectangle, namely $a \times b$, as shown in Figure N181.

Suhana mentally cut the parallelogram into two triangles along its diagonal and then she labelled the triangles as "I" and "K". Suhana mentally moved triangle "I" from one end of the parallelogram to the other end of the parallelogram to form a rectangle and wrote its area formula as $a \times b$, as shown in Figure N181. Suhana drew a parallelogram, as shown in Figure N182. She stated the formula for the area of a parallelogram as 'the *tapak* [base] times the *tinggi* [height]'.

Suhana could recall the formula for the area of a triangle, namely '$\frac{1}{2} \times t\ \times tapak\ [base]$', as shown in Figure N179. Nevertheless, she was unable to develop it. Suhana attempted to develop the formula but unsuccessful. Excerpt S54 is illustrative (Suhana/L1586-1618).

Excerpt S54

R: How would you develop (derive) the formula for calculating the area of a triangle?
S: Triangle. (Draws a triangle and then writes its area formula, as shown in Figure N183). Ok first find the *tinggi* [height], "a" and then the *tapak* [base]. So, it becomes half times *tinggi* [height] times *tapak* [base].
R: How do you get the formula?
S: …(Silent for a while) or this one. (Draws a rectangle and then writes its area formula, as shown in Figure N184). This is "J", "L" and then "J" is here. So, "L" is equal, "L" here …(silent for a while) which become rectangle. So, this *tapak* [base] I put it "b", *tinggi* [height] I put it "a" and then this *tinggi* [height] I put "a". So, equal to "a times b" (Writes rectangle = a x b, as shown in Figure N184).
R: How do you get the formula?

S: (Draws another triangle and then writes its area formula, as shown in Figure N185). Ok this formula rectangle, the *tinggi* [height] is "a" and then got *tapak* [base] one. Divide it into two. We got "b", *tapak* [base] divide 2. So, the area for this one (points to the triangle in Figure N185) is "a", is *tinggi* [height] times *tapak* [base] divide by 2 or times half.
R: How do you get that formula?
S: …(Silent for a while) just like that. I just memorized the formula.

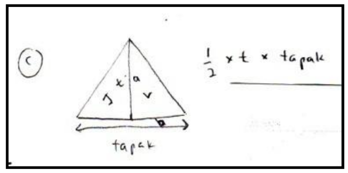


Figure N183. Suhana draws a triangle and then writes its area formula.

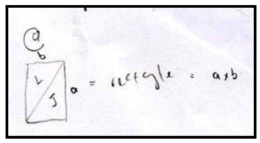


Figure N184. Suhana draws a rectangle and then writes its area formula.

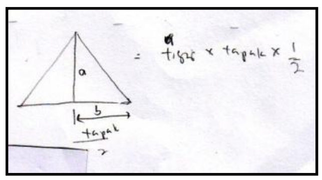


Figure N185. Suhana draws another triangle and then writes its area formula.

In Excerpt S54, Suhana could recall the formula for the area of a triangle. She stated that the formula for the area of a triangle is ‘half times *tinggi* [height] times *tapak* [base]’. Nevertheless, she was unable to develop it. When probed to develop the formula, Suhana attempted to develop the formula but unsuccessful. She mentally cut the isosceles triangle from Figure N183 along its symmetrical line and then rearranged it to be a rectangle, as shown in Figure N184. Suhana drew a rectangle and wrote its area formula as ‘$a \times b$’. When probed further to develop the formula, she mentally cut the rectangle from Figure N184 diagonally and then rearranged it to be an isosceles triangle, as shown in Figure N185. Suhana drew another triangle (isosceles triangle) and wrote its area formula as ‘*tinggi* [height] × *tapak* [base] × $\frac{1}{2}$’. When the researcher asked how she got that formula, Suhana expressed that the formula was just like that and she just memorized the formula.

Suhana could recall the formula for the area of a trapezium as $\frac{1}{2} \times tinggi\ [height] \times (a + b)$, as shown in Figure N186. She also knew how to develop the formula for the area of a trapezium. Excerpt S55 is illustrative (Suhana/L1619-1661).

Excerpt S55

R: How would you develop (derive) the formula for calculating the area of a trapezium?
S: Trapezium. (Draws a trapezium and then writes its area formula, as shown in Figure N186). This is "a", "b", and *tinggi* [height]. Ok half times *tinggi* [height] times (a plus b). Or divide into two: (a times *tinggi* [height]) plus ($b - a$) times *tinggi* [height] ("times half" missing).
R: Could you show me how it is?
S: Ok this is the original formula (points to first formula, $\frac{1}{2} \times$ *tinggi* [height] × (a + b), as shown in Figure N186).
R: Could you show me how it is?

S: It is half times *tinggi* [height], this one (points to "*a*") and this one (points to "*b*") we have to add on first, "*a*", "*b*" like this one. And then for the second part, we identify it into two parts: (*a* times *tinggi* [height]) plus (*b* – *a*) times *tinggi* [height] ("times half" missing).

R: Could you show me how it is?

S: So, eh (*b* – *a*) times *tinggi* [height] times half (realizes the missing part of "times half").

R: Could you show me how do you get this formula (points to the first formula, as shown in Figure N186) from here (points to the second formula, as shown in Figure N186)?

S: This one (points to the first formula, as shown in Figure N186) derive from this one (points to the second formula, as shown in Figure N186).

R: How?

S: Factorize strategy. (Develops the formula for the area of a trapezium, as shown in Figure N187). (First attempt to develop the formula): And then got "*a* plus *b* minus a times $\frac{1}{2}$…(silent for a while) (realizes a mistake and then cancels it, as shown in Figure N187). (Second attempt to develop the formula): *Tinggi* [height], "*a* plus $\frac{1}{2}$ *b* minus $\frac{1}{2}$ *a*" and then equal to *tinggi* [height], "*a* minus $\frac{1}{2}$ *a* equal to $\frac{1}{2}$ *a* plus $\frac{1}{2}$ *b*". Therefore, factorize $\frac{1}{2}$ times *tinggi* [height] times (*a* plus *b*).

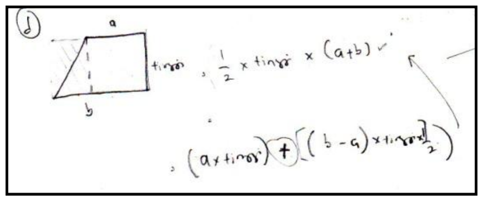


Figure N186. Suhana draws a trapezium and then writes its area formula.

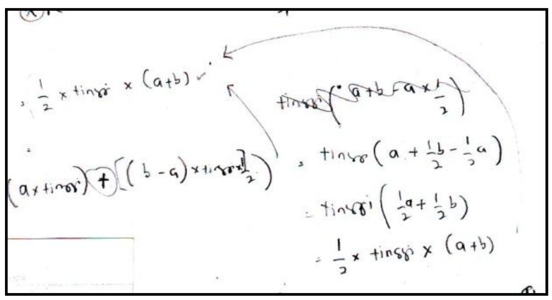


Figure N187. Suhana develops the formula for the area of a trapezium.

In Excerpt S55, Suhana drew a trapezium and then wrote its area formula as $\frac{1}{2} \times tinggi\ [height] \times (a + b),$ as shown in Figure N186. She used dotted line to partition the trapezium into a rectangle and a triangle and then wrote the formula for the area of a trapezium as the combination of the formulae for the area of a rectangle and a triangle, namely (*a* × *tinggi* [*height*]) + [(*b* – *a*) × *tinggi* [*height*] × $\frac{1}{2}$], as shown in Figure N186. Suhana developed the formula for the area of a trapezium from the combination of the formulae for the area of a rectangle and a triangle, namely (*a* × *tinggi* [*height*]) + [(*b* – *a*) × *tinggi* [*height*] × $\frac{1}{2}$] using algebraic method, as shown in Figure N187. In the first attempt, she had mistakenly simplified (*a* × *tinggi* [*height*]) + [(*b* – *a*) × *tinggi* [*height*] × $\frac{1}{2}$] as *tinggi* [*height*] (*a* + *b* – *a* × $\frac{1}{2}$). Suhana realized her mistake and cancelled it, as shown in Figure N187. In the second attempt, she has successfully developed the formula for the area of a trapezium. She correctly simplified (*a* × *tinggi* [*height*]) + [(*b* – *a*) × *tinggi* [*height*] × $\frac{1}{2}$] as follow: (*a* × *tinggi* [*height*]) + [(*b* – *a*) × *tinggi* [*height*] × $\frac{1}{2}$] = *tinggi* [*height*] (*a* + $\frac{1}{2}$ *b* – $\frac{1}{2}$ *a*) = *tinggi* [*height*]($\frac{1}{2}$ *a* + $\frac{1}{2}$ *b*) = $\frac{1}{2}$ × *tinggi* [*height*] × (*a* + *b*), which is the formula for the area of a trapezium.

Summary

In summary, Suhana could recall the formula for the area of a rectangle, parallelogram, triangle, and trapezium. Nevertheless, she was only able to develop the formulae for the area of a parallelogram and a trapezium. Suhana did not attempt to develop the formula for the area of a rectangle. She attempted to develop the formula for the area of a triangle but unsuccessful.

Conceptual Knowledge

Suhana could recall the formula for the area of a rectangle. Nevertheless, she was unable to develop the formula. It was apparent that Suhana lack of conceptual knowledge underpinning the formula for the area of a rectangle.

Suhana could recall the formula for the area of a parallelogram. She was able to develop the formula. Suhana mentally transformed the parallelogram to a rectangle by cutting the parallelogram into two triangles along its diagonal. Suhana mentally moved a triangle from one end of the parallelogram to the other end of the parallelogram to form a rectangle. It indicated that she understands the relationship between the formula for the area of a parallelogram and rectangle. A parallelogram can always be transformed into a rectangle with the same base, same height, and the same area. Thus, the formula for the area of a parallelogram is exactly the same as the formula for the area of a rectangle, namely 'base times height'.

Suhana could recall the formula for the area of a triangle. Nevertheless, she was unable to develop the formula. Suhana did not know the relationship between the area of a triangle and the area of the rectangle that encloses it. Had she been known of this relationship, Suhana would know how to develop the formula for the area of a triangle.

Suhana could recall the formula for the area of a trapezium. She was able to develop the formula. Suhana developed the formula using algebraic method. Suhana developed the formula for the area of a trapezium from the combination of the formulae for the area of a rectangle and a triangle, namely ($a \times$ *tinggi* [*height*]) + [($b - a$) $\times$ *tinggi* [*height*] $\times \frac{1}{2}$] using algebraic method. She correctly simplified it as $\frac{1}{2} \times$ *tinggi* [*height*] $\times$ ($a + b$), which is the formula for the area of a trapezium. It indicated that Suhana knew that the formula for the area of a trapezium is related to the formulae for the area of a rectangle and triangle.

Linguistic Knowledge

Suhana used appropriate mathematical symbols to write the formula for the area of a rectangle, namely '$a \times b$', as shown in Figure N180. Nevertheless, Suhana used inappropriate mathematical terms 'longer side' and 'shorter side' to explain the meaning of the symbols a and b that she employed. Suhana explained that "a for the longer side and then b is the shorter side." (Suhana/L1557). Actually, a and b represents the length and the width of the rectangle.

Suhana used appropriate mathematical symbols to write the formula for the area of a parallelogram, namely '$a \times b$', as shown in Figure N181. Suhana used appropriate mathematical terms '*tapak* [base]', 'times', and '*tinggi* [height]' to state the formula for the area of a parallelogram. Suhana stated the formula for the area of a parallelogram as "…the *tapak* [base] times the *tinggi* [height]." (Suhana/L1585).

Suhana used appropriate mathematical symbols to write the formula for the area of a triangle, namely '$\frac{1}{2} \times t \times tapak\ [height]$', as shown in Figure N183. Suhana used appropriate mathematical terms 'half', 'times', '*tinggi* [height]', and '*tapak* [base]' to state the formula for the area of a triangle. Suhana stated the formula as "…half times *tinggi* [height] times *tapak* [base]." (Suhana/L1594).

Suhana used appropriate mathematical symbols to write the formula for the area of a trapezium, namely $\frac{1}{2} \times tinggi\ [height] \times (a + b)$, as shown in Figure N186. Suhana did not explain the meaning of the mathematical symbols $(a + b)$ that she employed. Actually, $(a + b)$ in the formula for the area of a trapezium represents the sum of the length of the parallel sides of the trapezium.

Strategic Knowledge

Suhana used the cut and paste strategy to develop the formula for the area of a parallelogram. Suhana mentally cut the parallelogram into two triangles along its diagonal and then she labelled the triangles as "I" and "K". Suhana mentally moved triangle "I" from one end of the parallelogram to the other end of the parallelogram to form a rectangle and wrote its area formula as $a \times b$, as shown in Figure N181.

Suhana used algebraic method to develop the formula for the area of a trapezium from the combination of the formulae for the area of a rectangle and a triangle, namely $(a \times tinggi\ [height]) + [(b - a) \times tinggi\ [height] \times \frac{1}{2}]$, as shown in Figure N187. In the second attempt, she correctly simplified $(a \times tinggi\ [height]) + [(b - a) \times tinggi\ [height] \times \frac{1}{2}]$ as $\frac{1}{2} \times tinggi\ [height] \times (a + b)$, which is the formula for the area of a trapezium.

Ethical Knowledge

Suhana could recall the formula for the area of a rectangle but she did not attempt to develop the formula, as shown in Excerpt S52. Suhana had succeeded in developing the formula for the area of a parallelogram, as shown in Figure N181. Suhana could recall the formula for the area of a triangle. She attempted to develop the formula but unsuccessful, as shown in Excerpt S54. Suhana also succeeded in developing the formula for the area of a trapezium, as shown in Figure N187.

Level of Subject Matter Knowledge

In this section, Suhana' levels (low, medium, high) of subject matter knowledge of perimeter and area was analyzed in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

Suhana achieved a medium level of conceptual knowledge of perimeter and area when she obtained 68.0% of appropriate mathematical elements of conceptual knowledge of perimeter and area during the clinical interview. Suhana achieved a medium level of procedural knowledge of perimeter and area when she obtained 63.6% of appropriate mathematical elements of procedural knowledge of perimeter and area. Suhana secured a high level of linguistic knowledge of perimeter and area when she obtained 72.1% of appropriate mathematical elements of linguistic knowledge of perimeter and area. Suhana secured a high level of strategic knowledge of perimeter and area when she obtained 85.7% of appropriate mathematical elements of strategic knowledge of perimeter and area. Suhana achieved a medium level of ethical knowledge of perimeter and area when she obtained 65.3% of appropriate mathematical elements of ethical knowledge of perimeter and area. Suhana achieved an overall medium level of subject matter knowledge of perimeter and area when she obtained 69.7% of appropriate mathematical elements of subject matter knowledge of perimeter and area.

Tan

Tan lives in Sungai Petani, Kedah. Tan is 22 years 7 months old when he was interviewed. Currently, he is pursuing a 4-year Bachelor of Science with Education (B.Sc.Ed.) program at a public university. He majored and minored in chemistry and mathematics respectively. He obtained grade 1A in Mathematics and Additional Mathematics in his 2002 SPM examination (equivalent to O level examination). He also scored A in Mathematics T in the 2004 STPM examination (equivalent to A level examination). Tan performed quite well in his mathematics content courses at the university level when he secured two A, two A− and one B+ in five mathematics content courses he had completed during the first and second year of his studies. The detail of his performance is shown in Table N9.

Table N9

Mathematics Performance of Tan

Courses	Grades
1. Calculus for Science Students I	A−
2. Algebra for Science Students	A−
3. Statistics for Science Students	B+
4. Calculus for Science Students II	A
5. Differential Equation I	A

At the time of data collection, Tan was in his second semester of third year studies. He attained 3.69 in the Cumulative Grade Point Average (CGPA) for his first two years of studies at the public university. He does not have any teaching experience prior to this interview.

Notion of Perimeter

Conceptual Knowledge

Tan has successfully selected all the shapes that have a perimeter, namely "A", "C", "D",”F”, "H", "I", “J”, and "K". Excerpt T1 shows his choice of shapes that have a perimeter (Tan/L75-78).

Excerpt T1

R: (Puts a handout comprises 12 shapes in front of Tan). Tick the shapes that have a perimeter.
S: (Ticks shapes "A", "C", "D",”F”, "H", "I", “J”, and "K", as shown in Figure N188).

In Excerpt T1, Tan has selected all simple closed curves (A, C, H, K) as well as all closed but not simple curves (D, I) that have a perimeter. He also selected the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that his notion of perimeter was not only limited to simple closed curves, and closed but not simple curves, but also inclusive of 3-dimensional shapes. Tan did not select the two simple but not closed curves (B, G) as well as the two 1-dimensional shapes (E, L) that do not have a perimeter. In other words, Tan did not select an open shape (including the lines) as having a perimeter.

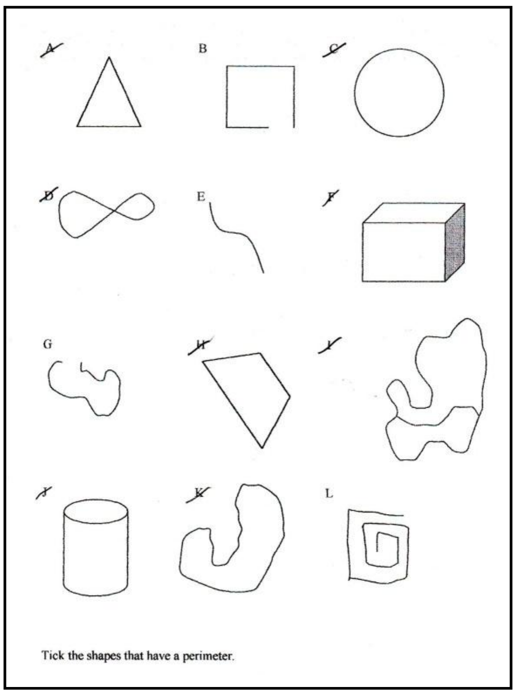


Figure N188. Tan’s selection of shapes that have a perimeter.

Excerpt T2 depicts his justification of selecting shapes "A", "C", "D","H", "I", and "K", (Tan/L176-206).

Excerpt T2

R: Why did you select shape "A"?
S: "A" is, for me is quite sure is a perimeter. I can calculate because closed, is a closed length object.
R: Why did you select shape "C"?
S: …If circle, we have to use something else like the *benang* [thread] to calculate the total length of the circumference.
R: Why did you select shape "D"?
S: "D" is the mixture of between a vertical length or straight length and a circumference. So, we can total it up to figure out the perimeter.
R: Why did you select shape "H"?
S: For "H" is trapezium. The case is almost the same like the triangle one (refers to shape "A"), that it is a closed object where we can calculate all length at the object. Total it up. Figure it out to find perimeter.
R: Why did you select shape "I"?

S: "I" is quite complex because it's like a route, like *Sepang* circuit for example where we need to use the *benang* [thread] to measure the total length of this route. …
R: Why did you select shape "K"?
S: "K" is almost same like "I". But "I", the route is more complex compared to "K".

In Excerpt T2, Tan explained that he selected shapes "A" and "H", because they are closed objects. Tan also explained that he selected shape “C”, "D", “I”, and “K” because the perimeters of these shapes can be measured using thread. It indicated that he appeared to associate the notion of perimeter with the measurement of perimeter (i.e., perimeter does not exist until it is measured).

Initially, Tan did not select shapes "F" and "J" as having a perimeter because they are 3-dimensional objects. Subsequently, Tan changed his mind to select "F" and "J" that have a perimeter as he realized that perimeter can be calculated on each surface of the 3-dimensional objects. It also indicated that Tan appeared to associate the notion of perimeter with the measurement of perimeter (i.e., perimeter does not exist until it is measured). He elaborated that shapes "F" and "J" fulfilled the basic definition of perimeter, namely total length that surrounds the surface of the 3-dimensional objects. Tan explained that the perimeter of the circular base of shape “J” can be measured using the thread and the perimeter of its curved surface can be measured using ruler or formula. Excerpt T3 is illustrative (Tan/L214-228, L233-239).

Excerpt T3

R: Why didn't you select shape "F"?
S: Actually, I am not quite sure for "F". Perimeter is also can be calculated for "F". But this object involves 3D dimension already. So, if I still calculate the total perimeter, I have to see each face in order to calculate the perimeter. So, if chosen can be accepted or not, I can accept the "F".
R: Why didn't you select shape "F" at the beginning?
S: …Usually at school, teacher gave us example more on 2D to calculate perimeter. I am sorry to say that I can't remember if teacher gave us 3D dimension to work out for the perimeter. But, em as I figure out when I explained just now, I think that "F" can be accepted to calculate the perimeter because it fulfills the basic definition of perimeter. It surrounds the object, the line, the length and then we can total up to find the total length for that 3D object.
.
.
.
R: Why didn't you select shape "J"?
S: Em "J". If I select "F", then "J" is not an excuse for me to not accept it. Because the circle (refers to circular base of the cylinder), we can still measure using the *benang* [thread] and then here (points to the curved surface of the cylinder) we can measure using ruler or by formula if we were given data of the length. We can still calculate the perimeter as well.
R: Why did you select shape "J"?
S: The reason same as "F".

Tan explained that he did not select shapes “B” and “G”, because they are incomplete to surround to become an object or a shape. Excerpt T4 demonstrates his justification for not selecting shapes “B” and “G” as having a perimeter (Tan/L207-210, L229-232).

Excerpt T4

R: Why didn't you select shape "B"?
S: "B" because the length is incomplete to surround, to become an object. Although it is similar like square but it doesn't like square because it is incomplete line.
.
.
.
R: Why didn't you select shape "G"?
S: "G" is also the same, too extreme. Although it is almost nearly completed the circuit but it is not counted as perimeter because it is incomplete surrounded to become an object or a certain shape.

Tan explained that he did not select shape “E” because the line is not connected. Tan also explained that he did not select shape “L” because it is incomplete to surround certain object. Excerpt T5 reveals his justification for not selecting shapes “E” and “L” as having a perimeter (Tan/L211-213, L240-243).

Excerpt T5

R: Why didn't you select shape "E"?
S: For "E" is totally wrong. The line is not connected. It is a too extreme for two ends.
.
.
.
R: Why didn't you select shape "L"?
S: "L" is just like, let's say I start from here. I walking to the library without ending. I didn't surround certain object. So, it can't form a perimeter.

Summary

In summary, Tan has selected all simple closed curves (A, C, H, K) and all closed but not simple curves (D, I) that have a perimeter. He also selected the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that his notion of perimeter was not only limited to simple closed curves, and closed but not simple curves, but also inclusive of 3-dimensional shapes. Tan justified his selection by explaining that he selected shapes "A" and "H", because they are closed objects. Tan explained that he selected shape “C”, "D", “I”, and “K” because the perimeters of these shapes can be measured using thread. It indicated that Tan appeared to associate the notion of perimeter with the measurement of perimeter (i.e., perimeter does not exist until it is measured). Tan also explained that he selected shapes "F" and "J" because perimeter can be calculated on each surface of the 3-dimensional objects. It also indicated that Tan appeared to associate the notion of perimeter with the measurement of perimeter (i.e., perimeter does not exist until it is measured).

Linguistic Knowledge

Tan used appropriate mathematical term ‘closed’ to justify his selection of shapes “A” and “H” that have a perimeter. Tan explained that he selected these shapes because they are closed objects. He used appropriate mathematical term ‘measure’ to justify his selection of shapes “C”, "D", “I”, and “K” that have a perimeter. Tan explained that he selected these shapes because their perimeters can be measured using thread, as shown in Excerpt T2. He used appropriate mathematical term ‘calculate’ to justify his selection of shapes “F” and “J” that have a perimeter. Tan explained that he selected shapes “F” and “J” because perimeter can be calculated on each surface of the 3-dimensional objects, as shown in Excerpt T3.

Tan used inappropriate negation ‘incomplete’ as his justification for not selecting shapes “B” and “G” as having a perimeter. Tan explained that he did not select shapes “B” and “G”, because they are incomplete to surround to become an object or a shape, as shown in Excerpt T4. Tan used inappropriate negation ‘not connected’ as his justification for not selecting shape “E” as having a perimeter. Tan explained that he did not select shape “E” because the line is not connected. Tan also used inappropriate negation ‘incomplete’ as his justification for not selecting shape “L” as having a perimeter. Tan explained that he did not select shape “L” because it is incomplete to surround certain object, as shown in Excerpt T5.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Tan had taken the effort to justify the selection of shapes that have a perimeter. He provided appropriate justification for selecting shapes "A", "C", "D",”F”, "H", "I", “J”, and "K" that have a perimeter, as shown in Excerpts T2 and T3.

Tan also had taken the effort to provide justification for not selecting other shapes that do not have a perimeter. Nevertheless, he provided inappropriate justification for not selecting shapes “B”, “E”, “G”, and “L” as having a perimeter, as shown in Excerpts T4 and T5.

Notion of Area

Conceptual Knowledge

Tan has successfully selected shapes “A”, “C”, “D”, “F”, “H”, “I”, “J”, and “K” as having an area. Excerpt T6 shows his choice of shapes that have an area (Tan/L268-271).

Excerpt T6

R: (Puts a handout comprises 12 shapes in front of Tan). Tick the shapes that have an area.
S: (Ticks shapes "A", "C", "D", “F”, "H", "I", “J”, and "K", as shown in Figure N189).

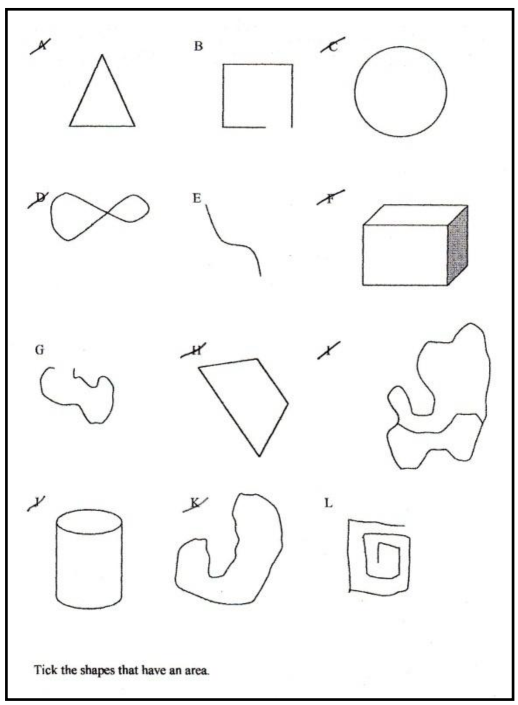


Figure N189. Tan’s selection of shapes that have an area.

In Excerpt T6, Tan has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. He also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that Tan had a static perspective of the notion of area. Based on this perspective, area can be viewed as the amount of surface enclosed within a boundary. It also indicated that his notion of area was not only limited to 2-dimensional shapes (closed plane shapes), but also inclusive of 3-dimensional shapes. Tan did not select the two open shapes (B, G) as well as the two 1-dimensional shapes (E, L) that do not have an area. In other words, Tan did not select an open shape (including the lines) as having an area. It can be inferred that he did not has a dynamic perspective of area or, at least, this knowledge was not accessible to him during the clinical interview.

When asked to justify his selection, Tan explained that he selected shape "A", because it is a closed length object and thus its area can be calculated. Tan elaborated the procedure to calculate the area of shape "A": Measure the length (of sides) and then calculates its area using formula. Tan explained that he selected shapes "C", "D", "H", "I", and "K" because their area can be calculated. It indicated that Tan appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). He incorrectly named shape "C" as circular (it should be circle).Tan elaborated that the area of shape "C" can be calculated using formula πr^2. He viewed shape "D" as two sectors and explained that its area can be calculated if the angle and radius of the sectors were given. Tan correctly named shape "H" as trapezium and explained that its area can be calculated using formula 'half times the total length of two parallel sides times height'. He viewed shapes "I" and "K" as irregular shapes and explained that their area can be calculated by partitioning them into many regions for which area measurement formulae were known. Excerpt T7 depicts his justification of selecting each of these shapes (Tan/L282-293, L297-306, L310-312).

Excerpt T7

R: Why did you select shape "A"?
S: Unlike perimeter. There is one common thing shared between these two concepts. One is the closed length object. Like triangle (refers to shape "A") for example, is a closed length object. So, we can measure the area by using the formula. So, we measure the length first and then we can figure out, we can calculate the area.
R: Why did you select shape "C"?
S: Circular (refers to shape "C"), we can use the πr^2.
R: Why did you select shape "D"?
S: In "D", it is quite the same. We can, actually this is the combination of two sectors of a circle. So, we can calculate if we were given angle inside and the length of the radius.
.
.
.
R: Why did you select shape "H"?
S: Trapezium (refers to shape "H"), we just need to know the two parallel lengths and then we total up, we times with the length of the height and then times half to get the trapezium.
R: Why did you select shape "I"?
S: "I" is quite difficult because the shape is irregular but we still can calculate the area by dividing the area into many regions to calculate. Because some area can be resembled like a circle, part of a circle, not totally circle. Whereas for other part is like a straight line, is more on shape triangle, something like this.
.
.
.
R: Why did you select shape "K"?
S: "K" also quite difficult because the shape is irregular but we still can calculate the area by dividing the area into many regions like "I".

Tan explained that he selected shapes "F" and "J" because their surface area can be calculated. It also indicated that Tan appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). He elaborated that the surface area of shape "F" can be calculated if its length, width, and height were known. Tan pointed out that the surface area of shape "F" can be calculated using formula '2πj times two plus πjt', where j and t represents the radius of the base and the height of the cylinder. He had provided the incorrect formula for calculating the surface area of the cylinder. The correct formula should be '$2\pi r^2 + 2\pi rh$', where r and h represents the radius of the base and the height of the cylinder. Excerpt T8 demonstrates his justification for selecting shapes "F" and "J" that having an area (Tan/L294-296, L307-309).

Excerpt T8

R: Why did you select shape "F"?
S: "F" is quite easy to calculate because if we know the length, the width, and the height, we can calculate the total surface of the area.
.
.
.
R: Why did you select shape "J"?
S: "J", we can calculate by using the formula of $2\pi j$ times two plus πjt, the t stands for height.

Tan explained that he did not select shapes “B”, “E”, “G”, and “L” because they are not surrounded to form an object. Excerpt T9 reveals his justification for not selecting shapes “B”, “E”, “G”, and “L” as having an area (Tan/L313-317).

Excerpt T9

R: Why didn't you select shape "B"?
S: For shapes "B", "E", "G", and "L", they shared the same common thing, that is incomplete closed surround object. That meaning that the length is not fulfill, not surrounded to form an object. It is almost same like a perimeter (refers to the previous task, Task 1.1).

Summary

In summary, Tan has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. He also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that Tan had a static perspective of the notion of area. His notion of area was not only limited to 2-dimensional shapes (closed plane shapes), but also inclusive of 3-dimensional shapes. Tan justified his selection by explaining that he selected shape "A" because it is a closed length object and thus its area can be calculated. Tan explained that he selected shapes "C", "D", "H", "I", and "K" because their area can be calculated. It indicated that Tan appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Tan also explained that he selected shapes “F” and “J” because their surface area can be calculated. It also indicated that Tan appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured).

Linguistic Knowledge

Tan used appropriate mathematical term ‘closed’ to justify his selection of shape “A” that have an area. Tan explained that he selected shape “A” because it is a closed length object and thus its area can be calculated. Tan used appropriate mathematical term ‘calculate’ to justify his selection of shapes “C”, “D”, “H”, “I”, and “K” that have an area. Tan explained that he selected shapes “C”, “D”, “H”, “I”, and “K” because their area can be calculated, as shown in Excerpt T7. Tan also used appropriate mathematical term ‘calculate’ to justify his selection of shapes “F” and “J” that have an area. Tan explained that he selected shapes “F” and “J” because their surface area can be calculated, as shown in Excerpt T8.

Tan used appropriate negation ‘not surrounded’ as his justification for not selecting shapes “B”, “E”, “G”, and “L” as having an area. Tan explained that he did not select shapes “B”, “E”, “G”, and “L” because they are not surrounded to form an object, as shown in Excerpts T9.

Tan used inappropriate mathematical term ‘circular’ to name shape “C”. In fact, shape “C” is a circle.

Ethical Knowledge

Tan had taken the effort to justify the selection of shapes that have an area, as shown in Excerpts T7 and T8. He provided appropriate justification for selecting shapes "A", "C", "D", "F", "H", "I", "J", and "K" that have an area.

Tan also had taken the effort to provide justification for not selecting other shapes that do not have an area. He provided appropriate justification for not selecting shapes "B", "E", "G", and "L" as having an area, as shown in Excerpt T9.

Notion of the Units of Area

Conceptual Knowledge

Tan stated that square, rectangle, and triangle can be used as the unit of area. Excerpt T10 is illustrative (Tan/L323-343).

Excerpt T10

R: (Puts a card written the following scenario in front of Tan). Ali, Chong, and David are discussing about the units of area. Ali says that we can use a square as the unit of area. Chong says that we can use a rectangle as the unit of area. David says that we can use a triangle as the unit of area. How would you respond to these students?
S: Em actually three of them are quite correct.
R: Ali says that we can use a square as the unit of area.
S: As the unit of area. Rectangle is another unit of area and then triangle, unit of area. Actually, they all can be used as the unit of area.
R: Why?
S: Because even though the way we calculate the area is different, but at the end, the unit is the power of two. Let's say centimeter square or meter square.

In Excerpt T10, Tan stated that square, rectangle, and triangle can be used as the units of area measurement. It indicated that his notion of the unit of area was not only limited to square, but also nonsquare (such as rectangle and triangle). He explained that even though the way the area being calculated is different, at the end, the unit is "the power of two" (square unit) such as square centimetre or square metre. Tan literally stated the units as centimeter square or meter square. It indicated that he was unable to provide the appropriate justification that any shape that tessellates a plane can be used as a unit of area measurement.

Summary

In summary, Tan stated that square, rectangle, and triangle can be used as the units of area measurement. It indicated that his notion of the unit of area was not only limited to square, but also nonsquare (such as rectangle and triangle). He explained that a square, rectangle, and triangle can be used as the unit of area because its unit is "the power of two" (square unit) such as square centimetre or square metre. It indicated that Tan was unable to provide the appropriate justification that any shape that tessellates a plane can be used as a unit of area measurement.

Linguistic knowledge

Tan used inappropriate mathematical term 'their unit is "the power of two" (square unit)' to justify that a square, rectangle, and triangle can be used as the unit of area. He explained that a square, rectangle, and triangle can be used as the unit of area because their unit is “the power of two” (square unit) such as square centimetre or square metre. Tan literally stated the units as centimeter square or meter square. He also incorrectly stated the square unit as ‘the power of two’, as shown in Excerpt T10.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Tan had taken the effort to justify the shapes that can be used as a unit of area measurement. Nevertheless, he was unable to provide an appropriate justification for the shapes that can be used as a unit of area measure. This can be seen in Excerpt T10. In reality, any shape that tessellates a plane can be used as a unit of area measurement.

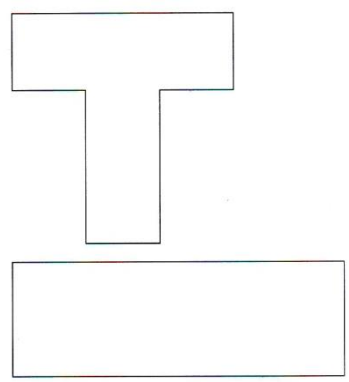
Comparing Perimeter (No Dimension Given)

Strategic Knowledge

Tan used a formal method of measuring the side by thread and ruler to determine whether the given pair of shapes had the same perimeter. Excerpt T11 shows how he used thread and ruler to compare the perimeter (Tan/ L412-430).

Excerpt T11

R: (Puts the following pair of shape in front of Tan). How would you find out whether they had the same perimeter?



S: …use this *benang* [thread]. Like geography, when we want to measure the length of the river, we use the *benang* [thread]. But, in mathematics, the same method can be applied.
R: Could you show me how it is?
S: We just use the *benang* [thread] to surround the object. (Measures the perimeter of the T-shape using the thread and ruler). When I reached the end, I will tell student to mark with pen. So, there is a mark there. Either you cut it out or you use this *benang* [thread], put at the ruler and then you measure the total length. So, the other object (refers to the given rectangle) can use the same way, use the *benang* [thread]. … After they measure this length, they also measure this length and if both are match, then it can be concluded they had the same perimeter.
R: What do you mean by "match"?
S: If they measure the same length, then both objects have the same perimeter, regardless the shapes of the object.

In Excerpt T11, Tan used the thread to surround the T-shape. He measured the length of each side of the T-shape by a piece of thread and then put it on a ruler to determine its total length (perimeter). Tan also measured the length of each side of the rectangle by a piece of thread and then put it on a ruler to determine its total length (perimeter). He explained that if both the given shapes had the same total length, then they had the same perimeter.

When probed for alternative method of comparing the perimeter, Tan suggested a semi-formal method of cut and paste both shapes on a 1-cm grid paper and then counts the number of unit on each side, to determine whether the given pair of shapes had the same perimeter. Excerpt T12 depicts how he used this semi-formal method to compare the perimeter (Tan/L431-450).

Excerpt T12

R: Could you think of other way of finding out whether they had the same perimeter?
S: I think the… the graph.
R: Could you tell me more about it?
S: We can cut this object, paste on the graph paper. Because each graph paper is on the scale of one to ten. It's one centimeter if I am not mistaken. (Takes a 1-cm grid paper). Actually, in actual graph, this one is divided into smaller scale. So, we tell the student, they cut and paste exactly to the line here. So, since they know that each cube here, eh square here. Excuse me, each square here has the same length. So, we can tell the student to calculate the total of the length using this graph paper. Because it is a general rule already, each square is 1 cm. So, it is easier for them to visualize when there is a lot of cube, square here compared they have to figure out like this mentally. Then, we calculate how much length for each object (refers to each shape), total up and then if they have the same length, come out with the same outcome that means they have same perimeter.
R: Could you think of other way of finding out whether they had the same perimeter?
S: I think that's all.

In Excerpt T12, Tan suggested to cut and paste the T-shape on the 1-cm grid paper and then counts the number of unit on each side. He also suggested to cut and paste the rectangle on the 1-cm grid paper and then counts the number of unit on each side. Tan explained that if both the given shapes had the same total length, then they had the same perimeter.

Summary

In summary, Tan produced one formal method and one semi-formal method of determining whether the given pair of shape had the same perimeter. In the first method, he used a formal method of measuring the side by thread and ruler. In the second method, Tan used a semi-formal method of cut and paste of both shapes on a 1-cm grid paper and then counts the number of unit on each side.

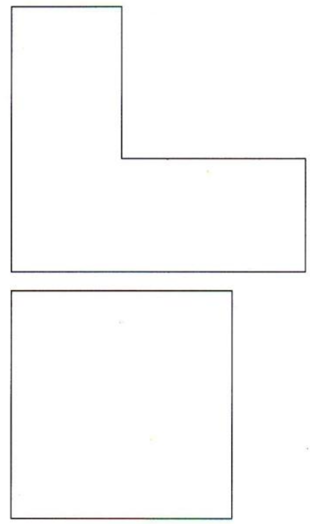
Comparing Area (No Dimension Given)

Strategic Knowledge

Tan used an informal method of cut-and-paste to compare the area of the given pair of shapes. Excerpt T13 shows how he used this informal method to compare the area (Tan/L484-492).

Excerpt T13

R: (Puts the following pair of shape in front of Tan). How would you find out whether they had the same area?



S: …I would cut it and then try to paste into other object to see whether it can completely superimpose into other object or not. If they happened to be superimposed completely into other object after I cut it fragmentally of the first object (refers to the given L-shape), then it can be concluded that these two objects (refers to the given pair of shapes) have the same area without even measure it.

In Excerpt T13, Tan suggested that he would cut the L-shape and then superimposed it on the square. He concluded that if the L-shape covered the square exactly, then they had the same area.

When probed for alternative method of comparing the area, Tan partitioned L-shape into two rectangles for which area measurement formulae were known. Excerpt T14 depicts the formal method of measuring the side and applying the area formula that he used to compare the area of the given pair of shapes (Tan/L524-567, L576-585).

Excerpt T14

R: Could you think of other way of finding out whether they had the same area?
S: …We used the ruler to calculate…
R: How would you use the ruler to find out whether they had the same area?
S: …Ok I cut into two, tell the student to cut into two where there are two distinctive rectangles (Partitions L-shape into two rectangles, as shown in Figure N190). So, once the rectangle has been identified, we just tell the student to measure length of the vertical one (refers to the length of the upper rectangle of the partitioned L-shape, as shown in Figure N190) and then the next thing to do is measure the horizontal line (refers to the width of the upper rectangle of the partitioned L-shape, as shown in Figure N190). …I will tell them to times these two lengths in order to get the area. So, this part (refers to the upper rectangle) of area is completed. Then we go the next part (refers to the lower rectangle). …The same thing goes for this square (points to the given square) to measure whether it is the same area with this one (points to the given L-shape) or not.

.
.
.

R: So, in this case, how would you find out whether they had the same area?
S: Em… after you have measure out the length, then you times to get the area. This one (points to the given L-shape) you divide it into two parts, find each one and then sum up. Then this whole area (points to the given L-shape) and this area (points to the given square) we make a comparison. If they happened to be distinguished and then they are not having the same area. But if they happened to be the same value of the final outcome, they all have, the both have the same area.

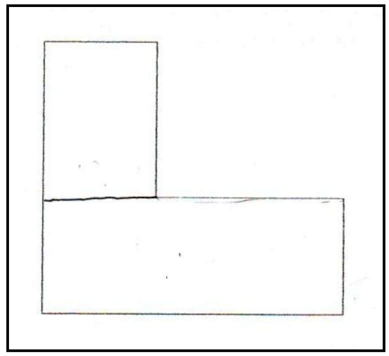


Figure N190. Tan partitions the L-shape into two rectangles.

In Excerpt T14, Tan partitioned L-shape into two rectangles, as shown in Figure N190. Tan suggested to measure the length and the width of each rectangle by ruler and then calculates its area using rectangle area formulae. He also suggested to measure the length of the two adjacent sides of the square by ruler and then calculates its area using square area formula. Tan explained that if they had the same value of the final outcome, then they have the same area.

When probed for other method of comparing the area, Tan used a semi-formal method of tracing both shapes on a 1-cm grid paper and then determine its length and width by counting the number of 1-cm square on its length and width. Excerpt T15 demonstrates how he used this semi-formal method to determine each area and then compare their measurements (Tan/L586-634).

Excerpt T15

R: Could you think of other way of finding out whether they had the same area?

S: The last method I am still like to use the same, using the graph. Cut it and paste it where we no need to use the ruler. …Let's say this, they can be cut and paste into here (Traces by free hand, the L-shape onto a 1-cm paper grid, as shown in Figure N191). …So, we tell the student to figure out, calculate out the length, total length of here (points to the length of the upper rectangle of the partitioned L-shape) and then divided like I said just now. Divided into two rectangles and then we total up the length (points to the length of the upper rectangle of the partitioned L-shape) and then this vertical one (points to the width of the upper rectangle of the partitioned L-shape). Then times out. This one also same (points to lower rectangle of the partitioned L-shape). We can get two areas and then plus it to get the final area. This one (points to the given square) still the same, we just paste it. …So, if this two (points to the given pair of shapes) match the same area, then it can be concluded that both have the same area.

R: What do you mean by "match"?

S: Match is if they calculate both the area respectively and happened to be same, that having the same value, then it can be concluded that they have the same area.

R: Could you think of other way of finding out whether they had the same area?

S: I don't think so (laugh).

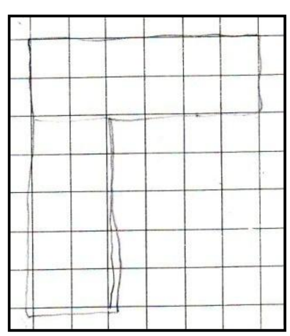


Figure N191. Tan traces the L-shape on the 1-cm grid paper.

In Excerpt T15, Tan traced the L-shape on the grid paper, as shown in Figure N191. He partitioned the traced L-shape into two rectangles. Tan suggested to count the number of 1-cm grid on the length and the width of each rectangle and then multiplied the length and the width respectively to get its area. He also would trace the square on the 1-cm grid paper. Tan would count the number of 1-cm grid on the length of the two adjacent sides of the traced square. He would multiply the length of the two adjacent sides to get its area. Tan explained that if they had the same value, then they have the same area.

Summary

In summary, Tan produced one informal method, one formal method, and one semi-formal method of determining whether the given pair of shapes had the same area. In the first method, Tan used an informal method of cut-and-paste. In the second method, he partitioned L-shape into two rectangles, as shown in Figure N190. Tan measured the length of side by ruler and applied area formulae. In the third method, he used a semi-formal method of tracing both shapes on a 1-cm grid paper and then determine its length and width by counting the number of 1-cm square on its length and width, as shown in Figure N191 and Excerpt T15.

Comparing Perimeter (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Tan explained that he was unable to determine which shape has the longer perimeter. Excerpt T16 shows the justification that he made (Tan/L669-685).

Excerpt T16

R: (Puts the following table in front of Tan). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 1	25 paper clips	12 sticks

S: …25 paper clips doesn't means that it might longer than 12 sticks. May be that the paper clip is very short. Some may be very long at the market. So, we might not know how long is it the paper clips for each one. The same goes to the stick. …May be these 12 sticks sum up might be longer than these 25 paper clips. So, to tell that both have the same or different perimeter or either one is longer than the other one, we can't tell because there are no units allocated for this two (refers to paper clip and stick, respectively).

In Excerpt T16, Tan explained that he was unable to determine which shape has the longer perimeter as the length of each paper clip and stick were not known. Tan elaborated that there were no units allocated for the paper clip and the stick. It indicated that he focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. Tan knew that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. He explained that 25 paper clips did not necessarily longer than 12 sticks and 12 sticks might longer than 25 paper clips.

In Set 2, Tan explained that two conclusions can be made in this case. Excerpt T17 depicts the justification that he made (Tan/L711-725).

Excerpt T17

R: (Puts the following table in front of Tan). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 2	10 paper clips	15 paper clips

S: This one almost similar to the Set 1, but the only different is, now two same things, paper clips but different in number. If the teacher provides the students these two kinds of paper clips happened to be the same kind, then we can conclude definitely that this 15 paper clips might have a longer perimeter compared to the 10 one, if each paper clip have the same length. Ok if these two paper clips, let's say the 10 paper clips the teacher gave different paper clips and then the 15, teacher gave another paper clip that is different in shape and length, then it can't be say that it has, the 15 paper clips might have a longer perimeter than 10 paper clips.

In Excerpt T17, Tan explained that if the paper clips for both shapes A and B were of the same length, then shape B has the longer perimeter as it has 15 paper clips compared to 10 paper clips in shape A. Tan also explained that if the paper clips for shapes A and B were of the different length, then he was unable to determine which shape has the longer perimeter. It indicated that he focused on the unit of measure when comparing perimeters in Set 2 with common nonstandard units. Tan knew that common nonstandard units (such as paper clips) are not reliable for comparing perimeters.

In another situation when shapes A and B had the same perimeter, Tan explained that the paper clips in shape A is longer than the paper clips in shape B. Excerpt T18 demonstrates his justification about their units of measurement (Tan/L726-730).

Excerpt T18

R: If shapes A and B had the same perimeter, what would you tell about their units of measure?
S: First thing is that if they had the same perimeter, of course these 10 paper clips, each paper clip has longer length compared to the 15 paper clips. That's why they can have the same perimeter.

In Excerpt T18, Tan explained that the paper clips in shape A is longer than the paper clips in shape B so that they had the same perimeter. It indicated that Tan understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length.

In Set 3, Tan stated that shape A has the longer perimeter. Excerpt T19 reveals his choice of shape that has the longer perimeter and the justification that he made (Tan/L786-796, L801-810).

Excerpt T19

R: (Puts the following table in front of Tan). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 3	16 cm	13 cm

S: This one ok centimeter is an international unit already. So, it has been recognized all around the world. …So, definitely a teacher or a student can tell with 100% sure that 16 cm of shape A definitely will longer than the shape B.

.
.
.

R: Why is it?

S: Because the number 16 is bigger than 13. For this case, we see the number only because the unit is the same for these two shapes, that is centimeter.
R: Can you tell me more about centimeter?
S: …Ok cm is a standard unit of measurement, is one of the measurement that is used to measure a certain object or length that is given.

In Excerpt T19, Tan explained that he was 100% sure that shape A has the longer perimeter because centimetre is a standard unit of length measurement and 16 is larger than 13. He elaborated that we focused on the number (of unit) only in this case as the unit (of measure) is the same for both shapes A and B, namely centimeter. It indicated that he focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Tan knew that common standard unit (such as cm) is reliable for comparing perimeters.

Summary

In summary, Tan focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. He knew that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. Tan focused on the unit of measure when comparing perimeters in Set 2 with common nonstandard units. He knew that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. Tan understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length. He focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Tan knew that common standard unit (such as cm) is reliable for comparing perimeters.

Comparing Area (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Tan explained that he was unable to determine which shape has the larger area. Excerpt T20 shows the justification that he made (Tan/L828-849).

Excerpt T20

R: (Puts the following table in front of Tan). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 1	25 triangles	12 squares

S: It is hard to tell because …I don't know shape A, each side has what length is it and then shape B, each side what length is it. I might not know. …So, even though there are 25 triangles in shape A whereas in shape B there are 12 squares. I can't tell that which one has the larger area compared to the other.
R: But in this case, 25 is larger than 12.
S: Yeah but triangle I don't know how big is it. How big is the area is? And then square, I don't know how big is the area for the square. So, even if we sum it up, may be this square is big and then even though the quantity is small. The 25 and 12 is quantity. Even though 12 is small but when they combine into it to form the shape B, it might bigger than the 25 triangles, if the triangle happened to be small.

In Excerpt T20, Tan explained that he was unable to determine which shape has the larger area as he did not know the length of each side of the triangle and square. Tan elaborated that even though 25 is larger than 12 but the area of the triangle and the square were not given. He emphasized that 25 and 12 is just the quantity of triangle and square in shapes A and B respectively. Tan expressed that if the square is big, then the area of 12 squares might larger than the area of 25 triangles. It indicated that Tan focused on the unit of measure when comparing area in Set 1 with nonstandard units. He knew that nonstandard units (such as triangle and square) are not reliable for comparing areas.

In Set 2, Tan explained that he was unable to determine which shape has the larger area. Excerpt T21 depicts the justification that he made (Tan/L870-884).

Excerpt T21

R: (Puts the following table in front of Tan). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 2	10 squares	15 squares

S: In this set, both have the same type of object. There are squares. Furthermore, it is different area. Given shape A has 10 squares, shape B has 15. but this almost like the same case as the Set 1 because the 10 and the 15, these two numbers are quantity, not the value of the area. If each square happened to be have the same particular area, so of course the shape B with 15 squares will be larger area than shape A with 10 squares only. If these two happened to have different square for each shape, we can't tell sure that whether shape B is bigger, has a bigger area compared to shape A even though the quantity of 15 is larger than 10.

In Excerpt T21, Tan explained that he was unable to determine which shape has the larger area because 10 and the 15 are just the quantities of the squares, not the area of the squares. Tan elaborated that if the area of each square were the same, then certainly shape B with 15 squares has the larger area than shape A with 10 squares only. He expressed that if the area of the squares were different, then he would unable to determine which shape has the larger area even though the quantity of 15 is larger than 10. It indicated that he focused on the unit of measure when comparing areas in Set 2 with common nonstandard units. Tan knew that common nonstandard units (such as squares) are not reliable for comparing areas.

In another situation when shapes A and B had the same area, Tan explained that the squares in shape A are bigger. Excerpt T22 demonstrates his justification about their units of measurement (Tan/L885-892).

Excerpt T22

R: If shapes A and B had the same area, what can you say about their units of measure?
S: If they happened to have same area, so I can deduce that shape A, each square in it is bigger compared to shape B because the quantity of square in shape A is small, is very few compared to shape B that is 15. If I draw out a 15…(draws a diagram to illustrate the size of squares in shapes A and B, as shown in Figure N192).

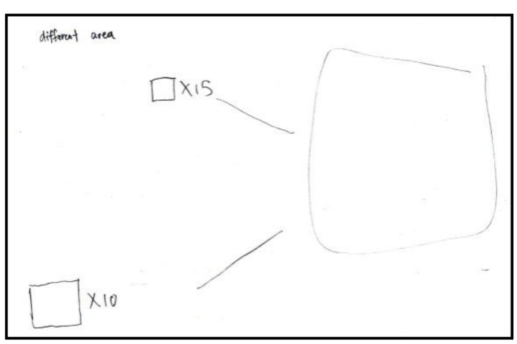


Figure N192. Tan draws a diagram to illustrate the size of squares in shapes A and B.

In Excerpt T22, Tan explained that if shapes A and B had the same area, then the squares in shape A are bigger compare to the squares in shape B as the quantity of square in shape A is smaller, namely 10, compared to 15 in shape B. He drew a diagram to illustrate the size of squares in shapes A and B, as shown in Figure N192. It indicated that Tan understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area.

In Set 3, Tan stated that shape A has the larger area. Excerpt T23 reveals his choice of shape that has the larger area and the justification that he made (Tan/L894-911).

Excerpt T23

R: (Puts the following table in front of Tan). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 3	16 cm^2	13 cm^2

S: Ok now just like Set 1, they compared triangle and square and then Set 2, they compared the squares for different shape and then this one, Set 3, they compared between the same unit.
R: What do you mean by "the same unit'?
S: The same unit is like shape A and shape B has the same unit, that is centimeter square (misreads cm^2 as centimeter square). Centimeter square is a unit, is not an object like just now. So, the unit is the standard unit like for the perimeter one. So, if same unit, now we can compare shape A and shape B with the assigned number only. So, the shape A has 16 centimeter square and then shape B has 13 centimeter square. So, definitely one can tell that 16 centimeter square is larger than 13 centimeter square. So, straight away they (refers to students) can deduce that shape A has a bigger area than shape B.

In Excerpt T23, Tan explained that shape A has the larger area because they used the same standard unit, namely cm^2, and 16 is larger than 13. Tan elaborated that since they used the same standard unit, he just have to compare their number (of unit) only, namely 16 and 13. It indicated that Tan focused on the number of unit when comparing areas in Set 3 with common standard unit. He knew that common standard unit (such as cm^2) is reliable for comparing areas.

Summary

In summary, Tan focused on the unit of measure when comparing areas in Set 1 with nonstandard units. He knew that nonstandard units (such as triangle and square) are not reliable for comparing areas. Tan focused on the unit of measure when comparing areas in Set 2 with common nonstandard units. He knew that common nonstandard units (such as squares) are not reliable for comparing areas. Tan understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area. He focused on the number of unit when comparing areas in Set 3 with common standard unit. Tan knew that common standard unit (such as cm^2) is reliable for comparing areas.

Linguistic Knowledge

Tan read 16 cm^2 and 13 cm^2 literally as '16 centimeter square' and '13 centimeter square' respectively, as shown in Excerpt T23. In another situation, Excerpt T24 exhibits how Tan wrote 16 cm^2 and 13 cm^2 in English words (Tan/L912-920).

Excerpt T24

R: (Puts a blank paper written the following measurements in front of Tan).
16 cm^2
13 cm^2
How would you write these measurements in English words?

S: (Writes the following).

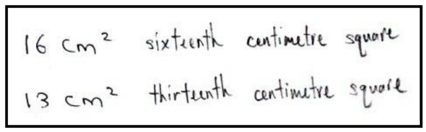


Figure N193. Tan writes 16 cm^2 and 13 cm^2 in English words.

In Excerpt T24, Tan wrote 16 cm^2 and 13 cm^2 literally as 'sixteenth centimetre square' and 'thirteenth centimetre square', as shown in Figure N193. The correct answer should be 'sixteen square centimetres' and 'thirteen square centimetres'. It indicated that he did not know about the conventions pertaining to writing and reading of Standard International (SI) area measurement units.

Converting Standard Units of Area Measurement

Procedural Knowledge

Tan realized that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2. Excerpt T25 shows the algorithms that Tan used when he was converting 3 cm^2 to mm^2 (Tan/L936-981).

Excerpt T25

R: (Puts a card written the following scenario in front of Tan). Some Form One teachers noticed that several of their students seemed to multiply by 10, 100, and 1000, respectively when they were converting units of area from cm^2 to mm^2, m^2 to cm^2, and km^2 to m^2:

$3 \text{ cm}^2 = 3 \times 10 \text{ mm}^2 = 30 \text{ mm}^2$
$4.7 \text{ m}^2 = 4.7 \times 100 \text{ cm}^2 = 470 \text{ cm}^2$
$1.25 \text{ km}^2 = 1.25 \times 1000 \text{ m}^2 = 1250 \text{ m}^2$

What would you do if you were teaching Form One and you noticed that several of your students were doing this?

S: Ok 3 cm^2. First, I would like to ask students whether they have learnt the *FSTB* (*faktor sepunya terbesar*) [HCF, highest common factor] and *GSTK* (*gandaan sepunya terkecil*) [LCM, least common multiple/lowest common multiple] during Form One. So, they will learn what factor for 3. Of course, they will tell is 1 times 3. This is the very basic. So, 1 times 3, I write like this. (Converts 3 cm^2 to mm^2, as shown in Figure N194).

R: Could you explain your solution?

S: So, this cm two (misreads cm^2 as cm two), yeah I will ask them: Can you break into two? Of course, they can if they learn the power, they will definitely say yes. I break it into two. So, after I break into two, then I convert each of them, like I did just now. 1 cm is equals to 10 mm. Then we times with 3, 30 mm. So, either they use calculator or times by mentally, they can get 300 mm^2. So, from this, we can point the mistake between what they did and what I show for them.

$$
\begin{aligned}
3\,cm^2 &= 1\,cm \times 3\,cm \\
&= 10\,mm \times 30\,mm \\
&= 300\,mm^2
\end{aligned}
$$

Figure N194. Tan converts 3 cm^2 to mm^2.

In Excerpt T25, Tan has successfully converted 3 cm^2 to mm^2. He stated that 1 and 3 are the factors of 3 and viewed 3 cm^2 as the product of 1 cm times 3 cm. Tan knew the relationship between the standard units of length measurement that 1 cm = 10 mm. He also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Tan times 10 twice when he converted 1 cm to mm and 3 cm to mm separately, as shown in Figure N194.

Tan found that the students made a mistake when they were converting unit of area from 4.7 m^2 to cm^2. Excerpt T26 depicts the algorithms that Tan used when he was converting 4.7 m^2 to cm^2 (Tan/L982-998).

Excerpt T26

R: What about the second one?

S: (Coughs). The second one can goes to 4.7 m^2. Ok I give them a tip this one. For the easiest one, we always start with 1 times 4.7. (Converts 4.7 m^2 to cm^2, as shown in Figure N195).

R: Could you explain your solution?

S: This is the easiest factor they can use. So, assign both same, meter (refers to units for 1 and 4.7, respectively) and then try to convert if they want to convert to cm. 1 m equal to 100 cm and then this one, 4.7. I will try to tell them to convert separately first. 1 m equal to 100 cm. So, the 4.7 m, how do they convert? They convert by 4.7 times with 100 cm. So, after that, it is 470 cm. So, after they convert, convert these two (refers to 1 and 4.7, respectively), substitute back into this, into the previous one, 4.7 m^2 and then 100 cm times 470 cm. Then only they will try to times these two to get the final form, 47 000 cm^2.

$$
\begin{aligned}
4.7\,m^2 &= 1\,m \times 4.7\,m \\
&\cancel{= 100\,cm \times 4.7} \\
1\,m &= 100\,cm \\
4.7\,m &= 4.7 \times 100\,cm \\
&= 470\,cm \\
4.7\,m^2 &= 100\,cm \times 470\,cm \\
&= 47000\,cm^2
\end{aligned}
$$

Figure N195. Tan converts 4.7 m^2 to cm^2.

In Excerpt T26, Tan has successfully converted 4.7 m^2 to cm^2. He stated that 1 and 4.7 are the easiest factors of 4.7 and viewed 4.7 m^2 as the product of 1 m times 4.7 m. Tan knew the relationship between the standard units of length measurement that 1 m = 100 cm. He also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Tan times 100 twice when he converted 1 m to cm and 4.7 m to cm respectively, as shown in Figure N195.

Tan also found that the students made a mistake when they were converting unit of area from 1.25 km^2 to m^2. Excerpt T27 demonstrates the algorithms that Tan used when he was converting 1.25 km^2 to m^2 (Tan/L999-1015).

Excerpt T27

R: What about the third one?
S: Em for 1.25 km^2. (Converts 1.25 km^2 to m^2, as shown in Figure N196).
R: Could you explain your solution?
S: This one is similar to the second case. Just assign 1 at the front and then times with 1.25. Then assign each one, km (refers to units for 1 and 1.25, respectively).1.25 km^2 = 1 km x 1.25 km. Do these two in separately to convert the unit first. 1 km = 1000 m and then 1.25 km = 1250 m and then substitute back, we get 1.25 km^2 = 1000 m x 1250 m = 125 000 m^2. The final answer is 125 000 m^2.

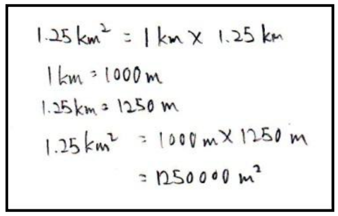


Figure N196. Tan converts 1.25 km^2 to m^2.

In Excerpt T27, Tan has successfully converted 1.25 km^2 to m^2. He stated that this one was similar to the second case and viewed 1.25 km^2 as the product of 1 km times 1.25 km. Tan knew the relationship between the standard units of length measurement that 1 km = 1000 m. He also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. Thus, Tan times 1000 twice when he converted 1 km to m and 1.25 km to m respectively, as shown in Figure N196.

Summary

In summary, Tan realized that the students made mistakes when they were converting 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2 respectively. He knew the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. Tan also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring. He viewed 3 cm^2 as the product of 1 cm times 3 cm, 4.7 m^2 as the product of 1 m times 4.7 m, and 1.25 km^2 as the product of 1 km times 1.25 km. Thus, Tan times 10, 100, and 1000 twice respectively when he converted 1 cm to mm and 3 cm to mm, 1 m to cm and 4.7 m to cm, and 1 km to m and 1.25 km to m, as shown in Figures N194, N195, and N196.

Conceptual Knowledge

Tan knew the relationships between the standard units of length measurement such as 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. These can be seen in Figures N194, N195, and N196. He also knew the relationship between area units and linear units of measurement that area units are derived from linear units based on squaring, as shown in Figures N194, N195, and N196.

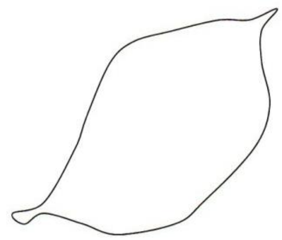
Relationship between Perimeter and Area
(Same Perimeter, Same Area?)

Conceptual Knowledge

Tan did not know that there is no direct relationship between perimeter and area. He did not know that two shapes with the same perimeter can have different areas. Thus, Tan thought that the student's method of calculating the area of the leaf was correct. Excerpt T28 shows Tan's responses to the Form One student (Tan/L1041-1087).

Excerpt T28

R: (Puts a card written the following scenario in front of Tan). This is a picture of a leaf. A Form One student said that he had found a way to calculate the area of the leaf. The student placed a thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle.



How would you respond to this student?

S: Firstly, I will give an applause to him because he figured out this way of calculating the area. Actually, there area many ways to calculate this area. After they measured using the thread, actually, he not necessary had to form the rectangle because the rectangle is easier to form. Actually, he can form many thing, many shapes like triangle. It depends on the comfortability of the student to calculate the area. Then em if I form a square also can, provided that all total the length happened to have the same side. So, It depends on the student. One might think that eh they will try to use eh even more circular shape because it is even more easier. You just apply the formula of circumference to get the area. Because you can find, figure out the radius and then from the radius, you can find the area. Furthermore, the rectangle you have form very, very vertical and then it's quite take a long time. Whereas this thread, you can form circle straight away, it's easier. Then measure the circumference, get the radius, you can get the area already.

R: Would the student's method works?

S: The method for me, it's correct.

R: Why is it?

S: Because correct but not accurate. Because the thread that you used to measure might not touch exactly the edge line of the leaf. So, he might over estimate the area of the leaf or under estimate the area of the leaf. So, this is the best. This is one of the best ways to measure area of the leaf.

In Excerpt T28, Tan thought that the student's method of calculating the area of the leaf was correct. He applauded this student for figuring out the method of calculating the area of the leaf. The student placed a piece of thread around the boundary of the leaf and then rearranged the thread to form a rectangle, and got the area of the leaf as the area of a rectangle. Tan explained that the student need not necessarily has to form a rectangle. He suggested that the student can also form other shapes such as triangle, squares, or circle depending on the comfort of the student to calculate the area.

Tan explained that even though the method of calculating the area of the leaf was correct but it may not be accurate as the thread that the student used might not touch exactly the outline of the leaf. Thus, the student might over or under estimate the area of the leaf. Nevertheless, Tan believed that this was one of the "best" ways to measure the area of the leaf.

Summary

In summary, Tan did not know that there is no direct relationship between perimeter and area. He did not know that two shapes with the same perimeter can have different areas. Thus, Tan thought that the student's method of calculating the area of the leaf was correct.

Ethical Knowledge

In Task 5.1, Tan thought that the student's method of calculating the area of the leaf was correct, as shown in Excerpt T28. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter has the same area.

Tan did not attempt to examine the possible pattern of the relationship between perimeter and area. He did not attempt to formulate generalization pertaining to the relationship between perimeter and area. Tan never tests the student's generalization that two shapes with the same perimeter have the same area.

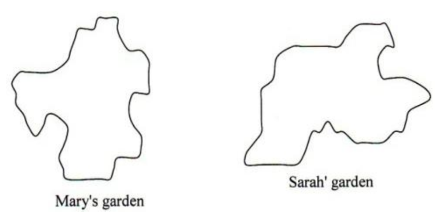
Relationship between Perimeter and Area
(Longer Perimeter, Larger Area?)

Conceptual Knowledge

Tan did not know that there is no direct relationship between perimeter and area. He did not know that the garden with the longer perimeter could have a smaller area. Thus, Tan thought that Mary's claim was correct. Excerpt T29 shows Tan's responses to the claim made by Mary that the garden with the longer perimeter has the larger area (Tan/L1200-1211).

Excerpt T29

R: (Puts a card written the following scenario in front of Tan). Mary and Sarah are discussing whose garden has the larger area to plant flowers. Mary claims that all they have to do is walk around the two gardens to get the perimeter and the one with the longer perimeter has the larger area. How would you respond to these students?



S: This situation is similar like what the student (refers to the student in the previous task, Task 5.1) did just now. Ok just now the student used the thread to measure the total length of the leaf. Actually, this is a perimeter, a part of perimeter. So, the longer perimeter will have the larger area. A larger perimeter might have larger area. So, this is one of the ways for Mary to do to calculate the area because they just used the total length. They used like thread just now, they get the total length. Em actually this is just extra.information. What they more emphasize to do is they shape into the shape that is easier to calculate.

R: Would Mary's method correct?

S: I can think so, yes. Because it is almost same like the thread method (in the previous task, Task 5.1).

In Excerpt T29, Tan stated that this situation is similar to the previous situation in Task 5.1 where the student placed a piece of thread around the boundary of the leaf and then rearranged the thread to form a rectangle, and got the area of the leaf as the area of a rectangle. Tan thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Tan stated that this is one of the ways for Mary to calculate the area (of the gardens) as it was similar to the thread method in the previous task, Task 5.1, where the student rearranged the thread to form a rectangle that was easier to calculate its area. He reiterated that Mary's method of determining whose garden has the larger area was correct as it was similar to the thread method in the previous task, Task 5.1.

Summary

In summary, Tan did not know that there is no direct relationship between perimeter and area. He did not know that the garden with the longer perimeter could have a smaller area. Thus, Tan thought that Mary's claim was correct.

Ethical Knowledge

In Task 5.2, Tan thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Tan did not attempt to examine the possible pattern of the relationship between perimeter and area. He did not attempt to formulate generalization pertaining to the relationship between perimeter and area. Tan never tests Mary's generalization that that the garden with the longer perimeter has the larger area.

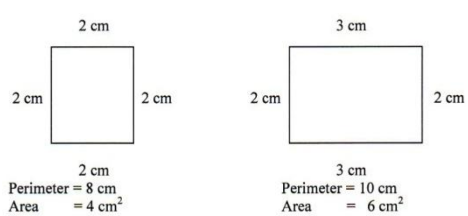
Relationship between Perimeter and Area
(Perimeter Increases, Area Increases?)

Conceptual Knowledge

Tan knew that there is no direct relationship between perimeter and area. He knew that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Tan knew that the student's "theory" was not correct. This is shown in Excerpt T30 (Tan/L1273-1323).

Excerpt T30

R: (Puts a card written the following scenario in front of Tan). Suppose that one of your Form One students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:



How would you respond to this student?

S: …Although the concept is correct, the perimeter increase and then the area also increase, but I would like to give an example for this one This one (points to the given rectangle as shown above) the perimeter is 10 if I'm not mistaken. 3 plus 3 and then plus 2 plus 2, 10. So, this one (points to the given rectangle as shown above) the area is 2 times 3 is 6. This one (points to the given square as shown above) 2 times 2 is 4. This one (points to the given rectangle as shown above) is 6. But I would like to show this one, this triangle (draws an isosceles triangle and then calculates its area, as shown in Figure N197). Ok given here perimeter is also 10 and then I make this 10 also, 3, 3, 4 and then this one is a isosceles triangle. Let's say I draw a line, 90^0 and then using Pythagoras theorem, this one is 2 and then this one is square root 5 (reads $\sqrt{5}$ as square root 5). Let's say I want to calculate this area is this one (point to the base of the above triangle) times this one (point to the height of the above triangle), 4 times $\sqrt{5}$. No, no, times half (writes the following).

$\frac{1}{2}$ x 4 x $\sqrt{5}$ = 4.472 cm^2.

Then I get 4.472. If this one is assigned cm, cm^2. So, I can show to this student that you can see although these two has same perimeter, but the area is different. This one (points to the given rectangle as shown above) is larger. This one (points to the isosceles triangle as shown in Figure N197) is smaller even though they have same perimeter. So, it depends on the situation. Now I can see that bigger or larger the value of perimeter doesn't guarantee that the area is also increase. This is what I can tell the student. You point this situation another new one to this student to correct her concept.

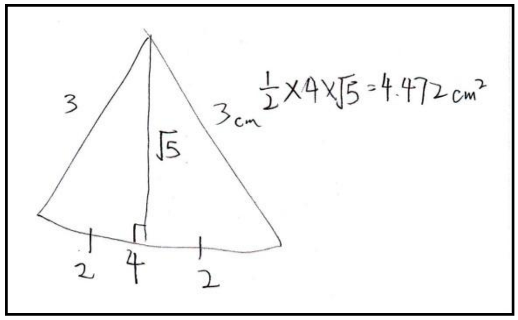


Figure N197. Tan draws an isosceles triangle and then calculates its area.

In Excerpt T30, Tan initially thought that the student's "theory" was correct that as the perimeter of a closed figure increases, the area also increases. He went through the example showed by the student that as the perimeter increases from 8 cm to 10 cm, its area also increases from 4 cm^2 to 6 cm^2. Tan drew an isosceles triangle with the perimeter of 10 cm then calculated its area as 4.472 cm^2, as shown in Figure N197. He found that although the rectangle and the triangle have the same perimeter (10 cm), their areas were different, namely 6 cm^2 and 4.472 m^2 respectively. Tan explained that the triangle has the smaller area even though they had the same perimeter. He realized that increases in perimeter did not guarantee that the area also increases. Subsequently, Tan knew that the student's "theory" was not correct.

Tan did not know that the student's claim about the relationship between perimeter and area is not a theory. The claim is a conjecture. He also did not know that an example is not a proof and a theory cannot be proved by an example.

Summary

In summary, Tan knew that there is no direct relationship between perimeter and area. He knew that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Tan knew that the student's "theory" was not correct.

Ethical Knowledge

In Task 5.3, the student formulated a generalization that as the perimeter of a closed figure increases, the area also increases. Tan generated an example to test the student's generalization, as shown in Excerpt T30. The example generated by him showed that although the rectangle and the triangle have the same perimeter (10 cm), their areas were different, namely 6 cm^2 and

4.472 m^2 respectively. He explained that the triangle has the smaller area even though they had the same perimeter. Tan realized that increases in perimeter did not guarantee that the area also increases. He knew that the student's "theory" was not correct.

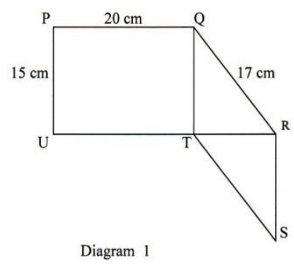
Calculating Perimeter and Area
(Rectangle and Parallelogram/Triangle)

Procedural knowledge

After read through Task 6.1, Tan labelled the missing sides of Diagram 1 that required for calculating the perimeter and area of the diagram, as shown in Figure T11 in Excerpt T31 (Tan/L1333-1362, L1372-1378).

Excerpt T31

R: (Puts a card written the following problem in front of Tan). Suppose that one of your Form One students asks you for help with the following problem:



In Diagram 1, PQTU is a rectangle and QRST is a parallelogram. UTR is a straight line.
Calculate
(m) the perimeter of the diagram,
(n) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 1, as shown in Figure N198). …So, they need to calculate the length of this one, TR. So, TR is, using the Pythagoras' theorem (calculates the length of TR, as shown in Figure N199. TR is 17^2 - 15^2, TR is 8.

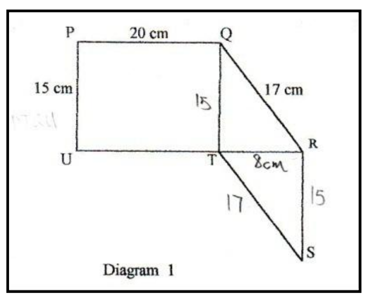


Figure N198. Tan labels the missing sides of Diagram 1.

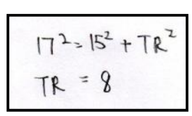


Figure N199. Tan calculates the length of TR.

In Excerpt T31, Tan labelled QT, RS, and ST as 15, 15, and 17 respectively on Diagram 1, as shown in Figure N198. Tan realized that he needed to find the length of TR in order to calculate the area of Diagram 1. Tan has successfully calculated the length of TR using Pythagoras' theorem, as shown in Figure N199. Excerpt T32 depicts how Tan has incorrectly calculated the perimeter of Diagram 1 (Tan/L1365-1370, L1378-1399).

Excerpt T32

S: …So, in this case I can tell the student, why didn't you cut this triangle at the out there (points to the triangle TRS of Diagram 1). You cut this (points to the triangle TRS of Diagram 1) and then you put it, paste it on the upper one (points to the triangle TQR of Diagram 1) so that it can form a long rectangle. …So, after they calculated the TR is 8 cm and then they have already cut this part (points to the triangle TRS of Diagram 1) and paste it. They will find that this exactly form the rectangle. So, once they are form the rectangle, this 8 plus 20 is 28, So, you form like this (draws a "long" rectangle, as shown in Figure N200). So, like this, 20 cm (points to PQ), 15 cm (points to PU). So, this one (points to TR) is 8 cm. Given that is a rectangular for PQTU and this one it will form also a rectangular TQSR and then this one you can total up for perimeter. (Calculates the perimeter of Diagram 1, as shown in Figure N201). You total, plus 8 and then plus 15. This one same, 20, 8, 15. So, plus all, plus 8, plus 15. So, total up is 86 cm or they can use a simpler method, 20 you times 2, 8 times 2 and 15 times 2, all add together you can still get the 86.

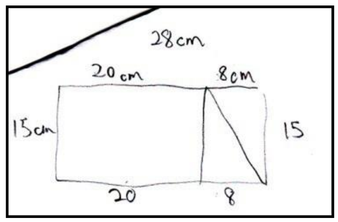


Figure N200. Tan draws a "long" rectangle.

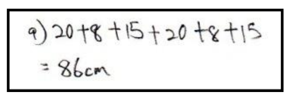


Figure N201. Tan calculates the perimeter of Diagram 1.

In Excerpt T32, Tan mentally cut the triangle TRS of Diagram 1 and pasted it next to the triangle TQR of Diagram 1 so that it formed a rectangle ("TQSR") with the dimension of 15 cm by 8 cm, as shown in Figure N200. He used the list all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N201. He listed all the length of sides that surrounded the "long" rectangle and then summed them up to get the perimeter of the diagram as 86 cm (the correct answer should be 104 cm). Tan did not know that the "cut and paste" transformation does not conserve the perimeter of a diagram. Thus, he incorrectly calculated the perimeter of the diagram as 86 cm based on the length of sides that surrounded the "long" rectangle formed (20 + 8 + 15 + 20 + 8 + 15) and not based on the length of sides that surrounded Diagram 1 (20 + 17 + 15 + 20 + 17 + 15 = 104).

Excerpt T33 demonstrates how Tan has successfully calculated the area of Diagram 1 (Tan/L1400-1410).

Excerpt T33

S: I continue, to find the area is the same. After you cut it and paste to a fine rectangle, this 20 cm plus 8 cm. You plus it up and then you get 28 cm. So, this is the horizontal total line and then plus the vertical line 15. So, (b) (Calculates the area of Diagram 1, as shown in Figure N202). The area is 15 times 28 and then 420 cm^2 (misreads cm^2 as centimeter square). Make sure, always make sure that tell student to write the unit at the final answer.

R: What did you get for the area?

S: 420.

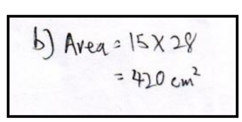


Figure N202. Tan calculates the area of Diagram 1.

In Excerpt T33, Tan used the "cut and paste" transformation to transform Diagram 1 into a "long" rectangle, as shown in Figure N200. He calculated the area of Diagram 1 as the area of the "long" rectangle using the area formula of a rectangle where its length and width is 28 cm and 15 cm respectively, as shown in Figure N202. Tan got the area of the diagram as 420 cm^2.

Summary

In summary, Tan has incorrectly calculated the perimeter of Diagram 1 using the list all-and-sum algorithm. He has successfully calculated the area of Diagram 1 using the "cut and paste" transformation.

Linguistic knowledge

Tan used the correct standard units of measurement for perimeter (cm) and area (cm^2) when he wrote the answers for these measurements, as shown in Figures N201, N202, and N203.

Strategic Knowledge

When probed to check the answer for the perimeter, Tan suggested that he would use the recalculating strategy to verify the answer. Excerpt T34 is illustrative (Tan/L1438-1443).

Excerpt T34

R: If you were asked to check the answer for the perimeter of the diagram, how would you check?

S: Check. So, after I defined that definition, well I tried to apply the definition by find all the length surround the object. Then excluded the line inside the object. Then I calculate the total length and that is what I called a perimeter already.

In Excerpt T34, Tan suggested that he would check the answer for perimeter by recalculating strategy that using the same method and calculate again. Tan defined perimeter as the length that surrounded an object. He explained that inner lengths (QT and RT) would not be included in the calculation of the perimeter.

When probed to check the answer for the area, Tan used an alternative procedure (alternative method), namely partition-and-sum algorithm to generate an answer which could be used to verify his original answer. Excerpt T35 is illustrative (Tan/L1411-1426).

Excerpt T35

R: How would you check your answer for the area?

S: Ok if we go back to the theory, rectangle is 15 (uses alternative method to calculate the area of Diagram 1, as shown in Figure N203). If we use the usual way is 15 times with 20 to get the PQTU area and then to find the area of parallelogram is the height times the vertical side. So, I will tell student to times the vertical. It's still the same. We have to find the Pythagoras' theorem for, in order to get TR. Then once you get TR, TR is 8 cm, so the area of parallelogram QRST is straight away you can 8 times 15. So, we will get 120 cm square (misreads cm^2 as centimeter square). Then only we plus these two areas and then we can get the total area of the picture (refers to Diagram 1). So, it's 300 plus 120, 420 cm square (misreads cm^2 as centimeter square). So, both methods is the same, can lead to the same answer. But the first method I introduced is a lot of easier to visualize.

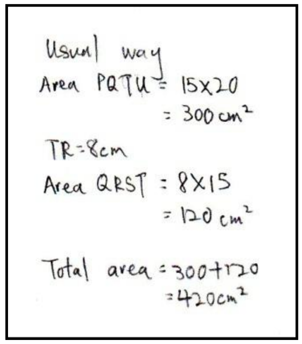


Figure N203. Tan uses alternative method to calculate the area of Diagram 1.

In Excerpt T35, Tan checked the answer for area using the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N203. He partitioned Diagram 1 into rectangle PQTU and parallelogram QRST. Tan calculated the area of the rectangle using the area formula of a rectangle as 300 cm^2. He calculated the area of the parallelogram using the area formula of a parallelogram as 120 cm^2. Tan then summed them up to get the area of the diagram as 420 cm^2. Tan explained that both methods gave the same answer, namely 420 cm^2. He believed that the first method that he "introduced" was easier to visualize, as shown in Figure N202.

Ethical Knowledge

Tan has incorrectly calculated the perimeter of Diagram 1. Nevertheless, he has successfully calculated the area of Diagram 1. Tan did not check the correctness of the answers for perimeter as well as area. When probed to check answers, then only Tan suggested the strategies that he would use to check the answers for perimeter and area. Tan wrote the measurement units (without probed) for the answers of the perimeter and area that he has calculated, as shown in Figures N201, N202, and N203.

Calculating Perimeter and Area
(Square and Trapezium/Triangle)

Procedural Knowledge

After read through Task 6.2, Tan labelled the missing sides of Diagram 2 that required for calculating the perimeter and area of the diagram, as shown in Figure N204 in Excerpt T36 (Tan/L1489-1534).

Excerpt T36

R: (Puts a card written the following problem in front of Tan). Suppose that one of your Form One students asks you for help with the following problem:

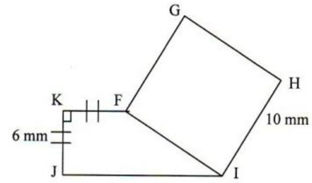


Diagram 2

In Diagram 2, FGHI is a square and FIJK is a trapezium.
Calculate
(m) the perimeter of the diagram,
(n) the area of the diagram.

How would you solve this problem?

S: Firstly, the picture (refers to Diagram 2) is two combination of object, square and trapezium. So, square they can visualize how to calculate the value but trapezium, the JI they didn't give, assigned a value of length on it. So, the only way is to cut the trapezium into fragment that can be used to determine length. From here (points to HI equal to 10 mm on Diagram 2), FI we know that 10 mm. So, given that this KF and KJ are same length, 6 mm both of them. So, I cut between the point F and the JI line. I cut the JI line into certain length and then I can get that this one, this point I assigned, let's say, A (labels A on Diagram 2, as shown in Figure N204). So, JA is the same as KF and KJ. So, I put 6. Since I put a line here (points to FA), it becomes a square already for KFJA. So, each of them has 6 mm. So, I can get FA, 6. FI is 10 and then to calculate IA, I need to use the Pythagoras' theorem. So, I can get AI, 8.

R: How did you get 8?

S: By using Pythagoras' theorem, is (calculates the length of AI using Pythagoras' theorem, as shown in Figure N205). 10^2, 6^2 plus AI^2. So, AI is 8.

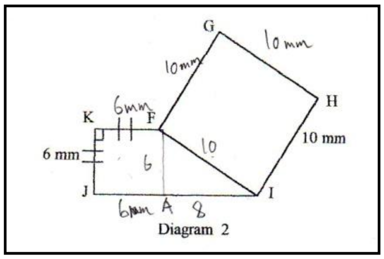


Figure N204. Tan labels the missing sides of Diagram 2.

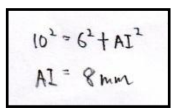


Figure N205. Tan calculates the length of AI using Pythagoras' theorem.

In Excerpt T36, Tan labelled FG, GH, FI, KF, FA, and JA as 10 mm, 10 mm, 10, 6 mm, 6, and 6 mm respectively on Diagram 2, as shown in Figure N204. Tan realized that he needed to find the length of AI, as shown in Figure N204. Tan partitioned trapezium FIJK into a square and a triangle, as shown in Figure N204. He has successfully calculated the length of AI as 8 (mm) using Pythagoras' theorem, as shown in Figure N205.

Excerpt T37 depicts how Tan has successfully calculated the perimeter of Diagram 2 (Tan/L1537-1546).

Excerpt T37

S: …So, the best way to calculate the perimeter is to label each side, regardless you know or don't know because one thing is you afraid that you might left out one length, particular length. This is what student commonly do in their mistake. So, list all the lengths. I will tell the student to make sure that you don't plus the internal length, plus what is outside the surface of the object of the length. So, (calculates the perimeter of Diagram 2, as shown in Figure N206).

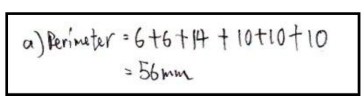


Figure N206. Tan calculates the perimeter of Diagram 2.

In Excerpt T37, Tan stated that the best way to calculate the perimeter is to label each side so that no particular length is left out. He expressed that this is the common mistake that student makes. Tan explained that the perimeter only involved the "outside" length, not the "internal" length. He used the list all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N206. Tan listed all the length of sides that surrounded the diagram and then mentally summed them up to get the perimeter of the diagram as 56 mm.

Excerpt T38 demonstrates how Tan has successfully calculated the area of Diagram 2 (Tan/L1554-1566).

Excerpt T38

S: Then for part (b), area is just like I said just now. You can divide into three parts because when you draw a line FA, the object (refers to Diagram 2) has become a three parts to be solved. One, two squares with different sides, side length and then one is the right-angled triangle. So, you will calculate separately first. (Calculates the area of Diagram 2, as shown in Figure N207). KFAJ is 6 times 6, 36 and then area of FAI is half times 6 times 8, 24. And then the area of FGHI 10 times 10, 100. After that, tell the student that total area, we sum all these three areas, 36 plus 24 plus 100, is 160. It is wise to tell the student, you do it separately first and then you join together. Ok this is one of the methods, Method One.

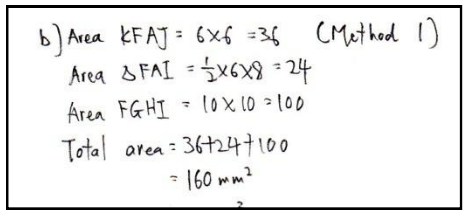


Figure N207. Tan calculates the area of Diagram 2.

In Excerpt T38, Tan used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N207. He partitioned Diagram 2 into small square KFAJ, triangle FAI, and large square FGHI, as shown in Figure N204. He calculated the area of the small square, triangle, and large square separately using the area formulae of a square, triangle, and square respectively and then summed them up to get the area of the diagram as 160 mm^2.

Summary

In summary, Tan has successfully calculated the perimeter of Diagram 2 using the list all-and-sum algorithm. He has also correctly calculated the area of Diagram 2 using the partition-and-sum algorithm.

Linguistic Knowledge

Tan used the correct units of measurement for perimeter (cm) and area (cm^2) when he wrote the answers of these measurements, as shown in Figures N206, N207, and N208.

Strategic Knowledge

When probed to check the answer for the perimeter, Tan suggested that he would use alternative method to verify the answer. Excerpt T39 is illustrative (Tan/L1577-1582).

Excerpt T39

R: How would you check your answer for the perimeter?

S: Ok perimeter just now I calculated using list all and then you sum it up. The other way, if you draw it at a full length, that means 10 mm, you exactly go and draw it 10 mm. Then after you draw it, you use the thread to measure it and then count the total length with the ruler. It's still the same. You can check it.

In Excerpt T39, Tan suggested that he would use the exact measurement to draw Diagram 2 and then use a piece of thread and ruler to measure its perimeter. He was confident that the answer is the same.

Tan used alternative method to verify the answer for the area. Excerpt T40 is illustrative (Tan/L1566-1576).

Excerpt T40

S: …Method Two, you want more faster way is, this one you no need to split, straight away you can tell that it's a trapezium. So, you used the trapezium for area KFIJ (uses alternative method to calculate the area of Diagram 2, as shown in Figure N208). It is half times 6 times 14, half times two parallel lines, 42 mm^2. Area of FGHI already calculated, 100. Total area is 100 plus 42, 142. (Realizes a mistake and recalculate the area of KFIJ as follow). Half times 6 times 20, 60 mm^2. So, total area is 100 plus 60, 160 mm^2. So, this is method two.

In Excerpt T40, Tan used the repartition-and-sum strategy to check the answer for the area of Diagram 2, as shown in Figure N208. He repartitioned Diagram 2 into trapezium KFIJ and square FGHI, as shown in Figure N208. Tan calculated the area of the trapezium and square separately using the area formulae of a trapezium and square respectively and then summed them up to get the area of the diagram as 160 mm^2.

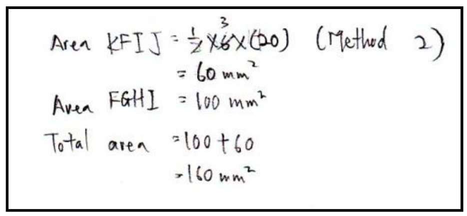


Figure N208. Tan uses alternative method to calculate the area of Diagram 2.

Ethical Knowledge

Tan has successfully calculated the perimeter and area of Diagram 2. Nevertheless, he did not check the correctness of the answer for perimeter. When probed to check answer, then only Tan suggested the strategy that he would use to check the answer for perimeter. Tan checked the correctness of the answer for area without being probed. Tan wrote the measurement units (without probed) for the answers of perimeter and area, as shown in Figure N206, N207, and N208.

Fencing Problem

Strategic Knowledge

Tan used the differentiation method to solve the fencing problem. Excerpt T41 is illustrative (Tan/L1619-1631, L1775-1780, L1789-1810).

Excerpt T41

R: (Puts a card written the following problem in front of Tan). Suppose that one of your students asks you for help with the following problem:

A gardener has 84 m of fencing to enclose a garden along three sides, with the fourth side of the garden being formed by a wall. (Assume that the wall is perfectly straight). What are the dimensions of a rectangular garden that will yield the largest area being enclosed?

How would you solve this problem?

S: (Draws a diagram to represents the fencing of the rectangular garden, as shown in Figure N209). …(Uses the differentiation method to solve the fencing problem, as shown in Figure N210).

.
.
.

R: Could you explain your solution?

S: Ok I explain from beginning. Given that the gardener has 84 m of fencing. So, to get a rectangle, given that the fence to a wall, I come out with a formula, $84 = 2x + y$. So, this is the perimeter that the fence will take, the general formula. So, the area is since the two sides of it must be the same, so I x times y, I can get the area. So, there are two equations. But that's a problem, got two variables. I need to eliminate one of the variables in order to perform differentiation. So, what I do is I substitute y to eliminate it and then left out x only. So, with area we get function of x only, I can perform the differentiation. So, what I get after differentiate it is $\frac{dA}{dx}$ = 84 - 4x. To find the critical point, the $\frac{dA}{dx}$ must be assigned to zero. After I assigned the zero, I get x is 21. After that I substitute this as 21 to the perimeter equation. So, I get the final y is 42. But before I find the largest area, I need to prove that this differentiation is maximum. So, I differentiate twice, I get minus 4 (refers to – 4). So, it's smaller than zero. So, it can be deduced that it is a maximum. Why is it a maximum? Because when you differentiate twice and you get a negative value, which is smaller than zero, then straight away the equation can be said that it's a minimum equation. It's a maximum equation, sorry. So, after confirming that it is a maximum, straight away I can find the maximum area by times the two variables, 42 times 21 and I get the final answer 882.

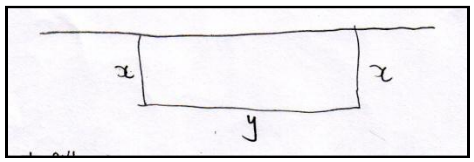


Figure N209. Tan draws a diagram to represents the fencing of the rectangular garden.

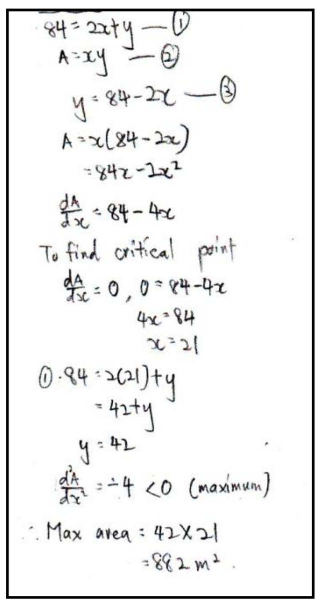


Figure N210. Tan uses the differentiation method to solve the fencing problem.

In Excerpt T41, Tan drew a diagram to represents the fencing of the rectangular garden, as shown in Figure N209. He used the differentiation method to solve the fencing problem, as shown in Figure N210. Tan wrote equation ① to represent the perimeter of the fencing. He wrote equation ② to represent the area of the rectangular garden. Tan explained that he needed to eliminate one of the variables, namely y, in order to find the derivative ($\frac{dA}{dx}$). Thus, Tan rewrote the equation ① as $y = 84 - 2x$ and labelled it as equation ③. He substituted $y = 84 - 2x$ into equation ② and simplified it as $A = 84x - 2x^2$. After differentiated with respect to x, Tan got the derivative $\frac{dA}{dx}$ = 84 – 4x. At the stationary point, $\frac{dA}{dx}$ = 0 and he got x = 21. Tan substituted the value of x into equation ① and got y = 42. Tan elaborated that he needed to find the value of $\frac{d^2A}{dx^2}$ at the stationary point. If $\frac{d^2A}{dx^2} < 0$, then the point is at a maximum. Tan found that $\frac{d^2A}{dx^2} = -4 < 0$ and thus (21, 42) is a maximum point. Tan concluded that 882 m^2 was the largest area being enclosed and 42 (m) by 21 (m) was the dimension of the rectangular garden that will yield the largest area being enclosed.

Summary

In summary, Tan has successfully solved the fencing problem using the differentiation method. Tan checked the answer of the fencing problem, without being probed, by calculating the value of $\frac{d^2A}{dx^2}$ at the stationary point.

Ethical Knowledge

Tan checked the answer of the fencing problem, without being probed, by calculating the value of $\frac{d^2A}{dx^2}$ at the stationary point. This can be seen in Excerpt T41. Tan wrote the area measurement unit, namely 882 m^2, as shown in Figure N210.

Developing Area Formulae

Procedural Knowledge

Tan could recall the formula for the area of a rectangle. Nevertheless, he was unable to develop it. Tan attempted to develop the formula for the area of a rectangle but unsuccessful. Excerpt T42 is illustrative (Tan/L1835-1863).

Excerpt T42

R: (Puts a card written the following scenario in front of Tan). Suppose that a Form One student comes to you and says that he does not know how to develop (derive) the formula for calculating the area of the following figures:
(a) Rectangle,
(b) Parallelogram,
(c) Triangle, and
(d) Trapezium.
How would you show him the way to develop (derive) the formula for calculating the area of these figures? Let's start with rectangle.

S: Derive the formula…(silent for a while) ok in our school time, the only thing that teacher teach us is memorizing. If I tell him to memorizing, it is one of the ways to memorize the formula. Have to think quite a hard way to derive this formula because all these formula is a very basic formula already. If you really want to derive for example the rectangle (draws a rectangle with the dimension of 3 cm by 2 cm, as shown in Figure N211). Let's say this is 2 cm, this is 3 cm. I can cut it into three parts where each part is 1 cm.

R: Could you tell me more about it?

S: Ok the way just now I don't think it is quite work. But the only way that I can think is I can tell him to remember, memorize the formula of rectangle and then let them understand what is area is all about. That is the surface, the total surface of a particular object. So, the rectangle is the horizontal side (refers to the length of the rectangle) times the vertical side (refers to the width of the rectangle).

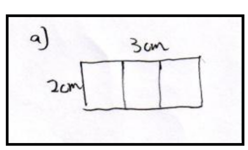


Figure N211. Tan draws with the dimension of 3 cm by 2 cm.

In Excerpt T42, Tan stated that his teacher only taught him and his classmates to memorize the formulae. Tan expressed that he had to think hard to develop the formulae as these formulae are very basic. Tan attempted to develop the formula for the area of a rectangle but unsuccessful. He drew a rectangle with the dimension of 3 cm by 2 cm, as shown in Figure N211. Tan explained that he could cut the rectangle into three parts where the width of each part was 1 cm. Tan expressed that he did not think that his method to develop the formula for the area of a rectangle work. Thus, Tan stated that the only way he could think of is to ask the student to memorize the formula and help him to understand the concept of area. Tan defined the area of an object as the total surface of the object. He stated the formula for the area of a rectangle as 'the horizontal side (refers to the length of the rectangle) times the vertical side (refers to the width of the rectangle).

Tan could recall the formula for the area of a parallelogram. He also knew how to develop the formula for the area of a parallelogram. Excerpt T43 is illustrative (Tan/L1864-1885).

Excerpt T43

R: How would you show him the way to develop (derive) the formula for calculating the area of a parallelogram?
S: So, from the area of rectangle, I can derive parallelogram. Parallelogram is same like this (draws a parallelogram and then develops its area formula, as shown in Figure N212). Let's say this is the general picture about parallelogram. Given that you know the area of rectangle is vertical times the horizontal one, So, parallelogram I can tell the student to cut one side here (points to the triangle ADE, as shown in Figure N212). So, it will become a rectangle, back to the rectangle initially to the (a) (refers to part (a) of this Task, rectangle). So, I can tell the student that you measure the length here (points to AE) and then measure some more length DC. You times the length of AE and DC, equal to area of parallelogram. Because you cut it and you paste to the side that I told just now, it will resemblying (sic) a rectangle.

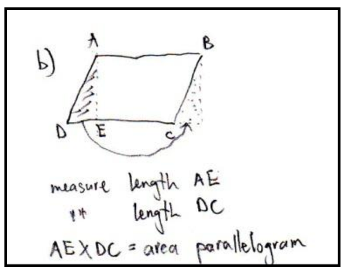


Figure N212. Tan draws a parallelogram and then develops its area formula.

In Excerpt T43, Tan drew a parallelogram and then developed its area formula, as shown in Figure N212. He stated that the formula for the area of a rectangle is 'vertical (side) times horizontal (side)'. Tan mentally cut out a right-angled triangle, namely triangle ADE, from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle, as shown in Figure N212. Thus, the area of the parallelogram equals to the area of the rectangle formed and its area formula is 'vertical (side) times horizontal (side)' or '$AE \times DC$'.

Tan could recall the formula for the area of a triangle. He also knew how to develop the formula for the area of a triangle. Excerpt T44 is illustrative (Tan/L1886-1901).

Excerpt T44

R: How would you show him the way to develop (derive) the formula for calculating the area of a triangle?
S: The triangle. Let's say I give a right-angled triangle. (Draws a right-angled triangle and then develop its area formula, as shown in Figure N213). You cut the area, the rectangle diagonally, you will get a right-angled triangle. So, once the area of rectangle is the vertical (refers to the width of the rectangle) times the horizontal one (refers to the length of the rectangle). So, to find the area of right-angled triangle, you must identify which one is right-angled first. Here is right-angled (points to

∠ABC). So, you take this measurement, you measure AB and BC. Then you times AB and BC to get an area. But, it is not complete yet because this way will lead to area of rectangle. So, you need to times half in order to get the area of triangle.

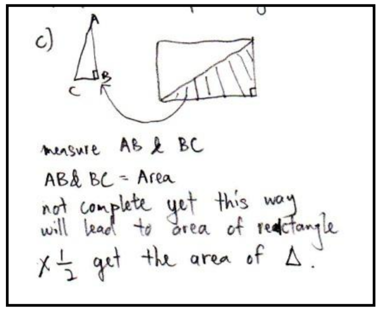


Figure N213. Tan draws a right-angled triangle and then develops its area formula.

In Excerpt T44, Tan used the partition strategy to develop the formula for the area of a triangle. He drew a right-angled triangle and then developed its area formula, as shown in Figure N213. Tan mentally cut a rectangle diagonally and then took out a right-angled triangle, namely triangle ABC. He stated that the formula for the area of a rectangle is 'the vertical (side) times the horizontal (side)'. Tan emphasized that it needed to times half in order to get the area of a triangle, namely 'half times the vertical (side) times the horizontal (side)' or 'half times *AB* times *BC*'.

Tan could recall the formula for the area of a trapezium. He also knew how to develop the formula for the area of a trapezium. Excerpt T45 is illustrative (Tan/L1904-1954).

Excerpt T45

R: How would you show him the way to develop (derive) the formula for calculating the area of trapezium?

S: (Draws a trapezium, as shown in Figure N214). Em trapezium, the picture is like this (points to Figure N214). I will tell the student it is a combination of triangle plus a rectangle or a square. So, I will tell the student you combine these two formula. (Develops the formula for the area of a trapezium, as shown in Figure N215).

R: Could you tell me how you get the formula?

S: Since the formula for a square or rectangle is the same, AB times AC. So, this is for rectangle or square (points to the area formula of rectangle/square, as shown in Figure N215). The formula of triangle we just form just now (refers to the previous task, Task (c)). Formula triangle is half *BE* times *ED*. So, the total area is *AB* times *AC* plus half of *BE* times *ED*. Like this you can stop already. But, if you want, really want to develop the formula, you can still continue. *BE* has the same length as *AC*. So, I convert into here (writes *BE* as *AC* in the area formula of triangle, as shown in Figure N215) and then since there are two *AC* same, I pull the *AC* out from the plus, left only *AB* plus half of *ED*. But *AB* actually you want to convert it or don't want to convert it, also same. I convert it to easily see *CE*. Ok then this *ED* is actually is *CD* minus *CE*. So, you times into it, you'll get like this (points to $AC\ [CE + \frac{1}{2}CD - \frac{1}{2}CE]$, as shown in Figure N215) and then the *CE* they are same. So, you can minus to get the final form here (points to $AC\ [\frac{1}{2}CE + \frac{1}{2}CD]$, as shown in Figure N215). Then you can pull the half out of the plus operation, then you get this final answer (points to $AC\ (\frac{1}{2})\ [CE + CD]$, as shown in Figure N215). Actually, you can still simplify it. *CE* is actually same as *AB*. So, the final form, you can get the formula is this one (points to $\frac{1}{2}AC\ (AB + CD)$, as shown in Figure N215). This is the area formula of trapezium. So, the basical (sic) thing, conclusion: You want or don't want, you have to memorize the formula of rectangle. So, from the rectangle, we can deduce the more complex object.

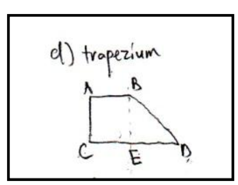


Figure N214. Tan draws a trapezium

formula of rectangle/square $= AB \times AC$

formula $\Delta = \frac{1}{2} \times BE \times ED$

$$\text{total area} = (AB \times AC) + (\tfrac{1}{2} \times BE \times ED)$$
$$= AB \times AC + (\tfrac{1}{2} \times AC \times ED)$$
$$= AC(AB + \tfrac{1}{2}ED)$$
$$= AC(CE + \tfrac{1}{2}ED)$$
$$= AC[CE + \tfrac{1}{2}(CD - CE)]$$
$$= AC[CE + \tfrac{1}{2}CD - \tfrac{1}{2}CE]$$
$$= AC[\tfrac{1}{2}CE + \tfrac{1}{2}CD]$$
$$= AC(\tfrac{1}{2})(CE + CD)$$
$$= \tfrac{1}{2}AC(AB + CD)$$

Figure N215. Tan develops the formula for the area of a trapezium.

In Excerpt T45, Tan used the algebraic method to develop the formula for the area of a trapezium. He drew a trapezium, as shown in Figure N214. Tan explained that a trapezium is a composite of a triangle and a rectangle or a square. He developed the formula for the area of a trapezium using the combination of the formula for the area of a triangle and a rectangle or a square, as shown in Figure N215. Tan wrote the formula for the area of a rectangle or a square, and a triangle as '$AB \times AC$' and '$\frac{1}{2} \times BE \times ED$' respectively. He wrote the formula for the total area of a rectangle or a square, and a triangle as '$(AB \times AC) + (\frac{1}{2} \times BE \times ED)$'. Tan then used the algebraic method to simplified it as '$\frac{1}{2} AC\ (AB + CD)$' which is the formula for the area of a trapezium, as shown in Figure N215. He reiterated that the formula for the area of a rectangle has to be memorized and other "more complex" area formulae can be deduced from it.

Summary

In summary, Tan could recall the formula for the area of a rectangle, parallelogram, triangle, and trapezium. He was able to develop the formulae for the area of a parallelogram, triangle, and trapezium. Tan attempted to develop the formula for the area of a rectangle but unsuccessful.

Conceptual Knowledge

Tan could recall the formula for the area of a rectangle. Nevertheless, he was unable to develop the formula. It was apparent that Tan lack of conceptual knowledge underpinning the formula for the area of a rectangle.

Tan could recall the formula for the area of a parallelogram. He was able to develop the formula. Tan mentally transformed the parallelogram to a rectangle by cutting out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle. It indicated that he understands the relationship between the formula for the area of a parallelogram and rectangle. A parallelogram can always be transformed into a rectangle with the same base, same height, and the same area. Thus, the formula for the area of a parallelogram is exactly the same as the formula for the area of a rectangle, namely 'base times height'.

Tan could recall the formula for the area of a triangle. He was able to develop the formula. Tan developed the formula for the area of a triangle based on the formula for the area of a rectangle. It indicated that he knew the relationship between the formulae for the area of a triangle and rectangle that encloses it. Tan understands the relationship that the area of a triangle is half of the area of the rectangle that encloses it.

Tan could recall the formula for the area of a trapezium. He was able to develop the formula. Tan developed the formula using algebraic method. Tan developed the formula for the area of a trapezium using the combination of the formula for the area of a triangle and a rectangle or a square. Tan wrote the formula for the total area of a rectangle or a square, and a triangle as '$(AB \times AC) + (\frac{1}{2} \times BE \times ED)$'. He then used the algebraic method to simplified it as '$\frac{1}{2}\ AC\ (AB + CD)$' which is the formula for the area of a trapezium. It indicated that Tan knew that the formula for the area of a trapezium is related to the formulae for the area of a rectangle and triangle.

Linguistic Knowledge

Tan used inappropriate mathematical terms 'horizontal side' and 'vertical side' to state the formula for the area of a rectangle. In Excerpt T42, he stated the formula as "…the horizontal side (refers to the length of the rectangle) times the vertical side (refers to the width of the rectangle)" (Tan/L1862-1863). Conventionally, the formula for the area of a rectangle is stated as 'length times width'.

Tan used appropriate mathematical symbols to write the formula for the area of a parallelogram, namely '$AE \times DC$'. Nevertheless, Tan used inappropriate mathematical terms 'vertical (side)' and 'horizontal (side)' to state the formula for the area of a parallelogram. In Excerpt T43, he stated the formula as "…vertical (side) times horizontal (side)…" (Tan/L1877). Conventionally, the formula for the area of a parallelogram is stated as 'base times height'.

Tan indicated the formula for the area of a triangle as '$\frac{1}{2} \times AB \times BC$', as shown in Excerpt T44. He used inappropriate mathematical terms 'vertical (side)' and 'horizontal (side)' to state the formula for the area of a triangle. In Excerpt T44, Tan indicated the formula as half times the vertical (side) times the horizontal (side) or half times *AB* times *BC*. Conventionally, the formula for the area of a triangle is stated as 'half times base times height'

Tan used appropriate mathematical symbols to write the formula for the area of a trapezium, as '$\frac{1}{2}\ AC\ (AB + CD)$', as shown in Figure N215. Nevertheless, Tan did not explain the meaning of the mathematical symbols that he employed. Conventionally, the formula for the area of a trapezium is stated as 'half times the sum of the parallel sides times the height'.

Strategic Knowledge

Tan attempted to develop the formula for calculating the area of a rectangle but unsuccessful, as shown in Figure T24. Tan used the cut and paste strategy to develop the formula for the area of a parallelogram. He mentally cut out a right-angled triangle from one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle, as shown in Figure N212.

Tan used the partition strategy to develop the formula for the area of a triangle. He developed the formula for the area of a triangle from the formula for the area of a rectangle. Tan stated that the formula for the area of a rectangle is 'the vertical (side) times the horizontal (side)'. He mentally cut a rectangle diagonally and then took out a right-angled triangle, namely triangle ABC. Tan emphasized that it needed to times half in order to get the area of a triangle, namely 'half times the vertical (side) times the horizontal (side)' or 'half times *AB* times *BC*', as shown in Figure N213.

Tan had also successfully developed the formula for the area of a trapezium using algebraic method. He developed the formula for the area of a trapezium using the combination of the formula for the area of a triangle and a rectangle or a square, as shown in Figure N215. Tan wrote the formula for the total area of a rectangle or a square, and a triangle as '$(AB \times AC) + (\frac{1}{2} \times BE \times ED)$'. He then used the algebraic method to simplified it as '$\frac{1}{2}\,AC\,(AB + CD)$' which is the formula for the area of a trapezium, as shown in Figure N215.

Ethical Knowledge

Tan attempted to develop the formula for the area of a rectangle but unsuccessful, as shown in Figure N211. He had succeeded in developing the formula for the area of a parallelogram, triangle, and trapezium, as shown in Figures N212, N213, and N215 respectively.

Level of Subject Matter Knowledge

In this section, Tan' levels (low, medium, high) of subject matter knowledge of perimeter and area was analyzed in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

Tan secured a high level of conceptual knowledge of perimeter and area when he obtained 88.0% of appropriate mathematical elements of conceptual knowledge of perimeter and area during the clinical interview. Tan secured a high level of procedural knowledge of perimeter and area when he obtained 81.8% of appropriate mathematical elements of procedural knowledge of perimeter and area. Tan achieved a medium level of linguistic knowledge of perimeter and area when he obtained 62.8%% of appropriate mathematical elements of linguistic knowledge of perimeter and area. Tan secured a high level of strategic knowledge of perimeter and area when he obtained 92.9% of appropriate mathematical elements of strategic knowledge of perimeter and area. Tan achieved a medium level of ethical knowledge of perimeter and area when he obtained 63.3% of appropriate mathematical elements of ethical knowledge of perimeter and area. Tan secured an overall high level of subject matter knowledge of perimeter and area when he obtained 71.8% of appropriate mathematical elements of subject matter knowledge of perimeter and area.

Usha

Usha lives in Johor Baharu, Johor. Usha is 21 years 9 months old when she was interviewed. Currently, she is pursuing a 4-year Bachelor of Science with Education (B.Sc.Ed.) program at a public university. She majored and minored in mathematics and biology respectively. She obtained grade 1A in Mathematics and 3B in Additional Mathematics in her 2003 SPM examination (equivalent to O level). She scored A in Mathematics in the 2004 Matriculation examination (equivalent to A level). Usha performed satisfactory in her mathematics content courses at the university level when she secured one B, four C+, one C, and one C− in seven mathematics content courses she had completed during the first and second year of her studies. The detail of her performance is shown in Table N10.

Table N10

Mathematics Performance of Usha

Courses	Grades
1. Calculus for Science Students I	C+
2. Algebra for Science Students	C+
3. Statistics for Science Students	C+
4. Calculus for Science Students II	C+
5. Differential Equation I	B
6. Programming for Scientific Applications	C−
7. Introduction to Analysis	C

At the time of data collection, Usha was in her second semester of third year studies. She attained 2.92 in the Cumulative Grade Point Average (CGPA) for her first two years of studies at the public university. She does not have any teaching experience prior to this interview.

Notion of Perimeter

Conceptual Knowledge

Usha has successfully selected all the shapes that have a perimeter, namely "A", "C", "D",”F”, "H", "I", “J”, and "K". Excerpt U1 shows her choice of shapes that have a perimeter (Usha/L127-130).

Excerpt U1

R: (Puts a handout comprises 12 shapes in front of Usha). Tick the shapes that have a perimeter.
S: (Ticks shapes "A", "C", "D",”F”, "H", "I", “J”, and "K", as shown in Figure N216).

In Excerpt U1, Usha has selected all simple closed curves (A, C, H, K) as well as all closed but not simple curves (D, I) that have a perimeter. She also selected the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that her notion of perimeter was not only limited to simple closed curves, and closed but not simple curves, but also inclusive of 3-dimensional shapes. Usha did not select the two simple but not closed curves (B, G) as well as the two 1-dimensional shapes (E, L) that do not have a perimeter. In other words, Usha did not select an open shape (including the lines) as having a perimeter.

When asked to justify her selection, Usha explained that she selected shapes "A", "C", "D",”F”, "H", "I", “J”, and "K" because all these surfaces are closed. Excerpt U2 depicts her justification of selecting each of these shapes (Usha/L137-145, 149-155, 161-162).

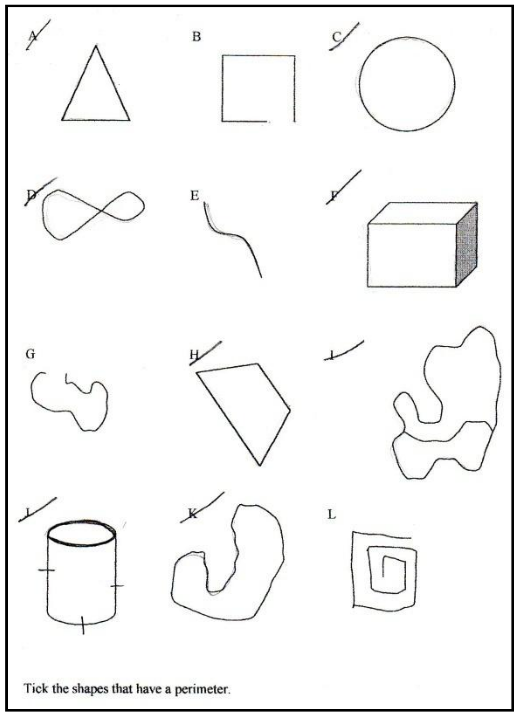


Figure N216. Usha’s selection of shapes that have a perimeter.

Excerpt U2

R: Why did you select shape "A"?
S: Because I think the surface is closed. So, there is a perimeter.
R: Why did you select shape "C"?
S: "C" because it is also closed. So, because in my concept perimeter is like a shape, an outer line. So, I just chose "C".
R: Why did you select shape "D"?
S: Because also same reason. Because it is closed.
R: Why did you select shape "F"?
S: "F" because it is also a shape. So, it is all closed.
.
.
.
R: Why did you select shape "H"?
S: Because of the closed surface.
R: Why did you select shape "I"?
S: "I" because I think can calculate the outer line here even it is not straight line but may be there is a length.
R: Why did you select shape "J"?
S: Because also it is closed.
.
.
.

R: Why did you select shape "K"?
S: Because it is also closed.

Usha explained that she did not select shape “B” because it is not closed. Usha also explained that she did not select shape “G” it is just a line. Excerpt U3 reveals her justification for not selecting shapes “B” and “G” as having a perimeter (Usha/L163-164, L169-170).

Excerpt U3

R: Why didn't you select shape "B"?
S: Because I think it is not closed.
.
.
.
R: Why didn't you select shape "G"?
S: It is also like a line, that's why.

Usha explained that she did not select shapes “E” and “L” because they are just a line. Excerpt U4 exhibits her justification for not selecting shapes “E” and “L” as having a perimeter (Usha/L165-168, L171-172).

Excerpt U4

R: Why didn't you select shape "E"?
S: "E" because it is just a line.
R: Could you tell me more about it?
S: So if line I think they just call the length of the line, not the perimeter.
.
.
.
R: Why didn't you select shape "L"?
S: Because that one also like a line.

Summary

In summary, Usha has selected all simple closed curves (A, C, H, K) and all closed but not simple curves (D, I) that have a perimeter. She also selected the two 3-dimensional shapes (F, J) that have a perimeter. It indicated that her notion of perimeter was not only limited to simple closed curves, and closed but not simple curves, but also inclusive of 3-dimensional shapes. Usha justified her selection by explaining that all these surfaces are closed.

Linguistic Knowledge

Usha used appropriate mathematical term ‘closed’ to justify her selection of shapes that have a perimeter. Usha explained that she selected shapes “A”, “C”, “D”, “F”, “H”, “I”, “J”, and “K” because all these surfaces are closed, as shown in Excerpt U2.

Usha used appropriate negation ‘not closed’ as her justification for not selecting shape “B” as having a perimeter. Usha explained that she did not select shape “B” because it is not closed, as shown in Excerpt U3. Usha also used appropriate mathematical term ‘line’ as her justification for not selecting shape “E”, “G”, and “L” as having a perimeter. Usha explained that she did not select shapes “E”, “G”, and “L” because it is just a line, as shown in Excerpts U3 and U4.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Usha had taken the effort to justify the selection of shapes that have a perimeter, as shown in Excerpt U2. She provided appropriate justification for selecting shapes "A", "C", "D", "F", "H", "I", "J", and "K" that have a perimeter.

Usha also had taken the effort to provide justification for not selecting other shapes that do not have a perimeter. She provided appropriate justification for not selecting shapes "B" and "G" as having a perimeter, as shown in Excerpt U3. Usha also provided appropriate justification for not selecting shapes "E" and "L" as having a perimeter, as shown in Excerpt U4.

Notion of Area

Conceptual Knowledge

Usha has successfully selected all the shapes that having an area, namely "A", "C", "D","F", "H", "I", "J", and "K". Excerpt U5 shows her choice of shapes that have an area (Usha/L127-130).

Excerpt U5

R: (Puts a handout comprises 12 shapes in front of Usha). Tick the shapes that have an area.
S: (Ticks shapes "A", "C", "D","F", "H", "I", "J", and "K", as shown in Figure N217).

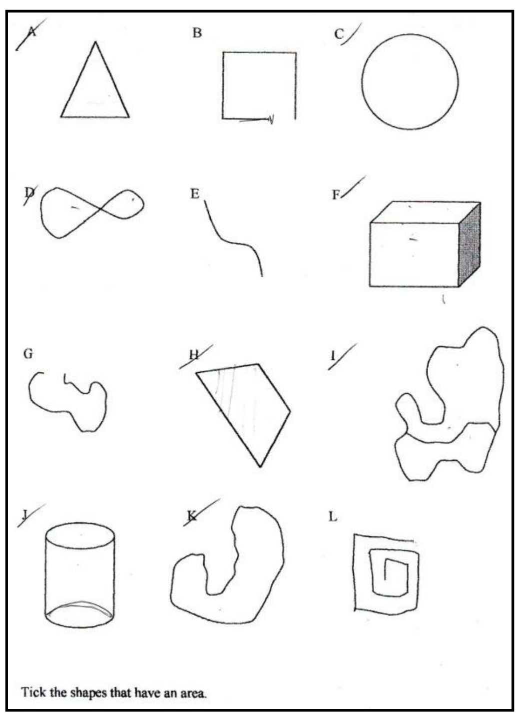


Figure N217. Usha's selection of shapes that have an area.

In Excerpt U5, Usha has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. She also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that Usha had a static perspective of the notion of area. Based on this perspective, area can be viewed as the amount of surface enclosed within a boundary. It also indicated that her notion of area was not only limited to 2-dimensional shapes (closed plane shapes), but also inclusive of 3-dimensional shapes. Usha also did not select the two open shapes (B, G) as well as the two 1-dimensional shapes (E, L) that do not have an area. In other words, Usha did not select an open shape (including the lines) as having an area. It can be inferred that she did not has a dynamic perspective of area or, at least, this knowledge was not accessible to her during the clinical interview.

When asked to justify her selection, Usha explained that she selected shapes "A", "C", "D", "H", and "K" because they are closed and have surface inside. Usha explained that she selected shape “I” because its area can be calculated. It indicated that Usha appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Usha explained that she selected shapes “F” and “J” because their surface area can be calculated. It also indicated that Usha appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Excerpt U6 depicts her justification of selecting each of these shapes (Usha/L190-212).

Excerpt U6

R: Why did you select shape "A"?
S: Because area means the surface inside. So, there is a surface here. So, that's why I chose "A".
R: Why did you select shape "C"?
S: Because it is closed.
R: Why did you select shape "D"?
S: "D" because also have surface inside.
R: Why did you select shape "F"?
S: "F" I still can calculate the surface.
R: Which surface are you referring to?
S: The surface means for this one, May be it is one, two, three, four, five, six, six surface.
R: Why did you select shape "H"?
S: "H" also there is an area that covered inside.
R: Why did you select shape "I"?
S: This one "I". So, there is also parts, two parts that I can calculate.
R: Could you tell me more about it?
S: May be for this area, we can use the graph paper to calculate the area.
R: Why did you select shape "J"?
S: "J" is also area. It is two circles of sphere. Then the side, the surface covered side.
R: Why did you select shape "K"?
S: "K" because there is surface inside.

Usha explained that she did not select shape “B” because it is open. Usha also explained that she did not select shape “G” because it is not closed. Excerpt U7 reveals her justification for not selecting shapes “B” and “G” as having an area (Usha/L213-215, L218-219).

Excerpt U7

R: Why didn't you select shape "B"?
S: Because it is not area actually because it is open here. So, we don't know how to calculate because the line here is just until here.
.
.
.
R: Why didn't you select shape "G"?
S: "G" because this shape is not closed.

Usha explained that she did not select shapes “E” and “L” because they are just a line. Excerpt U8 exhibits her justification for not selecting shapes “E” and “L” as having an area (Usha/L216-217, L220-222).

Excerpt U8

R: Why didn't you select shape "E"?
S: There is no area here. It is just a line.
.
.
.

R: Why didn't you select shape "L"?
S: "L" because it is also like a line. Actually this shape we can stretch it to a line. It is not closed.

Summary

In summary, Usha has selected all 2-dimensional shapes (A, C, D, H, I, K) that have an area. She also selected the two 3-dimensional shapes (F, J) that have an area. It revealed that Usha had a static perspective of the notion of area. Her notion of area was not only limited to 2-dimensional shapes (closed plane shapes), but also inclusive of 3-dimensional shapes. Usha justified her selection by explaining that she selected shapes "A", "C", "D", "H", and "K" because they are closed and have surface inside. Usha explained that she selected shape “I” because its area can be calculated. It indicated that Usha appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured). Usha also explained that she selected shapes “F” and “J” because their surface area can be calculated. It also indicated that Usha appeared to associate the notion of area with the measurement of area (i.e., area does not exist until it is measured).

Linguistic Knowledge

Usha used appropriate mathematical term ‘closed’ to justify her selection of shapes that have an area. Usha explained that she selected shapes "A", "C", "D", "H", and "K" because they are closed and have surface inside, as shown in Excerpt U6. Usha used appropriate mathematical term ‘calculate’ to justify her selection of shape “I” that have an area. Usha explained that she selected shape “I” because its area can be calculated, as shown in Excerpt U6. Usha also used appropriate mathematical term ‘calculate’ to justify her selection of shapes “F” and “J” that have an area. Usha explained that she selected shapes “F” and “J” because their surface area can be calculated, as shown in Excerpt U6.

Usha used appropriate mathematical term ‘open’ as her justification for not selecting shape “B” as having an area. Usha explained that she did not select shape “B”, because it is open, as shown in Excerpt U7. Usha also used appropriate negation ‘not closed’ as her justification for not selecting shape “G” as having an area. Usha explained that she did not select shape “G”, because it is not closed, as shown in Excerpts U7. Usha also used appropriate mathematical term ‘line’ as her justification for not selecting shapes “E” and “L” as having an area. Usha explained that she did not select shapes “E” and “L” because it is just a line, as shown in Excerpt U8.

Ethical Knowledge

Usha had taken the effort to justify the selection of shapes that have an area, as shown in Excerpt U6. She provided appropriate justification for selecting shapes “A”, “C”, “D”, ”F”, “H”, “I”, “J”, and “K” that have an area.

Usha also had taken the effort to provide justification for not selecting other shapes that do not have an area. She provided appropriate justification for not selecting shapes “B” and “G” as having an area, as shown in Excerpt U7. Usha also provided appropriate justification for not selecting shapes “E” and “L” as having an area, as shown in Excerpt U8.

Notion of the Units of Area

Conceptual Knowledge

Usha stated that a square, rectangle, and triangle can be used as the unit of area. It indicated that her notion of the unit of area was not only limited to square, but also nonsquare (such as rectangle and triangle). She expressed that if we used graph paper to measure the area of any shapes, it is also involved a square inside it. Usha explained that a square, rectangle, and triangle can be used as the unit of area because they represent the area of a particular shape with some shape. It indicated that she was unable to provide the appropriate justification that any shape that tessellates a plane can be used as a unit of area measurement. Excerpt U9 is illustrative (Usha/L235-251).

Excerpt U9

R: (Puts a card written the following scenario in front of Usha). Ali, Chong, and David are discussing about the units of area. Ali says that we can use a square as the unit of area. Chong says that we can use a rectangle as the unit of area. David says that we can use a triangle as the unit of area. How would you respond to these students?
S: If I respond to Ali, I would say em it is true, we can.
R: Why?
S: We can used square as a unit of area. Because if we used graph paper to calculate any shape, that is also a square inside. So, the same like this.
R: Chong says that we can use a rectangle as the unit of area. How would you respond to Chong?
S: May be the shape is quite different, square and rectangle, but it is also same. They represent the area of a particular shape with some shape.
R: David says that we can use a triangle as the unit of area. How would you respond to David?
S: Also the same, we can still use triangle.

Summary

In summary, Usha stated that a square, rectangle, and triangle can be used as the unit of area. It indicated that her notion of the unit of area was not only limited to square, but also nonsquare (such as rectangle and triangle). Usha explained that a square, rectangle, and triangle can be used as the unit of area because they represent the area of a particular shape with some shape. It indicated that she was unable to provide the appropriate justification that any shape that tessellates a plane can be used as a unit of area measurement.

Linguistic knowledge

Usha used inappropriate mathematical term ‘represents’ to justify that a square, rectangle, and triangle can be used as the unit of area. She explained that a square, rectangle, and triangle can be used as the unit of area because they represent the area of a particular shape with some shape.

Ethical Knowledge

Knowledge and justification of knowledge is an important aspect in any discipline. Usha had taken the effort to justify the shapes that can be used as a unit of area measurement. Nevertheless, she was unable to provide an appropriate justification for the shapes that can be used as a unit of area measure. This can be seen in Excerpt U9. In reality, any shape that tessellates a plane can be used as a unit of area measurement.

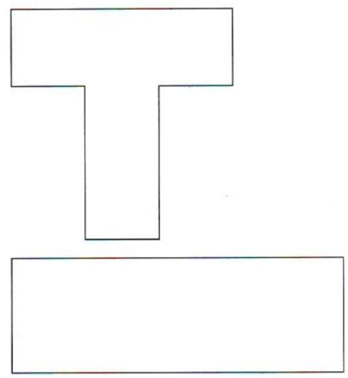
Comparing Perimeter (No Dimension Given)

Strategic Knowledge

Usha used the formal method of measuring the side and applying the definition of perimeter to determine whether the given pair of shapes had the same perimeter. Excerpt U10 shows the formal method that she used to compare the perimeter of the given pair of shapes (Usha/L360-381, 388-391)

Excerpt U10

R: (Puts the following pair of shape in front of Usha). How would you find out whether the following pair of shapes had the same perimeter?



S: Measure, use ruler.
R: Could you show me how it is?
S: We measure using ruler. (Measure the length of each side of the T-shape using ruler. Labels its lengths and then writes the total length, as shown in Figure N218).
R: Could you explain your solution?
S: This is 6. This one is 2. This one is 2. So, this all combine will be 6. This is 4, 4. So, it is 10, 14.
R: How do you get 14?
S: 14. 6, this one combine with this one, 6. Then this one 4. This one 2, 2. 10 eh no, no, sorry. This is 10 (laughs). 10, 24 actually.
R: What about the other shape?
S: (Measure the length of each side of the rectangle using ruler. Labels its lengths and then writes the total length, as shown in Figure N219). 9, 9. This is about 3, 3. So, 3, 3 plus 6. 24 cm.

.
.
.

R: So, do they have the same perimeter?
S: Yeah, same perimeter.
R: What is the perimeter?
S: 24 cm.

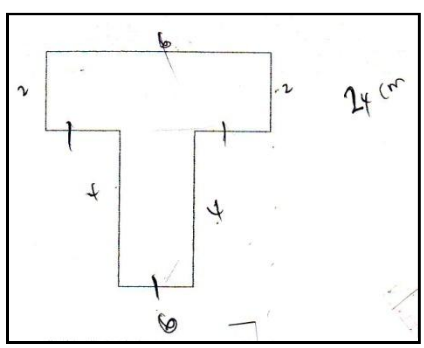


Figure N218. Usha measures the length of each side of the T-shape by ruler and then calculates its perimeter.

Figure N219. Usha measures the length of each side of the rectangle by ruler and then calculates its perimeter.

In Excerpt U10, Usha measured the length of each side of the given T-shape by ruler. She then labelled the lengths and calculated its perimeter as 24 cm, as shown in Figure N218. Usha measured the length of each side of the given rectangle by ruler. She then labelled the lengths and calculated its perimeter as 24 cm, as shown in Figure N219.

When probed for alternative method of comparing the perimeter, Usha used a formal method of measuring the side by thread and ruler. Excerpt U11 depicts how she used thread and ruler to determine each perimeter and then compare their measurements (Usha/L392-408, L411-412).

Excerpt U11

R: Could you think of other way of finding out whether they had the same perimeter?
S: Use this one, what is this actually they call?
R: We called it thread in English or *benang* in Malay language.
S: Thread, thread. This one is quite long method. This is a long method. I think the only method is very suitable using this one (refers to the first method, measures using ruler). Because even though we use thread, we have to, find the ruler then to calculate (it).
R: Could you show me how it is?
S: (Using thread, measures the lengths of each side of the T-shape and marks it on the thread. Cuts it and puts it on the ruler to get the total length). So, 24.
R: What about the other one?
S: The other one also the same way. (Using thread, measures the lengths of each side of the rectangle and marks it on the thread. Cuts it and puts it on the ruler to get the total length). Somewhere 24. So, near to 24. So, that's the way.
.
.
.
R: What do you get?
S: Both is same, 24.

In Excerpt U11, Usha measured the length of each side of the T-shape by thread and then put it on a ruler to determine its total length (perimeter). She also measured the length of each side of the rectangle by thread and then put it on a ruler to determine its total length (perimeter). Usha found that both of them measured 24 (she did not write the unit of measure).

When probed for other method of comparing the perimeter, Usha used a semi-formal method of tracing both shapes on a 1-cm grid paper and then counts the number of unit on each side, as shown in Figures N220 and N221. Excerpt U12 demonstrates how she used this semi-formal method to determine each perimeter and then compare their measurements (Usha/L413-432).

Excerpt U12

R: Could you think of other way of finding out whether they had the same perimeter?
S: (Takes a piece of 1-cm grid paper). So, in case we don't have ruler, just cover to this (laughs) paper (grid paper). Then to calculate. (Traces the rectangle on the 1-cm grid paper and then counts the number of unit on each side, as shown in Figure N221). (Counting) one, one, one. So, this one is three. This side is three. So, (counting) one, two, three, four, five, six, seven, eight, nine. So, nine, nine. So, (total up the lengths for the rectangle) this 24. (Traces the L-shape on the 1-cm grid paper and then counts the number of unit on each side, as shown in Figure N222). (Counting) one, two, three, four, five, six cm. So, (Counting) one, two, three, four, five, six. One, two. One, two. Two. Here also two. So, this is 24.

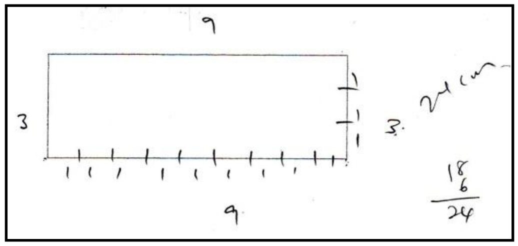


Figure N220. Usha traces the rectangle on the 1-cm grid paper and then counts the number of unit on its length and width.

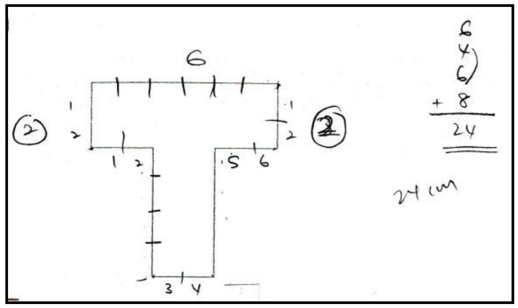


Figure N221. Usha traces the T-shape on the 1-cm grid paper and then counts the number of unit on each side.

In Excerpt U12, Usha traced the rectangle on the 1-cm grid paper and then counted the number of unit on its length and width. She labelled its length and width and then calculated its perimeter as 24 cm, as shown in Figure N220. Usha also traced the T-shape on the 1-cm grid paper and then counted the number of unit on each side. She then calculated its perimeter as 24 cm, as shown in Figure N221.

When probed further for other method of comparing the perimeter, Usha used a formal method of measuring the side by a compass and ruler. Excerpt U13 reveals how she used a compass and ruler to determine each perimeter and then compare their measurements (Usha/L433-456).

Excerpt U13

R: Could you think of other way of finding out whether they had the same perimeter?

S: May be using this one (points to the compasses). It is just same as that thread. (Uses a compass to measure the length of each side of the T-shape and then puts it on the ruler to determine its length after measuring each side. Totals up its lengths, as shown in Figure N222).

R: Could you explain your solution?

S: First, we have to measure here. Then put it on the ruler. So, 6 first one, second one is 2. This one also two. Two. Four. Four. Two. Two. So, 12, 24 cm (total up the length of each side for the T-shape).
(Uses a compass to measure the length of each side of the rectangle and then puts it on the ruler to determine its length after measuring each side. Totals up its lengths, as shown in Figure N223). This one 3, 3. This is also 24 cm (total up the length of each side for the rectangle). So, that's all *lah.*

R: Could you think of other way of finding out whether they had the same perimeter?

S: I think that's all.

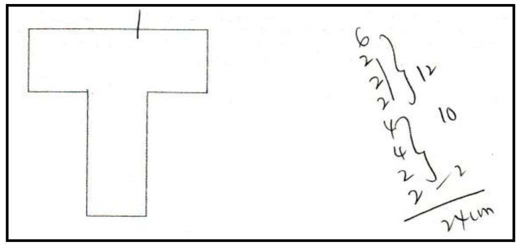


Figure N222. Usha measures the length of each side of the T-shape by compasses and then calculates its perimeter.

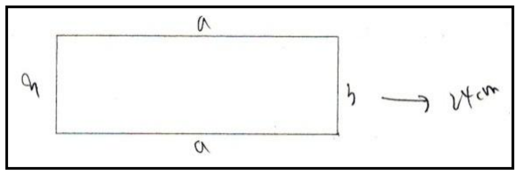


Figure N223. Usha measures the length of each side of the rectangle by compasses and then calculates its perimeter.

In Excerpt U13, Usha measured the length of each side of the T-shape by a compass and then puts it on the ruler to determine its length after measuring each side. She then totalled up its lengths, as shown in Figure N222. Usha also measured the length of each side of the rectangle by a compass and then puts it on the ruler to determine its length after measuring each side. She then totalled up its lengths, as shown in Figure N223.

Summary

In summary, Usha produced three formal methods and one semi-formal method of determining whether the given pair of shape had the same perimeter. In the first method, Usha used the formal method of measuring the side and applying the definition of perimeter. In the second method, she used a formal method of measuring the side by thread and ruler. In the third method, Usha used a semi-formal method of tracing both shapes on a 1-cm grid paper and then counts the number of unit on each side. In the fourth method, Usha used a formal method of measuring the side by a compass and ruler.

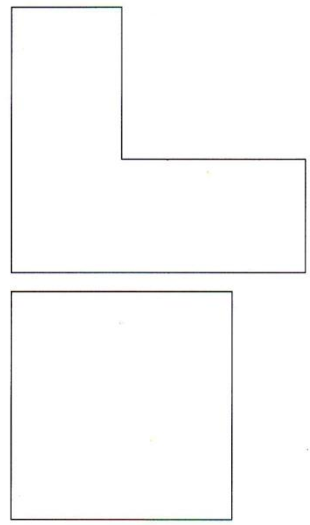
Comparing Area (No Dimension Given)

Strategic Knowledge

Usha partitioned L-shape into two rectangles for which area measurement formulae were known. Excerpt U14 shows the formal method of measuring the side and applying the area formula that she used to compare the area of the given pair of shape (Usha/L501-524).

Excerpt U14

R: (Puts the following pair of shape in front of Usha). How would you find out whether the following pair of shapes had the same area?



R: How would you find out whether they had the same area?

S: …(Silent for a while) we measure (by ruler). Then calculate. (Partitions the L-shape into two rectangles. Measures the lengths and widths of rectangles respectively by ruler. Labels its measurement on the respective sides and then calculate its area using area formula of rectangle, as shown in Figure N224). (Measures two adjacent lengths of the square respectively by ruler. Labels its measurement on the respective sides and then calculate its area using formula of square, as shown in Figure N225).

R: Could you explain your solution?

S: For this one (points to the L-shape), I divide it into two shapes which is two also rectangles. So, to calculate this, the first rectangle area, I measure first. Seven, three. Then times. This area 21. And this one 3, 5. Times, 15. Then I plus, 36. For this one (points to the square), we have to times the two sides to get the area. This 6 cm. This also 6. Then I times, 36.

R: So, what's your conclusion?

S: Both are same.

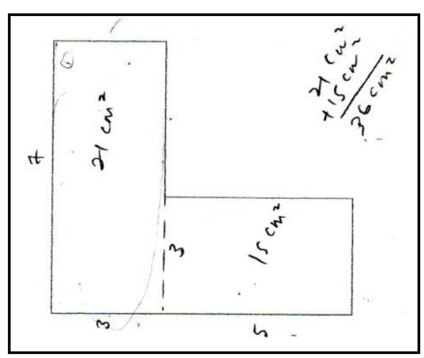


Figure N224. Usha measures the length and width of each rectangle by ruler and then calculates its area.

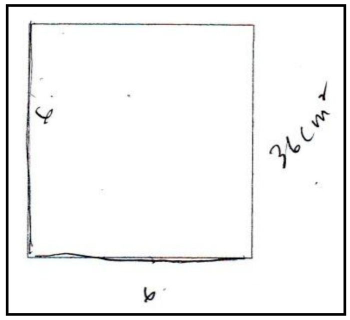


Figure N225. Usha measures the length of two adjacent sides of the square by ruler and then calculates its area.

In Excerpt U14, Usha partitioned L-shape into two rectangles. She measured its lengths and widths by ruler respectively. Usha labelled its measurement on the respective sides and then calculated its area using rectangle area formula, as shown in Figure N224. She also measured the length of two adjacent sides of the square by ruler. Usha labelled its measurement on the respective sides and then calculated its area using square area formula, as shown in Figure N225. She concluded that the given pair of shape had the same area.

When probed for alternative method of comparing the area, Usha used a semi-formal method of tracing both shapes on a 1-cm grid paper and then counts the number of 1-cm square covered by each shape, as shown in Figures N226 and N227. Excerpt U15 depicts how she used this semi-formal method to determine each area and then compare their measurements (Usha/L525-546).

Excerpt U15

R: Could you think of other way of finding out whether they had the same area?

S: Use the graph paper. (Traces the L-shape on the 1-cm grid paper. Counts the number of 1-cm square covered by the shape in the following way, as shown in Figure N226). 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36. (Traces the square on the 1-cm grid paper. Counts the number of 1-cm square covered by the shape in the following way, as shown in Figure N227). 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36.

R: Could you explain your solution?

S: Actually the graph paper here, the square is, the area of one square is 1 cm^2 (misreads 1 cm^2 as 1 cm square). So, I just want to know how many squares here to cover the whole, the all surface. So, I put here (puts the L-shape on the 1-cm grid paper). Then I count *lah*. I calculate the total of the square.

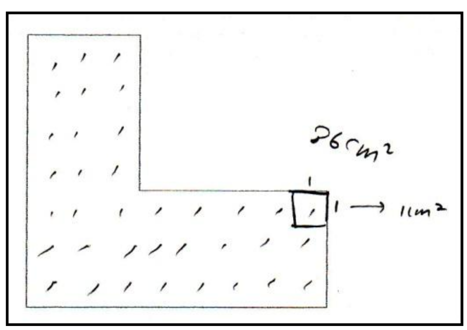


Figure N226. Usha traces the L-shape on the 1-cm grid paper and counts the number of 1-cm square covered by the shape.

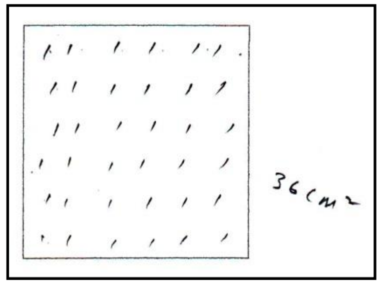


Figure N227. Usha traces the square on the 1-cm grid paper and counts the number of 1-cm square covered by the shape.

In Excerpt U15, Usha traced the L-shape on the 1-cm grid paper. She counted the number of 1-cm square covered by the shape, as shown in Figure N226. Usha also traced the square on the 1-cm grid paper. She counted the number of 1-cm square covered by the shape, as shown in Figure N227. Usha explained that the area of one 1-cm square is 1 cm^2 (she reads 1 cm^2 literally as one centimetre square).

When probed for other method of comparing the area, Usha used the formal method of measuring the side by thread and ruler and applying the area formula to compare the area of the given pair of shape. Excerpt U16 demonstrates the formal method that she used to compare the area of the given pair of shape (Usha/L547-564).

Excerpt U16

R: Could you think of other way of finding out whether they had the same area?

S: Use the thread *lah.* Actually I am using the thread just to know the length. (Partitions the L-shape into two rectangles implicitly. Using the thread to measures the lengths and widths of rectangles respectively and then put it on the ruler to determine its length. Labels its measurement on the respective sides and then calculate its area using area formula of a rectangle, as shown in Figure N228). (Using the thread to measures two adjacent lengths of the square respectively and then put it on the ruler to determine its length. Labels its measurement on the respective sides and then calculate its area using area formula of a square, as shown in Figure N229).

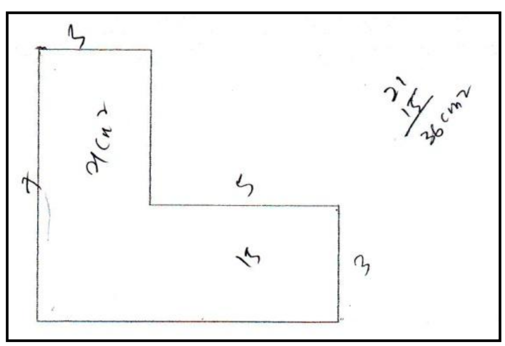


Figure N228. Usha measures the length and width of each rectangle by thread and ruler and then calculates its area.

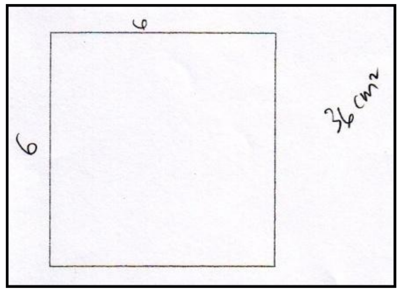


Figure N229. Usha measures the length of two adjacent sides of the square by thread and ruler and then calculates its area.

In Excerpt U16, Usha mentally partitioned L-shape into two rectangles for which area measurement formulae were known. She measured the lengths and widths of the rectangles respectively by thread and then put it on the ruler to determine its length. Usha labelled its measurement on the respective sides and then calculated its area using area formula of a rectangle, as shown in Figure N228. She also measured the two adjacent lengths of the square respectively and then put it on the ruler to determine its length. Usha labelled its measurement on the respective sides and then calculated its area using area formula of a square, as shown in Figure N229.

When probed further for other method of comparing the area, Usha used the formal method of measuring the side by compass and ruler and applying the area formula to compare the area of the given pair of shape. Excerpt U17 reveals the formal method of measuring the side and applying the area formula that she used to compare the area of the given pair of shape (Usha/L570-577).

Excerpt U17

R: Could you think of other way of finding out whether they had the same area?
S: Other method. It is also same like using this one (points to the compass). Then measure. Then put it on the ruler. It just the same method *lah*. Actually we can just straight away using the ruler.
R: Could you think of other way of finding out whether they had the same area?
S: Other method. No, that's all.

In Excerpt U17, Usha suggested that she would measure the lengths and widths of the rectangles respectively by a compass, put it on the ruler to determine its length and then calculated its area using area formula of a rectangle, same as the first method. It indicated that Usha would also measure the two adjacent lengths of the square respectively by a compass, put it on the ruler to determine its length and then calculated its area using area formula of a square, same as the first method.

Summary

In summary, Usha produced three formal methods and one semi-formal method of determining whether the given pair of shape had the same area. In the first method, Usha used the formal method of measuring the side and applying the area formula. In the second method, she used a semi-formal method of tracing both shapes on a 1-cm grid paper and then counts the number of 1-cm square covered by each shape. In the third method, Usha used a formal method of measuring the side by thread and ruler, and applying the area formula. In the fourth method, Usha used a formal method of measuring the side by a compass and ruler, and applying the area formula.

Comparing Perimeter (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Usha explained that she was unable to determine which shape has the longer perimeter. Excerpt U18 shows the justification that she made (Usha/L594-607).

Excerpt U18

R: (Puts the following table in front of Usha). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 1	25 paper clips	12 sticks

S: I can not tell.
R: Why?
S: Because I don't know the size of the paper clip and the stick.
R: But in this case, 25 is larger than 12.
S: No. Paper size, the paper clip may be small. The stick may be longer than the paper clip. So, we don't know. They have to give some more, extra information.
R: What extra information do you need?
S: The size of the paper clip and the stick.

In Excerpt U18, Usha explained that she was unable to determine which shape has the longer perimeter as she did not know the size of the paper clip and the stick. It indicated that Usha focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. Usha knew that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. She elaborated that even though 25 is larger than 12 but the paper clip may be small and the stick may be longer than the paper clip. Thus, Usha stated that she needed extra information, namely the size of the paper clip and the stick, for her to determine which shape has the longer perimeter.

In Set 2, Usha explained that she was unable to determine which shape has the longer perimeter. Excerpt U19 depicts the justification that she made (Usha/L631-648).

Excerpt U19

R: (Puts the following table in front of Usha). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 2	10 paper clips	15 paper clips

S: (Draws the following diagrams).

Figure N230 Usha draws two paper clips.

I can not use the measurement because I don't know they are using the same paper clip for both shape or not. Some paper clip may be this one, small (points to the smaller paper clip that she has drawn, as shown in Figure N230).
R: So, what's your conclusion?
S: My conclusion is I can not predict which is the big.
R: But shape A has 10 paper clips and shape B has 15 paper clips.
S: But if in this case we are using same size of paper clip, shape B should be a big perimeter.

In Excerpt U19, Usha explained that she was unable to determine which shape has the longer perimeter as she did not know whether they used the same paper clips for shapes A and B. Usha stated that some paper clip is small, as shown in Figure N230. It indicated that she focused on the unit of measure when comparing perimeters in Set 2 with common nonstandard unit. Usha knew that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. She explained that shape B has the longer perimeter if the same size of paper clips were used for both shapes.

In another situation when shapes A and B had the same perimeter, Usha explained that the paper clips in shape A is longer than the paper clips in shape B. Excerpt U20 demonstrates her justification about their units of measurement (Usha/L649-660).

Excerpt U20

R: If shapes A and B had the same perimeter, what would you tell about their units of measure?
S: Oh! May be paper clip they used for shape A is a bit big, the height is big. Then this one is small (points to the smaller paper clip that she has drawn, as shown in Figure N230). So, it needs more than this (points to the bigger paper clip that she has drawn, as shown in Figure N230).
R: Which one needs more (paper clips)?
S: This one, 15.
R: Why?
S: Because it is 15. So, may be it is a bit shorter then this one.
R: Which one is shorter?
S: This one (points to shape B).

In Excerpt U20, Usha explained that the paper clips in shape A is longer than the paper clips in shape B. She elaborated that the paper clip in shape B is shorter and thus it needed more paper clips (15) than shape A (10 paper clips) to get the same perimeter as shape A. It indicated that Usha understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length.

In Set 3, Usha stated that shape A has the longer perimeter. Excerpt U21 reveals her choice of shape that has the longer perimeter and the justification that she made (UshaL668-687).

Excerpt U21

R: (Puts the following table in front of Usha). In the following set, shape A has a different perimeter from shape B. Could you tell, from the measurement given, which shape has the longer perimeter?

	Shape A	Shape B
Set 3	16 cm	13 cm

S: Shape A.
R: Why?
S: Longer perimeter because the measurement here is larger, 16 cm.
R: Could you tell me more about it?
S: If the measurement is larger, sure that perimeter is bigger than shape B.
R: Why shape A has the longer perimeter?
S: Because of the value here.
R: Could you tell me more about it?
S: 16 cm, 13 cm. So, 16 is more than 13. So, this (point to shape A) should be larger.
R: But just now the case (refers to the previous task, Task 3.3 (b)) where you have 15 paper clips and 10 paper clips, you said you were not sure.
S: That one represents something, other thing. This is standardized, cm, cm they measured. That one just state paper clips. So, I don't know.

In Excerpt U21, Usha explained that shape A has the longer perimeter because it has the larger measurement, namely 16 cm, compared to shape B (13 cm). She elaborated that 16 is more than 13 and centimeter (cm) is a standard unit of measurement. Thus, Usha concluded that shape A has the longer perimeter. It indicated that she focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Usha knew that common standard unit (such as cm) is reliable for comparing perimeters.

Summary

In summary, Usha focused on the unit of measure when comparing perimeters in Set 1 with nonstandard units. She knew that nonstandard units (such as paper clip and stick) are not reliable for comparing perimeters. Usha focused on the unit of measure when comparing perimeters in Set 2 with common nonstandard unit. She knew that common nonstandard units (such as paper clips) are not reliable for comparing perimeters. Usha understands the inverse proportion between the number of units and the unit of measure: the longer the unit of measure, the smaller the number of units required to get the same length. She focused on the number of unit when comparing perimeters in Set 3 with common standard unit. Usha knew that common standard unit (such as cm) is reliable for comparing perimeters.

Comparing Area (Nonstandard and Standard Units)

Conceptual Knowledge

In Set 1, Usha explained that she was unable to determine which shape has the larger area. Excerpt U22 shows the justification that she made (Usha/L755-769).

Excerpt U22

R: (Puts the following table in front of Usha). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 1	25 triangles	12 squares

S: I can not tell.
R: Why?
S: Because they are, they have different shape, among triangles and squares. Then I also don't know which one is bigger or smaller. They didn't state the area of each one, squares and triangles.
R: In this case, 25 is larger than 12.
S: Even like that but I still can not. They are different shape and I also don't know, may be the triangles each of it can be same with square, here the area and may be not. I don't know because there is no value stated.

In Excerpt U22, Usha explained that she was unable to determine which shape has the larger area as they were different shape, triangle and square, and she did not know the area of each triangle and square. It indicated that Usha focused on the unit of measure when comparing area in Set 1 with nonstandard units. She knew that nonstandard units (such as triangle and square) are not reliable for comparing areas. Usha elaborated that even though 25 is larger than 12 but they were different shape and the area of a triangle might be or might not be same as the area of a square. Furthermore, the area of the triangle and the square were not given.

In Set 2, Usha explained that she was unable to determine which shape has the larger area. Excerpt U23 depicts the justification that she made (Usha/L797-805).

Excerpt U23

R: (Puts the following table in front of Usha). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 2	10 squares	15 squares

S: I still can not (tell). This is because even they are both are squares but I don't know, this might be a bigger square than this one. This also might be bigger. So, they still have to put the value, the area of the square that they used here.

In Excerpt U23, Usha explained that she was unable to determine which shape has the larger area as she did not know the area of the squares in shapes A and B. Usha elaborated that a square from one shape might be larger than a square from another shape. It indicated that she focused on the unit of measure when comparing areas in Set 2 with common nonstandard units. Usha knew that common nonstandard units (such as squares) are not reliable for comparing areas.

In another situation when shapes A and B had the same area, Usha explained that shape B used smaller squares. Excerpt U24 demonstrates her justification about their units of measurement (Usha/L808-816).

Excerpt U24

R: If shapes A and B had the same area, what can you say about their units of measure?
S: Same area. May be the square used here are smaller squares.
R: Which one smaller?
S: Shape B. For shape B, they used the smaller squares. So, they need more squares, 15 squares. Then this one is 10 squares, this one is more bigger (sic). The shape, the unit that they used is different *lah*. This one is smaller (points to squares in shape B) and this one is bigger (points to squares in shape A).

In Excerpt U24, Usha explained that shape B used smaller squares and thus it needed more squares, namely 15 squares. She elaborated that shape A used 10 squares and its squares are larger. It indicated that Usha understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area.

In Set 3, Usha stated that shape A has the larger area. Excerpt U25 reveals her choice of shape that has the larger area and the justification that she made (Usha/L839-858).

Excerpt U25

R: (Puts the following table in front of Usha). In the following set, shape A has a different area from shape B. Could you tell, from the measurement given, which shape has the larger area?

	Shape A	Shape B
Set 3	16 cm^2	13 cm^2

S: Shape A.
R: Why?
S: Because the area for shape A is 16 cm^2 (misreads 16 cm^2 as 16 cm square). Then shape B is 13 cm^2 (misreads 13 cm^2 as 13 cm square). 16 cm^2 is bigger than 13 cm^2.
R: How would you explain to your student which one is larger?
S: I will explain that 16 cm^2 is larger than 13 cm^2. So, this (points to shape A) is bigger.
R: Why?
S: The A is larger…(silent for a while) because its like they used the value. Here is the same value, cm^2 and cm^2. So, that's why we can compared it. We didn't use like, we didn't represent other thing like 16 squares or 13 squares. If like that, it is

like too general, like we don't know square is big or small. If like this cm^2, cm^2, we know it is like same unit. So, we just compared the 16 and 13.

In Excerpt U25, Usha explained that shape A has the larger area because 16 cm^2 is larger than 13 cm^2. Usha elaborated that shapes A and B used the same unit, namely cm^2. Thus, she just compared the number of units, namely 16 and 13. It indicated that Usha focused on the number of unit when comparing areas in Set 3 with common standard unit. Usha knew that common standard unit (such as cm^2) is reliable for comparing areas.

Summary

In summary, Usha focused on the unit of measure when comparing areas in Set 1 with nonstandard units. She knew that nonstandard units (such as triangle and square) are not reliable for comparing areas. Usha focused on the unit of measure when comparing areas in Set 2 with common nonstandard units. She knew that common nonstandard units (such as square) are not reliable for comparing areas. Usha understands the inverse proportion between the number of units and the unit of measure: the larger the unit of measure, the smaller the number of units required to get the same area. She focused on the number of unit when comparing areas in Set 3 with common standard unit. Usha knew that common standard unit (such as cm^2) is reliable for comparing areas.

Linguistic Knowledge

Usha read 16 cm^2 and 13 cm^2 literally as ’16 centimeter square’ and ‘13 centimeter square’ respectively, as shown in Excerpt U25. In another situation, Excerpt U26 exhibits how Usha wrote 16 cm^2 and 13 cm^2 in English words (Usha/L865-871).

Excerpt U26

R: (Puts a blank paper written the following measurements in front of Usha).
16 cm^2
13 cm^2
How would you write these measurements in English words?
S: (Writes the following).

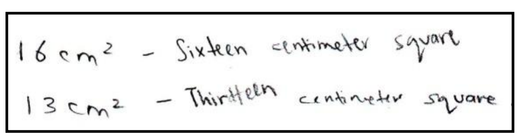


Figure N231. Usha writes 16 cm^2 and 13 cm^2 in English words.

In Excerpt U26, Usha wrote 16 cm^2 and 13 cm^2 literally as ‘sixteen centimetre square’ and ‘thirdteen (sic) centimetre square’, as shown in Figure N231. The correct answer should be ‘sixteen square centimetres’ and ‘thirteen square centimetres’. It indicated that she did not know about the conventions pertaining to writing and reading of Standard International (SI) area measurement units.

Converting Standard Units of Area Measurement

Procedural Knowledge

Usha did not realize that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2. Excerpt U27 is illustrative (Usha/L875-902).

Excerpt U27

R: (Puts a card written the following scenario in front of Usha). Some Form One teachers noticed that several of their students seemed to multiply by 10, 100, and 1000, respectively when they were converting units of area from cm^2 to mm^2, m^2 to cm^2, and km^2 to m^2:

3 cm^2 = 3 x 10 mm^2 = 30 mm^2
4.7 m^2 = 4.7 x 100 cm^2 = 470 cm^2
1.25 km^2 = 1.25 x 1000 m^2 = 1250 m^2

What would you do if you were teaching Form One and you noticed that several of your students were doing this?

S: Em (writes the relationship between the standard units of area measurement, as shown in Figure N232). It is just correct I think because 10 mm^2 (misreads 10 mm^2 as 10 mm square) equals to 1 cm^2 (misreads 1 cm^2 as 1 cm square).

R: Since you were given three questions here, you may start with the first question.

S: Ok from the first one, I can see that 10 mm^2 equals to 1 cm^2. So, if they do like this, it is correct. I think it is correct.

R: What about the second question?

S: Second is the 100 cm^2 (misreads 100 cm^2 as 100 cm square) equal to 1 m^2 (misreads 1 m^2 as 1 m square). Em then the third one is 1000 m^2 (misreads 1000 m^2 as 1000 m square) equal to 1 km^2 (misreads 1 km^2 as 1 km square).

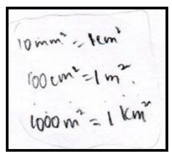


Figure N232. Usha writes the relationship between the standard units of area measurement.

In Excerpt U27, Usha thought that 1 cm^2 = 10 mm^2, 1 m^2 = 100 cm^2, and 1 km^2 = 1000 m^2, as shown in Figure N232. It indicated that She did not know the relationships between the standard units of area measurement such as 1 cm^2 = 100 mm^2, 1 m^2 = 10 000 cm^2, and 1 km^2 = 1 000 000 m^2. Usha did not know the relationships between the standard units of length measurement such as 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. She also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring. Consequently, Usha did not realize that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2. The students also thought that 1 cm^2 = 10 mm^2, 1 m^2 = 100 cm^2, and 1 km^2 = 1000 m^2. Thus, she concluded that the students had correctly converted the unit of area for the first question, namely 3 cm^2 to 30 mm^2, because she thought that 1 cm^2 = 10 mm^2.

When probed further for the correctness of the students' solutions, Usha stated that she was not sure whether the students' first solution was correct or not. Excerpt U28 shows her responses for the correctness of the students' solutions to each question (Usha/L903-919).

Excerpt U28

R: Do the students' first solution correct?

S: Actually I am not very sure if this one (points to the students' first solution) is correct or not. But if this (points to 10 mm^2 = 1 cm^2 as shown in Figure N232) is correct, then here (points to the students' first solution) should be correct. Because the students times the value of centimeter with this (points to 10 mm^2). The cm^2 is bigger than mm^2. So, if they want to change 3 cm^2 to mm^2, which is smaller. So, they have to know how many mm^2 equal to cm^2. So, it is 10 mm^2. So, they have to times the value of cm^2.

R: What about the students' second solution?

S: The second one also same. They want to change m^2 to cm^2. So, if the value is 100 cm^2 = 1 m^2, they have to simply times the value of m^2 that they want to change to cm^2 with 100.
R: What about the students' third solution?
S: Then the third one also, 1000 m^2 = 1 km^2. So, they have to times the km^2 here, the value of km^2 with 1000 so that they can get, they can convert this km^2 to m^2.

In Excerpt U28, Usha stated that she was not sure whether the students' first solution was correct or not. She explained that the students' first solution was correct if 1 cm^2 = 10 mm^2. Usha elaborated that the students have to times 10 when they wanted to convert 3 cm^2 to mm^2. She reiterated that the same goes for the second and third solutions. Usha explained that the students have to times 100 when they wanted to convert m^2 to cm^2 as she thought that 1 m^2 = 100 cm^2. Usha also explained that the students have to times 1000 when they wanted to convert km^2 to m^2 as she thought that 1 km^2 = 1000 m^2.

Summary

In summary, Usha did not realize that the students made a mistake when they were converting unit of area from 3 cm^2 to mm^2, 4.7 m^2 to cm^2, and 1.25 km^2 to m^2, as shown in Figure N232, Excerpts U27, and U28. She thought that 1 cm^2 = 10 mm^2, 1 m^2 = 100 cm^2, and 1 km^2 = 1000 m^2, as shown in Figure N232. It indicated that Usha did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2, 1 m^2 = 10 000 cm^2, and 1 km^2 = 1 000 000 m^2. She did not know the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. Usha also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring.

Conceptual Knowledge

Usha thought that 1 cm^2 = 10 mm^2, 1 m^2 = 100 cm^2, and 1 km^2 = 1000 m^2, as shown in Figure N232. It indicated that She did not know the relationships between the standard units of area measurement that 1 cm^2 = 100 mm^2, 1 m^2 = 10 000 cm^2, and 1 km^2 = 1 000 000 m^2. Usha did not know the relationships between the standard units of length measurement that 1 cm = 10 mm, 1 m = 100 cm, and 1 km = 1000 m. She also did not know the relationships between area units and linear units of measurement that area units are derived from linear units based on squaring.

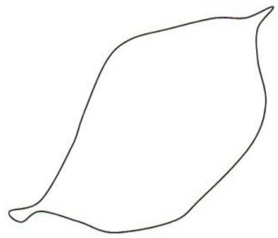
Relationship between Perimeter and Area
(Same Perimeter, Same Area?)

Conceptual Knowledge

Usha did not know that there is no direct relationship between perimeter and area. She did not know that two shapes with the same perimeter can have different areas. Thus, Usha thought that the student's method of calculating the area of the leaf was correct. Excerpt U29 shows Usha's responses to the Form One student (Usha/L959-988).

Excerpt U29

R: (Puts a card written the following scenario in front of Usha). This is a picture of a leaf. A Form One student said that he had found a way to calculate the area of the leaf. The student placed a piece of thread around the boundary of the leaf. Then he rearranged the thread to form a rectangle and got the area of the leaf as the area of a rectangle.



How would you respond to this student?

S: …(Silent for a while) it is correct. I think the method is correct because he just convert the shape of the leaf to rectangle. So, he just measure the area of rectangle. So, will same with the area of the leaf.

R: Could you tell me more about it?

S: …(Silent for a while) he just, from the perimeter of this leaf, he convert it to other shape, any other shape that is easier for him to calculate the area. So, the area will be same, the area of the leaf.

In Excerpt U29, Usha thought that the student's method of calculating the area of the leaf was correct. She agreed with the student that the area of the leaf same as the area of the rectangle formed. Usha explained that the student used the perimeter of the leaf to form other shape, namely rectangle, which was easier for him to calculate the area and thus the area of the leaf same as the area of the rectangle.

Summary

In summary, Usha did not know that there is no direct relationship between perimeter and area. She did not know that two shapes with the same perimeter can have different areas. Thus, Usha thought that the student's method of calculating the area of the leaf was correct.

Ethical Knowledge

In Task 5.1, Usha thought that the student's method of calculating the area of the leaf was correct. The student's method of calculating the area of the leaf was derived from his generalization that two shapes with the same perimeter have the same area. Usha did not attempt to examine the possible pattern of the relationship between perimeter and area. She did not attempt to formulate generalization pertaining to the relationship between perimeter and area. Usha never tests the student's generalization that two shapes with the same perimeter have the same area.

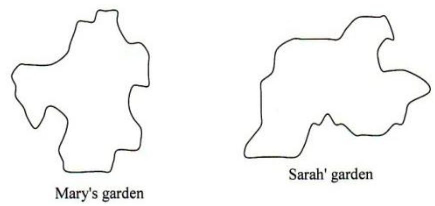
Relationship between Perimeter and Area
(Longer Perimeter, Larger Area?)

Conceptual Knowledge

Usha did not know that there is no direct relationship between perimeter and area. She did not know that the garden with the longer perimeter could have a smaller area. Thus, Usha thought that Mary's claim was correct. Excerpt U30 shows Usha's responses to the claim made by Mary that the garden with the longer perimeter has the larger area (Usha/L1005-1039).

Excerpt U30

R: (Puts a card written the following scenario in front of Usha). Mary and Sarah are discussing whose garden has the larger area to plant flowers. Mary claims that all they have to do is walk around the two gardens to get the perimeter and the one with the longer perimeter has the larger area. How would you respond to these students?



S: Em…(silent for a while) I think this may be correct, correct (laughs).
R: Why?
S: Because if they change the perimeter from, ok just like that before (refers to the previous task, Task 5.1), the leaf. If they change the perimeter to other shape like rectangle. (Draws two rectangles with the perimeters of 24 cm and 26 cm respectively and then calculates its respective area, as shown in Figure N233). So, let's say this garden (points to Mary's garden) has perimeter 24 cm. So, if they change to a rectangle shape. Symmetry you know, 12, 12. Then you know right 2, 2, 10, 10. The make up is rectangle. So, they times, they will get 20 cm^2 (misreads 20 cm^2 as 20 cm square), the area. If a bigger, if they have 26 (refers to perimeter, 26 cm), they can also change to 3, 3, 10, 10. So, the area will be 30. So, the longer the perimeter, the area also will be larger.

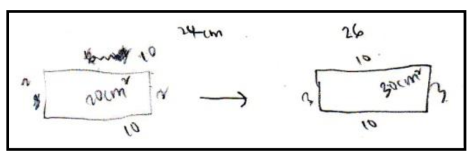


Figure N233. Usha draws two rectangles and calculates its perimeter and area respectively.

In Excerpt U30, Usha thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Usha drew two rectangles with the perimeters of 24 cm and 26 cm respectively. She labelled its dimensions as 10 (cm) by 2 (cm) and 10 (cm) by 3 (cm) respectively and then calculates its area as 20 cm^2 and 30 cm^2 respectively, as shown in Figure N233. The example generated by Usha showed that the rectangle with the longer perimeter has the larger area. Thus, Usha concluded that the longer the perimeter of a shape, the larger the area of the shape.

Summary

In summary, Usha did not know that there is no direct relationship between perimeter and area. She did not know that the garden with the longer perimeter could have a smaller area. Thus, Usha thought that Mary's claim was correct.

Ethical Knowledge

In Task 5.2, Usha thought that Mary's claim was correct. Mary's method of comparing the areas of two gardens was derived from her generalization that the garden with the longer perimeter has the larger area. Usha generated an example to examine the possible pattern of the relationship between perimeter and area. She drew two rectangles with the perimeters of 24 cm and 26 cm respectively. Usha labelled its dimensions as 10 (cm) by 2 (cm) and 10 (cm) by 3 (cm) respectively and then calculates its area as 20 cm^2 and 30 cm^2 respectively, as shown in Figure U18. The example generated by Usha showed that the rectangle with the longer perimeter has the larger area. Thus, Usha concluded that the longer the perimeter of a shape, the larger the area of the shape.

Usha found that the rectangle with the longer perimeter has the larger area. Thus, she formulated a generalization pertaining to the relationship between perimeter and area that the longer the perimeter of a shape, the larger the area of the shape that concurred with Mary's generalization. Usha tested Mary's generalization that the garden with the longer perimeter has the larger area with the example that she generated. Her example concurred with Mary's claim that the garden with the longer perimeter has the larger area. Nevertheless, she did not know that an example could not be used to determine the truth of a generalization. A counterexample can be used to refute the truth of a generalization.

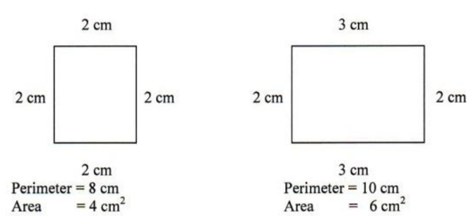
Relationship between Perimeter and Area
(Perimeter Increases, Area Increases?)

Conceptual Knowledge

Usha did not know that there is no direct relationship between perimeter and area. She did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Usha thought that the student's "theory" was correct. This is shown in Excerpt U31 (Usha/L1090-1116).

Excerpt U31

R: (Puts a card written the following scenario in front of Usha). Suppose that one of your Form One students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:



How would you respond to this student?

S: Em I'll say it is true because when the area in a shape is big, so sure, of course the perimeter that cover the outline of area will be big. So, if the perimeter is big, the area should be big.

R: Why?

S: If a shape with the smaller side, it will have small perimeter and also small area. Compare to if a shape have a large perimeter, the area also will be large.

In Excerpt U31, Usha thought that the student's "theory" was correct. Usha agreed with the student that as the perimeter of a closed figure increases, the area also increases. She explained that when the area of a shape is large, the perimeter that surrounded the outline of the area would be longer. Usha elaborated that a shape with the smaller side would have small perimeter and also small area. Thus, she concluded that a shape with the longer perimeter have the larger area.

Usha did not know that the student's claim about the relationship between perimeter and area is not a theory. The claim is a conjecture. She also did not know that an example is not a proof and a theory cannot be proved by an example.

Summary

In summary, Usha did not know that there is no direct relationship between perimeter and area. She did not know that when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same. Thus, Usha thought that the student's "theory" was correct.

Ethical Knowledge

In Task 5.3, the student formulated a generalization that as the perimeter of a closed figure increases, the area also increases. She explained that when the area of a shape is large, the perimeter that surrounded the outline of the area would be longer. Usha elaborated that a shape with the smaller side would have small perimeter and also small area. Thus, she concluded that a shape with the longer perimeter have the larger area. Usha thought that the student's "theory" was correct. She did not attempt to test the student's generalization, as shown in Excerpt U31. Usha accepted the student's generalization without attempting to generate an example or counterexample to test it. In reality, when the perimeter of a figure increases, the area of the figure may increases, decreases, or remains the same.

Calculating Perimeter and Area
(Rectangle and Parallelogram/Triangle)

Procedural knowledge

After read through Task 6.1, Usha labelled the missing sides of Diagram 1 that required for calculating the perimeter and area of the diagram, as shown in Figure N234 in Excerpt U32 (Usha/L1150-1182).

Excerpt U32

R: (Puts a card written the following problem in front of Usha). Suppose that one of your Form One students asks you for help with the following problem:

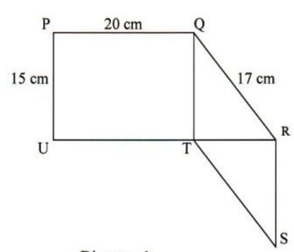


Diagram 1

In Diagram 1, PQTU is a rectangle and QRST is a parallelogram. UTR is a straight line.
Calculate
(o) the perimeter of the diagram,

(p) the area of the diagram.

How would you solve this problem?

S: (Labels Diagram 1 and then circles the lengths to be counted for the perimeter, as shown in Figure N234). Ok the perimeter is outer line. So, as usual we have to calculate all.

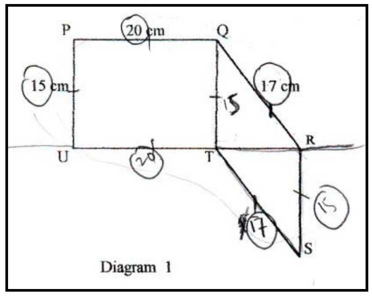


Figure N234. Usha labels the missing sides of Diagram 1.

In Excerpt U32, Usha Beng labelled UT, TS, QT, and RS as 20, 17, 15, and 15 respectively on Diagram 1. She then circled the lengths to be used to calculate the perimeter of the diagram, as shown in Figure N234. Excerpt U33 depicts how Usha has successfully calculated the perimeter of Diagram 1 (Usha/L1197-1202).

Excerpt U33

S: ...So, for the perimeter, we have to calculate (Calculates the perimeter, as shown in Figure N235). So, the answer is 104 for (a).

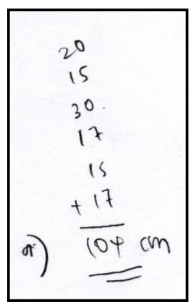


Figure N235. Usha calculates the perimeter of Diagram 1.

In Excerpt U33, Usha used the list all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N235. She listed all the length of sides that surrounded the diagram and then sum them up to get the perimeter of the diagram as 104 cm.

Excerpt U34 demonstrates how Usha has incorrectly calculated the area of Diagram 1 (Usha/L1202-1213)).

Excerpt U34

S: …Then for (b), (Calculates the area, as shown in Figure N236). First we calculate the area of this rectangle which is 20 times 15. So, 300 cm^2 (misreads 300 cm^2 as 300 cm square). For parallelogram, the area of parallelogram is 17 times 15 (It should be 15 times 8, not 17 times 15).

R: What do you get?

S: 255 (for 17 times 15). Ok 555 cm^2 (The correct answer should be 420 cm^2, not 555 cm^2).

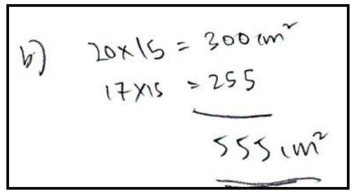


Figure N236. Usha calculates the area of Diagram 1.

In Excerpt U34, Usha used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N236. She partitioned Diagram 1 into a rectangle PQTU and a parallelogram QRST. Usha correctly calculated the area of the rectangle as 300 cm^2. Nevertheless, Usha confused with the slanted side and the height of the parallelogram that she used the slanted side QR as the height (TR = 8 cm) of the parallelogram. Thus, Usha incorrectly calculated the area of the parallelogram as '17 x 15 = 255 cm^2' (The area of the parallelogram should be '15 x 8 = 120 cm^2'). Consequently, she got the area of the diagram as 555 cm^2 (The correct answer should be 420 cm^2, not 555 cm^2).

.

Summary

In summary, Usha has successfully calculated the perimeter of Diagram 1 by using the list all-and-sum algorithm. She incorrectly calculated the area of Diagram 1. Usha confused with the slanted side and the height of the parallelogram that she used the slanted side QR as the height (TR) of the parallelogram to calculate the area of Diagram 1.

Linguistic knowledge

Usha used the correct standard units of measurement for perimeter (cm) and area (cm^2) when she wrote the answers for these measurements, as shown in Figures N236 and N237.

Strategic Knowledge

When probed to check the answer for the perimeter, Usha suggested that she would use the recalculating strategy to verify the answer. Excerpt U35 is illustrative (Usha/1214-1215).

Excerpt U35

R: How would you check your answer for the perimeter?
S: Em I just add again. Add the perimeter again.

In Excerpt U35, Usha suggested that she would check the answer for the perimeter by the recalculating strategy that using the same method and calculate again.

When probed to check the answer for the area, Usha suggested that she would use the recalculating strategy to verify the answer. Excerpt U36 is illustrative (Usha/L1216-1217).

Excerpt U36

R: How would you check your answer for the area?
S: Em I will just calculate again. If I want to check, I will calculate again.

In Excerpt U36, Usha suggested that she would check the answer for the area by the recalculating strategy that using the same method and calculate again.

Ethical Knowledge

Usha has successfully calculated the perimeter of Diagram 1. Nevertheless, she incorrectly calculated the area of Diagram 1. Usha did not check the correctness of the answers for the perimeter as well as the area. When probed to check answers, then only Usha suggested the strategies that she would use to check the answers for the perimeter and area. Usha wrote the measurement units (without probed) for the answers of the perimeter and area that she has calculated, as shown in Figures N235 and N236.

Calculating Perimeter and Area
(Square and Trapezium/Triangle)

Procedural Knowledge

After read through Task 6.2, Usha labelled the missing sides of Diagram 2 that required for calculating the perimeter and area of the diagram, as shown in Figure N237. She then used a calculator to sum up the total length of sides that she has circled and wrote the answer of the perimeter as 56 mm. Excerpt U37 is illustrative (Usha/L1251-1280, L1286-1298).

Excerpt U37

R: (Puts a card written the following problem in front of Usha). Suppose that one of your Form One students asks you for help with the following problem:

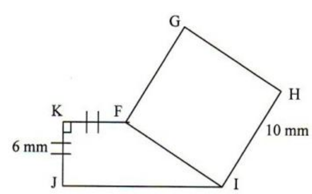


Diagram 2

In Diagram 2, FGHI is a square and FIJK is a trapezium.
Calculate
(o) the perimeter of the diagram,
(p) the area of the diagram.

How would you solve this problem?

S: (Labels the missing sides of Diagram 2, as shown in Figure N237. Uses Pythagoras' theorem to find the length of ZI. Circles the lengths to be counted for the perimeter). (Uses calculator to sum up the total length of sides that she has circled and writes the perimeter as 56 mm.)

.
.
.

R: Could you explain your solution?
S: For the perimeter, we know it is outer line. So, we have to find all the outer line. Since this is a square, so all of the side will be same. They have give one of the side, 10 mm. So, we know that GH, FG will be 10. Then for this one (points to KF), they have put the sign (╫) that it is same with this side (points to KJ). This is 6 mm (points to KF). So, this one also 6 mm (points to JZ). Then it should be all the side also same. We just assume that this is Z. So, we know that JZ is 6 also. So, we

have to find ZI. For ZI, we use that theorem, theorem Pythagoras I think. So, if we want to find ZI, we ten square the hypotenuse minus six square. Then all of it we squared and we get 8. So, after that we plus all the outer line. We will get 56.

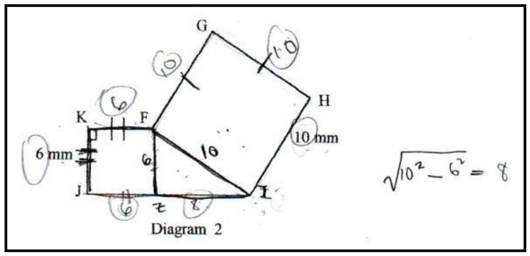


Figure N237 Usha labels the missing sides of Diagram 2.

In Excerpt U37, Usha labelled GH, FG, FI, KF, JZ, and FZ as, 10 mm, 10 mm, 10 mm, 6 mm, 6 mm and 6 mm respectively on Diagram 2. Usha realized that she needed to find the length of ZI. Usha partitioned trapezium FIJK into a square and a triangle, as shown in Figure N238. She has successfully calculated the length of ZI as 8 (mm) by using Pythagoras' theorem.

Usha used the circle all-and-sum algorithm to calculate the perimeter of the diagram, as shown in Figure N237. She circled all the length of sides that surrounded the diagram and then used a calculator to sum them up to get the perimeter of the diagram as 56 mm.

Excerpt U38 demonstrates how Usha has successfully calculated the area of Diagram 2 (Usha/L1280-1285, L1303-1307).

Excerpt U38

S: …(Calculates the area of Diagram 2, as shown in Figure N238).

.

.

.

R: Could you explain your solution for the area?

S: Area for square, times the two sides. 10, 10, 100. Then for trapezium, the formula is one over two times their height then times the sum of the two sides, KF and JI. So, after that I will get 60. Then, for both different shape, plus the two area. Then 160.

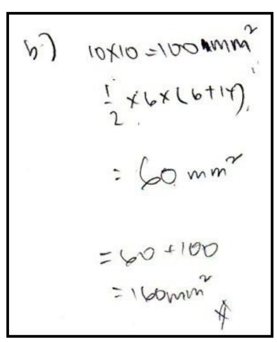


Figure N238. Usha calculates the area of Diagram 2.

In Excerpt U38, Usha used the partition-and-sum algorithm to calculate the area of the diagram, as shown in Figure N238. She partitioned Diagram 2 into square FGHI and trapezium FIJK. Usha calculated the area of the square and trapezium separately using the area formulae of square and trapezium respectively and then summed them up to get the area of the diagram as 160 mm^2.

Summary

In summary, Usha has successfully calculated the perimeter of Diagram 2 using the circle all-and-sum algorithm. She has also correctly calculated the area of Diagram 2 using the partition-and-sum algorithm.

Linguistic Knowledge

Usha used the correct standard units of measurement for perimeter (mm) and area (mm^2) when she wrote the answer of these measurements, as shown in Excerpt U40 and Figure N239.

Strategic Knowledge

When probed to check the answer for the perimeter, Usha suggested that she would use the recalculating strategy to verify the answer. Excerpt U39 is illustrative (Usha/L1299-1302).

Excerpt U39

R: How would you check your answer for the perimeter?
S: Check my answer for the perimeter…(silent for a while) I'll make sure that all the outer line that I have calculate is correct. Then we just plus it. So, that's the way.

In Excerpt U39, Usha suggested that she would check the answer for the perimeter by the recalculating strategy that using the same method and calculate again. She would make sure that all the lengths used for calculating the perimeter were correct.

When probed to check the answer for the area, Usha suggested that she would use the recalculating strategy to verify the answer. Excerpt U40 is illustrative (Usha/L1308-1310).

Excerpt U43

R: How would you check your answer for the perimeter?
S: If involved the formula, I will check that whether the formula that I used is correct. Then the value to calculate. Then calculate again.

In Excerpt U40**,** Usha suggested that she would check the answer for the area by the recalculating strategy. She would check whether the formula that she used was correct and then the lengths used for calculating the area were correct.

Ethical Knowledge

Usha has successfully calculated the perimeter and area of Diagram 2. Nevertheless, she did not check the correctness of the answers for perimeter and area. When probed to check answers, then only Usha suggested the strategy that she would use to check the answers for perimeter and area. Usha wrote the measurement units (without probed) for the answers of perimeter and area, as shown in Excerpt U37 and Figure N238.

Fencing Problem

Strategic Knowledge

Usha used looking for a pattern strategy to solve the fencing problem. Excerpt U41 is illustrative (Usha/L1312-1326).

Excerpt U41

R: (Puts a card written the following problem in front of Usha). Suppose that one of your students asks you for help with the following problem:

A gardener has 84 m of fencing to enclose a garden along three sides, with the fourth side of the garden being formed by a wall. (Assume that the wall is perfectly straight). What are the dimensions of a rectangular garden that will yield the largest area being enclosed?

How would you solve this problem?

S: (Uses looking for a pattern strategy to solve this problem. Writes the following, as shown in Figure N239).

In Excerpt U41, Usha started off with the length and the width of the rectangular garden as 70 m and 7 m respectively and this yielded the area being enclosed as 490 m^2. She then reduced the length of the rectangular garden, ten metres at a time, to 60 m and increased the width of the rectangular garden accordingly to 12 m. Consequently, the area increased to 720 m^2. Usha saw a pattern that area increases as she reduces the length of the rectangular garden while increases its width accordingly. Usha noticed that when she reduced the length of the rectangular garden to 50 m and increased its width to 17 m, the area increased to 850 m^2, as shown in Figure N239.

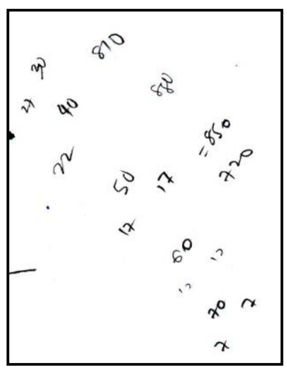


Figure N239. Usha uses looking for a pattern strategy to solve the fencing problem.

Excerpt U42 further illustrates how Usha used this strategy to solve the fencing problem (Usha/L1327-1337).

Excerpt U42

R: Could you explain your solution?

S: So I'm …(silent for a while) so for the largest area, the side that we have to use is 50 m and 17 m to get the largest area which is eh, no, no (realizes a mistake). Wait, wait, wait, 40 times 22. (Draws a diagram and then writes its dimension and area, as shown in Figure N240). As you know this is 40, then 22. 40 m, 22 m. So, we will get a largest area which is 880 m^2 (misreads 880 m^2 as 880 m square).

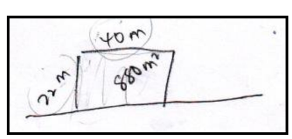


Figure N240. Usha draws a diagram and then writes its dimensions and area.

In Excerpt U42, Usha initially thought that 50 m by 17 m was the dimension of the rectangular garden that will yield the largest area being enclosed. Subsequently, Usha realized her mistake when she reduced the length of the rectangular garden to 40 m and increased its width to 22 m. She found that the area increased to 880 m^2. Usha concluded that 880 m^2 was the largest area being enclosed. She drew a diagram that displayed the dimension of a rectangular garden that will yield the largest area being enclosed, as shown in Figure N240.

Excerpt U43 shows how Usha justified that 880 m^2 was the largest area being enclosed (Usha/L1338-1341).

Excerpt U43

R: How do you know 880 m^2 is the largest area?
S: Because if I put 30 for this side of the rectangle, here will be 27. So if 30 times 27 (uses calculator) is 810. So, if we want to get the largest area, we should put 40 m and 22 m.

In Excerpt U43, Usha justified her answer that 880 m^2 was the largest area being enclosed by reducing the length of the rectangular garden to 30 m and increasing its width to 27 m. She found that the area decreased to 810 m^2. Usha concluded that 40 m by 22 m was the dimension of the rectangular garden that will yield the largest area being enclosed, namely 880 m^2. Table N11 summarizes the dimensions of the rectangular garden and its area that Usha has figured out. Usha did not aware that 880 m^2 was not the largest area being enclosed and 40 m by 22 m was not the dimension of the rectangular garden that will yield the largest area being enclosed. In fact, 882 m^2 is the largest area being enclosed and 42 m by 21 m is the dimension of the rectangular garden that will yield the largest area being enclosed.

Table N11

Dimensions of Rectangular Garden and its Area That Usha has Figured out

Length (cm)	Width (cm)	Width (cm)	Area (cm^2)
70	7	7	490
60	12	12	720
50	17	17	850
40	22	22	880
30	27	27	810

Summary

In summary, Usha used looking for a pattern strategy to solve the fencing problem. Nevertheless, she did not find the dimension of the rectangular garden that will yield the largest area being enclosed. Usha used the same strategy, namely the looking for a pattern strategy, to check the answer for the fencing problem without being probed.

Ethical Knowledge

Usha used the same strategy, namely the looking for a pattern strategy, to check the answer for the fencing problem without being probed. This can be seen in Excerpts U42 and U43. Usha wrote measurement units for the dimension (40 m by 22 m) that she thought would yield the largest area being enclosed and its area (880 m^2). Excerpts U42 and U43 are illustrative.

Developing Area Formulae

Procedural Knowledge

Usha could recall the formula for the area of a rectangle. Nevertheless, she was unable to develop it. Excerpt U44 is illustrative (Usha/L1383-1405, L1418-1420).

Excerpt U44

R: (Puts a card written the following scenario in front of Usha). Suppose that a Form One student comes to you and says that he does not know how to develop (derive) the formula for calculating the area of the following figures:

(q) Rectangle,
(r) Parallelogram,
(s) Triangle, and
(t) Trapezium.

How would you show him the way to develop (derive) the formula for calculating the area of these figures? Let's start with rectangle.

S: For rectangle, we have to multiply the height, the length, the height and the width of that shape. So, if we calculate both of this side, we will get the area.

R: What is the formula for the area of a rectangle?

S: (Draws a rectangle and then writes its area formula, as shown in Figure N241). It can simply put any x, y. So, xy.

R: What do your "x" and "y" stands for?

S: "x" is height. Then "y" is the width, "y" is the width.

.
.
.

R: How would you develop (derive) the formula for calculating the area of rectangle?

S: Derive em…(silent for a while) I also don't know.

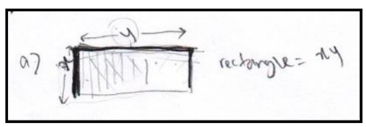


Figure N241. Usha draws a rectangle and writes its area formula.

In Excerpt U44, Usha stated the formula for the area of a rectangle as ‘height times width’. She drew a rectangle and wrote its area formula as xy, where x and y is the height and the width of the rectangle, as shown in Figure N241. Nevertheless, Usha was unable to develop it. She just memorized the formula. Usha did not attempt to develop the formula.

Usha neither could recall the formula for the area of a parallelogram nor able to develop it. Excerpt U45 is illustrative (Usha/L1421-1432).

Excerpt U45

R: How would you develop (derive) the formula for calculating the area of a parallelogram?

S: Parallelogram…(silent for a while) actually I don't know the formula of parallelogram. I forgot already.

R: Never mind. Would you like to try?

S: So, I think we have to multiply the two sides of the parallelogram to get the area. It just same as rectangle. (Draws a parallelogram and writes its area formula, as shown in Figure N242). Times x and y. I think its like that, area.

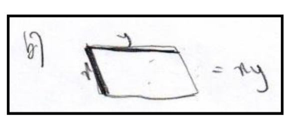


Figure N242. Usha draws a parallelogram and writes its area formula.

In Excerpt U45, Usha admitted that she could not recall the formula for the area of a parallelogram. When probed to try, Usha thought that the formula for the area of a parallelogram is same as the formula for the area of a rectangle. She stated that the area of a parallelogram can be found by multiplying the lengths of two adjacent sides of the parallelogram. Thus, Usha drew a parallelogram and wrote its area formula as ‘x times y’, as shown in Figure N242.

Usha neither could recall the formula for the area of a triangle nor able to develop it. Excerpt U46 is illustrative (Usha/L1433-1451).

Excerpt U46

R: How would you develop (derive) the formula for calculating the area of a triangle?

S: Triangle. (Draws two triangles and writes its area formula, as shown in Figure N243. Initially, the area formula was written as $\frac{1}{2}$ *y* x *x*. Later, she shaded the half to become *y* x *x*, as shown in Figure N243). Triangle, we have to one over two times y times x. Ah the area of triangle, y times x…(silent for a while) y times x. I think y times x, height times the length of the base.

R: What's the formula for calculating the area of a triangle?

S: For triangle, it is like this rectangle (points to the rectangle in Figure N241). So, we have to find the height of the triangle then the base triangle. So, we times the base and the height to get all the area of the triangle.

R: How do you get the formula as "y times x" for the area of a triangle?

S: Since we want to cover all the surface, we have to multiply with, I also don't know how to explain, but we have to multiply the height and the base to get all the surface we need.

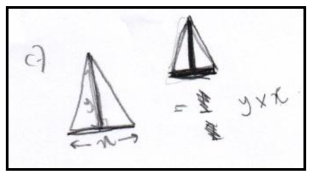


Figure N243. Usha draws two triangles and writes its area formula.

In Excerpt U46, Usha drew two triangles, as shown in Figure N243. Initially, she wrote the formula for the area of a triangle as $\frac{1}{2} y \times x$. Later, she shaded the half to become $y \times x$, as shown in Figure N243. Usha thought that the formula for the area of a triangle is ‘$y \times x$’, where x and y is the base and the height of the triangle. She elaborated that a triangle is just like a rectangle and we have to multiply the base and the height of the triangle to get its area. Usha was unable to develop the formula for the area of a triangle.

Usha could recall the formula for the area of a trapezium as ‘$\frac{1}{2} \times h \times (a+b)$’. Nevertheless, she was unable to develop it. Excerpt U47 is illustrative (Usha/L1493-1499, L1527-1530).

Excerpt U47

R: What is the formula for calculating the area of trapezium?

S: (Draws a trapezium and writes its area formula, as shown in Figure N244). So, trapezium is one over two times h times "a plus b".

.
.
.

R: How would you develop (derive) the formula for calculating the area of a trapezium?

S: Actually I don't know how to derive but I just memorized from my prior lesson in school.

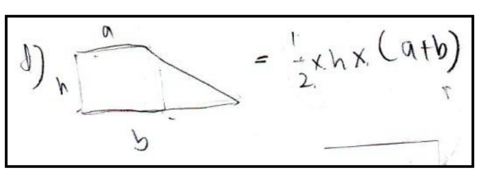


Figure N244. Usha draws a trapezium and writes its area formula.

In Excerpt U47, Usha drew a trapezium and wrote its area formula, as shown in Figure N244. She stated that the formula for the area of a trapezium is ‘one over two times h times (a + b)’. Nevertheless, Usha was unable to develop the formula for the area of a trapezium. Usha admitted that she just memorized it from her prior lesson in school. Usha did not attempt to develop the formula.

Summary

In summary, Usha could recall the formula for the area of a rectangle and trapezium. Nevertheless, she was unable to develop these formulae. Usha did not attempt to develop the formula for the area of a rectangle and trapezium.

Conceptual Knowledge

Usha could recall the formula for the area of a rectangle. Nevertheless, she was unable to develop the formula. It was apparent that Usha lack of conceptual knowledge underpinning the formula for the area of a rectangle.

Usha could not recall the formula for the area of a parallelogram. She was unable to develop the formula. It was apparent that she did not know the relationship between the area of a parallelogram and the area of a rectangle. Had Usha been known of this relationship, she would know how to develop the formula for the area of a parallelogram.

Usha could not recall the formula for the area of a triangle. She was unable to develop the formula. It was quite clear that Usha did not know the relationship between the area of a triangle and the area of the rectangle that encloses it. Had she been known of this relationship, Usha would know how to develop the formula for the area of a triangle.

Usha could recall the formula for the area of a trapezium. Nevertheless, she was unable to develop the formula. It was quite clear that Usha did not know the relationship between the area formulae of a rectangle, parallelogram, triangle, and trapezium. Had she been known of this relationship, Usha would know how to develop the formula for the area of a trapezium.

Linguistic Knowledge

Usha used appropriate mathematical symbols to write the formula for the area of a rectangle, namely ‘xy’, as shown in Figure N241. Nevertheless, Usha used inappropriate mathematical terms ‘height’ and ‘width’ to explain the meaning of the symbols *x* and *y* that she employed. Usha explained that “"x" is height. Then "y" is the width, "y" is the width.” (Usha/L1405). Actually, *x* and y in her formula represents the height and the base, or the width and the length of the rectangle. Conventionally, the formula for the area of a rectangle is stated as ‘length times width’, ‘length times breadth’, or ‘base times height’.

Usha used appropriate mathematical symbols to write the formula for the area of a trapezium, namely ‘$\frac{1}{2} \times h \times (a + b)$’, as shown in Figure N244. Nevertheless, Usha did not explain the meaning of the mathematical symbols that she employed.

Strategic Knowledge

Usha was unable to develop the formulae for the area of a rectangle, parallelogram, triangle, and trapezium. Usha could recall the formulae for the area of a rectangle and trapezium but she did not attempt to develop the formulae. Thus, the researcher was not able to trace the strategy that she might used to develop these formulae.

Ethical Knowledge

Usha could recall the formula for the area of a rectangle but she did not attempt to develop the formula, as shown in Excerpt U44. Usha could not recall the formula for the area of a parallelogram and she did not attempt to develop the formula, as shown in Excerpt U45. Usha could not recall the formula for the area of a triangle and she did not attempt to develop the formula, as shown in Excerpt U46. Usha could recall the formula for the area of a trapezium but she did not attempt to develop the formula, as shown in Excerpt U47.

Level of Subject Matter Knowledge

In this section, Usha' levels (low, medium, high) of subject matter knowledge of perimeter and area was analyzed in terms of its level of each of the five basic types of knowledge, namely levels of conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge as well as the overall level of SMK that were identified from the clinical interview.

Usha achieved a medium level of conceptual knowledge of perimeter and area when she obtained 44.0% of appropriate mathematical elements of conceptual knowledge of perimeter and area during the clinical interview. Usha gained a low level of procedural knowledge of perimeter and area when she obtained 27.3% of appropriate mathematical elements of procedural knowledge of perimeter and area. Usha achieved a medium level of linguistic knowledge of perimeter and area when she obtained 69.8%% of appropriate mathematical elements of linguistic knowledge of perimeter and area. Usha obtained a medium level of strategic knowledge of perimeter and area when she obtained 71.4% of appropriate mathematical elements of strategic knowledge of perimeter and area. Usha obtained a medium level of ethical knowledge of perimeter and area when she obtained 71.4% of appropriate mathematical elements of ethical knowledge of perimeter and area. Usha achieved an overall medium level of subject matter knowledge of perimeter and area when she obtained 62.7% of appropriate mathematical elements of subject matter knowledge of perimeter and area.