The basic mechanisms contributing to the hollow cathode effect as described in Section 1.3 together with description of experiments proving the existence and importance of these mechanisms shall be reviewed in this chapter. The relations between these mechanisms and the operating conditions and parameters of the hollow cathode discharge shall also be discussed. The review of the hollow cathode discharge characteristics shall begin with a discussion of the breakdown characteristic of a hollow cathode geometry before delving into the electrical and optical characteristics of the hollow cathode discharge after breakdown and the behaviour of the charged and neutral current-sustaining carriers in the discharge. Lastly, the development of the numerical modelling of the hollow cathode discharge in relation to some of its characteristics will be given.

2.1 Breakdown characteristics

The Paschen plots relating the breakdown voltages as a function of the product of the gas pressure and the electrode separation $pd$ has long been used to characterize the breakdown phenomena in a system with planar electrodes, or
otherwise, under uniform field configuration[ENG65]. These plots are somewhat changed when applied to systems of nonplanar geometries such as the coaxial electrode arrangement, point-to-plane configuration as well as in the hollow cathode discharge configuration with various shapes of electrodes. This is due to the distortion of the electric field and the ambiguity in the distance between the electrodes.

Deviations from the Paschen law have been shown for coaxial cylindrical electrode arrangement. Raju & Hackam[RAJ74] postulated that the higher breakdown voltages at and near the Paschen minimum obtained in a concentric coaxial field as compared with a uniform field is attributed to non-equilibrium ionization effects leading to a departure from the similarity theorem. However, Heisen[HEI76] refuted this explanation and pointed out that violations of the similarity theorem cannot be caused by non-equilibrium ionization. He further showed that the larger the ratio of the radius of the outer electrode to that of the inner one \((R_O/R_I)\), the higher the breakdown voltage at the Paschen minimum and this behaviour is confined to the vicinity of the Paschen minimum. At higher \(pd\) values, the situation is reversed. He explained these behaviours in relation to the Stoletow constant (corresponding to optimum ionizing power at an optimum reduced field value).

Much investigations has been carried out on the breakdown characteristics in non-uniform fields, but mostly in a coaxial cylindrical electrode geometry. Thus, the present work also aims to investigate the breakdown characteristic of a cylindrical electrode geometry but with the anode at one of the open end of the cylindrical cathode. Explanation in terms of the Stoletow constant in relation to the mapped electric field distribution shall be presented in Section 4.1.
In a different class of discharge such as the transient hollow cathode discharge in pseudosparks with single-gap which are operated to the left of the Paschen minimum, it has been shown[LIU95] that if the cathode-anode gap separation $d$ is larger than three times the hollow cathode cavity diameter $a$, the Paschen plot is valid. Otherwise, for $(d/a)<3$, the breakdown voltage is found to be a function of $p^2da$. It is postulated that the pseudospark breakdown is not determined only by the Townsend breakdown criterion, but mechanisms in which the effects of the time-varying and space-charge fields are dominant may also be responsible. Furthermore, typical pseudospark operates in a reduced field $E/p$ up to hundreds of $kV cm^{−1}torr^{−1}$ as compared to dc hollow cathode discharge which are usually operated in approximately two orders of magnitude less.

2.2 Electrical characteristics

2.2.1 Current density

The current enhancement was demonstrated by Güntherschulze[GUN30] when he compared the current $(i)$ to a double parallel plate cathode to the sum $(i_0)$ of the currents, measured successively, to two single planar cathodes, at a chosen voltage. This comparison was carried out over a range of pressures in several gases. The ratio of the currents $(i/i_0)$ was then plotted as a function of the product of cathode separation and pressure $(ap)$ as shown in Figure 2.2.1. Similar curves were shown by Badareu & Wächter[BAD58b], Badareu & Popescu[BAD60b] and Cioboratu[CIO64] for other gases. The curving at the upper parts was attributed to heating effects and it could be seen that the current $i$ depends strongly on the cathode fall potential $V_K$. 

15
These curves manifest a linear portion independent of the discharge voltage and the filling gas; and the current magnification \( q = i/i_0 \) is deduced to be approximately proportional to \((ap)^{2.5}\). It was further approximated that for small values of \( ap \) when the hollow cathode effect is optimum, the reduced current density \( j/p^2 \) is proportional to \((ap)^{2.5}\) and Little and von Engel\([LIT54]\) showed that it is consistent with the Townsend similarity relation, specifically \( V = f(ap, j/p^2) \) as mentioned in Section 1.4. For large \( ap \), it is equivalent to two independent conventional glow discharges and \( j/p^2 \) is then proportional to \((pd_k)^{2.5}\) where \( d_k \) is the width of the cathode dark space. Thus the upper limit of the optimum range of operation of a hollow cathode discharge could possibly be deduced at the value of \( ap \) when the curve deviates from the linear relation (occurring usually at the value of \( q \) between 2 and 1; corresponding to the splitting into two independent discharges in the double planar cathode configuration). Table 2.2.1 lists these upper limits of \( ap \) for some gases estimated from the plotted results of various workers. It is seen that this upper limit of \( ap \) shifts to higher values when the cathode fall potentials is increased for the same gas. On the other hand, at fixed cathode fall potential, it decreases with heavier gases (comparing only the rare gases).

<table>
<thead>
<tr>
<th>Gas</th>
<th>Cathode fall potential ( V ) (V)</th>
<th>High limit of ( ap ) ( cm )-torr</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>250</td>
<td>3.2</td>
<td>GUN30</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>4.5</td>
<td>GUN30</td>
</tr>
<tr>
<td>Kr</td>
<td>350</td>
<td>0.7</td>
<td>CIO64</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>0.86</td>
<td>CIO64</td>
</tr>
<tr>
<td>Xe</td>
<td>400</td>
<td>0.41</td>
<td>CIO64</td>
</tr>
<tr>
<td></td>
<td>450</td>
<td>0.57</td>
<td>CIO64</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.78</td>
<td>CIO64</td>
</tr>
<tr>
<td>( N_2 )</td>
<td>300</td>
<td>0.55</td>
<td>LIT54</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1.0</td>
<td>GUN30</td>
</tr>
<tr>
<td>Hg vapour</td>
<td>350</td>
<td>0.50</td>
<td>BAD58</td>
</tr>
</tbody>
</table>

*Table 2.1.1* List of the upper limits of \( ap \) deduced from the point of deviation from the linear relation in the logarithmic plots of \( q \) against \( ap \) for various gases at different cathode fall potentials by various workers.
The axial current distribution for a sectionalised cylindrical hollow cathode (each section 2cm long of diameter 3cm) with one end closed was reported by Lompe, Seeliger & Wolter[LOM39]. While the distribution is fairly even at lower current(100mA), with increasing current(250mA) this is true only at high pressure(10torr). Generally the current maximum tends to occur near the closed end. Similar variations were also observed by Howorka & Pahl[HOW72] using a double open-ended cathode (2cm diameter; 1.4cm length), but at much lower pressure of (0.05-1)torr and current at (1-30)mA. Kirichenko et al.[KIR76] also demonstrated inhomogeneous axial current distribution in the double open-ended Ni hollow cathode (3cm diameter; 20cm length) with an anode at both ends operated in He at
(0.1-10) torr and 300 mA. However, minimal inhomogeneity (maybe fair homogeneity) could be seen to exist at the mid-pressure range of (0.2-0.5) torr. Maximum current is collected from the cathode sections nearest the anodes at high pressure (8-10) torr, but switches to the mid-sections of the cathode at low pressure (0.1-0.2) torr. On investigating the penetration depth of the current as a function of the operating pressure with only one anode at one of the open ends of the hollow cathode (similar in arrangement to the setup in the present work), a resonant-type of behaviour was observed. The ‘resonant’ peak (corresponds to maximum penetration depth) is observed to occur at around 2 torr independent of the discharge current; the penetration depth generally increases with the current for the entire range of pressure studied.

2.2.2 Voltage-current (V-j) characteristics

The voltage-current density (V-j) relationship was studied as a function of cathode length at fixed pressure of 1 torr in a He-Al hollow cathode discharge (1.8 cm diameter) by Lompe et al. [LOM39]. For the same current density, the sustaining voltage decreases with increasing hollow cathode length. The sustaining voltage in the hollow cathode geometry is much lower than that in the conventional planar cathode discharge; the difference being greater as the current density increases. The voltage rises more sharply with increasing current in the planar cathode than in the hollow cathode geometry, which has the an additional feature of approaching a limiting voltage as the current increases. This could be observed from the V-j curves.
of Lompe et al. shown in Figure 2.2.2. Grimm[GR68] and Döpel[DOP69] also demonstrated similar trends in their hollow cathode discharges with other gases.

Figure 2.2.2 The voltage-current density characteristics for a He-Al hollow cathode discharge[LOM39] (1.8cm diameter; variable lengths) at 1torr. Zero length corresponds to the planar cathode discharge.

If the ratio of the cathode length to its diameter is sufficiently large, such as in the hollow cathode discharge in Figure 2.2.2 (at 5cm length with $l_K:a=2.8$), negative resistance is manifested at the higher current density ($\geq 3mAcm^{-2}$). Musha[MUS62] attributed the occurrence of negative resistance at high current in a Ne-Mo hollow cathode discharge ($1\times1cm^2$ parallel plates at separation (0.05-0.16)cm) and $p=40torr$ to the copious sputtering of metal atoms from the cathode. This feature is to be distinguished from the occurrence of negative resistance at low current which was attributed to the presence of anode glow instead. Other workers have also reported this latter feature and attributed it to various phenomena: Zhiglinskii & Khlopina[ZHI72] to the transition from a Townsend discharge to the
glow discharge regime; White[WHI59] to the moment when the discharge jumped from the plane face of the cathode block into the interior of the nearly spherical hollow cavity; and Shuker et al.[SHU83] to Penning ionization effect in a Ne-Ca hollow cathode discharge by correlating with observations of the optogalvanic effect.

Another peculiar feature in the \( V-i \) curves is pointed out by Simon et al.[SIM91] in the Ar-Ni and Ar-Ti uncooled hollow cathode discharges (0.7cm diameter; 2.1cm length) operated at \((0.1-10)\text{torr}\) but in current range of \((15mA-1A)\) which envelops the glow until arc regimes. \( Ni \) has a magnetic phase transformation at \( 623K \) while \( Ti \) exhibits crystallographic \( \alpha \rightarrow \beta \) phase transformation at \( 1155K \). By monitoring the temperature at the inner surface of the hollow cathode simultaneously with the \( V-i \) curves, the magnetic phase transformation with \( Ni \) hollow cathode is identified at a minimum point at \(~30mA\) while the crystallographic phase transformation with \( Ti \) hollow cathode at a maximum point at \(~230mA\).

As mentioned in Section 1.3, the voltage-pressure(\( V-p \)) at fixed current can be used to determine the optimum pressure range of the hollow cathode discharge operation as defined by Kirichenko et al.[KIR76]. This range is identified to lie between points at which \( V_{\text{MIN}} \) and \( V_{\text{MAX}} \) occur. They observed that the optimum pressure range is independent of the cathode material, being governed solely by the filling gas. This optimum range gets smaller and shifts to occur at lower pressures for rare gases of higher \( Z \). It is deduced that the optimum pressure range which corresponds to a fully developed hollow cathode discharge is approximately \((0.5-0.6)\text{cm-torr} < p < (2.4-3.6)\text{cm-torr}\) in the He-Ni hollow cathode discharge (3cm diameter; 20cm length); and the range shifts slightly towards higher \( p \) as \( i \) increases.

\(^{\dagger}Z \) is the quantum number.
from 25mA to 200mA. This trend is consistent with those discussed in Section 2.2.1 on the basis of current magnification factor (assuming that higher discharge current is maintained at higher potentials). The two different definition of a fully developed hollow cathode discharge operation can thus be correlated positively and it shall be confirmed with further investigation carried out in the present work.

2.2.3 Radial and axial potential distribution

The electric field within the cathode dark space in a N$_2$-Al hollow cathode discharge (double cathode disks of diameter 2.4cm at separation 1.6cm) at 0.3torr was first measured using a beam deflection method by Little and von Engel[LIT54]. It was shown to fall linearly, as in the conventional discharge, from a very high value at the cathode surface to near zero in the glow and a major part of the discharge voltage appears across this dark space(see Figure 2.2.3). Badareu et al.[BAD60a] used probe measurements instead to compare the potential distribution in front of the hollow cathode with that at a planar cathode. Similar field distribution was also obtained by Takiyama et al.[TAK86] using the Stark-splitting method in a He-Fe hollow cathode discharge (0.4cm diameter; 1.8cm length) operated at 6torr, 15mA and 220V; with $E=4.4kVcm^{-1}$ at cathode surface falling to a small constant value of 0.25kVcm$^{-1}$ in the negative glow ($d_K$ estimated to be 0.95cm and $V_K$ at 210V). More recently, Sakai et al.[SAK91] also measured the electric field in a He hollow cathode discharge (2×2cm$^2$ parallel plates at separation 0.6cm) operated at 1.2torr, 40mA and 390V via laser-induced fluorescence detection and obtained similar variation with the maximum of 3.9kVcm$^{-1}$ at the cathode surface($d_K=0.17cm$ and $V_K=330V$).
Figure 2.2.3 The electric field as a function of position within the cathode separation for two cathode fall potentials deduced from measured beam deflexion[LIT54] at \( N_2 \) gas pressure 0.3torr; full line: \( V_K=376V, j=6\times10^{-3}Acm^{-2} \) and dashed line: \( V_K=318V, j=2\times10^{-3}Acm^{-2} \). The positions of the two cathodes are shown. The width of the dark space \( d_K \) is seen to be 0.42cm and 0.56cm respectively.

From the field distribution, the cathode dark space width \( d_K \) taken as the distance from the cathode surface to the edge of the negative glow can be determined. The boundary of the negative glow with the cathode dark space can then be defined at the point when the field reaches zero rather than by luminous inspection at which the gradient of glow intensity is steepest. The width \( d_K \) is independent[GUN30] of the gas pressure for a given cathode fall \( V_K \) and cathode diameter(or separation in the case of parallel cathode plates) under a fully developed hollow cathode discharge condition; and is later confirmed by Little & von Engel[LIT54] and Kirichenko et al.[KIR76]. \( d_K \) in a hollow cathode discharge is smaller than that in a conventional planar cathode discharge. This reduction in thickness results from a change of the space charge distribution (under hollow cathode discharge operation) which reduces the cathode fall potential when the
cathode is operated at constant current density; or increases the current density when operated at constant cathode fall voltage.

Figure 2.2.4 Radial profile of the plasma potential in a He-Ni cylindrical hollow cathode (3cm diameter; 20cm length) at 50mA[KIR76].

Depending on the operating conditions, the field in the negative glow of a hollow cathode discharge can be non-zero. This was shown by Pahl et al.[PAH72] and Kirichenko et al.[KIR76] in their determination of the radial potential distribution in the plasma of a cylindrical hollow cathode using probe measurements (see Figure 2.2.4). At low values of ap(0.9cm-torr), the potential has its maximum at the axis whereas at high values of ap(≥3.6cm-torr), the potential maximum moves outwards from the centre, generating a slight depression in the centre. These would affect the motion of the positive ions. The potential maximum at centre generates a force on the positive ions, causing them to move towards the cathode wall while the slight depression at centre would act as a weak trap for them. However, at ap values corresponding to the range of operation of a fully developed hollow cathode effect defined by Kirichenko et al., the potential still exhibits a spatial plateau reflecting a
near-zero field in the negative glow. On the other hand, according to Borodin et al. [BOR67a], the radial fields in the luminous region of a He hollow cathode discharge (2 cm diameter; 10 cm length) are small ($\leq 0.5V/cm$) which is confirmed by the $E$ field measurements of Takiyama et al. [TAK86] mentioned earlier.

Kagan and co-workers [BOR67a; BOR67b; GOF69] have investigated the axial potential distribution in the hollow cathode discharges (2 cm diameter; 10 cm length) in He at (0.5-5) torr and Ar at (0.1-2) torr and found that better homogeneity is achieved at higher pressures and currents. The longitudinal field is estimated to be very small in the central region, but increases slightly towards the edges of the hollow cathode. More recently, Mizeraczyk [MIZ87] investigated the axial distribution in a He-stainless steel hollow cathode discharge (0.5 cm diameter; variable lengths of 0.77, 1.55, 3.11, 4.93 cm) at (2.3-20) mbar and (10-150) mA as normally used for lasers. No axial electric field was observed along nearly the whole length of the hollow cathode when shorter cathodes (0.77 and 1.55 cm lengths) were used and operated at high pressures ($ap \geq 2.5 cm$-torr).

2.3 Electron energy distribution function and electron density

The first experiments to determine the electron energy distribution function in a double parallel plate cathode were performed by Badareu and Popescu [BAD58a]. Since then, many more related works have been reported especially in the 1960s and 1970s using different hollow cathode dimensions in various gases (or gas mixtures) and operating conditions. The measurements were mostly made using the Langmuir probe which limits the electron energy range up to (25-40) eV only. For energies up
to the cathode fall potential, electrostatic analyser was utilised[BOR67a] and the electron energy distribution function obtained for the whole range of electron energy is shown in Figure 2.3.1.

(a) Electron energy distribution function obtained from Langmuir probe measurement at 1torr.

(b) Electron energy distribution function obtained from electrostatic analyser measurement at 0.5torr and 1) 100mA; 2) 50mA in the energy range close to the cathode fall potentials indicated by the vertical lines.

Figure 2.3.1 An exemplary electron energy distribution function obtained for the whole range of the electron energy in a He hollow cathode discharge (2cm diameter; 10cm length)[BOR67a].

The electron energy distribution function measured in the hollow cathode discharge deviates considerably from Maxwellian. Generally, three main groups of electrons can be identified; namely (i) the very fast ones which undergo negligible collisions giving the peak close to the cathode fall potential; (ii) a medium energy
group, which are subdivided into roughly two ranges, the lower range from $5eV$ to
the first excitation energy of the filling gas and the higher range up to the detectable
limit by the Langmuir probe method; and (iii) a slow group $(0-5)eV$, more or less
thermalised at about $(0.5-2)eV$.

The very fast electron group was examined by Okmatovski[OKH78] and was
described as pendular electrons responsible for the high discharge currents attainable
with hollow cathode configuration. These are made up of those electrons which are
emitted from one side of the cathode surface and accelerated in the cathode fall to
reach the opposite cathode fall region without significant energy loss. Their
influence on the space charge and potential distribution is strongest inside the
cathode fall itself where these electrons are decelerated down to nearly zero velocity
upon reversal of the direction of motion[GUN23, SCH26]. They are capable of making
two or more traverses of the glow within the hollow cathode cavity. The effect of the
pendular electrons is enhanced through ionization inside the cathode fall, generating
secondary electrons which fall through part of the cathode fall and become pendular
electrons themselves. Experimental verification of the pendular effect was obtained
by Helm[HEL72] by incorporating a small hole into the surface of a cylindrical
hollow cathode. The electron energy distribution function of those electrons
penetrating the hole was measured by the electrostatic analyser. He showed that the
ratio of the number of pendular electrons to discharge electrons(of comparatively
much lower energy) decreased as the product of cathode diameter and pressure $ap$
increased as shown in Figure 2.3.2. Another independent proof of the existence of
the pendular electrons was given by Metel & Nastyukha[MET81] using a cylindrical
hollow cathode with a cylindrical foreign body in the centre of the cathode which
served to block the motion of the beam electrons. With an axial magnetic field $B$ the electron trajectories could be bent such that the electrons passed the foreign body. Above a certain threshold value of $B$, the discharge voltage at constant current was observed to drop significantly.

![Graph showing the ratio $F$ of pendular electrons to discharge electrons versus pressure in relative units.](image)

**Figure 2.3.2** Ratio $F$ of pendular electrons to discharge electrons versus pressure in relative units[HEL72].

Depending on the operation conditions, the lower energy range of the fast electron group sometimes show a plateau or slight peak in its electron energy distribution function. The appearance of the plateau/peak depends on $p$ as reported by Zhiglinskii & Khlopina[ZHI72] in an $Ar$ hollow cathode discharge (($0.75-0.8)cm$ diameter; $(1.8-2.0)cm$ length) and Mizeraczyk[MIZ83] in a $He$-stainless steel transverse-type hollow cathode discharge $(0.6cm$ diameter; $6cm$ length). Zhiglinskii & Khlopina claimed that the peak is at its highest (equivalent to largest number of fast electrons) at the optimum pressure of $0.3torr$ while Mizeraczyk observed the plateau at $(10-20)mbar$. It is also a function of the axial distance from the cathode centre; occurring usually at the edge of the cathode (nearest to the anode side) as in a
longitudinal *He-stainless steel* hollow cathode discharge\[MIZ87\] (0.5cm diameter; 3.11cm length) at \( p \leq 5\text{torr} \) and 50mA or with a coaxial anode as in a *He* hollow cathode discharge\[BOR67b\] (2cm diameter; 5cm length) at 0.5torr and 40mA. This leads to the problem of spatial inhomogeneity especially when a hollow cathode discharge is used for laser excitations. Fast electrons at this lower energy range are usually larger in number in a positive column than in the hollow cathode under the same conditions. At the higher energy range (that is, from the first excitation energy of the filling gas to the limit detectable by the Langmuir probe method), the reverse is true as shown in Figure 2.3.3 for the *He* hollow cathode discharge\[BOR66\] (2cm diameter; 10cm length) at 1.2torr and 20mA and their number generally decreases with increasing energy. *Borodin & Kagan* further showed that a peak in the variation of the number of the fast electrons (19-24eV) with pressure exists at \( ap \approx 2\text{cm-torr} \) and this was defined as the optimum mode in the hollow cathode discharge. Generally, it was shown that the number of fast electrons \( n_f \) in these energy ranges decreases as \( p \) increases\[TKA76, MIZ83\]; but contradictory results were shown on its variation with current \( i \) by different workers. *Tkachenko & Tyutyunnik*\[TKA76\] showed \( n_f \) to decrease as \( i \) increases in a *He-Ni* hollow cathode discharge (3cm diameter; 20cm length) at (0.3-0.5)torr and (10-50)mA while *Mizeraczyk & Urbanik*\[MIZ83\] reported \( n_f \) to increase with \( i \) in a *He-stainless steel* hollow cathode discharge (0.6cm diameter; 6cm length) at 7.5torr and (20-100)mA. It is likely that this is due to their operation of different types of hollow cathode discharge; the former of the longitudinal type whilst the latter in a transverse configuration (its ratio of length to diameter of cathode is larger than 7 which is out
of the range of the condition of hollow cathode discharge operation discussed in
Section 1.1).

![Graphs showing electron energy distribution functions in the negative glow of a He hollow cathode discharge (2cm diameter; 10cm length) and the positive column under similar conditions (p=1.2torr and i=20 & 40mA).[BOR66].](image)

*Figure 2.3.3* Comparison of the electron energy distribution functions in the negative glow of a He hollow cathode discharge (2cm diameter; 10cm length) and the positive column under similar conditions (p=1.2torr and i=20 & 40mA).[BOR66].

The slow electron group usually exhibits narrower peak and is larger in number density in the hollow cathode than in a similar positive column as evident from Figure 2.3.3. The mean energy $\bar{e}_s$ of these slow electrons depends on $p$ and $i$ as shown by Tkachenko & Tyutyunnik[TKA76] in a He-Ni hollow cathode discharge (3cm diameter; 20cm length) at (0.1-2)torr; $\bar{e}_s$ initially falls with increasing $p$ until a minimum at $p_{\text{MIN}}$=1torr; after which it increases with $p$ instead. This trend differs from the fast electron group(10-25eV) discussed earlier whose mean energy $\bar{e}_f$ exhibited a minimum at 0.15torr and a maximum at 0.4torr; after which it decreases with increasing $p$. $\bar{e}_s$ decreasing with $p$ is reflected in the change of the shape of the electron energy distribution function in the energy range(0–5eV) whereby the peak
gets higher and narrower as $p$ increases at fixed $i$[GOF69, MIZ83]. On increasing the
discharge current $i$, the peak of the electron energy distribution function broadens
and gets lower[GOF69, TKA76] and the mean energy $\bar{e}_s$ was also shown to
increase(except for low $p$ at 1.5 & 5torr which exhibited a valley at about 50-
60mA)[MIZ83], too

In fact, it is in the latter two energy ranges(slow and fast electrons) of the
electron energy distribution function that the importance of some collision processes
in the discharge could be deduced as shown by Mizeraczyk & Urbanik[MIZ83]. With
the use of only the Langmuir probe in the present thesis work, the electron energy
distribution function to be studied shall be limited to these energy ranges. Relating
these to the spectral emission characteristics, various important collision processes
can then be deduced for different hollow cathode parameters and conditions of
operation which may be linked to an efficient hollow cathode discharge. The
electron density $n_e$ measured in various hollow cathode discharge for $i$ up to 100mA
lies in the range of $(10^6-10^{13})cm^{-3}$ as shown in Table 2.3.1 and generally increases
linearly with $i$ at constant $p$. At constant $i$, the density $n_e$ increases with $p$ until a
maximum at $p_{\text{MAX}}$ (the measured values in some hollow cathode are listed in Table
2.3.1); after which it then decreases with increasing $p$. However, Belal &
Dunn[BEL78] using the laser heterodyne measurement of the electron density $n_e$,
observed a minimum at the lower pressure region as well and indicated the similarity
in behaviour to the relation between the cathode fall potential with pressure which
could perhaps be linked to the optimum pressure range. The results of Tkachenko &
Tyutyunnik[TKA76] also seemed to link the high end of the optimum pressure range
to the occurrence of maximum $n_e$. In comparison to the positive column under
similar operating conditions, Borodin et al.[BOR66] showed that the electron density in the hollow cathode is approximately three to seven times higher. The higher orders of magnitude of the charged particle density achievable in a hollow cathode discharge is definitely favourable for the cumulative ionization processes[STU64, STU67] as mentioned in Section 1.3.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Hollow cathode discharge</th>
<th>Range of i / mA</th>
<th>Range of n_e / cm⁻³ measured</th>
<th>(a p_{\text{MAX}} / \text{cm-torr at } i / \text{mA})</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOR66</td>
<td>(\text{He - ?} ) length:diameter=10.2 &amp; 5.1 (p=(0.9-3.5)\text{torr})</td>
<td>15 - 60</td>
<td>(10^{11} - 4.8\times10^{11})</td>
<td>(-2.5) at 20-60</td>
</tr>
<tr>
<td>GOF69</td>
<td>(\text{Ar - ?} ) length:diameter=10.2 (p=(0.1-2)\text{torr})</td>
<td>20 - 100</td>
<td>(5\times10^{10} - 10^{11})</td>
<td>2 at 50-100</td>
</tr>
<tr>
<td>TKA76</td>
<td>(\text{He - Ni} ) length:diameter=20.3 (p=(0.1-2)\text{torr})</td>
<td>10 - 50</td>
<td>(5\times10^{10} - 8\times10^{10})</td>
<td>2.4 at 50</td>
</tr>
<tr>
<td>BEL78</td>
<td>(\text{He - ?} ) length:diameter=10.0.6 (p=(1-15)\text{torr})</td>
<td>10-150</td>
<td>(3\times10^{12} - 4\times10^{13})</td>
<td>5.4 at 100†</td>
</tr>
<tr>
<td>MIZ87</td>
<td>(\text{He - stainless steel} ) length=0.77, 1.55 &amp; 3.11\text{cm}; diameter=0.5\text{cm} (p=(2.3-20)\text{mbar})</td>
<td>20 - 50</td>
<td>(10^{10} - 5\times10^{12})</td>
<td>5 at 40</td>
</tr>
</tbody>
</table>

Table 2.3.1  Range of \(n_e\) measured in various hollow cathode discharge and the value of \(a p_{\text{MAX}}\) (at which \(n_e\) is maximum obtained from the plot of \(n_e\) against \(p\).)  † A minimum is observed in this case at \(a p=1.8\text{cm-torr}\).  ? Hollow cathode material was not mentioned in the reference.

As was with the potential distribution, axial and radial inhomogeneities in the electron energy distribution function and, therefore, the average electron energy \(\varepsilon\) (usually estimated from the slow and fast electron groups only) and its density \(n_e\) within the hollow cathode have been shown to exist under certain operating conditions. On the radial profiles, Tkachenko & Tyutyunnik showed that within the optimum pressure range, \(\varepsilon\) is quite constant' at the central region but increases towards the wall of the cathode. The reverse is true beyond the defined-optimum range. The electron density \(n_e\), however, exhibits maximum value at the axis, decreasing towards the wall in the optimum range. Beyond that, the maximum
moves outwards forming a slight dip at the centre of the radial profile of $n_e$. These radial profiles of $n_e$ is rather similar to the radial profiles of the spectral intensity to be discussed in Section 2.6; but it is difficult to link them as they are obtained with different hollow cathode discharge operated under different conditions. It is hoped that the present work would be able to reconcile this. On the axial homogeneity, Mizeraczyk[MIZ87] concluded that it is achieved only in short cathodes (0.77 and 1.55 cm) for $ap \geq 2.5\,\text{cm-torr}$ in terms of the shape of the electron energy distribution function, the mean energy and density of electrons as well as the plasma potential. He also typified the electron energy distribution function in hollow cathode as having a narrow peak with low mean energy as compared to the ones obtained in the positive column which show broad peak with higher mean energy.

2.4 Ion energy distribution and ion density

Not many early workers have investigated the ion density and its energy distribution in a hollow cathode discharge; among the few who did is Bodarenko[BOD76] who examined the energy distribution of those ions falling on the base of a cylindrical cup cathode in an Ar hollow cathode discharge (3.2 cm diameter; 3.8 cm length) at 0.7 torr and 590 V. He found that a high proportion of $Ar^+$, and $Ar^{2+}$, attained more than 80% of the cathode fall potential. In contrast, the normal conventional glow discharge reported by Davis & Vanderslice[DAV63] and Bodarenko[BOD75] and the abnormal planar cathode discharge by Bodarenko[BOD73] exhibited a majority of the $Ar^+$ ions (and a smaller group of $Ar^{2+}$ in the latter case) having energies of less than half the cathode fall potential with only just a few able
to attain the maximum available. The principal factor in preventing ions from reaching the cathode with full accelerating voltage (upon passage through the cathode dark space) was considered to be due to symmetrical charge exchange [Lit54], in which, an electron could be transferred between a fast moving ion and an atom of the same gas resulting in a new ion of only thermal speed, and a fast atom having no electrical effect. Thus, in a hollow cathode discharge with presumably much shorter dark space and correspondingly much fewer collisions at any given pressure, the ions were able to attain high energies having undergone very little symmetrical charge exchange. The large proportion of high energy ions is probably a contributing factor to an increase of the production of secondary electrons of higher energy due to bombardment of ions on the cathode and this leads to the enhancement of the current density in the hollow cathode discharge as mentioned in Section 1.3.

Radial inhomogeneity is also manifested in the positive ion density distribution as shown by Howorka et al. [HOR73] in an Ar hollow cathode discharge (2 cm diameter) at 0.35 torr. The radial distribution of Ar$^+$ density exhibited a valley at the axis, then rose to a maximum before falling sharply somewhere mid-way between the axis and the wall of the cathode. The maximum moves outwards from the axis with increasing discharge current (small range studied: 2-4 mA) at constant pressure. The range of the Ar$^+$ density measured was $(0.5-2.5) \times 10^{10} \text{ cm}^{-3}$. Increase of pressure at constant current also shifts the maximum outwards. Kuen et al. [KUE81] showed a bell-shaped radial ion density distribution of the atomic He$^+$ and molecular He$_2^+$ ions with a peak at the axis in a He hollow cathode discharge (2 cm diameter; 4 cm length) at 1.7 torr and 25 mA. The bell-shaped density distribution was attributed
the losses suffered by ions, mainly by diffusion. Upon increase of the gas pressure to \( ap=8 \text{ cm-torr} \) at a fixed current, a change in the bell-shaped density distribution occurred only in the \( \text{He}^+ \) resulting in one with a flattened top. This led to the inference of additional loss mechanisms of \( \text{He}^+ \) being termolecular reaction to form \( \text{He}_2^+ \) and recombination with electrons. The density distribution of \( \text{He}_2^+ \) remained bell-shaped but showing a lower peak.

2.5 Spectral emission characteristics

As has been discussed, a central column of brightness from the coalesced negative glow would be expected under the hollow cathode effect regime. In a cylindrical hollow cathode, annular distribution of brightness of emission would be observed. Upon further increasing the current at constant pressure, visual impressions indicated that the emission intensified towards the edge of the annular dark space and the axial region became relatively less bright as shown by Lompe et al.[LOM39] in a gas mixture of \( \text{He-Ne-Hg} \) hollow cathode discharge (1.7cm diameter; 5cm length) at (0.5-20)torr and (2-75) mA. This spread is likewise observed upon increasing the gas pressure at fixed current. The different behaviour of different spectral lines (originating from different levels whose excitation function \( f_{\text{EXC}} \) are appreciable at different electron energies) adumbrates the connection between the electron energy distribution function and radial coordinates. For example, the lines \( \text{HeI 501.6 nm} \) (\( f_{\text{EXC}} \) appreciable at \( e>23 \text{eV} \)) whose profile shows radial spread and \( \text{Hg 435.8 nm} \) (\( f_{\text{EXC}} \) peak at \( e>8 \text{eV} \)) which dominates the axial region implies the preference of a larger proportion of the fast electrons inhabiting the regions at the
edge of the glow in the hollow cathode discharge (except at low gas pressure). Similar observation is reported by Gofmeister & Kagan [GOF68] for Ne I 667.8nm and 640.2nm in a Ne hollow cathode discharge (2cm diameter; 10cm length) at (1-5)torr but constant current. This deduction is confirmed with the measured electron energy distribution function at the axis and at 0.7cm off-axis in a He hollow cathode discharge of similar dimension at 1torr by Borodin et al. [BOR67a]. (However, it should be cautioned that in view of the depth of the light source as well as of possible variations in current density along the hollow cathode length, the manner of focussing the image will to some extend affect the accuracy of the resulting picture of the radial light distribution, though probably leaving the main features fairly clear.) In a later study on the behaviour of the radial profiles of a series of spectral lines involving excited He states (3≤n≤8) by Kuen et al. [KUE81] in a He hollow cathode discharge (2cm diameter; 4cm length) at (0.6-4)torr and (10-25)mA coupled with the time-resolved measurements (afterglows) and investigations on its ion balance, the dominating processes that lead to the population of excited states under different operating conditions had been inferred. The radial profiles of the ionic lines, on the other hand, is indicative of the spatial distribution of the very fast electrons in the discharge described in Section 2.4.

It is known that neutral energetic particles (photons and metastables) also play a role on the efficiency of a hollow cathode discharge [LIT54, STU67]. As their motion is independent on the electric field, the efficiency of utilizing these particles therefore depends on the solid angle under which the cathode surface is seen from different positions inside the negative glow. Intensity and spectral distribution of UV radiation from the negative glow and the material of the cathode contributes to
photo-emission. Due to the electron energy distribution function with high energy tail and higher electron density in a hollow cathode, it is expected that the influence of photoemission is more important in a hollow cathode discharge than in a conventional planar cathode discharge. Strong emission of VUV from hollow cathode discharge has been demonstrated by Holmgren et al. [HOL84] and Danzmann et al. [DAN85]. The importance of metastable atoms for the emission of electrons from the cathode was shown by Lawler et al. [LAW83] and high metastable densities (up to $10^{12} \text{cm}^{-3}$) in hollow cathode discharge [HOO77] have been measured. However, Sturges and Oskam [STU66] showed that the presence of metastables is not essential for the occurrence of a pronounced hollow cathode effect in the hollow cathode discharges of $H_2$ and noble gases with molybdenum cathodes ($3 \times 3 \text{cm}^2$ parallel plates of variable separation up to $3.2 \text{cm}$) at $(1-25)\text{torr}$.

The changing annular pattern should be fully taken into account and integrated in order to obtain the total emission $I_{\text{TOT}}$ at any chosen wavelength. Musha [MUS62] studied the variation of the total emission intensity with current and obtained a relationship of the form of $I_{\text{TOT}} \propto i^n$ in a Ne-Mo hollow cathode discharge ($4 \times 4 \text{cm}^2$ parallel plates at $0.4\text{cm}$ separation) at $6\text{torr}$. He showed that $n \equiv 1$ for Ne. This linear form is probable if the dominant excitation process is direct excitation to the upper level of the line by single electron collisions and that the electron energy distribution function does not change with current (though the total number at each energy may be greater). In cases where excitation occurs by multi-steps, the relationship would take quadratic (as in $I_{\text{TOT}} = ai + bi^2$) or higher terms. If the integration of the intensity over the cathode can be assumed to be satisfactory, a form for $I_{\text{TOT}}$ which is less than linear with current $i$ would suggest that a current-
dependent process quenches certain lines. Possible quenching was suggested (somewhat inconclusive due to insufficient data) by de Hoog & et al. [HOO77] on the observed saturation of emission at rather high currents in a situation of laser-type interaction with sputtered Cu.

Though much work has been carried out on the spectral emission aspect of the hollow cathode discharge, it can be seen that various conclusions as regard to the processes which dominate have been made in hollow cathode of different configurations and operating conditions. It is thus difficult to correlate this to the other characteristics of the hollow cathode discharge operation. It is hoped that this can be resolved with this thesis work.

2.6 Sputtering action

The hollow cathode discharge has been realised as ideal sources for the spectra of metals at temperatures much lower than required to evaporate these materials. The mechanism that generates the vapour pressure of these metals (either from the cathode material or a sample on the inside wall of the hollow cathode) is the sputtering action. The momentum of a bombarding ion or atom is transferred in a wall collision to one or a few surface atoms of the cathode material. These atoms are then reflected in the asymmetric potential at the surface. It is obvious that the sputtering coefficient has some threshold with respect to the momentum of the incoming ion and increases strongly with increasing atomic weight of the ion and ion energy as can be seen in Figure 2.6.1. The sputtering coefficient also varies significantly for different materials.
Figure 2.6.1 Yield curve for sputtering of Cu atoms by Xe\textsuperscript{+} ions (other metal-gas combinations give similar forms.)\cite{STU62}

In a hollow cathode these sputtered metal atoms either diffuse back to the wall or are ionized and return to the cathode surface as positive ions, thus contributing to further sputtering of the cathode. In cathodes of material with high sputtering coefficient (such as copper and molybdenum), this process can change the geometry until the cathode approaches a stable configuration observed by White\cite{WHi59} to be a near spherical cavity with aperture to sphere radius ratio of \(~0.25\). Such geometry has been applied to improve the lifetime of Cu\textit{II} lasers using Cu hollow cathode\cite{HOE84}. The sputtering rate is also a direct consequence of the current density in the hollow cathode. Areas in which material is removed have current densities larger than the average current density and the final stable geometry supports an almost constant current density inside the cathode.

The dominant influence of the sputtered metal vapour on the hollow cathode characteristic stems from the fact that the metal atoms have much lower ionization energies than the atoms or molecules of most buffer gases. If operated with rare
gases, Penning ionization can become a major contributing ionization mechanism[WHI59, MUS62].

Sputtering can cause the $V$-$i$ characteristic of the discharge to manifest negative resistance under higher current regime. This has been observed in a hollow cathode discharge operated in rare gases-$Mo$ combination by Musha[MUS62]. With increasing current, the density of sputtered atoms and ions increases until the dominant sputtering rate is caused by sputtering through the metal ions which are much heavier than the rare gas ions and resulting in the manifestation of negative $V$-$i$ characteristic. This behaviour is observed in the dependence of total radiation on the current as $I_{\text{TOT}} \propto i^3$ in a hollow cathode discharge (similar to the dependence of $Mo$ lines on current) as compared to that in a conventional planar cathode discharge as $I_{\text{TOT}} \propto i$ (similar to the dependence of $Ne$ lines on current). Sputtering action also causes hysteresis in the $V$-$i$ curves as observed by Salk & Schaefer[SAL77] in a discharge system with four parallel cylindrical $Cu$ cathodes.

Metal vapour densities have been measured for several metal-gas combinations in hollow cathode discharge and $Cu$ vapour densities up to $10^{14} \text{cm}^{-3}$ was detected in $Cu$ hollow cathode[HOO77]. These metal vapour densities also manifest radial profiles which are dependent on the pressure of the filling gas and discharge current. At low pressure the metal vapour density is almost uniform within the hollow cathode with a slight depression at the axis with maxima at the cathode surface[VEL84]. As the pressure is increased, the depression deepens sharply reaching zero density at high pressure. This differs from the $Ne$ metastable radial density profiles which showed peaks at the edges of the negative glow, dropping to low value next to the cathode surface with a depression at the axis(more pronounced at
higher pressures). Comparatively, the $Cu^+$ ion density is one order of magnitude lower than that of the $Ne^+$ ion and their sum is equal to the electron density as measured at the axis of the hollow cathode.

Metal atom sputtering is essential in a hollow cathode discharge intended for use in chemical analysis, but the conditions required to produce and control this in a variety of commercial lamp sizes (usually a few mm in diameter with length two to three times as large) have not been discussed sufficiently. On the other hand, much investigation of metal atom and ion sputtering in hollow cathode discharge for metal-vapour laser excitation has been carried out to determine and understand the essential processes governing the operation and performance of the particular laser. Diffusion of sputtered vapour is quite complicated, being dependent on changes of cathode temperature and on the nature of the metal more strongly than is that of an inert buffer gas.

2.7 Numerical modelling of the hollow cathode discharge

A review of the hollow cathode discharge would not be complete without a discussion on the numerical modelling of the basic processes responsible for the hollow cathode effect. Basically, the negative glow and the cathode fall regions are of interest in a hollow cathode discharge. Much modelling work has been performed in these regions using various numerical methods under different approximations as could be found in a review article on low-pressure gas discharge modelling by Lister[1982].
To model the hollow cathode discharge, the various contributing mechanisms to the hollow cathode effect as discussed in Section 1.3 has to be taken into account. A model by Kagan[KAG85] for a rectangular hollow cathode assumed that the pendular electrons are injected from an infinitely thin cathode fall into a negative glow with zero electric field. In this case, the ‘beam’ is modelled using only the first (continuity) moment of the Boltzmann equation and the electron energy distribution function and the Townsend coefficient $\alpha$ are computed.

Other workers preferred the Monte Carlo collision method which provides flexibility in describing the particles’ motion (at the expense of being rather computer-intensive). In their Monte Carlo collision simulations of He hollow cathode discharge with parallel plate cathodes, Boeuf et al.[BOE88,BOE90] showed that the pendular electrons explained the current enhancement as a function of gap separation quite well. The Monte Carlo collision computations by Schoenbach et al.[SCH90] in similar discharges at 0.15 and 0.5torr with 1cm gap separation showed that a large fraction of the electrons leaving one cathode strike the second cathode.

A sophisticated multi-temperature model (incorporating a collisional-radiative model) of a He cylindrical hollow cathode discharge (1cm diameter; 3.2cm length) at (1-3)torr was presented by Hashiguchi & Hasikuni[HAS87,HAS89]. Radial density profiles of excited states and charged particles as well as the $V$-$i$ relations are computed; the latter model made comparison with experimental results and uses the electron energy distribution function calculated via the Monte Carlo collision method[HAS88]. Good agreement with experiment is obtained for densities only at the axis of the hollow cathode. The computed densities have larger peak near the edge of the glow than the measured ones. Discrepancy is attributed to the assumed
linear electric field and consideration of collisions with ground state atoms only in calculating the electron energy distribution function. In an improved version, Hashiguchi\cite{HAS91} included the electron-electron collisions and collisions with atoms in the excited states, as well as collisions with atoms in the ground state to span over practically the entire range of the electron energy in the calculation of the electron energy distribution function in a helium hollow cathode discharge. He expects to obtain all kinds of rate constants from this electron energy distribution function that are more exact than from the two-temperature model, and the numerical modelling of the hollow cathode discharge can be improved.