4.1 Breakdown characteristics of the hollow cathode discharge

The relationship between the breakdown voltage $V_B$ and the product of the pressure and the anode-cathode separation $pd$ for various hollow cathode configurations is shown in Figure 4.1.1.

*Figure 4.1.1* $V_B$ as a function of the $pd$ for various cathode configurations as compared to the calculated *Paschen*’s relation.
The anode-cathode separation is assumed to be the shortest distance between the electrodes and fixed at 0.95cm (see inset of Figure 3.1.1). (This is not altogether true as the paths traced by electrons emerging from different locations on the hollow cathode surfaces vary in length upon reaching the anode.) Thus $pd$ is varied by changing the operating pressures of the helium gas. Plotted on the same graph for comparison is the Paschen's curve calculated from the Townsend theory of ionization[ENG65,WAR62] for planar parallel electrodes. It can be seen that the occurrence of the minimum voltage required for breakdown (Paschen's minimum) at $(pd)_{\text{MIN}}$ is independent of the cathode configuration, being constant at 3torr-cm. This $(pd)_{\text{MIN}}$ value is close to the calculated value given in Section 3.2.1.1. However, the corresponding minimum breakdown voltage $V'_{B(\text{MIN})}$ obtained for the various cathodes are found to increase in the following order:

$$V'_{B(\text{MIN})-\text{CALC}} < V'_{B(\text{MIN})-\text{PC}} < V'_{B(\text{MIN})-\text{HC6512}} < V'_{B(\text{MIN})-\text{HC1212}} < V'_{B(\text{MIN})-\text{HC1612}},$$

each at magnitude of 150V, 190V, 200V, 210V and 220V respectively. The subscript -CALC denotes the calculated plane-parallel electrodes configuration, -PC the planar cathode configuration with tubular anode, -HC6512, -HC1212 and -HC1612 are the respective cylindrical hollow cathodes with diameter 0.65, 0.8 and 1.6cm each, their lengths being fixed at 1.2cm. These observations confirm Miller's[MIL88] postulate that the breakdown voltage near and at the Paschen minimum, irrespective of electrode configuration, cannot be less than that obtained for planar electrodes. It is generally observed that the calculated breakdown voltages are lower than that obtained experimentally within the range of $pd$ values evaluated. This is expected as uniform electric field has been assumed in the calculation whereas the fields in the
Figure 4.1.2 The 2-D spatial distributions of the electric field mapped out between the electrodes for different configurations. The directional lines drawn on the upper half of each diagram represents the most likely paths to be taken by the electrons leaving various equally-spaced locations on the cathode surfaces. The assigned initial anode potential is fixed at 150V.
experimental configurations are non-uniform even in the case of the planar cathode as a tubular anode is used.

The non-uniformities of the electric field in the various cathode configurations used can be seen in the 2-D spatial electric field distribution mapping (shown in Figures 4.1.2(a)-(c) for PC, HC6512 and HC1612) carried out using iteration by the relaxation method as described in Section 3.2.1.2. The initially assigned anode potential of each map is $150V = V_{B(MIN)-CALC}$ whilst the cathode is at $0V$. This will yield a uniform electric field of $154Vcm^{-1}$ in the plane-parallel gap which corresponds to the maximum ionizing power $\eta_{max}$ (its reciprocal is the Stoletow constant) of the electrons occurring at an optimum reduced field of $E/p = 49Vcm^{-1}torr^{-1}$ (or $E = 155Vcm^{-1}$ at $pd = 3torr-cm$). The value of $\eta_{MAX}$ is deduced from the plot of the electron ionization coefficient $\alpha/p$ against the reduced field $E/p$ at the point where its tangent passes through the origin[ENG65] as shown in Figure 4.1.3.

![Figure 4.1.3](image)

Figure 4.1.3  Electron ionization coefficient $\alpha/p$ as a function of the reduced field $E/p$ for He. Inset (in linear scale) shows the maximum ionizing power $\eta_{MAX} = 1.2 \times 10^4$ ion pairs/V occurring at a point whose tangent to the curve passes through the origin[ENG65].
The upper half of each map traces the electric field lines. It is assumed that any electron which emerges from the cathode surface would generally traverse along the field lines until a collision occurs. It can be seen from Figures 4.1.2(a)-(c) that the spaces in between the anodes and the cathodes do not have continuous electric field regions of at least $155V_{cm^{-1}}$. These configurations become inefficient in causing breakdown. Thus, higher applied voltages are required to build sufficiently high electric fields to initiate breakdown.

Comparing the planar cathode PC with the hollow cathodes HC6512, HC1212 (not shown in the figure) and HC1612, the hollow cathode configurations have larger regions of low electric field ($<100V_{cm^{-1}}$) in the spaces between the electrodes. This contributes to lesser amount of ionizing collisions in the hollow cathodes when compared to the planar cathode. On the other hand, if comparison is made only among the hollow cathodes, the regions of low electric field ($<100V_{cm^{-1}}$) are found to increase with the hollow cathode diameter, thereby, resulting in $V_{B_{(MIN)}-HC6512} < V_{B_{(MIN)}-HC1212} < V_{B_{(MIN)}-HC1612}$.

The manifested tendency of the electric field line to converge onto the axis, especially in the case of the hollow cathode with the smallest diameter HC6512, may enhance the development of an electron beam which can contribute to more ionization along its path. (It has been demonstrated by Zeng et al. [ZEN83] that a glow discharge electron beam can be generated in helium at pressures $(0.15-0.8)torr$ with discharge currents at $(20-700)mA$ without differential pumping. In this case, the concave-shaped cathode served to shape the electric field in the cathode fall region so that a converging electron beam results.) This compliments the lower breakdown
voltage obtained in the HC6512 configuration. However, this effect is notably not sufficiently large enough to cause $V_{B(MIN)\,HC6512}$ to be lower than $V_{B(MIN)\,FC}$ in the present system.

When the initial anode potential is assigned according to the experimentally obtained value of the voltage at the Paschen's minimum for the respective cathode configurations, it can be shown that breakdown is then attainable. This can be deduced from Figures 4.1.4(a)-(c) which show the spaces linking the anodes and the cathodes in the various configurations to have continuous electric field regions larger than 155V cm$^{-1}$. Thus, the electrons are able to achieve at least the condition for maximum ionizing power, if not exceeding it.

At large $pd$ ($\geq 14$ cm-torr), the hollow cathode configurations tend to require lower voltages for breakdown when compared to the planar cathode arrangement. This is likely to be due to the space charges distortion to the electric field being more pronounced in the hollow cathodes, especially at high pressures. In the event of $E_0$ being a given applied field across a gap, then introducing a small space charge will change it to $E_0 + \Delta$, where $\Delta$ can either be positive or negative. The potential of the gap then changes from $V$ to $V + \Delta V$, where $\Delta V = \int_0^\infty \Delta dx$ and the function of the ionization coefficient (equation 3.2.4) becomes

$$\frac{\alpha_s}{p} = A \exp \left[ - B \left( \frac{p}{E_0 + \Delta} \right)^{1/2} \right]. \quad (4.1.1)$$
Figure 4.1.4 The 2-D spatial distributions of the electric field mapped out between the electrodes for different configurations. The directional lines drawn on the upper half of each diagram represents the most likely paths to be taken by the electrons leaving various equally-spaced locations on the cathode surfaces. The assigned initial anode potentials are varied according to their respective $V_{B_{\text{MIN}}}$.
Expanding equation (4.1.1) yields

\[
\frac{\alpha_s}{p} = \frac{\alpha_0}{p} \left[ 1 + \frac{1}{2} \left( \frac{B}{E_0} \right) \left( \frac{p}{E_0} \right)^{1/2} \Delta + \frac{1}{8} \left( \frac{B}{E_0} \right) \left( \frac{p}{E_0} \right)^{1/2} \left\{ \left( \frac{p}{E_0} \right)^{1/2} - 3 \right\} \Delta^2 \right] . \tag{4.1.2}
\]

Upon integrating the above equation, the change in the total number of ion pairs in the gap caused by the small space charge is then given by

\[
\int_0^t \alpha_s dx - \alpha_0 \Delta = \frac{1}{2} \alpha_0 B \left( \frac{p}{E_0^3} \right)^{1/2} \Delta V + \frac{1}{8} \alpha_0 B \left( \frac{p}{E_0^3} \right)^{1/2} \left\{ \left( \frac{p}{E_0} \right)^{1/2} - 3 \right\} \int_0^t \Delta^2 dx . \tag{4.1.3}
\]

As \( \int_0^t \Delta^2 dx \) is always positive irrespective of the polarity of the space charge, expression (4.1.3) will be positive only if \( B(p/E_0)^{1/2} - 3 > 0 \), that is, \( E_0/p < B^2/9 \).

The net result is that the total ionization in a gap increases through space charges when \( E/p \) is smaller than a value \( B^2/9 = 22 \text{Vcm}^{-1}\text{torr}^{-1} \) in helium gas with \( B \) taken as \( 14 \text{V}^{1/2} \text{cm}^{-1/2} \text{torr}^{1/2} \), thereby, facilitating breakdown of the gap. This corresponds to the point where the Paschen's curves for the hollow cathodes intersect with that of the planar cathode. At approximately \( 14 \text{cm-torr} \) and \( 280 \text{V} \) for HC6512 and PC, it gives an average \( E/p \) of \( 20 \text{Vcm}^{-1}\text{torr}^{-1} \). In the HC1212 and PC case, the average \( E/p \) value obtained at the intersection is \( 19 \text{Vcm}^{-1}\text{torr}^{-1} \) and that extrapolated from HC1612 and PC gives \( 18 \text{Vcm}^{-1}\text{torr}^{-1} \). Thus, at the extreme right side of the Paschen's minimum where the reduced field is low, the more pronounced presence of space charges in the hollow cathodes which facilitates breakdown dictates lower breakdown voltages than that for the planar cathode case.

It is observed that the breakdown voltage depends also on whether the laboratory room is lighted or not. With the fluorescent lamp switched on,
breakdown occurs at less than 330\textit{V} for all the conditions and cathode configurations studied. When the room is dark, the applied voltage can exceed 1\textit{kV} without initiating any breakdown. Thus it is likely that the first few electrons emitted from the cathode surfaces are due to the UV radiations from the fluorescent lamps in the laboratory and it is imperative that the conditions in the laboratory be kept constant throughout the experimental investigation of the breakdown characteristics.

The condition of the surface of the cathode is known to affect the breakdown characteristics greatly. Thus it is also important to keep the surfaces of the electrodes (especially the cathodes) and the glass tube (the inner surface in contact with the gas) free from any coating due to the sputtering of the cathode material or deposition of impurities from the gas. Otherwise, the $V_b - pd$ curves will be greatly distorted as shown in \textit{Figure 4.1.5}. The coating or contamination on the surface of the electrode changes its work function which in turn will affect the second Townsend ionization coefficient $\gamma$, a contributing factor to the breakdown criterion. ($\gamma$ is also a strong function of the bombarding ion energy; but it is nearly constant at energies less than a few hundred $eV$ as in the present setup.) The electric field distribution will be affected with the coating of the electrode material on the glass surface making it slightly conducting and this will also change the breakdown characteristics. A qualitative analysis of the coating on the glass window is discussed in \textit{Section 5.3}. It is also known\cite{FRA56} that the presence of patches of charges on the walls of the chamber, perhaps remaining over from a previous discharge and prevented from dispersing by their mutual attractions or surface conditions can give rise to capricious behaviour of the breakdown voltages.
However, this could be remedied by running a rod or the hand over the outside of the vessel.

\[ V_B / V \]

\[ \text{"coated" cathode} \]

\[ \text{--- calc} \]
\[ \text{--- PC} \]
\[ \text{--- HC6512} \]
\[ \text{--- HC1612} \]

\[ pd / \text{cm-torr} \]

Figure 4.1.5 Paschen's relation for the various cathode configurations as compared to the calculated one. In this case, the cathode surfaces are 'coated'.

4.2 Voltage-current (V-i) characteristics of the hollow cathode discharge after breakdown

The relationship between the sustaining discharge voltage \( V_A \) and its current \( I_A \) after breakdown is investigated at different operating pressures for all the hollow cathode configurations listed in Table 3.1.1. These \( V-i \) curves are then compared to
that obtained for the planar cathode discharge with the anode-cathode separation kept constant at 0.95cm. The discharge current is not allowed to exceed 60mA as serious distortion may occur to the $V$-$i$ characteristics. The sustaining voltage at a particular discharge current is highly dependent on the cathode temperature and the dependence is more pronounced at higher currents[STU64]. This can be expected as variation in cathode temperature results, for a fixed gas pressure, in a local change in gas density in the cathode region, where production of majority of charged particles occur[WHI59]. This in turn reduces the value of $E/p$ requiring a higher voltage to maintain the same amount of ionization; which probably accounts for the positive $dV_A$ obtained (discussed in the next paragraph). The problem of the cathode temperature fluctuation is further minimised by using water-cooled cathodes.

However, from the observed "hysteresis loop" shown in Figures 4.2.1 for cathode configurations HC812, HC848 and PC as the difference $dV_A (= V_{A-DCR} - V_{A-INC})$, where $V_{A-INC}$ is recorded when $i_A$ is increased and $V_{A-DCR}$ on decreasing $i_A$), the problem of the cathode being heated up still persists. (Other hollow cathode configurations show similar trend.) Large differences $|dV_A|$ (>5%) are found to occur mostly at low operating current and pressure; being extremely large for the planar cathode case. Sturges and Oskam[STU64] have shown that the sustaining voltage increases with the cathode temperature in a He-stainless steel hollow cathode discharge (9cm diameter disc at 1.5cm separation) at $ap=6cm$-torr and discharge currents at 25, 90 and 185mA; the rate being larger for higher current. (†At 25mA, a decrease in $V_A$ is recorded when cathode temperature increases from 30°C to 50°C exhibiting an initial negative rate.) In the present case, as the discharge current $i_A$ is increased to a maximum of 60mA, the cathode may have acquired a relatively high
Figure 4.2.1 Variation of the difference \( dV_A = V_{A,\text{DCR}} - V_{A,\text{INC}} \) in % of the average sustaining voltage with the discharge current \( i_A \) at different \( He \) pressures as indicated for cathode configurations HC812, HC848 and PC. Other hollow cathode configurations are not shown as they exhibit similar behaviour.
temperature which is not dissipated fast enough on the decreasing path of the current though it is water-cooled. In other words, the cathode is at a higher temperature on the current reducing path and it would indeed be expected to obtain larger temperature differences at the lower current range than at the high end due to the procedure in taking the measurement. (No attempt was made to measure the cathode temperature; but the outer surface of the hollow cathode arrangement felt hot to touch after a particular set of $V$-$i$ at a fixed pressure was measured - the inner surface expected to be hotter.) This implies that the design for the dissipation of heat through water-cooling is not efficient enough; being especially so for the planar cathode arrangement (as larger $|dV/A|$ is exhibited when compared to the hollow cathodes). The gas which is enclosed in the hotter cathode cavity may also have attained a higher temperature which could slow down the mobility of the ions (which is governed by the thermal velocity of the gas atoms and expected to vary inversely as the square root of the absolute temperature as shown by Chanin & Biondi[CHA57] for $He^+$) especially at low values of $E/p$. The sluggishness of the ions increases the resistivity which may lead to higher voltages required in sustaining the same current; but this effect is considerably small as positive difference $dV_A$ is negligible at the high pressures. On the other hand, the negative difference $dV_A$ observed (occurring mostly at approximately $(2-12)mbar$) may be due to the negative rate as demonstrated by Sturges & Oskam. Hotter cathodes may also enhance the emission of electrons upon bombardment by positive ions, resulting in lower voltages being required to sustain the same current. It is thus important to allow the discharge to attain equilibrium at each current so that high reproducibility of conditions may be achieved.
Another contributing factor to the variation of the sustaining voltage at fixed discharge current (at the same operating pressure) is the sputtering process of the cathode\cite{WH69, SAL77} as discussed in Section 2.6. Though the choice of the filling gas, helium in this case, has minimized this process (as gas atoms of mass close to that of the metal atoms are the most efficient sputters), it is still substantial especially at low pressures in the hollow cathodes as evident by the thickness of the deposition on the glass windows which will be discussed in Section 5.3. White correlated the changes in the sustaining voltage with the appearance of fresh surfaces of different work functions as sputtering proceeds and major transformations in the internal configuration of the cathode. He then developed a dimensionally stable hollow cathode configuration which balances between sputtering and deposition.

It is observed that at some of the very low pressure operation of the discharge (especially in the planar cathode case), the discharge prefers to strike a longer path from the anode to the buffer chamber (which is close to ground potential as it is connected to the vacuum pump via metal connectors) about 20cm away when the sustaining voltage is increased beyond a certain value. Thus, the discharge path no longer exists between the anode and the designated cathode. This preference shows that the shorter discharge path is much too resistive because it provides less area density of gas for the likelihood of ionizing collisions in sustaining the discharge current.

On the \textit{V-i} relationships for all the cathode configurations at helium pressure range of $p=(0.5-25)\text{mbar}$, it is generally observed that the range of sustaining voltages for a fixed hollow cathode length increases with its diameter $2r_K$. This pressure range together with the hollow cathode dimensions \((0.24\text{torr-cm} \leq 2pr_K \leq \)
30torr-cm) would ensure the study of its characteristics to include the range of applicability of hollow cathode discharge operation (see Section 1.1) and beyond. On increasing the length or depth of the hollow cathode cavity at fixed diameter, the range of the sustaining voltage tends to decrease. This can be explained by the fact that as the hollow cathode diameter increases (or its depth decreases), the hollow cathode discharge tends to assume the characteristics of the planar cathode discharge as has been established by Little & von Engel[2]. Thus, a longer but narrower hollow cathode is favourable for an efficient discharge in terms of lower potential required to sustain the same current. This is due to the better ability of the hollow cathode geometry to trap the particles (charged and neutral) contributing to more ionization. The resulting field distribution in the hollow cathode geometry is also conducive for pendular electrons to form.

The $V-i$ characteristics of the hollow cathode discharge as compared to that of a planar cathode discharge are shown in Figures 4.2.2 for 0.5($V-i$ for PC was not obtained), 2, 8 and 25mbar on a semi-logarithmic scale. For $p<8$mbar, the $V-i$ curves for the hollow cathode discharge shift to higher sustaining voltages (at constant current) or to larger discharge currents (at constant voltage) as their diameters are increased but their lengths kept constant (or on shortening their lengths at fixed diameters). This shift is most distinctive at the lowest pressure studied (0.5mbar). At the higher pressure range (8 - 25)mbar, the $V-i$ curves for the various hollow cathode discharges tend to merge together and are close to the $V-i$ curve for the planar cathode discharge, especially at low currents. It can be deduced that the hollow cathode discharge is not prominent at the higher pressure range of $p\geq$
Figure 4.2.2  Semi-logarithmic plot of the V-i curves for the various hollow cathodes as compared to the planar one PC at 0.5, 2, 8 and 25 mbar. Distinguishable shift of the V-i curves with respect to the hollow cathode diameters is exhibited until 8 mbar, beyond which the curves tend to merge together.
8mbar (equivalent to maximum $2pr_K$ of 9.6torr-cm which is near the upper limit of applicability of hollow cathode discharge operation) in the present setup.

The $V$-$i$ curves obtained can generally be sorted into various shapes. They are (a) $V_A$ rises with $i_A$, the slope getting more gradual as $i_A$ becomes larger; (b) similar to (a) except that a plateau (and the slope may go negative) at the higher current region is observed; and (c) $V_A$ decrease as $i_A$ increases initially showing a minimum, after which $V_A$ then increases with $i_A$. These categorized shapes can also be clearly discerned in terms of its dynamic resistance $dV/di$ variation to be discussed in Section 4.2.3. The range of operating conditions in relation to these categorized shapes are listed in Table 4.2.1. When compared to the optimum pressure range to be discussed in Section 4.2.1, shapes (a) and (b) are more likely to be characteristic of a fully developed hollow cathode discharge. The voltage variation typified by a minimum in shape (c) is quite unlikely to indicate magnetic phase transformation [SIM91] mentioned in Section 2.2.2 though stainless steel hollow cathode is used in this setup ($Fe$ and some of its alloys are known to display transition between ferromagnetism and paramagnetism at the Curie temperature - $700^\circ C$ for pure $Fe$ and at slightly lower values for some of its alloys) as the hollow cathode is water-cooled and would not have been able to attain such a high Curie temperature. As these minima usually occur at low currents, they are more likely to be related to the transition from a Townsend-to-normal-to-glow discharge regimes. Comparatively, the shapes of the $V$-$i$ curves displayed by the planar cathode discharge differ from that of the hollow cathode discharge. Its voltage is observed to increase steeply with current within the range of operating conditions studied; the
slope getting less steep with increasing pressure, but is still steeper than those obtained with hollow cathode.

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Table 4.2.1 Range of operating pressures under which the three categorized shapes of the $V$-$i$ curves are observed for the various hollow cathode configurations.

↑ takes shape (c) with minimum occurring at lower current.

↑↑ approaches the behaviour of the said shape.

### gives the value of the discharge current at the minimum of shape (c); and the value the discharge current beyond which a plateau is observed in shape (b).

Increasing the interelectrode distance $d_{AK}$ to 1.95cm with the HC6512 did not seem to significantly change the $V$-$i$ characteristics for the same current and pressure range observed. However, upon very large increment of $d_{AK}$ to 10cm as in a hollow cathode HC1620 (1.6cm diameter; 2cm length[CH187]), the $V$-$i$ curves display different behaviour in which at sufficiently high pressure, the sustaining voltage decreases with increasing current. These are shown in Figures 4.2.3. Comparison is made here with a shorter separation at $d_{AK}=5.8cm$ and the influence of the extended length of the positive column on the $V$-$i$ curves is observed. It is clear that an extended length of the positive column requires higher sustaining voltage(almost
Figure 4.2.3  \( V-i \) (\( p \) fixed) curves of other hollow cathode configurations for comparison.
doubled) which suggests the existence of an axial potential gradient (or axial electric field) in the positive column. This was confirmed in Reference [CHI87] whereby the floating potential at different axial points along the positive column was measured and the axial electric field was found to increase with pressure, $0.5V/cm$ at $1torr$ to $60V/cm$ at $20torr$ in the HC1620 ($d_{Ak}=100mm$). The presence of the extended positive column may have resulted in negative gradient in the $V-i$ curves (over the entire range of current measured) especially at sufficiently high operating pressure which is a conducive condition for the growth of the positive column[LOE39]. This helium-stainless steel hollow cathode discharge ($1.6cm$ diameter; $2cm$ length) operated at $30cm-torr$ and $(4-60)mA$ should be distinguished from the negative resistance obtained in the Ne-Mo one ($1cm^2$ parallel plates at $(0.05-0.08)cm$ separation) operated at $(2-3.2)cm-torr$ and $(1-80)mA$ by Musha[MUS62] which was attributed to sputtering action. (Also to be noted is the operation condition of $2prK=30torr-cm$ which exceeds the upper limit of the applicability condition for hollow cathode discharge described in Section 1.1.) It is therefore shown that the presence of positive column in the discharge can mask the true properties of the hollow cathode discharge as was reported by Sturges & Oskam[STU84] on the presence of anode fall.

4.2.1 **Identifying the fully developed hollow cathode discharge regime**

*from the V-p (i fixed) plots*

It is believed that the definition of the optimum pressure range for the hollow cathode discharge should be based on the fundamental properties or behaviour (mentioned briefly in Section 1.4) adopted for other gas discharges such as $V-i$
characteristics and the cathode fall potential $V_K$, which in turn, is governed by the ratio of the cathode current density $j$ to the square of the pressure $p^2$. Though an unambiguous relationship cannot be specified among these three parameters in a hollow cathode discharge, the function $j(p)$ can be examined at a fixed value of $V_K$ or function $V_K(p)$ at a fixed value of $j$ of which the latter one is easier to manage. (At a fixed value of $V_K$ a change in the gas pressure is accompanied by a marked change in the power drawn by the discharge and, thus, by marked changes in the gas temperature and the state of the cathode surface. This makes it comparatively difficult to manage experimentally.) Thus, if the discharge is maintained to cover the entire interior of the hollow cathode and with the short interelectrode separation ensuring the sustaining voltage $V_A$ to be close to the cathode fall potential $V_K$, the hollow cathode effect can be manifested clearly by plotting the sustaining voltages against the operating pressures at fixed discharge currents as shown in Figures 4.2.4.

It has been identified by Kirichenko et al.[KIR76] that the hollow cathode effect is most prominent at the mid-range described as the optimum pressure range mentioned in Section 2.2.2 where the sustaining voltage $V_A$ increases with increasing pressure $p$. This feature distinguishes it from the planar cathode discharge in which the sustaining voltage always decreases on increasing the pressure at fixed currents within the pressure range studied (0.5–25) mbar. The boundary limits of this optimum pressure range and its corresponding sustaining voltages are shown in Figures 4.2.5 for the various discharge currents and hollow cathode configurations used in the present system. The lower limits of the optimum pressure range especially for HC1248 and HC1648 cannot be defined because the minima in the $V$-
Figure 4.2.4 Plots of $V_A$ against $p$ at fixed discharge currents at 0.5, 5, 20 and 50 mA for various cathode configurations showing the defined 'optimum pressure range' corresponding to the region where $V_A$ increases with $p$. 
Figure 4.2.5  Boundaries of the defined 'optimum pressure range' and its corresponding sustaining voltage limits.
$p$ curves are not clearly manifested. The following general observations concerning
the dependence of the *optimum pressure range* (in terms of $ap$/torr-cm) and the
corresponding sustaining voltage $V_A$ range (in $V$) on the hollow cathode
configuration and discharge current are made (excluding that for HC1248 and
HC1648).

(l) **Fixed cathode length $l_K$ :-**

(a) Variation of upper and lower limits of $ap$ range{or corresponding $V_A$
range};

(i) shift to higher values as cathode diameter $a$ is increased {similarly
for the corresponding $V_A$ range},

(ii) shift to higher values as discharge current $i$ is increased (agrees with
results of *Kirichenko et al.*[KIR76]) - **exception** at $l_K=4.8cm$ where
lower limit value does not change significantly {similarly for the
corresponding $V_A$ range, approaches saturation at high current
region}.

(b) Variation of width of $ap$ range{or corresponding $V_A$ range};

(i) expands as cathode diameter $a$ is increased - **exception** for HC1612
whose ratio of cathode length to its diameter $l_K:a < 1$, smallest width
is manifested {the corresponding $V_A$ range shrinks as cathode
diameter is increased},

(ii) expands as discharge current $i$ is increased (agrees with results of
*Kirichenko et al.* [KIR76]) {similarly for the corresponding $V_A$
range}.
(II) Fixed cathode diameter \( a \): 

(a) Upper and lower limits of \( ap \) range shift to lower values as cathode length \( l_k \) is increased (similarly observed for the upper and lower limits of the corresponding \( V_A \) range);

(b) Width of \( ap \) range expands for longer cathode length \( l_k \) (similarly observed for the width of the corresponding \( V_A \) range; but at \( a=0.65 \text{cm} \), the width is approximately the same for both cathode lengths).

(These observations are confined to \( i \geq 5 \text{mA} \) for the shorter hollow cathode and \( i \geq 20 \text{mA} \) for the longer hollow cathode in order to ensure that the glow within the hollow cathode is fully formed.) The lower limit of the defined optimum pressure range is observed to take \( ap \) values from \( 0.5 \text{cm-torr} \) to \( 5.4 \text{cm-torr} \) while the upper limit from \( 3.7 \text{cm-torr} \) to \( 10 \text{cm-torr} \) for all the hollow cathode configurations and discharge currents \( (0.5-60) \text{mA} \) investigated. Though the upper limit agrees with the applicability conditions for hollow cathode discharge operation discussed in Section 1.1, the lowest limit extends a little beyond. This disagreement of the lower limit is also observed in the results shown by Kirichenko et al. [KIR76] in a He-Ni hollow cathode discharge (3cm diameter; 20cm length) exhibiting optimum range at \( (0.5-3.6) \text{cm-torr} \) with discharge currents at \( (25-200) \text{mA} \).

On plotting the width of the optimum pressure range \( W_p \) (in torr) \{or the width of the corresponding \( V_A \) range \( W_V \) (in \( V \)} against the current density \( j \) (in \( \text{mA/cm}^2 \)) results in exponential relationships estimated as

\[
j \propto \exp \left[ (0.53 \pm 0.02)W_p \right], \tag{4.2.1}
\]

\[
j \propto \exp \left[ (0.044 \pm 0.002)W_p \right], \tag{4.2.2}
\]
from *Figures 4.2.6(a)* and (b) respectively. The widths $W_p$ and $W_V$ are observed to approach saturation values as the current density $j$ is increased. ($W_p$ and $W_V$ values for $i<5mA$ at hollow cathode length $l_K=1.2cm$ and $i<20mA$ at $l_K=4.8cm$ are excluded as their current density values cannot be accurately estimated; bands of $W_p$ values are indicated for HC1248 and HC1648 as their lower limits are expected to be non-zero but slightly lower than 0.38torr {whereas $W_V$ for HC1248 and HC1648 are excluded as the lower limits could not be estimated}. Including on the graphs for comparison are the results obtained from Kirichenko et al.[KIR76]. Their width $W_p$ values are lower and do not coincide with the above-deduced relationship (the current density range studied was small). Contrary to their claim that the optimum pressure range is independent of the cathode material, these differences could be attributed to the different hollow cathode materials used. This could be observed at $j=14.1mAcm^{-2}$ where the Fe hollow cathode shows larger width $W_p$ than the Mo or Ni cathodes. In comparison, stainless steel hollow cathodes are used in the present system although the same filling gas is used for all. This deduction is, however, inconclusive as too few results obtained using other cathode materials are compared with. {The corresponding width $W_V$ from the results Kirichenko et al. also do not agree with the present experimental except for Fe hollow cathode. This is expected as the sustaining voltage corresponding to the optimum pressure range can be affected by the hollow cathode material.}

Thus, it can be seen that the features above that distinguish the hollow cathode discharge from the planar cathode discharge seem to be governed by the dimensions and material of the cathode as well as the gas pressure. These, in turn, would control the conditions under which the electrons and ions move in the cathode.
Figure 4.2.6  Semi-logarithmic plot of:
(a) width of the optimum pressure range $W_p$,
(b) corresponding width of the sustaining voltage $W_V$,
against the current density $j$ for the various hollow cathode configurations
(He-stainless steel) as indicated.  
Results from Kirichenko et al. [KIR76] are included for comparison.  (He-Ni
combination used unless indicated otherwise.)
cavity. Making a numerical estimate for the helium hollow cathode discharge, the minimum mean free path against ionization for helium $\lambda_{ei}$ (in cm) at room temperature (corresponding to the maximum ionization probability\cite{BRO59}) is given as

$$\lambda_{ei} = \frac{0.86}{p},$$ (4.2.3)

where $p$ is in torr. For helium pressure of 2mbar (in the case of HC6512 at 5mA), this mean free path is 0.57cm which is almost three times larger than the width of the cathode dark space $d_k$ at 0.21cm (deduced from the radial intensity distribution measurement in Section 5.2.3). Thus at the observed minimum in the function $V_A(p)$, electrons from the cathode are assumed to traverse the cathode dark space with little or no collision, entering the negative glow with an energy close to $eV_k$ and causing ionization primarily here. Assuming that the ratio $R$ of the exciting to ionizing collisions is approximately constant (say 10\%) and that all excitations are to the first excited level $V_{exc}$ at 19.82V, the number of ionization events $n_i$ caused by one such electron in the cathode cavity can be written as

$$n_i = \frac{V_k (1-k_i)}{V_i + RV_{exc}},$$ (4.2.4)

where $k_i$ is the fraction of the energy carried off by fast electrons as the result of their escape from the cathode cavity (related to the probability of escape of the electron) and $V_i$ the first ionization potential of helium at 24.58V. Denoting the fraction of ions which arrive at the cathode from the negative glow as $\delta$, the condition for steady-state discharge can then be expressed as

$$\gamma \delta \frac{V_k (1-k_i)}{V_i + RV_{exc}} = 1.$$ (4.2.5)
or rearranging, \[ V_k = \frac{V_i + RV_{ec}}{\gamma \delta (1 - k_i)} \] \hspace{1cm} (4.2.6)

where \( \gamma \) is the second Townsend ionization coefficient. The minimum of the function \( V_A(p) \) apparently corresponds to the optimum conditions for ionization in the cavity and for secondary processes at the cathode. Setting \( \delta = 1 \) and \( k_i = 0.27 \) (ratio of the open ended area to the enclosed surface area of the cathode) for HC6512 and \( \gamma \approx 0.2 \) for helium ions, \( V_A \approx 182V \). The minimum sustaining voltage obtained experimentally is \( V_A = 220V \). In assuming zero electric field in the negative glow of the hollow cathode discharge, \( V_k \) is estimated at an average of \( 25V \) (from Figure 5.1.6(f)) lower than the sustaining voltage \( V_A \). It is likely that the fraction \( \delta \) of ions arriving at cathode from the negative glow is less than unity as some could have escaped the cathode cavity. Furthermore, uncertainties also exist in the values of \( R \) and \( \gamma \). Within these adjustments, equation \( (4.2.6) \) may then be satisfied. Since \( k_i \) increases with the diameter of the hollow cathode at fixed length, the corresponding value of \( V_k \) will also increase and this agrees with the variation of the lower limit of the sustaining voltage corresponding to the optimum pressure range with cathode diameter described earlier. On the other hand, increasing the length of the hollow cathodes (from HC#12 to HC#48) would decrease the chances of escape of the electrons, thereby resulting in lower values of \( V_k \) confirming the experimental observations.

As will be discussed in Section 5.2.3, the cathode dark space width \( d_k \) varies as a function of the pressure \( p \) (or \( ap \)). \( d_k \) generally decreases with \( p \) except when the hollow cathode discharge is fully developed, where it remains constant. Little & von Engel[LI54] and Takiyama et al[TAK86] have shown that the electric field \( E \) falls
linearly from the cathode to zero at the boundary between the cathode dark space and the negative glow in the hollow cathode discharge. In order to maintain a constant rate of ionization as at fixed discharge current $i_A$, the reduced field $E/p$ should also be kept at the same value. Since $d_K$ is constant in the optimum pressure range, the sustaining voltage $V_A$ (assumed to be close to the cathode fall potential and the average field is taken as $\bar{E} = V_A/d_K$) would increase with $p$ in order to keep the reduced field $E/p$ constant. Though $V_A$ falls as $p$ is increased outside the optimum pressure range, $d_K$ also decreases so as to keep $E/p$ constant in order to maintain the same discharge current.

4.2.2 Current density magnification $j/j_0$ in the hollow cathode discharge

The current density magnification $q = j/j_0$ is obtained for various fixed discharge voltages $V_A$ at an interval of $5V$ or $10V$ from $200V$ to $325V$ for hollow cathodes of lengths $l_K = 1.2cm$ and $4.8cm$. $j$ is the hollow cathode discharge current density (= ratio of the discharge current $i_A$ to the effective surface area of the hollow cathode) whilst $j_0$ is the current density drawn when the hollow cathode is replaced by a planar cathode keeping the anode-cathode separation constant and using the same anode, both being operated at the same discharge voltage $V_A$ and pressure $p$. The current density magnifications $q$ at $V_A = 200$ & $300V$ plotted as functions of the product of the hollow cathode diameter and operating pressure $ap$ for both cathode lengths on the log-log scale are shown in Figure 4.2.7. Two regions of dependence can be distinguished in the graph, namely, the linear dependence region which exists at the lower $ap$ values and a low constant value region at the higher end. The linear
dependence region shows that the current density magnification decreases linearly as the diameter-pressure product $ap$ increases on the logarithmic scale for low $ap$ and becomes independent of $ap$ at large $ap$. This implies that smaller hollow cathode diameter at constant pressure is more efficient in enhancing the current density magnification. This is because at constant cathode fall potential $V_k$ (it is assumed that the anode fall potential is negligibly small such that most of the voltage $V_A$ applied across the electrodes constitutes $V_k$), smaller hollow cathode diameter at constant $p$ reduces the cathode dark space width $d_k$, thereby increases the field and hence the density of ions. It also causes the ions to strike the cathode with larger velocity and the current density rises rapidly. It is believed that the hollow cathode discharge is fully developed in this linear region. Generally, the slopes tend to a constant mean value of $-(2.64 \pm 0.06)$ which agree closely with the mean value of $-2.56$ given in Figure 7 of Reference[GLO64] as well as the computed universal value of $-2.50$ given in equation (3.2.19) (differing only by 6%). From Figure 4.2.7, the upper limits of these linear regions(defined at either the point of deviation from the linear portion or when $j/j_0 \leq 1$) are estimated at $ap=4.5cm$-torr for $V_A=200V$, increasing to $6cm$-torr for higher sustaining voltage at $300V$. The increase of this upper limit with the sustaining voltage agrees with those observed in a He-Fe hollow cathode discharge ($50cm^2$ parallel plates at 1cm separation) at $250V$ and $400V$ by Güntherschulze[GUN30] shown in Table 2.2.1 and is consistent with the observations made in the upper limit determined from the $V-p$ curves in Section 4.2.1 assuming that the higher sustaining voltage is required to maintain higher current. The values of the upper limits are well within the specified limit of the condition of applicability for hollow cathode discharge operation given in Section 1.1.
The low constant value region occurring at high $ap$ values indicates that the hollow cathode effect is minimal as the negative glow breaks up (it becomes annular in a cylindrical hollow cathode and can no longer sustain a full central column with maximum intensity at the axis as could be seen in Section 5.2.1). It is then said that the planar cathode discharge characteristics dominates here.

From the relationship between the intercept values $inct$ (of the curves of $\log(j/j_0)$ against $\log(ap)$ at $2rp = 1$) and the discharge voltages $V_A$ shown in Figure 4.2.8, the current density magnification $j/j_0$ is generally found to increase with the discharge voltage $V_A$ at fixed ($ap$).
4.2.3 Dynamic resistance $dV/di$ in the hollow cathode discharge

The dynamic resistance $dV/di$ is determined from the slope of the $V-i$ characteristic curves. Its variation with current for various operating gas pressures and hollow cathode configurations (shown for the smallest and largest hollow cathode diameters as in HC6512 & HC1612 and HC6548 & HC1648) together with the planar cathode arrangement are shown in Figures 4.2.9.

The planar cathode discharge exhibits no negative slope in the current and pressure range studied. Its dynamic resistance decreases steeply with increasing current at the low current range (<10mA) for a fixed gas pressure. At larger currents, it tends to fall very gradually (being almost of constant value). It is also observed that the slope decreases with increasing gas pressure at fixed currents. Within the discharge current range (0.022–60)mA and pressure range (1–25)mbar,
Figure 4.2.9: Variation of the dynamic resistance with the discharge current at fixed pressure for HC6512, HC1612, HC6548, and HC1648 as compared to that for PC.
its slope values vary from $6\,\Omega$ at low current and pressure $(0.022\,mA, 1\,\text{mbar})$ to $2\,k\Omega$ at larger currents and pressures $(i>55\,mA, p\geq 20\,\text{mbar})$. This differs from the hollow cathode configurations operated in the current range $(0.5\sim 60)\,mA$ and pressure range $(1\sim 25)\,\text{mbar}$, where large positive dynamic resistances (up to $76k\Omega$) occur usually at low currents, mid-pressure range $(0.75\,mA, p=(4\sim 8)\,\text{mbar})$ and larger hollow cathode diameter at fixed length (or shorter hollow cathode length at fixed diameter) supports higher positive value. Low positive values (sometimes close to zero) usually occur at higher currents irrespective of pressure. It can generally be concluded that the discharge gets more resistive as it approaches the planar cathode discharge.

At high pressures $(ap\geq 10\,\text{cm-torr};$ beyond the range for hollow cathode discharge operation) and low currents $(i_A\leq 4\,mA)$ for the hollow cathode configurations, negative dynamic resistances are manifested (up to $-26k\Omega$). It is likely that these negative slopes stem from the operation of the discharge in the subnormal region (corresponding to the transition from the Townsend discharge to the glow discharge regimes mentioned in Section 2.2.2). Visual inspection of the glow indicates that the discharge is partially formed - filling only part of the hollow cathode and may even be displaced towards one side of the wall. At low pressures $(0.24\,\text{torr-cm} \leq ap \leq 0.9\,\text{torr-cm})$, negative resistance (up to $-7k\Omega$) occur at higher currents up to $20\,mA$ (higher for smaller hollow cathode diameter and longer hollow cathode length.) This could likely be attributed to sputtering action described by Mushi[MUS62] as sputtering (deduced from the thickness of the coating on the glass window at one end of the hollow cathode discussed in Section 5.2.4) is found to be intense at low pressures and high currents. However, this is not conclusive as the
emission lines (only the visible range was scanned) resulting from the material of the hollow cathode (constituents of stainless steel) is not detected although some of these elemental constituents did show positive results upon a qualitative analysis utilizing the EDAX (Energy-Dispersive X-ray Analysis) method via the SEM (Scanning Electron Microscope) of the deposition on the glass window (discussed in Section 5.3). Very low negative dynamic resistances (up to $-0.55k\Omega$) are manifested at the mid-pressure range ($1.2\text{cm-torr} \leq ap \leq 7.2\text{cm-torr}$) and high currents $\geq 35mA$. This $ap$ range which is within the hollow cathode discharge operation (already discussed in Section 4.2.1) plus the high current condition is more compatible with those described by Musha[MUS62] which was related to sputtering action (discussed in Section 2.6). However, it is not supported by the sputtering observations whereby the deposition on the glass window within these conditions are not as intense as those at low $p$ but high $i_A$; nor by the spectral emission measurements which exhibits mostly $He_I$ lines (shown in Section 5.2.2) showing variation of total intensity being approximately proportional to $i_A$. 

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