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**INVESTIGATION OF MASS DEPENDENCE IN PARTICLE-
ANTIPARTICLE MIXING WITHIN THE FRAMEWORK OF
THE STANDARD MODEL**

by

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ABSTRACT

In the Standard Model, the mixing present in a general meson-antimeson $P^0 - \bar{P}^0$ system occurs at second order in the weak interactions through the box diagrams. The box diagrams also play an important role in explaining the observed $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings and CP violation in the $K^0 - \bar{K}^0$ system. In order to study mixing, a calculation of the effective Hamiltonian for the box diagram amplitude describing the $P^0 \leftrightarrow \bar{P}^0$ transition is required.

Earlier calculations of the effective Hamiltonian for the $K^0 - \bar{K}^0$ system was first done for two generations of quarks by making the approximation of zero external momentum and negligible internal quark masses compared to the W boson mass. Later, this calculation was extended to include three generations of quarks. For the $B_d^0 \leftrightarrow \bar{B}_d^0$ and $B_s^0 \leftrightarrow \bar{B}_s^0$ transitions, the effective Hamiltonian was first calculated directly from the W^\pm exchange box diagrams by ignoring the momentum dependence of the W^\pm propagators. A more detailed calculation of the effective Hamiltonian by taking into account the momentum and mass of the external b quark has also been considered.

The aim of this study is to investigate the mass dependence in particle antiparticle mixing in the $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ systems within the framework of the Standard Model. In order to do this, we have to first calculate the amplitudes of all the diagrams that contribute to the $P^0 \leftrightarrow \bar{P}^0$ transition amplitude, where P is either B_d or B_s . The internal quarks for the $P^0 \leftrightarrow \bar{P}^0$ transition are the u , c and t quarks. The B_d^0 meson is composed of a b quark and a \bar{d} quark, while

the B_s^0 consists of a b quark and a \bar{s} quark These calculations are performed in the 't Hooft-Feynman gauge, and the amplitudes of the diagrams are obtained by neglecting the mass and momentum of the light d or s quark.

The effective Hamiltonian for the $P^0 \leftrightarrow \bar{P}^0$ transition is then obtained by summing up the individual amplitudes and the final expression contains two form factors, B_{ij} and C_{ij} which are expressed as integrals over the x variable. These form factors are respectively attached to a $V - A$ and $S + P$ type operator. From the effective Hamiltonian, the off-diagonal mass matrix and decay matrix elements, $M_{12} + \frac{1}{2}\Gamma_{12}$ are obtained by sandwiching the effective Hamiltonian between the P^0 and \bar{P}^0 states. M_{12} and Γ_{12} are respectively the absorptive and dispersive parts for the $P^0 \leftrightarrow \bar{P}^0$ transition. The hadronic matrix elements of the $S + P$ and $V - A$ operators appearing between the P^0 and \bar{P}^0 states are estimated using the Vacuum Saturation Method and it is found that the form factors come in the following linear combination: $A_{ij}^{(\alpha)} = B_{ij}^{(\alpha)} - \frac{5}{8}C_{ij}^{(\alpha)}$, $\alpha = a, d$ where a and d respectively stand for the absorptive and dispersive parts of the effective Hamiltonian arising from the box diagrams and the indices i, j represent the internal u, c or t quark. The absorptive and dispersive parts are evaluated analytically. An absorptive part is generated whenever the quadratic functions in the logarithms become negative and this occurs when $i, j = u, c$. The calculation of the absorptive part is straightforward and the final expression for $A_{ij}^{(a)}$ is symmetric with respect to i and j .

On the other hand, a dispersive part is generated whenever the quadratic functions in the logarithms are positive. The resulting expression for $A_{ij}^{(d)}$ is also

symmetric but is found to be more lengthy. Both $A_{ij}^{(\alpha)}$ and $A_{ij}^{(d)}$ demonstrate a strong dependence on the internal quark masses.

The analytical expressions for M_{12} and Γ_{12} are then obtained and evaluated and it is found that $|\Gamma_{12}| \ll |M_{12}|$. The dominant contribution to M_{12} occurs when the top quarks are the internal quarks. It is also seen that the mass difference, ΔM reduces to $\Delta M = 2|M_{12}|$ and that the width difference, $\Delta\Gamma$ is very small. The parameters r and χ which describe particle-antiparticle mixing in the $B_d^0 \leftrightarrow \bar{B}_d^0$ and $B_s^0 \leftrightarrow \bar{B}_s^0$ transitions are calculated. For the $B_d^0 - \bar{B}_d^0$ system, the values are found to be $r_d = 0.34$ and $\chi_d = 0.25$. However for the $B_s^0 - \bar{B}_s^0$ system, $r_s \approx 1$ and $\chi_s \approx 0.50$. Thus it can be concluded that mixing is more substantial in the $B_s^0 - \bar{B}_s^0$ system than in the $B_d^0 - \bar{B}_d^0$ system.

ABSTRAK

Di dalam Model Piawai, percampuran yang hadir di dalam suatu sistem am meson-antimeson $P^0 - \bar{P}^0$, berlaku pada peringkat kedua dalam tindak balas lemah menerusi gambarajah-gambarajah kotak. Gambarajah - gambarajah kotak memainkan peranan penting dalam menerangkan pencerapan percampuran dalam $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$ dan $B_s^0 - \bar{B}_s^0$ perlanggaran CP dalam sistem $K^0 - \bar{K}^0$. Untuk mengkaji percampuran, satu pengiraan Hamiltonian berkesan bagi amplitud gambarajah kotak yang memerlukan peralihan $P^0 \leftrightarrow \bar{P}^0$ perlu dilakukan..

Pengiraan terdahulu Hamiltonian berkesan bagi sistem $K^0 - \bar{K}^0$ telah dibuat bagi dua generasi kuark dengan membuat andaian momentum luaran sifar dan jisim dalaman kuark boleh diabaikan berbanding jisim W boson. Kemudian, pengiraan ini telah dilanjutkan untuk merangkumi tiga generasi kuark. Untuk peralihan $B_d^0 \leftrightarrow \bar{B}_d^0$ dan $B_s^0 \leftrightarrow \bar{B}_s^0$, Hamiltonian berkesan telah dikira secara langsung daripada gambarajah kotak W^\pm exchange dengan mengabaikan pergantungan momentum W^\pm propagator. Satu pengiraan yang lebih terperinci untuk Hamiltonian berkesan dengan mengambil kira momentum dan jisim kuark luaran b juga telah dibuat.

Tujuan kajian ini ialah untuk menyiasat persandaran jisim dalam percampuran zarah antizarah di dalam sistem $B_d^0 - \bar{B}_d^0$ dan $B_s^0 - \bar{B}_s^0$ dalam rangka Model Piawai. Untuk melakukannya, kita perlu mengira amplitud semua gambarajah kotak yang menyumbang kepada peralihan amplitud $P^0 \leftrightarrow \bar{P}^0$, di mana P ialah B_d atau B_s . kuark dalaman untuk peralihan $P^0 \leftrightarrow \bar{P}^0$ adalah kuark-kuark u , c dan t .

Meson B_d^0 adalah terdiri daripada satu kuark b dan satu kuark \bar{d} , manakala meson B_s^0 terdiri daripada satu kuark b dan satu \bar{s} kuark. Pengiraan ini dilakukan dalam tolok 't Hooft-Feynman dan amplitud gambarajah kotak didapati dengan mengabaikan jisim dan momentum kuark ringan d atau s .

Hamiltonian berkesan untuk peralihan $P^0 \leftrightarrow \bar{P}^0$ kemudiannya didapati dengan menjumlahkan semua amplitud individu dan ungkapan akhir didapati mengandungi dua faktor iaitu B_{ij} and C_{ij} yang ditinggalkan dalam bentuk kamiran terhadap pembolehubah x . Kedua-dua faktor tersebut masing-masing terikat kepada operator jenis $V - A$ and $S + P$. Daripada Hamiltonian berkesan, unsur-unsur *off-diagonal mass matrix* dan *decay matrix*, $M_{12} + \frac{1}{2}\Gamma_{12}$ didapati dengan meletakkan Hamiltonian berkesan antara keadaan-keadaan P^0 dan \bar{P}^0 . M_{12} dan Γ_{12} masing-masing dikenali sebagai bahagian absorptif dan dispersive untuk peralihan $P^0 \leftrightarrow \bar{P}^0$. Unsur-unsur matriks hadronik untuk operator-operator $S + P$ dan $V - A$ yang hadir antara keadaan-keadaan P^0 dan \bar{P}^0 dianggarkan menggunakan Kaedah *Vacuum Saturation* dan didapati bahawa kedua-dua faktor wujud sebagai kombinasi linear seperti berikut: $A_{ij}^{(\alpha)} = B_{ij}^{(\alpha)} - \frac{5}{8}C_{ij}^{(\alpha)}$, $\alpha = a, d$ dimana a dan d masing-masing dikenali sebagai bahagian absorptive dan dispersive untuk Hamiltonian berkesan yang dihasilkan daripada gambarajah-gambrajah kotak dan indeks i, j mewakili kuark dalaman u, c, t . Bahagian absorptif and dispersif dikira secara analitik.

Bahagian absorptif dihasilkan apabila fungsi kuadratik dalam logaritma mengambil nilai negatif dan keadaan ini berlaku apabila $i, j = u, c$.

Pengiraan untuk bahagian absorptif adalah mudah dan ungkapan akhir $A_{ij}^{(a)}$ adalah bersimetri untuk i dan j .

Bahagian dispersif pula dihasilkan apabila fungsi kuadratik dalam logaritma menjadi positif. Ungkapan akhir untuk $A_{ij}^{(d)}$ juga bersimetri untuk i dan j tetapi lebih rumit. Kedua-dua $A_{ij}^{(a)}$ dan $A_{ij}^{(d)}$ menunjukkan pergantungan yang kuat terhadap jisim kuark dalaman.

Ungkapan-ungkapan analitik bagi M_{12} dan Γ_{12} kemudiannya dihasilkan dan dikira dan didapati bahawa $|\Gamma_{12}| \ll |M_{12}|$. Sumbangan utama kepada M_{12} berlaku bila kuark atas menjadi kuark dalaman. Juga didapati beza jisim, ΔM boleh dipermudahkan kepada $\Delta M = 2|M_{12}|$ dan *width difference*, $\Delta\Gamma$ adalah sangat kecil. Parameter-parameter r dan χ yang memerihalkan percampuran zarah antizarah dalam peralihan $B_d^0 \leftrightarrow \bar{B}_d^0$ dan $B_s^0 \leftrightarrow \bar{B}_s^0$ dikira. Bagi sistem $B_d^0 - \bar{B}_d^0$, nilai-nilai yang didapati adalah $r_d = 0.34$ dan $\chi_d = 0.25$. Tetapi untuk sistem $B_s^0 - \bar{B}_s^0$, $r_s \approx 1$ dan $\chi_s \approx 0.50$. Oleh itu, boleh disimpulkan bahawa percampuran adalah lebih diutamakan di dalam sistem $B_s^0 - \bar{B}_s^0$ berbanding sistem $B_d^0 - \bar{B}_d^0$.

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