

APPENDIX

DIMENSIONAL REGULARIZATION

Dimensional regularization [27] is widely used in calculations of radiative corrections. Since an analytic continuation in the space-time dimensions is not unique, there is a variety of conventions in this method. We adopt the following convention:

$$Tr(1) = 4 \quad (A.1)$$

$$\{\gamma_\mu, \gamma_5\} = 0 \quad (A.2)$$

and $\int \frac{d^n k}{(2\pi)^4}$ in n dimensional space. Below we list basic formulas in our convention.

The basic algebra is

$$\{\gamma_\mu, \gamma_5\} = 2g_{\mu\nu} \quad (A.3)$$

The metric tensor satisfies

$$g_{\mu\nu}g^{\mu\nu} = n. \quad (A.4)$$

Combining (A.1) and (A.2), we obtain

$$\gamma_\lambda \gamma^\lambda = n \quad (A.5)$$

$$\gamma_\lambda \gamma_\mu \gamma^\lambda = (2-n)\gamma_\mu \quad (A.6)$$

$$\gamma_\lambda \gamma_\mu \gamma_\nu \gamma^\lambda = 4g_{\mu\nu} + (n-4)\gamma_\mu \gamma_\nu \quad (A.7)$$

$$\gamma_\lambda \gamma_\mu \gamma_\nu \gamma_\rho \gamma^\lambda = -2\gamma_\rho \gamma_\nu \gamma_\mu + (4-n)\gamma_\mu \gamma_\nu \gamma_\rho. \quad (A.8)$$

Further, by using our convention on unit matrix 1,

$$Tr(1) = 4 \quad (A.9)$$

we find

$$Tr(\lambda_\mu \lambda_\nu) = 4g_{\mu\nu} \quad (A.10)$$

$$Tr(\lambda_\mu \lambda_\nu \lambda_\lambda \lambda_\rho) = 4(g_{\mu\nu}g_{\lambda\rho} + g_{\mu\rho}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\rho}) \quad (A.11)$$

As mentioned above, the γ_5 matrix is defined so that it satisfies Eq.(A.2). There occurs no trouble concerning the γ_5 matrix in the present case, since the Weinberg-Salam theory is an anomaly-free theory.

The Feynman parametrization is needed to combine a product of several different quadratic factors appearing in the denominators of the momentum integral. For an arbitrary number of factors, the Feynman parametrization is given by :

$$\frac{1}{a_1 a_2 \dots a_n} = (n-1)! \int_0^1 U_1^{n-2} dU_1 \int_0^1 U_2^{n-3} dU_2 \dots \int_0^1 dU_{n-1} \times \left[(a_1 - a_2)U_1 \dots U_{n-1} + (a_2 - a_3)U_1 \dots U_{n-2} + \dots + a_n \right]^{-n} \quad (A.12)$$

A special case of Eq.(A.11) used in our calculation is

$$\frac{1}{ab} = \int_0^1 dx [b + (a-b)x]^{-2} \quad (A.13)$$

After a Feynman parametrization of the propagators and a shift of the momentum variables, the momentum integrals reduce to an integral of the form:

$$I(m, r) = \int \frac{d^n \tilde{k}}{(2\pi)^n} \frac{(\tilde{k}^2)^r}{(\tilde{k}^2 - R^2)^m} \quad (A.14)$$

This Minkowski space integral is performed after a Wick rotation into Euclidean space and we obtain the basic formula:

$$I(m, r) = \int \frac{d^n \tilde{k}}{(2\pi)^n} \frac{(\hat{k}^2)^r}{(\tilde{k}^2 - R^2)^m} = \frac{i}{(16\pi^2)^{n/4}} (-1)^{r-m} (R^2)^{r-m+n/2} \\ \times \frac{\Gamma(r+n/2)\Gamma(m-r-n/2)}{\Gamma(n/2)\Gamma(m)} \quad (\text{A.15})$$

By symmetrical integration, it can easily be proved that:

$$\int \frac{d^n \hat{k}}{(2\pi)^n} \frac{\tilde{k}_\mu \tilde{k}_\nu}{(\tilde{k}^2 - R^2)^m} = \frac{1}{n} g_{\mu\nu} \int \frac{d^n \tilde{k}}{(2\pi)^n} \frac{\tilde{k}^2}{(\tilde{k}^2 - R^2)^m} \quad (\text{A.16})$$

and

$$\int \frac{d^n \tilde{k}}{(2\pi)^n} \frac{\tilde{k}_\mu \hat{k}_{\mu_1} \hat{k}_{\mu_2} \dots \tilde{k}_{\mu_q}}{(\tilde{k}^2 - R^2)^m} = 0 \quad \text{for } q \text{ odd.} \quad (\text{A.17})$$

The Gamma function $\Gamma(x)$ has the following properties:

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma + O(\varepsilon) \quad (\text{A.18})$$

$$\Gamma(\varepsilon - 1) = -\frac{1}{\varepsilon} - (1 - \gamma) + O(\varepsilon) \quad (\text{A.19})$$

where $\gamma = 0.5772$ is the Euler constant.