

## CHAPTER 1

### INTRODUCTION

Since the original observation of  $CP$  violation in the neutral kaon system by Christenson, Cronin, Fitch and Turlay in 1964 [1], neutral pseudoscalar systems such as  $K^0 - \bar{K}^0$ ,  $D^0 - \bar{D}^0$ ,  $B_d^0 - \bar{B}_d^0$  and  $B_s^0 - \bar{B}_s^0$  have been studied extensively [2-3]. Besides attempting to understand  $CP$  violation in itself, one of the motivations for this is to study the parameters which account for particle-antiparticle mixing in neutral meson systems. In the Standard Model, the mixing phenomenon occurs when a neutral meson,  $P^0$  transforms into its antiparticle,  $\bar{P}^0$  through the well known box diagrams. In order to study mixing, we need to first calculate the effective Hamiltonian for the box diagram amplitude describing the  $P^0 \leftrightarrow \bar{P}^0$  transition. From the effective Hamiltonian, the off-diagonal mass matrix and decay matrix elements can be found and the mixing parameters can then be calculated.

For the  $K^0 - \bar{K}^0$  system, the effective Hamiltonian was first calculated by Gaillard and Lee [4] for two generations of quarks. They also made the approximation of zero external momentum and negligible internal quark masses compared to the  $W$  boson mass. Later, Inami and Lim [5] extended their calculation to three generations of quarks and gave the exact analytical form of the effective Hamiltonian in the  $R_\xi$  gauge, which was very useful for analyzing the  $K^0 \leftrightarrow \bar{K}^0$  and  $D^0 \leftrightarrow \bar{D}^0$  transitions. However it was not obvious whether the Inami-Lim effective

Hamiltonian can be applied to the  $B_d^0 \leftrightarrow \bar{B}_d^0$  and  $B_s^0 \leftrightarrow \bar{B}_s^0$  transitions because the external  $b$  quark mass is quite heavy and may not be neglected.

For the  $B_d^0 \leftrightarrow \bar{B}_d^0$  and  $B_s^0 \leftrightarrow \bar{B}_s^0$  transitions, the effective Hamiltonian was first calculated from the  $W^\pm$  exchange box diagrams with the approximation that the  $W$  mass is very large [6-7]. Detailed calculations without such approximation were carried out by several authors subsequently [8, 9, 10].

In this thesis, we extend on these calculations to investigate the mass dependence of the mixing parameters in the  $B_d^0 - \bar{B}_d^0$  and  $B_s^0 - \bar{B}_s^0$  systems. To begin with, an expression of the effective Hamiltonian for the  $B_d^0 \leftrightarrow \bar{B}_d^0$  and  $B_s^0 \leftrightarrow \bar{B}_s^0$  transitions is first obtained in the 't Hooft-Feynman gauge. The off-diagonal mass matrix and decay matrix elements,  $M_{12} + \frac{1}{2}\Gamma_{12}$  are later obtained from the effective Hamiltonian using a procedure due Gaillard and Lee [4]. The absorptive and dispersive parts arising from these transitions are evaluated analytically. These are then applied to study the mixing phenomena in the  $B_d^0 \leftrightarrow \bar{B}_d^0$  and  $B_s^0 \leftrightarrow \bar{B}_s^0$  transitions.

The organisation of this thesis is as follows. In Chapter 2, we review and summarize the  $SU(2) \times U(1)$  Weinberg-Salam theory and its application to the leptonic sector. A brief discussion of the extension of the  $SU(2) \times U(1)$  theory to the quark sector, to include the GIM mechanism [30] and the three generations of quarks is also presented. The Feynman rules in the 't Hooft-Feynman gauge, which is required for our subsequent calculations is also presented at the end of this chapter. In Chapter 3, we consider in detail the amplitudes of the Feynman diagrams which

contribute to the  $B_d^0 \leftrightarrow \overline{B}_d^0$  and  $B_s^0 \leftrightarrow \overline{B}_s^0$  transitions in the 't Hooft-Feynman gauge. The effective Hamiltonian is then obtained from these amplitudes.

In Chapter 4, we demonstrate the technique of obtaining the off-diagonal mass matrix and decay matrix elements from the effective Hamiltonian. Details of the procedure used to obtain analytical expressions for the dispersive and absorptive parts are also shown. The behaviour of the form factors is also demonstrated. In Chapter 5, we discuss the application of our calculation to study mixing in the  $B_d^0 - \overline{B}_d^0$  and  $B_s^0 - \overline{B}_s^0$  systems. In Chapter 6, we present the conclusions of this study. The Dirac algebra is listed in the Appendix. Also collected in the Appendix is a list of useful general formulae for loop momentum integration in  $n$ -dimensional Euclidean space.