CHAPTER 3

THE EFFECTIVE HAMILTONIAN FOR THE $P^0 \leftrightarrow \overline{P}^0$ TRANSITION

3.1 INTRODUCTION

In this chapter, we consider in detail the $P^0 \leftrightarrow \overline{P}^0$ transition of neutral meson systems in the Standard Model where P is either B_d or B_s. The B_d^0 meson is composed of a b quark and a $\overline{d}$ quark while the B_s^0 meson is composed of a b quark and a $\overline{s}$ quark. In Section 3.2, we calculate the amplitudes arising from the Feynman diagrams for the $P^0 \leftrightarrow \overline{P}^0$ transition in the 't Hooft-Feynman gauge. These calculations are performed in $n = 4 - \varepsilon$ space time dimensions, adopting the dimensional regularization procedure to control divergences. Finally in Section 3.3, we derive the expression for the effective Hamiltonian for the $P^0 \leftrightarrow \overline{P}^0$ transition.

3.2 THE AMPLITUDES FOR THE $P^0 \leftrightarrow \overline{P}^0$ TRANSITION

The P^0 meson is composed of one heavy quark Q, and one light quark q. Box diagrams contributing to the $P^0 \leftrightarrow \overline{P}^0$ transition in the 't Hooft-Feynman gauge are shown in Fig. 3.1

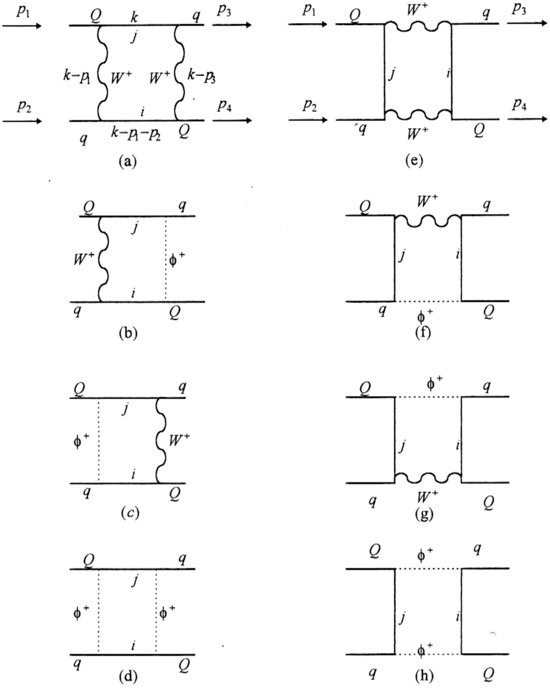

Fig. 3.1. Box diagrams contributing to the $P^0 \leftrightarrow \overline{P}^0$ transition in the 't Hooft-Feynman gauge

In Fig. 3.1, Q represents the external b quark while q denotes either the d or s quark. p_1, p_2 are the incoming momenta while p_3, p_4 are the outgoing momenta. Both the Q quark and q quark are on mass shell. The indices i, j represent the internal u, c or t quark. As our calculations are being done in the 't Hooft-Feynman gauge, we have to take into account the contribution from the unphysical charged Higgs scalar, $\phi^{\pm}$ (Fig. 3.1(b), (c), (d), (f), (g) and (h)) in addition to the W exchange diagrams (Fig. 3.1(a) and (b)).

We shall now calculate the amplitude for the box diagram in Fig. 3.1(a). From the Feynman rules in Chapter 2, the amplitude, M_a is given by

$$M_a = \frac{ig^4}{4} \sum_{i,j} \lambda_i \lambda_j \int \frac{d^4k}{(2\pi)^4} \bar{u}(3) \gamma_\lambda L \frac{1}{\not{k} - m_j} \gamma^\tau L u(1) \bar{v}(2) \gamma^\tau L \frac{1}{\not{k} - \not{p}_1 - \not{p}_2 - m_i} \gamma^\lambda L v(4)$$

$$\times \quad \frac{1}{[(k-p_1)^2 - M_w{}^2][(k-p_3)^2 - M_w{}^2]} \tag{3.1}$$

where $L = \frac{1-\gamma^5}{2}$ and $R = \frac{1+\gamma^5}{2}$. In Eq. (3.1), $\lambda_i = V_{iQ} V_{iq}$, $\lambda_j = V_{jQ} V_{jq}$ where V is the KM matrix element and $i, j = u, c$ or t quark. In both the $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ systems, the momentum and mass of the light quark, q is much smaller than those of the heavy quark, Q and can be neglected. Eq.(3.1) then becomes:

$$M_a = \frac{ig^4}{4} \sum_{i,j} \lambda_i \lambda_j \, [\bar{v}(2) \gamma^\lambda \gamma^\mu \gamma^\tau \, L u(1)][\bar{v}(2) \gamma_\tau \gamma_\alpha \gamma_\lambda L v(2)] I \tag{3.2}$$

where

$$I = \int \frac{k_\mu (k-p_1)^\alpha d^4k}{(k^2 - M_W{}^2)(k^2 - m_j{}^2)([k-p_1]^2 - M_W{}^2)([k-p_1]^2 - m_i{}^2)} \tag{3.3}$$

The denominators in Eq. (3.3) can be cast in the following form:

$$I = \frac{1}{(M_W{}^2 - m_j{}^2)(M_W{}^2 - m_i{}^2)} \int d^4 k k_\mu (k - p_1)^\alpha \left\{ \frac{1}{(k^2 - M_W{}^2)(k - p_1)^2} + \right.$$
$$+ \frac{1}{(k^2 - m_j{}^2)([k - p_1]^2 - m_i{}^2)} - \frac{1}{\left(k^2 - M_W^2\right)\left([k - p_1]^2 - m_i^2\right)}$$
$$\left. - \frac{1}{(k^2 - m_j{}^2)([k - p_1]^2 - M_W{}^2)} \right\} \tag{3.4}$$

Feynman parametrizing the denominators of the integral in Eq. (3.4) with the help of Eq. [A.13] given in the Appendix and using the on shell condition for Q:

$$p_1^2 = M_Q^2, \tag{3.5}$$

we obtain:

$$I = \frac{1}{(M_W{}^2 - m_j{}^2)(M_W{}^2 - m_i{}^2)} \int \frac{d^4 k}{(2\pi)^4} k_\mu (k - p_1)^\alpha \left\{ \int_0^1 \frac{dx}{([k - p_1 x]^2 - D_1)^2} \right.$$
$$\left. + \int_0^1 \frac{dx}{\left([k - p_1 x]^2 - D_2\right)^2} - \int_0^1 \frac{dx}{\left([k - p_1 x]^2 - D_3\right)^2} - \int_0^1 \frac{dx}{\left([k - p_1 x]^2 - D_4\right)^2} \right\} \tag{3.6}$$

where

$$D_1 = M_W{}^2\left(1 + x_h\left[x^2 - x\right]\right)$$

$$D_2 = M_W{}^2\left(x_h\left[x^2 - x\right] + x_j[1 - x] + x x_i\right)$$

$$D_3 = M_W{}^2\left(1 + x_h\left[x^2 - x\right] + \left[x_j - 1\right]x\right)$$

$$D_4 = M_W{}^2\left(x_h\left[x^2 - x\right] + x_j[1 - x] + x\right) \tag{3.7}$$

In Eq. (3.6), $x_h = {m_Q{}^2}/{M_W{}^2}$ and $x_i = {m_i{}^2}/{M_W{}^2}$. Generalizing to $n = 4 - \varepsilon$ space time dimensions and making the shift $\hat{k} = k_1 - p_1 x$ in Eq. (3.6), we obtain

$$I = \int_0^1 dx \left\{ \frac{d^n k}{(2\pi)^n} \left(\left[k_\mu k^\alpha \right] + [x^2 - x] p_{1\mu} p_1^\alpha + x \left[k_\mu p_1^\alpha + p_{1\mu} k^\alpha \right] - k_\mu p_1^\alpha \right) F_{\hat{k},D} \right\}$$

$$\times \frac{1}{\left(M_W^2 - m_i^2\right)\left(M_W^2 - m_j^2\right)} \tag{3.8}$$

where

$$F_{\hat{k},D} = \frac{1}{\left(\hat{k}^2 - D_1\right)^2} + \frac{1}{\left(\hat{k}^2 - D_2\right)} - \frac{1}{\left(\hat{k}^2 - D_3\right)^2} - \frac{1}{\left(\hat{k}^2 - D_4\right)^2} \tag{3.9}$$

Carrying out the momentum integration in Eq. (3.8) using Eq. [A.15] and omitting all terms that contain odd powers in $\hat{k}$, we get after some algebra

$$I = \frac{i}{32\pi^2 \left(M_W^2 - m_i^2\right)\left(M_W^2 - m_j^2\right)} \int_0^1 dx \left\{ g_\mu^\alpha \left[\sum_{i=3}^{4} D_i \ln D_i - \sum_{i=1}^{2} D_i \ln D_i \right] \right.$$

$$\left. + 2 p_{1\mu} p_1^\alpha \left(x^2 - x\right) \left[\sum_{i=3}^{4} \ln D_i - \sum_{i=1}^{2} \ln D_i \right] \right\} \tag{3.10}$$

Substituting Eq. (3.10) into Eq.(3.2) and noting that

$$\bar{u}(3)\left[\gamma^\lambda \gamma^\mu \gamma^\tau L\ \right] u(1) \bar{v}(2) \left[\gamma_\tau \gamma_\alpha \gamma_\lambda L\right] v(4) = 4 \bar{u}(3) \left[\gamma_\alpha L\right] u(1) \bar{v}(2) \left[\gamma^\mu L\right] v(4) \tag{3.11}$$

and using the Dirac equations

$$\left.\begin{aligned} \bar{u}(3)\left(\not{p}_1 - m_Q\right) &= 0 \\ \bar{v}(2)\left(\not{p}_1 + m_Q\right) &= 0 \end{aligned}\right\} \tag{3.12}$$

we obtain

$$M_a = \frac{g^4 \sum \lambda_i \lambda_j}{32\pi^2 M_W^2 (1-\lambda_i)(1-\lambda_j)} \Bigg\{ [\bar{u}(3)\gamma_\mu L u(1)][\bar{v}(2)\gamma^\mu L v(4)] \int_0^1 dx \left[\sum_{i=1}^{2} D_i \ln D_i - \sum_{i=3}^{4} D_i \ln D_i \right]$$

$$- 2[\bar{u}(3) R u(1)][\bar{v}(2) R v(4)] \lambda_h \int_0^1 dx (x^2 - x) \left[\sum_{i=1}^{2} \ln D_i - \sum_{i=3}^{4} \ln D_i \right] \Bigg\} \tag{3.13}$$

The amplitudes for the remaining diagrams in Fig. 1 are calculated in a similar manner.

We obtain :

$$M_b = \frac{g^4 \sum \lambda_i \lambda_j x_j}{64\pi^2 M_W^2 (1-x_i)(1-x_j)} [\bar{u}(3)\gamma_\mu L u(1)][\bar{v}(2)\gamma^\mu L v(4)] \int_0^1 dx \left[x_h (1-x) - x_i \right]$$

$$\times \left[\sum_{n=1}^{2} \ln D_n - \sum_{n=3}^{4} \ln D_n \right] \tag{3.14}$$

$$M_c = \frac{g^4 \sum \lambda_i \lambda_j}{64\pi^2 M_W^2 (1-x_i)(1-x_j)} [\bar{u}(3)\gamma_\mu L u(1)][\bar{v}(2)\gamma^\mu L v(4)]$$

$$\times \int_0^1 dx \left[x x_h x_i - x_i x_j \right] \left[\sum_{n=1}^{2} \ln D_n - \sum_{n=3}^{4} \ln D_n \right] \tag{3.15}$$

$$M_d = \frac{g^4 \sum \lambda_i \lambda_j x_i x_j}{64\pi^2 M_W^2 (1-x_i)(1-x_j)} \Bigg\{ [\bar{u}(3) R u(1)][\bar{v}(2) R v(4)]$$

$$\times x_h \int_0^1 dx x (1-x) \left[\sum_{n=1}^{2} \ln D_n - \sum_{n=3}^{4} \ln D_n \right]$$

$$+ \frac{1}{2} [\bar{u}(3)\gamma_\mu L u(1)][\bar{v}(2)\lambda^\mu L v(4)] \int_0^1 dx \left[\sum_{n=1}^{2} D_i \ln D_n - \sum_{n=3}^{4} D_i \ln D_n \right] \Bigg\}$$

(3.16)

Making use of the following Fiertz transforms:

$$\left.\begin{array}{l}[\bar{u}(3)\gamma_\mu Lv(4)][\bar{v}(2)\gamma^\mu Lu(1)] = [\bar{u}(3)\gamma_\mu Lu(1)][\bar{v}(2)\gamma^\mu Lv(4)] \\ [\bar{u}(3)Rv(4)][\bar{v}(2)Ru(1)] = [\bar{u}(3)Ru(1)][\bar{v}(2)Rv(4)]\end{array}\right\} \tag{3.17}$$

we find that the amplitudes of diagrams (e) to (h) in Fig. 1 are related to the amplitudes of diagrams of diagrams (a) to (d) as follows:

$$\left.\begin{array}{l}M_e = M_a \\ M_f = M_b \\ M_g = M_c \\ M_h = M_d\end{array}\right\} \tag{3.18}$$

3.3 THE EFFECTIVE HAMILTONIAN FOR THE $P^0 \leftrightarrow \overline{P}^0$ TRANSITION

The effective Hamiltonian, H_{eff}, is the sum over all the amplitudes, $\sum_{\alpha=a}^{h} M_\alpha$, calculated in Sect. (3.2). It is given by

$$H_{eff} = \frac{G_F{}^2 M_W{}^2}{16\pi^2}\left\{\left(\bar{q}\gamma_\alpha[1-\gamma_5]Q\right)^2 \sum_{i,j}\lambda_i\lambda_j B_{ij} + \left(\bar{q}[1+\gamma_5]Q\right)^2 \sum_{i,j}\lambda_i\lambda_j C_{ij}\right\} \tag{3.19}$$

In Eq. (3.19), B_{ij} and C_{ij} are the form factors and are given by

$$B_{ij} = \frac{1}{(1-x_i)(1-x_j)}\int_0^1 dx\left\{\left(2+\frac{x_i x_j}{2}\right)\left(\Lambda_1 \ln\Lambda_1 + \Lambda_2 \ln\Lambda_2 - \Lambda_3 \ln\Lambda_3 - \Lambda_4 \ln\Lambda_4\right)\right.$$

$$\left. + \left(x_h\left[x x_i + (1-x)x_j\right] - 2x_i x_j\right)\left(\ln(\Lambda_1\Lambda_2) - \ln(\Lambda_3\Lambda_4)\right)\right\} \tag{3.20}$$

and

$$C_{ij} = \frac{x_h\left(4 + x_i x_j\right)}{(1-x_i)(1-x_j)}\int_0^1 dx x(1-x)\left(\ln(\Lambda_1\Lambda_2) - \ln(\Lambda_3\Lambda_4)\right) \tag{3.21}$$

where

$$
\left.\begin{aligned}
\Lambda_1 &= x_h x^2 + \left(x_i - x_j - x_h\right)x + x_j \\
\Lambda_2 &= x_h x^2 - x_h x + 1 \\
\Lambda_3 &= x_h x^2 + \left(1 - x_j - x_h\right)x + x_j \\
\Lambda_4 &= x_h x^2 + \left(x_i - x_h - 1\right)x + 1
\end{aligned}\right\} \qquad (3.22)
$$

The H_{eff} derived in Eq. (3.19) is composed of two form factors, B_{ij} and C_{ij} and is in agreement with that obtained by previous authors [9-10]. As given in Eqs. (3.20) and (3.21), these form factors are expressed as integrals over the x variable. Another feature is the presence of both the $V-A$ and $S+P$ type of operators. In the next chapter, we will show how the off-diagonal mass matrix and decay matrix elements can be obtained from the effective Hamiltonian.