CHAPTER 6

CONCLUSION

In this study, our objective was to investigate the mass dependence in particle antiparticle mixing in the $B_d^0 \leftrightarrow \overline{B}_d^0$ and $B_s^0 \leftrightarrow \overline{B}_s^0$ transitions within the framework of the Standard Model. We began our study by first calculating the amplitudes of all the box diagrams which contribute to the $B_d^0 \leftrightarrow \overline{B}_d^0$ and $B_s^0 \leftrightarrow \overline{B}_s^0$ transitions in the 't Hooft-Feynman gauge by neglecting the momentum and mass of the light external d and s compared to that of the external b quark. The effective Hamiltonian was then obtained from these amplitudes and was found to contain two form factors, B_{ij} and C_{ij}, both expressed as integrals over the x variable and respectively attached to a $V-A$ and $S+P$ operator. Our expressions for the form factors agrees with earlier calculations [9-10].

We next considered in detail the off-diagonal mass and decay matrix elements, $M_{12} + i\!/\!_2\Gamma_{12}$, which are respectively the dispersive and absorptive parts of the $B_d^0 \leftrightarrow \overline{B}_d^0$ and $B_s^0 \leftrightarrow \overline{B}_s^0$ transitions. These were obtained by sandwiching the effective Hamiltonian between the P^0 and $\overline{P}^0$ states. The hadronic matrix elements for the $V-A$ and $S+P$ operators were estimated using the Vacuum Saturation Method and parametrized in terms of the meson mass, the decay constant, and the bag parameter. From our calculations, we have also established that the form factors actually come in the following linear combination: $A_{ij}^{\alpha} = B_{ij}^{\alpha} - 5\!/\!_8 C_{ij}^{\alpha}, \alpha = a,d$ where a and d are respectively the absorptive and dispersive parts and the indices i,j represent the internal quarks.

We have also shown in detail the procedure used to obtain the absorptive and dispersive parts from $A_{ij}^{(\alpha)}$ by the method of analytical integration. It is seen that the calculation of the absorptive part, $A_{ij}^{(a)}$, is straightforward. The absorptive part is developed whenever $x_h^2 > 4x_h x_j$. Also, the expression obtained for A_{ij}^{a} is symmetric with respect to i and j.

On the contrary, the computation of the dispersive part is more complicated. Depending on the quark masses, two cases had to be considered, (i) $D(x_i, x_j) > 0$ and $D(x_i, x_j) < 0$, where $D(x_i, x_j) = (x_i - x_j - x_h)^2 - 4x_h x_j$. The first case arises for $i, j = u, c, t$ but excludes the case when $i, j = t$. The second case occurs when $i, j = t$. The final expressions for $A_{ij}^{(d)}$ resulting from both these cases are also symmetric for i and j. It was also found that the form factors show a strong dependence on the internal quark masses.

We next examined the behaviour of M_{12} and Γ_{12}. By exploiting the unitarity condition of the KM matrix elements, it was found that the dominant contribution to M_{12} comes when the two internal quarks are the top quarks. It was also shown that $\Gamma_{12} << M_{12}$, a result which allows for the mass difference the width difference to be written as $\Delta M = 2\,\mathrm{Re}\, M_{12}$ and $\Delta\Gamma \approx 0$ respectively.

We then proceeded to calculate the values of the mass differences and the mixing parameters, r and λ, for the $B_d^0 \leftrightarrow \overline{B}_d^0$ and $B_s^0 \leftrightarrow \overline{B}_s^0$ transitions. For the $B_d^0 \leftrightarrow \overline{B}_d^0$ transition, we obtained $r_d = 0.34$, $\chi_d = 0.25$ and $\Delta M_d = 4.27 \times 10^{-13}$ GeV. The values for the $B_s^0 \leftrightarrow \overline{B}_s^0$ transition were found to be $r_s \approx 1$, $\chi_s \approx 0.5$ and $\Delta M_s = 9.35 \times 10^{-12}$ GeV. We find no significant difference between our results and that of experiment, if we make allowances for the combined uncertainties in the hadronic matrix elements and the KM matrix elements. The value of the mixing parameter for the $B_s^0 \leftrightarrow \overline{B}_s^0$ transition, r_s was found to be close to its maximum value of one. We may then conclude that that mixing is more substantial in the $B_s^0 - \overline{B}_s^0$ .system than in the $B_d^0 - \overline{B}_d^0$ system.