Appendix

Dimensional Regularization

The first ingredient for renormalization prescription in quantum field theory is a regularization procedure which isolates infinities that appear in the individual Feynman diagrams. The regularization procedure is arbitrary provided that the local gauge invariance of the theory is preserved. For gauge theories and especially for the Yang-Mills theories, the dimensional regularization appears as a simple and elegant regularization scheme. It regularises Feynman diagrams by analytic continuation to $4\text{-}\varepsilon$ space-time dimensions and isolates infra-red and ultra-violet divergence as poles in ε, and as such, is widely used in calculations of radiative corrections.

Since an analytic continuation in the space-time dimensions is not unique, there is a variety of conventions in this method. We adopt the following convention:

$$Tr(I)=4, \qquad \{\gamma_\mu, \gamma_5\} = 0 \tag{A.1}$$

Below, we list various formulas in our convention:

The basic algebra is

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \tag{A.2}$$

The metric tensor $g_{\mu\nu}$ satisfies

$$g_{\mu\nu}g^{\mu\nu} = n \tag{A.3}$$

combining (A.2) and (A.3) we obtain

$$\gamma_\rho \gamma^\rho = n$$

$$\gamma_\rho \gamma_\mu \gamma^\rho = (2-n)\gamma_\mu$$

$$\gamma_\rho \gamma_\mu \gamma_\nu \gamma^\rho = 4 g_{\mu\nu} + (n-4)\gamma_\mu \gamma_\nu$$

$$\gamma_\rho \gamma_\mu \gamma_\nu \gamma_\sigma \gamma^\rho = -2\gamma_\sigma \gamma_\nu \gamma_\mu + (4-n)\gamma_\mu \gamma_\nu \gamma_\sigma \tag{A.4}$$

Using (A.1), we find that

$$Tr(\gamma_\mu \gamma_\nu) = 4 g_{\mu\nu}$$

$$Tr(\gamma_\mu \gamma_\nu \gamma_\lambda \gamma_\sigma) = 4\left[g_{\mu\nu} g_{\lambda\rho} + g_{\mu\rho} g_{\nu\lambda} - g_{\mu\lambda} g_{\nu\rho}\right]$$

$$Tr(\gamma_{\mu_1} \cdots \gamma_{\mu_m}) = 0 \text{ for } m \text{ odd} \tag{A.5}$$

There occurs no trouble concerning the λ_5 matrix in the present case, as the Weinberg-Salam theory is an anomaly-free theory.

The Feynman parametrization is needed to combine the product of denominators appearing in the momentum integral. The Feynman parametrization is given by:

$$\frac{1}{a_1 a_2 \ldots a_n} = (n-1)! \int_0^1 U_1^{n-2} dU_1 \int_0^1 U_2^{n-3} dU_2 \ldots \int_0^1 dU_{n-1} \times \left[(a_1 - a_2)U_1 \ldots U_{n-1} + (a_2 - a_3)U_1 \ldots U_{n-2} + \ldots + a_n\right]^{-n} \tag{A.6}$$

which is very convenient as it allows possible cancellations of the terms of two different propagators and also the advantage of finite bounds of integration (useful in numerical calculations, e. g. various QED calculations). Special cases of (A.6) used in our calculation are:

$$\frac{1}{ab} = \int_0^1 dx\left[b + (a-b)x\right]^{-2} \tag{A.7}$$

$$\frac{1}{abc} = 2\int_0^1 dx \int_0^{1-x} dy\left[a + (b-a)x + (c-a)y\right]^{-3} \tag{A.8}$$

After a Feynman parametrization of the propagators and a shift of the momentum variable, the momentum integrals for the one-loop diagram reduce usually to an integral of the form:

$$I(m,r)=\int\frac{d^n\tilde{q}}{(2\pi)^n}\frac{\left(\tilde{q}^2\right)^r}{\left[\tilde{q}^2-R^2\right]^m}=\frac{i}{\left(16\pi^2\right)^{n/4}}(-1)^{r-m}\left(R^2\right)^{r-m+\frac{n}{2}}\times$$
$$\times\frac{\Gamma\left(r+\frac{n}{2}\right)\Gamma\left(m-r+\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma(m)} \tag{A.10}$$

By symmetrical integration, it is easily shown that:

$$\int\frac{d^n\tilde{q}}{(2\pi)^n}\frac{\tilde{q}_\mu\tilde{q}_\nu}{\left[\tilde{q}^2-R^2\right]^m}=\frac{1}{n}g_{\mu\nu}\int\frac{d^n\tilde{q}}{(2\pi)^n}\frac{\tilde{q}^2}{\left[\tilde{q}^2-R^2\right]^m} \tag{A.11}$$

and

$$\int\frac{d^n\tilde{q}}{(2\pi)^n}\frac{\tilde{q}_{\mu_1}\tilde{q}_{\mu_2}\cdots\tilde{q}_{\mu_k}}{\left[\tilde{q}^2-R^2\right]^m}=0\ \text{ for } k \text{ odd} \tag{A.12}$$

The Gamma function $\Gamma(x)$ has the following properties:

$$\Gamma(\varepsilon)=\frac{1}{\varepsilon}-\gamma+0(\varepsilon) \tag{A.13}$$

$$\Gamma(\varepsilon-1)=\frac{1}{\varepsilon}-(1-\gamma)+0(\varepsilon) \tag{A.14}$$

where $\gamma=0.5772$ is the Euler constant.