Chapter 1

Introduction

The Standard Model[1-3] based on the gauge group $SU_c(3) \times SU(2) \times U(1)$ has by now been firmly established against rather stringent experimental tests despite the fact that we still do not quite understand the Higgs sector from a fundamental point of view.[4-8] The existence of weak neutral current has been confirmed in the $\nu_\mu N$ scattering done in CERN in 1973.[9] Gluon jets observed in 1979 and 1980 are evidence in support of QCD.[10-15] The conclusive evidence in favour of the model comes from the observation of *W* and Z^o bosons at CERN in 1984 with masses as predicted by the model.[16] Precision measurements of the Z^o properties,[17] such as M_Z, Γ_Z, $\mathrm{Br}(Z^o \to f\bar{f})$ and $sin^2\,\theta_w$ are in good agreement with theoretical calculation from the model. The most striking evidence comes from the recent observation of the long-awaited *t* quark with a mass that agrees well with what one deduces from the precision measurements at LEP1 and SLC.[18]

Nevertheless, the structure of Standard Model has to be tested in great details, particularly the mixing of quark flavours. With six quark flavours, the flavours can be parametrized with a 3 × 3 Kobayashi-Maskawa (KM) mixing matrix.[19] Besides three mixing matrix angles θ_1, θ_2 and θ_3, the KM matrix also contains a phase angle δ that would give rise to *CP* violating effects in a natural way, via the complexity of the $W^{\pm}$ couplings in the quark sector. One consequence of this flavour mixing is the neutral conversion of quark flavours at one-loop order, such as from a *s* quark to a *d* quark with emission of a gluon,[20-24] a photon,[25-30] or a Z boson[28,31,32].

Within the Standard Model with one Higgs doublet, all neutral current couplings are naturally diagonal. As a result, the off-diagonal couplings are absent at the tree level. They appear only at the one loop level, and are therefore strongly suppressed due to the so-called GIM mechanism. [33]

Flavour-changing neutral current (FCNC) at the quark sector has aroused intense theoretical interest as a place for observing higher-order electroweak interactions. Flavour-changing neutral current is a key to test the influence of ultraheavy quarks on higher-order electroweak corrections to Born amplitudes and to determine the strength of processes forbidden in lowest order. [34] Furthermore, flavour-changing processes, such as $b \to s l^- l^+$, can serve as excellent 'window' to test the finer structure of the Standard Model or to see if there is new physics beyond it. [35,36]

Another important aspect of the flavour-changing neutral current is its relation to *CP* violation. *CP* violation, which was first observed experimentally in 1964 by Christenson *et al.* [37] remains as one of the least understood aspect of the Standard Model. Despite strenuous effort, physicists still cannot claim to possess a proven description of this phenomenon. However, measurement of *CP* violation effect allows us to address three fundamental questions [38] : (i) Is the KM phase of the three generation SM the only source of *CP* violation? (ii) What are the exact values of the CKM parameters? (iii) Is there any new physics in the quark sector if we find deviations from SM predictions? All these questions are yet to be answered convincingly. More investigations into the origin of *CP* violation and calculation of its empirical behaviour still need to be done before we come out with an universally acceptable physical explanation.

In 1979, Bander *et al.* [39] had shown that flavour-changing neutral current can give rise to a difference between the decay rate of a particle into a definite final state with the rate of the antiparticle decaying into the corresponding charge-conjugated state, namely $\Gamma(i \rightarrow f) \neq \Gamma(\bar{i} \rightarrow \bar{f})$, where i and f refer to their respective initial and final states, and the bar refers to their respective *CP* conjugated states. This is a possible candidate for direct *CP* violation.

The three conditions for this *CP* decay asymmetry to arise are:[40] (1)two amplitudes with different KM phases must contribute to the same process, (2) there should be a complex phase in the KM matrix, and (3) at least one of the amplitudes must have an absorptive part (this is some times referred to as final state interaction).

Flavour-changing neutral current involving conversion of a quark flavour to another quark flavour with emission of virtual gluon or virtual photon have been studied extensively in the literature.[20–32,39,41] But a sufficiently complete calculation of flavour-changing neutral current involving emission of virtual Z boson, especially their explicit form factors, is still lacking. A more comprehensive study of the so-called 'electroweak penguins', which include both emissions of virtual photon and virtual Z boson, is awaiting for a complete gauge invariant calculation on the flavour-changing $Zq_1\bar{q}_2$ vertex, and this is the aim of this project.

A better understanding of the calculation of flavour-changing $Zq_1\bar{q}_2$ vertex at the quark level is relevant to several physical situations. The rare decays of B mesons[36,40,42–48] with expected large *CP* asymmetries will require a good knowledge of the $Zq_1\bar{q}_2$ vertex. Another closely related process is the flavour-changing Z^o decay, $Z^o \rightarrow q_2\bar{q}_1$.[34,49–51] *CP* asymmetry is also expected in this process from the Standard Model. Such Z^o decays can serve as sensitive window to test the existence of ultraheavy quarks.[34] The new generation of e^+e^- colliding-beam machines will permit production of large number of Z^o that make the search for rare flavour-changing decays of Z^o worthwhile. The calculation on $Zq_1\bar{q}_2$ vertex is directly applicable to address such theoretical investigation. The $Zq_1\bar{q}_2$ vertex is also of importance in higher-order processes such as $K^+ \to \pi^+ \mu^+ \mu^-$, $K_L \to \mu^+ \mu^-$ and $K^+ \to \nu\bar{\nu}$,[38] which are relevant to the determination of the element $|V_{ub}|$ in the KM matrix.

Motivated by these possible physical applications, especially *CP* violation, we make the calculation of flavour-changing $Zq_1\bar{q}_2$ vertex as the prime objective of this present study. The calculation will be carried out in the 't Hooft-Feynman gauge.

Several authors have treated the $Zq_1\bar{q}_2$ vertex in the literature, the pioneer works being those of Vainshtein and Khriplovich[52] and Gaillard *et al.*[53,54] These authors assume that all quark masses are small compared to M_W. The calculations of Inami and Lim[28] and Ma and Pramudita[32], on the other hand, have relaxed this assumption for internal quark to take into account of the possible existence of ultraheavy quark. Their work however assumes vanishing external quark masses and external momentum. The short coming of their calculations is that the *CP* violation effect in the flavour-changing transition cannot be extracted. (No absorptive part, and therefore *CP* violation effect, is generated due to their approximation of vanishing invariant mass k of the virtual Z.) To extract out the *CP* violation effects manifested in these flavour-changing transitions, we have to retain the k-dependence of the vertex function.

Soares and Barroso[31] has attempted to compute the $Zq_1\bar{q}_2$ vertex without assuming vanishing external quark masses and external momentum. However, their work emphasises the renormalization aspect of the vertex. Furthermore, their vertex function is expressed in forms of integrals over Feynman parameters, rendering the behaviour of the vertex form factors not easily apprehensible. The absorptive parts of the vertex form factors which are implicitly embedded in their general expressions are also not transparent. To extract their numerical behaviour of both the dispersive and absorptive parts, we have to carry out this double integration explicitly.

The aim of this project is to perform a general calculation of the $Zq_1\bar{q}_2$ vertex function, along a line similar to Soares and Barroso. Instead of calculating the counter terms using Ward-Takahasi identity, we shall extract the counter terms using a simple renormalization prescription.[55] The general expression for the vertex function is expressed in a form advocated by Chia.[24] The vertex form factors will be explicitly evaluated numerically. The k-dependence of both the dispersive and absorptive parts of the form factors is explicitly displayed. We also apply our calculation to estimate the CP violation effects in processes like $q_1 \to q_2 Z^*$ and $Z^0 \to q_2\bar{q}_1$.

The organisation of the thesis is as followed. In Chapter 2, a brief review of the Standard Model is presented. At the end of this chapter, the Feynman rules in the 't Hooft-Feynman gauge are listed.

In Chapter 3, the calculation of the off-shell unrenormalized $Zs\bar{d}$ vertex is presented. Dimensional regularization is used to isolate the divergent part of the unrenormalized vertex. This unrenormalized vertex will then be renormalized through a renormalization scheme proposed by Chia and Chong.[55] The renormalized vertex function is then put on-shell. They shall be expressed in term of various form factors in a systematic manner adopted by Chia[24] to facilitate their numerical manipulation in the following chapter.

Numerical evaluation of the renormalized on-shall vertex function is treated in Chapter 4, which details the integration over the two Feynman parameters. The first parameter can be integrated analytically, while the other parameter is evaluated numerically using the Romberg's method. The behaviours of the form factors are displayed explicitly.

In Chapter 5, the result of our calculation is applied to several *CP* violating processes of interest, namely $q_1 \to q_2 Z^*$ and $Z^0 \to q_2 \bar{q}_1$. The decay rates and *CP* violating decay rate asymmetries are obtained. The dependence of the decay rates and *CP* asymmetries on the KM matrix elements is investigated.

In Chapter 6, the conclusions on the result of the calculation and their physical inference is given.

The Dirac algebra is listed in the Appendix. Also collected in the Appendix is a list of useful general formulae for loop momentum integration in *n*-dimensional Euclidean space.