Chapter 2

The Standard $SU(2) \times U(1)$ Weinberg-Salam Model

2.1 Introduction

In this chapter, we shall first provide in Section 2.2 a brief review of electroweak interaction theories in gauge models where we outline the essentials of the gauge theory of Standard Model. Then in Section 2.3, we shall present the full Lagrangian and Feynman rules in the 't Hooft-Feynman gauge.

2.2 The Standard $SU(2) \times U(1)$ Weinberg-Salam Model for Quarks and Leptons

In constructing the Standard $SU(2) \times U(1)$ Weinberg-Salam Model, the following gauge fields are introduced:

\[ W^1_\mu, W^2_\mu, W^3_\mu \quad \text{for } SU(2) \]  \hspace{1cm} (2.1)

\[ B_\mu \quad \text{for } U(1). \]  \hspace{1cm} (2.2)

The corresponding Lagrangian is

\[ L_1 = -\frac{1}{4} F_{\mu\nu}^i F^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \]  \hspace{1cm} (2.3)

where the field-strength tensors are

\[ F_{\mu\nu}^i F^{i\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g e^{ik} W^k_\mu W^i_\nu \]  \hspace{1cm} (2.4)

\[ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \]  \hspace{1cm} (2.5)
The Lagrangian of Eq. (2.3) contains four massless gauge bosons. There is no mass term due to the constrain by gauge symmetry. To implement the Higgs mechanism, we introduce an SU(2) doublet of complex scalar fields, the Higgs fields:

\[ \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \]  

(2.6)

The Lagrangian for \( \Phi \) is

\[ L_2 = -\left( D^\mu \phi \right)^\dagger \left( D^\mu \phi \right) - V(\phi) \]  

(2.7)

where the covariant derivative for \( \Phi \) is

\[ D^\mu \Phi(x) = \left( \partial^\mu - ig \frac{\tau^}{2} \cdot \vec{\phi} - i \frac{g'}{2} \cdot B^\mu \right) \Phi(x) \]  

(2.8)

and the Higgs potential is

\[ V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda \left( \Phi^\dagger \Phi \right)^2, \]  

(2.9)

with \( \mu^2 < 0 \) and \( \lambda > 0 \).

After spontaneous symmetry breaking of both SU(2) \( \times \) U(1) symmetries into a U(1) symmetry which is identified as the electromagnetic gauge group, \( V(\Phi) \) acquires a non zero vacuum expectation value

\[ \langle \Phi \rangle = \begin{pmatrix} 0 \\ v / \sqrt{2} \end{pmatrix} \]  

(2.10)

with

\[ v = \sqrt{- \frac{\mu^2}{\lambda}}. \]  

(2.11)

Expending the Lagrangian about the minimum of the Higgs potential in unitary gauge, and introducing
\[ W^\pm_\mu = \left( W^1_\mu \mp W^2_\mu \right) / \sqrt{2} \tag{2.12} \]

\[ A_\mu = B_\mu \cos \theta_w + W^3_\mu \sin \theta_w \tag{2.13} \]

\[ Z_\mu = -B_\mu \sin \theta_w + W^3_\mu \cos \theta_w \tag{2.14} \]

with

\[ \sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}} \tag{2.15} \]

\[ \cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}} \tag{2.16} \]

we will then find that mass terms for the weak bosons are generated in \( L_2 \), namely

\[ M^\nu_\mu = g \nu / 2, \tag{2.17} \]

\[ M_Z = \frac{1}{2} \left[ g^2 + (g')^2 \right]^{1/2} \nu = M_\mu / \cos \theta_w \tag{2.18} \]

while leaving the other massless,

\[ M_A = 0. \tag{2.19} \]

\( W^\pm_\mu \) are identified as the massive charged boson fields, \( Z_\mu \) the neutral weak boson field and \( A_\mu \) the photon field.

In the Unitary gauge, three degrees of freedom associated with \( \phi \) have been absorbed into \( W^\pm_\mu \) and \( Z_\mu \) as their longitudinal components, and the remaining one \( \eta \), the Higgs scalar, acquires a mass \( m^2_\eta = -2\mu^2 \).

By the above mechanism, we have succeeded in construction an \( SU(2) \times U(1) \) gauge theory in which three gauge bosons \( (W^\pm_\mu \text{ and } Z_\mu) \) are massive while the remaining one, the photon field \( A_\mu \), is massless. \( L_3 \) and \( L_2 \) are invariant but the ground state solution varies with the gauge (i.e. breaks gauge symmetry spontaneously).
For three generation of fermions, the physical lepton fields relevant to weak
interaction form a left-handed $SU(2)$ doublet
\[ L_L = \begin{pmatrix} \nu_e^* \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \] (2.20)

and the right-handed fermions form a $SU(2)$ singlet
\[ R_R = e_R, \mu_R, \tau_R. \] (2.21)

The hadronic sector of the theory is built upon a single left-handed weak isospin doublets
\[ L_u = \begin{pmatrix} u \\ d \end{pmatrix}_L, L_c = \begin{pmatrix} c \\ s \end{pmatrix}_L, L_t = \begin{pmatrix} t \\ b \end{pmatrix}_L \] (2.22)

and right-handed weak isospin singlets
\[ R_u = u_R, R_c = c_R, R_t = t_R \] (2.23a)
\[ R_d = d_R, R_s = s_R, R_b = b_R. \] (2.23b)

The down-type quarks are mixed states related by a general Kobayashi-Maskawa\textsuperscript{19} rotation, described by the matrix $V$ through
\[
\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix},
\] (2.24)

where \( c_j = \cos \theta_j \), \( s_j = \sin \theta_j \), \( f = 1, 2, 3 \) is the family index. The fermion Lagrangian in the lepton sector is therefore given by
\[
L_\text{s} = -\bar{\nu}_\mu \left[ \partial_\mu + ig' B_\mu \right] R - \bar{\nu}_\mu \left[ \partial_\mu - ig \tilde{\tau} \cdot \bar{\nu}_\mu / 2 + ig' B_\mu / 2 \right] L.
\] (2.25)

The coupling constant for the weak-isospin group $SU(2)\text{\textsubscript{L}}$ is $g$, and the coupling constant for the weak-hypercharge group $U(1)$ is denoted as $g'/2$.

To generate the lepton masses, we include the following $SU(2) \times U(1)$ gauge invariant interaction term, which involve Yukawa couplings of the scalar to the leptons
\[ L_4^L = - \sum_{l=\mu, \tau} G_l \left[ (L_l \phi) R_l + h.c. \right]. \]  

(2.26)

Whereas for the scalar-quarks interaction Lagrangian, we have

\[ L_4^Q = \sum_{l, j=1}^{3} -Y_{u}^d(\bar{u}_l \rightarrow d_j) \phi d_{jr} - Y_{d}^u(\bar{u}_l \rightarrow d_j) \phi d_{jr} + h.c. \]  

(2.27)

with charge-conjugated Higgs doublet

\[ \phi_e = -i \tau_2 \phi^* = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}. \]  

(2.28)

Here, \( G_t, Y_u^d \) and \( Y_d^u \) being some arbitrary Yukawa couplings that are chosen so that the leptons and quarks are mass eigen states with the correct masses. Putting everything together, the final Lagrangian after spontaneous symmetry breaking in Unitary gauge is summarised as below:

Gauge bosons kinetic energies and interactions:

\[ L_1 = L_0^G + L_1^{GG} \]

\[ = -\left\{ F_{\mu \nu}^\mu \mu + Z_{\mu \nu}^\nu Z_{\mu \nu} + 2W_{\mu \nu}^+ W_{\mu \nu}^- \right\} / 4 \]

\[ - i e \left[ F_{\mu \nu}^\mu \mu W_{\nu}^+ + \left( W_{\mu \nu}^+ W_{\mu \nu}^- - W_{\mu \nu}^- W_{\mu \nu}^+ \right) A_{\mu} \right] - i e \cot \theta_w \times \]

\[ \times \left\{ Z_{\mu \nu}^\mu \mu W_{\nu}^+ + \left( W_{\mu \nu}^+ W_{\mu \nu}^- - W_{\mu \nu}^- W_{\mu \nu}^+ \right) \right\} Z_{\nu} - e^2 \left\{ W_{\mu \nu}^+ W_{\mu \nu}^- \left( A_{\nu} + \cot \theta_w Z_{\nu} \right) \right\} \times \]

\[ \times \left( A_{\nu} + \cot \theta_w Z_{\nu} \right) - W_{\mu \nu}^+ W_{\nu}^- \left( A_{\mu} + \cot \theta_w Z_{\mu} \right) \]

\[ - \frac{1}{2} \left( e / \sin \theta_w \right)^2 \left( W_{\mu \nu}^+ W_{\mu \nu}^- - W_{\mu \nu}^- W_{\mu \nu}^+ \right) \]

(2.29)

where

\[ e = g \sin \theta_w \]  

(2.30)

\[ F_{\mu \nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \]  

(2.31)

\[ Z_{\mu \nu} = \partial_{\mu} Z_{\nu} - \partial_{\nu} Z_{\mu} \]  

(2.32)

\[ W_{\mu \nu}^\pm = \partial_{\mu} W_{\nu}^\pm - \partial_{\nu} W_{\mu}^\pm. \]  

(2.33)
Higgs kinetic energy, mass and couplings with weak bosons and boson masses:

\[ L_2 = L^H + L^{HQ}_1 + L^{HH}_1 + L^Q_M + \mu^4 / 4\lambda \]

\[ L^H = \partial^\mu \eta \partial_\mu \eta / 2 - m_\eta^2 \eta^2 / 2 \]  

\[ L^{HQ}_1 + L^{HH}_1 = -\frac{\lambda}{4} \left( \eta^4 + 4 v \eta^3 \right) \left( \frac{1}{8} \left[ (2 v \eta + \eta^2) \right] \left[ 2 g^2 W^\mu_\mu W^{\mu -} + \left[ g^2 + (g')^2 \right] Z^\mu Z^{\mu} \right] \right) \]  

\[ L^Q_M = -\frac{1}{8} v^2 \left\{ 2 g^2 W^\mu_\mu W^{\mu -} + \left[ g^2 + (g')^2 \right] Z^\mu Z^{\mu} \right\} . \]  

Lagrangian for Lepton and quark kinetic energies, and their weak interactions with gauge bosons:

\[ L_3 = L^L_0 + L^Q_0 + L^{LQ}_1 + L^{QQ}_1 \]

\[ = - \sum_{l=e,\mu,\tau} e_l \partial^\mu \gamma^\mu e_l + v_1 \partial^\mu \gamma^\mu v_1 - \sum_{j=1,2,3} u_j \partial^\mu \gamma^\mu u_j + d_j \partial^\mu \gamma^\mu d_j \]

\[ + \frac{1}{2 \sqrt{2}} \frac{e}{\sin \theta_W} \left\{ J^{(-)}_\mu (x) W^\mu (x) + h.c. \right\} + \frac{1}{2} \frac{e}{\sin \theta_W \cos \theta_W} Z^\mu (x) \bar{N}^\mu (x) \]

\[ - e \sum_{l=e,\mu,\tau} \bar{e}_l \gamma^\mu e_l A_\mu . \]  

with

\[ e = g \sin \theta_W = g' \cos \theta_W \]

\[ J^{(-)}_\mu = l^{(-)}_\mu + \bar{\nu}^{(-)}_\mu \]

and

\[ \bar{N}^{(-)}_\mu = \bar{\nu}^{(0)}_\mu + N_\mu . \]  

Here we have

\[ l^{(-)}_\mu = i e \gamma^\mu (1 + \gamma_5) \nu_e + i \bar{\nu}^\mu (1 + \gamma_5) \nu_e + i \bar{\nu}^\mu (1 + \gamma_5) \nu_\tau . \]
\[ J_{\mu}^{(-)} = i \bar{d} \gamma_{\mu} (1 + \gamma_5) u + i \bar{s} \gamma_{\mu} (1 + \gamma_5) c + i \bar{b} \gamma_{\mu} (1 + \gamma_5) t \] (2.43)

\[ I_{\mu}^{(\mu)} = \frac{i}{2} \left\{ \nu_e \gamma_{\mu} (1 + \gamma_5) \nu_e - \bar{e} \gamma_{\mu} (1 + \gamma_5) e \right\} + 2 \sin^2 \theta W i e \gamma_{\mu} e \\
+ \left\{ e \to \mu \right\} + \left\{ e \to \tau \right\} \] (2.44)

and

\[ N_{\mu} = \frac{i}{2} \left[ \bar{u} \gamma_{\mu} (1 + \gamma_5) u - \bar{d} \gamma_{\mu} (1 + \gamma_5) d + \bar{c} \gamma_{\mu} (1 + \gamma_5) c - \bar{s} \gamma_{\mu} (1 + \gamma_5) s \\
+ i \gamma_{\mu} (1 + \gamma_5) t - \bar{b} \gamma_{\mu} (1 + \gamma_5) b \right] - 2 \sin^2 \theta W J_{\mu}^{em}, \] (2.45)

where the hadronic electromagnetic current \( J_{\mu}^{em} \) is given by

\[ J_{\mu}^{em} = \frac{2}{3} i \bar{u} \gamma_{\mu} u - \frac{1}{3} i \bar{d} \gamma_{\mu} d + \frac{2}{3} i \bar{c} \gamma_{\mu} c - \frac{1}{3} i \bar{s} \gamma_{\mu} s + \frac{2}{3} i \bar{t} \gamma_{\mu} t - \frac{1}{3} i \bar{b} \gamma_{\mu} b. \] (2.46)

The lepton and quark masses and couplings to Higgs, in the canonical form, is given by the Lagrangian

\[ L_4 = L_4^L + L_4^Q \]

\[ = \left( L_4^L + L_4^\mu \right) + \left( L_4^Q + L_4^Q \right) \]

\[ = - \left( \bar{e} \gamma_{\mu} e + \bar{\mu} \gamma_{\mu} \mu + \bar{\tau} \gamma_{\mu} \tau \right) \bar{\nu}(e) \left( \begin{array}{c} e \\ \mu \\ \tau \end{array} \right) \bar{\nu}(u) \left( \begin{array}{c} u \\ c \\ t \end{array} \right) \bar{\nu}(d) \left( \begin{array}{c} d \\ s \\ t \end{array} \right) \bar{\nu}(b) \left( \begin{array}{c} b \end{array} \right) \left( 1 + \frac{\eta}{\nu} \right) \] (2.47)

where

\[ \bar{\nu}(e) = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad \bar{\nu}(u) = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad \bar{\nu}(d) = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}. \] (2.48)
2.3 Lagrangian and Feynman Rules in the 't Hooft-Feynman Gauge

Since the calculation in this project shall be done in the 't Hooft-Feynman (HF) gauge\textsuperscript{56–58}, it is proper to present the full Lagrangian in this gauge. In this gauge the Higgs doublet is parametrized in a more general form,

\[
\phi = \frac{\phi^+}{\sqrt{2}} \left( \nu + \eta + i\phi_4 \right)
\]

(2.49)

where \(\phi_4\) and \(\phi^+\) are the neutral and charged unphysical scalar boson. The expression in Eq. (2.49) is then substituted into the Lagrangian \(L_1, L_2, L_3\) and \(L_4\) to break the symmetry. The Lagrangian not involving any Higgs field remains explicitly the same as in the Unitary gauge, whereas the Lagrangians involving Higgs fields are modified. List below are the related modified Lagrangians, namely \(L^H, L_1^{HO} + L_1^{NH}\) in \(L_2\) and \(L_1^{LH}, L_1^{OH}\) in \(L_4\):

\[
L^H = \frac{1}{2} \partial^\mu \nu \partial_\mu \eta + \partial^\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2} \partial^\mu \phi^4 \partial_\mu \phi^4 - \frac{1}{2} m_H^2 \eta^2
\]

(2.50)

\[
L_1^{BH} = -\lambda \nu \left( \eta^3 + 2 \eta \phi^+ \phi^- + \eta \phi_4^2 \right)
- \frac{1}{4} \lambda \left( 4 \phi^+ \phi^+ \eta^2 + 4 \phi^+ \phi^- \eta^2 + \phi^4 + 4 \phi_4^2 \phi^+ \phi^- + 2 \eta^2 \phi_4^2 \right)
\]

(2.51)

\[
L_1^{LH} = \sum_{\ell=e,\mu,\tau} \left[ - \frac{m_\ell}{\sqrt{2}} e_\ell (1 - \gamma_3) v_\ell \phi^- - m_\ell \bar{e}_\ell e_\ell \eta - i m_\ell \bar{e}_\ell \gamma_5 e_\ell \phi_4 - \frac{m_\ell}{\sqrt{2}} v_\ell (1 + \gamma_5) e_\ell \phi^+ \right]
\]

(2.52)
\begin{equation}
L_{1}^{GH} = - \frac{g}{\sqrt{2} M_w} \left[ \bar{u} \left[ \hat{V} \hat{m}(d) R - \hat{m}(u) V L \right] d \phi^* + \bar{d} \left[ \hat{m}(d)^\dagger V^* L - V^* \hat{m}(u) R \right] V \phi \right.
+ \frac{1}{\sqrt{2}} \bar{u} \hat{m}(d) d \eta + \frac{1}{\sqrt{2}} \bar{u} \hat{m}(u) u \eta - \frac{i}{\sqrt{2}} \bar{d} \hat{m}(d) \gamma_5 d \phi_4 - \frac{i}{\sqrt{2}} \bar{u} \hat{m}(u) \gamma_5 u \phi_4 \right].
\end{equation}

(2.53)

\begin{equation}
L_{1}^{10} = g M_w W_\mu^- W^{\mu \eta} \eta + \frac{g M_w}{2 \cos \theta_w} Z_\mu Z_\mu \eta - \frac{g M_w}{\cos \theta_w} \sin^2 \theta_w \left( Z_\mu W^{\mu \eta} \phi^- + Z_\mu W^{\mu -i} \phi^+ \right)
+ g M_w W^{+ \mu} \left( \eta^2 + \phi_4^2 \right) + \frac{g^2}{8 \cos^2 \theta_w} Z_\mu Z_\mu \left( \eta^2 + \phi_4^2 \right) + \frac{g^2}{4} \cos^2 2 \theta_w Z_\mu Z_\mu
+ 4 \cos 2 \theta_w \tan \theta_w Z_\mu A^\mu + 4 \sin^2 \theta_w A_\mu A^\mu + 2 W_\mu^+ W^{\mu -} \right] \phi^+ \phi^-
+ \frac{ig \cos 2 \theta_w}{2 \cos \theta_w} \left[ Z_\mu \left( \partial^\mu \phi^- \right) \phi^+ - Z_\mu \left( \partial^\mu \phi^+ \right) \phi^- \right]
+ ig \sin \theta_w \left[ A_\mu \left( \partial^\mu \phi^- \right) \phi^+ - A_\mu \left( \partial^\mu \phi^+ \right) \phi^- \right]
+ \frac{ig}{2} \left[ W_\mu^+ \left( \partial^\mu \phi^- \right) - W_\mu^- \left( \partial^\mu \phi^+ \right) \right] \eta - \frac{g^2}{2 \cos \theta_w} \sin^2 \theta_w \left( Z_\mu W^{\mu \eta} \phi^- + Z_\mu W^{\mu -i} \phi^+ \right) \eta
+ \frac{g^2}{2} \sin \theta_w \left( A_\mu W^{\mu \eta} \phi^- + A_\mu W^{\mu -i} \phi^+ \right) \eta - \frac{g^2}{2} \left[ W_\mu^+ \left( \partial^\mu \phi^- \right) + W_\mu^- \left( \partial^\mu \phi^+ \right) \right] \phi_4
+ \frac{ig^2 \sin^2 \theta_w}{2 \cos \theta_w} \left( Z_\mu W^{- \mu \eta} \phi^- + Z_\mu W^{- \mu -i} \phi^- \right) \phi_4 + \frac{ig^2}{2} \sin \theta_w \left( A_\mu W^{+ \mu \eta} \phi^- - A_\mu W^{+ \mu -i} \phi^- \right) \phi_4
+ \frac{ig}{2} \left( W_\mu^- \phi^- - W_\mu^+ \phi^- \right) \left( \partial^\mu \eta \right) + \frac{g}{2} \left( W_\mu^+ \phi^+ + W_\mu^- \phi^- \right) \left( \partial^\mu \phi_4 \right)
+ \frac{g}{2 \cos \theta_w} Z_\mu \left[ \phi_4 \left( \partial^\mu \eta \right) - \eta \left( \partial^\mu \phi_4 \right) \right].
\end{equation}

(2.54)

In addition to the above Lagrangian, there emerges another term \(L_{\text{mixing}}\) in this gauge, given by

\begin{equation}
L_{\text{mixing}} = i M_W \left[ W_\mu^+ \partial^\mu \phi^- - W_\mu^- \partial^\mu \phi^+ \right] - M_Z Z_\mu \partial^\mu \phi_4
\end{equation}

(2.55)

The presence of the mixing terms in \(L_{\text{mixing}}\) show that \(W_\mu^\pm, \phi_4, Z_\mu\) and \(\phi_4\) are not independent normal coordinates. They can be removed by fixing the gauge, i. e. by adding a
gauge fixing term to the Lagrangian. This gauge fixing term also gives rise to mass term of $M_w$ and $M_z$ to the unphysical Higgs fields $\phi^+$ and $\phi^-$ respectively.

We list below the set of Feynman rules in the 't Hooft-Feynman gauge, relevant to our subsequent calculation of the flavour-changing $Zq, \bar{q}$ vertex:

**Propagators:**

- **Quark:**
  \[ \frac{i}{q \cdot \gamma - m} \]

- **$W^\pm$ boson:**
  \[ \frac{-ig_{\mu\nu}}{p^2 - M_w^2} \]

- **Unphysical $\phi^+$ boson:**
  \[ \frac{i}{p^2 - M_w^2} \]

**Vertices:**

\[ i \rightarrow W^- \rightarrow j \]

\[ -i \frac{g}{\sqrt{2}} \gamma^\mu \frac{1}{2} (1 - \gamma_5) W^\mu \]

\[ j \rightarrow \phi^- \rightarrow i \]

\[ \frac{ig}{\sqrt{2}M_w} (m_R - m_L) V_y \]
\[ \frac{\imath g}{\sqrt{2} M_W} (m, L - m_R) \gamma_\mu \]

\[ \frac{\imath g \gamma_\mu}{\cos \theta_W} \left( L - \frac{2}{3} \sin^2 \theta_W \right) \]

\[ - \frac{\imath g \gamma_\mu}{2 \cos \theta_W} \left( L - \frac{4}{3} \sin^2 \theta_W \right) \]

\[ \imath g \cos \theta_W \left[ (k - 2q)_\mu g_{\nu\lambda} + (q + k)_\nu g_{\lambda\mu} + (q - 2k)_\lambda g_{\mu\nu} \right] \]

\[ - \frac{\imath g \sin^2 \theta_W}{\cos \theta_W} M_W g_{\mu\nu} \]
In the vertices shown above, $m_i$ and $m_j$ refer to the masses of quark $i$ and $j$ respectively, where $i = d, s, b; j = u, c, t$. 

\[
\frac{\tan \cos 2\theta^\ast}{2 \cos \theta^\ast} (2q - k)^{\mu}
\]