Chapter 3

The Flavour-Changing $Zs\bar{d}$ Vertex

3.1 Introduction

In this chapter we shall perform the calculation for the vertex function of the conversion of a down-type quark to a different down-type quark with an emission of a virtual Z boson, specifically,

$$s \rightarrow d + Z^{*}. \tag{3.1}$$

The calculation shall be carried out in 't Hooft-Feynman gauge. In Section 3.2, the complete expression for the unrenormalized $Zs\bar{d}$ vertex shall be presented explicitly as double integrals over the Feynman parameters. We shall use dimensional regularization[56-58] to tame the divergence encountered in the loop-induced vertex. The renormalization of the vertex function shall be performed in Section 3.3. The vertex function shall be put on-shell in Section 3.4

3.2 Unrenormalized Loop-Induced $Zs\bar{d}$ Vertex Function

At the one loop level, the Feynman diagrams contributing to the unrenormalized flavour-changing $Zs\bar{d}$ vertex are listed in Fig. 3.1. Since we perform the calculation in the 't Hooft – Feynman gauge, in addition to the two diagrams in Figs. 3.1(a) and (c), we have also four additional diagrams, Figs. 3.1(b), (d), (e) and (f), arising from the contribution of the unphysical charged Higgs scalar.[53-54]

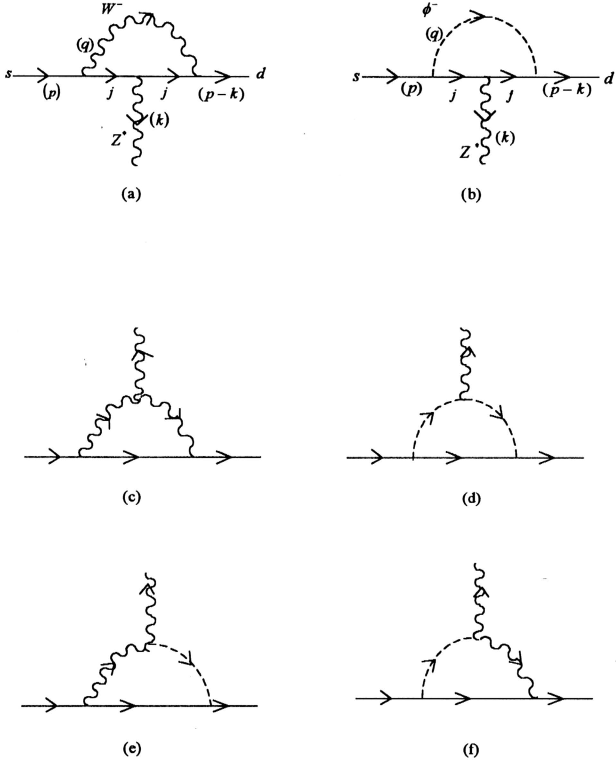

Figure 3.1. The one-loop diagrams contributing to the induced $Zs\bar{d}$ vertex in 't Hooft-Feynman gauge. Diagrams (a) and (c) involve only W (wavy line) in the loop; whereas diagrams (b), (d), (e) and (f) are supplementary diagrams which involve the unphysical scalar ϕ (dashed line).

The vertex function of $Z s \bar{d}$ is the sum of the contributions from the six Feynman diagrams in Fig. 3.1:

$$\Gamma_\mu(p,k) = \sum_{i=a}^{f} \Gamma_\mu^{(i)}(p,k) \qquad i = a, b, \ldots, f. \tag{3.2}$$

These individual contributions are easily written down, and they are

$$\Gamma_\mu^{(a)}(p,k) = -\frac{ig^3}{4\cos\theta_w}\sum_j \lambda_j \int\frac{d^4q}{(2\pi)^4}\left\{\gamma_\alpha L \frac{1}{p-q\cdot\gamma-k-m_j}\gamma_\mu(L-\frac{4}{3}s^2)\times \right. \\ \left. \times\frac{1}{p-q\cdot\gamma-m_j}\gamma_\beta L\frac{g^{\alpha\beta}}{q^2-M_w^2}\right\} \tag{3.3}$$

$$\Gamma_\mu^{(b)}(p,k) = -\frac{ig^3}{4M_w^2\cos\theta_w}\sum_j \lambda_j \int\frac{d^4q}{(2\pi)^4}\left\{(m_d L - m_j R)\frac{1}{p-q\cdot\gamma-k-m_j}\times \right. \\ \left. \times\gamma_\mu(L-\frac{4}{3}s^2)\frac{1}{p-q\cdot\gamma-m_j}(m_j L - m_s R)\frac{1}{q^2-M_w^2}\right\} \tag{3.4}$$

$$\Gamma_\mu^{(c)}(p,k) = -\frac{ig^3(1-s^2)}{2\cos\theta_w}\sum_j \lambda_j \int\frac{d^4q}{(2\pi)^4}\times \\ \times\left\{\gamma_\alpha L\frac{1}{p-q\cdot\gamma-m_j}\gamma_\beta L\frac{g^{\alpha\nu}}{(q-k)^2-M_w^2}\times \right. \\ \times[-g_{\nu\lambda}(2q-k)_\mu + g_{\lambda\mu}(q+k)_\nu + g_{\mu\nu}(q-2k)_\lambda]\times \\ \left. \times\frac{g^{\beta\lambda}}{q^2-M_w^2}\right\} \tag{3.5}$$

$$\Gamma_\mu^{(d)}(p,k)=\frac{ig^3(1-2s^2)}{4M_w^2\cos\theta_w}\sum_j\lambda_j\int\frac{d^4q}{(2\pi)^4}\left\{(m_dL-m_jR)\frac{1}{p-q\cdot\gamma-m_j}\times\right.$$
$$\left.\times(m_jL-m_sR)\frac{1}{(q-k)^2-M_w^2}(2q-k)_\mu\frac{1}{q^2-M_w^2}\right\}$$
(3.6)

$$\Gamma_\mu^{(e)}(p,k)=-\frac{ig^3s^2}{2\cos\theta_w}\sum_j\lambda_j\int\frac{d^4q}{(2\pi)^4}\left\{(m_dL-m_jR)\frac{1}{p-q\cdot\gamma-m_j}\gamma_\alpha L\times\right.$$
$$\left.\times\frac{g_{\mu\beta}}{(q-k)^2-M_w^2}\frac{g^{\alpha\beta}}{q^2-M_w^2}\right\} \tag{3.7}$$

$$\Gamma_\mu^{(f)}(p,k)=\frac{ig^3s^2}{2\cos\theta_w}\sum_j\lambda_j\int\frac{d^4q}{(2\pi)^4}\left\{\gamma_\alpha L\frac{1}{p-q\cdot\gamma-m_j}\times(m_jL-m_sR)\times\right.$$
$$\left.\times\frac{g^{\alpha\beta}}{(q-k)^2-M_w^2}\frac{g_{\mu\beta}}{q^2-M_w^2}\right\}. \tag{3.8}$$

In the above equations, g is the coupling constant, θ_W is the Weinberg angle, m_j ($j = u, c, t$) is the mass of the internal quark, $L=\frac{1}{2}(1-\gamma_5)$ and $R=\frac{1}{2}(1+\gamma_5)$ are respectively the left and right projection operator, and

$$s=\sin\theta_W \tag{3.9}$$

$$\lambda_j=V_{jd}^*V_{js}, \tag{3.10}$$

where V_{ij} are the matrix elements of the KM mixing matrix. The momentum assignments can be directly recognised from Fig 3.1.

These individual contributions to the vertex function fall into two categories according to their denominators. For diagrams 3.1(a) and (b), their denominators are grouped together as

$$F^{(Q)} = [(p-q-k)^2 - m_j^2]^{-1}[(p-q)^2 - m_j^2]^{-1}[q^2 - M_w^2]^{-1} \tag{3.11}$$

which can be expressed, after introducing Feynman parameters, as

$$F^{(Q)} = 2\int_0^1 dx \int_0^{1-x} dy [\tilde{q}^2 - D_Q]^{-3} \tag{3.12}$$

where

$$\tilde{q} = q - (1-x)p + yk \tag{3.13}$$

and

$$D_Q = xM_w^2 + (1-x)m_j^2 - x(1-x)p^2 - y(1-y)k^2 + 2xy(p\cdot k). \tag{3.14}$$

For diagrams 3.1(c) to (f), their denominators are

$$F^{(w)} = [(p-q)^2 - m_j^2]^{-1}[(q-k)^2 - M_w^2]^{-1}[q^2 - M_w^2]^{-1} \tag{3.15}$$

which can be expressed as

$$F^{(w)} = 2\int_0^1 dx \int_0^{1-x} dy [\tilde{q}^2 - D_w]^{-3} \tag{3.16}$$

where

$$\tilde{q} = q - (xp + yk) \tag{3.17}$$

and

$$D_w = (1-x)M_w^2 + xm_j^2 - x(1-x)p^2 - y(1-y)k^2 + 2xy(p\cdot k). \tag{3.18}$$

We generalise the integrals in Eqs. (3.3) to (3.8) from dimension $4 \to n = 4 - \varepsilon$ and shift the integration over the loop momentum, $\int \frac{d^n q}{(2\pi)^n} \to \int \frac{d^n \tilde{q}}{(2\pi)^n}$. The integration over $\tilde{q}$ is then easily evaluated, with the aid of Eq. (A.10) of Appendix A. After integration, Eqs. (3.3) to (3.8) become

$$\Gamma_\mu^{(a)}(p,k)=\int_0^1 dx\int_0^{1-x} dy\frac{g^3}{32\pi^2\cos\theta_W}\sum_j\lambda_j\times$$
$$\times\left\{\left(\frac{4}{3}s^2-1\right)\ln\hat{D}_Q\gamma_\mu+\left[\left(1-\frac{4}{3}s^2\right)\left[x^2\not{p}\gamma_\mu\not{p}+x(y-1)\not{p}\gamma_\mu\not{k}\right.\right.\right.$$
$$\left.\left.\left.+xy\not{k}\gamma_\mu\not{p}+y(y-1)\not{k}\gamma_\mu\not{k}\right]+\left(\frac{4}{3}m_j^2s^2\gamma_\mu\right)\right]D_Q^{-1}\right\}L$$

(3.19)

$$\Gamma_\mu^{(b)}(p,k)=\int_0^1 dx\int_0^{1-x} dy\frac{g^3}{32\pi^2\cos\theta_w}\sum_j\lambda_j\times$$
$$\times\left\{\frac{2}{3}s^2\hat{m}_j^2\xi\gamma_\mu L+\frac{2}{3}s^2\hat{m}_j^2\gamma_\mu L+\right.$$
$$\left[\frac{1}{2}\hat{m}_s\hat{m}_d(\frac{4}{3}s^2-1)\gamma_\mu R+\frac{2}{3}\hat{m}_j^2\gamma_\mu L\right]\ln\hat{D}_Q$$
$$-\frac{1}{2}D_Q^{-1}\left[\ [x^2\not{p}\gamma_\mu\not{p}+xy\not{p}\gamma_\mu\not{k}+x(y-1)\not{k}\gamma_\mu\not{p}+y(y-1)\not{k}\gamma_\mu\not{k}]\times\right.$$
$$\times[\hat{m}_s\hat{m}_d(\frac{4}{3}s^2-1)R+\frac{4}{3}\hat{m}_j^2s^2L]$$
$$+(x\gamma_\mu\not{p}+y\gamma_\mu\not{k})[m_s\hat{m}_j^2(1-\frac{4}{3}s^2)R-\frac{4}{3}m_d\hat{m}_j^2s^2L]$$
$$+[x\not{p}\gamma_\mu+(y-1)\not{k}\gamma_\mu][m_d\hat{m}_j^2(1-\frac{4}{3}s^2)L-\frac{4}{3}s^2m_s\hat{m}_j^2R]$$
$$\left.\left.+\frac{4}{3}s^2m_sm_d\hat{m}_j^2\gamma_\mu R-\hat{m}_j^2m_j^2(1-\frac{4}{3}s^2)\gamma_\mu\ \right]\right\}$$

(3.20)

$$\Gamma_\mu^{(c)}(p,k)=\int_0^1 dx\int_0^{1-x} dy\frac{g^3}{32\pi^2\cos\theta_W}\sum_j\lambda_j(1-s^2)\times$$
$$\times\left\{6\gamma_\mu\ln\hat{D}_w-\left[2x(1-x)\not{p}\gamma_\mu\not{p}+(1-x)(2y-1)\not{p}\gamma_\mu\not{k}\right.\right.$$
$$\left.\left.-2xy\not{k}\gamma_\mu\not{p}+y(1-2y)\not{k}\gamma_\mu\not{k}+2[2x(1-x)p^2+y(1-2y)k^2]\gamma_\mu\right]D_w^{-1}\right\}$$

(3.21)

$$
\begin{aligned}
\Gamma_\mu^{(d)}(p,k) = & \frac{1}{2}(1-2s^2)\frac{g^3}{32\pi^2\cos\theta_W}\sum_j \lambda_j \times \left\{ \frac{1}{2}\hat{m}_j^2 \xi \gamma_\mu L \right. \\
& + \int_0^1 dx \int_0^{1-x} dy \Big[(\hat{m}_s \hat{m}_d L + m_j^2 R)[2x(x-1) p\!\!\!/ \gamma_\mu p\!\!\!/ \\
& + (1-x)(1-2y) p\!\!\!/ \gamma_\mu k\!\!\!/ + 2xy k\!\!\!/ \gamma_\mu p\!\!\!/ + y(2y-1) k\!\!\!/ \gamma_\mu k\!\!\!/ + 2x(x-1)p^2\gamma_\mu \\
& + y(2y-1)k^2\gamma_\mu] + \hat{m}_j^2 (m_d L + m_s R)[2x(\gamma_\mu p\!\!\!/ + p\!\!\!/ \gamma_\mu) \\
& + (2y-1)(\gamma_\mu k\!\!\!/ + k\!\!\!/ \gamma_\mu)] \Big] D_w^{-1} \\
& \left. + 2\int_0^1 dx \int_0^{1-x} dy (\hat{m}_s \hat{m}_d + \hat{m}_j^2 R) \ln \hat{D}_w \gamma_\mu L \right\}
\end{aligned}
\tag{3.22}
$$

$$
\Gamma_\mu^{(e)}(p,k) = \frac{g^3}{32\pi^2\cos\theta_W}\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy\, s^2 \left\{ m_j^2 \gamma_\mu L - m_d\left[(1-x)p\!\!\!/ - y k\!\!\!/\right]\gamma_\mu R \right\} D_w^{-1}
\tag{3.23}
$$

$$
\Gamma_\mu^{(f)}(p,k) = \frac{g^4}{32\pi^2\cos\theta_W}\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy\, s^2 \left\{ m_j^2 \gamma_\mu L - m_s \gamma_\mu \left[(1-x)p\!\!\!/ - y k\!\!\!/\right] R \right\} D_w^{-1} ,
\tag{3.24}
$$

where we have introduced $\xi = \ln\left(\frac{M_w^2 e^\gamma}{4\pi\mu^2}\right) - \frac{2}{\varepsilon}$, with the Euler's constant $\gamma = 0.5772$, $\hat{m}_s = \frac{m_s}{M_w}$, $\hat{m}_d = \frac{m_d}{M_w}$, $\hat{m}_j = \frac{m_j}{M_w}$, $\hat{D}_Q = \frac{D_Q}{M_w^2}$ and $\hat{D}_W = \frac{D_W}{M_w^2}$. In arriving at the above expressions, we have dropped terms independent of m_j due to the unitarity property of the KM matrix. The divergence in the limit of $n \to 4$ has been singled out in terms of ξ. Putting everything together, the unrenormalized vertex is then given by

$$\Gamma_\mu(p,k)=\frac{g^3}{32\pi^2\cos\theta_W}\sum_j\lambda_j\Bigg\{-\frac{1}{4}\left(\frac{2}{3}s^2-1\right)\xi\hat{m}_j^2\gamma_\mu L+\frac{1}{3}\hat{m}_j^2 s^2\gamma_\mu L$$
$$+\Bigg[E_1^L\gamma_\mu+\frac{1}{M_w^2}\left(E_2^L p\gamma_\mu p+E_3^L k\gamma_\mu k+E_4^L p\gamma_\mu k+E_5^L k\gamma_\mu p+E_6^L pk\gamma_\mu\right)$$
$$+\frac{1}{M_w}\left(E_7^L p\gamma_\mu+E_8^L\gamma_\mu p+E_9^L k\gamma_\mu+E_{10}^L\gamma_\mu k\right)\Bigg]L$$
$$+\Bigg[E_1^R\gamma_\mu+\frac{1}{M_w^2}\left(E_2^R p\gamma_\mu p+E_3^R k\gamma_\mu k+E_4^R p\gamma_\mu k+E_5^R k\gamma_\mu p+E_6^R pk\gamma_\mu\right)$$
$$+\frac{1}{M_w}\left(E_7^R p\gamma_\mu+E_8^R\gamma_\mu p+E_9^R k\gamma_\mu+E_{10}^R\gamma_\mu k\right)\Bigg]R\Bigg\}$$

(3.25)

where

$$E_i^{L,R}=\int_0^1 dx\int_0^{1-x}H_i^{L,R}\,dy \qquad i=1,2,\ldots,10 \tag{3.26}$$

$$H_1^L=\Bigg\{\left(2(1-s^2)+\frac{1}{4}(1-2s^2)\hat{m}_j^2\right)\left(2x(x-1)p^2+y(2y-1)k^2\right)$$
$$+[xy(1-2s^2)\hat{m}_j^2-2(x+2y-4xy-1)(1-s^2)]k\cdot p+2m_j^2s^2\Bigg\}D_w^{-1}$$
$$+\left[6(1-s^2)+\frac{1}{2}(1-2s^2)\hat{m}_j^2\right]\ln\hat{D}_w-m_j^2\left[\frac{4}{3}s^2+\frac{1}{2}\left(\frac{4}{3}s^2-1\right)\hat{m}_j^2\right]D_Q^{-1}$$
$$+\left(\frac{4}{3}s^2-1+\frac{2}{3}s^2\hat{m}_j^2\right)\ln\hat{D}_Q \tag{3.27a}$$

$$H_2^L=2x(x-1)\left[\frac{1}{4}(1-2s^2)\hat{m}_j^2+1-s^2\right]\hat{D}_w^{-1}-x^2\left[\frac{4}{3}s^2-1+\frac{2}{3}s^2\hat{m}_j^2\right]\hat{D}_Q^{-1} \tag{3.27b}$$

$$H_3^L=y(2y-1)\left[\frac{1}{4}(1-2s^2)\hat{m}_j^2+1-s^2\right]\hat{D}_w^{-1}+y(y-1)\left[\frac{4}{3}s^2-1+\frac{2}{3}s^2\hat{m}_j^2\right]\hat{D}_Q^{-1}$$

(3.27c)

$$H_4^L = \left[\frac{1}{4}(1-2y)(1-x)(1-2s^2)\hat{m}_j^2 + (x+2xy-y-1)(1-s^2)\right]\hat{D}_w^{-1} + x\left[(1-y)\left(\frac{4}{3}s^2-1\right) - \frac{2}{3}ys^2\hat{m}_j^2\right]\hat{D}_Q^{-1} \tag{3.27d}$$

$$H_5^L = \left[\frac{1}{2}xy(1-2s^2)\hat{m}_j^2 + (2xy-y+2-2x)(1-s^2)\right]\hat{D}_w^{-1} + x\left[y\left(1-\frac{4}{3}s^2\right) + \frac{2}{3}(1-y)s^2\hat{m}_j^2\right]\hat{D}_Q^{-1} \tag{3.27e}$$

$$H_6^L = \frac{1}{4}(1-x-2y)(1-2s^2)\hat{m}_j^2\hat{D}_w^{-1} \tag{3.27f}$$

$$H_7^L = \hat{m}_d\left[(x-1)s^2 + \frac{1}{2}x(1-2s^2)\hat{m}_j^2\right]\hat{D}_w^{-1} - \frac{x}{2}\left(1-\frac{4}{3}s^2\right)\hat{m}_d\hat{m}_j^2\hat{D}_Q^{-1} \tag{3.27g}$$

$$H_8^L = \frac{x}{2}\hat{m}_d\hat{m}_j^2\left[(1-2s^2)\hat{D}_w^{-1} + \frac{4}{3}s^2\hat{D}_Q^{-1}\right] \tag{3.27h}$$

$$H_9^L = \hat{m}_d\left[ys^2 + \frac{1}{4}(2y-1)(1-2s^2)\hat{m}_j^2\right]\hat{D}_w^{-1} + \frac{1}{2}(1-y)\left(1-\frac{4}{3}s^2\right)\hat{m}_d\hat{m}_j^2\hat{D}_Q^{-1} \tag{3.27i}$$

$$H_{10}^L = \frac{1}{4}\hat{m}_d\hat{m}_j^2\left[(2y-1)(1-2s^2)\hat{D}_w^{-1} + \frac{8}{3}ys^2\hat{D}_Q^{-1}\right] \tag{3.27j}$$

$$H_1^R = \frac{1}{2}(1-2s^2)\hat{m}_s\hat{m}_d\left\{\left[x(x-1)p^2 + y(y-\frac{1}{2})k^2 + 2xyk\cdot p\right]D_w^{-1} + \ln\hat{D}_w\right\} - \hat{m}_s\hat{m}_d\left[\frac{2}{3}s^2\hat{m}_j^2\hat{D}_Q^{-1} + \left(\frac{1}{2}-\frac{2}{3}s^2\right)\ln\hat{D}_Q\right] \tag{3.28a}$$

$$H_2^R = \frac{x}{2}\hat{m}_s\hat{m}_d\left[(x-1)(1-2s^2)\hat{D}_w^{-1} + x\left(1-\frac{4}{3}s^2\right)\hat{D}_Q^{-1}\right] \tag{3.28b}$$

$$H_3^R = \frac{y}{2}\hat{m}_s\hat{m}_d\left[\left(y-\frac{1}{2}\right)\left(1-2s^2\right)\hat{D}_w^{-1} + (1-y)\left(\frac{4}{3}s^2-1\right)\hat{D}_Q^{-1}\right] \tag{3.28c}$$

$$H_4^R = \frac{1}{2}\hat{m}_s\hat{m}_d\left[(y-\frac{1}{2})(x-1)(1-2s^2)\hat{D}_w^{-1} + xy(1-\frac{4}{3}s^2)\hat{D}_Q^{-1}\right] \tag{3.28d}$$

$$H_5^R = \frac{x}{2}\hat{m}_s\hat{m}_d\left[y(1-2s^2)\hat{D}_w^{-1} + (1-y)\left(\frac{4}{3}s^2-1\right)\hat{D}_Q^{-1}\right] \tag{3.28e}$$

$$H_6^R = \frac{1}{4}(1-x-2y)(1-2s^2)\hat{m}_s\hat{m}_d\hat{D}_w^{-1} \tag{3.28f}$$

$$H_7^R = \frac{x}{2}\hat{m}_s\hat{m}_j^2\left[(1-2s^2)\hat{D}_w^{-1} + \frac{4}{3}s^2\hat{D}_Q^{-1}\right] \tag{3.28g}$$

$$H_8^R = \hat{m}_s\left[(x-1)s^2 + \frac{x}{2}(1-2s^2)\hat{m}_j^2\right]\hat{D}_w^{-1} + \frac{x}{2}(\frac{4}{3}s^2 - 1)\hat{m}_s\hat{m}_j^2\hat{D}_Q^{-1} \tag{3.28h}$$

$$H_9^R = \frac{1}{2}\hat{m}_s\hat{m}_j^2\left[(y-\frac{1}{2})(1-2s^2)\hat{D}_w^{-1} + \frac{4}{3}(y-1)s^2\hat{D}_Q^{-1}\right] \tag{3.28i}$$

$$H_{10}^R = \hat{m}_s\left[ys^2 + \frac{1}{4}(2y-1)(1-2s^2)\hat{m}_j^2\right]\hat{D}_w^{-1} + \frac{y}{2}(\frac{4}{3}s^2 - 1)\hat{m}_s\hat{m}_j^2\hat{D}_Q^{-1}. \tag{3.28j}$$

The vertex function is divergent and renormalization is needed. The scheme for renormalization is discussed in Section 3.3.

3.3 Renormalization of the $Zs\bar{d}$ Vertex Function

The renormalization scheme for $Zs\bar{d}$ vertex has been treated formally by Soares and Barroso[31] by generating the corresponding counter term through the on-shell renormalization scheme developed by Sakakibara.[59] Recently, Chia and Chong[55] have given a simple prescription for renormalizing the $Zs\bar{d}$ vertex, where the counter term is generated as the on-shell vertices arising from the off-diagonal self-energy as depicted in Fig. 3.2. Pictorially, this renormalization scheme is shown in Fig. 3.3.

The unrenormalized flavour-changing $Zs\bar{d}$ vertex function arising from the off-diagonal self-energy can be written as

$$\Omega_\mu(p,k) = \sum_i \Omega_\mu^{(i)}(p,k) \qquad i = a,b,c,d \tag{3.29}$$

where index i refers to the corresponding diagram in Fig. 3.2. Explicitly, we have

$$\Omega_\mu^{(a)}(p,k) = -\frac{ig^3}{4\cos\theta_w}\sum_j \lambda_j \int\frac{d^4q}{(2\pi)^4}\left\{\gamma_\mu(\frac{2}{3}s^2 - L)\frac{1}{p-m_d}\gamma_\alpha L \times \right. \\ \left. \times\frac{1}{p-q\cdot\gamma - m_j}\gamma_\beta L\frac{g^{\alpha\beta}}{q^2-M_w^2}\right\} \tag{3.30a}$$

$$\Omega_\mu^{(b)}(p,k) = \frac{ig^3}{4\cos\theta_w}\sum_j \lambda_j \int\frac{d^4q}{(2\pi)^4}\left\{\gamma_\alpha L\frac{1}{p-k-q\cdot\gamma - m_j}\gamma_\beta L\frac{g^{\alpha\beta}}{q^2-M_w^2}\times \right. \\ \left. \times\frac{1}{p-k-m_s}\gamma_\mu(L-\frac{2}{3}s^2)\right\} \tag{3.30b}$$

$$\Omega_\mu^{(c)}(p,k) = -\frac{ig^3}{4M_w^2\cos\theta_w}\sum_j \lambda_j \int\frac{d^4q}{(2\pi)^4}\left\{\gamma_\mu(L-\frac{2}{3}s^2)\frac{1}{p-m_d}(m_jR - m_dL)\times \right. \\ \left. \times\frac{1}{p-q\cdot\gamma - m_j}(m_jL - m_sR)\frac{1}{q^2-M_w^2}\right\}$$

(3.30c)

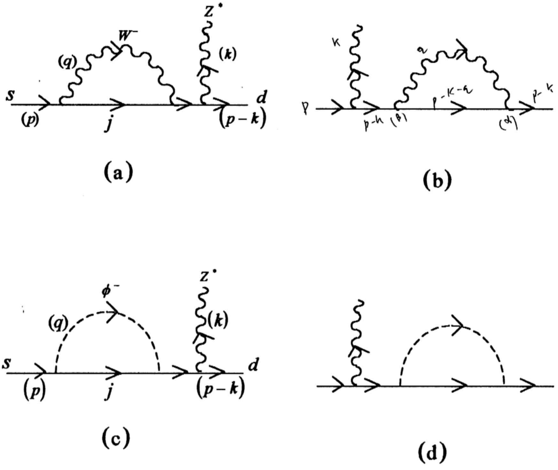

Figure 3.2. One-particle reducible diagrams involving flavour-changing self-energy.

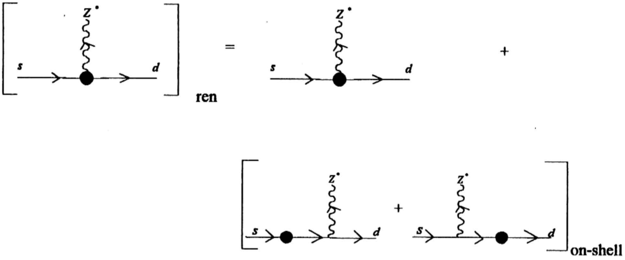

Figure 3.3. The renormalization prescription of Chia and Chong.

$$\Omega_\mu^{(d)}(p,k) = -\frac{ig^3}{4M_w^2\cos\theta_w}\sum_j \lambda_j \int\frac{d^4q}{(2\pi)^4}\left\{(m_jR - m_dL)\frac{1}{\not p - \not k - q\cdot\gamma - m_j}\times\right.$$
$$\left.\times(m_jL - m_sR)\frac{1}{q^2 - M_w^2}\frac{1}{\not p - \not k - m_s}\gamma_\mu(L - \frac{2}{3}s^2)\right\}.$$

(3.30d)

Again, the denominators are parametrized by introducing Feynman parameters

$$\left[q^2 - M_w^2\right]^{-1}\left[(p-q)^2 - m_j^2\right]^{-1} = \int_0^1 dx\left[\tilde{q}_1^2 - D_1\right]^{-2} \tag{3.31a}$$

$$\left[q^2 - M_w^2\right]^{-1}\left[(p-k-q)^2 - m_j^2\right]^{-1} = \int_0^1 dx\left[\tilde{q}_2^2 - D_2\right]^{-2} \tag{3.31b}$$

where

$$\tilde{q}_1 = q - (1-x)p \tag{3.32a}$$

$$\tilde{q}_2 = q - (1-x)(p-k) \tag{3.32b}$$

and

$$D_1 = xM_w^2 + (1-x)m_j^2 - x(1-x)p^2 \tag{3.33a}$$

$$D_2 = xM_w^2 + (1-x)m_j^2 - x(1-x)(p-k)^2. \tag{3.33b}$$

Proceeding as before, the integration over the loop momentum $\tilde{q}$ in n-dimension space is performed. After collecting the divergent terms, we obtain, after some manipulation,

$$\Omega_\mu^{(a)}(p,k) = \left(p^2 - m_d^2\right)^{-1}\frac{g^3}{32\cos\theta_W}\sum_j \lambda_j \times$$
$$\times\int_0^1\left\{x\left(\xi + 1 + \ln\hat{D}_1\right)\gamma_\mu\left[(\frac{2}{3}s^2 - 1)p^2L + \frac{2}{3}s^2 m_d R\not p\right]\right\}dx$$

(3.34a)

$$\Omega_\mu^{(b)}(p,k) = \left[(p-k)^2 - m_s^2\right]^{-1}\frac{g^3}{32\cos\theta_W}\sum_j \lambda_j \int_0^1\left\{x\left(\xi + 1 + \ln\hat{D}_2\right)\left[(\frac{2}{3}s^2 - 1)(p-k)^2 R\right.\right.$$
$$\left.\left.+\frac{2}{3}s^2 m_s(\not p - \not k)L\right]\gamma_\mu\right\}dx \tag{3.34b}$$

$$\Omega_\mu^{(c)}(p,k)=\left[p^2-m_d^2\right]^{-1}\frac{g^3}{32\pi^2\cos\theta_W M_w^2}\sum_j\lambda_j\int_0^1\left(\xi+\ln\hat{D}_1\right)\left\{\left(\frac{1}{3}s^2-\frac{1}{2}\right)\gamma_\mu L\times\right.$$
$$\times\left[m_j^2(xp^2-m_d^2)+m_s(xm_d^2-m_j^2)\not{p}\right]$$
$$\left.-\frac{1}{3}s^2m_d\gamma_\mu R\left[m_s(m_j^2-xp^2)+(1-x)m_j^2\not{p}\right]\right\}dx \qquad (3.34c)$$

$$\Omega_\mu^{(d)}(p,k)=\left[(p-k)^2-m_s^2\right]^{-1}\frac{g^3}{32\pi^2\cos\theta_W M_w^2}\sum_j\lambda_j\int_0^1\left(\xi+\ln\hat{D}_2\right)\left\{\left(\frac{1}{3}s^2-\frac{1}{2}\right)\times\right.$$
$$\times\left[m_j^2[x(p-k)^2-m_s^2]+m_d(xm_s^2-m_j^2)(\not{p}-\not{k})\right]\gamma_\mu L$$
$$\left.-\frac{1}{3}s^2m_s\left[m_d[m_j^2-x(p-k)^2]+(1-x)m_j^2(\not{p}-\not{k})\right]\gamma_\mu R\right\}dx. \qquad (3.34d)$$

Putting the external quarks on mass shell,

$$\not{p}u_s(p)=m_su_s(p) \quad , \quad p^2=m_s^2 \qquad (3.35a)$$

$$\bar{u}_d(p-k)(\not{p}-\not{k})=m_d\bar{u}_d(p-k) \quad , \quad (p-k)^2=m_d^2 \qquad (3.35b)$$

and omitting terms independent of m_j, we obtain the on-shell vertex function arising from the self-energy,

$$\tilde{\Omega}_\mu(\text{on}-\text{shell})=\frac{g^3}{32\pi^2\cos\theta_W(m_d^2-m_s^2)}\sum_j\lambda_j\left\{\left(\frac{2}{3}s^2-1\right)\left[\frac{1}{4}(m_d^2-m_s^2)\hat{m}_j^2\xi\right.\right.$$
$$+\left(1+\frac{\hat{m}_j^2}{2}\right)\left[m_d^2F_1(m_d^2)-m_s^2F_1(m_s^2)\right]+\frac{1}{2}\hat{m}_s^2m_d^2\left[F_1(m_d^2)-F_1(m_s^2)\right]$$
$$\left.+\frac{1}{2}(m_s^2+m_d^2)\hat{m}_j^2\left[F_0(m_s^2)-F_0(m_d^2)\right]\right]\gamma_\mu L$$
$$+\frac{2}{3}s^2m_sm_d\left[\frac{1}{2}\left[\hat{m}_d^2F_1(m_d^2)-\hat{m}_s^2F_1(m_s^2)\right]\right.$$
$$\left.\left.+\left(1+\frac{\hat{m}_j^2}{2}\right)\left[F_1(m_d^2)-F_1(m_s^2)\right]+\hat{m}_j^2\left[F_0(m_s^2)-F_0(m_d^2)\right]\right]\gamma_\mu R\right\}$$

(3.36)

where

$$F_0(p^2) = \int_0^1 \ln\left[x + (1-x)\hat{m}_j^2 + x(x-1)\frac{p^2}{M_w^2}\right]dx \tag{3.37a}$$

$$F_1(p^2) = \int_0^1 x\ln\left[x + (1-x)\hat{m}_j^2 + x(x-1)\frac{p^2}{M_w^2}\right]dx . \tag{3.37b}$$

Eq. (3.36) is then the counter term desired. The $Zs\bar{d}$ vertex function is now readily renormalized to give

$$\begin{aligned}
\Gamma_{\mu,R}(p,k) &= \Gamma_\mu(p,k) + \tilde{\Omega}_\mu(\text{on}-\text{shell}) \\
&= \frac{g^3}{32\pi^2\cos\theta_W}\sum_j \lambda_j\Big\{\beta_L\gamma_\mu L + \beta_R\gamma_\mu R + \frac{1}{3}\hat{m}_j^2 s^2\gamma_\mu L \\
&+ \Big[E_1^L\gamma_\mu + \frac{1}{M_w^2}\left(E_2^L p\gamma_\mu p + E_3^L k\gamma_\mu k + E_4^L p\gamma_\mu k + E_5^L k\gamma_\mu p + E_6^L pk\gamma_\mu\right) \\
&+ \frac{1}{M_w}\left(E_7^L p\gamma_\mu + E_8^L\gamma_\mu p + E_9^L k\gamma_\mu + E_{10}^L\gamma_\mu k\right)\Big]L \\
&+ \Big[E_1^R\gamma_\mu + \frac{1}{M_w^2}\left(E_2^R p\gamma_\mu p + E_3^R k\gamma_\mu k + E_4^R p\gamma_\mu k + E_5^R k\gamma_\mu p + E_6^R pk\gamma_\mu\right) \\
&+ \frac{1}{M_w}\left(E_7^R p\gamma_\mu + E_8^R\gamma_\mu p + E_9^R k\gamma_\mu + E_{10}^R\gamma_\mu k\right)\Big]R\Big\}
\end{aligned} \tag{3.38}$$

where

$$\begin{aligned}
\beta_L = \frac{(\frac{2}{3}s^2 - 1)}{m_d^2 - m_s^2}\Big\{&(1+\frac{\hat{m}_j^2}{2})\left[m_d^2 F_1(m_d^2) - m_s^2 F_1(m_s^2)\right] \\
&+ \frac{1}{2}\hat{m}_s^2 m_d^2\left[F_1(m_d^2) - F_1(m_s^2)\right] + \frac{\hat{m}_j^2}{2}(m_s^2 + m_d^2)\left[F_0(m_s^2) - F_0(m_d^2)\right]\Big\}
\end{aligned} \tag{3.39}$$

and

$$\beta_R = \frac{2}{3} s^2 \frac{m_s m_d}{m_d^2 - m_s^2} \left\{ \frac{1}{2} \left[\hat{m}_d^2 F_1(m_d^2) - \hat{m}_s^2 F_1(m_s^2) \right] \right.$$
$$\left. + (1 + \frac{1}{2} \hat{m}_j^2) \left[F_1(m_d^2) - F_1(m_s^2) \right] + \hat{m}_j^2 \left[F_0(m_s^2) - F_0(m_d^2) \right] \right\}. \tag{3.40}$$

The divergent term in $\Gamma_\mu(p,k)$ is exactly cancelled by the divergent term in $\widetilde{\Omega}_\mu$, giving Eq. (3.38) as the renormalized vertex function.

3.4 The On-Shell Vertex Function

The vertex function is put on-shell by applying the on-shell condition of Eq. (3.35). By utilising the following identities,

$$\gamma_\mu \not{k} = k_\mu + i\sigma_{\mu\nu} k^\nu \tag{3.41}$$

$$\not{k} \gamma_\mu = k_\mu - i\sigma_{\mu\nu} k^\nu \tag{3.42}$$

where

$$\sigma_{\mu\nu} = \frac{1}{2i} \left[\gamma_\mu, \gamma_\nu \right], \tag{3.43}$$

we obtain the following expression for the renormalized on-shell vertex function

$$\widetilde{\Gamma}_{\mu,R}(\text{on}-\text{shell}) = \frac{gG_F}{4\sqrt{2}\pi^2 \cos\theta_w} \sum_j \lambda_j \left\{ (k_\mu \not{k} - k^2 \gamma_\mu)(A_j^L L + A_j^R R) \right.$$
$$\left. + i\sigma_{\mu\nu} k^\nu (m_d B_j^L L + m_s B_j^R R) + M_w^2 (C_j^L \gamma_\mu L + C_j^R \gamma_\mu R) \right\} \tag{3.44}$$

with

$$G_F = \left(g / M_W \right)^2 / 4\sqrt{2}. \tag{3.45}$$

The form factors A_j^{LR}, B_j^{LR} and C_j^{LR} are given by

$$A_j^L = \frac{1}{\hat{m}_s^2 - \hat{m}_d^2}\Big[\hat{m}_d^2\left(\widetilde{E}_4^L + \widetilde{E}_6^L\right) + \hat{m}_s^2\left(\widetilde{E}_2^L + \widetilde{E}_5^L\right)$$
$$+ \hat{m}_s\hat{m}_d\left(\widetilde{E}_2^R + \widetilde{E}_4^R + \widetilde{E}_5^R + \widetilde{E}_6^R\right) + \hat{m}_d\left(\widetilde{E}_7^L + \widetilde{E}_9^L + \widetilde{E}_{10}^L\right) \tag{3.46}$$
$$+ \hat{m}_s\left(\widetilde{E}_7^R + \widetilde{E}_9^R + \widetilde{E}_{10}^R\right)\Big] + 2\left(\widetilde{E}_3^L + \widetilde{E}_4^L\right)$$

$$A_j^R = \frac{1}{\hat{m}_s^2 - \hat{m}_d^2}\Big[\hat{m}_d^2\left(\widetilde{E}_4^R + \widetilde{E}_6^R\right) + \hat{m}_s^2\left(\widetilde{E}_2^R + \widetilde{E}_5^R\right)$$
$$+ \hat{m}_s\hat{m}_d\left(\widetilde{E}_2^L + \widetilde{E}_4^L + \widetilde{E}_5^L + \widetilde{E}_6^L\right) + \hat{m}_d\left(\widetilde{E}_7^R + \widetilde{E}_9^R + \widetilde{E}_{10}^R\right) \tag{3.47}$$
$$+ \hat{m}_s\left(\widetilde{E}_7^L + \widetilde{E}_9^L + \widetilde{E}_{10}^L\right)\Big] + 2\left(\widetilde{E}_3^R + \widetilde{E}_4^R\right)$$

$$B_j^L = \widetilde{E}_4^L - \widetilde{E}_6^L - \frac{m_s}{m_d}\left(\widetilde{E}_2^R + \widetilde{E}_5^R\right) + \frac{1}{\hat{m}_d}\left(\widetilde{E}_{10}^L - \widetilde{E}_7^L - \widetilde{E}_9^L\right) \tag{3.48}$$

$$B_j^R = \frac{m_d}{m_s}\left(\widetilde{E}_4^R - \widetilde{E}_6^R\right) - \widetilde{E}_2^L - \widetilde{E}_5^L + \frac{1}{\hat{m}_s}\left(\widetilde{E}_{10}^R - \widetilde{E}_7^R - \widetilde{E}_9^R\right) \tag{3.49}$$

$$C_j^L = \widetilde{E}_1^L + \frac{k^2}{M_w^2}\left(\widetilde{E}_3^L + \widetilde{E}_4^L - \widetilde{E}_6^L\right) + \hat{m}_s\hat{m}_d\widetilde{E}_2^R + \hat{m}_d\widetilde{E}_7^L + \hat{m}_s\widetilde{E}_8^R$$
$$+ \frac{\hat{m}_d k^2}{m_s^2 - m_d^2}\left[\hat{m}_d\left(\widetilde{E}_4^L + \widetilde{E}_6^L\right) + \hat{m}_s\left(\widetilde{E}_2^R + \widetilde{E}_5^R\right) + \widetilde{E}_7^L + \widetilde{E}_9^L + \widetilde{E}_{10}^L\right]$$
$$+ \frac{\hat{m}_s k^2}{m_s^2 - m_d^2}\left[\hat{m}_d\left(\widetilde{E}_4^R + \widetilde{E}_6^R\right) + \hat{m}_s\left(\widetilde{E}_2^L + \widetilde{E}_5^L\right) + \widetilde{E}_7^R + \widetilde{E}_9^R + \widetilde{E}_{10}^R\right] \tag{3.50}$$
$$+ \beta_L + \frac{1}{3}s^2\hat{m}_j^2$$

$$C_j^R = \widetilde{E}_1^R + \frac{k^2}{M_w^2}\left(\widetilde{E}_3^R + \widetilde{E}_4^R + \widetilde{E}_6^R\right) + \hat{m}_s\hat{m}_d\widetilde{E}_2^L + \hat{m}_d\widetilde{E}_7^R + \hat{m}_s\widetilde{E}_8^L$$
$$+ \frac{\hat{m}_d k^2}{m_s^2 - m_d^2}\left[\hat{m}_d\left(\widetilde{E}_4^R + \widetilde{E}_6^R\right) + \hat{m}_s\left(\widetilde{E}_2^L + \widetilde{E}_5^L\right) + \widetilde{E}_7^R + \widetilde{E}_9^R + \widetilde{E}_{10}^R\right] \tag{3.51}$$
$$+ \frac{\hat{m}_s k^2}{m_s^2 - m_d^2}\left[\hat{m}_d\left(\widetilde{E}_4^L + \widetilde{E}_6^L\right) + \hat{m}_s\left(\widetilde{E}_2^R + \widetilde{E}_5^R\right) + \widetilde{E}_7^L + \widetilde{E}_9^L + \widetilde{E}_{10}^L\right] + \beta_R$$

with

$$\tilde{E}_i^L = E_i^L(\text{on}-\text{shell}) \text{ and } \tilde{E}_i^R = E_i^R(\text{on}-\text{shell}). \tag{3.52}$$

In Eqs. (3.46) - (3.51) the form factors are expressed as double integrals over the Feynman parameters x and y. The integration over the Feynman parameters will be carried out in the next chapter. We wish to point out here that our result is more general than that obtained by earlier calculation.[28,32] As expressed in Eq. (3.44), the on-shell vertex function contains the form factors $A_j^{L,R}$ and $B_j^{L,R}$ in addition to $C_j^{L,R}$. In contrast, earlier calculations have obtained only C_j^L. Our result for the vertex function reduces to that of earlier calculations in the limit of vanishing external momentum and quark masses.