Chapter 4

The $Z s \bar{d}$ Vertex Form Factors

4.1 Introduction

In the previous chapter, we have derived an explicit expression for the on-shell $Zs\bar{d}$ vertex function in terms of six form factors. The form factors are expressed in terms of double integrals over the Feynman parameters. In this chapter the double integration over the Feynman parameters of the form factors will be treated in detail.

The Feynman diagrams that contribute to the Z-penguin at the one-loop level fall naturally into two types. Type I consists of diagrams in which the external Z boson is emitted from the internal quark line and type II consists of diagrams in which the emitted Z boson comes from the W line or the unphysical Higgs line. We shall refer to these 2 types of diagrams as the Q-diagrams and the W-diagrams respectively. The Q-diagrams of Figs. 3.1(a) and (b) involve the integration over a W propagator and two internal j quark propagators (the unphysical Higgs scalar propagator has the same denominator as the W propagators). The corresponding contributions to the vertex function therefore are expected to develop absorptive parts when $k^2 > (2m_j)^2$. Similarly, the W-diagrams of Figs. 3.1(c) - (f) involve the integration over two W-propagators and one internal j quark propagator, and therefore give rise to absorptive parts when $k^2 > (2M_W)^2$.

To obtain the absorptive parts and the dispersive parts of the form factors of the vertex function, a systematic scheme of carrying out the integration is developed. First of all, the integration over parameter y is performed analytically. The integration over parameter x is then carried out numerically using Romberg's method of integration. The numerical integration scheme is similar to that used by S. P. Chia and P. C. Chn'g[60] and P. C. Chn'g.[61] In their integration, the denominator involved is

$$\widetilde{D}_Q(x,y) \approx m_1^2 x^2 + \left[\left(m_1^2 - m_2^2 + k^2\right)y + \left(M_W^2 - m_1^2 - m_j^2\right)\right]x \\ + k^2 y^2 + k^2 y + m_j^2. \tag{4.1}$$

For our case at hand, the external masses, m_1^2 and m_2^2, are neglected compared to M_W^2, so that the quadratic term in x in Eq. (4.1) is dropped to facilitate an easier treatment of the integration, yet do not sacrifice the validity of the numerical results. Care has to be taken, however, while carrying out the numerical integration because there are poles to be handled within the range of integration. A small numerical trick is devised to overcome the problem associated with these singularities. We shall also discuss in details how the absorptive parts of the form factors are generated. In Section 4.3, The behaviour of the form factors, both the dispersive and the absorptive parts, shall be presented.

4.2 Numerical Evaluation of the Form Factors

The $\widetilde{E}$'s in Eq. (3.26) that make up the form factors in Eq. (3.46) - (3.51) are expressible in the following general form

$$\widetilde{E} = \int_0^1 dx \int_0^{1-x} dy\, \widetilde{H}(x,y) = \int_0^1 dy \int_0^{1-y} dx\, \widetilde{H}(x,y) \tag{4.2}$$

where $\widetilde{H}(x,y)$ are generally a double variable function of x and y,

$$\widetilde{H}(x,y) = \frac{h_W^{(2)}(y)x^2 + h_W^{(1)}(y)x + h_W^{(0)}(y)}{\widetilde{D}_W(x,y)} + h_W^{(\ln)}(0)\ln\left[\frac{\hat{D}_w(x,y)}{M_w^2}\right] \\ + \frac{h_Q^{(2)}(y)x^2 + h_Q^{(1)}(y)x + h_Q^{(0)}(y)}{\widetilde{D}_Q(x,y)} + h_Q^{(\ln)}(0)\ln\left[\frac{\hat{D}_Q(x,y)}{M_w^2}\right]. \tag{4.3}$$

The terms involving $\ln\frac{\hat{D}_{W,Q}}{M_W^2}$ in Eq. (4.3) are present only in the expressions of H_1^L and H_1^R.

The explicit dependence on y in $h_{W,Q}^{(i)}(y)$ in Eq. (4.3) are quadratic at the most. In fact we may find that the expressions for $h_{W,Q}^{(i)}(y)$ as explicit functions of y are generally of the form as listed below:

$$h_W^{(2)}(y) = h_{W0}^{(2)} \qquad h_Q^{(2)}(y) = h_{Q0}^{(2)}$$

$$h_W^{(1)}(y) = h_{W0}^{(1)} + yh_{W1}^{(1)} \qquad h_Q^{(1)}(y) = h_{Q0}^{(1)} + yh_{Q1}^{(1)}$$

$$h_W^{(0)}(y) = h_{W0}^{(0)} + yh_{W1}^{(0)} + y^2h_{W2}^{(0)} \qquad h_Q^{(0)}(y) = h_{Q0}^{(0)} + yh_{Q1}^{(0)} + y^2h_{Q2}^{(0)}. \tag{4.4}$$

The explicit expressions for $h_{Wj}^{(i)}$ and $h_{Qj}^{(i)}$, for $j = 0, 1, 2$ in Eqs. (4.4) can be easily deduced from Eqs. (3.27) and Eq. (3.28). They are explicit functions of the particle masses m_s, m_d, m_j, M_W^2 and $\sin^2\theta_W$ without x or y dependence. Before proceed, we shall make an approximation in the denominator functions $\tilde{D}_Q(x,y)$ and $\tilde{D}_W(x,y)$ in Eqs. (3.14) and (3.18) by neglecting the external quark masses in their expressions, i. e. $m_s^2, m_d^2 << M_W^2$. With this approximation, $\tilde{D}_{W,Q}(x,y)$ then become

$$\tilde{D}_W(x,y) \approx \left(m_j^2 - M_W^2 + k^2y\right)x + k^2y^2 + k^2y + M_W^2 \tag{4.5a}$$

$$\tilde{D}_Q(x,y) \approx \left(M_W^2 - m_j^2 + k^2y\right)x + k^2y^2 + k^2y + m_j^2. \tag{4.5b}$$

Now we may start to integrate over the variable x analytically, so that

$$\begin{aligned}\tilde{E} = \int_0^1 dy \Big\{&\left[h_W^{(2)}(y)S_W^{(2)}(y) + h_W^{(1)}(y)S_W^{(1)}(y) + h_W^{(0)}(y)S_W^{(0)}(y) + h_W^{(\ln)}(0)S_W^{(\ln)}(y)\right] \\ &+\left[h_Q^{(2)}(y)S_Q^{(2)}(y) + h_Q^{(1)}(y)S_Q^{(1)}(y) + h_Q^{(0)}(y)S_Q^{(0)}(y) + h_Q^{(\ln)}(0)S_Q^{(\ln)}(y)\right]\Big\}\end{aligned} \tag{4.6}$$

where

$$S_Q^{(2)}(y) = \int_0^{1-y} \frac{x^2}{\widetilde{D}_Q(x,y)}dx$$
$$= \frac{(1-y)}{2\left(M_W^2 - m_j^2 + k^2 y\right)^2}\left[M_W^2 - 3m_j^2 + y\left(3k^2 + m_j^2 - M_W^2\right) - 3k^2y^2\right]$$
$$+ \frac{\left(k^2y^2 - k^2y + m_j^2\right)^2}{\left(M_W^2 - m_j^2 + k^2 y\right)^3}\left\{\ln\left[y\left(m_j^2 - M_W^2\right) + M_W^2\right] - \ln\left[\left(k^2y^2 - k^2y + m_j^2\right)\right]\right\}$$

(4.7a)

$$S_Q^{(1)}(y) = \int_0^{1-y} \frac{x}{\widetilde{D}_Q(x,y)}dx$$
$$= \frac{(1-y)}{M_W^2 - m_j^2 + k^2 y}$$
$$- \frac{\left(k^2y^2 - k^2y + m_j^2\right)}{\left(M_W^2 - m_j^2 + k^2 y\right)^2}\left\{\ln\left[y\left(m_j^2 - M_W^2\right) + M_W^2\right] - \ln\left[\left(k^2y^2 - k^2y + m_j^2\right)\right]\right\}$$

(4.7b)

$$S_Q^{(0)}(y) = \int_0^{1-y} \frac{1}{\widetilde{D}_Q(x,y)}dx$$
$$= \frac{\ln\left[y\left(m_j^2 - M_W^2\right) + M_W^2\right] - \ln\left[\left(k^2y^2 - k^2y + m_j^2\right)\right]}{\left(M_W^2 - m_j^2 + k^2 y\right)} \qquad (4.7c)$$

$$S_W^{(i)}(y) = S_Q^{(i)}(y)(M_W^2 \leftrightarrow m_j^2), \quad i = 0, 1, 2, \qquad (4.7d\text{-}f)$$

whereas

$$S_Q^{(\ln)}(y) = \int_0^{1-y} \ln\left[\frac{\widetilde{D}_Q(x,y)}{M_W^2}\right]dx$$

$$= \frac{\left(m_j^2 - M_W^2\right)y + M_W^2}{\left(M_W^2 - m_j^2 + k^2 y\right)^2}\ln\left[\left(\hat{m}_j^2 - 1\right)y + 1\right] \qquad (4.7g)$$

$$- \frac{\left(k^2 y^2 - k^2 y + m_j^2\right)}{\left(M_W^2 - m_j^2 + k^2 y\right)}\ln\left(\hat{k}^2 y^2 - \hat{k}^2 y + \hat{m}_j^2\right) - (1 - y)$$

and

$$S_W^{(\ln)}(y) = \int_0^{1-y} \ln\left[\frac{\widetilde{D}_W(x,y)}{M_W^2}\right]dx$$

$$= \frac{\left(M_W^2 - m_j^2\right)y + m_j^2}{\left(m_j^2 - M_W^2 + k^2 y\right)^2}\ln\left[\left(1 - \hat{m}_j^2\right)y + \hat{m}_j^2\right] \qquad (4.7h)$$

$$- \frac{\left(k^2 y^2 - k^2 y + M_W^2\right)}{\left(m_j^2 - M_W^2 + k^2 y\right)}\ln\left(\hat{k}^2 y^2 - \hat{k}^2 y + 1\right).$$

In the above equations we have introduced the notation $\hat{k}^2 = k^2 / M_W^2$. For easy reference, we shall refer to the functions defined in Eqs. (4.7a) to (4.7h) as the S functions. Before we evaluate the numerical values of these integrals, there is one more step to be taken. Note that by the unitarity condition of the KM matrices, terms independent of m_j are to be dropped. Therefore we have to subtract from the integral these m_j - independent terms, i. e.

$$\widetilde{E} \to \widetilde{E} - \widetilde{E}(k, m_j = 0) \qquad (4.8)$$

In actual computation, we have written a computer program using FORTRAN to evaluate the numerical values of the form factors. We define the form factors in terms of the E's as in Eqs. (3.46) to (3.51), whereas the E's in turn are defined in terms of the functions of h and S as in Eq. (4.6) after the x-integration. A subroutine of numerical integration using Romberg's method is called to evaluate the integration of $\widetilde{E}$'s in Eq. (4.6), and thus the form factors which are combinations of these $\widetilde{E}$'s are obtained numerically.

However, in Eqs. (4.6) and (4.7a) to (4.7h), we see that the S functions that comprise the integrand of $\widetilde{E}$'s contain two types of poles, i.e. (i) pole in the logarithmic functions and (ii) pole in their denominator functions. For case (i), the logarithmic part of the S functions, $\ln(k^2y^2 - k^2y + m^2)$ becomes undefined at $y_{1,2} = \left[1 \pm \sqrt{1-4m^2/k^2}\right]/2$ when $k^2 \geq 4m^2$ within the range of $0 \leq y \leq 1$, where $m = m_j$ or M_W. At these points the argument of the logarithm become zero. Whereas in case (ii), when $k^2 \geq \left|M_W^2 - m_j^2\right|$, at the point of $y = \left(M_W^2 - m_j^2\right)/k^2$, the S functions blow up and become undefined because their denominators become zero. Nevertheless, these singularities are superficial and vanish after the integration over y from 0 to 1 is made. Reverting to numerical method, we can avoid these complication due to these poles by inserting some vanishingly small but finite parameter, ε, into the relevant logarithmic argument and denominator functions in the S functions to avoid them from becoming zero. This will prevent the computer from seeing floating point errors caused by these singularities, i.e.

$$\int_0^1 \frac{F(y)}{(y-a)^p} dy \to \int_0^1 \frac{F(y)}{(y-a+\varepsilon)^p} dy \tag{4.9a}$$

and

$$\int_0^1 G(y)\ln(k^2y^2 - k^2y + m^2)dy \to \int_0^1 G(y)\ln(k^2y^2 - k^2y + m^2 + \varepsilon)dy. \tag{4.9b}$$

As mentioned, the form factors shall contain absorptive parts. The absorptive parts emerge from the logarithmic terms with quadratic argument in the S functions. If the quadratic argument have no real roots, meaning that it is always in the positive domain, we evaluate it straight-forwardly using numerical method. There are not much complication involved in this case. But when $k^2 > 4m^2$, the argument of the logarithm will become negative between the real roots of the quadratic argument, y_1 and y_2, and an imaginary part will be generated. This is exactly where the absorptive part coming from:

$$\int_0^1 G(y)\ln(k^2y^2 - k^2y + m^2)dy \rightarrow \int_0^1 G(y)\ln\left|k^2y^2 - k^2y + m^2 + \varepsilon\right|dy + i\pi\int_{y_1}^{y_2} G(y)dy \tag{4.10}$$

when $k^2 > 4m^2$.

Within the physical region of $s \rightarrow dZ^*$, $b \rightarrow sZ^*$ and $b \rightarrow dZ^*$, k^2 will only pass through the thresholds at $k^2 = 4m_u^2$ and $k^2 = 4m_c^2$, whereas the others threshold at $k^2 = 4m_t^2$ and $k^2 = 4M_W^2$ will not be exceeded. Therefore, for the region of k^2 beyond the thresholds at $k^2 = 4m_{u,c}^2$ but below the physical limit at $k^2 = (m_1 - m_2)^2$, the $\widetilde{E}$ functions in Eq. (3.26) will carry an absorptive part in addition to the dispersive part:

$$\widetilde{E} = \mathrm{Re}\,\widetilde{E} + i\,\mathrm{Im}\,\widetilde{E}. \tag{4.11}$$

The real part of $\widetilde{E}$ reads

$$\begin{aligned}\mathrm{Re}\,\widetilde{E} = \int_0^1 dy\,\{&h_W^{(2)}(y)\,\mathrm{Re}\,S_W^{(2)}(y) + h_W^{(1)}(y)\,\mathrm{Re}\,S_W^{(1)}(y) + h_W^{(0)}(y)\,\mathrm{Re}\,S_W^{(0)}(y)\\ &+ h_W^{(\ln)}(0)\,\mathrm{Re}\,S_W^{(\ln)}(y) + h_Q^{(2)}(y)\,\mathrm{Re}\,S_Q^{(2)}(y) + h_Q^{(1)}(y)\,\mathrm{Re}\,S_Q^{(1)}(y)\\ &+ h_Q^{(0)}(y)\,\mathrm{Re}\,S_Q^{(0)}(y) + h_Q^{(\ln)}(0)\,\mathrm{Re}\,S_Q^{(\ln)}(y)\}\end{aligned} \tag{4.12}$$

where

$$\begin{aligned}\mathrm{Re}\,S_Q^{(2)}(y) = &\frac{(1-y)}{2\left(M_W^2 - m_j^2 + k^2y\right)^2}\left[M_W^2 - 3m_j^2 + y\left(3k^2 + m_j^2 - M_W^2\right) - 3k^2y^2\right]\\ &+ \frac{\left(k^2y^2 - k^2y + m_j^2\right)^2}{\left(M_W^2 - m_j^2 + k^2y\right)^3}\left\{\ln\left[y\left(m_j^2 - M_W^2\right) + M_W^2\right] - \ln\left|k^2y^2 - k^2y + m_j^2\right|\right\}\end{aligned} \tag{4.13}$$

$$\mathrm{Re}\,S_Q^{(1)}(y) = \frac{(1-y)}{M_W^2 - m_j^2 + k^2 y} - \frac{\left(k^2y^2 - k^2y + m_j^2\right)}{\left(M_W^2 - m_j^2 + k^2y\right)^2}\left\{\ln\left[y\left(m_j^2 - M_W^2\right) + M_W^2\right] - \ln\left|k^2y^2 - k^2y + m_j^2\right|\right\} \tag{4.14}$$

$$\mathrm{Re}\,S_Q^{(0)}(y) = \frac{\ln\left[y\left(m_j^2 - M_W^2\right) + M_W^2\right] - \ln\left|k^2y^2 - k^2y + m_j^2\right|}{\left(M_W^2 - m_j^2 + k^2y\right)} \tag{4.15}$$

$$\mathrm{Re}\,S_W^{(1)}(y) = \mathrm{Re}\,S_Q^{(1)}(y)(M_W^2 \leftrightarrow m_j^2) \tag{4.16}$$

$$\mathrm{Re}\,S_Q^{(\ln)}(y) = \frac{\left(m_j^2 - M_W^2\right)y + M_W^2}{\left(M_W^2 - m_j^2 + k^2y\right)^2}\ln\left[\left(\hat{m}_j^2 - 1\right)y + 1\right] - \frac{\left(k^2y^2 - k^2y + m_j^2\right)}{\left(M_W^2 - m_j^2 + k^2y\right)}\ln\left|\hat{k}^2y^2 - \hat{k}^2y + \hat{m}_j^2\right| - (1-y) \tag{4.17}$$

$$\mathrm{Re}\,S_W^{(\ln)}(y) = \frac{\left(M_W^2 - m_j^2\right)y + m_j^2}{\left(m_j^2 - M_W^2 + k^2y\right)^2}\ln\left[\left(1 - \hat{m}_j^2\right)y + \hat{m}_j^2\right] - \frac{\left(k^2y^2 - k^2y + M_W^2\right)}{\left(m_j^2 - M_W^2 + k^2y\right)}\ln\left|\hat{k}^2y^2 - \hat{k}^2y + 1\right|. \tag{4.18}$$

The imaginary part of $\widetilde{E}$ reads

$$\mathrm{Im}\,\widetilde{E} = \int_{y_1}^{y_2} dy\left\{h_Q^{(2)}(y)\,\mathrm{Im}\,S_Q^{(2)}(y) + h_Q^{(1)}(y)\,\mathrm{Im}\,S_Q^{(1)}(y) + h_Q^{(0)}(y)\,\mathrm{Im}\,S_Q^{(0)}(y) + h_Q^{(\ln)}(0)\,\mathrm{Im}\,S_Q^{(\ln)}(y)\right\} \tag{4.19}$$

where

$$y_1 = \left[1 - \sqrt{1 - 4m_j^2 / k^2}\right] / 2 \tag{4.20a}$$

$$y_2 = \left[1 + \sqrt{1 - 4m_j^2 / k^2}\right] / 2, \tag{4.20b}$$

$$\mathrm{Im}\, S_Q^{(2)}(y) = -\pi \frac{\left(k^2 y^2 - k^2 y + m_j^2\right)^2}{\left(M_W^2 - m_j^2 + k^2 y\right)^3} \tag{4.21}$$

$$\mathrm{Im}\, S_Q^{(1)}(y) = \pi \frac{\left(k^2 y^2 - k^2 y + m_j^2\right)}{\left(M_W^2 - m_j^2 + k^2 y\right)^2} \tag{4.22}$$

$$\mathrm{Im}\, S_Q^{(0)}(y) = \frac{-\pi}{\left(M_W^2 - m_j^2 + k^2 y\right)} \tag{4.23}$$

$$\mathrm{Im}\, S_Q^{(\ln)}(y) = -\pi \frac{\left(k^2 y^2 - k^2 y + m_j^2\right)}{\left(M_W^2 - m_j^2 + k^2 y\right)} \tag{4.24}$$

so that the absorptive part of the form factors reads

$$\begin{aligned} \mathrm{Im}\, A_j^L = \frac{1}{\hat{m}_s^2 - \hat{m}_d^2} \Big[& \hat{m}_d^2 \left(\mathrm{Im}\, \widetilde{E}_4^L + \mathrm{Im}\, \widetilde{E}_6^L\right) + \hat{m}_s^2 \left(\mathrm{Im}\, \widetilde{E}_2^L + \mathrm{Im}\, \widetilde{E}_5^L\right) \\ & + \hat{m}_s \hat{m}_d \left(\mathrm{Im}\, \widetilde{E}_2^R + \mathrm{Im}\, \widetilde{E}_4^R + \mathrm{Im}\, \widetilde{E}_5^R + \mathrm{Im}\, \widetilde{E}_6^R\right) \\ & + \hat{m}_d \left(\mathrm{Im}\, \widetilde{E}_7^L + \mathrm{Im}\, \widetilde{E}_9^L + \mathrm{Im}\, \widetilde{E}_{10}^L\right) + \hat{m}_s \left(\mathrm{Im}\, \widetilde{E}_7^R + \mathrm{Im}\, \widetilde{E}_9^R + \mathrm{Im}\, \widetilde{E}_{10}^R\right) \Big] \\ & + 2\left(\mathrm{Im}\, \widetilde{E}_3^L + \mathrm{Im}\, \widetilde{E}_4^L\right) \end{aligned} \tag{4.25}$$

$$\begin{aligned} \mathrm{Im}\, A_j^R = \frac{1}{\hat{m}_s^2 - \hat{m}_d^2} \Big[& \hat{m}_d^2 \left(\mathrm{Im}\, \widetilde{E}_4^R + \mathrm{Im}\, \widetilde{E}_6^R\right) + \hat{m}_s^2 \left(\mathrm{Im}\, \widetilde{E}_2^R + \mathrm{Im}\, \widetilde{E}_5^R\right) \\ & + \hat{m}_s \hat{m}_d \left(\mathrm{Im}\, \widetilde{E}_2^L + \mathrm{Im}\, \widetilde{E}_4^L + \mathrm{Im}\, \widetilde{E}_5^L + \mathrm{Im}\, \widetilde{E}_6^L\right) \\ & + \hat{m}_d \left(\mathrm{Im}\, \widetilde{E}_7^R + \mathrm{Im}\, \widetilde{E}_9^R + \mathrm{Im}\, \widetilde{E}_{10}^R\right) + \hat{m}_s \left(\mathrm{Im}\, \widetilde{E}_7^L + \mathrm{Im}\, \widetilde{E}_9^L + \mathrm{Im}\, \widetilde{E}_{10}^L\right) \Big] \\ & + 2\left(\mathrm{Im}\, \widetilde{E}_3^R + \mathrm{Im}\, \widetilde{E}_4^R\right) \end{aligned} \tag{4.26}$$

$$\mathrm{Im}\, B_j^L = \mathrm{Im}\, \widetilde{E}_4^L - \mathrm{Im}\, \widetilde{E}_6^L - \frac{m_s}{m_d}\left(\mathrm{Im}\, \widetilde{E}_2^R + \mathrm{Im}\, \widetilde{E}_5^R\right) + \frac{1}{\hat{m}_d}\left(\mathrm{Im}\, \widetilde{E}_{10}^L - \mathrm{Im}\, \widetilde{E}_7^L - \mathrm{Im}\, \widetilde{E}_9^L\right) \tag{4.27}$$

$$\mathrm{Im}\,B_J^R = \frac{m_d}{m_s}\left(\mathrm{Im}\,\widetilde{E}_4^R - \mathrm{Im}\,\widetilde{E}_6^R\right) - \mathrm{Im}\,\widetilde{E}_2^L - \mathrm{Im}\,\widetilde{E}_5^L + \frac{1}{\hat{m}_s}\left(\mathrm{Im}\,\widetilde{E}_{10}^R - \mathrm{Im}\,\widetilde{E}_7^R - \mathrm{Im}\,\widetilde{E}_9^R\right) \tag{4.28}$$

$$\begin{aligned}\mathrm{Im}\,C_J^L = {} & \mathrm{Im}\,\widetilde{E}_1^L + \frac{k^2}{M_w^2}\left(\mathrm{Im}\,\widetilde{E}_3^L + \mathrm{Im}\,\widetilde{E}_4^L - \mathrm{Im}\,\widetilde{E}_6^L\right) \\ & + \hat{m}_s\hat{m}_d\,\mathrm{Im}\,\widetilde{E}_2^R + \hat{m}_d\,\mathrm{Im}\,\widetilde{E}_7^L + \hat{m}_s\,\mathrm{Im}\,\widetilde{E}_8^R \\ & + \frac{\hat{m}_d k^2}{m_s^2 - m_d^2}\Big[\hat{m}_d\left(\mathrm{Im}\,\widetilde{E}_4^L + \mathrm{Im}\,\widetilde{E}_6^L\right) + \hat{m}_s\left(\mathrm{Im}\,\widetilde{E}_2^R + \mathrm{Im}\,\widetilde{E}_5^R\right) \\ & + \mathrm{Im}\,\widetilde{E}_7^L + \mathrm{Im}\,\widetilde{E}_9^L + \mathrm{Im}\,\widetilde{E}_{10}^L\Big] \\ & + \frac{\hat{m}_s k^2}{m_s^2 - m_d^2}\Big[\hat{m}_d\left(\mathrm{Im}\,\widetilde{E}_4^R + \mathrm{Im}\,\widetilde{E}_6^R\right) + \hat{m}_s\left(\mathrm{Im}\,\widetilde{E}_2^L + \mathrm{Im}\,\widetilde{E}_5^L\right) \\ & + Im\widetilde{E}_7^R + \mathrm{Im}\,\widetilde{E}_9^R + \mathrm{Im}\,\widetilde{E}_{10}^R\Big]\end{aligned} \tag{4.29}$$

$$\begin{aligned}\mathrm{Im}\,C_J^R = {} & \mathrm{Im}\,\widetilde{E}_1^R + \frac{k^2}{M_w^2}\left(\mathrm{Im}\,\widetilde{E}_3^R + \mathrm{Im}\,\widetilde{E}_4^R + \mathrm{Im}\,\widetilde{E}_6^R\right) \\ & + \hat{m}_s\hat{m}_d\,\mathrm{Im}\,\widetilde{E}_2^L + \hat{m}_d\,\mathrm{Im}\,\widetilde{E}_7^R + \hat{m}_s\,\mathrm{Im}\,\widetilde{E}_8^L \\ & + \frac{\hat{m}_d k^2}{m_s^2 - m_d^2}\Big[\hat{m}_d\left(\mathrm{Im}\,\widetilde{E}_4^R + \mathrm{Im}\,\widetilde{E}_6^R\right) \\ & + \hat{m}_s\left(\mathrm{Im}\,\widetilde{E}_2^L + \mathrm{Im}\,\widetilde{E}_5^L\right) + \mathrm{Im}\,\widetilde{E}_7^R + \mathrm{Im}\,\widetilde{E}_9^R + \mathrm{Im}\,\widetilde{E}_{10}^R\Big] \\ & + \frac{\hat{m}_s k^2}{m_s^2 - m_d^2}\Big[\hat{m}_d\left(\mathrm{Im}\,\widetilde{E}_4^L + \mathrm{Im}\,\widetilde{E}_6^L\right) + \hat{m}_s\left(\mathrm{Im}\,\widetilde{E}_2^R + \mathrm{Im}\,\widetilde{E}_5^R\right) \\ & + \mathrm{Im}\,\widetilde{E}_7^L + \mathrm{Im}\,\widetilde{E}_9^L + \mathrm{Im}\,\widetilde{E}_{10}^L\Big].\end{aligned} \tag{4.30}$$

Above is the scheme we have utilised to evaluate the numerical values of the form factors. Although we only show the calculation for the transition of $s \to d\,Z^*$, nevertheless the results of the form factors are directly applicable to other similar decays of any loop-induced decay of a down-type quark to another down-type quark by simply substituting the corresponding external quark masses.

4.3 Behaviour of the Form Factors

The dispersive and absorptive parts of the Z-penguin vertex form factors are computed numerically from Eqs. (3.46) - (3.51) using the scheme outlined in Section 4.2. We have carried out the computation for two such vertices, namely $Zs\bar{d}$ and $Zb\bar{d}$, to see the effect of the external mass dependence. We have also computed the form factors separately for internal u, c and t quarks. The range of k is the physical region of the above process. The following set of values are used for quark masses and W mass:

$$m_u = 0.005 \text{ GeV}$$

$$m_c = 1.3 \text{ GeV}$$

$$m_t = 176 \text{ GeV}$$

$$m_d = 0.01 \text{ GeV}$$

$$m_s = 0.2 \text{ GeV}$$

$$m_b = 4.3 \text{ GeV}$$

$$M_W = 80.22 \text{ GeV}.$$

The value of $\sin^2\theta$ is taken to be

$$\sin^2\theta = 0.23.$$

The numerical values of these form factors are not presented here due to page limitation. We only display the behaviour of these form factors, as functions of k, graphically in Fig. 4.1 to 4.6 for

$$s \to dZ^*$$

$$b \to dZ^*$$

transitions. Our results show that

$$A_j^R << A_j^L(=A_j), \quad C_j^R << C_j^L(=C_j), \quad B_j^R \approx B_j^L(=B_j). \tag{4.31}$$

These dominant form factors A_j, B_j and C_j are almost independent of the external quark masses. Our result also agrees with that of Inami and Lim[28] and Ma and Pramudita[32] when we let $k \to 0$.

Within the physical region $k^2 < (m_s - m_d)^2 = (0.2 - 0.01)^2 \mathrm{GeV}^2$ for $s \to d Z^*$, only at the threshold of $k^2 = 4m_u^2$ do the form factors develop absorptive parts, namely $\mathrm{Im}\, A_u$, $\mathrm{Im}\, B_u$ and $\mathrm{Im}\, C_u$. However, for $b \to d Z^*$, absorptive parts are developed at thresholds of $k^2 = 4m_u^2$ and $4m_c^2$, which are within the physical region of $k^2 < (m_b - m_d)^2 = (4.3 - 0.01)^2 \mathrm{GeV}^2$. When the internal quark j is a top quark, none of these form factors develop absorptive parts since the threshold for internal t quark at $k^2 = 4m_t^2$ is way outside the physical region for all the above processes.

The k-dependence of these form factors is displayed in Figs. 4.1 to 4.6. It is noted that at the threshold values, $k = 2m_j$, the dispersive parts of the form factors display cusp behaviour. The absorptive parts of the form factors display distinctive threshold behaviour, rising abruptly when the corresponding thre sholds are traversed. The explicit development of absorptive parts in the form factors of such loop-induced vertex is essential to give rise to direct *CP* violation effect.[39]

With the numerical behaviour of these form factors in hand, we shall proceed to the next chapter where applications are made.

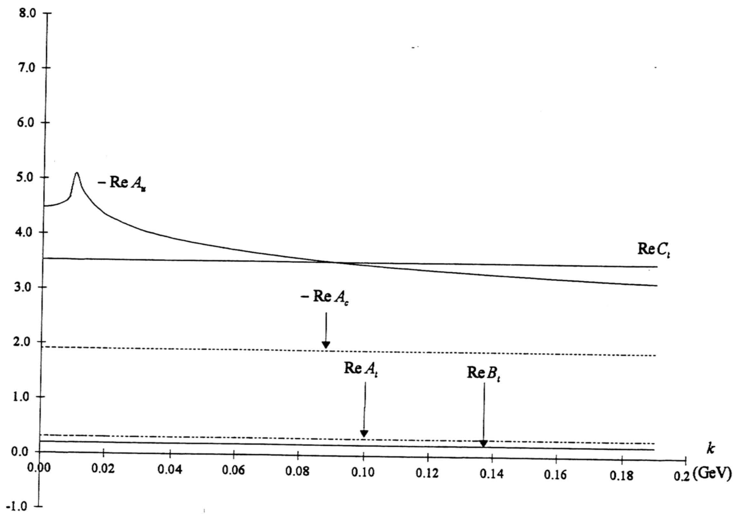

Figure 4.1. Real parts of the dominant form factors Re A_u, Re A_c, Re A_t, Re B_t and Re C_t for the $Zs\bar{d}$ vertex versus invariant mass k of the virtual Z boson.

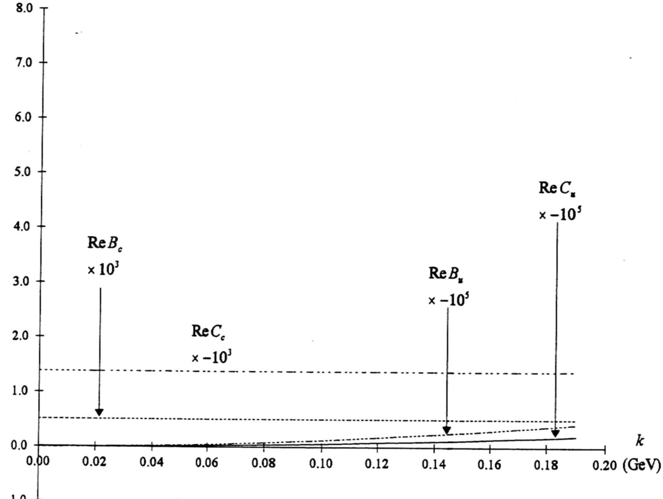

Figure 4.2. Real parts of the less dominant form factors Re B_u, Re B_c, Re C_u and Re C_c for the $Zs\bar{d}$ vertex versus invariant mass k of the virtual Z boson.

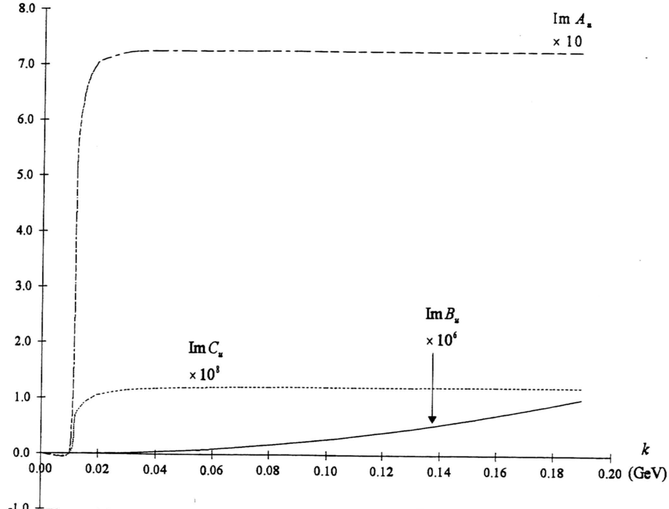

Figure 4.3. Imaginary parts of the form factors for the $Z s \bar{d}$ vertex versus invariant mass k of the virtual Z boson. Here Im A_z is dominant.

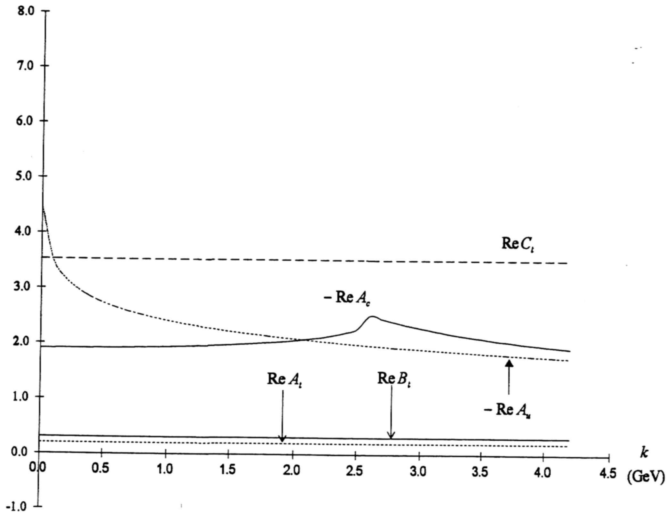

Figure 4.4. Real parts of the dominant form factors Re A_u, Re A_c, Re A_t, Re B_t and Re C_t for the $Zb\bar{d}$ vertex versus invariant mass k of the virtual Z boson.

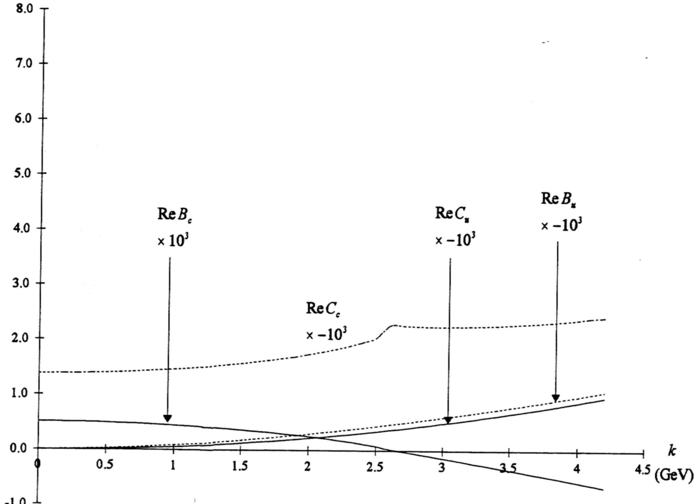

Figure 4.5. Real parts of the less dominant form factors $\mathrm{Re}\,B_u$, $\mathrm{Re}\,B_c$, $\mathrm{Re}\,C_u$ and $\mathrm{Re}\,C_c$ for the $Zb\bar{d}$ vertex versus invariant mass k of the virtual Z boson.

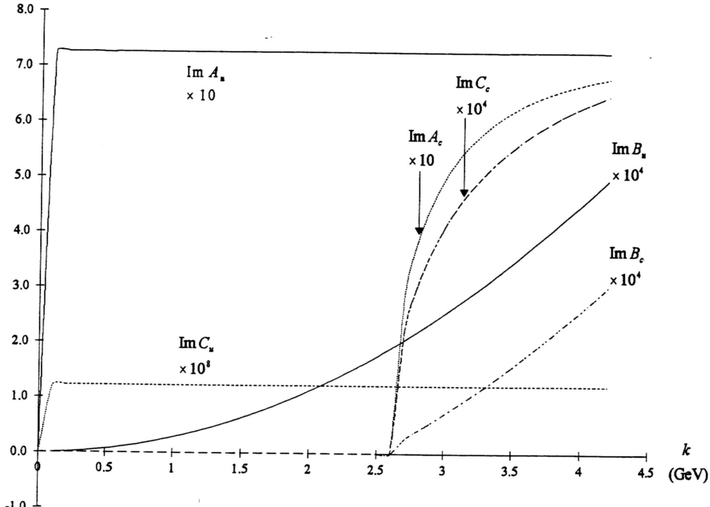

Figure 4.6. Imaginary parts of the form factors for $Zb\bar{d}$ vertex versus invariant mass k of the virtual Z boson. Here Im A_u and Im A_c are dominant.