Chapter VI

Conclusions

We have performed the calculation of the Z-penguin vertex function in the 't Hooft-Feynman gauge. The complete expression for the Z-penguin vertex is presented explicitly as double integrals over the Feynman parameters. The vertex function is found to be divergent, and the divergence is isolated by dimensional regularization. Renormalization of the vertex is effected by a simple prescription, in which the counter term is extracted from the on-shell amplitude of one-particle reducible diagrams involving flavour-changing self-energy. The general expression for the on-shell vertex function is then obtained, and is found to be more general than that obtained by earlier calculations. The vertex function obtained are expressed in terms of the form factors $A_j^{L,R}$ and $B_j^{L,R}$ and $C_j^{L,R}$, with $j = u$, c, t quarks. Earlier calculations contain only C_j form factors. Our result for the vertex function reduces to that of earlier calculations for small external momenta and quark masses.

The main purpose of this thesis is to investigate the behaviour of the Z-penguin vertex function for non-zero invariant mass k of the virtual Z boson. The Feynman diagrams contributing to the Z-penguin vertex at the one-loop level, as illustrated in Fig. 3.1, can be divided into two types, type Q and type W. Diagrams of type Q involve one W propagator and two internal j quark propagators. Integrating over the internal momentum brings out the characteristic feature that this type of diagrams develops an absorptive part whenever $k^2 > (2m_j)^2$. Similarly, type W diagrams which involve two W propagators and one j quark propagator, will develop an absorptive part whenever $k^2 > (2M_W)^2$. Explicit computation of the dispersive parts and the absorptive parts of the various form factors of the vertex function is carried out by introducing Feynman parameters, and performing the final integration numerically.

The dispersive and absorptive parts of the form factors are computed for internal *u*, *c* and *t* quarks. Since the *W* boson threshold is very far away from the physical region of the process considered, we have not computed the absorptive parts of the form factors associated with this threshold. The form factors are found to have little dependence on external quark masses.

We have applied the *Z*-penguin vertex to the process $q_1 \to q_2 Z^*$. The development of the absorptive part of the *Z*-penguin vertex function is directly related to the non-zero value if the decay rate asymmetry parameter a between $q_1 \to q_2 Z^*$ and $\bar{q}_1 \to \bar{q}_2 Z^*$. Thress processes are considered explicitly, namely $s \to d\,Z^*$, $b \to s\,Z^*$ and $b \to d\,Z^*$. For each of the processes, we have also analysed its dependence on the KM matrix elements. The decay rate for the process $q_1 \to q_2 Z^*$ is found to be predominately given by the C_j from factors, agreeing with the findings of earlier calculations. However, for the decay rate asymmetry, the A_j form factors play an equally important role as the C_j form factors. This is because $\mathrm{Im}A_j$ is usually much larger than $\mathrm{Im}C_j$, especially for $j = u$ and c, so as to overcome the kinematic suppression.

For $s \to dZ^*$, the decay rate is found to be of order 10^{-12} GeV. There is a sensitive dependence on $|V_{cs}V_{cd}^*|^2$. As $|V_{cs}V_{cd}^*|^2$ crosses the value of 0.046, the decay rate drops by as much as 4 orders of magnitude. The decay rate asymmetry $|a|$, rises from zero sharply above the internal *u* quark threshold, $k = 2m_u$. The internal *c* quark threshold, $k = 2m_c$, is outside of the physical region. $|a|$ is of order 10^{-7}. However, as the decay rate decreases sharply as $|V_{cs}V_{cd}^*|^2$ approaches 0.046, $|a|$ also increases to about 10^{-4}.

For $b \to s Z^*$, the decay rate is of order 10^{-9} GeV, and the asymmetry parameter is of the order 10^{-5}. There is little dependence on the KM matrix elements.

For $b \to d Z^*$ the decay rate is around 10^{-11} GeV, whereas the asymmetry is of order 10^{-4}. There is only slight dependence on the KM matrix elements.

We have also applied the Z-penguin vertex to the flavour-changing decay $Z^0 \to \bar{q}_1 q_2$. The calculation is similar to that for the process $q_1 \to q_2 Z^*$, but with $k^2 = M_Z^2$. At this value of k^2, we find that $A_{ut} - C_{ut} \approx A_{ct} - C_{ct}$. Hence the decay rate for $Z^0 \to \bar{q}_1 q_2$ is approximately given by $\Gamma \approx (3.026\text{keV})|\lambda_t|^2 |A_{ct} - C_{ct}|^2$ whereas the asymmetry parameter by $a \approx -(4.85 \times 10^{-4}) J / |\lambda_t^*|^2$.

The process $Z^0 \to \bar{q}_1 q_2$ shows similar dependence on the KM matrix elements sd that for $q_1 \to q_2 Z^*$. For example, the decay rate for $Z^0 \to \bar{s}d$ drops sharply as $|V_{cs} V_{cd}^*|^2$ crosses 0.046. The other decay modes, $Z^0 \to \bar{s}b$ and $Z^0 \to \bar{b}d$ show a rather flat variation with the KM matrix elements.

A good knowledge of the relevant KM matrix elements allows us to predict the branching ratio of $Z^0 \to \bar{b}s$ from the Z-penguin to be 1.8×10^{-8}. The branching ratios for $Z^0 \to \bar{s}d$ and $Z^0 \to \bar{b}d$ cannot, however, be predicted with the same certainty, because of the uncertain phases of the KM matrix elements involved. They are found of order (or less than) 10^{-9}. For $Z^0 \to b\bar{s}$, $a \sim -8.9 \times 10^{-6}$. For $Z^0 \to s\bar{d}$ and $Z^0 \to b\bar{d}$, the asymmetry parameter can be larger, $a \sim 10^{-3}$ and $a \sim 10^{-4}$, because of the smallness of the corresponding decay rates. It is to be stressed that the active participation of A_j form factors in the subtle cancellation of $A_{ut} - C_{ut} \approx A_{ct} - C_{ct}$ renders the asymmetry parameter very small.

A possible extension of this work is to apply the *Z*-penguin vertex to the process $e^+e^- \to q_1\bar{q}_2$ at C. M. energy above the $2M_W$ threshold. The advantage of this is that it is expected that the $2M_W$ threshold to yield an absorptive contribution much larger than those from the $2m_j$ thresholds analysed in this thesis. The corresponding direct *CP* violation effect may be reasonably expected to be large. It may be detectable in e^+e^- collision process.