CHAPTER 2

REVIEW OF RELATED LITERATURE

2.1 Introduction

The literature review focuses on four main areas, namely, the definitions of understanding, the difficulties of Algebra, the findings of some studies on errors and misunderstandings in quadratic equations, and writing in the mathematics classroom. The last section also includes the methods of analysis of writing in mathematics.

2.2 Understanding

Understanding of a concept, a group of concepts, or symbols, according to Skemp (1979), was to connect it with an appropriate schema. Understanding differs from knowledge, as understanding was the ability to apply the knowledge in order to find ways of achieving a desired goal.

Skemp further differentiated between three kinds of understanding which were instrumental understanding, relational understanding and logical understanding.

> "Instrumental understanding is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works. Relational understanding is the ability to deduce specific rules or procedures from more general mathematical relationships. Logical understanding is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning."
> (page 45)

The goals of each type of understanding and the methods to achieve them differed. The goal in instrumental understanding was to give the right answer according to rules that had been learnt and by manipulating symbols to make connections. The goal in relational understanding was to construct relations between schemas. This might be achieved by quiet reflection, alternated with discussion, but never discussion alone. The function of the symbols was now for manipulating and communicating mathematical concepts. Logical understanding would be achieved when the person used the understanding that he already has to convince others. The goal in this form of understanding was to ensure that the schemas that have been constructed were sound and accurate.

To assist understanding, Bisanz and LeFevre (1992) noted that there were various behaviors associated with understanding. These behaviors could provide a measure of understanding. They proposed a model of two-dimensional conceptual space consisting of "type of activities" and "degree of generalization" to measure understanding. "Type of activities" in understanding referred to application, justification or evaluation of solution methods while "degree of generalization" referred to how these activities are used. A profile was then constructed to illustrate the contexts where a person shows understanding and a measure of his understanding was made.

The difficulty in this model is that the identification of the type of activities and the measurement of the degree of generalization are somewhat arbitrary. Comparing to the three different types of understanding suggested by Skemp, the classification is a simpler and more useful way to determine the level of understanding that a student possesses.

2.3 Symbolism in Algebra

Algebra is a language in which one is able to express and analyze relationships between quantities that change with the use and manipulation of symbols (Nunn, 1927; Van Dyke & Craine, 1997). The use of symbols has enabled the reasoning processes in mathematics to be simplified and clarified. Hence, when learning Algebra, students were first taught the laws of Algebra. With the knowledge of these laws, and through the processes of reasoning, students would be able to make conclusions, with the aid of symbols (Everett, 1966). However, because of the use of these symbols, Algebra was considered to be a complex area in mathematics. Students had many preconceptions about these symbols together with their possible meanings (Barbeau & Brown, 1997).

Stacey and MacGregor's (1997) study showed that these symbols may be misinterpreted because they have been used to symbolize many different items in different situations. A letter like *c* for example, might sometimes represent a number or the intercept of the y-axis. Other letters would be used to denote quantities, example, *m* for mass and *t* for time. On the other hand, letters could also be used as abbreviations, for example, *cm* for centimeters and *m* for meters. In view of the various meanings associated with letters as symbols in Algebra, students could misinterpret the meanings of these letters.

Terms like "10*h*" and "x^4" were also commonly misinterpreted as "10 plus *h*" and "*x* times 4" respectively.

According to Stacey and MacGregor (1997) there is interference from prior knowledge in learning Algebra. When students first learnt Algebra, letters were used to denote unknown numbers. However, in a question like the following: "a = 28 + b, which is the larger number, a or b?", students were not able to determine whether "a" or "b" was the larger number. Some students interpreted the equation as "a equals 28, and then b was added in". Other students thought that these letters denoted unknown numbers, and that they could not tell which was the larger number. These misconceptions occurred because of interference from their prior knowledge of arithmetic where they had learnt that the equal sign meant to "give" a solution. However, in equations, the equal sign was used to show equality of both sides.

Esty (1992) explained the difficulties faced by students were because they took mathematics to be a foreign language that was only required in the mathematics class. According to him, in order to be proficient in Algebra, students needed to master the rules of Algebra. With this, and the knowledge of the mathematics vocabulary, Esty claimed that students could then understand high school Algebra fully. The difficulties in the language aspects of mathematics lead Esty to recommend that the emphasis in homework and examination questions in mathematics should shift from that of doing mathematics to understanding mathematics.

As Algebra is a language of symbols, it has its own syntax and meaning which has to be mastered before understanding can take place. The syntax and meaning refer to the basic rules of Algebra and need to be considered as a separate language in order to achieve understanding in Algebra.

2.4 Quadratic Equations

The study of Algebra includes solutions of quadratic equations. Although there has been much research on errors and misconceptions in Algebra (Barbeau & Brown, 1997; Meyerson & McGinty, 1978; Stacey & MacGregor, 1997), little has been done on quadratic functions and quadratic equations. In this section, findings concerning quadratic equations are discussed.

Equations are important concepts in high school mathematics because they are considered a useful tool (Whitcraft, 1980). However, several common errors have been identified by Whitcraft in ninth grade Algebra involving quadratic equations and their solutions. They were:

- Errors in factoring.
- Errors in forming the equation.
- Errors in completing the square in quadratics.
- Errors in solution of equation.
- Failure to perform the same operation upon both sides of an equation.
- Incomplete solution.

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Zaslavsky (1997) indicated that students could not differentiate between a quadratic function and a quadratic equation. For example, equivalent equations such as $x^2 + 2x - 3 = 0$ and $2x^2 + 4x - 6 = 0$ could be treated as same equations when solving them. However, functions such as $y = x^2 + 2x - 3$ and $y = 2x^2 + 4x - 6$ are not equivalent and cannot be simplified in a similar manner. In this study, when students were asked for examples of quadratic functions, quadratic equations were given instead. Generally, he stated that students seemed to prefer the standard form of the equation $y = ax^2 + bx + c$ of a quadratic function rather than the canonical form $y = a(x - m)^2 + n$ or the multiplicative form $y = a(x - x_1)(x - x_1)$.

Kaur and Sharon (1994) showed that first year college students were unaware of the difference between quadratic equations and expressions. They were confused over mathematical rules and principles. The researchers listed the misconceptions and errors in the following areas:

- Misinterpretations of conventions such as $|\ |$, $\sqrt{}$ and $\pm$.
- Conceptual misunderstanding of identities such as $(x + y)^2 = x^2 + y^2 + 2xy$.
- Conceptual misunderstandings of procedures such as "squaring both sides".
- Failure in interpreting inequalities involving unknowns.
- Wrong application of cancellation principles.
- Failure in differentiating between mathematical terminology such as "equation" and "expression".
- Failure in relating the behavior of a quadratic function and its discriminant.
- Mistranslation of instructions, due to language incompetence.
- Inability to interpret statements, and evaluate whether specific criteria are adequate or have been met.
- Ineffective construction of counter-examples, due to the disregard of negative numbers.
- Confusion over established mathematical rules and principles.
- Formulation of false generalizations from known laws.

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The findings indicated that these misconceptions and errors were related to the linguistic aspects of mathematics because the errors were related to interpretation, meaning and terminology.

Esty (1992) also noted that students were confused with the terms: equation and expression. An equation could be regarded as an open sentence. Hence, when solving an equation, the sequence of equivalent equations should be shown and each step justified in each step by algebraic theorems and application of logic. The last sequence would show the solution. Hence, he suggested more time be spent in teaching the patterns of logic and the justification of these processes.

Related to classroom instruction, researchers (Meyerson & McGinty, 1978; Whitcraft, 1980) cautioned that the errors and misconceptions that occurred in solving quadratic equations might be a result of teaching and learning processes which did not promote understanding.

2.4 Writing in Mathematics

Writing could take different forms in mathematics. Spika (1992) categorized the types of writing assignments in the mathematics class into two types; formal and informal. In formal writing, the content is important in terms of conveying the writer's thoughts. Examples included the writing of proofs, solutions to journal problems and lecture notes. On the other hand, informal writing such as in-class writing and reading logs gave an insight to the writer's thoughts rather than the content.

There were many benefits of using writing in the mathematics classroom. Writing improved the students' understanding of mathematical concepts and their ability to express their understanding (Nahrgang & Petersen, 1986). This is because during writing, connections between ideas were being made and the difficulties the student may be facing were analyzed (Miller, 1992; Clarke, Waywood & Stephens, 1993; Dougherty, 1996).

Students' writing in mathematics also provided useful feedback to teachers on the students' instructional needs and the common difficulties encountered (Miller & England, 1989; Miller, 1992). Hence, students did not only improve their communication skills, but also increase their understanding of mathematical concepts (Meyer & Hillman, 1996).

Writing is also beneficial as a research tool as it could provide data on diagnostic details of error patterns and clarification of students' understanding of mathematics concepts and procedures (Drake & Amspaugh, 1994; Clarke, Waywood & Stephens, 1993).

Students' writing had been used to assess students' understanding in mathematics and specifically in Algebra (Doughtery, 1996). Writing had provided the context in which algebraic concepts could be integrated, reflected upon and communicated. However, the analysis of students' understandings in most of the research done is of a qualitative nature (Nahrgang & Petersen, 1986; Borasi & Rose, 1989; Dougherty, 1996; Mayer & Hillman, 1996,).

Shield & Galbraith (1998) however, developed an objective method of analyzing writing, by providing a coding scheme for the description of the content of writing. They made use of Leinhardt's (1987) features of an explanation and van Dormolen's (1985) definition of "aspects of mathematics" to identify parts of students' writings to determine their level of understanding.

Leinhart's (1987) features of an explanation were as follows:

- Identification of a goal.
- Signal monitors indicating progress towards the goal.
- Examples of the case or instance.
- Demonstrations that include parallel representations, some levels of linkage of these representations, and identification of conditions of use and nonuse.
- Legitimization of the new concept or procedure in terms of one or more of the following - known principles, crosschecks of representations and compelling logic.
- Linkage of new concepts to old through identification of familiar, expanded, and new elements.

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These features were adopted in this study as possible characteristics to identify in students' writings. The presence of one or more of these features in the students' written response would indicate understanding of the concepts.

van Dormolen (1985) had analyzed the content of mathematics textbooks for their "aspect of mathematics". The aspects of mathematics identified were theoretical, algorithmic, logical, methodological and conventional. A theoretical aspect of mathematics meant that the written content consisted of theorems, definitions and axioms. The writing would have an overall mathematical structure. Algorithmic meant that the methods on how specific operations or procedures were performed were stated explicitly. The logical aspect of mathematics would be present in the writing when there were statements about the way one should work using a theory or an extension of the theory. The methodological aspect was different from the algorithmic aspect ever though it had rules on how to perform an operation or procedure. These rules were more general and heuristic in nature. Lastly, the conventional aspect included conventions in mathematics such as how to name a diagram or write a proof.

In students' written responses, the aspect of mathematics shown would be an indicator to the type of understanding the student possesses. In Shield and Galbraith's (1998) study, students' written responses were analyzed by observing characteristics in their writing according to Leinhart's features of explanation and van Dolomon's aspects of mathematics to determine understanding.

Writing tasks in mathematics have potential benefits to the student who engages in it as well as to the teacher and researcher who intend to use it as a diagnostic tool to obtain feedback. It is also an invaluable research tool in exploring subjects' understanding of particular concepts. In this study, the researcher makes use of the qualitative analysis methods in previous research to explore students' understanding of quadratic equations.