CHAPTER 5

SUMMARY AND CONCLUSION

5.1 Introduction

This chapter summarizes the findings of the written responses and interviews by outlining the students' understanding of quadratic equations, their roots, methods of obtaining the roots and general understanding of quadratic equations. This is followed by the implications of this study and the recommendations for the use of writing in the classroom and suggestions for future research.

5.2 Summary of Findings

The following are the general findings obtained from the analysis of data of the understanding of quadratic equations, their roots and methods of obtaining the roots.

Understanding of Quadratic Equations

All the students understood that quadratic equations had an unknown with the highest power, two. However, one student, S4, believed quadratic equations involved terms with powers higher than two. She considered equations with terms to the third and fourth powers as quadratic and insisted

that quadratic equations could have three terms. Equations with an unknown to the second degree but with only two terms were not considered quadratic.

Not all the students mentioned the use of fractions in quadratic equations. S1 and S3 had explained that the equation with the term $1/x^2$, was not a quadratic equation and that there were no fractions in quadratic equations. However, S5 had identified that equations, which had the term $1/x^2$ and $1/x$, were still quadratic equations.

Though the equal sign was used in all the examples of quadratic equations given by the students, two of the students specifically mentioned that the equation of the general form must be equated to zero.

One of the students, S3, had tried to explain the use of the quadratic equation by saying that it was used to find the roots of the equation. She also explained that when the roots were substituted into the equation, the equation was true, that is, an identity was formed.

The students' understanding of quadratic equations was that it had to be equated to zero and generally had an unknown to the power of two.

Understanding Roots of Quadratic Equations

All the students were of the opinion that the roots of a quadratic equation were the solutions of the quadratic equation, x. They were able to list three methods of finding the roots to the quadratic equation, namely factorization, completing the square and applying the general formula. S1 went into great

detail when explaining factorization and the importance of identifying the signs involved when performing factorization.

All the students explained that the roots of the equation were obtained from the quadratic equation. However, there were some differences in their written responses. Two students, S2 and S4, identified the three different types of roots and the conditions for these roots. Further to this, S4 had also explained in detail how the type of roots in an equation could be determined. S3 had showed that a quadratic equation could be obtained from a pair of roots.

In general, none of the students mentioned the use of quadratic equations or its application in real-life situations. The meaning of roots of quadratic equations were also not related to real-life situations or even to graphical functions. This showed that the students had little relational understanding of quadratic equations and roots of quadratic equations.

Understanding of the Methods of Obtaining Roots of Quadratic Equations

Most of the students knew that there were three methods of obtaining roots of quadratic equations. However, the written responses were mainly on factorization. On another method of obtaining the solution to the equation, all the students had chosen the general formula method.

The method used by the students for factorization was similar, that is the "cross-multiplication method". Firstly, the factors of the constant and x^2 were listed. Then, the correct signs for the factors were determined. This was done either with the aid of a table, by multiplying the terms to check the expansion, or

by using a certain rule remembered by the student. This rule was based on the fact that if the constant in the quadratic equation was positive, their factors had either both positive, or, both negative signs. If the constant was negative, one of the factors was positive and the other negative. One student, S1 had explained three methods of factorization. However, when her work was analyzed, all her methods, whether she used a "cross" or a table to show the position of the factors or signs, were basically the same.

There were several differences in the written response of students. Students S3, S4 and S5 who had identified the procedure to obtain the products of the factors as "cross multiplication". One student, S2, had given an explanation with examples of the term in the highest power, ax^2, with $a \neq 1$.

Another student, S5, had started her written response on factorization by explaining the factorization of common factors from an expression. Though she did not explain it in detail, this explanation indicated that she had put thought on the concept of factorization and had linked it to her previous mathematics knowledge. However, this student had also explained that quadratic equations had to have three terms before factorization could be done. S4, understood that factorization and expansion were related as she explained the importance of knowing expansion before factorization.

When they were asked to factorize and solve the quadratic equation, not all the subjects completed the task. Only three of the students, S2, S4 and S5 had solved the equation. However, none of them explained the Zero Product

Rule nor the reason why the factors should be equal to zero. There was no justification of the process of solving quadratic equations.

All the students had performed the process of factorization and solution of equation as an algorithm and had chosen the general formula method as the alternative method to solving quadratic equations. Two students, S2 and S3, had explained that with the general formula, one had to be careful with the multiplication of the different signs. Almost all the students, except S2, changed their answers from the surd form during their calculation to the decimal form, using a calculator. Only S2 preferred to leave her answer in surd form while S4, gave the answer in both the decimal and surd form. Most of the students preferred the decimal form as the values could be easily computed with the calculator. One of them even claimed that the decimal form is more accurate. However, none of them made any error when they produced the answers in decimal form. Furthermore, the students seemed to be of the opinion that surds were more difficult to simplify. This may seem to be true as both of these students who used surds made mistakes while simplifying the surds.

All of the students had given their answers to three or four decimal places. They seemed to have memorized the formula and substituted the appropriate values to obtain the answer. Although three of the students did include some explanation while the substitution and computation of the formula was done, none of them explained the origins of the general formula.

General Understanding

The students' understanding on the meaning of quadratic equations and roots of quadratic equations and the methods for obtaining the solutions to quadratic equations were summarized according to the kinds of understanding (Skemp, 1979). Table 4 gives a summary of the findings for each student.

Table 4 : Types of understandings inferred from students' responses on areas in quadratic equations

	S1	S2	S3	S4	S5
Meaning of Quadratic Equations	I	I	I	I	I
Meaning of Roots of Quadratic Equations	I	-	R	I	I
Solving Quadratic Equations: Factorization	R	R	R	R	I
Solving Quadratic Equations: General formula	I	R	I	I	I

Note: I Instrumental Understanding
R Relational Understanding

The students seem to posses instrumental understanding on the meaning of quadratic equations. They were able to apply the definition of a quadratic equation to form new quadratic equations. With reference to the meaning of roots of quadratic equations, one of the students seem to posses relational understanding as she deduced the conditions for the different types of roots and showed how to obtain them. The other students however were more concerned in stating the methods of obtaining the roots as a solution to the equation. For one student, the type of understanding she possessed on roots of

equation could not be determined as she had merely stated the types of roots and their condition without any elaboration.

In explaining factorization as a method of solving quadratic equations, students seem to have relational understanding. They reasoned how the sign of the factors in the expressions were determined. In the general formula method, understanding was mainly instrumental as it involved the application of a remembered formula.

In general, the type of understanding demonstrated was more instrumental understanding, except for factorization where most of the students showed relational understanding.

In studying the content of the writing from Table 5, the students' preference for the algorithmic aspect of mathematics was noted. Only when the students wrote about the meaning of quadratic equations a theoretical aspect of mathematics was used. They were giving definitions of quadratic equations in this case. When explaining roots of quadratic equations, most of the students were more concerned with the procedure of determining the roots rather than the definitions of the roots of quadratic equations. Only one student used theoretical aspect in his written response. When the procedures involving factorization and the general formula were explained, all the responses were algorithmic. The emphasis in the written responses was on the procedure for finding solutions.

Table 5 : Aspect of mathematics inferred from students' responses on areas in quadratic equations

	S1	S2	S3	S4	S5
Meaning of Quadratic Equations	T	T	T	T	T
Meaning of Roots of Quadratic Equations	A	T	A	A	A
Solving Quadratic Equations: Factorization	A	A	A	A	A
Solving Quadratic Equations: General formula	A	A	A	A	A

Note: T Theoretical
A Algorithmic

Hence, in general students seemed to write in an algorithmic manner and were more concerned with outlining what should be done according to the correct procedure than in other aspects of mathematics.

5. 3 Implications and Recommendations

Students' understanding of quadratic equations, roots of quadratic equations and the methods of solving quadratic equations was mainly instrumental (Skemp, 1979) and the aspect of mathematics, algorithmic (van Dormolen, 1989). In order to encourage a higher level of understanding the teaching-learning method would have to be reviewed. The following recommendations may help to improve students' understanding in quadratic equations.

- Teaching should place more emphasis on the terminology, definitions and basic concepts. Mathematical terms like variable, constant, identity, roots, equation and expression should be used explicitly to familiarize students with

these terms. The correct use of language in instruction is important so as not to confuse the students. This is because students' proficiency in the language affects their understanding.

- The relationship between roots of quadratic equations and the quadratic equation should be stressed upon. Students do not see the link between the different types of roots and their conditions, and the quadratic equation. Classroom instruction should provide ample opportunities for students to realize that these conditions can be derived from the general form of the quadratic equation. This is because the discriminant, $b^2 - 4ac$ takes different values for different conditions.
- There are various forms of expressions that can be factorized. Some expressions have a single unknown and may have a common factor. Other expressions may not have any common factors and can still be factorized, for example $a^2 - b^2$. Factorization is an important aspect in finding the solution to quadratic equations. Hence, opportunities for students to be exposed to different forms of equations that can be factorized in different ways should be planned and provided.
- Surds are important as they are commonly used when simplifying the general formula. However, students are not confident in simplifying expressions that contain surds and prefer to use the calculator. The topic should be emphasized to provide students with more practice to build their confidence and their skills.

- The scope and importance of Algebra and its usefulness in many areas and topics in mathematics need to be considered in classroom instruction. This would enable students to realize its importance and to pay more attention in mastering Algebra.
- Opportunities should be provided for students to make connections and links between what had been previously learnt in Algebra by using writing and other such non-standard tasks should be given. This would enable students to progress to a higher level of understanding. When students write about rules, theorems and conventions in Algebra, they are making connections with what they have learnt before and using it in a meaningful way. They are able to reflect upon their writing and in this way build their understanding.

5.4 Suggestions for Further Research

The study is a part of research that can be done in the field of Algebra and in using writing as a means of communication in mathematics. Future research that can be carried out are as follows:

- Different tasks, which can be a combination of multiple choice and structured questions can be given to students to determine their understanding of quadratic equations.
- A teaching experiment can be carried out to determine whether students would have a better understanding of quadratic equations when guided in a systematic manner in writing in Mathematics.

5.5 Conclusion

In this study writing played a role in data collection as it had encouraged students to communicate in mathematics. The written responses were used to determine students' understanding of quadratic equations, roots of quadratic equations and methods of solving quadratic equations. From the written responses and interviews conducted, it was seen that the students' understanding of quadratic equations was mainly instrumental, as the students generally followed rules and procedures to solve quadratic equations.