CHAPTER 6

NUMERICAL WORK

6.1 Introduction

This chapter presents the sequential and parallel results of the experiments conducted using various schemes under consideration for 1-D, 2-D Parabolic and Bio-Heat Equations with similar sequential and parallel results for 1-D, 2-D and 3-D Telegraphic Equations. Approximation solutions were compared to that of other alternating methods for the Parabolic and Telegraphic Equations. The acceleration parameter was chosen to provide the most rapid convergence. Unless otherwise stated the convergence criterion was taken as $eps = 10^{-4}$. The parallel schemes were implemented using MPI and PVM message passing along with C programming language. The schemes used in this Chapter are the schemes treated in Chapters 3, 4 and 5.

6.2 Sequential Results for 1-D Parabolic & Bio-Heat Equations

The 1-D problem considered was taken from (Saulev (1964),) given as

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, \quad for \quad 0 < x < 1, \quad 0 < t < T$$

$$\left.\begin{aligned} U(x,0) &= f(x), && 0 \le x \le 1 \\ U(0,t) &= g_o(t), && 0 < t \le T \\ U(1,t) &= g_1(t), && 0 < t \le T \end{aligned}\right\}$$

and subject to initial condition

$$U(x,0) = 4x(1-x) \qquad 0 \le x \le 1 \tag{6.1}$$

and the boundary conditions

$$U(0,t) = U(1,t) = 0 \qquad t \ge 0\,. \tag{6.2}$$

The exact solution was given by

$$U(x,t) = \frac{32}{\pi^2} \sum_{k=1,(2)}^{\infty} \frac{1}{k^3} e^{-\pi^2 k^2 t} \sin(k\pi x) \tag{6.3}$$

in this experiment, the values of $U(x_i,t)$ where i = 1 and m, were taken to satisfy the exact solution Eq. (6.3). An energy balance for a control volume of tissue with volumetric blood flow and metabolism yields the Bio-Heat equation given by (Pennes 1948 and Deng & Liu 2002). We consider the 1-D Bio-Heat equation of the form:

$$\frac{\partial U}{\partial t} = c\left(\frac{\partial^2 U}{\partial x^2}\right) + bU - bU^*. \tag{6.4}$$

The boundary condition and initial condition posed are:

$$U(0,t) = 0, \quad U(1,t) = 0 \tag{6.5}$$

$$U(x,0) = 0.7(1+3x), \qquad 0 \le x \le 1 \tag{6.6}$$

The values of the physical properties in our test cases are chosen to be $\rho = 1000kg/m^3$, $c = c_b = 4200J/kg^oC$, $\omega_b = 0.5kg/m^3$, the step increase of the temperature is set to be $U_o = 12^oC$.

Table 6.1 Sequential results for 1-D Parabolic Equation with various schemes

Absolute error of numerical solutions, $\lambda = 0.5,\ t = 0.25,\ \Delta t = 0.005,\ \Delta x = 0.1,\ eps = 10^{-4}$

X												
method	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*Av. Abs. Err*	*RMS**	*It.*
GS	8.58x10^{-3}	5.82x10^{-2}	6.92x10^{-3}	8.06x10^{-3}	7.11x10^{-2}	8.34x10^{-2}	8.16x10^{-2}	7.69x10^{-3}	7.36x10^{-2}	3.99x10^{-1}	3.08x10^{-3}	7
SOR	6.65x10^{-3}	4.60x10^{-2}	4.72x10^{-3}	7.28x10^{-3}	6.24x10^{-2}	7.57x10^{-2}	7.02x10^{-2}	6.57x10^{-3}	6.84x10^{-2}	3.48x10^{-1}	2.39x10^{-3}	5
IADE-DY	7.53x10^{-4}	1.36x10^{-3}	8.50x10^{-5}	9.74x10^{-4}	1.71x10^{-3}	2.05x10^{-3}	1.97x10^{-3}	1.52x10^{-3}	8.00x10^{-4}	1.12x10^{-2}	1.93x10^{-6}	3
IADE-MF	5.37x10^{-4}	1.02x10^{-3}	1.41x10^{-3}	1.65x10^{-3}	1.74x10^{-3}	1.65x10^{-3}	1.41x10^{-3}	1.02x10^{-4}	5.38x10^{-4}	1.22x10^{-3}	1.68x10^{-6}	3
IADE 4th order	-	2.61x10^{-7}	2.98x10^{-7}	5.89x10^{-6}	2.12x10^{-6}	1.09x10^{-7}	7.54x10^{-7}	5.77x10^{-7}	-	1.12x10^{-6}	4.47x10^{-12}	2
AGE 4th order	-	2.54x10^{-7}	2.61x10^{-7}	5.65x10^{-6}	2.04x10^{-6}	1.03x10^{-7}	7.27x10^{-7}	5.51x10^{-7}	-	1.07x10^{-6}	4.12x10^{-12}	2
EXACT SOLN	2.70461 x10^{-2}	5.14447 x10^{-2}	7.08075 x10^{-2}	8.32392 x10^{-2}	8.7522 x10^{-2}	8.323992 x10^{-2}	7.08075 x10^{-2}	5.14447 x10^{-2}	2.70461 x10^{-2}	-	-	-

Table 6.2 Sequential results for 1-D Bio-Heat Equation with various schemes

Method	***GS***	***SOR***	***IADE-DY***	***IADE-MF***	***IADE 4th order***	***4th AGE***
Av. Abs.						
Err.	4.52×10^{-2}	3.98×10^{-2}	2.63×10^{-3}	2.47×10^{-4}	3.44×10^{-7}	3.11×10^{-7}
RMS	4.39×10^{-4}	3.28×10^{-4}	2.54×10^{-7}	2.28×10^{-7}	5.82×10^{-13}	5.43×10^{-13}
It	*9*	*7*	*5*	*4*	*3*	*3*
Δx	1×10^{-1}	1×10^{-1}	1×10^{-1}	1×10^{-1}	1×10^{-1}	1×10^{-1}
Δt	5×10^{-3}	5×10^{-3}	5×10^{-3}	5×10^{-3}	5×10^{-3}	5×10^{-3}
λ	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}
t	2.5×10^{-1}	2.5×10^{-1}	2.5×10^{-1}	2.5×10^{-1}	2.5×10^{-1}	2.5×10^{-1}
eps	1×10^{-4}	1×10^{-4}	1×10^{-4}	1×10^{-4}	1×10^{-4}	1×10^{-4}

6.3 Parallel Results for 1-D Parabolic and Bio-Heat Equations

Here, we present parallel results for various schemes under consideration using MPI and PVM for various mesh sizes for the 1-D Parabolic and Bio-Heat Equations. Table 6.3 shows various performance timing for 300 meshes for the Parabolic case, while Fig. 6.1 to 6.8 show the parallel performance for 100 to 300 meshes on MPI and PVM platform for Parabolic and Bio-Heat case respectively. From Table 6.3, we can see that the master time T_M is constant when the number of processors increases for a given grid size and number of sub-domains. The master program is responsible for (1) sending updated variables to slaves (T_1), (2) assigning task to slaves (T_2), (3) waiting for the slaves to execute tasks (T_3), (4) receiving the results (T_4).

Table 6.3 Parallel Performance for various schemes with MPI and PVM for 300 mesh

(1 –D Parabolic case)

		PVM					MPI		
Scheme	***N***	T_w	T_m	T_{sd}	S_{par}	E_{par}	***N***	S_{par}	E_{par}
	1	*258.6*	*13.8*	*9.3*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*231.2*	*13.8*	*9.1*	*1.431*	*0.716*	*2*	*1.624*	*0.898*
	6	*193.5*	*13.6*	*9.1*	*1.728*	*0.288*	*6*	*1.813*	*0.302*
GS	*8*	*168.7*	*13.6*	*9.1*	*2.108*	*0.264*	*8*	*2.296*	*0.287*
	10	*118.2*	*13.6*	*9.1*	*2.624*	*0.262*	*10*	*2.865*	*0.287*
	12	*98.7*	*13.6*	*9.1*	*2.938*	*0.245*	*12*	*3.208*	*0.267*
	14	*85.6*	*13.6*	*9.1*	*3.382*	*0.242*	*14*	*3.468*	*0.248*
	16	*82.4*	*13.6*	*9.1*	*3.764*	*0.235*	*16*	*3.861*	*0.241*
	1	*279.4*	*14.5*	*10.7*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*256.8*	*14.3*	*10.4*	*1.528*	*0.764*	*2*	*1.808*	*0.904*
	6	*211.4*	*14.3*	*10.4*	*1.876*	*0.313*	*6*	*1.928*	*0.321*
SOR	*8*	*180.2*	*14.3*	*10.4*	*2.284*	*0.286*	*8*	*2.387*	*0.298*
	10	*168.4*	*14.3*	*10.4*	*2.721*	*0.272*	*10*	*2.962*	*0.296*
	12	*114.6*	*14.3*	*10.4*	*3.084*	*0.257*	*12*	*3.381*	*0.282*
	14	*97.7*	*14.3*	*10.4*	*3.574*	*0.255*	*14*	*3.711*	*0.265*
	16	*91.5*	*14.3*	*10.4*	*4.012*	*0.251*	*16*	*4.108*	*0.257*
	1	*328.4*	*15.8*	*11.5*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*304.7*	*15.6*	*11.4*	*1.612*	*0.806*	*4*	*1.879*	*0.940*
	6	*258.6*	*15.6*	*11.4*	*1.928*	*0.321*	*6*	*1.948*	*0.325*
IADE-DY	*8*	*223.8*	*15.6*	*11.4*	*2.408*	*0.301*	*8*	*2.518*	*0.315*
	10	*211.3*	*15.6*	*11.4*	*2.896*	*0.290*	*10*	*3.096*	*0.310*
	12	*193.7*	*15.6*	*11.4*	*3.286*	*0.274*	*12*	*3.421*	*0.285*
	14	*170.8*	*15.6*	*11.4*	*3.714*	*0.265*	*14*	*3.822*	*0.273*
	16	*120.5*	*15.6*	*11.4*	*4.208*	*0.263*	*16*	*4.325*	*0.270*
	1	*335.7*	*16.2*	*11.5*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*310.2*	*16*	*11.4*	*1.786*	*0.893*	*2*	*1.902*	*0.951*
	6	*263.5*	*16*	*11.4*	*2.324*	*0.387*	*6*	*2.424*	*0.404*
IADE-MF	*8*	*235.4*	*16*	*11.4*	*2.589*	*0.324*	*8*	*2.687*	*0.336*

	10	*228.7*	*16*	*11.4*	*2.949*	*0.295*	*10*	*3.118*	*0.312*
	12	*203.8*	*16*	*11.4*	*3.398*	*0.283*	*12*	*3.513*	*0.293*
	14	*180.5*	*16*	*11.4*	*3.826*	*0.273*	*14*	*3.911*	*0.279*
	16	*130.2*	*16*	*11.4*	*4.311*	*0.269*	*16*	*4.426*	*0.277*
	1	*361.4*	*18.8*	*11.5*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*331.8*	*18.6*	*11.4*	*1.824*	*0.912*	*2*	*1.913*	*0.957*
	6	*284.6*	*18.6*	*11.4*	*2.463*	*0.411*	*6*	*2.584*	*0.431*
IADE-4th	*8*	*253.7*	*18.6*	*11.4*	*2.628*	*0.329*	*8*	*2.789*	*0.349*
	10	*239.6*	*18.6*	*11.4*	*3.256*	*0.326*	*10*	*3.396*	*0.340*
	12	*212.6*	*18.6*	*11.4*	*3.491*	*0.291*	*12*	*3.611*	*0.301*
	14	*193.5*	*18.6*	*11.4*	*3.988*	*0.285*	*14*	*4.209*	*0.301*
	16	*148.6*	*18.6*	*11.4*	*4.538*	*0.284*	*16*	*4.714*	*0.295*
	1	*386.7*	*20.1*	*11.5*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*351.5*	*20.1*	*11.4*	*1.908*	*0.954*	*2*	*1.927*	*0.964*
	6	*299.3*	*20*	*11.4*	*2.521*	*0.420*	*6*	*2.712*	*0.452*
AGE-4th	*8*	*271.1*	*20*	*11.4*	*2.714*	*0.339*	*8*	*2.866*	*0.358*
	10	*248.8*	*20*	*11.4*	*3.318*	*0.332*	*10*	*3.429*	*0.343*
	12	*229.9*	*20*	*11.4*	*3.521*	*0.293*	*12*	*3.713*	*0.309*
	14	*211.5*	*20*	*11.4*	*4.029*	*0.288*	*14*	*4.261*	*0.304*
	16	*169.9*	*20*	*11.4*	*4.604*	*0.288*	*16*	*4.821*	*0.301*

Table 6.4 Effectiveness of the various schemes with PVM and MPI for 300 mesh size

(1-D Parabolic case)

	N	***PVM T(s)***	***L_n***	***MPI T(s)***	***L_n***
	1	*201.9*	*0.50*	*181.7*	*0.55*
	2	*141.1*	*0.51*	*111.9*	*0.80*
GS	*6*	*116.8*	*0.25*	*100.2*	*0.30*
	8	*95.8*	*0.28*	*79.1*	*0.36*
	10	*76.9*	*0.34*	*63.4*	*0.45*
	12	*68.7*	*0.36*	*56.6*	*0.47*
	14	*59.7*	*0.41*	*52.4*	*0.47*
	16	*53.6*	*0.44*	*47.1*	*0.51*
	1	*238.3*	*0.42*	*221.6*	*0.45*
SOR	*2*	*155.96*	*0.49*	*122.57*	*0.74*
	6	*127.0*	*0.25*	*114.94*	*0.28*

	8	*104.33*	*0.27*	*92.84*	*0.32*
	10	*87.58*	*0.31*	*74.8*	*0.40*
	12	*77.27*	*0.33*	*65.54*	*0.43*
	14	*66.68*	*0.38*	*59.71*	*0.44*
	16	*59.40*	*0.42*	*53.94*	*0.48*
	1	*322.6*	*0.31*	*307.4*	*0.33*
IADE-DY	*2*	*200.12*	*0.40*	*163.60*	*0.57*
	6	*167.32*	*0.19*	*157.80*	*0.21*
	8	*133.97*	*0.22*	*122.08*	*0.26*
	10	*114.40*	*0.26*	*99.29*	*0.31*
	12	*98.17*	*0.28*	*89.86*	*0.32*
	14	*86.86*	*0.31*	*80.43*	*0.34*
	16	*76.66*	*0.34*	*71.08*	*0.38*
	1	*366.1*	*0.27*	*324.9*	*0.31*
	2	*204.98*	*0.44*	*170.82*	*0.56*
IADE-MF	*6*	*157.53*	*0.25*	*134.03*	*0.30*
	8	*141.41*	*0.23*	*120.92*	*0.28*
	10	*124.14*	*0.24*	*104.20*	*0.30*
	12	*107.74*	*0.26*	*92.49*	*0.32*
	14	*95.69*	*0.29*	*83.07*	*0.34*
	16	*84.92*	*0.32*	*73.41*	*0.38*
	1	*421.8*	*0.24*	*409.5*	*0.24*
IADE-4th	*2*	*231.25*	*0.39*	*214.06*	*0.45*
	6	*171.25*	*0.24*	*158.48*	*0.27*
	8	*160.50*	*0.20*	*146.83*	*0.24*
	10	*129.55*	*0.25*	*120.58*	*0.28*
	12	*120.82*	*0.24*	*113.40*	*0.27*
	14	*105.77*	*0.27*	*97.29*	*0.31*
	16	*92.95*	*0.31*	*86.87*	*0.34*
	1	*487.6*	*0.19*	*496.9*	*0.20*
AGE-4th	*2*	*240.51*	*0.35*	*257.86*	*0.37*
	6	*174.55*	*0.20*	*183.22*	*0.25*
	8	*159.9*	*0.18*	*173.38*	*0.21*
	10	*125.08*	*0.21*	*144.91*	*0.24*
	12	*116.06*	*0.20*	*133.83*	*0.23*
	14	*97.47*	*0.22*	*116.62*	*0.26*
	16	*81.37*	*0.26*	*103.07*	*0.29*

Table 6.5 Performance Improvement for 1-D Parabolic with various schemes.

N	***PVM T(s)***							***(%)***		
	GS	***SOR***	***IADE-DY***	***IADE-MF***	***IADE-4TH***	***AGE-4TH***	***GS+SOR***	***IADE-DY+IADE-MF***	***IADE-4TH + AGE-4TH***	***SOR+ AGE-4TH***
1	*201.9*	*238.3*	*322.6*	*366.1*	*421.8*	*519.2*	*15.3*	*11.9*	*18.8*	*54.1*
2	*141.1*	*155.9*	*200.1*	*204.9*	*231.3*	*272.0*	*9.5*	*2.4*	*15.0*	*42.7*
6	*116.8*	*127.0*	*167.3*	*177.5*	*171.3*	*205.9*	*8.0*	*5.8*	*16.8*	*38.3*
8	*95.8*	*104.3*	*133.9*	*141.4*	*160.5*	*191.3*	*8.2*	*5.3*	*16.1*	*45.5*
10	*76.9*	*87.6*	*111.4*	*124.1*	*129.6*	*156.5*	*12.2*	*10.3*	*17.2*	*44*
12	*68.7*	*77.27*	*98.17*	*107.7*	*120.8*	*147.5*	*11.1*	*8.9*	*18.1*	*47.6*
14	*59.7*	*66.7*	*86.9*	*95.7*	*105.8*	*128.9*	*10.5*	*9.2*	*17.9*	*48.3*
16	*53.6*	*59.4*	*76.7*	*84.9*	*92.95*	*112.8*	*9.8*	*9.7*	*17.6*	*47.3*

Table 6.6 Performance Improvement for 300 mesh size 1-D Parabolic with various schemes.

N	MPI T(s)							***(%)***		
	GS	***SOR***	***IADE-DY***	***IADE-MF***	***IADE-4TH***	***AGE-4TH***	***GS+SOR***	***IADE-DY+IADE-MF***	***IADE-4TH + AGE-4TH***	***SOR+ AGE-4TH***
1	*181.7*	*221.6*	*307.4*	*324.9*	*409.5*	*496.9*	*18.0*	*5.4*	*17.6*	*55.4*
2	*111.9*	*122.6*	*163.6*	*170.8*	*214.1*	*257.9*	*8.7*	*4.2*	*17.0*	*52.5*
6	*100.2*	*114.9*	*157.8*	*164.0*	*158.5*	*183.2*	*12.8*	*3.8*	*13.5*	*37.3*
8	*79.1*	*92.84*	*122.1*	*138.9*	*146.8*	*173.4*	*14.8*	*12.1*	*15.3*	*46.5*
10	*63.4*	*74.8*	*99.3*	*104.2*	*120.6*	*144.9*	*15.2*	*4.7*	*16.8*	*48.4*
12	*56.6*	*65.54*	*89.9*	*92.5*	*113.4*	*133.8*	*13.6*	*2.8*	*15.3*	*51.0*
14	*52.4*	*59.7*	*80.4*	*83.1*	*97.3*	*116.6*	*12.2*	*3.2*	*16.6*	*48.8*
16	*47.1*	*53.9*	*71.1*	*73.4*	*86.9*	*103.1*	*12.7*	*3.2*	*15.7*	*47.7*

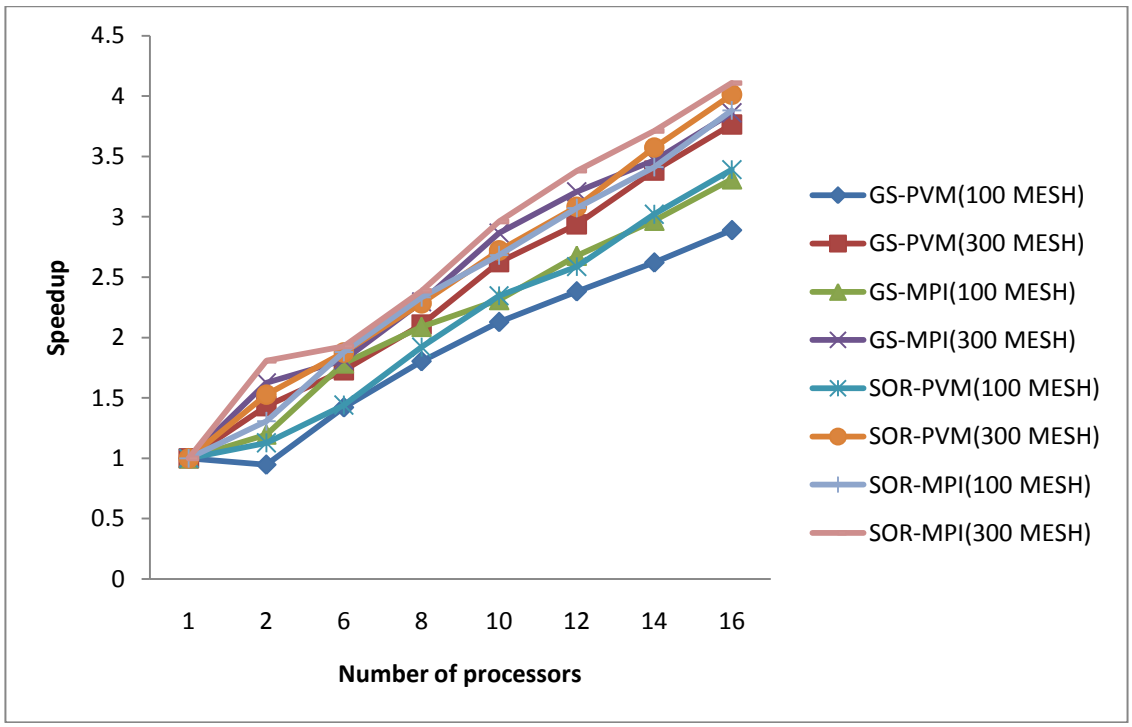

Fig. 6.1 Speed-up of GS, SOR $(\omega = 1.25)$ for 100 & 300 meshes with MPI and PVM (1-D Parabolic)

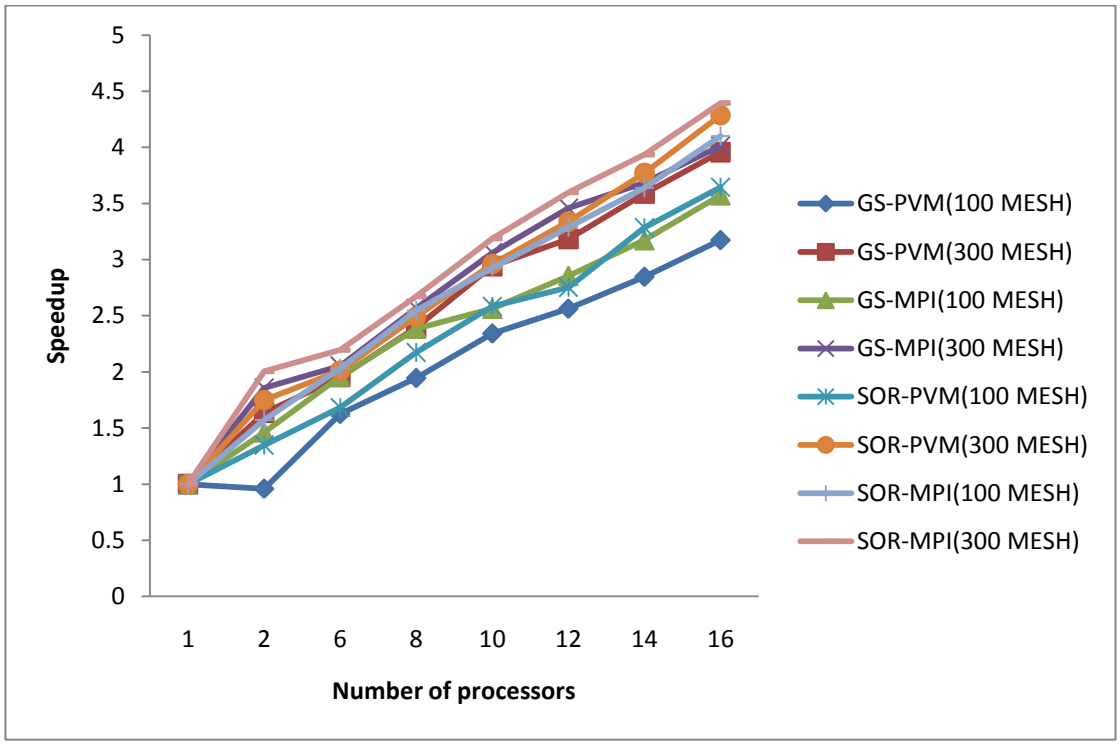

Fig. 6.2 Speed-up of GS & SOR $(\omega = 1.25)$ for 100 & 300 meshes with MPI and PVM (1-D Bio-Heat)

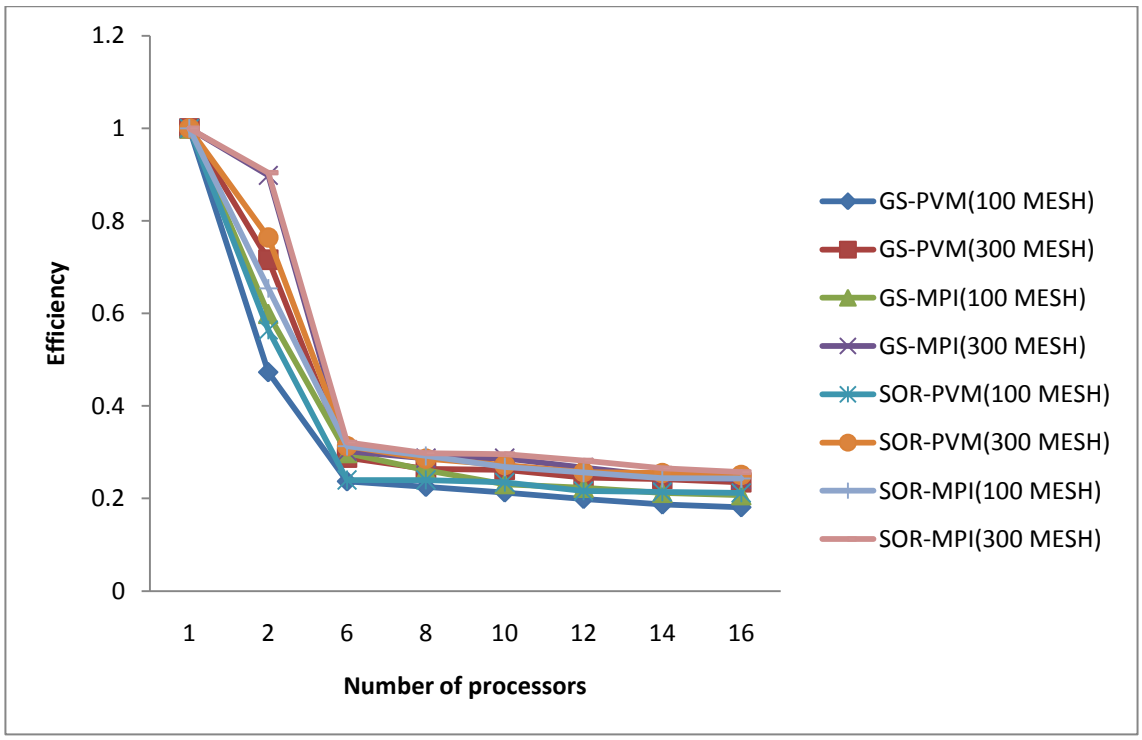

Fig. 6.3 Efficiency of GS & SOR ($\omega = 1.25$) for 100 & 300 meshes with MPI and PVM

(1-D Parabolic)

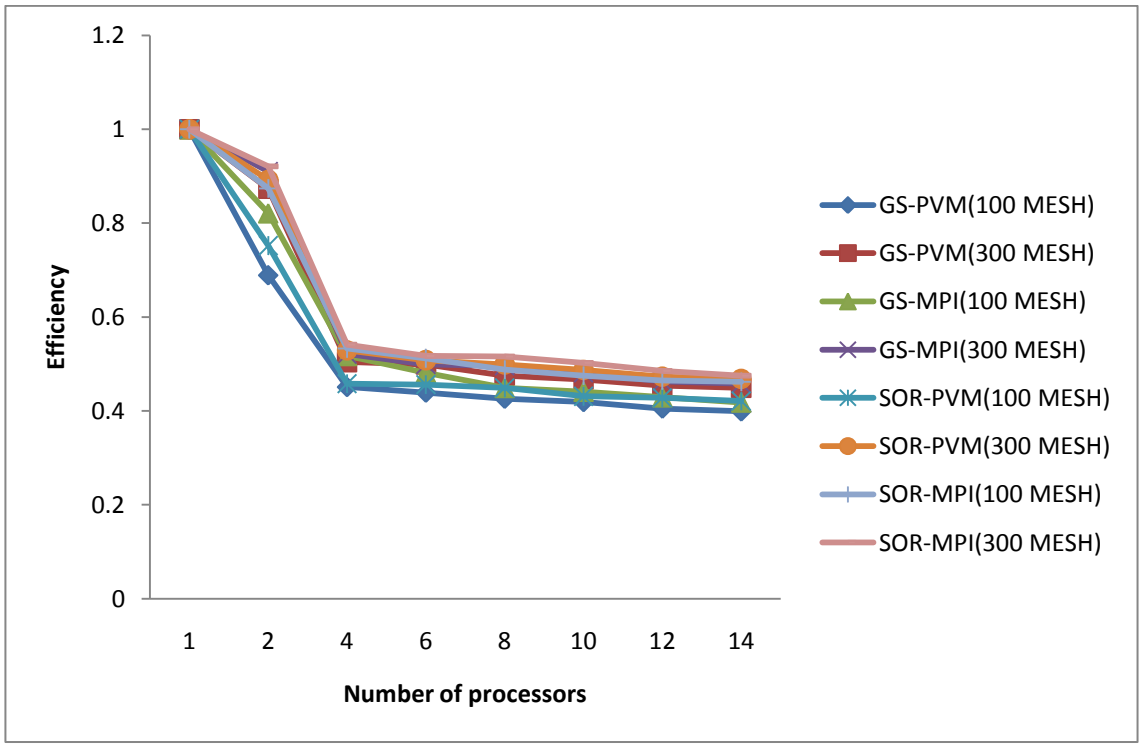

Fig. 6.4 Efficiency of GS & SOR ($\omega = 1.25$) for 100 & 300 meshes with MPI and PVM

(1-D Bio-Heat)

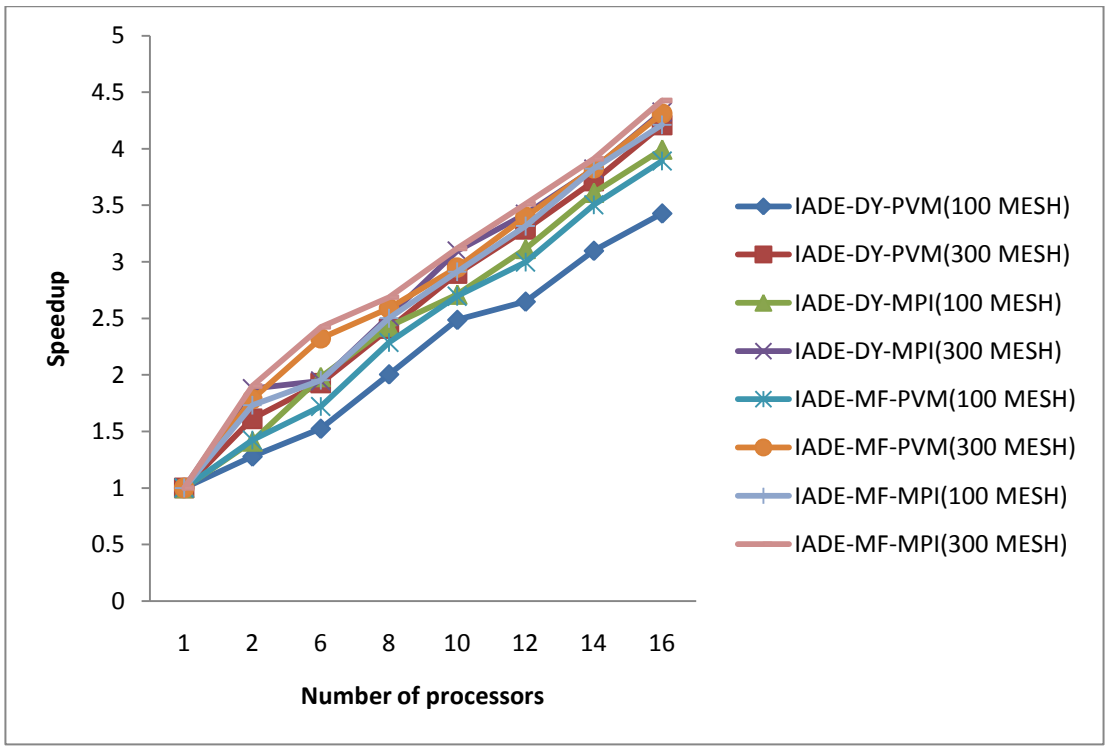

Fig. 6.5 Speedup of IADE-DY & IADE-MF for 100 & 300 meshes with MPI and PVM

(1-D Parabolic)

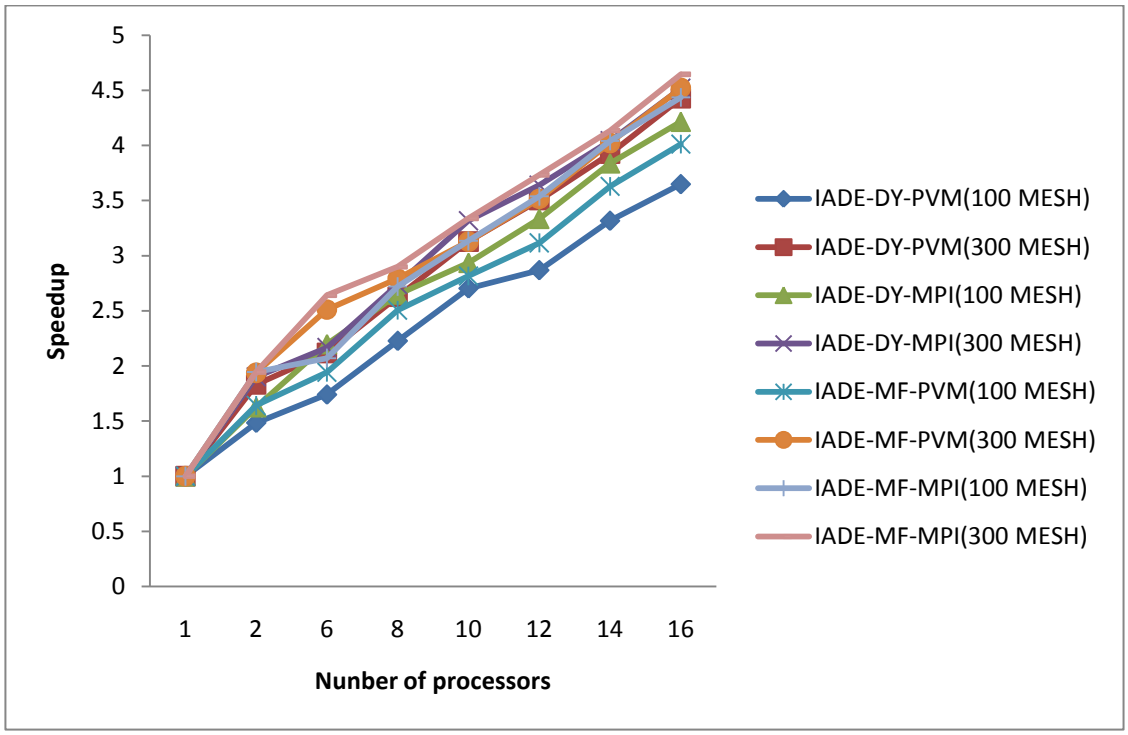

Fig. 6.6 Speedup of IADE-DY & IADE-MF for 100 & 300 meshes with MPI and PVM

(1-D Bio-Heat)

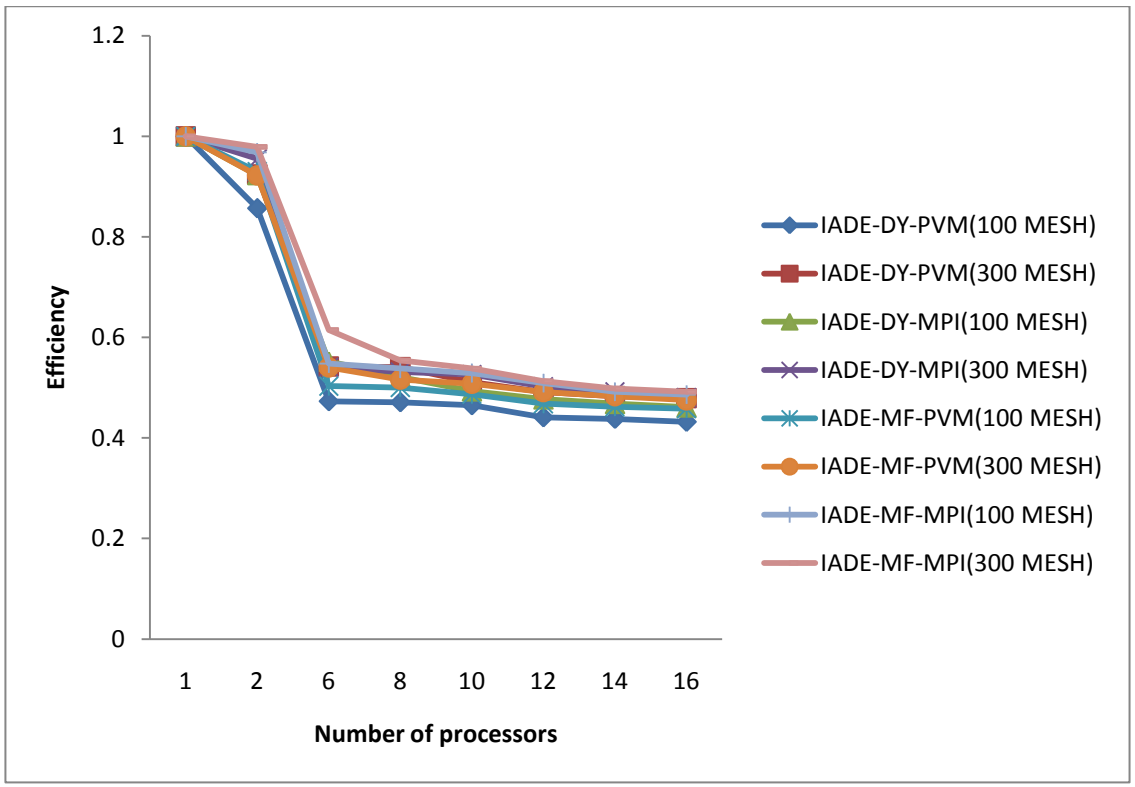

Fig. 6.7 Efficiency of IADE-DY & IADE-MF for 100 & 300 meshes with MPI and PVM (1-D Bio-Heat)

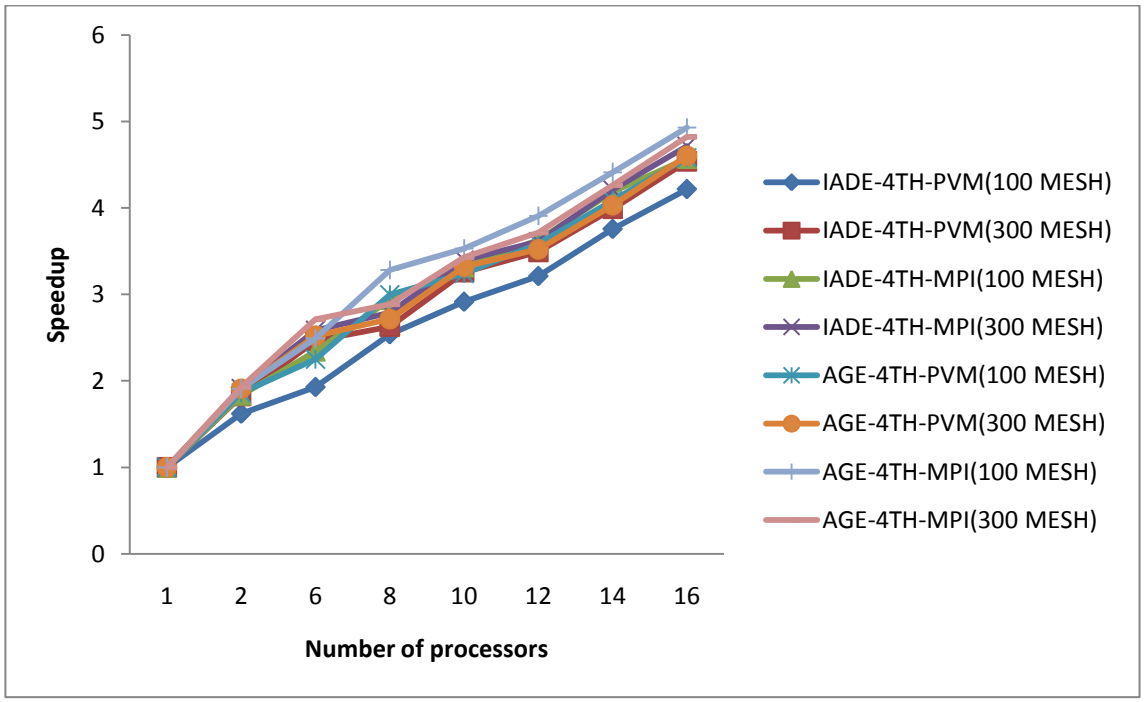

Fig. 6.8 Speedup of IADE-DY-4th & AGE-4th for 100 & 300 meshes with MPI and PVM (1-D Parabolic)

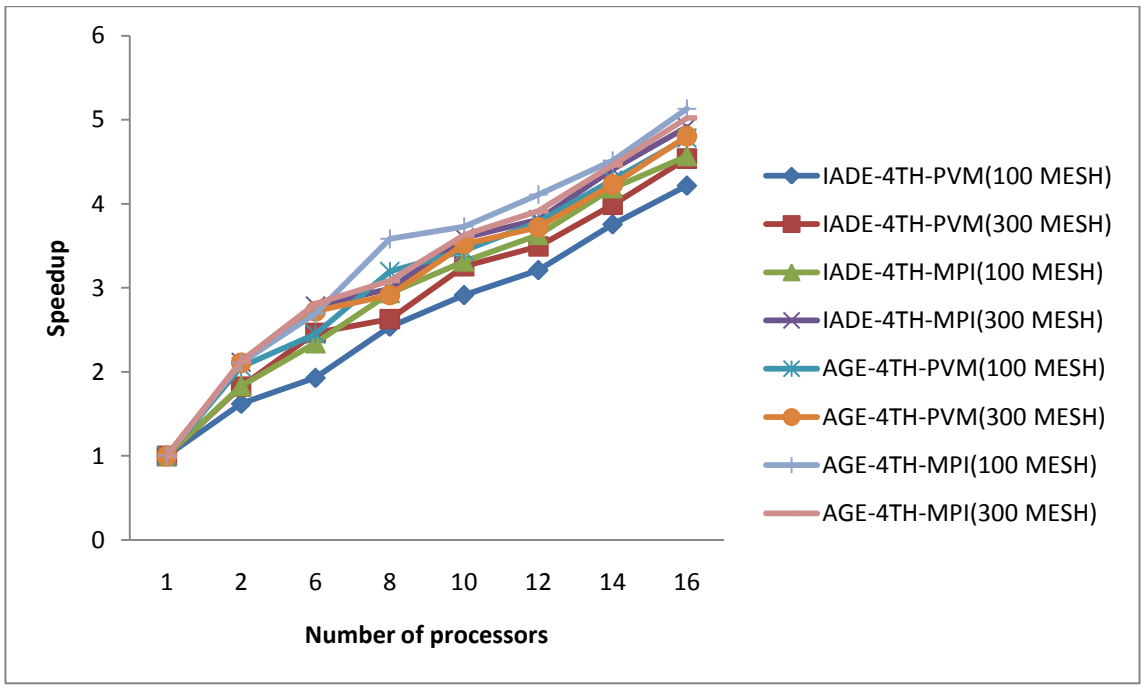

Fig. 6.9 Speedup of IADE-DY-4th & AGE-4th for 100 & 300 meshes with MPI and PVM (1-D Bio-Heat)

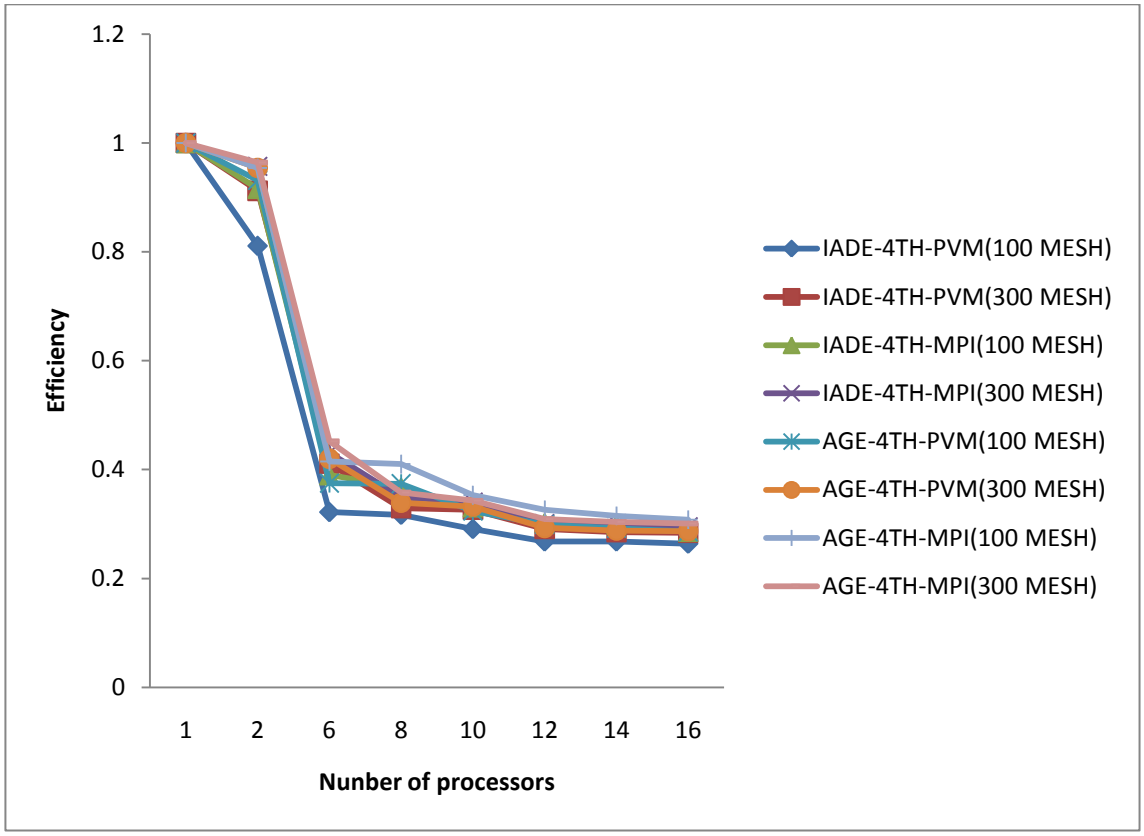

Fig. 6.10 Efficiency of IADE-DY-4th & AGE-4th for 100 & 300 meshes with MPI and PVM (1-D Parabolic)

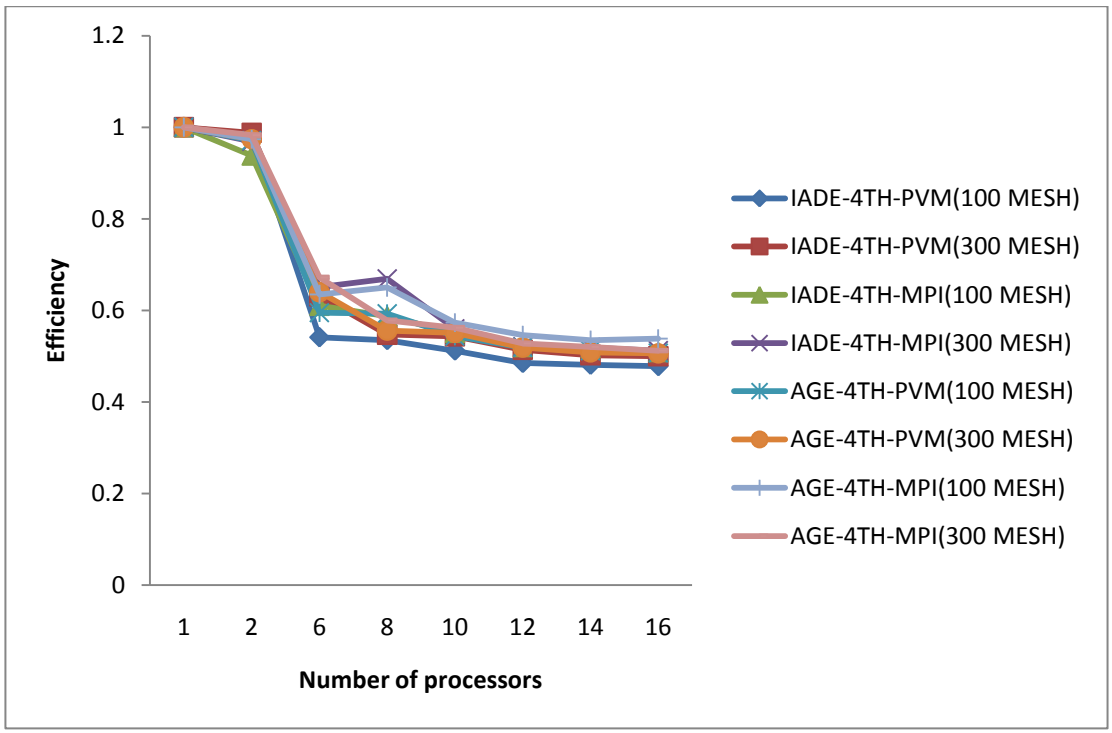

Fig. 6.11 Efficiency of IADE-DY-4th & AGE-4th for 100 & 300 meshes with MPI and PVM (1-D Bio-Heat)

6.4 Sequential Results for 2-D Parabolic and Bio-Heat Equations

The algorithms were tested on 2-D Parabolic Equation and the numerical results of the various schemes are tabulated in the Table 6.7 below. We consider the PDE by (Johnson & Riess (1982)) given as

$$\frac{\partial u(x,y,t)}{\partial t} = \alpha^2 \left[\frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2} \right] \quad 0 \le x \le 2, 0 \le y \le 1,\ t \ge 0.$$

The boundary condition and initial condition posed are:

$$\left.\begin{aligned} u(0,y,t) &= 0 \\ u(2,y,t) &= 0 \\ u(x,0,t) &= 0 \\ u(x,1,t) &= 0 \end{aligned}\right\} \tag{6.7}$$

$$u(x,y,0) = Sin(\pi x)Sin(2\pi y) \quad (0 \le x \le 1,\ 0 \le y \le 1 \quad . \tag{6.8}$$

The exact solution is given by

$$u(x,y,t) = e^{-5\pi^2 t} Sin(2\pi x) Sin(\pi y)\,.$$

We evaluated the schemes on Bio-Heat Equation given by (Jennifer et. al. 2002 and Dai & Zhang 2002)) and Table 6.8 shows the sequential computational results. The 2-D Bio-Heat equation of the form:

$$\frac{\partial U}{\partial t} = c\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) - bU + bU^{\oplus}, \tag{6.9}$$

The boundary condition and initial condition posed are:

$$\left.\begin{aligned} U(0,y,t) &= 0 \\ U(1,y,t) &= 0 \\ U(x,0,t) &= 0 \\ U(x,1,t) &= 0 \end{aligned}\right\} \quad t \geq 0 \tag{6.10}$$

$$U(x,y,0) = x(\pi - x - 1)y(\pi - y - 1) \quad (0 \leq x \leq 1,\ 0 \leq y \leq 1)$$

The cell size was chosen as $\Delta x = \Delta y$. The values of the physical properties in our test cases are chosen to be $\rho = 1000kg/m^3$, $c = c_b = 4200J/kg^oC$, $\omega_b = 0.5kg/m^3$, temperature is set to be $U_o = 12^oC$.

Table 6.7 Sequential results for 2-D Parabolic Equation with various schemes

Absolute error of numerical solutions,

$\lambda = 0.5, \Delta x = \Delta y = 0.02, t = 0.0018,\ \Delta t = 0.0002,\ r = 1.0,\ eps = 10^{-4}, y = 0.02$

X											
method	*0.02*	*0.3*	*0.58*	*0.86*	*1.14*	*1.42*	*1.7*	*1.98*	*Av. Abs. Err*	*RMS**	*It.*
GS	4.08x10^{-3}	5.92x10^{-3}	6.21x10^{-3}	8.65x10^{-3}	8.92x10^{-3}	8.65x10^{-3}	6.21x10^{-3}	5.92x10^{-3}	6.82x10^{-3}	4.91x10^{-5}	9
SOR	2.93x10^{-3}	4.79x10^{-3}	5.91x10^{-3}	7.31x10^{-3}	7.50x10^{-3}	7.31x10^{-3}	5.91x10^{-3}	4.79x10^{-3}	5.81x10^{-3}	3.20x10^{-5}	7
IADE-DY	2.31x10^{-3}	4.23x10^{-3}	5.52x10^{-3}	6.21x10^{-3}	6.43x10^{-3}	6.21x10^{-3}	5.52x10^{-3}	4.23x10^{-3}	5.08x10^{-3}	2.47x10^{-5}	5
MF-DS	5.58x10^{-6}	2.14x10^{-5}	1.73x10^{-5}	1.62x10^{-5}	2.21x10^{-5}	6.35x10^{-6}	2.42x10^{-5}	1.54x10^{-6}	1.43x10^{-5}	2.71x10^{-10}	4
AGE	1.45x10^{-6}	9.88x10^{-6}	5.28x10^{-6}	7.9x10^{-6}	8.24x10^{-6}	4.82x10^{-6}	1.0x10^{-5}	1.28.x10^{-6}	6.11x10^{-6}	3.54x10^{-11}	4
EXACT SOLN	7.2008 x10^{-3}	5.46416 x10^{-2}	-2.76785 x10^{-2}	-4.42688 x10^{-2}	4.42688 x10^{-2}	2.76785 x10^{-2}	-5.46416 x10^{-2}	-7.2008 x10^{-3}	-	-	-

Table 6.8 Sequential results for 2-D Bio-Heat Equation with various schemes

Method	*GS*	*SOR*	*IADE-DY*	*MF-DS*	*AGE*
Av. Abs.					
Err.	10.5×10^{-4}	8.7×10^{-4}	5.8×10^{-5}	4.2×10^{-6}	5.1×10^{-7}
RMS	5.6×10^{-4}	4.3×10^{-7}	5.6×10^{-6}	4.7×10^{-7}	5.2×10^{-13}
It	11	9	7	5	5
Δx	1×10^{-1}	1×10^{-1}	1×10^{-1}	1×10^{-1}	1×10^{-1}
Δt	2×10^{-4}	2×10^{-4}	2×10^{-4}	2×10^{-4}	2×10^{-4}
λ	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}
t	1.8×10^{-3}	1.8×10^{-3}	1.8×10^{-3}	1.8×10^{-3}	1.8×10^{-3}
eps	1×10^{-4}	1×10^{-4}	1×10^{-4}	1×10^{-4}	1×10^{-4}

6.5 Parallel Results for 2-D Parabolic and Bio-Heat Equations

The speedup and efficiency obtained for various grid sizes for the PVM and MPI implementation are given in the figures below and Table 6.9 list the values for 300 x 300 mesh sizes for 2-D Parabolic case.

Table 6.9: 300 x 300 mesh sizes using various schemes

		PVM					***MPI***		
Scheme	***N***	T_w	T_m	T_{sd}	S_{par}	E_{par}	***N***	S_{par}	E_{par}
	1	*528.9*	*33.6*	*8.7*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*496.1*	*33.4*	*8.5*	*1.648*	*0.824*	*2*	*1.67*	*0.835*
	6	*413.8*	*33.4*	*8.5*	*2.448*	*0.408*	*6*	*2.481*	*0.414*
GS	*8*	*378.5*	*33.4*	*8.5*	*3.076*	*0.385*	*8*	*3.196*	*0.400*
	10	*321.5*	*33.4*	*8.5*	*3.635*	*0.364*	*10*	*3.765*	*0.377*
	12	*252.7*	*33.4*	*8.5*	*4.11*	*0.343*	*12*	*4.278*	*0.357*
	14	*210.5*	*33.4*	*8.5*	*4.312*	*0.308*	*14*	*4.788*	*0.342*
	16	*189.7*	*33.4*	*8.5*	*4.164*	*0.260*	*16*	*5.024*	*0.314*
	1	*685.4*	*38.6*	*9.5*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*536.8*	*38.5*	*9.3*	*1.786*	*0.893*	*2*	*1.798*	*0.899*
	6	*482.1*	*38.5*	*9.3*	*2.625*	*0.438*	*6*	*2.658*	*0.443*
SOR	*8*	*413.8*	*38.5*	*9.3*	*3.284*	*0.411*	*8*	*3.476*	*0.435*

	10	*386.9*	*38.5*	*9.3*	*4.07*	*0.407*	*10*	*4.24*	*0.424*
	12	*251.8*	*38.5*	*9.3*	*4.752*	*0.396*	*12*	*4.926*	*0.411*
	14	*210.1*	*38.5*	*9.3*	*5.089*	*0.364*	*14*	*5.572*	*0.398*
	16	*189.6*	*38.5*	*9.3*	*5.584*	*0.349*	*16*	*5.976*	*0.374*
	1	*691.8*	*41.2*	*9.7*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*543.7*	*41*	*9.6*	*1.798*	*0.899*	*2*	*1.812*	*0.906*
	6	*487.1*	*41*	*9.6*	*2.661*	*0.444*	*6*	*2.685*	*0.448*
ADI	*8*	*408.5*	*41*	*9.5*	*3.46*	*0.433*	*8*	*3.524*	*0.441*
	10	*398.8*	*41*	*9.5*	*4.155*	*0.416*	*10*	*4.315*	*0.432*
	12	*262.6*	*41*	*9.5*	*4.806*	*0.401*	*12*	*5.082*	*0.424*
	14	*208.9*	*41*	*9.5*	*5.474*	*0.391*	*14*	*5.733*	*0.410*
	16	*199.3*	*41*	*9.5*	*5.792*	*0.362*	*16*	*6.176*	*0.386*
	1	*702.5*	*46.3*	*10.1*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*556.1*	*46.2*	*9.7*	*1.816*	*0.908*	*2*	*1.822*	*0.911*
IADE-DY	*6*	*491.5*	*46.2*	*9.7*	*2.688*	*0.448*	*6*	*2.715*	*0.453*
	8	*412.8*	*46.2*	*9.7*	*3.528*	*0.441*	*8*	*3.564*	*0.446*
	10	*401.9*	*46.2*	*9.7*	*4.305*	*0.431*	*10*	*4.39*	*0.439*
	12	*271.6*	*46.2*	*9.7*	*5.07*	*0.423*	*12*	*5.196*	*0.433*
	14	*211.5*	*46.2*	*9.7*	*5.719*	*0.409*	*14*	*5.873*	*0.420*
	16	*201.4*	*46.2*	*9.7*	*6.296*	*0.394*	*16*	*6.512*	*0.407*
	1	*702.5*	*51.3*	*10.1*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*556.1*	*51.2*	*9.7*	*1.834*	*0.917*	*2*	*1.922*	*0.961*
MF-DS	*6*	*491.5*	*51.2*	*9.7*	*2.728*	*0.455*	*6*	*2.749*	*0.458*
	8	*412.8*	*51.2*	*9.7*	*3.671*	*0.459*	*8*	*3.744*	*0.468*
	10	*401.9*	*51.2*	*9.7*	*4.396*	*0.440*	*10*	*4.51*	*0.451*
	12	*271.6*	*51.2*	*9.7*	*5.108*	*0.426*	*12*	*5.268*	*0.439*
	14	*211.5*	*51.2*	*9.7*	*5.827*	*0.416*	*14*	*5.95*	*0.425*
	16	*201.4*	*51.2*	*9.7*	*6.374*	*0.398*	*16*	*6.544*	*0.409*
	1	*702.5*	*54.2*	*10.1*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*686.1*	*54*	*9.7*	*1.896*	*0.948*	*4*	*1.933*	*0.967*
AGE	*6*	*591.5*	*54*	*9.7*	*2.889*	*0.482*	*6*	*2.921*	*0.487*
	8	*512.8*	*54*	*9.7*	*3.738*	*0.467*	*8*	*3.861*	*0.483*
	10	*501.9*	*54*	*9.7*	*4.415*	*0.442*	*10*	*4.568*	*0.457*
	12	*471.6*	*54*	*9.7*	*5.188*	*0.432*	*12*	*5.364*	*0.447*

14	*411.5*	*54*	*9.7*	*5.906*	*0.422*	*14*	*6.018*	*0.430*
16	*361.4*	*54*	*9.7*	*6.495*	*0.406*	*16*	*6.628*	*0.414*

Table 6.10 Effectiveness of the various schemes with PVM and MPI for 300x300 mesh size (Parabolic case)

	N	***PVM T(s)***	***L_n***	***MPI T(s)***	***L_n***
	1	*752.4*	*0.13*	*718.3*	*0.14*
	2	*456.6*	*0.18*	*430.1*	*0.19*
GS	*6*	*307.4*	*0.13*	*289.5*	*0.14*
	8	*244.6*	*0.15*	*224.75*	*0.18*
	10	*206.9*	*0.18*	*190.8*	*0.20*
	12	*183.1*	*0.19*	*167.9*	*0.21*
	14	*174.5*	*0.18*	*150.0*	*0.23*
	16	*180.7*	*0.14*	*142.97*	*0.22*
	1	*876.5*	*0.11*	*841.8*	*0.12*
SOR	*2*	*490.8*	*0.18*	*468.2*	*0.19*
	6	*333.9*	*0.13*	*316.7*	*0.14*
	8	*266.9*	*0.15*	*242.2*	*0.18*
	10	*215.4*	*0.19*	*198.5*	*0.21*
	12	*184.5*	*0.21*	*170.9*	*0.24*
	14	*172.2*	*0.21*	*151.1*	*0.26*
	16	*156.9*	*0.22*	*140.9*	*0.27*
	1	*992.4*	*0.10*	*928.7*	*0.11*
ADI	*2*	*551.9*	*0.16*	*512.5*	*0.18*
	6	*372.9*	*0.12*	*345.9*	*0.13*
	8	*286.8*	*0.15*	*263.5*	*0.17*
	10	*238.8*	*0.17*	*215.2*	*0.20*
	12	*206.5*	*0.19*	*182.74*	*0.23*
	14	*181.3*	*0.22*	*161.9*	*0.25*
	16	*171.3*	*0.21*	*150.4*	*0.26*
	1	*1241.7*	*0.08*	*1108.2*	*0.09*
	2	*683.8*	*0.13*	*608.2*	*0.15*
IADE-DY	*6*	*461.9*	*0.10*	*408.2*	*0.11*
	8	*351.9*	*0.13*	*310.9*	*0.14*
	10	*288.4*	*0.15*	*252.4*	*0.17*
	12	*244.9*	*0.17*	*213.3*	*0.20*
	14	*217.1*	*0.19*	*188.7*	*0.22*
	16	*197.2*	*0.20*	*170.2*	*0.24*
	1	*1508.1*	*0.07*	*1312.7*	*0.08*

MF-DS	*2*	*822.3*	*0.11*	*682.9*	*0.14*	
	6	*552.8*	*0.08*	*477.5*	*0.10*	
	8	*410.8*	*0.11*	*350.6*	*0.13*	
	10	*343.1*	*0.13*	*291.1*	*0.15*	
	12	*295.2*	*0.14*	*249.2*	*0.18*	
	14	*258.8*	*0.16*	*220.6*	*0.19*	
	16	*236.6*	*0.17*	*200.6*	*0.20*	
	1	*1933.3*	*0.05*	*1798.5*	*0.06*	
AGE	*2*	*1019.7*	*0.09*	*930.42*	*0.10*	
	6	*669.19*	*0.07*	*615.71*	*0.08*	
	8	*517.20*	*0.09*	*465.81*	*0.10*	
	10	*437.89*	*0.10*	*393.72*	*0.12*	
	12	*372.65*	*0.12*	*335.29*	*0.13*	
	14	*327.35*	*0.13*	*298.85*	*0.14*	
	16	*297.66*	*0.14*	*271.35*	*0.15*	

Table 6.11 Performance Improvement for 300x300 2-D Parabolic with various schemes.

N	***PVM T(s)***						***(%)***			
	GS	***SOR***	***ADI***	***IADE-DY***	***MF-DS***	***AGE***	***GS+ SOR***	***ADI+IADE-DY***	***MF-DS + AGE***	***AGE+ SOR***
1	*752.4*	*876.5*	*992.4*	*1242*	*1508*	*1933*	*14.2*	*20.1*	*22*	*54.7*
2	*456.6*	*490.8*	*551.9*	*683.8*	*822.3*	*1019*	*7.0*	*19.3*	*19.4*	*51.9*
6	*307.4*	*333.9*	*372.9*	*461.9*	*552.8*	*669.2*	*8.0*	*19.3*	*17.4*	*50.1*
8	*244.6*	*266.9*	*288.8*	*351.9*	*410.8*	*517.2*	*8.4*	*18.5*	*20.6*	*48.4*
10	*206.9*	*215.4*	*238.8*	*288.4*	*343.1*	*437.9*	*3.9*	*17.2*	*21.6*	*50.9*
12	*183.1*	*184.5*	*206.5*	*244.9*	*295.2*	*372.7*	*0.8*	*15.7*	*20.8*	*50.5*
14	*174.5*	*179.2*	*181.3*	*217.1*	*258.8*	*327.4*	*2.6*	*16.5*	*20.9*	*45.3*
16	*180.7*	*196.9*	*171.3*	*197.2*	*236.6*	*297.7*	*8.3*	*13.1*	*20.5*	*33.8*

Table 6.12 Performance Improvement for 300x300 2-D Parabolic with various schemes.

N	***MPI T(s)***						***(%)***			
	GS	***SOR***	***ADI***	***IADE-DY***	***MF-DS***	***AGE***	***GS+ SOR***	***ADI+IADE-DY***	***MF-DS +AGE***	***AGE+ SOR***
1	*718.3*	*841.8*	*928.7*	*1108*	*1313*	*1799*	*14.7*	*16.2*	*27.0*	*53.2*
2	*430.1*	*468.2*	*512.5*	*608.2*	*683*	*930.4*	*8.1*	*15.7*	*26.6*	*49.7*
6	*289.5*	*316.7*	*345.9*	*408.2*	*477.5*	*615.7*	*8.6*	*15.3*	*22.4*	*48.6*
8	*224.8*	*242.2*	*263.5*	*310.9*	*350.6*	*465.8*	*7.2*	*15.2*	*24.7*	*48.0*
10	*190.8*	*198.5*	*215.2*	*252.4*	*291.1*	*393.7*	*3.9*	*14.7*	*26.1*	*49.6*
12	*167.9*	*170.9*	*182.7*	*213.3*	*249.2*	*335.3*	*1.7*	*14.3*	*25.7*	*49.0*
14	*150.0*	*151.1*	*161.9*	*188.7*	*220.6*	*298.9*	*0.7*	*14.2*	*26.2*	*49.4*
16	*142.9*	*149.9*	*150.4*	*170.2*	*200.6*	*271.4*	*4.6*	*11.6*	*26.1*	*44.8*

Table 6.13 Slave computational time for 100 iterations as a function of block numbers for PVM
(2-D Parabolic case)

NI x NJ	***B = 50***	***B = 100***	***B = 200***
100 x 100	*589*	*723*	*846*
200 x 200	*1012*	*1342*	*1528*
300 x 300	*1869*	*2225*	*2586*

Table 6.14 Slave computational time for 100 iterations as a function of block numbers for MPI
(2-D Parabolic case)

NI x NJ	***B = 50***	***B = 100***	***B = 200***
100 x 100	*511*	*689*	*803*
200 x 200	*983*	*1208*	*1493*
300 x 300	*1528*	*2108*	*2414*

Table 6.15 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 100 x 100 PVM (2-D Parabolic case)

N	***B = 50***	***B = 100***	***B = 200***
1	*4528*	*4912*	*3208*
2	*4704*	*5028*	*3416*
6	*4813*	*5217*	*3727*
8	*4984*	*5308*	*3921*
10	*5203*	*5604*	*4219*
12	*5321*	*5638*	*4307*
14	*5408*	*5673*	*4321*
16	*5421*	*5907*	*4434*

Table 6.16 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 100 x 100
MPI (2-D Parabolic case)

N	***B = 50***	***B = 100***	***B = 200***
1	*4243*	*4718*	*2948*
2	*4628*	*4976*	*3418*
6	*4721*	*5108*	*3639*
8	*4797*	*5211*	*3741*
10	*5108*	*5416*	*3968*
12	*5219*	*5527*	*4114*
14	*5337*	*5599*	*4225*
16	*5377*	*5682*	*4311*

Table 6.17 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 200 x 200 PVM (2-D Parabolic case)

N	***B = 50***	***B = 100***	***B = 200***
1	*5012*	*5128*	*3764*
2	*5247*	*5389*	*3824*
6	*5299*	*5463*	*3918*
8	*5437*	*5511*	*4210*
10	*5558*	*5793*	*4328*
12	*5789*	*5912*	*4479*
14	*5921*	*5993*	*4528*
16	*5921*	*6037*	*4611*

Table 6.18 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 200 x 200
MPI (2-D Parabolic case)

N	***B = 50***	***B = 100***	***B = 200***
1	*4821*	*4978*	*3582*
2	*4962*	*4998*	*3608*
6	*5108*	*5224*	*3773*
8	*5219*	*5328*	*3994*
10	*5307*	*5515*	*4205*
12	*5438*	*5708*	*4324*
14	*5776*	*5834*	*4494*
16	*5776*	*5938*	*4502*

Table 6.19 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 300 x 300 PVM (2-D Parabolic case)

N	***B = 50***	***B = 100***	***B = 200***
1	*5343*	*5734*	*3912*
2	*5938*	*6214*	*4328*
6	*6234*	*6373*	*4494*
8	*6432*	*6498*	*4528*
10	*6539*	*6677*	*4621*
12	*6683*	*6721*	*4696*
14	*6878*	*6808*	*4701*
16	*6878*	*6874*	*4799*

Table 6.20 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 300 x 300
MPI (2-D Parabolic case)

N	***B = 50***	***B = 100***	***B = 200***
1	*5321*	*5582*	*3781*
2	*5911*	*5987*	*4084*
6	*6024*	*6118*	*4212*
8	*6133*	*6228*	*4314*
10	*6341*	*6393*	*4582*
12	*6417*	*6528*	*4609*

14	*6613*	*6782*	*4693*
16	*6621*	*6811*	*4724*

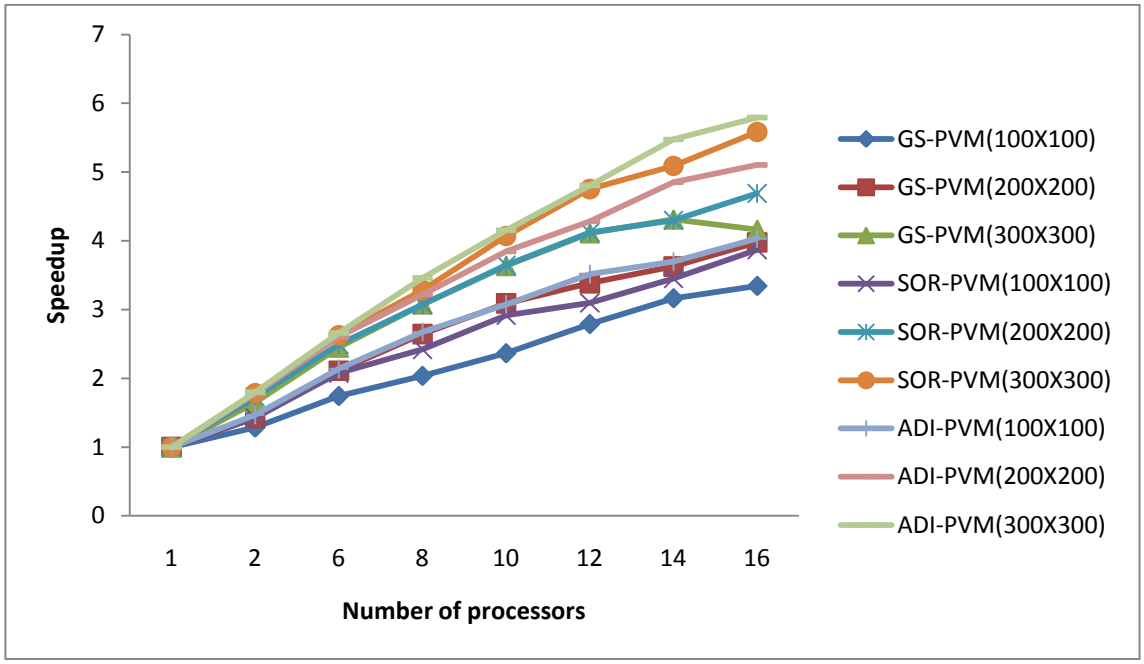

Fig. 6.12 Speedup for 100x100 to 300x300 meshes with PVM (2-D Parabolic)

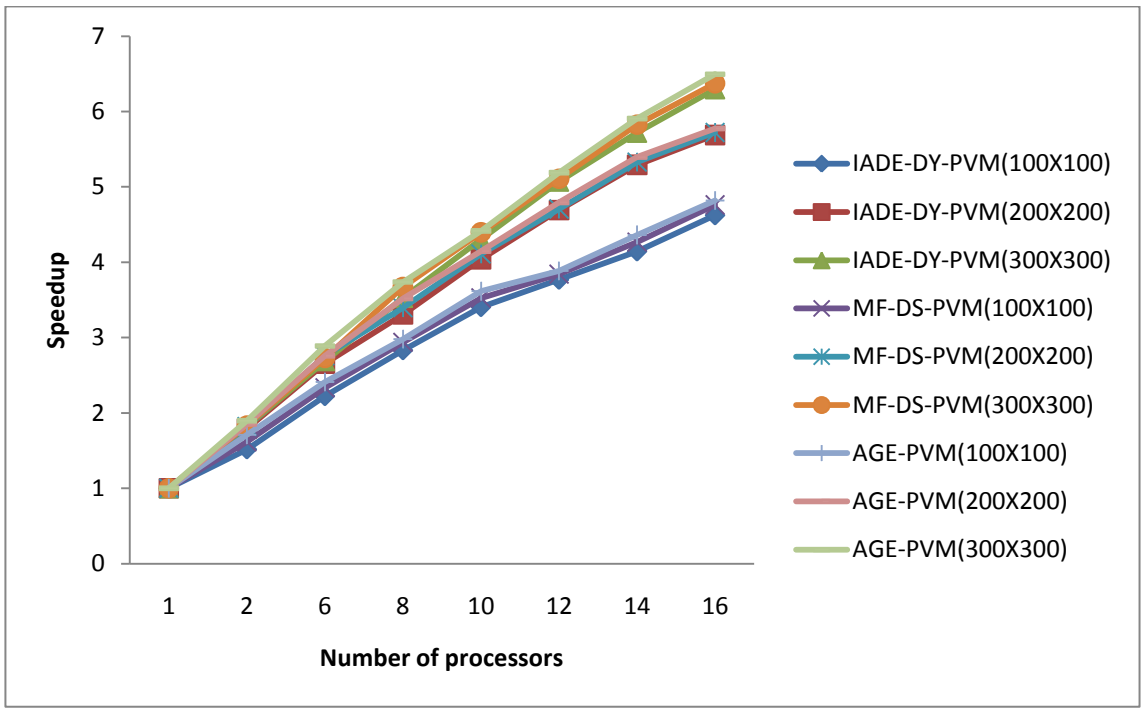

Fig. 6.13 Speedup for 100x100 to 300x300 meshes with PVM (2-D Bio-Heat)

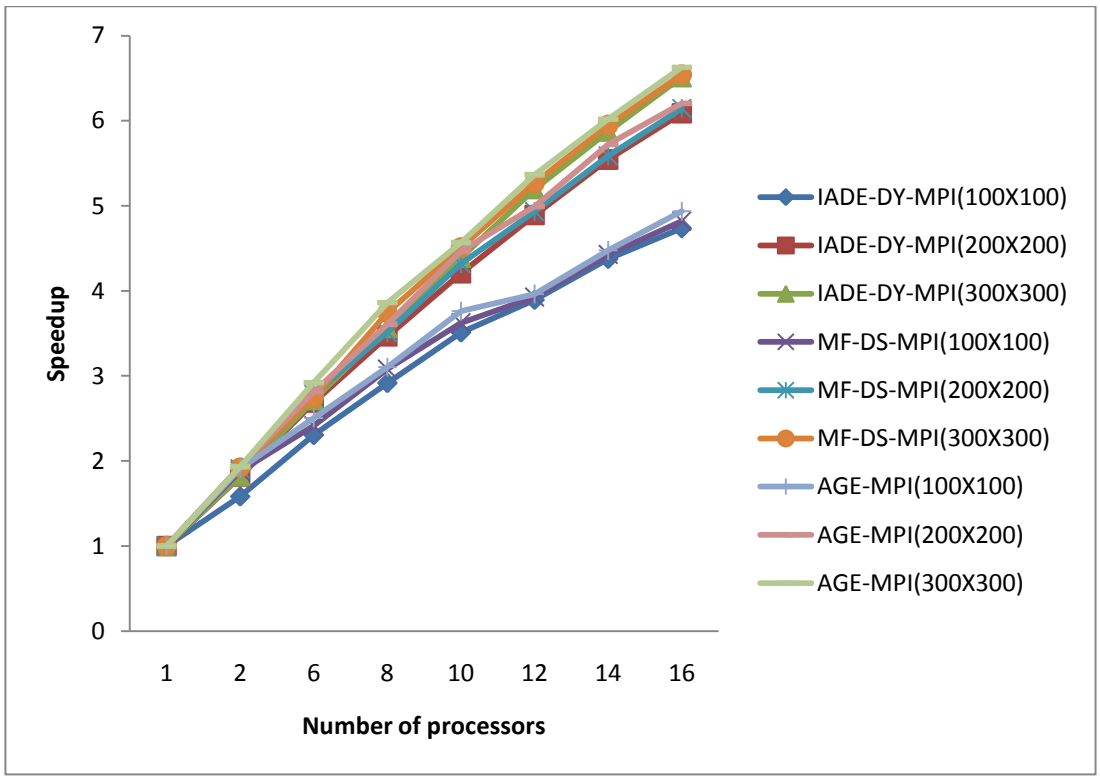

Fig. 6.14 Speedup for 100x100 to 300x300 meshes with MPI (2-D Parabolic)

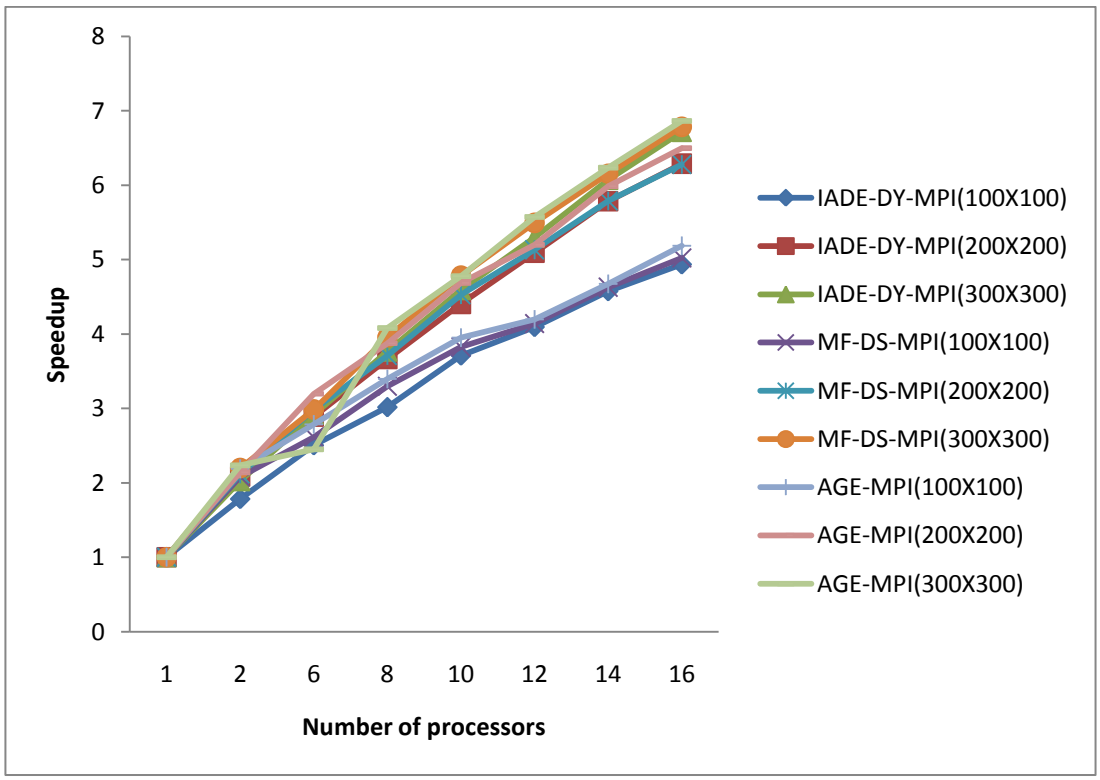

Fig. 6.15 Speedup for 100x100 to 300x300 meshes with MPI (2-D Bio-Heat)

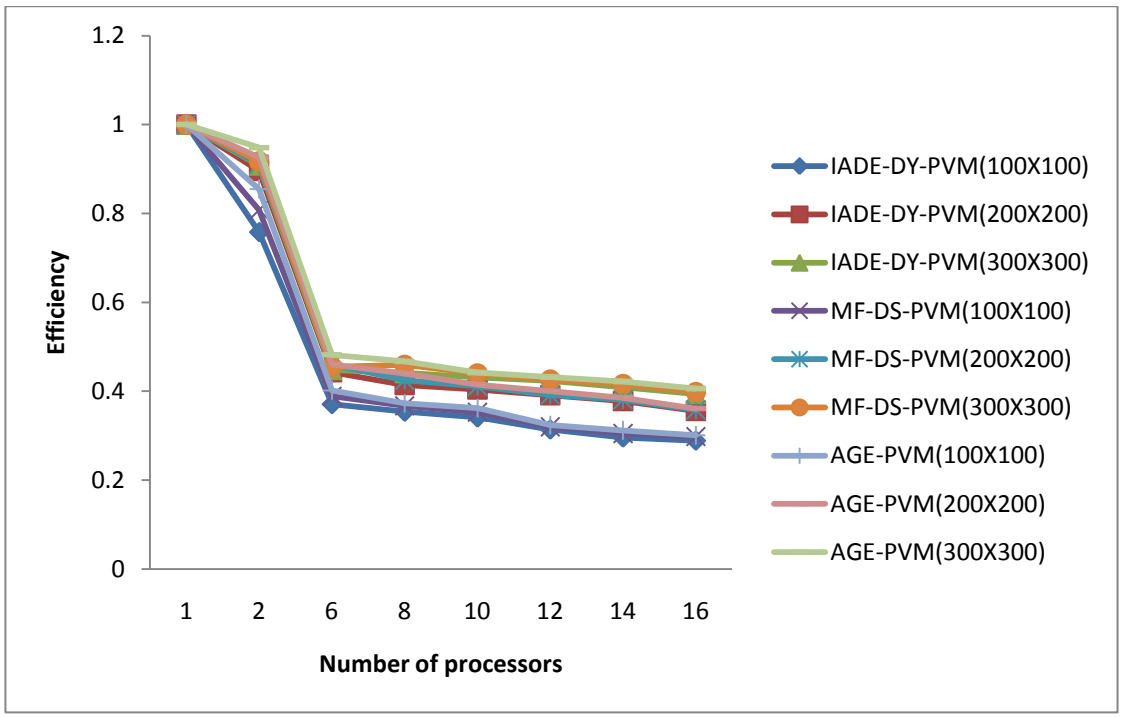

Fig. 6.16 Efficiency for 100x100 to 300x300 meshes with PVM (2-D Parabolic)

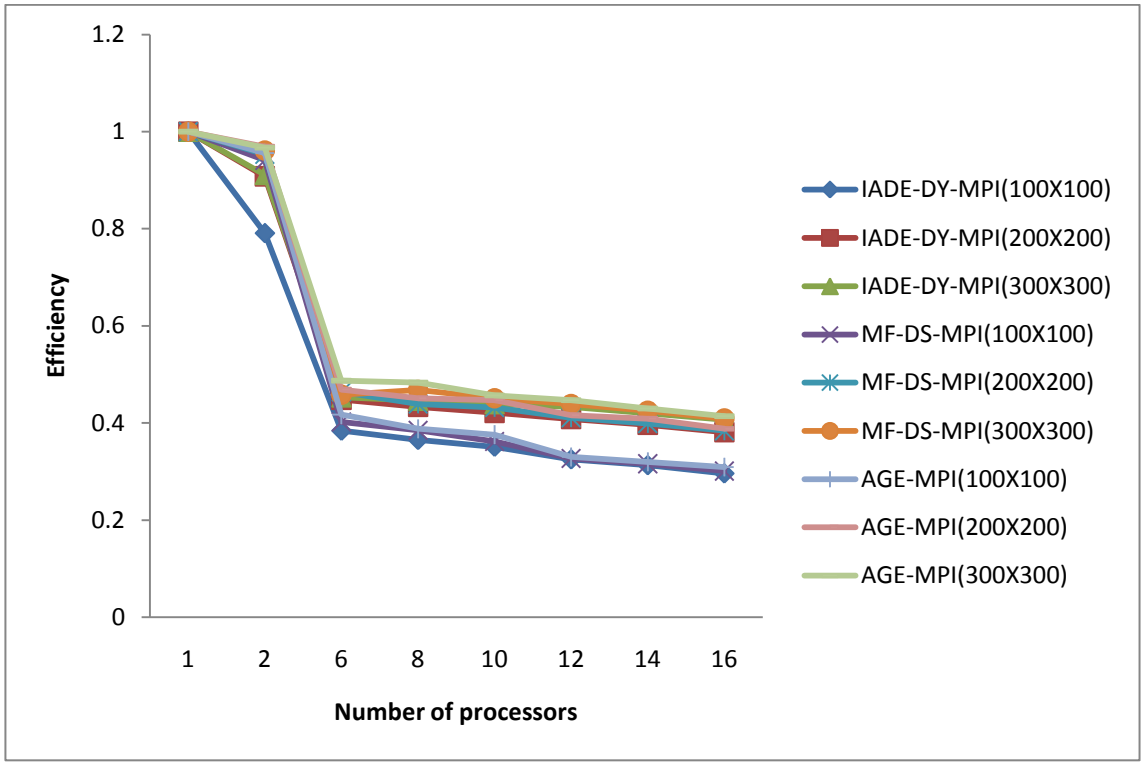

Fig. 6.17 Efficiency for 100x100 to 300x300 meshes with MPI (2-D Parabolic)

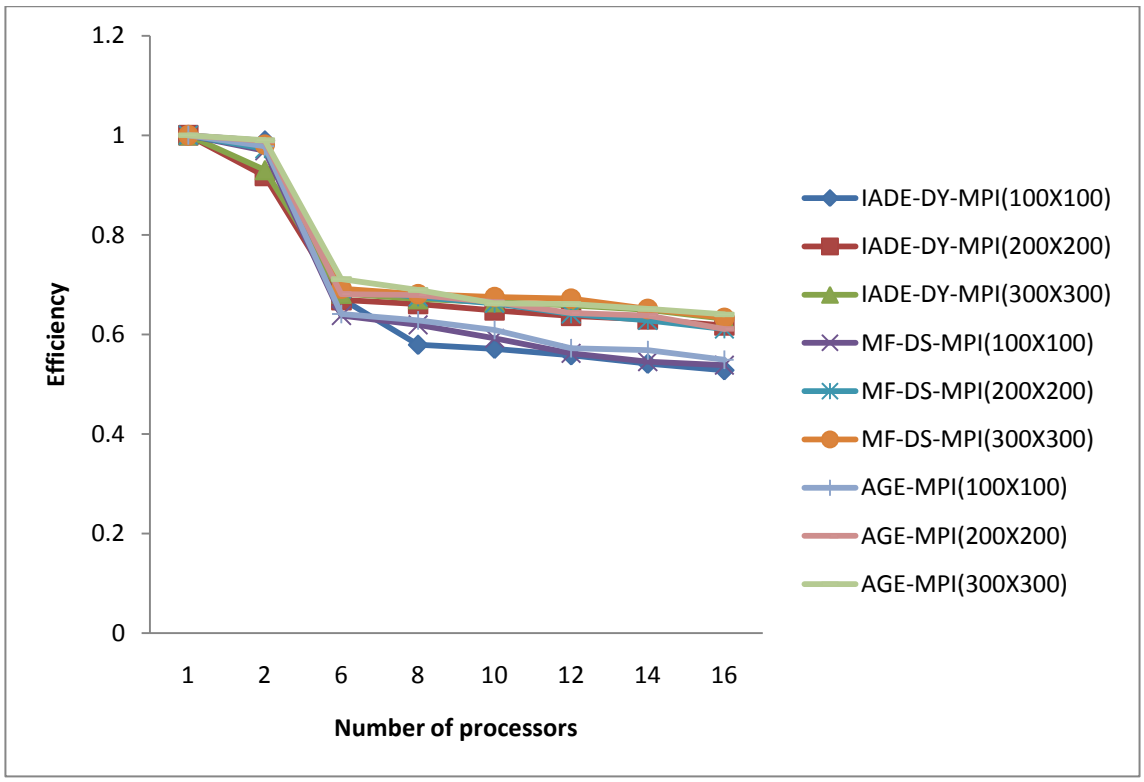

Fig. 6.18 Efficiency for 100x100 to 300x300 meshes with PVM (2-D Bio-Heat)

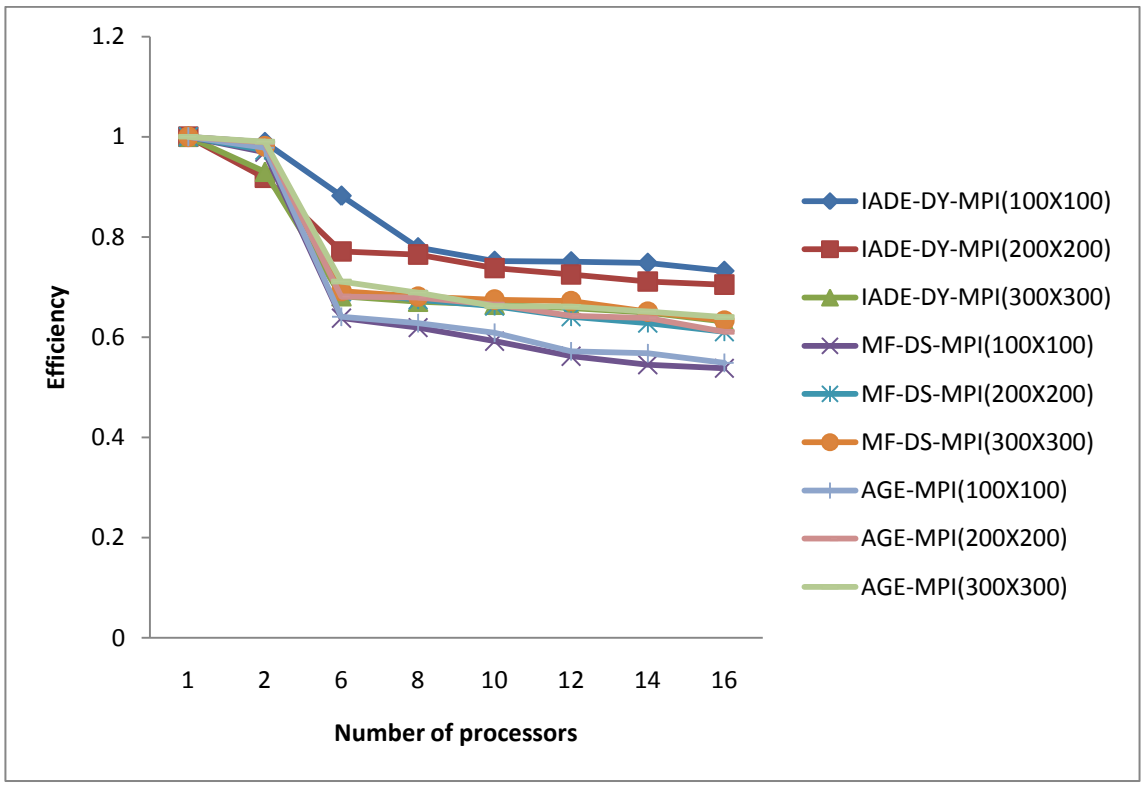

Fig. 6.19 Efficiency for 100x100 to 300x300 meshes with MPI (2-D Bio-Heat)

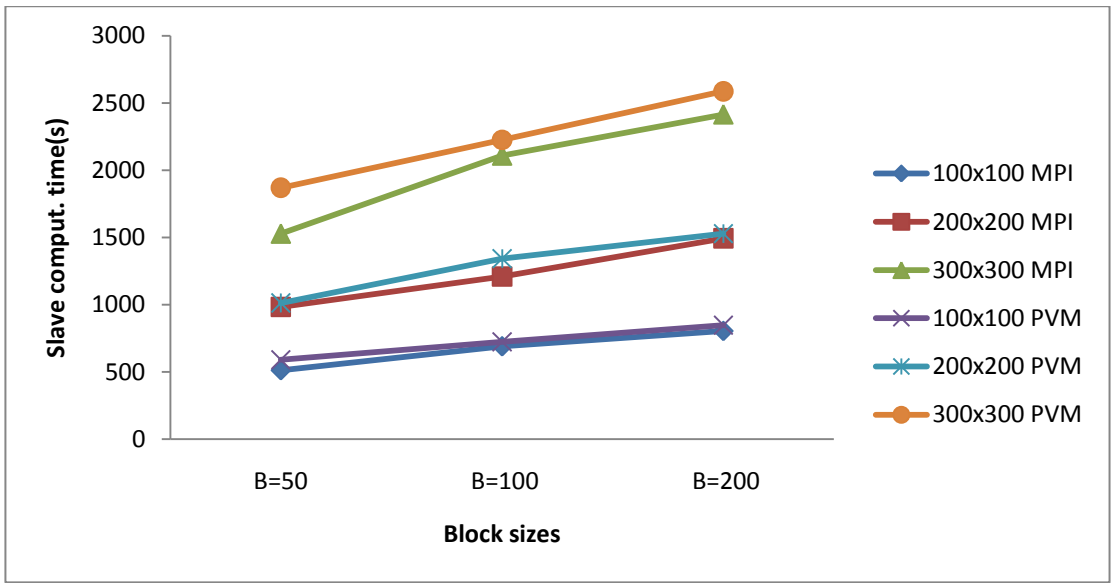

Fig.6.20 Slave computational time for various block sizes (2-D Parabolic MPI/PVM)

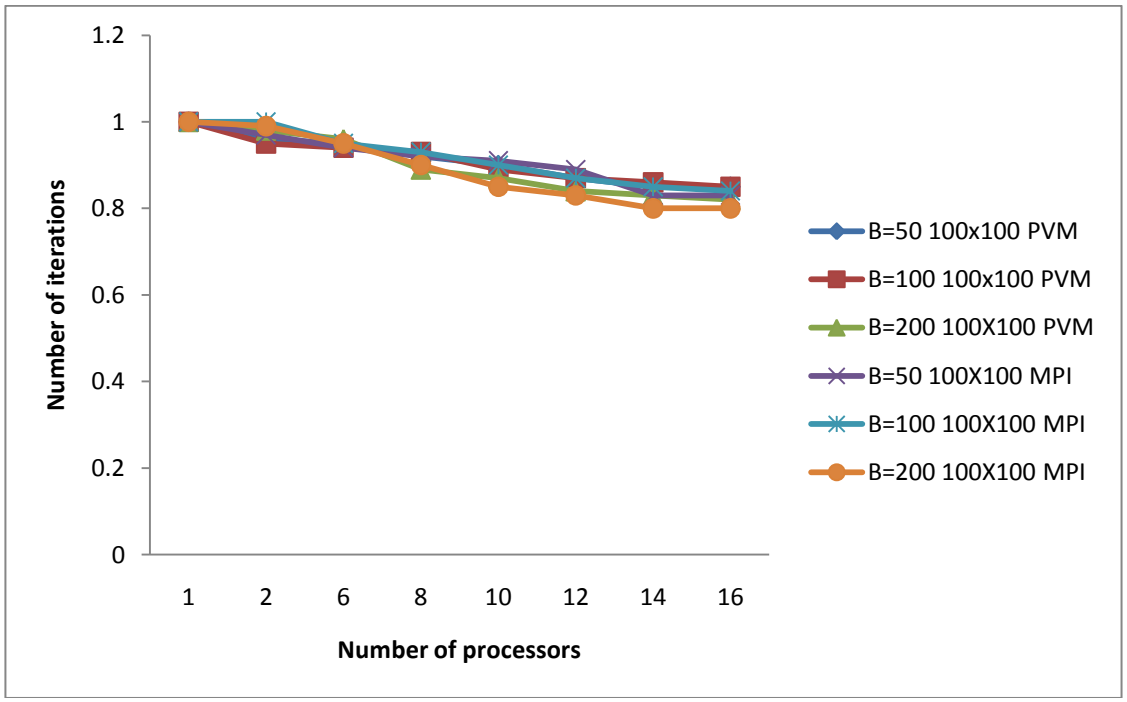

Fig.6.21 Number of iterations versus block sizes for 100x100 MPI and PVM

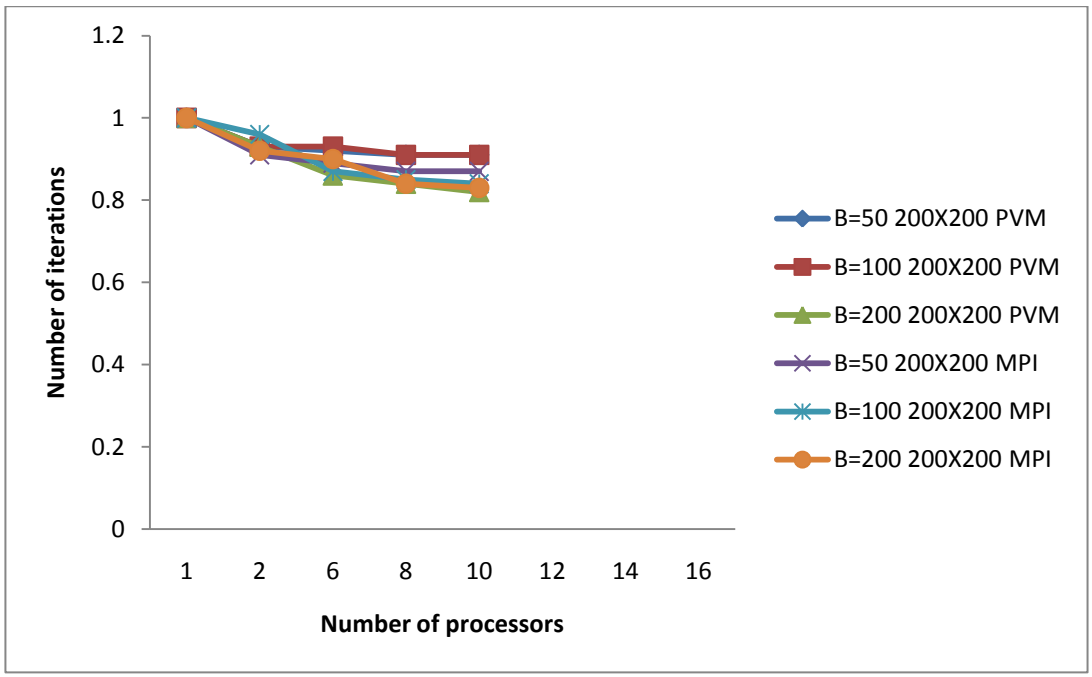

Fig.6.22 Number of iterations versus block numbers for 200x200 PVM and MPI.

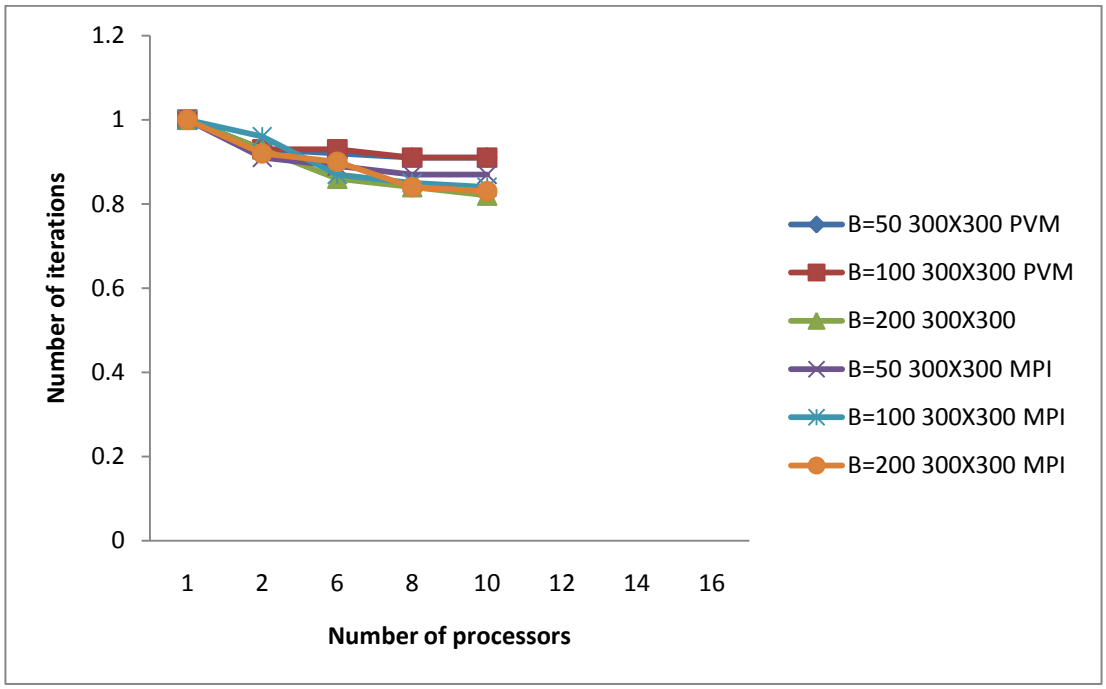

Fig.6.23 Number of iterations versus block numbers for 300x300 PVM AND MPI.

6.6 Sequential Computational Results for 1-D Telegraph Equation

To demonstrate the implicit formulation on the Telegraph Equation and using the *LU* algorithm to solve the resulting linear system and also using the IAGE-MF to solve the

solution of the linear system, numerical tests have been carried out. The values of the physical properties in our test case are chosen in such a way that LC, *(RC + GL)*, are equal to one. Table 6.21 shows the various numerical results on Eq. (4.1) with the boundary condition and initial condition posed as:

$$\frac{\partial^2 U}{\partial x^2} = LC\frac{\partial^2 U}{\partial t^2} + (RC + GL)\frac{\partial U}{\partial t} + RGU, \qquad 0 \le x \le 1,\ 0 \le t \le 1$$

$$\left.\begin{aligned} U(0,t) &= 0 \\ U(1,t) &= 100 \end{aligned}\right\} \quad t \ge 0 \tag{6.11}$$

and

$$\begin{aligned} U(x,0) &= Sin(\pi x) \quad 0 \le x \le 1 \\ U_t(x,0) &= Sin(\pi x) \quad 0 \le x \le 1 \end{aligned} \tag{6.12}$$

The exact solution is given by

$$U(x, y, t) = e^{-\pi t} Sin(\pi x).$$

Table 6.21 Numerical results for 1-D Telegraph Equation with various schemes

Method	***Av. Abs. Err.***	***RMS***	***It***	Δx	Δt	λ	t	eps
3-L IMP	6.3×10^{-4}	3.7×10^{-4}	5	1×10^{-1}	5×10^{-3}	5×10^{-1}	2.5×10^{-1}	1×10^{-4}
IADE-MF	3.2×10^{-4}	2.3×10^{-7}	3	1×10^{-1}	5×10^{-3}	5×10^{-1}	2.5×10^{-1}	1×10^{-4}

6.7 Parallel Results for 1-D Telegraph Equation

The CPU time for the master task and the data communication is constant for a given grid size and sub-domain. Therefore, the task in the inner loop should be made as large as possible to maximize the efficiency. The speedup and efficiency obtained for 300 mesh sizes for the MPI and PVM implementation on 1-D Telegraph Equation is listed in Table 6.22 for 100 iterations. In the Table, we also listed the elapsed time for the

master task, T_w, the master CPU time, T_m, the average slave computational time, T_{sc}, and the slave data communication time, T_{sd}, all in seconds.

Table 6.22: 300 mesh size using two schemes (1-D Telegraph)

		PVM						MPI	
Scheme	**N**	T_w	T_m	T_{sd}	T_{sc}	S_{par}	E_{par}	**N**	S_{par}	E_{par}
	1	476.9	15.2	3.6	270.8	1.000	1.000	1	1.000	1.000
	2	318.5	14.8	3.8	142.6	1.797	0.899	2	1.912	0.956
	8	189.9	14.6	3.6	36.3	5.314	0.664	8	5.486	0.686
Imp	10	163.7	14.6	3.6	28.21	6.240	0.624	10	6.410	0.641
	14	149.9	14.6	3.6	23.99	6.864	0.490	14	6.928	0.495
	16	136.8	14.6	3.6	21.6	7.108	0.444	16	7.211	0.451
	1	529.8	29.5	7.4	361.8	1.000	1.000	1	1.000	1.000
	2	412.3	28.2	7.4	171.0	1.930	0.965	2	2.246	0.968
	8	218	28.2	7.3	23.69	6.736	0.842	8	6.813	0.852
IADE-MF	10	199.5	28.2	7.3	14.78	7.930	0.793	10	8.113	0.811
	14	178.1	28.2	7.3	11.64	8.458	0.604	14	9.004	0.688
	16	166.2	28.2	7.3	10.52	8.924	0.558	16	9.325	0.653

Table 6.23 Effectiveness of the various schemes with PVM and MPI for 300 mesh size (1-D Telegraphic case)

	N	**PVM T(s)**	L_n	**MPI T(s)**	L_n
	1	289.6	0.35	261.2	0.38
	2	161.1	0.56	136.6	0.70
IMP	8	54.50	1.22	47.62	1.44
	10	46.41	1.34	40.76	1.58
	14	42.19	1.16	37.71	1.31
	16	40.74	1.09	36.23	1.24
	1	398.7	0.25	371.6	0.27
IADE-MF	2	206.58	0.47	165.5	0.59
	8	59.19	1.42	54.54	1.56
	10	50.28	1.58	45.80	1.77
	14	47.4	1.28	41.27	1.67
	16	44.68	1.25	39.85	1.64

Table 6.24 Improvement of performance for two schemes on 1-D Telegraph Equation.

Size	*N*	*T(s) PVM Imp*	*IADE-MF*	*MPI Imp*	*T(s) IADE-MF*	*PVM(%) Improvement*	*MPI(%) Improvement*
	1	*289.6*	*398.7*	*261.24*	*371.6*	*27.4*	*29.7*
	2	*161.1*	*206.6*	*136.6*	*165.5*	*22*	*17.4*
300 x 300	*8*	*54.50*	*59.19*	*47.62*	*54.54*	*7.9*	*12.7*
	10	*46.41*	*50.28*	*40.76*	*45.80*	*7.7*	*11.0*
	14	*42.19*	*47.14*	*37.71*	*41.27*	*10.5*	*8.6*
	16	*40.74*	*44.68*	*36.23*	*39.85*	*8.8*	*9.1*

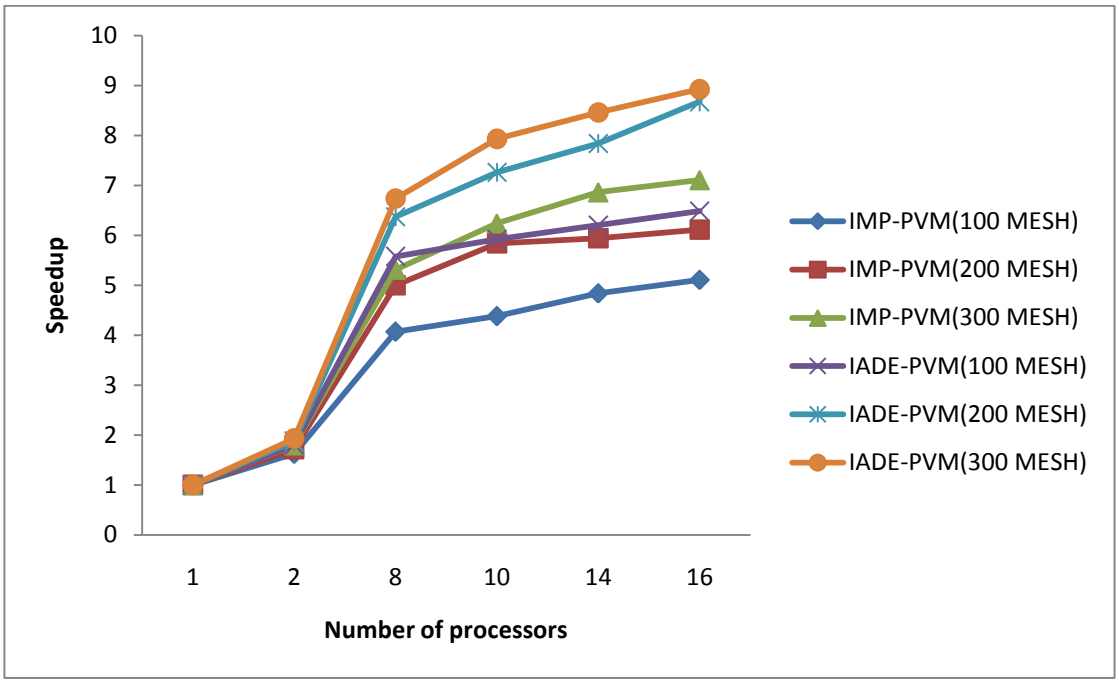

Fig. 6.24 1-D Telegraph Speedup of various mesh sizes with PVM

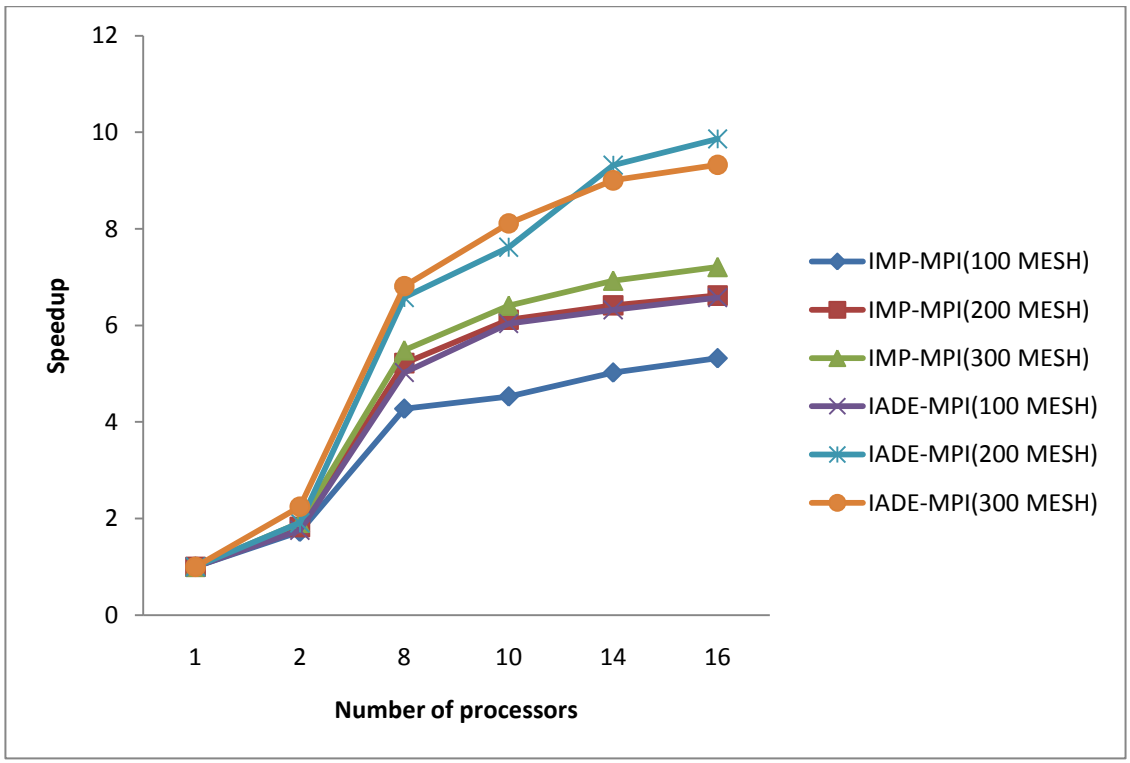

Fig. 6.25 1-D Telegraph Speedup of various mesh sizes with MPI

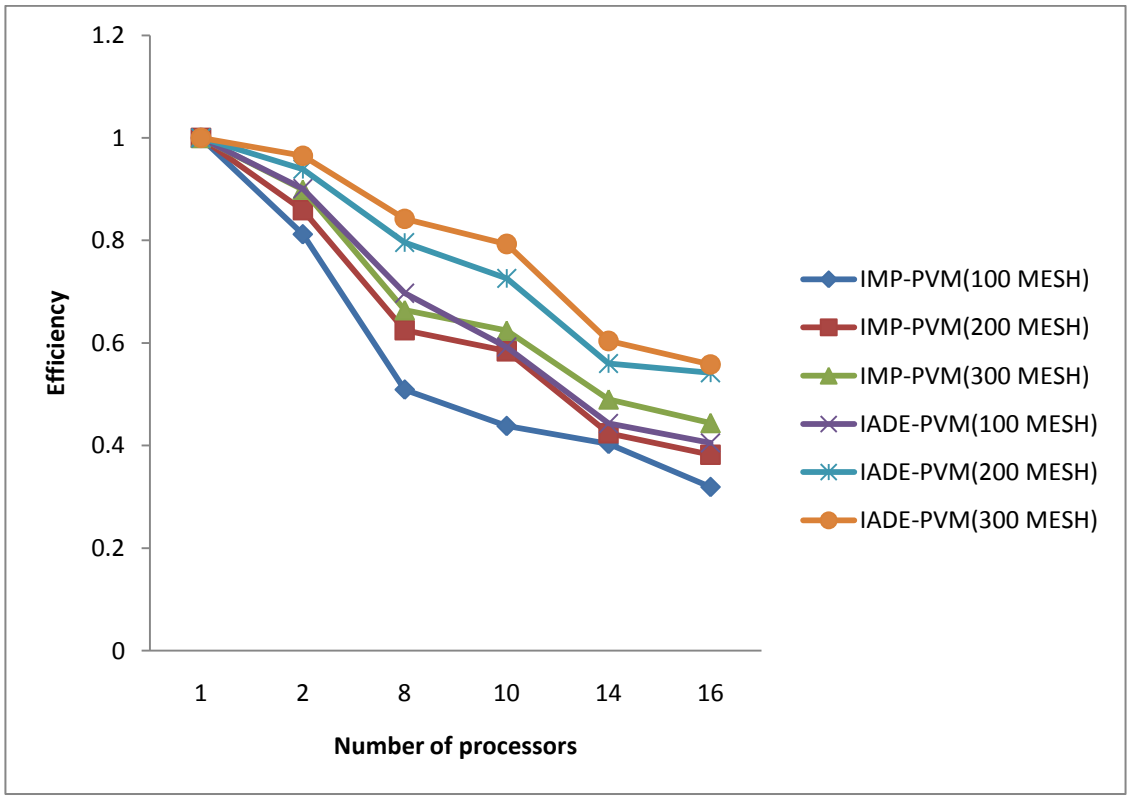

Fig. 6.26 1-D Telegraph Efficiency of various mesh sizes with PVM

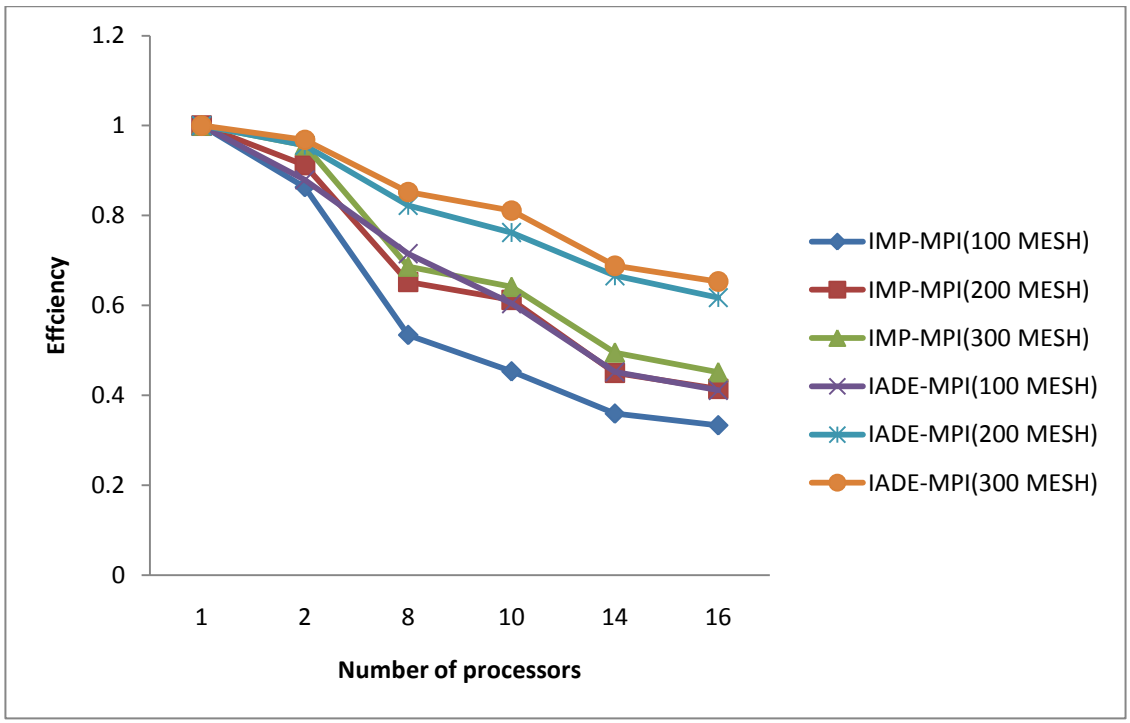

Fig. 6.27 1-D Telegraph Efficiency of various mesh sizes with MPI

6.8 Sequential Results for 2-D Telegraph Equation

We evaluate the various schemes discussed in Chapter 4 on the Telegraphic Equation with the values of the physical properties in our test case chosen in such a way that LC, *(RC + GL)*, are equal to one. The numerical solutions are graphed in Fig. 6.28 for average of absolute errors and in Fig. 6.29 for the root mean squares (rms), with the values $\lambda = 0.5, \Delta t = 0.05, \Delta x = 0.5, \Delta y = 0.5, r = 0.5,$ and $\varepsilon = 10^{-4}$.

According to (Mohanty et. al. (1995)), Eq. (4.4) is considered with the boundary condition and initial condition posed as:

$$\frac{\partial^2 v}{\partial t^2} + a\frac{\partial v}{\partial t} = b\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad 0 \le x \le 1,\ 0 \le y \le 1,\ t > 0$$

$$\left.\begin{aligned} v(0, y, t) &= 0 \\ v(1, y, t) &= 100 \\ v(x, 0, t) &= 0 \\ v(x, 1, t) &= 100 \end{aligned}\right\} \quad t \ge 0 \qquad (6.13)$$

$$v(x, y, 0) = e^{xy}, \tag{6.14}$$

Exact solution: $v(x, y, t) = e^{xy-t}$

Table 6.25: The absolute errors of the schemes solution to the 2-D telegraph equation $\lambda = 0.5,\ \Delta t = 0.05,\ \Delta x = 0.5,\ \Delta y = 0.5,\ r = 0.5,\ eps = 10^{-4}$.

t	*method*	*0.5*	*1.0*	*1.5*	*2.0*
0.5	***ADI***	*0.2341873E-4*	*0.1934786E-4*	*0.1637992E-4*	*0.1466718E-4*
	IADE-DY	*0.3452816E-5*	*0.2134631E-5*	*0.1946342E-5*	*0.1784694E-5*
	MF-DS	*0.5568412E-6*	*0.2675781E-6*	*0.2045187E-6*	*0.1845328E-6*
1.0	***ADI***	*0.6876821E-3*	*0.5284172E-3*	*0.8246118E-3*	*0.7124638E-3*
	IADE-DY	*0.4790324E-4*	*0.3467814E-4*	*0.5841241E-3*	*0.5249378E-4*
	MF-DS	*0.1125567E-5*	*0.1011284E-5*	*0.5784373E-6*	*0.4125481E-5*
1.5	***ADI***	*0.6978924E-3*	*0.8246283E-3*	*0.7921183E-3*	*0.5621836E-3*
	IADE-DY	*0.5671284E-3*	*0.6794285E-3*	*0.6257897E-3*	*0.3478971E-3*
	MF-DY	*0.3895562E-4*	*0.6124373E-4*	*0.4825721E-4*	*0.2568478E-4*
2.0	***ADI***	*0.5256482E-2*	*0.5086438E-2*	*0.4123464E-2*	*0.7984769E-2*
	IADE-DY	*0.3784672E-2*	*0.3467942E-2*	*0.2452894E-2*	*0.5947825E-2*
	MF-DY	*0.1845857E-3*	*0.1457824E-3*	*0.1257824E-3*	*0.5486392E-3*

Table 6.26: The average of absolute errors, root mean squares of 2-D telegraph $\lambda = 0.5,\ \Delta t = 0.05,\ \Delta x = 0.5,\ \Delta y = 0.5,\ r = 0.5,\ eps = 10^{-4}$.

		Av. of absolute errors	***Root mean square***
t	*method*		
0.5	*ADI*	*1.84534225E-5*	*3.515511374E-10*
	IADE-DY	*2.32962075E-6*	*2.345196769E-11*
	MF-DS	*3.03367711E-7*	*1.143876036E-13*
1.0	*ADI*	*6.88293725E-4*	*1.218937027E-5*
	IADE-DY	*1.79799815E-4*	*8.686346373E-8*
	MF-DS	*1.71019232E-6*	*4.910944898E-12*
1.5	*ADI*	*7.19205651E-4*	*5.276418591E-7*
	IADE-DY	*5.55060921E-4*	*3.239757124E-7*
	MF-DY	*4.35353352E-5*	*2.064200259E-9*
2.0	*ADI*	*5.61278825E-3*	*3.356548647E-5*
	IADE-DY	*3.91333325E-3*	*1.693591877E-5*
	MF-DY	*2.51197425E-4*	*3.721505727E-7*

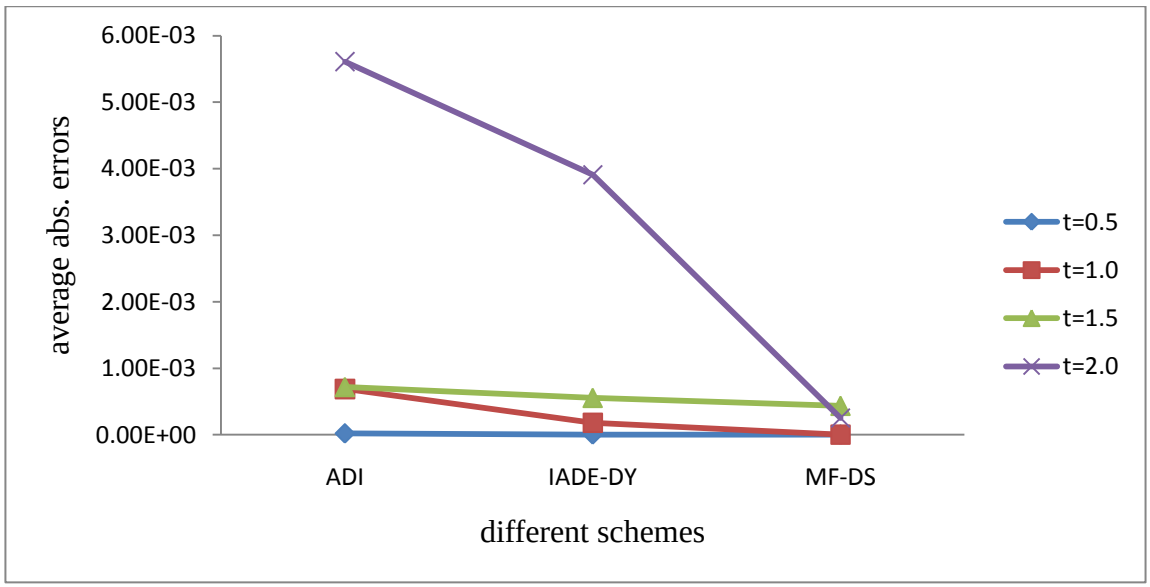

Fig. 6.28 average abs. errors for 2-D Telegraph Equation

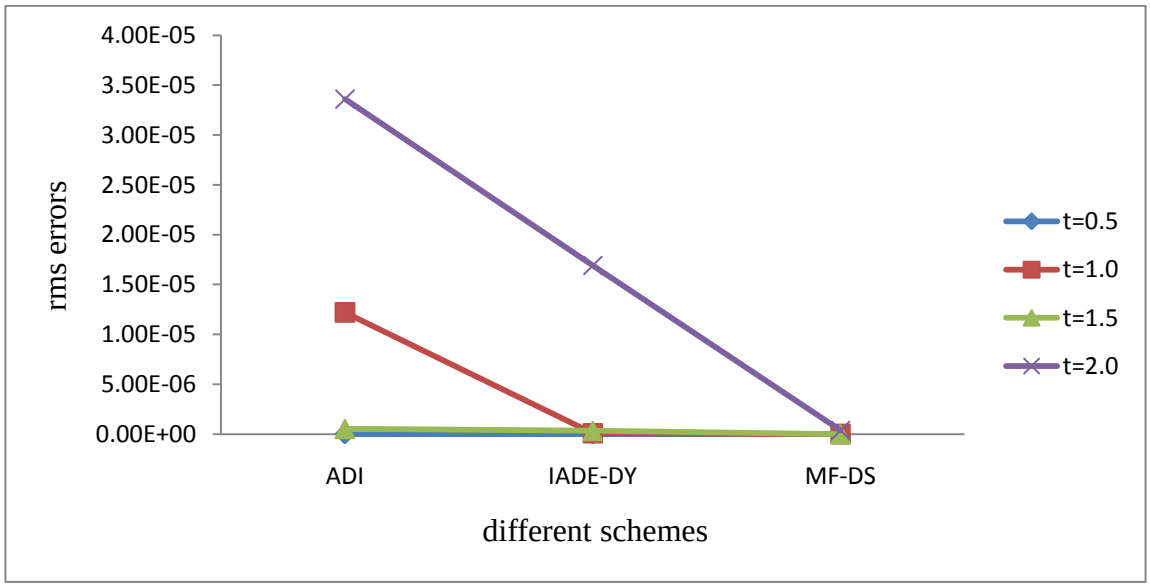

Fig. 6.29 rms errors for 2-D Telegraph Equation

6.9 Parallel Results for 2-D Telegraph Equation

The speedup and efficiency obtained for various schemes with 300 x 300 grid sizes for PVM and MPI implementation are listed in the Table below for 2-D Telegraph case.

Table 6.27: 300 x 300 meshes using three schemes

		PVM					*MPI*		
Scheme	***N***	T_w	T_m	T_{sd}	S_{par}	E_{par}	***N***	S_{par}	E_{par}
	1	*699.1*	*31.4*	*9.3*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*621.6*	*31.2*	*9.1*	*1.882*	*0.941*	*2*	*1.896*	*0.948*
ADI	*8*	*496.9*	*31.2*	*9.1*	*6.976*	*0.872*	*8*	*7.080*	*0.885*
	10	*438.5*	*31.2*	*9.1*	*8.48*	*0.848*	*10*	*8.550*	*0.855*
	14	*325.4*	*31.2*	*9.1*	*11.214*	*0.801*	*14*	*11.452*	*0.818*
	16	*299.3*	*31.2*	*9.1*	*12.624*	*0.789*	*16*	*12.752*	*0.797*
	1	*793.8*	*35.6*	*10.7*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*658.4*	*35.5*	*10.4*	*1.906*	*0.953*	*2*	*1.922*	*0.961*
IADE	*8*	*535.7*	*35.5*	*10.4*	*7.048*	*0.881*	*8*	*7.144*	*0.893*
	10	*499.2*	*35.5*	*10.4*	*8.560*	*0.856*	*10*	*8.620*	*0.862*
	14	*321.6*	*35.5*	*10.4*	*11.368*	*0.812*	*14*	*11.606*	*0.829*
	16	*293.7*	*35.5*	*10.4*	*12.72*	*0.795*	*16*	*13.072*	*0.817*
	1	*828.4*	*41.8*	*11.5*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*696.9*	*41.6*	*11.4*	*1.924*	*0.962*	*2*	*1.946*	*0.973*
MF-DS	*8*	*562.5*	*41.6*	*11.4*	*7.192*	*0.899*	*8*	*7.312*	*0.914*
	10	*512.3*	*41.6*	*11.4*	*8.750*	*0.875*	*10*	*8.81*	*0.881*
	14	*346.1*	*41.6*	*11.4*	*11.676*	*0.834*	*14*	*11.774*	*0.841*
	16	*311.8*	*41.6*	*11.4*	*13.136*	*0.821*	*16*	*13.36*	*0.835*

Table 6.28 Effectiveness of the various schemes with PVM and MPI for 300 x 300 mesh size (2-D Telegraphic case)

	N	***PVM T(s)***	L_n	***MPI T(s)***	L_n
	1	*1269*	*0.08*	*1148*	*0.09*
	2	*674.3*	*0.14*	*605.5*	*0.16*
ADI	*8*	*181.9*	*0.48*	*162.2*	*0.55*
	10	*149.7*	*0.57*	*134.3*	*0.64*
	14	*113.2*	*0.71*	*100.2*	*0.82*
	16	*60.22*	*0.78*	*90.03*	*0.89*
	1	*1482*	*0.07*	*1378*	*0.07*
IADE	*2*	*777.5*	*0.12*	*716.9*	*0.13*
	8	*210.3*	*0.42*	*192.9*	*0.46*
	10	*173.1*	*0.49*	*159.9*	*0.54*
	14	*130.4*	*0.62*	*118.7*	*0.70*

	16	*116.5*	*0.68*	*105.4*	*0.76*
MF-DS	*1*	*1695*	*0.06*	*1528*	*0.07*
	2	*880.9*	*0.11*	*785.2*	*0.12*
	8	*235.7*	*0.38*	*208.9*	*0.44*
	10	*193.7*	*0.45*	*173.4*	*0.51*
	14	*145.2*	*0.57*	*129.8*	*0.65*
	16	*75.73*	*0.64*	*114.4*	*0.73*

Table 6.29 Performance improvement of different schemes for 300x300 mesh size on 2-D Telegraphic Equation

N	***PVM***	***Time(s)***		***%***	***%***	***MPI***	***Time(s)***		***%***	***%***
	ADI	***IADE***	***MF-DS***	***ADI+ IADE***	***IADE+ MF-DS***	***ADI***	***IADE***	***MF-DS***	***ADI+ IADE***	***IADE+ MF-DS***
1	*126.9*	*1482*	*1695*	*14.4*	*12.6*	*1148*	*1378*	*1528*	*16.7*	*9.8*
2	*674.3*	*777.5*	*880.9*	*13.3*	*11.7*	*605.5*	*716.9*	*785.2*	*15.5*	*8.7*
8	*181.9*	*210.3*	*235.7*	*13.5*	*10.8*	*162.2*	*192.9*	*208.9*	*15.9*	*7.7*
10	*149.7*	*173.1*	*193.7*	*13.6*	*10.6*	*134.3*	*159.9*	*173.4*	*16.0*	*7.8*
14	*113.2*	*130.4*	*145.2*	*13.2*	*10.2*	*100.2*	*118.7*	*129.8*	*15.6*	*8.5*
16	*100.5*	*116.5*	*129.0*	*13.7*	*9.7*	*90.03*	*105.42*	*114.4*	*14.6*	*7.8*

Table 6.30 Slave computational time for 100 iterations as a function of block numbers for PVM (2-D Telegraphic case)

NI x NJ	***B = 50***	***B = 100***	***B = 200***
100 x 100	*196.8*	*243.7*	*308.2*
200 x 200	*584.3*	*698.1*	*719.9*
300 x 300	*1641.7*	*2017.5*	*2572.1*

Table 6.31 Slave computational time for 100 iterations as a function of block numbers for MPI (2-D Telegraph case)

NI x NJ	***B = 50***	***B = 100***	***B = 200***
100 x 100	*169.3*	*217.6*	*293.5*
200 x 200	*499.6*	*578.7*	*698.4*
300 x 300	*1599.8*	*1962.9*	*2132.7*

Table 6.32 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 100 x 100 PVM (2-D Telegraphic case)

N	***B = 50***	***B = 100***	***B = 200***
2	*4421*	*4873*	*3184*
8	*4923*	*5214*	*3871*
10	*5118*	*5573*	*4126*
14	*5361*	*5571*	*4261*
16	*5361*	*5824*	*4328*

Table 6.33 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 100 x 100
MPI (2-D Telegraphic case)

N	***B = 50***	***B = 100***	***B = 200***
2	*4118*	*4610*	*2834*
8	*4583*	*4924*	*3118*
10	*4977*	*5328*	*3722*
14	*5185*	*5418*	*3971*
16	*5185*	*5421*	*4113*

Table 6.34 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 200 x 200 PVM (2-D Telegraphic case)

N	***B = 50***	***B = 100***	***B = 200***
2	*4962*	*5013*	*3628*
8	*5014*	*5338*	*3921*
10	*5311*	*5621*	*4218*
14	*5428*	*5681*	*4329*
16	*5428*	*5681*	*4478*

Table 6.35 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 200 x 200
MPI (2-D Telegraphic case)

N	***B = 50***	***B = 100***	***B = 200***
2	*4721*	*4816*	*3347*
8	*4926*	*5211*	*3767*
10	*5183*	*5487*	*3863*
14	*5219*	*5513*	*4108*
16	*5303*	*5513*	*4215*

Table 6.36 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 300 x 300 PVM (2-D Telegraphic case)

N	***B = 50***	***B = 100***	***B = 200***
2	*5824*	*5938*	*3729*
8	*6273*	*6397*	*3996*
10	*6318*	*6413*	*4321*
14	*6417*	*6528*	*4443*
16	*6417*	*6528*	*4527*

Table 6.37 The number of iteration to achieve a given tolerance of 10^{-4} for a grid of 300 x 300
MPI (2-D Telegraphic case)

N	***B = 50***	***B = 100***	***B = 200***
2	*5318*	*5324*	*3581*
8	*5821*	*5538*	*3897*
10	*5993*	*6119*	*3976*
14	*6086*	*6236*	*4274*
16	*6127*	*6312*	*4308*

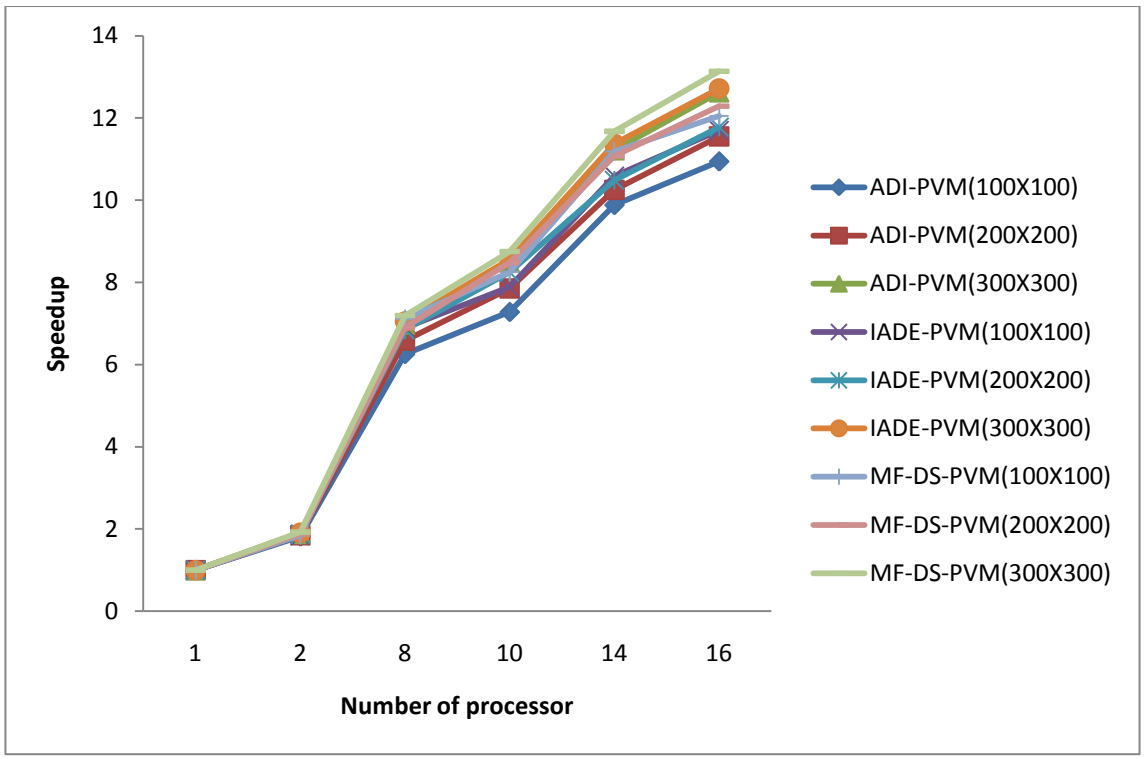

Fig. 6.30 Speedup of various mesh sizes with PVM for 2-D Telegraph

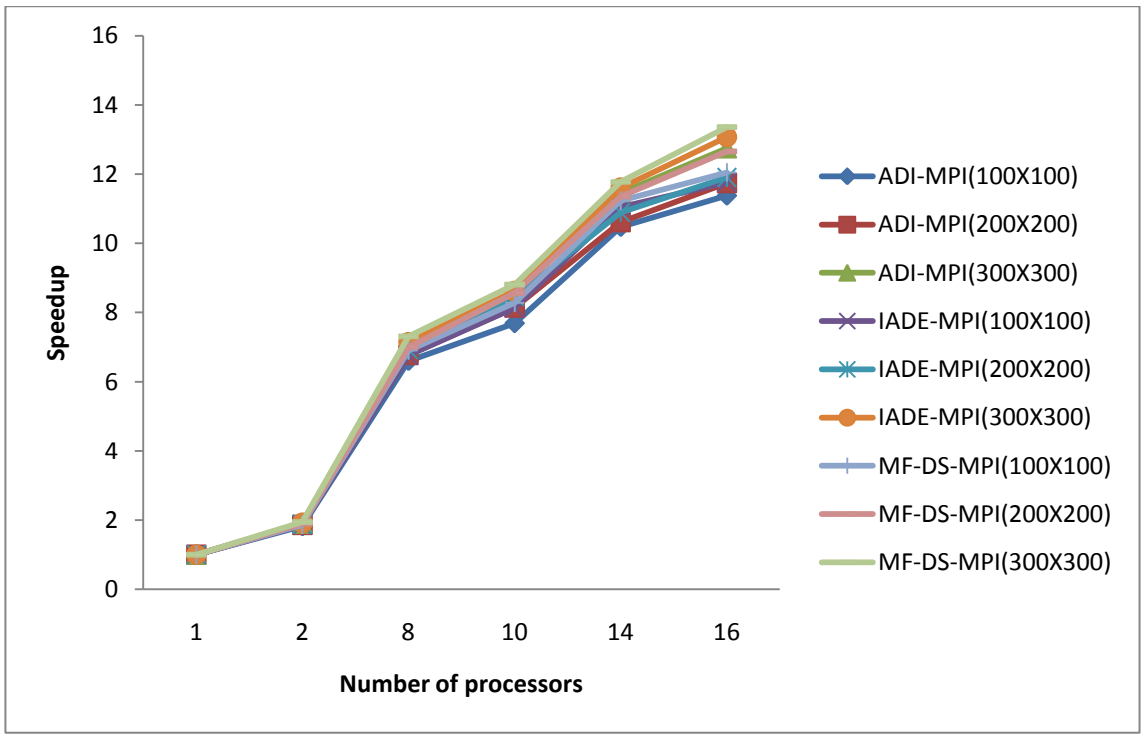

Fig. 6.31 Speedup of various mesh sizes with MPI for 2-D Telegraph

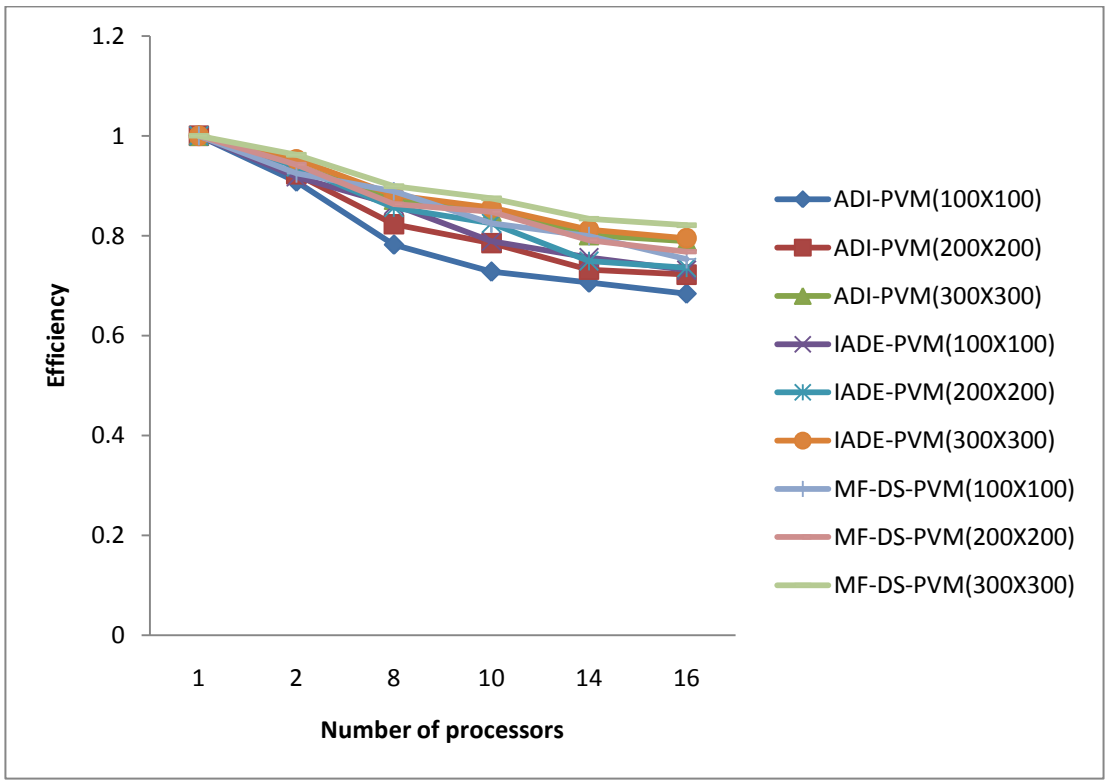

Fig. 6.32 Efficiency of various mesh sizes with PVM for 2-D Telegraph

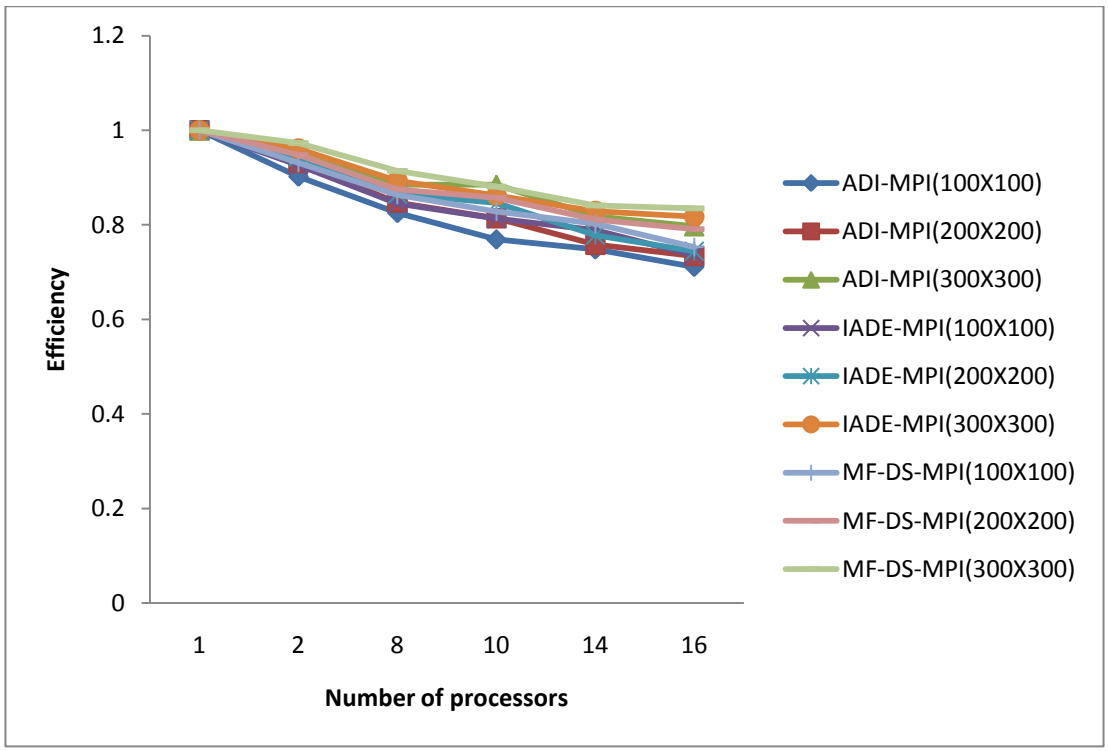

Fig. 6.33 Efficiency of various mesh sizes with MPI for 2-D Telegraph

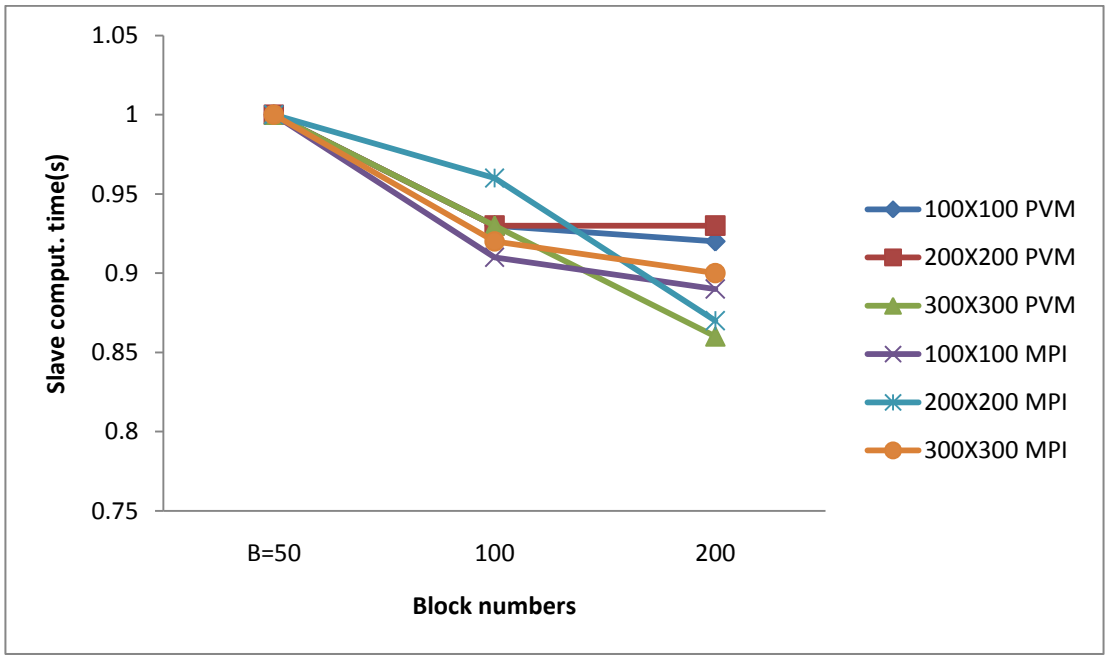

Fig.6.34 Slave computational time as function of block numbers for PVM and MPI

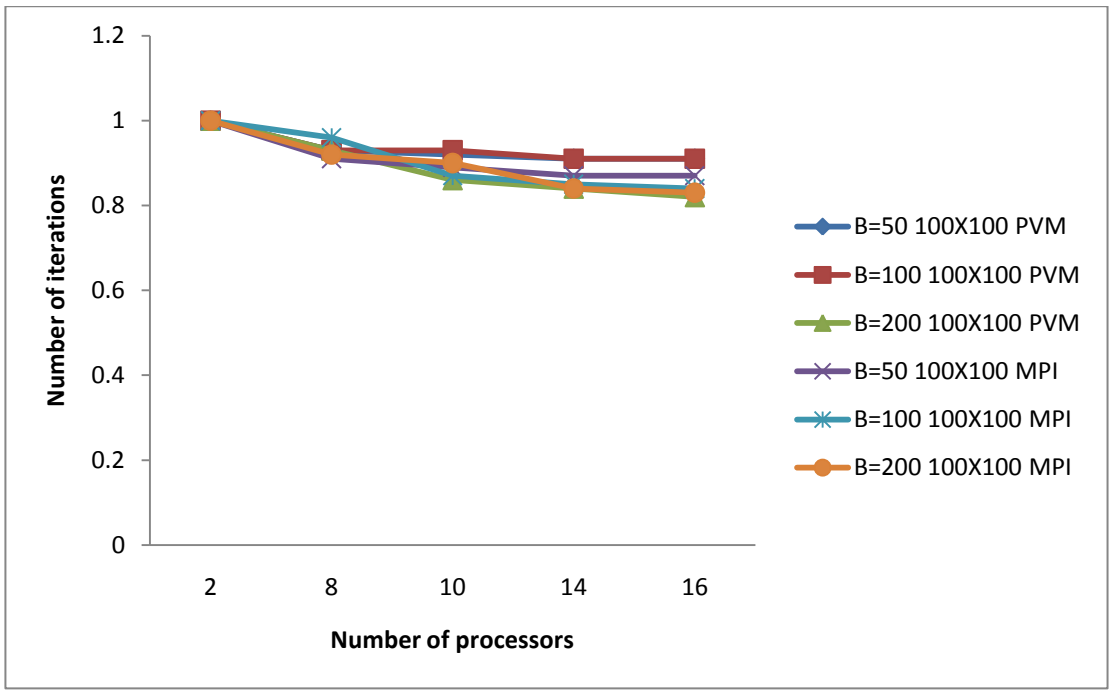

Fig.6.35. The number of iterations for different block sizes for 100 x 100 PVM & MPI

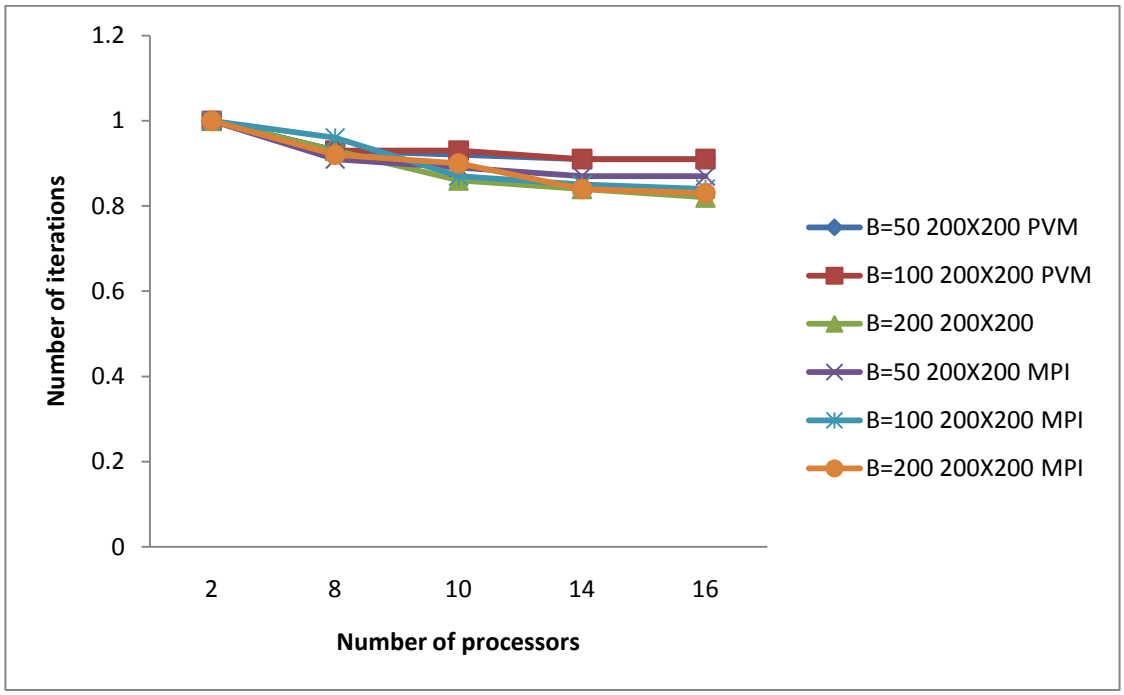

Fig.6.36. The number of iterations for different block sizes for 200x200 PVM & MPI

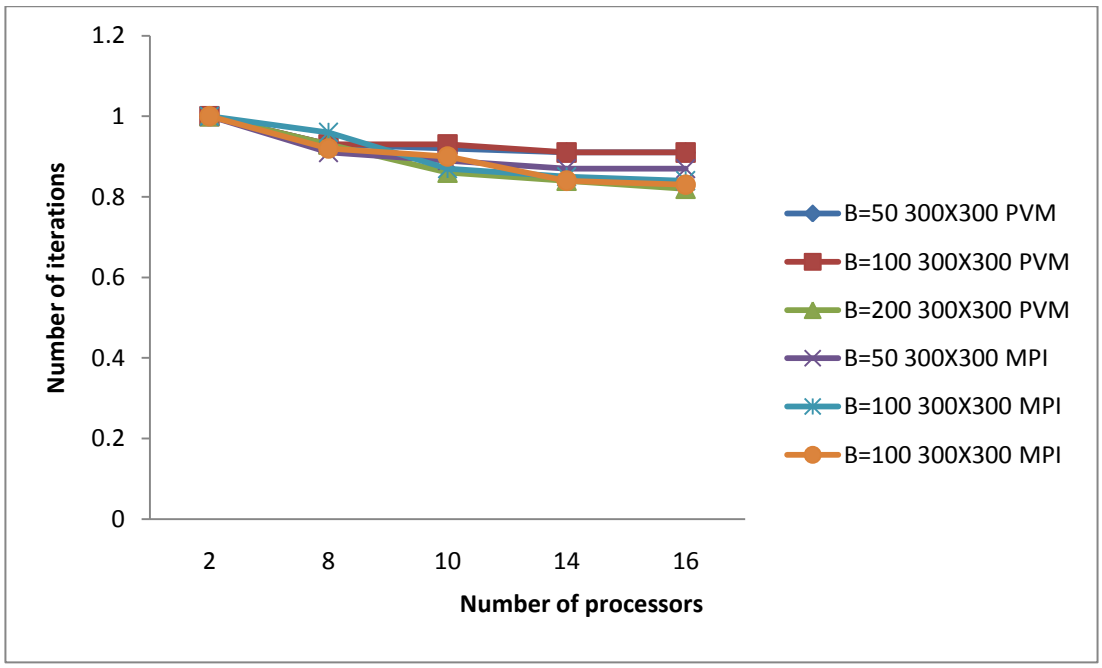

Fig.6.37 The number of iterations for different block sizes for 200x200 PVM & MPI

6.10 Sequential Results for 3-D Telegraph Equation

The 3-D ADI method outlined in this thesis is used to solve the proposed Eq. (5.1) in the region $0 < x, y, z < 1, t > 0$ whose exact solution is given by $v = e^{-t} SinxSinySinz$. The initial and boundary conditions and right hand side function can be obtained using the exact solution as a test procedure. Note that, the proposed difference scheme is a three level implicit scheme. To start any calculation, it is necessary to know the values of v at $t = k$ as well as $t = 0$. The accuracy of the solution at the grid points is determined by the root mean square (rms) errors.

Table 6.38: The RMS errors when $\lambda = 1.6$

	$R_x = 0.25, R_y = 0.5, R_z = 0.75$		$R_x = R_y = R_z = 1.0$		$R_x = 0.75, R_y = 0.5, R_z = 2.5$	
h	*t = 1.0*	*t = 2.0*	*t = 1.0*	*t = 2.0*	*t = 1.0*	*t = 2.0*
1/16	*0.3086(-03)*	*0.1311(-03)*	*0.1091(-02)*	*0.4586(-03)*	*0.3627(-03)*	*0.2430(-03)*
1/32	*0.6673(-04)*	*0.2714(-04)*	*0.1453(-03)*	*0.5738(-04)*	*0.7866(-04)*	*0.5010(-04)*
1/64	*0.1587(-04)*	*0.6384(-05)*	*0.2841(-04)*	*0.1007(-04)*	*0.1876(-04)*	*0.8802(-05)*

Table 6.39: The RMS errors when $\lambda = 3.2$

	$R_x = 0.5, R_y = 0.75, R_z = 1.0$		$R_x = R_y = R_z = 0.5$		$R_x = 0.5, R_y = 0.75, R_z = 1.0$	
h	*t = 1.0*	*t = 2.0*	*t = 1.0*	*t = 2.0*	*t = 1.0*	*t = 2.0*
1/16	*0.4100(-02)*	*0.1184(-02)*	*0.1853(-02)*	*0.1185(-02)*	*0.1189(-02)*	*0.6224(-03)*
1/32	*0.3068(-03)*	*0.1128(-03)*	*0.3403(-03)*	*0.1762(-03)*	*0.3061(-03)*	*0.1222(-03)*
1/64	*0.4792(-04)*	*0.1763(-04)*	*0.7473(-04)*	*0.3666(-04)*	*0.7441(-04)*	*0.2829(-04)*

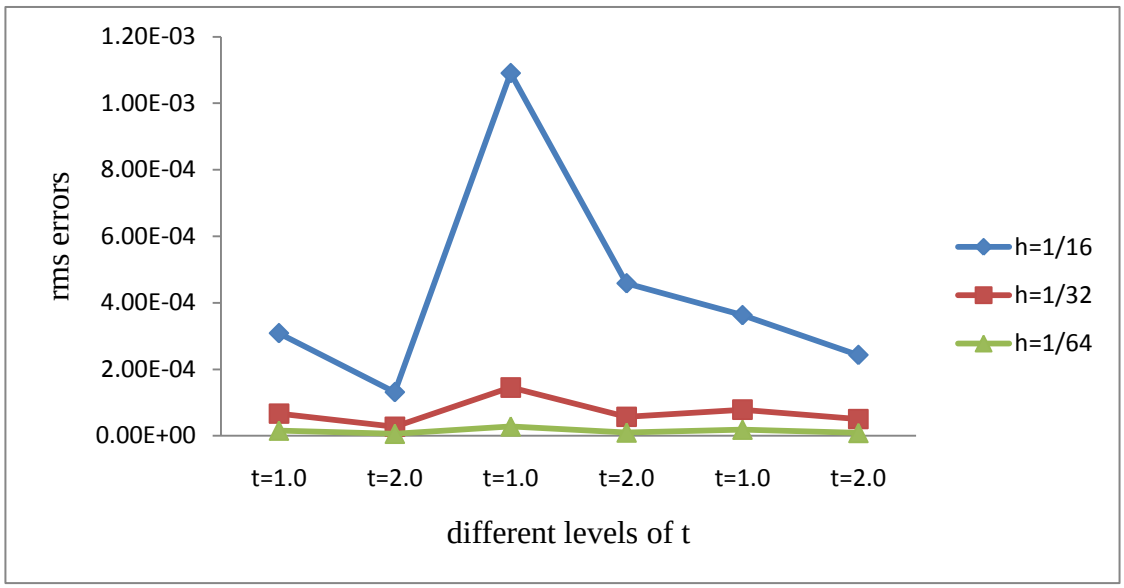

Fig. 6.38 rms errors for $\lambda = 1.6$

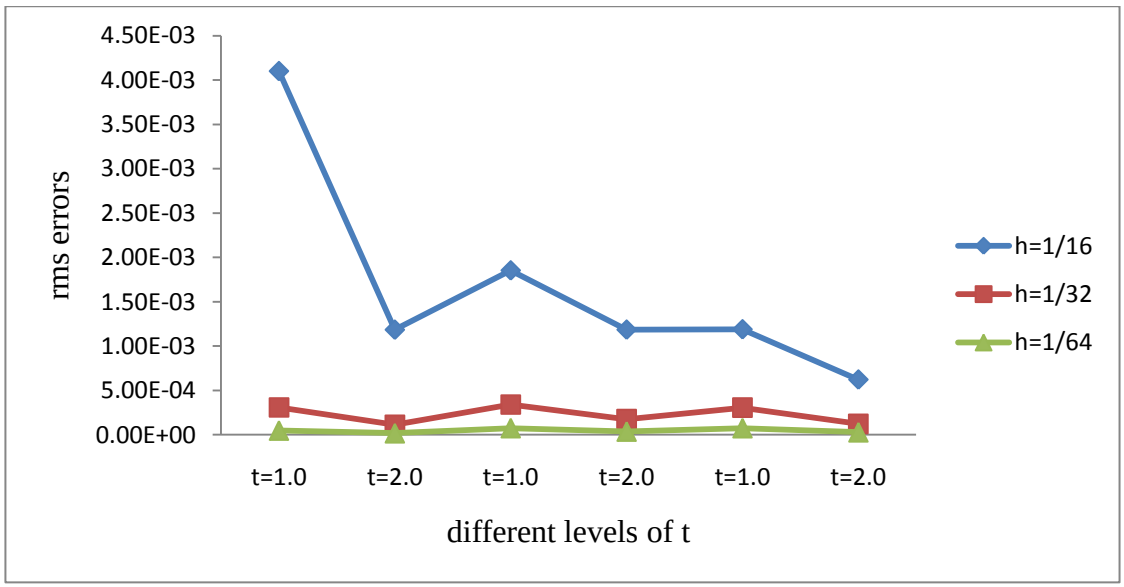

Fig.6.39 rms errors when $\lambda = 3.2$

Results for the problem using the 3-D ADI method showing the root mean square errors for $\lambda = 1.6$ and 3.2, are given in Fig. 6.38 and Fig. 6.39, respectively.

6.11 Parallel Results for 3-D Telegraph Equation

The speedup and efficiency obtained for various schemes with 300 x 300 grid sizes for PVM and MPI implementation are listed in the Table and figures below for 3-D Telegraph case.

Table 6.40: 300 x 300 x 300 meshes using three schemes

			PVM					***MPI***	
Scheme	***N***	T_w	T_m	T_{sd}	S_{par}	E_{par}	***N***	S_{par}	E_{par}
	1	*962.5*	*62.7*	*19.1*	*1.000*	*1.000*	*1*	*1.000*	*1.000*
	2	*896.9*	*62.6*	*19*	*1.988*	*0.994*	*2*	*1.996*	*0.998*
3D-ADI	*8*	*728.1*	*62.6*	*19*	*8.213*	*0.978*	*8*	*7.928*	*0.991*
	10	*596.6*	*62.6*	*19*	*9.26*	*0.926*	*10*	*9.84*	*0.984*
	14	*499.3*	*62.6*	*19*	*12.866*	*0.919*	*14*	*13.608*	*0.972*
	16	*387.4*	*62.6*	*19*	*14.496*	*0.906*	*16*	*15.296*	*0.956*

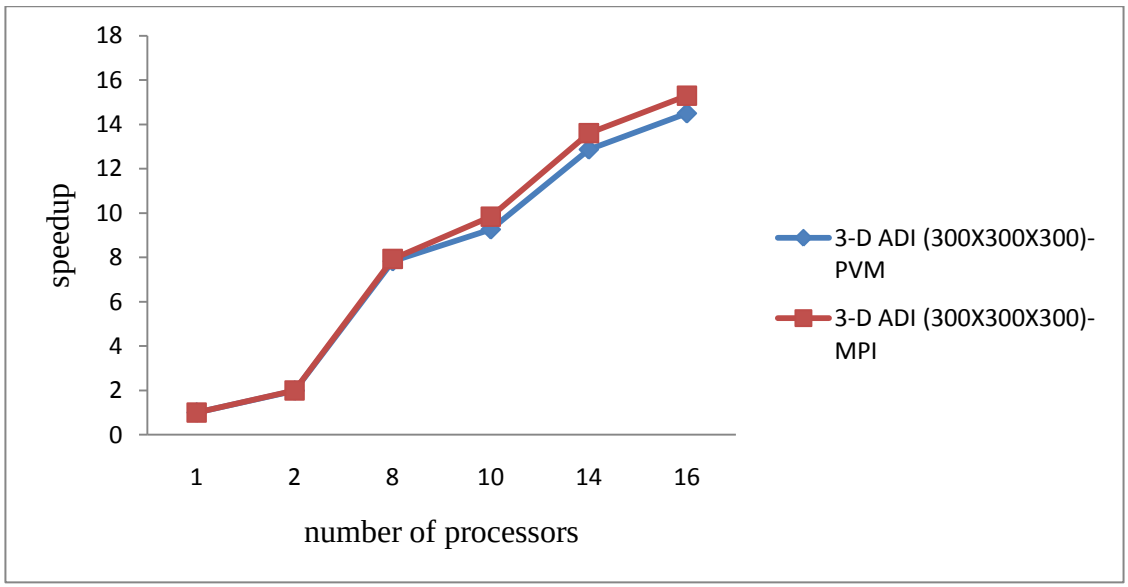

Fig. 6.40 Speedup for the 3-D ADI 300 x300x300 mesh sizes with PVM and MPI

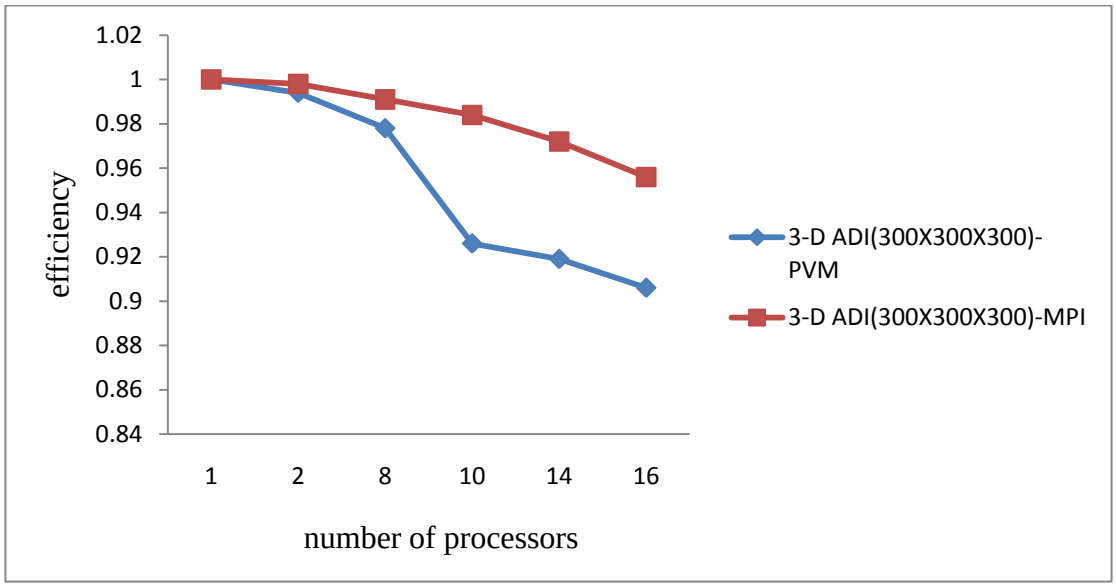

Fig 6.41 Efficiency for 3-D ADI (300x300x300) mesh sizes with PVM and MPI