CHAPTER 8

SUMMARY AND CONCLUSIONS

In this chapter, a summary of the present research will be given. We will then present a list of conclusions drawn from the research. Finally, a brief note on future work that can be done as a consequence of this research is presented.

8.1 Summary

1. Chapter 2 summarized the parallel machine models and interconnects as well as methodology for designing and building parallel programs. Basic notions of parallel computing placing emphasis on the differences between parallel and distributed computing has been presented. Various network topologies, the setting up of Armadillo cluster for the PVM implementation and the description of the Geranium Cadcam cluster were given. However, we went further to explain the parallel programming models used for this thesis work with the process creation, sending and receiving messages in parallel platform were established. We focused on features and differences of the considered software tools in this chapter.
2. We gave the finite-differences method for 1-D, 2-D Parabolic Equations and related numerical methods in Chapter 3. Here, we gave the various finite-difference schemes starting from the explicit method to implicit schemes then to the stationary methods and finally the alternating schemes on the equations. The fourth-order IADE and AGE scheme were developed for the 1-D Parabolic

Equation in this chapter. The double sweep two-stage IADE schemes on 2-D Parabolic Equation and the AGE class were formulated.

3. We also treated the finite difference schemes for 1-D and 2-D Bio-Heat Equations and related numerical methods in Chapter 4. The parallel performance analyses were fully treated in this chapter, given various parallel strategies for our design and the details of the implementation for all the dimensional differential problems . The effect of the DD, granularity and load balancing were also explained. Parallel performance measurement used in this thesis were stated and explained in this chapter.
4. The difference between the hyperbolic and parabolic equations were stated, finite-differences method for the 1-D Telegraphic Equation with the three level implicit schemes and the formulation of the IADE-MF scheme for 1-D Telegraph Equation accompanied with three different alternating schemes to solve the 2-D Telegraph Equation were formulated in Chapter 5. Here, we treated explicitly the parallel implementations of the algorithms and the data allocation in the algorithms. Numerical scheme for 3-D Telegraph Equation has been carried out using the ADI method. Stability analyses for the explicit and implicit schemes on the 2-D Telegraph Equation were mentioned. Similarly, stability analysis for 2-D and 3-D ADI scheme on 2-D and 3-D Telegraph Equations with their respective linear runtime were given in Chapter 5.
5. Sequential results for computing 1-D Parabolic and 1-D Bio-Heat Equations are presented in Chapter 6. In this same chapter, we have presented sequential and parallel results for 1-D Parabolic, 1-D Bio-Heat, 2-D Parabolic, 2-D Bio-Heat, 1-D, 2-D and 3-D Telegraph Equations. Comparisons of the various iterative schemes on the above differential equations show that the AGE class of method has extended range of efficiency and speedup for both MPI and PVM

implementation. Furthermore, the superiority of the sequential results of the AGE class of schemes indicate by the highest value of the temporal performance, accuracy and convergence on solving large-scale linear algebraic equations on a distributed memory multiprocessors platform superior for all number of processors. The parallel results in Chapter 6 show that the AGE class of schemes is more efficient algorithm than the others in comparison (see Table 6.3 and 6.6). The method is inherently explicit, the DD strategy is efficiently utilized straightforward to implement on our parallel platforms. The results in Chapter 6 present the numerical properties of the parallel solver both for MPI and PVM. The speedup and efficiency for various mesh sizes are plotted in figures. As expected, the execution time decreases with the increasing number of processors.

6. Chapter 7 has presented the discussions on the various results (sequential and parallel) of Chapter 6 implemented on 1-D, 2-D Parabolic and Bio-Heat, 1-D, 2-D and 3-D Telegraph Equation.

8.2 Conclusions

By reconstructing computation, we proposed the AGE class of method as an improved version of other class of iterative alternating methods. The proposed methods combine elements of numerical stability and parallel algorithm design without increasing computational costs. The performance analysis in Tables and Figures show that the AGE class has better parallelism and scalability. The schemes used in this thesis have been proved to be unconditionally stable with respect to initial values. The performance of the proposed schemes are examined experimentally (sequential and parallel). The stationary methods are not very scalable with respect to the problem size, and thus not recommended as compared to the class of AGE techniques. The accelerated convergence rate of the AGE class is appreciated in terms of effectiveness and performance improvement for solving large scale linear systems arising from discretizations. Stated below are summarized summary for the thesis:

1. We have discussed the parallel implementations of the various stationary and alternating iterative schemes on the Parabolic (example on the Bio-heat Equation) and Telegraph Equation using AGDS and Geranium Cadcam cluster. The implicit algorithms presented show significant improvement when implemented on the number of processors and the AGE class of methods further improves the results as observed in terms of speedup and efficiency. Our investigation was done on the SPMD programming model.
2. We have deduced the fact that implementation on MPI expresses communication slightly faster than PVM on a homogeneous platform. MPI is focused on message passing and explicitly states that resource management and the concept of a virtual machine are outside the scope of the MPI standard (Foster et al., (1998), Group et al., (1999)). Hence, MPI is a much richer communication method than PVM. We also notice that PVM provides only simple message passing. From our platform, we have come to the conclusions that as the mesh sizes increases for the various differential equations, the speedup of the MPI gets closer to linearity than the PVM and the efficiency for the MPI gets closer to unity than the PVM. Even as the processors increases, the MPI is seen to get slightly faster than PVM because of its many point to point and collective communication options than PVM.
3. The MPI has shown the ability to specify a logical communication topology. The improve performance of the MPI is due to the concepts of context and a

group of processes into a communicator and is suitable for non-blocking send used in our experiments.

4. We implemented the serial computations on one of the processors and calculated the total efficiencies. The efficiency for the different mesh sizes have been shown in figures and Tables. We found that the variation of the of the efficiency with processors are governed by the parallel efficiency, E_{par} and N_o / N_1. When the number of processor is small, the efficiency is far below unity. This is due to the influences of the domain decomposition efficiency. When the processor number becomes large, $E(n)$ decreases with processor due to the effect of both the convergence rate and the parallel efficiency.
5. The investigation on speedup shows that, speedup increases with the number of processors both for MPI and PVM. Their relative performance show that MPI perform slightly better than PVM. The convergence rate behavior N_o / N_1, the ratio of the iteration number for the best sequential CPU time on one processor and the iteration number for the parallel CPU time on n processors, describe the increase in the number of iteration required by the parallel method to achieve a specified accuracy, as compared to the serial method. This increase is caused mostly by the deterioration in the rate of convergence with increasing number of processors and sub-domains.
6. We emphasized the significant differences in the scheduling of the n independent tri-diagonal systems using various schemes to effect parallel performance on our platform with integrate memory and communication resources in an effective manner.
7. Our platform was designed to solve a wide variety of time-dependable PDE for various applications. We reported the study on speedup and efficiency for the scheduling of n independent tri-diagonal system of equations under the

computational environment of MPI and PVM. Hence, our approach shows conformity of the algorithms to scalability.

8. The sequential computational results of our experiments for the various schemes show stability and are accurate, agree with our parallel results for various schemes having elements of stability and are also accurate. For the stability analysis where Δt satisfies the stability criterion, sequential computational results prove that the AGE class of scheme is accurate. However, we found from the stability of the schemes in Chapter 4 and 5, that the proposed numerical scheme is unconditionally stable. The results from of the iterative schemes compares well.
9. Hence, we have come to some conclusions that: (1) the parallel efficiency is strongly dependant on the problem size and the number of processors, (2) a high parallel efficiency can be obtained with large scale problems, (3) the decomposition of domain greatly influences the performance of the parallel computation and (4) the convergence rate depends upon the number of processors for a given mesh size. On the basis of the current parallelization strategy, more sophisticated models can be attacked efficiently.

8.3 Future Work

Further work needs to be done on the 3-D implicit finite-difference algorithm for 3-D Telegraph Equation. Further improvement on alternating schemes to solve 3-D Telegraph Equation sequentially and on the parallel platform should be developed. In this thesis, we have shown stability analysis for the 3-D ADI scheme for 3-D Telegraph Equation that it is unconditionally stable. Discrete energy method to show stability should be investigated. The 3-D implicit algorithm has to be modified to accommodate temperature dependent thermal properties. Parallel implementation using the Bag of Task should be looked into to solve higher dimensional differential equation both in rectangular and polar coordinates.