CHAPTER 7

CONCLUSSION

7.1 Summary of the study

This study looks at the problem of detecting outliers in circular data and circular-circular regression models. The first part of the study focuses on understanding different distributions for circular variable and different regression models for bivariate circular data. The definition of various circular statistics such as circular mean and concentration parameter are presented. At the same time, we specifically choose the DM circular regression model proposed by Down and Mardia (2002) due to its interesting and useful properties. We employ the maximum likelihood estimation method to estimate the parameters of the model. The parameter estimates are further shown to be sensitive to the occurrence of outliers.

The second part of the study consider the occurrence of outliers in circular data generated from a family of α-stable wrapped distribution, that is, the wrapped normal distribution. Four numerical methods; the *C, M, D* and *A* statistics are considered to detect the outliers. Through a simulation study, in all cases considered, we show that the *A* statistic shows a better performance in terms of P1 and P5 than the other statistics. This is results are almost similar to that observed for the von Mises samples, except that the M statistics performs the best for the case of small sample size for the von Mises sample. The methods have been applied on the Kuantan wind direction data set and are able to detect observations further away from the rest as outliers.

Thirdly, we consider the problem of detecting outliers in the DM circular regression based on two different statistics; the *COVRATIO* statistic and *DMCEs* statistic. Since we employ the row deletion approach, such outliers are usually called influential observations. The cut-off points and the performance of both procedures are studied via simulation. For the *COVRATIO* statistic, it can be seen that the power is an increasing function of the concentration parameter κ and the power shows almost identical performance for larger sample size. As for the *DMCEs* statistic, the power of performance is an increasing function of concentration parameter κ while the sample size has a slight effect on the performance of the *DMCEs* statistic. The application of the procedures is illustrated by the ocean wind direction data and circadian biological rhythm data respectively. Again, the procedures are able to identify outlying pair of observations as possible influential observations.

7.2 Contributions

The study has contributed to circular data analysis in the following ways:

1. We show that the maximum likelihood estimates of the DM circular regression models are not robust toward the occurrence of outliers. Thus, it is important to identify outliers or influential observations for further investigation purposes.

2. Four outlier detection methods, namely the *C*, *M*, *D* and *A* statistics, have been successfully applied on data from the von Mises distributions. Here, we employ the statistics on data generated from the wrapped normal distribution. Via simulation, we generate a full table of cut-off points for the statistics specifically to be used for the wrapped normal cases. Almost similar results are observed except for the case of small sample size.

3. Using the row deletion approach, two statistics, namely the *COVRATIO* and *DMCEs* statistics are used to identify influential observations in the DM circular regression models. Via simulation, we generate a full table of cut-off points for both statistics. We

show that the statistics perform well in identifying the influential observations that exist in the data.

4. We apply the outlier detection methods on three different data sets. The methods are able to identify outlying observations in the data as outliers/influential observations in all cases.

7.3 Further Research

There are various possibilities for further research in this area. Some suggestions are given as follows:

i. To develop some effective procedures to detect multiple outliers as in circular regression models.

ii. To develop a new circular regression model that is more efficient and flexible for any circular data set.

We recognize that there are still many problems ready to be explored in circular statistics, and it is fascinating for statisticians to work on them.

Appendix 1

The generated data set of von Mises distribution

Case 1: $n = 20, \kappa = 2$

n	x	n	x
1	0.27677	11	5.45421
2	0.31527	12	0.43751
3	0.04937	13	5.52866
4	0.26373	14	0.94288
5	0.26016	15	0.35068
6	5.90887	16	5.54507
7	5.93816	17	0.84449
8	5.86406	18	6.01005
9	0.55044	19	0.32158
10	0.11379	20	5.88506

Case 2: $n = 20, \kappa = 5$

n	x	n	x
1	0.37612	11	0.13217
2	0.27021	12	5.35907
3	0.49733	13	5.16572
4	0.04846	14	6.00333
5	1.61604	15	6.10049
6	0.59267	16	5.01609
7	0.80265	17	0.53004
8	6.22198	18	0.82851
9	0.45967	19	6.09061
10	0.15739	20	1.42296

Case 3: $n = 20, \kappa = 10$

n	x	n	x
1	6.26330	11	0.04256
2	0.41279	12	0.27758
3	5.73247	13	0.67641
4	6.21777	14	6.03838
5	0.24974	15	0.29350
6	0.52480	16	0.34341
7	0.33469	17	0.15846
8	5.83638	18	5.82909
9	6.05267	19	0.34518
10	0.34138	20	0.05527

Appendix 2

The generated data set of wrapped Cauchy distribution

Case 1: $n = 20, \rho = 0.3$

n	X	n	X
1	5.64773755	11	3.57198417
2	6.19727632	12	0.45371010
3	6.05139250	13	0.79324420
4	0.17675869	14	1.19959351
5	4.95961113	15	1.15162695
6	4.94056895	16	0.06697139
7	5.80206673	17	6.13296379
8	5.14497244	18	0.48517796
9	6.21363881	19	6.14882225
10	5.97266512	20	0.55533236

Case2: $n = 20, \rho = 0.7$

n	x	n	x
1	6.2753531	11	0.7968829
2	6.2090926	12	2.4934442
3	4.9926923	13	5.3565528
4	3.9936748	14	5.1605534
5	5.6464979	15	6.1481802
6	0.1497620	16	4.8767233
7	4.0494362	17	4.9602459
8	4.8425346	18	5.4459013
9	5.7440898	19	4.9717175
10	5.9875046	20	1.2846745

Case 3: $n = 20, \rho = 0.975$

n	X	n	x
1	0.0019690	11	0.0900789
2	0.0524310	12	0.1008974
3	6.2825100	13	0.0553334
4	6.2700811	14	6.2504654
5	6.2175691	15	0.0038638
6	6.2517124	16	6.2809610
7	0.0058315	17	0.0291512
8	0.0365534	18	6.1667036
9	0.0534329	19	6.1290523
10	0.1762002	20	6.2735171

Appendix 3

The generated data set of wrapped normal distribution

Case 1: $n = 20$, $\rho = 0.3$

n	x	*n*	x
1	2.1970050	11	5.2223550
2	4.7821523	12	4.9634439
3	1.8074489	13	5.8328094
4	5.7977057	14	5.2760157
5	0.7980120	15	0.4653396
6	0.2400745	16	4.7871970
7	4.8340267	17	0.6756791
8	2.6109068	18	5.3259396
9	2.3135700	19	3.3214734
10	4.0284502	20	3.8777938

Case 2: $n = 20$, $\rho = 0.7$

n	x	*n*	x
1	5.35034436	11	0.64953686
2	5.81097030	12	0.55604486
3	5.68497181	13	0.19884046
4	6.05226770	14	5.64729733
5	0.02705889	15	0.71727671
6	6.22846619	16	0.01010159
7	6.00012016	17	0.87859142
8	5.27622679	18	0.22938378
9	6.04889647	19	1.21272681
10	6.14190795	20	0.99182426

Case 3: $n = 20$, $\rho = 0.975$

n	x	*n*	x
1	6.03453374	11	6.03475875
2	5.90301325	12	0.02616267
3	0.29366280	13	0.08133514
4	0.35915407	14	6.23881168
5	6.23707336	15	0.23408141
6	5.97558892	16	6.25658815
7	0.01176111	17	6.05104388
8	0.36666521	18	6.21995802
9	0.05465904	19	6.04755979
10	6.06079503	20	0.14974410

Appendix 4

The relation of concentration parameter κ and ρ

ρ	κ	ρ	κ
0.001	0.002	0.600	1.520
0.010	0.020	0.650	1.740
0.050	0.100	0.700	2.010
0.100	0.201	0.750	2.370
0.150	0.303	0.800	2.870
0.200	0.408	0.850	3.740
0.250	0.516	0.900	5.300
0.300	0.629	0.950	10.300
0.350	0.748	0.975	20.300
0.400	0.874	0.990	50.300
0.450	1.010	0.995	100.000
0.500	1.160	0.999	500.000
0.550	1.330	1.000	1000.000

Appendix 5

Ocean wind direction data

HF	AB	TimeHF	TimeAB	HF	AB	TimeHF	TimeAB
0.79	1.154	1.615	1.618	1.325	1.693	2.948	2.951
0.715	1.154	1.656	1.66	1.103	1.325	2.99	2.993
0.975	1.007	1.698	1.701	6.131	6.062	3.406	3.41
0.97	1.178	1.74	1.743	5.719	5.988	3.448	3.451
0.993	0.859	1.781	1.785	5.713	5.988	3.49	3.493
0.902	1.007	1.823	1.826	5.487	5.498	3.531	3.535
0.943	1.056	1.837	1.847	5.742	5.276	3.573	3.576
1.728	1.4	2.406	2.41	5.728	5.302	3.615	3.618
1.445	1.497	2.448	2.451	5.61	5.62	3.656	3.66
1.679	1.693	2.49	2.493	5.463	5.744	3.698	3.701
1.703	2.012	2.531	2.535	5.427	5.644	3.74	3.743
1.862	1.792	2.573	2.576	5.418	5.669	3.781	3.785
1.726	1.766	2.615	2.618	5.406	5.744	3.823	3.826
1.79	1.669	2.656	2.66	5.472	5.547	3.865	3.868
1.831	1.4	2.698	2.701	5.401	5.498	3.906	3.91
1.719	1.4	2.726	2.743	5.42	5.4	3.948	3.951
1.646	1.375	2.781	2.785	5.276	5.449	3.99	3.993
1.622	1.056	2.823	2.826	1.728	4.786	4.031	4.035
1.342	1.178	2.865	2.868	5.512	5.449	4.406	4.41
1.176	1.276	2.906	2.91	5.486	5.178	4.448	4.451

HF	AB	TimeHF	TimeAB	HF	AB	TimeHF	TimeAB
5.444	5.62	4.49	4.493	5.46	5.351	10.531	10.535
5.518	5.13	4.531	4.535	5.364	5.571	10.573	10.576
5.505	4.541	4.559	4.576	5.444	5.376	10.615	10.618
5.558	5.571	9.573	9.576	5.35	5.327	10.656	10.66
5.42	5.62	9.615	9.618	5.202	4.983	10.698	10.701
5.398	5.473	9.656	9.66	5.161	4.786	10.74	10.743
5.334	5.327	9.698	9.701	5.062	4.908	10.781	10.785
5.418	4.835	9.781	9.785	5.145	4.517	10.823	10.826
5.418	5.032	9.823	9.826	5.212	4.835	10.865	10.868
5.338	5.842	9.892	9.91	5.238	4.417	10.906	10.91
5.47	5.571	9.948	9.951	4.97	5.007	10.948	10.951
5.455	5.522	9.99	9.993	4.947	5.473	10.99	10.993
5.555	5.473	10.073	10.076	4.887	5.4	11.031	11.035
5.462	5.522	10.115	10.118	4.872	4.859	11.073	11.076
5.401	5.522	10.156	10.16	4.589	4.859	11.115	11.118
5.316	5.376	10.198	10.201	4.51	4.761	11.156	11.16
5.439	5.081	10.24	10.243	4.319	4.639	11.281	11.285
5.408	5.473	10.406	10.41	4.427	4.664	11.323	11.326
5.431	5.449	10.448	10.451	4.436	4.664	11.337	11.347
5.473	5.915	10.49	10.493	4.451	4.074	11.406	11.41

HF	AB	TimeHF	TimeAB	HF	AB	TimeHF	TimeAB
3.84	4.295	12.198	12.201	0.234	0.393	20.823	20.826
3.819	4.098	12.24	12.243	0.275	0.271	20.865	20.868
4.159	4.173	12.281	12.285	0.237	0.171	20.906	20.91
3.987	4.122	12.323	12.326	0.045	0.295	20.948	20.951
5.506	5.817	19.823	19.826	6.241	6.259	20.99	20.993
5.509	5.571	19.865	19.868	0.248	0.319	21.031	21.035
5.643	5.571	19.906	19.91	0.578	0.539	21.073	21.076
5.707	5.596	19.948	19.951	0.627	0.81	21.087	21.097
5.727	5.964	19.99	19.993	0.251	6.161	21.406	21.41
5.685	5.547	20.031	20.035	5.299	5.473	21.448	21.451
5.696	6.161	20.073	20.076	3.749	5.62	21.49	21.493
5.745	6.037	20.115	20.118	1.876	2.012	21.531	21.535
5.837	5.915	20.142	20.16	1.776	1.963	21.573	21.576
1.146	1.546	20.531	20.535	1.786	1.841	21.615	21.618
1.074	1.866	20.573	20.576	1.658	1.89	21.656	21.66
1.201	1.717	20.615	20.618	1.377	1.497	21.684	21.701
1.253	1.89	20.656	20.66	1.305	1.669	21.74	21.743
1.032	1.89	20.698	20.701	1.309	1.325	21.781	21.785
1.093	1.988	20.74	20.743	1.337	1.644	21.823	21.826
0.505	6.137	20.781	20.785	1.198	1.571	21.865	21.868

HF	AB	TimeHF	TimeAB
1.15	1.08	21.906	21.91
1.047	1.129	21.948	21.951
0.97	0.466	21.99	21.993
0.998	0.981	22.031	22.035
1.071	1.007	22.073	22.076
0.793	0.834	22.531	22.535
0.753	1.056	22.573	22.576
0.573	0.932	22.615	22.618
0.437	0.761	22.656	22.66

Appendix 6

Circadian data

Student	S1(radian)	S2(radian)
1	4.502949	4.572763
2	5.986479	6.038839
3	5.88176	5.602507
4	5.602507	0.017453
5	4.939282	4.956735
6	5.462881	5.445427
7	5.148721	5.497787
8	3.385939	3.228859
9	0.034907	6.056293
10	5.51524	5.72468

Appendix 7

Partial results of the cut-off points of $\left|COVRATIO_{(-i)} - 1\right|$

A1 . When $\alpha_{true} = 3, \beta_{true} = 3$ and $\omega_{true} = 0.95$

n	Level of percentiles	κ				
		2	4	5	7	10
30	1%	0.95	0.71	0.74	0.68	0.70
	5%	0.63	0.61	0.62	0.58	0.60
	10%	0.55	0.56	0.57	0.53	0.55
40	1%	0.52	0.60	0.64	0.62	0.65
	5%	0.45	0.52	0.53	0.51	0.50
	10%	0.43	0.47	0.46	0.45	0.44
50	1%	0.39	0.46	0.49	0.54	0.52
	5%	0.37	0.42	0.43	0.46	0.40
	10%	0.35	0.40	0.38	0.40	0.37
70	1%	0.30	0.39	0.43	0.45	0.43
	5%	0.27	0.34	0.39	0.37	0.35
	10%	0.26	0.31	0.34	0.33	0.31
100	1%	0.21	0.30	0.33	0.33	0.30
	5%	0.20	0.27	0.31	0.29	0.28
	10%	0.19	0.24	0.26	0.25	0.23
150	1%	0.15	0.20	0.23	0.25	0.33
	5%	0.14	0.18	0.21	0.22	0.20
	10%	0.13	0.17	0.19	0.18	0.19

A2 . When $\alpha_{true} = 1, \beta_{true} = 1$ and $\omega_{true} = 0.9$

n	Level of percentiles	κ				
		2	4	5	7	10
30	1%	0.81	0.74	0.75	0.68	0.79
	5%	0.62	0.63	0.63	0.62	0.65
	10%	0.56	0.58	0.57	0.55	0.56
40	1%	0.51	0.61	0.65	0.62	0.65
	5%	0.46	0.51	0.54	0.53	0.51
	10%	0.43	0.47	0.49	0.45	0.45
50	1%	0.42	0.57	0.55	0.55	0.55
	5%	0.38	0.44	0.48	0.45	0.45
	10%	0.36	0.41	0.42	0.41	0.40
70	1%	0.31	0.38	0.44	0.44	0.43
	5%	0.29	0.35	0.38	0.37	0.36
	10%	0.27	0.31	0.34	0.34	0.31
100	1%	0.22	0.29	0.32	0.31	0.31
	5%	0.20	0.27	0.28	0.28	0.27
	10%	0.20	0.25	0.27	0.25	0.24
150	1%	0.14	0.18	0.23	0.26	0.31
	5%	0.14	0.18	0.21	0.25	0.22
	10%	0.13	0.17	0.20	0.20	0.20

A3 . When $\alpha_{true} = 1.5, \beta_{true} = 1.5$ and $\omega_{true} = 0.4$

n	Level of percentiles	κ				
		2	4	5	7	10
30	1%	5.46	2.67	2.25	2.37	1.99
	5%	1.72	1.12	1.11	1.09	1.21
	10%	1.07	0.87	0.83	0.87	0.87
40	1%	1.72	1.15	1.19	0.88	0.79
	5%	0.71	0.67	0.65	0.61	0.63
	10%	0.55	0.57	0.59	0.54	0.54
50	1%	1.10	0.56	0.71	0.60	0.63
	5%	0.49	0.47	0.50	0.52	0.50
	10%	0.42	0.43	0.45	0.46	0.44
70	1%	0.40	0.39	0.44	0.45	0.45
	5%	0.31	0.35	0.38	0.37	0.39
	10%	0.29	0.33	0.34	0.33	0.33
100	1%	0.23	0.30	0.34	0.33	0.32
	5%	0.21	0.27	0.30	0.29	0.28
	10%	0.20	0.25	0.27	0.27	0.25
150	1%	0.15	0.20	0.24	0.26	0.27
	5%	0.14	0.19	0.21	0.22	0.21
	10%	0.14	0.17	0.19	0.19	0.19

A4 . When $\alpha_{true} = 1.5, \beta_{true} = 1.5$ and $\omega_{true} = 0.7$

n	Level of percentiles	κ				
		2	4	5	7	10
30	1%	0.90	0.75	0.76	0.74	0.83
	5%	0.64	0.66	0.66	0.63	0.65
	10%	0.59	0.60	0.60	0.56	0.59
40	1%	0.56	0.61	0.64	0.65	0.65
	5%	0.47	0.53	0.55	0.53	0.52
	10%	0.44	0.49	0.49	0.47	0.46
50	1%	0.41	0.51	0.54	0.58	0.54
	5%	0.38	0.44	0.48	0.47	0.44
	10%	0.36	0.41	0.42	0.42	0.40
70	1%	0.32	0.37	0.44	0.44	0.44
	5%	0.29	0.35	0.38	0.37	0.36
	10%	0.28	0.33	0.33	0.33	0.32
100	1%	0.21	0.29	0.33	0.37	0.30
	5%	0.20	0.26	0.30	0.30	0.26
	10%	0.19	0.25	0.26	0.27	0.23
150	1%	0.15	0.20	0.24	0.22	0.27
	5%	0.14	0.18	0.22	0.20	0.21
	10%	0.14	0.17	0.20	0.18	0.19

Appendix 8

Partial results of the cut-off points of *DMCEs*

B1 . When $\alpha_{true} = 3, \beta_{true} = 3$ and $\omega_{true} = 0.95$

n	Level of percentiles	κ			
		5	10	20	30
10	10%	0.0999	0.0998	0.0994	0.0988
	5%	0.1080	0.1042	0.1000	0.0999
	1%	0.1176	0.1110	0.1111	0.1105
20	10%	0.0524	0.0500	0.0500	0.0499
	5%	0.0528	0.0526	0.0525	0.0508
	1%	0.0574	0.0553	0.0554	0.0526
30	10%	0.0345	0.0341	0.0333	0.0333
	5%	0.0356	0.0345	0.0344	0.0343
	1%	0.0370	0.0367	0.0357	0.0353
50	10%	0.0208	0.0204	0.0200	0.0200
	5%	0.0213	0.0208	0.0204	0.0204
	1%	0.0217	0.0216	0.0213	0.0208
60	10%	0.0172	0.0169	0.0167	0.0131
	5%	0.0175	0.0172	0.0169	0.0174
	1%	0.0182	0.0178	0.0175	0.0179
70	10%	0.0149	0.0145	0.0104	0.0061
	5%	0.0151	0.0147	0.0146	0.0145
	1%	0.0156	0.0152	0.0151	0.0149
80	10%	0.0130	0.0128	0.0125	0.0052
	5%	0.0132	0.0130	0.0128	0.0127
	1%	0.0135	0.0133	0.0130	0.0130
90	10%	0.0115	0.0112	0.0089	0.0046
	5%	0.0117	0.0115	0.0115	0.0114
	1%	0.0119	0.0118	0.0117	0.0117
100	10%	0.0103	0.0102	0.0052	0.0040
	5%	0.0104	0.0103	0.0101	0.0101
	1%	0.0106	0.0105	0.0104	0.0104

B2 . When $\alpha_{true} = 1.5, \beta_{true} = 1.5$ and $\omega_{true} = 0.2$

n	Level of percentiles	κ			
		5	10	20	30
10	10%	0.0685	0.0535	0.0396	0.0338
	5%	0.0818	0.0674	0.0495	0.0421
	1%	0.0999	0.0999	0.0996	0.0421
20	10%	0.0340	0.0252	0.0184	0.0147
	5%	0.0386	0.0294	0.0229	0.0187
	1%	0.0498	0.0488	0.0472	0.0367
30	10%	0.0230	0.0167	0.0116	0.0092
	5%	0.0252	0.0189	0.0135	0.0106
	1%	0.0325	0.0307	0.0268	0.0185
50	10%	0.0141	0.0100	0.0070	0.0057
	5%	0.0153	0.0110	0.0077	0.0063
	1%	0.0187	0.0163	0.0119	0.0096
60	10%	0.0118	0.0086	0.0059	0.0048
	5%	0.0128	0.0094	0.0066	0.0053
	1%	0.0159	0.0129	0.0114	0.0099
70	10%	0.0103	0.0074	0.0051	0.0042
	5%	0.0111	0.0079	0.0056	0.0045
	1%	0.0135	0.0101	0.0079	0.0088
80	10%	0.0091	0.0065	0.0046	0.0038
	5%	0.0097	0.0070	0.0050	0.0042
	1%	0.0116	0.0085	0.0082	0.0081
90	10%	0.0081	0.0059	0.0042	0.0034
	5%	0.0088	0.0063	0.0046	0.0038
	1%	0.0104	0.0086	0.0077	0.0076
100	10%	0.0074	0.0053	0.0038	0.0030
	5%	0.0079	0.0057	0.0041	0.0034
	1%	0.0092	0.0071	0.0072	0.0071

B3 . When $\alpha_{true} = 1.5, \beta_{true} = 1.5$ and $\omega_{true} = 0.9$

n	Level of percentiles	κ			
		5	10	20	30
10	10%	0.0990	0.0980	0.0954	0.0955
	5%	0.1000	0.0997	0.0991	0.0993
	1%	0.1111	0.1098	0.1062	0.1008
20	10%	0.0500	0.0496	0.0462	0.0450
	5%	0.0517	0.0500	0.0499	0.0499
	1%	0.0540	0.0525	0.0525	0.0517
30	10%	0.0333	0.0329	0.0141	0.0097
	5%	0.0344	0.0333	0.0333	0.0185
	1%	0.0357	0.0345	0.0345	0.0333
50	10%	0.0200	0.0115	0.0072	0.0059
	5%	0.0204	0.0200	0.0084	0.0064
	1%	0.0208	0.0208	0.0204	0.0200
60	10%	0.0167	0.0096	0.0063	0.0048
	5%	0.0169	0.0167	0.0070	0.0052
	1%	0.0175	0.0172	0.0169	0.0067
70	10%	0.0143	0.0077	0.0052	0.0042
	5%	0.0145	0.0095	0.0058	0.0045
	1%	0.0149	0.0147	0.0145	0.0051
80	10%	0.0112	0.0069	0.0046	0.0037
	5%	0.0127	0.0125	0.0050	0.0039
	1%	0.0131	0.0128	0.0125	0.0046
90	10%	0.0099	0.0060	0.0042	0.0034
	5%	0.0114	0.0069	0.0045	0.0036
	1%	0.0116	0.0115	0.0085	0.0042
100	10%	0.0079	0.0054	0.0037	0.0030
	5%	0.0100	0.0058	0.0040	0.0033
	1%	0.0103	0.0102	0.0046	0.0038

B4 . When $\alpha_{true} = 0.5, \beta_{true} = 0.5$ and $\omega_{true} = 0.5$

n	Level of percentiles	κ			
		5	10	20	30
10	10%	0.0905	0.0809	0.0737	0.0638
	5%	0.0966	0.0930	0.0850	0.0776
	1%	0.1000	0.1000	0.0986	0.0986
20	10%	0.0442	0.0369	0.0198	0.0131
	5%	0.0480	0.0435	0.0358	0.0275
	1%	0.0500	0.0500	0.0472	0.0439
30	10%	0.0281	0.0184	0.0113	0.0090
	5%	0.0322	0.0264	0.0143	0.0100
	1%	0.0333	0.0330	0.0299	0.0291
50	10%	0.0151	0.0102	0.0070	0.0057
	5%	0.0176	0.0117	0.0075	0.0061
	1%	0.0200	0.0192	0.0140	0.0121
60	10%	0.0123	0.0083	0.0059	0.0048
	5%	0.0138	0.0091	0.0064	0.0052
	1%	0.0166	0.0120	0.0110	0.0077
70	10%	0.0107	0.0075	0.0051	0.0041
	5%	0.0118	0.0082	0.0055	0.0044
	1%	0.0142	0.0104	0.0067	0.0078
80	10%	0.0096	0.0065	0.0044	0.0037
	5%	0.0105	0.0070	0.0047	0.0040
	1%	0.0123	0.0094	0.0058	0.0045
90	10%	0.0084	0.0057	0.0040	0.0034
	5%	0.0091	0.0062	0.0042	0.0036
	1%	0.0109	0.0082	0.0049	0.0041
100	10%	0.0077	0.0052	0.0038	0.0031
	5%	0.0081	0.0056	0.0040	0.0032
	1%	0.0096	0.0064	0.0049	0.0037