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A COMPARATIVE STUDY OF AL-BĪRŪNĪ'S AND NEWTON'S PHILOSOPHIES OF MATHEMATICS

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A COMPARATIVE STUDY OF AL-BĪRŪNĪ'S AND NEWTON'S PHILOSOPHIES OF MATHEMATICS

BY

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LIST OF CONTENTS

ABSTRACT

Title: A Comparative Study Of al-Bīrūnī's And Newton's Philosophies of Mathematics
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The mathematization of nature has always been an important theme in the development of Islamic science and Western science. The flowering of Islamic science is closely related to the development of Islamic mathematics. Likewise, mathematics is extremely important to Western science to the extent that mathematics is dubbed the 'Queen of science'.

Underlying the development of both mathematics is its philosophy. In this dissertation, we have decided to examine the mathematical philosophy of al-Bīrūnī (973-1051 A.D.) and Isaac Newton (1642-1727 A.D.). We have chosen to investigate three aspects of their philosophy related to their concept of the mathematization of nature, namely; their concepts of nature, God and religion since these three aspects are the 'essence' of Islam and Christianity in the most general sense of the word. In more specific term, we will examine their view of the relationship between nature, science and religion, their concept of mathematization of nature and their view of mathematical knowledge. The main thesis is that there are more similarities than differences in their philosophy of mathematics with regard to the three aspects. Discussions are made in this dissertation in so far as they could illustrate the main thesis. An attempt is made towards the end to construct a 'shared philosophy of mathematics' based on these similarities.

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Transliteration

Arabic Letter	Transliteration	Short Vowels	
ء	'	ـَ	a
ب	b	ـُ	u
ت	t	ـِ	i
ث	th		
ج	j	Long Vowels	
ح	ḥ		
خ	kh	ا	ā
د	d	ـُو	ū
ذ	dh	ـِي	ī
ر	r		
ز	z	Diphthongs	
س	s		
ش	sh	ـَوْ	aw
ص	ṣ	ـَيْ	ay
ض	ḍ	ـِيْ	īy
ط	ṭ	ـُوْ	uww
ظ	ẓ		
ع	'		
غ	gh		
ف	f		
ق	q		
ك	k		
ل	l		
م	m		
ن	n		
ه	h		
و	w		
ي	y		
ى	ā		

PROLOGUE

1. Statement of the Problem

The mathematization of nature has always been an important theme in the development of Islamic science and its modern counterpart, Western science. In order to cast some light on the process of mathematization of nature which is instrumental to the development of both sciences, it is instructive to examine the different concepts of mathematization generally embraced by these two sciences as well as their precursor.

The flowering of Islamic science is closely related to the development of Islamic mathematics and it is impossible to understand Islamic mathematics in its totality, particularly its philosophy, without mentioning the obvious contribution of the Greeks because preceeding the advent of Islamic science there was Greek science.

Central to Greek mathematics is the Pythagorean school. This particular Greek school, according to one of Professor Nasr's pioneering works on Islamic science, "had disciples until the very end of the Graeco-Roman period, and was so influential in the

formation of Muslim intellectual sciences."[1] The Pythagorean school embraces the view that both the qualitative and the quantitative aspects of mathematics are important. Numbers and geometrical figures are more than merely quantitative representations. They are both qualitative and quantitative entities. For example, geometrical figures bear images of Unity, and numbers are projections of Unity.

In Islamic philosophy of mathematics, it is well known that the Ikhwān al-Ṣafā' adopts a similar philosophical position as the Pythagorean with regard to mathematics. Their concept of Unity underscores their concept of the mathematization of nature. Since aspects of Unity are so crucial to their concept of mathematization of nature, mathematics function as a 'Jacobs Ladder' for them to internalize the concept of Divine Unity.

The case of Western science is quite different. Mathematics is extremely important to Western science is dubbed the 'Queen of Science', echoing Carl Friedrich Gauss. However, unlike Islamic philosophy of mathematics, the qualitative aspects of mathematical entities are generally not viewed as significant as we will show in subsequent chapters. Consequently, its concept of mathematization is void of any 'qualitative' import. One needs only to skim through the early issues of Philosophical Transactions of the Royal Society of London to see that mathematics becomes merely symbols of quantities. For example, the reader cannot discern any link between

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[1]See S.H. Nasr, An Introduction to Islamic Cosmological Doctrines, (Massachusetts, 1964) p.47.

numbers or geometrical figures with anything other than gross matter. Numbers and geometrical figures are nothing save representations of the material world.

Our preliminary observation of the different and somewhat opposing concepts of mathematization stated above suggested to us that there must be a 'true' concept of mathematization. In more specific terms, we want to know whether in fact there is such a big difference after all between Islamic and earlier Western philosophy of mathematics. We want to examine the philosophical basis of their concepts of the mathematization of nature and to investigate whether there is any kind of similarity between them that is connected to their religious and spiritual consciousness. It is this philosophical curiousity that motivates and leads the author to the present comparative study.

2. Mathematization of Nature Chosen for Study

For the purpose of this dissertation, we have decided to examine the mathematical philosophy of two natural philosophers, al-Bīrūnī (973-1051 A.D.) and Isaac Newton (1642-1727 A.D.) In particular, we have chosen to investigate three aspects of their philosophy of mathematics connected to their concept of the mathematization of nature, namely; their concepts of nature, God and religion.

In our opinion, the mathematization of nature must necessarily focus on these three aspects. It is obvious that God and religion are the 'essence' of Islam and Christianity in the most general sense of the word. Also, both Islam and Christianity take nature

very seriously. The Qur̄an and the Bible deal with nature extensively. Yet the concept of nature must be related to their concept of God and religion. Nasr has shown the importance of the three aspects in studying nature. According to him, the Muslim mind embraces the concept that "man discovers the harmony and beauty of Nature not by projecting his own limited perspective upon the cosmos but by realizing his weakness and submitting to the Wisdom of the Creator".[2] In fact, to a Muslim, "the wonderful design, the innumerable patterns which He has built nature, God has declared to be His signs, His ā̄yā̄ts" which man is to study, investigate and "to make the necessary deduction, and thus to recognize, worship and serve Him".[3]

Interestingly, some aspects of the medieval Christian view of nature are influenced by the Islamic view. For example, the most well-known medieval European philosopher, St. Thomas Aquinas (1244/25-1274 A.D.) "[draws] heavily from the writings of Ibn Sī̄nā̄, al-Ghazzā̄lī̄ and other Islamic thinkers..."[4].

It remains to be seen whether there are some similarities in their philosophies of mathematics.

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[2]See his "Role of Nature and Methods of Its Study", in An Introduction to Islamic Cosmological Doctrines, ibid.

[3]See Ismail R. Al-Faruqi, "Islam and The Theory of Nature", The Islamic Quarterly, xxvi(1): 1982, pp.18-20.

[4]See S.H. Nasr, A Young Muslim's Guide To The Modern World, (Kuala Lumpur: Mekar Publishers, 1994) p.155.

3. Reasons for Choosing al-Bīrūnī and Newton

The choice of al-Bīrūnī and Newton is based on several considerations.

Al-Bīrūnī flourished during the rise of Islam in Central Asia, an era which "can only be compared with the mode of civilisation in the 19th Century Europe" and which "excelled all others in one respect, [that] here the scientific method was developed which was to revolutionize human history in subsequent times".[5] Furthermore it has been argued that al-Bīrūnī was one of the first Muslims to "develop the scientific and empirical methodology for the study of nature".[6]Likewise Newton, the icon of Western science in the 18th as well as 19th Century Europe and a progeny of Baconian inductive science, was a multidimensional scholar who contributed extensively to mathematics, dynamics, celestial mechanics, and astronomy.[7] Both mathematicians enjoy everlasting influences in subsequent scientific developments related to their fields. So comparing both of them is comparing two giants of science, each representing their own perceptions of the ideals and realities of science.

Interestingly enough, even though they are separated in time, there are notable similarities between them. Both were strong

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[5]See Inamullah Jan, "The Rise of Islam in Central Asia with special Reference to Educational Activities," The Islamic Quarterly xvii(3), 1984, p.187.

[6]Ibid, p.189.

[7]For a brief but reasonably authoritative introduction to his works, see Bernard Cohen, "Newton, Isaac" in Dictionary of Scientific Biography, (USA; Charles Scribner's Sons Publications, 1972), vol.x, pp.42-103.

adherents to their religion,[8] were at home in many different fields of knowledge and had a multitude of interests,[9] wrote on chronology of the ancients[10] and published monumental works on mathematics.[11]Incidentally, the natural question to ask is whether they did share some common philosophical positions with respect to the mathematization of nature.

Our other consideration in choosing the two sages concerns the historical span of six centuries separating them. Islamic mathematics was appropriated by the West as early as the 1200 AD. It would be an interesting venture to investigate the kind of

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[8] See for example, Bruce B. Lawrence, "Al-Bīrūnī and Islamic Mysticism", Hamdard Islamicus, 1(1):1978, pp.53-65; M.S.H. Masumi "Al-Bīrūnī's Devotion to the Qurān", Islamic Studies, xiii(4):1974, pp.45-59. Newton's interest in theology is evident from his writings in theology. Please refer to Isaac Newton, Newton: Theological Manuscripts (London; H. McLachlan, 1950) hereafter cited as Theological Manuscripts. It is a matter of record that in Newton's library there were 30 bibles. See Harrison, J. The Library of Isaac Newton, (London, 1978).

[9]For instance, both are not only well versed in mathematics, but also in instrument making, politics (al-Bīrūnī as court astronomer, Newton as Warden of the Mint) and history. See Hakim Mohamed Said and A.Z. Khan, Al-Bīrūnī: His Life Time and Works, (Karachi, 1981), and F.A. Manuel, A Potrait of Isaac Newton, (USA: DeCapo Press, 1968).

[10]See for example, Al-Bīrūnī, Alāthār al-Bāqiya, edited and translated by Sachau, E.and Newton, I, The Chronology of Ancient Kingdoms Amended, (London, 1728) edited by John Conduitt which contains the most comprehensive account of Newton's chronological theories. See also Newton, I. A Short Chronicle from the First Memory of Things in Europe to the Conquest of Persia by Alexander the Great, (London, 1728).

[11]I have in mind here al-Bīrūnī's Al-Tafhīm wa Sinā'at al-Tanjīm and his Al-Qānūn al-Masūdī. In the case of Newton, it is his Sir Isaac Newton's Mathematical Principles of Natural Philosophy and His System of the World, Translated into English by Andrew Motte in 1729. The translation revised, and supplied with an historical appendix, by Florian Cajori, (USA: Berkeley, 1934).

changes, if any, that had occurred in the philosophy of mathematics; particularly the concept of the mathematization of nature. So far we have not come across anywork that deals with this issue. Our present endeavour is partly an attempt to cast some light on it.

4. The Nature and Scope of the Present Study

It is generally accepted nowadays that one of the means to undertake a comparative study of scientific thought in different cultures is to study their beliefs, assumptions and work ethics underlying their philosophies using textual analysis of their various works.[12] Such an approach is especially helpful to this study since neither al-Bīrūnī nor Newton left any comprehensive philosophical work with repect to their concept of the mathematization of nature. For instance, in the case of al-Bīrūnī, we share Nasr's view that "from what remains of his writing, we can draw several conclusions of major importance" since none of his particular work on philosophy is extant.[13]

There are, however, al-Bīrūnī's work on astrology and astronomy. His Kitāb al-Tafhīm wa Ṣināʿat al-Tanjīm is invaluable to this study because in it he defines basic mathematical entities and states his opinion on the relationship between geometry, arithmetic

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[12]For example, Kuhn's notion of paradigm and scientific change is consonant with this approach. See Kuhn, T.S. The Structure Of Scientific Revolution, (Chicago, 1962).

[13]See S.H.Nasr, An Introduction to Islamic Cosmological Doctrines, op. cit., p. 166.

and astrology. In addition to Kitāb al-Tafhīm, his magnum opus Al-Qānūn Masʿūdī which represents his greatest achievement in mathematical astronomy and is the product of assiduous research is extant. Also available is his masterpiece on astronomical geography, Kitāb Taḥdīd -i-Nihāyat-il- Amākin li Tashiḥ-il Masāfāt-al-Masākin in which he not only calculates the longitudes and latitudes of different countries, but includes a normative account on what it takes to be a good mathematician. Equally important is his book A Treatise on Shadows wherein he clearly integrates organically his belief in the hereafter into his mathematical work.

To construct his philosophy by considering only these works would do injustice to the great, original thinker. There are also bits and pieces elsewhere reflecting his philosophical position which are related to his philosophy of mathematics. Notable among them are his Kitāb fī taḥqīq mā liʾl Hind min maqbūlat wa mardhūla and Al-Āthar al-Bāqiya'an al-Qurūn al-Khāliya, particularly under the chapter entitled Masāʾil sa'ala ʿanha Ibn Sīnā in which he poses interesting questions on the notion of the infinite. The concept of infinity is deeply rooted in mathematics. So in order to have a comprehensive insight into his mathematical philosophy, we will take into account these writings as well.

In the case of Newton, four of his works are extremely important in so far as the mathematization of nature is concerned. These are his Principia, Opticks, Theological Manuscripts and his letters to Dr. Richard Bentley. Throughout these writings, Newton made several interesting comments which are related to his

philosophy of mathematics in general and the three aspects (his conception of nature, God and Religion) in particular.

In the following seven chapters, we have undertaken a detailed treatment of the two mathematicians we have chosen. The main objective of this study is to construct their concept of the mathematization of nature with reference to the underlying philosophical positions. We also examine the distinguishing features of both al-Bīrūnī and Newton's concept of the mathematization of nature in addition to focussing on their similarities and differences. Discussions are made in so far as they could illustrate the main thesis in this study. The thesis is that there are more similarities than differences in their philosophy of mathematics with regard to nature, God and religion.

Chapter 1 is an investigation of al-Bīrūnī's 'overall' philosophy of science pertaining to his conception of nature, God and religion since these are the aspects that we are interested in. Questions about the manner in which he viewed the ontological status of scientific knowledge will be answered here. We need to find out what counts as scientific explanation, on the epistemological status of scientific theories and on the legitimacy of scientific method in procuring knowledge. Since the subject matter of philosophy of science necessarily includes discussion about the status of observable and non observable entities, this chapter is also an inquiry into al-Bīrūnī's perception on this matter. In short, Chapter 1 serves as the foundation for the next two chapters on al-Bīrūnī.

Chapter II and III deal with al-Bīrūnī's conception of mathematical abstraction and the status of mathematical knowledge respectively. Included in these chapters is al-Bīrūnī's view of numbers, since numbers are one of the major criteria that demarcate mathematics and non-mathematical disciplines. We will also examine his perception on the place of numbers and mathematics vis a' vis the sciences in unravelling nature.[14] What, if any, is the connection between numbers, mathematics, and nature? What is the status of mathematical laws, entities, theories, and infinity in his mathematical works such as the aforementioned Al-Tafhīm? How can mathematics increase our knowledge about God? What is the limitation of mathematical method? In what sense is mathematical knowledge uncertain? These are some of the pressing issues treated.

Chapter IV, V and VI are devoted to Newton. Just as Chapter I is about al-Bīrūnī's view of the relationship between nature, science and religion, Chapter IV deals with Newton's position on these topics. Issues discussed here include Newton's perception on matter, motion and force in ether, light, space and time and his position on the ontological and epistemological status of scientific knowledge. These issues are examined in so far as they are related to the three aspects (God, nature and religion) under study.

Issues discussed in Chapter V and VI correspond to those of

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[14]That the concept of numbers has profound effects in the Islamic philosophy of mathematics is clear from the work of the Ismā'īlī sect, Al-Ikhwān al-Safā' (c.960). See for an example, B.R. Goldstein, "A Treatise on Number Theory from a Tenth Century Arabic Source", Centaurus, 10:1964, pp. 129-60.

Chapter II and III. The contents of Chapter V and VI include a treatment of Newton's view of mathematics as a method of inquiry and a source of knowledge. In addition to that, there is also a discussion on the status of mathematical entities and the limitation of mathematics.

In Chapter VII which is the final chapter, an in depth comparison is made. Notable similarities and differences are discussed. An examination on the extent of the affinity between 'both philosophies' is carried out. Pressing issues that touch on the legitimacy of the mathematical method, as embraced by the two men, in procuring knowledge; that nature is knowable through mathematical inquiry, is likewise treated. An inquiry is made on whether their mathematical philosophies entail that mathematical knowledge is constructed; that mathematical truth is made but not found, whether mathematical theories are indeed nomological, whether a work ethic varies with mathematicians from different religious background and whether there must be some kind of objectivity in the flowering of mathematical knowledge. The notion of uncertainty of mathematics is also studied in greater detail.

5. Contemporary Scholarship on the Subject

In so far as we know, there is no comparative study of al-Bīrūnī and Newton's philosophy of mathematics within the scope that we have described above.

Let us consider the case of al-Bīrūnī. Although there have been numerous studies conducted about him, most of these works

underscore the historical perspectives. We have not come across many articles that investigate his philosophical stand. Worth mentioning among those that do examine some aspects of his philosophy are works by Nasr,[15] Lawrence,[16] Ma'sumi,[17] Nadvi,[18] Togan[19] and Barani[20]. From a more general point of view, Nasr has established some general conclusions and penetrating insights on al-Bīrūnī's philosophy of science and this only motivates us to further examine his philosophy of mathematics.[21]Others have seriously overlooked the qualitative aspect of his mathematics.[22]

With regard to Newton, definitely his philosophy of science draws much attention especially from western scholars. Yet to the

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[15]See the Chapter entitled "Al-Bīrūnī" in S.H. Nasr, An Introduction to Islamic Cosmological Doctrine, (USA: Cambridge, 1964).

[16]See Bruce B. Lawrence, "Al-Bīrūnī and Islamic Mysticism", Hamdard Islamicus, I(1):1978, pp.53-70.

[17]M.S.H. Masumi "Al-Bīrūnī's Devotion to the Quran", Islamic Studies, xiii(4): 1974, pp.45-59.

[18]See S.H.H. Nadvi, "Al-Bīrūnī and his Kitāb al-Jamāhir fī Ma'rifat al-Jawāhir: Ethical Reflections and Moral Philosophy", Islamic Studies, xiii(4):1974, pp.253-268.

[19]See Ahmat Zaki Velidi Toghan, "Bīrūnī's Picture of the World", Memoirs of Archaelogical Survey of India, No.53 (New Delhi, 1941).

[20]See S.H. Barani, "Al-Bīrūnī's Scientific Achievements", Indo-Iranica, 5(4):1951-52, pp. 37-48.

[21]Nasr's study of al-Bīrūnī include his al-Bīrūnī: An Annotated Bibliography, (Tehran: High Council of Culture and the Arts, 1973).

[22]For examples, see M.S. Khan, "Aryabatha I and al-Bīrūnī", Indian Journal of History of Science, 12(1977), pp. 237-244 and I Boolaky, "The Mathematical Geography of al-Biruni", Hamdard Islamicus, 7(2)(1984), pp. 63-76.

best of our knowledge, no systematic study specifically devoted to the three aspects of his philosophy of mathematics has been attempted. Manuel, for example, has made some general conclusions concerning the organic relationship between Newton's religious belief and his works but he did not offer a philosophical or a comparative study of his philosophy of mathematics, what more from the three aspects mentioned.[23] The significance of God, Nature and Religion in their philosophies of mathematics must be treated in a proper perspective. Modern interpretation of their philosophies of mathematics by the intuitionists, formalists and logicists who exclude metaphysics from mathematics are cases in point.[24]

All of these considerations have convinced me that it would be an interesting investigation and a worthwhile effort to examine and compare their philosophies of mathematics.

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[23]F.E. Manuel, The Religion of Isaac Newton (Oxford, 1974). See also D.T. Whiteside, The Mathematical Principles Underlying Newton's Principia Mathematica. (Univ. of Glassgow, 1970). Whiteside totally disregards the qualitative aspects of Newton's mathematics.

[24]See R.L. Wilder. Introduction to the Foundations of Mathematics, (John Wiley and Sons,1965). See also E.W. Beth. The Foundations of Mathematics. (North Holland, 1968).