Chapt. VII

A COMPARISON BETWEEN AL-BĪRŪNĪ AND NEWTON ON THE NATURE AND ROLE OF MATHEMATICS

Al-Bīrūnī and Newton were mathematicians of different religious traditions. One was a Muslim and the other a Christian. Not only were they devotees of different religions, they were also thinkers of different epochs. They grew up in societies having different norms and ideals. Al-Bīrūnī spent his life totally under Muslim governments which by and large follow the Sharī'ah. On the other hand, Newton lived in a country in which Christianity was the dominant religion.

These differences in cultural and historical background notwithstanding, there are many similarities in their philosophies of mathematics. In what follows, we will summarise these similarities as well as their differences and also point out the significance of our present study based on the findings of the previous chapters.

7.1 SIMILARITIES

i. The Nature and Role of Mathematics

Both al-Bīrūnī and Newton believe that mathematics is a primary link that connects nature, science and religion. Guided by their belief that everything is rooted in the Divine, mathematicians' contemplation of nature are facilitated by mathematics through

which they can know and internalize the levels of reality and the qualitative aspects of God in the world of quantities.

Above everything else, both of them construe mathematics as a way of knowing about this world to the end that man can know more about God and himself. It is both a theoretical and practical activity of solving problems by using symbols and manipulating them according to certain rules.

They look at mathematics as a very powerful tool of studying nature. However to say that they were instrumentalists as the word is understood today would not do justice to their philosophies of mathematics. Instrumentalists believe that in the case of mathematics, the latter is nothing more than a tool in our quest of knowledge whereas in our findings in Capter 1 and IV, al-Bīrūnī and to a lesser extent, Newton, believe that mathematics has an important role in man's understanding of the relationship between nature, science and religion. Nature can be scientifically analyzed through mathematics and religion plays a critical role in someparts of the process. Al-Bīrūnī's problems are circumscribed by religion whereas Newton's 'phenomena', above all, results from observing God's handiworks. In more specific terms, mathematics as practiced by al-Bīrūnī and Newton must be viewed from the perspective of contemplation wherein the mathematician is immersed in deciphering nature with the consequence of knowing more about his mode of existence and as a matter of fact, about Existence Itself. Mathematics is never merely an instrument void of metamathematical significance.

The relationship between nature, science and religion is grounded in mathematics. Nature is deciphered in several ways depending upon the field of study; sometimes by means of observation and experiments and also by way of witnesses and transmitters (as in the case of history). In both instances, both al-Bīrūnī and Newton maintain that mathematics has an integral role to play. In observations and experiments, the least that mathematics could offer is to validate their accuracy and in the case whereby so much depends on the reliability of transmitters, mathematics is used to check the accuracy of reports especially with regards to dates and locations (mathematical geography).

In the view of al-Bīrūnī and Newton, the usefulness and significance of mathematics is not confined to problem solving. Both of them believe that mathematics can sharpen man's intuitive capability at least in so far as precision and exactness is concerned. As an important consequence, mathematics helps mathematicians to study the abstractness of his object of study. The ability to contemplate the abstractness of things bears also another significant consequence; mathematics increases the mathematicians' knowledge of one important plane of reality, the abstract world which circumscribes the material world. Thus mathematics functions as a nexus and an invaluable bridge between the material world and the angelic world and ultimately to God who is the Most Abstract (al-laṭīf) of all.

With regard to their mathematical conception of nature, they have come to affirm that nature is not only simple but there is

also harmony and order. Simplicity, however, should not be understood in a vulgar sense. That the world is simple means that man are endowed with the faculty to know the world. And the fact that the world is created in six days as revealed in the Holy Qurān and Bible alludes to the orderliness of nature; that from the very beginning there is order in creation.

There is another similarity in al-Bīrūnī and Newton's view on the nature and role of mathematics. In the case of al-Bīrūnī, his mathematical quest begins by focussing on nature as the object of study. The external world which is the world of multiplicity appears in all variety of forms. Yet through mathematics he could find the common factors underlying the multiplicity. There is an underlying theme connecting the world of brute facts. It is not the case that all that exists are accidents which are devoid of any higher purposes. Rather, subjecting them to mathematical scrutiny will reveal that their existence point to an important aspect; that they are manifestations of the eternal and the actually infinite.

If al-Bīrūnī was to begin his mathematical study of nature by observing and experimenting with the sensibles in relation to problems, Newton begins his by contemplating on the phenomena. Mathematics to Newton is likewise an essential tool in deciphering nature and in solving problems as manifested in the phenomena. That mathematics is more than a tool is clear when we examine Newton's work closely, for an example the Principia. Man can unravel the abstract aspect of the phenomena and thereafter knows more about himself, nature and God by way of mathematics. More than anything

else, mathematics according to Newton provides a valuable linkage between the study of phenomena of nature, religion and God.

Although his Principia was written with this relationship in mind, the message was not recieved by some of his contemporaries like Hobbes and the other so-called atheists so much so that he had to explicitly state this underlying theme later in the Scholia of the Principia. And the Principia, which to us was an apologia for theology considering his over-emphasis on the mechanical and the quantitative aspects, was and still is his most well-known mathematical work that earns him a respectable place in the history of science, notwithstanding the history of the world.

ii. Mathematics and God

Central to both al-Biruni and Newton's philosophies of mathematics is their conception of God. Both mathematicians view the sensibles and mathematical objects as related to God in a manner corresponding to their mode of existence. God is the center for all mathematical objects there is.

In point of fact, it is their notion of God which dominates their conception of mathematics. According to both of them, by doing mathematics one should in the end knows more about God. Newton argues that true steps in natural philosophy will lead the philosopher to Pure Being whereas for al-Bīrūnī, the total worthiness of mathematics corresponds to the extent that mathematics can bring the mathematician closer to God, to the degrees that it can improve his 'taqwa'.

By this time, we may ask whether the 'Godhead' referred to by both of them in their mathematical treatises are indeed the same God. Newton's descriptions of God's Qualities given in the Principia and Opticks are not exactly the same with the descriptions stated by al-Bīrūnī throughout his writings. Al-Bīrūnī's God creates and destroys unceasingly whereas that is not the case with Newton's. Both however would agree very much that to know the essence of God is beyond human capability. In like manner, they would share the same belief that God is both the 'most' and the 'more'. For an example, they would agree to the statement that God is both 'greater' and 'the greatest'. He is the greatest of all and yet He is definitely greater than whatever list of Divine Qualities that they can think of.

iii. Mode of Mathematization

The first major similarity that comes to mind in their concept of the mathematization of nature is the function of the rational soul. Inspite of the importance of the external senses, it is the rational soul that can find meanings associated with the results of mathematical interpolation.

Interpolation of mathematical objects are carried out chiefly by the internal senses. Prior to arriving at a particular mathematical model, mathematical images are conveyed to the internal senses by the various sense organs. Once processed, mathematical meanings are abstracted by the rational soul.

Since both mathematicians realize the significance of the

rational soul, it is not surprising that both stress the need to purify the rational soul. Newton's so-called 'moral philosophy' and al-Bīrūnī's ethics sought to purify the rational soul of the mathematician. They believe that since it is God that imparts mathematical meanings to the soul and since God is the Most Pure, consequently having a 'pure soul' will facilitate the process.

Not all people have the same power of abstraction because of the discrepancy in the 'power' of the rational soul. Different people have different degrees of innate mathematical faculties and capacities. Thus with regard to the peoples' attitude in acquiring mathematical knowledge, both uphold the view that there are two 'classes' of people; those that have the 'knack' of mathematics (using Newton's terminology) and those who 'scream' at the sight of calculations and geometrical figures (paraphrasing al-Bīrūnī).

There is yet another striking parallelism between both al-Bīrūnī and Newton's mode of mathematization. If we look at their overall patterns of mathematization, we will find out that imbedded in that process is their belief in levels of reality as we have shown in Chapter III and VI respectively. There is a hiearchy of reality, so to speak. In the case of al-Bīrūnī, material objects and the infinite divisions of its constituents, the nature of light as represented in his Treatise On Shadows, the belief in the existence of Angels, the frequent mentions of God's Divine Qualities and the stated humility of not-knowing the Divine Essence correspond respectively to the levels of reality consisting of the material, subtle and angelic world circumscribed by the world of

Divine Qualities and Divine Essence. In similar vein, we have demonstrated that Newton likewise espouses the same belief with respect to the existence of the hierarchy although their view on how the levels operate are not the same.

As a consequence of their belief in the existence of various planes of reality, we can chart a one to one mapping between the faculties involved in the process of mathematization and the levels of reality. The external senses map into the world of brute facts which is the terrestrial world. The mind and other internal senses are mapped into the so-called intermediate world or the subtle world. Finally the soul who attains the mathematical meanings and ultimately the spirit, each corresponds to the celestial world and the world of infinity, which is none other than the world which includes Divine Qualities and the Divine Essence.

As a corollary to the one-to-one relationship between man (the microcosm) and the cosmos (macrocosm) manifested in both al-Bīrūnī and to a lesser extent in Newton's mode of mathematization (Newton believes that God's instrument such as gravity has its own innate power), we can explain the reason mathematics functions as a bridge connecting the world of sensibles to the world of intelligibles; simply because by mathematizing, we facilitate our comprehension of the abstract world. We brought ourselves yet closer to the world of the infinite and ultimately to the world of Divine Qualities and Divine Essence.

iv. Mathematical Knowledge

Both al-Bīrūnī and Newton believe that mathematical knowledge bears various degrees of certainty. In other words, mathematical knowledge in the form of mathematical models (solutions to mathematical problems) at the level of sense experience are not indubitable because they can either always be improved or corrected. At the level of sense experience, mathematical models are approximations. The improvements or corrections are subjected to the acuteness of the mathematicians external and internal senses, the accuracy of instruments (measurements), the inability to perform complete induction and the insolvability of other problems connected to the problem which the mathematician is solving. Both also believe that mathematical objects exist objectively in the realm of imagination.

Underlying both al-Bīrūnī and Newton's conception of mathematical knowledge is their belief in the existence of the levels of reality and the corresponding levels of truth. Whatever mathematical knowledge that they have acquired at the level of sense experience are approximations of truth. At a higher level, mathematical truths are truths <u>simpliciter</u>. All of these mathematical truths can be discovered.

Essential to al-Bīrūnī's 'sophisticated conjectures' and Newton's so-called 'mathematical reasoning' or 'mathematical demonstration' is the belief that these are improvements on other mathematical models. They are the 'better solutions' to problems and mathematician cannot say with <u>absolute</u> certainty that these

solutions are the best.

At the meta-mathematical level, implicit in their view of mathematical knowledge is that mathematics is open ended in the sense that it can never be final. It is the nature of mathematics pertaining to the world of sensibles to remain incomplete. Both also maintain that mathematical knowledge is acquired by man in the sense that it issues forth from God, and that at the level of gross matter, mathematical truth are relative.

7.2 DIFFERENCES

i. The Nature and Role of Mathematics

Essentially there are no major differences between al-Bīrūnī's observables and Newton's phenomena in their mathematical study of nature. Just as al-Bīrūnī emphasizes observation and experimentation, so does Newton. The thing worth underlining is that al-Bīrūnī precedes Newton in emphasizing observation and experiments by a time span of six hundred years, a fact which is really remarkable. But what is more astounding is that al-Bīrūnī precedes Newton not only in stressing the significance of observations and experiments, but also in justifying mathematics as a noble religious endeavour by expounding the function of mathematics as a vital nexus tying science, nature and religion together. In our opinion, this is indeed a noteworthy academic achievement of al-Bīrūnī.

Solving problems is considered as an essential part of mathematics and one of the major differences between al-Bīrūnī's

and Newton's philosophies of mathematics is al-Bīrūnī's treatment of scriptural inspiration of mathematical problems whereas for Newton, the problems solved are related to his religious belief to the extent that they are arguments for the existence of God. For instance, there is not much direct linkage between Newton's scientific problems and the needs of the society as prescribed in the Bible. It is to the end of arguing for the existence and uniqueness of God that his qualitative arguments in mathematics is directed.

There is a notable difference in their mathematical approach to integrate the three themes of God, Nature and Man. The difference lies not so much in emphasis but in degrees. For example, let us consider the role of the Scripture. Al-Bīrūnī's so-called scriptural input is explicitly evident throughout his mathematical works. We can find not only Quranic verses, but also their interpretations scattered in almost every chapter of his books.

These scriptural verses have three notable functions. First of all, the verses serve as reminders to the mathematician himself who is the 'seeker of knowledge' concerning the 'sacracity' of his endeavour. Secondly, the verses provide the raison d' etre for the problems. Problems are not solved simply as academic exercises. Rather they are solved because the mathematicians are religiously obliged to solve them. Solving these problems amount to fullfilling the concept of '*amal ṣāliḥ'. In part, it is a response to elevate the well-being of the society. (Here we can see that in accord with

al-Bīrūnī's view, mathematicians should be sensitive to the needs of the Muslim 'ummah'.) The third function of the verses is to justify the solution to the problems, not so much to show that the solution is right as to show that the solution is reasonable. (For example, we have in mind the solution to the problem of finding the exact direction of the Qiblah).

Such interwining of verses from the Scriptures and mathematical arguments are not easily discernible in Newton's mathematical endeavour. In fact, we can hardly find a single verse from the Bible quoted in the Principia or in the Opticks. In my point of view, this is so due to the nature of the teaching of Christianity as far as it is compared to Islam. Islam is the religion of knowledge in contrast to Christianity as a religion of love and ethics.[1] If al-Bīrūnī quotes more of the Scripture, that is because of the Quranic perspective.

But more important than that is Newton's understanding of Christianity. In his view, the role of scripture is only in the form of broad general principles. According to Newton, a principal part of Christianity is "the loving God and neighbour"[2] wherein he says that "the love of neighbour is that charity where no man can be saved".[3] In fact, Newton believes that "the other part of true

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[1]See S.H. Nasr, A Young Muslim's Guide to the Modern World, (Kuala Lumpur, 1993), pp.76-77. Unlike Islam, Christianity does not have the equivalent of Shari'ah.

[2]See Theological Manuscripts, p.28.

[3] Ibid, p. 28.

religion is our duty to man"[4] such that we should "love our neighbours as ourselves, we must be charitable to all men, for charity is the greatest of graces...[and] we must be righteous, and do to all men as we would that they should do to us".[5] These are the themes of Newton's ethics. That these themes serve as guiding notions in solving mathematical problems is sufficient to Newton.

Therefore inspite of the scarcity of explicit scriptural emphasis in his Principia or Opticks, to a certain extent his understanding of mathematical problems wanting to be solved is influenced by them. Still, we maintain that the organic synthesis between Scriptural verses and mathematical works are more evident in the case of al-Bīrūnī because the synthesis is done at a more comprehensive scale. For instance, we have demonstrated earlier that the latter's Treatise on Shadows incorporates not only ethics and discussions about God or shadows of this world, but also about the Hereafter. Remarks on life after death are almost 'literally' absent in Newton's mathematical works notwithstanding the fact that he considers mathematics as a ladder for the mathematician's spiritual ascent.

ii. Mode of mathematization

In so far as mathematization is concerned, al-Bīrūnī upholds the position that the process of mathematization is part of the act of contemplation. He subscribes to the view that everything is

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[4] Ibid., p.52.

[5] Ibid., p.52.

countable and that there is an inner drive of man to count. The mathematician can mathematize nature which is the object of study by utilizing his various faculties equipped by God. According to al-Bīrūnī, in addition to the sense of hearing and seeing there is the internal senses. The external senses perceive the particulars from the external world. Mathematical information passes from the external senses to the intellect which is the seat of mathematical knowledge. The intellect is the seat for the fruit of mathematization.

Al-Bīrūnī also maintains that numbers are inherent in the external world. Numbers are everywhere. The world of multiplicity as depicted in the series of natural numbers is in the final analysis a manifestation of the oneness of God and nowhere is this aspect more properly manifested than in his conception of the number One. Just as the other numbers issue forth from the number One, so are the contents of the external world. They originate from God.

In Newton's mode of mathematization, the process begins from contemplating the phenomena. Nature which is initially created by God can be studied mathematically because Newton believes that everything in the external world can be quantified. The "frame and operations of Nature" are reducible to "general rules or Laws" by way of mathematics. Essential to mathematization is the basic knowledge of arithmetic, geometry and mechanics. These three branches of mathematics coupled with the belief that God likewise is the perfect geometer and mechanic form the components of

Newton's "rational mechanics" which paved the way for the quantification of mathematics because of his emphasis on that aspect.

In the case of Newton, mathematization is interspersed with observations and experiments for he argues that who is the best teacher if not nature herself? In the course of observation and experiments, both the external and internal senses are involved. When sensations are excited by the spirit, mathematical images from the external world are brought into the sensorium via the organs of senses until they reach the brain where the mathematical images are processed by the internal faculties. The soul attains mathematical knowledge which ultimately issues forth from God. God will reveal the natural causes, which in reality are his 'instruments'. The mathematician will know the natural causes as mathematical notions which do not have equal power to God. Still, with the amount of natural power which these natural causes have, the distinction between the natural and supernatural in Newton's view of nature becomes more evident.

Akin to al-Bīrūnī's position, Newton believes that mathematization definitely involves numbers and geometry. But Newton includes what he called 'mechanics' which he deems is equally essential with geometry and number. The inclusion of mechanics and consequently the integration of the three fields; mechanics, geometry and numbers into his 'rational mechanics', by and large fashions his mode of mathematization. Accordingly, we find that Newton's mathematization and consequently his

mathematical findings are more structured and quantified than al-Bīrūnī.

On the other hand, al-Bīrūnī's mathematization is more versatile. For example, let us consider his Exhaustive Treatise on Shadows. It is not as structured as the Principia or Opticks although both observations and experiments are no less important to al-Bīrūnī.

For that matter, what Newton gains in 'depth' al-Bīrūnī profits in 'scope'. Al-Bīrūnī based his mode of mathematization upon the belief that numbers are the foundation of mathematics, and essential to the process of mathematization are the stages of comparison and quantification. Accordingly, he is more at ease in mathematizing not only problems associated with astronomy and light, but also geographical and astrological problems wherein the importance of experiments is not so evident.[6]

There is yet another disparity pertaining to their mode of mathematization. It concerns the orientation of problems that warrant immediate attention. Since there is a major difference between their religions for example with respect to the ritual exoteric part, we maintain that this difference in turn affects their mode of mathematization in particular when it is related to justifying research priorities or more specifically, in problem evaluation. Al-Bīrūnī's mathematical problems are problems circumscribed in the Holy Qur'ān and the tradition of the Holy

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[6]For an indepth explanation of Muslims' involvement in astrology, see A. Sayili, The Observatory in Islam, (Ankara, 1960).

Prophet (pbuh). Problems connected with finding the direction of the 'qiblah' which to al-Bīrūnī are religiously sanctioned was of no practical value to Newton. Yet it is sufficient for Newton that existence of God is central to the process of mathematization and that all problems which "bring mathematicians a step closer to God" are deemed legitimate.

Newton would agree that mathematical problems related to ascertaining the direction of the 'qiblah' are also important problems if it did bring one closer to God. Let us note that the justifications in arriving at the same conclusion (the importance of solving the problem) are not really the same. There is an additional requirement in the case of al-Bīrūnī and it is related to the role of the Scripture. It is not enough that a problem is deemed to be important merely on the ground that mathematizing the problem will bring the mathematician closer to God. Al-Bīrūnī also justifies his position based on another additional requirement, that the problem is circumscribed in the Scripture.

This requirement of finding justification from the Scripture, is not as strong in the case of Newton. For example, there is hardly any verse from the Scripture in his Principia of Opticks. There is an explanation for their dissimilar attitude towards the Scripture.

In brief, al-Bīrūnī accepts the authenticity of the Holy Qurān. He might have questions regarding few transmitted sayings of the Holy Prophet (pbuh) which affects his position regarding several 'madhabs'. The case of Newton, however, is totally

different. In addition to his quasi-mechanical view of nature, Newton was a non-conformist Christian. He spent more time ascertaining the authenticity of the Bible and the validity of the concept of trinity than writing the Principia or the Opticks. We claim more than anything else, it is his view of nature and his uncertainty with regard to the Bible which makes him exclude 'justification from the Scripture' as an integral component of problem evaluation. Thus his more unrestricted attitude to mathematical problems. It is sufficient for Newton that mathematization in the end functions as arguments about existence of God.

As a corollary to the above argument that the Scripture plays a more fundamental role to al-Bīrūnī than Newton in so far as research priorities (problems evaluation in the course of mathematization) is concerned, we maintain that in the case of al-Bīrūnī, the Scripture does in fact facilitate mathematization more so than Newton.

iii. Mathematical Knowledge

According to al-Bīrūnī, mathematical knowledge is always ultimately part of that knowledge about God's Name and Attributes. In the course of acquiring mathematical knowledge, the mathematician experiences two levels of perception. The first level corresponds to the level of sense experience wherein the mathematician sees mainly mathematical objects before him and nothing more. At the second level of perception, he will see the One characterizing the

Many, without ever imperfecting Itself. The mathematician sees that the existence of the Many is equivalent to that of the natural numbers. The Many are multiples of Unity. They are diversities within Unity. The mathematician experiences the Quranic verse, "Wheresoever you turn, there is the Aspect of God" since he witnesses the pervasive Divine Presence.

In addition to the above, al-Bīrūnī maintains that counting, which is a basic mathematical knowledge, is innate to man. For that matter, man has undergone the first mathematical experience which is the proto-quantification stage when he was in his primordial self. Man has a perfect understanding of the One because he has witnessed Him. This unique mathematical experience is the most innate mathematical intuition of all. Consequently in his earthly life, man has the ability to count, beginning with the elementary (but not trivial) ability to intuit the number one (aḥad) which is nothing save a pale reflection of the One (al-wāḥid). In this sense, we posit that mathematical knowledge is a consequence of the primordially innate mathematical experience.

Central to al-Bīrūnī's understanding of the status of mathematical knowledge is his conception of mathematical truth. In his view, there are levels of mathematical truth. The highest level of mathematical experience occurs at the highest level of mathematical truth. It is when the mathematician gains the illuminative experience of uniting with Divine Truth, wherein he transcends the veil of multiplicity. God, whose other name is al-Ḥaq, individuates truth from the level of absoluteness to the level

of sense experience. At the lowest level of mathematical truth which corresponds to the level of sense experience, mathematical truth which is an incomplete and imperfect copy of The Truth is viewed as a sophisticated conjecture.

Unlike al-Bīrūnī's emphasis on the notion of One, fundamental to Newton's conception of mathematical knowledge is his understanding of the ontological status of mathematical objects. At the level of sense experience, they are said to be relative, apparent and common. These features correspond to the outward aspect of nature. From the inner aspect of nature in the dimension of the abstract world, mathematical objects are associated with the concept of truth and absolute. In the ultimate analysis, mathematical objects are manifestations of some aspects of Being.

The arrival at mathematical knowledge by the soul is enhanced by the usage of mathematical entities such as infinity, numbers, points and lines. These entities are imbued with metaphysical principles. It is not the case that they are merely notations on papers. Rather, each of them is intricately connected to his cosmology.

According to Newton, mathematics is the best way of deciphering nature but he believes that mathematical reasonings or 'demonstrations' are neither complete nor final. Mathematical truths are bounded by the truth of its principles. Their certainty cannot exceed the certainty of their axioms (which he does not consider as absolute since they are verified by means of observation and experiments).

Moreover, Newton believes that truth separates understanding and imagination. Man can only understand things that are true. He likewise believes in levels of truth. In accord with his belief in the existence of planes of reality, Newton distinguishes between physical truth and mathematical truth. Physical truth concerns the domain of the material world whereas mathematical truth concerns more the domain of the abstract world.

We submit that the differences underlying al-Bīrūnī and Newton's conception of mathematical knowledge are more in terms of degrees. For example, numbers have a more fundamental role to al-Biruni than to Newton so much so that the former based his mathematics on the belief that each man has the innate ability to count. Thus man's capacity to mathematize and consequently acquiring mathematical knowledge. On the other hand, Newton includes basic understanding of mechanics as equally important in addition to knowledge of arithmetic (which also deals with numbers), resulting in a more quantitative orientation of mathematics.

Apart from numbers, al-Bīrūnī's conception of mathematical truth is more comprehensive. We have demonstrated earlier that at the level of sense experience, truth to al-Bīrūnī can fit several connotations. Truth can carry the meaning that the mathematical model 'works' (in solving physical problems). It can also mean that the model is 'consistent' with the axioms (his solutions to geometrical problems) or that the model corresponds to rigorous observations (particularly mathematical models functioning as

solutions to astronomical problems).

On the other hand, Newton's concept of mathematical truth at the level of brute facts is more restricted. In so far as mathematics is concerned, mathematical truth is underdetermined by its axioms (the principles) more so than the method. As we have shown earlier, Newton's method furnishes mathematical models which are structured. Thus structured, mathematical truth to Newton means that there is no contradiction between the mathematical models and the axioms.

In addition to the above, Newton upholds the position that a mathematical axiom cannot be true merely because its competitors are false. Al-Bīrūnī, however, does not limit himself by subscribing to this strict position. Although a mathematician will never know all the competing axioms, it is reasonable to accept a particular axiom or assumption as true if all the known competing axioms are shown to be false. For example, in the case of the heliocentric versus the geocentric system, al-Bīrūnī chooses the geocentric rather than the heliocentric system because according to him, the latter is less consistent with terrestrial phenomena. It is instructive in this case to bear in mind al-Bīrūnī's more versatile concept of mathematical truth (or falsity).

Apart from the variations in their conception of mathematical truth, there is also a difference with regard to their conception of infinity. Although both of them believe that only God is absolutely infinite, Newton has a more extensive conception of infinity. There are levels of infinity and that infinity can be

predicated of all sort of things; ignorance, power, intellect, et cetera. Al-Bīrūnī's abstraction of infinity is not as extensive. In fact, it is quite elementary. According to him, infinity belongs to God and all creations are finite.

This difference in their positions is interesting indeed. Newton's conception of infinity is heavily influenced by results of Wallis Arithmetica Infinitorum. Unfortunately in the time of al-Bīrūnī, the mathematical concept of infinity is not that developed.[7] Therefore inspite of al-Bīrūnī's profound 'rationality', it is difficult for him to discern that there are levels of infinity. (Here again we can see how mathematics does facilitate and to certain extent, functions as a catalyst in sharpening one's capability of knowing the abstract world. Mathematics can certainly functions not only as an important bridge between the sensibles and the intelligibles, but also between the intelligibles of different classes of abstractness).

Both al-Bīrūnī and Newton likewise have differences imbedded in their conception of zero. Al-Bīrūnī maintains that zero refers to a situation wherein something 'is not there yet'. It alludes to the emptiness created by God in order to be filled. While Newtons shares al-Bīrūnī's view that zero symbolizes the precreation stage, he differs from al-Bīrūnī by perceiving that zero corresponds to the state of absence of anything identifiable. Both Newton and al-Bīrūnī appeal to their interpretations of the Scripture to buttress their views.

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[7]See Chapter III, footnote 36.

There is also a difference in the way they view the concept of One. While it is crystal clear that their conception of One is related to their knowledge of Divine Unity, al-Bīrūnī has a more penetrating insight of the One. He has the discernment and acuteness to differentiate explicitly between the numerical one and the more encompassing concept of One. The numerical one is but a 'subset' of the higher concept of One. It is a pale reflection of the One.

7.3 SIGNIFICANCE OF STUDY

The significance of God in both al-Bīrūnī and Newton's philosophies of mathematics must be treated in a proper perspective. Modern interpretation of their philosophies of mathematics does not amply demonstrate this important aspect.

Let us first consider some recent analysis of al-Biruni's mathematical works. Apart from professor Nasr's pioneering studies, the qualitative aspects of al-Bīrūnī's philosophy of mathematics are not even mentioned.[8] Al-Bīrūnī's mathematics are presented as merely quantitative interpolations where signs are processed according to certain formal rules. Never is there any treatment on

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[8]I have in mind professor Nasr's An Introduction..., Science and Civilisation and Islam, and his al-Bīrūnī: An Annotated Bibliography. Example of studies that have overlooked this aspect are M. Anas, "Al-Bērūnī's Mathematics and Astronomy", Afghanistan, 26(1973), pp.76-85, M.S. Khan, "Aryabatha I and al-Bīrūnī", Indian Journal of History of Science, 12(1977), pp.237-244, A.K. Bag "Al-Bīrūnī on Indian Arithmetic", Indian Journal of History of Science, 10(2)(1975), pp.174-184, I. Boolaky, "The Mathematical Geography of Al-Bīrūnī", Hamdard Islamicus, 7(2)(1984), pp.63-76, and S.H. Barani, "Muslim Researches in Geodesy",in Al-Bīrūnī Commemoration, op. cit.

the qualitative aspects of mathematics although al-Bīrūnī's conception of numbers, for example, are imbued by them. As we have shown, his definition of the number one is clearly a manifestation of his understanding of Divine Unity.[9]

The same analysis applies to Newton. Few understand the Principia to include the qualitative aspects of his mathematics,[10] regardless of how much it might have. Most of modern interpreters of the Principia uphold the position as if there is no qualitative aspects at all in Newton's mathematics. As a corollary, this position bears the consequence that God is not central to his philosophy of mathematics. Just to cite an example, D.T. Whiteside's Mathematical Principles Underlying Newton's **Principia Mathematica** [11] consists chiefly of formal manipulation of symbols which is hardly the mathematical principles of the world as understood by Newton wherein the premises such as God as the source of mathematical knowledge and that by doing mathematics one can know more about God are clearly manifested in his mathematics. According to Whiteside; "...Newton's 'Propositions', 'Theorems', 'Problems', 'Lemmas' and 'Scholia' are mere expository frameworks inherited from his enforce study,...and they are manifestly

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[9]B. B. Lawrence has argued that there is a connection between al-Bīrūnī and 'mysticism' but the manner it is related to mathematics is not examined. See B.B. Lawrence, "Al-Bīrūnī and Islamic Mysticism", Hamdard Islamicus, 1(1)(1978), pp. 53-70.

[10]For example, see B. Stewart and P.G. Tait, The Unseen Universe or Physical Speculations on a Future State. (London, 1881).

[11]See D.T. Whiteside, The Mathematical Principles Underlying Newton's Principia Mathematica, (University of Glassgow, 1970).

retained in his subsequent mathematical writings purely as a literary convenience".[12] Although Whiteside shares Truesdell's claim that Newton's Principia is 'a book dense with the theory and application of the infinitesimal calculus',[13] the intricate connection between Newton's conception of the infinite and his conception of God as explicated in the Scholium of the Principia is not considered at all.

7.3 (i) Relevance to Contemporary Philosophies of Mathematics

Current investigations in the foundation of mathematics, in particular with regard to mathematics and cognition, are based on the assumption that learning mathematics has its own mode of reasoning which is defined by several variables. They hold the view that experience, intuition, emotion and motivation are the important variables. There are those who even believe that intuition is scientifically analyzable.[14] There are also those who subscribe to the view that as far as mathematization is concerned, a person's cognition about cognition is important. The assumption of this approach is that a person cognizes about cognition itself apart from cognizing objects and events. They form conceptions of the manner the mind works, "about their own mental states and

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[12] Ibid., p.8.

[13] Ibid., p.10.

[14] For example, see E. Fischbeim. Intuition in science and mathematics: An educational approach. (The Netherlands, 1987).

processes".[15] Yet, in none of these analyses does the role of the internal senses, the purification and the involvement of the soul (not to mention Divine Transcendence and Divine Unity) as subscribed by al-Bīrūnī and to a lesser extent, Newton, is considered.

The concept of levels of reality has not recieved much attention in contemporary popular researches in the foundation and philosophy of mathematics.[16] Currently, one of the major assumptions is that mathematization is an internal process concerning an external world which is void of any extra-mental realities.[17] The Divine Essence and the Divine Qualities bear almost no influence on the process. Like wise the Angelic World has no relevance at all. Man is capable of knowing all by himself.[18]

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[15]See H. Wellman, "The Origins of Metacognition", in D.L. Forrest-Pressley, G.E. MacKinnon, & T.G. Waller (eds.), Metacognition, cognition, and Human Performance (London, 1985).

[16]For a representative discussion of the various philosophies of mathematics, see R.L. Wilder, Introduction to Foundations of Mathematics, (John Wiley and Sons, 1965)

[17]For their other assumptions and a critical analysis of them, the reader can consult Shaharir Mohamed Zain, "Beberapa Kritikan Awal Terhadap Premis Ilmu Sains Tabii", in Kesturi, Jurnal Akademi Sains Islam Malaysia, (1)(1)(1991), pp.81-93.

[18]See for example, discussions in P. Benacerraf & H. Putnam, (eds.) Philosophy of mathematics: Selected Readings., (New Jersey, 1964); P. Ernest, "The Philosophy of mathematics and mathematics education", International Journal of Mathematical Education in Science and Technology, (16)(5), pp. 603-612; A. Heyting, Intuitionism: An Introduction, (Amsterdam, 1956) and D. Wheeler, "The World of Mathematics: Dream, myth or reality?" in J.C. Bergeron, N. Herscovics, & C.Kieran (eds.) Proceedings of the 11th International Conference for the Psychology of Mathematics Education, Vol.1, (Canada, 1987), pp.55-66.

As amply stated by profesor Osman Bakar:

> Fundamental premises of modern science are products of philosophical inquiries claimed to be supported by the external senses and bearing empirical import.[19]

In short, although the flowering of mathematics begins as early as the first man on earth and that the concept of the existence of levels of reality which is subscribed to by mathematicians of various cultures and from different religious traditions (as demonstrated in the case of both al-Bīrūnī and Newton) has brought forth tremendous developments of mathematics, popular modern analysis on its foundation has either overlooked or denied this fundamental aspect. They have either failed or missed the sacred mathematical connection between the heaven and the earth and the fact that Divine Immanence and Divine Transcendence is everywhere.

Formalistic conception (for example that propounded by David Hilbert) in the modern study of the foundation of mathematics requires that mathematics be expressed formally since they believe that the ultimate goal of mathematics is to reduce mathematical truth to a formal and coherent, symbolic system. Our study of Newton and al-Bīrūnī's conception of mathematical knowledge show that such a programme envisaged by them is impossible to be carried out chiefly because at the level of sense experience, mathematical truth changes over a period of cognitive development through a variety of problems situation and that particularly in the case of Newton, axiomatized knowledge is evidently only the last developed

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[19]See O. Bakar, "Sains Dalam Perspektif Islam", Dewan Budaya, (12)(1991), p.40.

state of the mathematicians' knowledge.

For that matter, one should never lose sight of the fact that the axiomatized presentation (as in the Principia) is only the 'external part' of his mathematical knowledge and that it can never be viewed as his mathematical knowledge per se. For al-Bīrūnī and to a lesser extent, Newton, mathematics is much more than stated definitions and propositions. Both 'internal' and 'external' aspects of mathematics are important.

Another aspect on which we wish to comment is the ontological status of geometrical entities as part of the ingredients of mathematical knowledge. Basically there are three common positions upheld by current modern propounders of philosophies of mathematics. The formalists maintain that mathematics is derived from axioms and thus no mathematical reality is assumed (geometrical objects have no objective existence); the realists argue that geomerical entites are abstract objects existing in the abstract world; and the constructivists subscribe to the view that geometrical entities are constructs formed in the human mind. In this case, it is clear that neither the formalists nor the constructivists' position fit squarely with both al-Bīrūnī and Newton. Both of them believe that geometrical entities exist objectively in the realm of imagination. Therefore the realists' position is the closest to that held by both al-Bīrūnī and Newton.

Unlike differences between the intuitionists, formalists, and constructivists in contemporary secular philosohies of mathematics, it is worthy to re-emphasize that there are interesting

similarities between Newton and al-Bīrūnī's philosophies of mathematics. There is an explanation for this uniformity. In our opinion, their differences are shaped by their basic religious belief and yet there are so much similarities in it. For example, in principle they share the belief in the existence of God, Prophets, angels and the Hereafter. However, the intuitionists, formalists and constructivists alike never consider these universal religious tenets as having any relevance at all to the foundation and philosophy of mathematics. Each of the group bases their philosophies on assumptions divorced from these basic religious tenets. In the absence of these tenets, there are no unifying themes underlying their philosophies. Consequently, there are marked contrasts between their philosophies of mathematics, not withstanding their variegated and secular philosophies.

Bereft of these universal tenets, it is understandable that the philosophies of mathematics propounded by the intuitionists, formalists and constructivists do not bear mathematical experience that help mathematicians in their spiritual ascend to the intelligibles and ultimately to God.

More importantly, in accord with these metamathematical tenets espoused by al-Bīrūnī and to a lesser extent, Newton, is the belief that man is a microcosm. He is a reflection of the macrocosm. It is the incognizance of this 'forgotten truth' (using Huston Smith's terminology), that man is the microcosm enjoying a one-to-one correspondence to the macrocosm, that the heaven and the earth are ontologically related and that God is the Lord of both; become the

principle cause of the secularization of mathematical experience. The mathematical experiences of the mathematicians are no longer part of that illuminative experience with Divine Unity and Aspects of Existence. Instead, the mathematical experiences they undergo are only fruits of their descent to the dry and morbid world of sophisticated quantification. It is unfortunate that Newton has some share in this by urging philosophy and religion 'to be preserved distinct'.

7.3(ii) A Shared philosophy of Mathematics

From our study of of al-Bīrūnī's philosophy of mathematics and after comparing its essentials with those of Newton's, we discover that there are more underlying agreements between them than there are differences. Accordingly, there is actually a 'shared' philosophy of mathematics espoused by them which demonstrates that contrary to modern philosophies of mathematics, philosophical and foundational problems in mathematics must be solved metamathematically.

Mathematics as understood and practiced by al-Bīrūnī and Newton can be regarded as a huge and comprehensive research programme imbedded with various levels of operation. We claim that basically these hiearchy of operation can be subsumed under three distinguishable worlds (for a lack of a better term). The three worlds can be presented geometrically as three concentric circles wherein the most important world which functions as the kernel of the research programme lies not in the innermost but at the

outermost layer. We will name the outermost layer 'World 1'.

World 1 is the world of metamathematics. It contains the metaphysical principles determining the nature of the mathematics produced. These metaphysical principles are not assumptions or axioms or conventionalists' claims. They function as the foundations of mathematics and its overall guiding principles. From our findings of al-Bīrūnī and Newton, there is a taxonomy of metamathematical principles situated in this so-called World 1. The first principle is the concept of Divine Unity (al-Bīrūnī's tawḥīd and Newton 'oneness of God'), followed by the concept of levels of reality and levels of truth. (We have demonstrated in the foregoing chapters that their philosophical positions, especially with regard to mathematization and the status of mathematical knowledge, are overshadowed chiefly by these metamathematical principles.)

Circumscribed and underdetermined by World 1 which is the outermost circle is World 2. It consists of assumptions, premises and axioms.[20] Inherent in Newton's and al-Bīrūnī's conception of mathematics is the position that there are two classes of assumptions; those that deal with the material world and those that deal with the subtle world. For example, al-Bīrūnī's and Newton's assumption that there is harmony in nature clearly deals more with the material world whereas the Euclidean postulates, that deal with mathematical objects residing in the realm of imagination, concerns more the subtle world. Likewise with the assumption of entities

[20]These assumptions are to be understood in the sense of the Euclidean postulates (which are subscribed to by both al-Bīrūnī and Newton).

such as ether and gravity. These assumptions (except gravity which is not yet formulated as such in the time of al-Bīrūnī), to both scholars are unlike the principles situated in World 1. They definitely are not infallible. For example, implicit in their belief in the plurality of worlds is the position that Euclidean geometry is not the only geometry possible.

World 3 is the world of mathematical models. It is a world overshadowed by both World 1 and World 2. The contents of World 3, as al-Bīrūnī's 'sophisticated conjectures' or Newton's 'structured models', are more readily subject to change than the contents of World 2. These mathematical models results from the application of various methods. In as much as mathematical models in World 3 are derivable from World 1 and World 2, they can be competitors or complementers. For example, geocentric and heliocentric models are competing mathematical models. The epicycles, however, are complementers. They are constructed as improvements to the geocentric model. (In view of these examples, note that although the construction of these mathematical models are influenced by World 1 and World 2, their end results are not necessarily compatible).

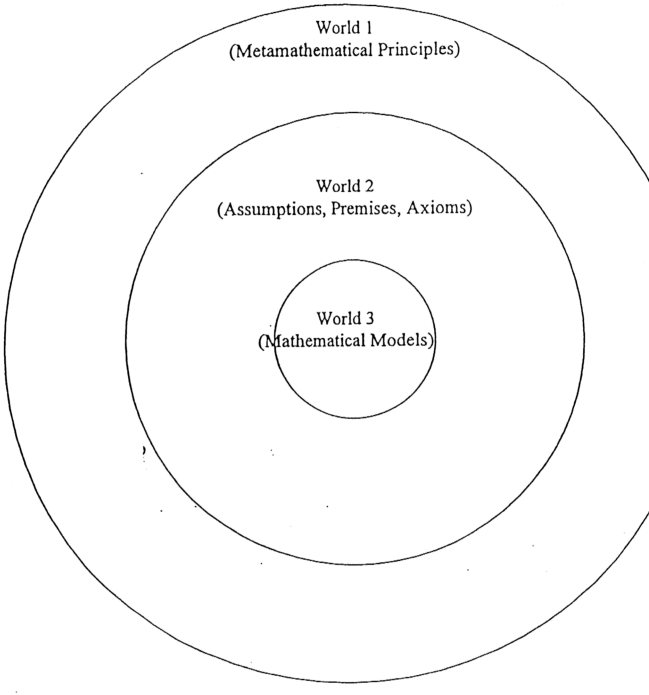
When we say that the contents of World 3 is derivable from World 2 and ultimately World 1, we mean that the elements of World 2 serves as the heuristic factor for World 3.

In all events, World 1 provides the overriding <u>regulative principles</u> for the other worlds. For instance, the discovery of anomalies with regard to a mathematical model shows not only that

there are inconsistencies in World 3 but also reinforce their belief in the incompleteness and uncertainty of mathematics at the level of sense experience and eventually, the 'external' part of mathematical knowledge.

We can see in this case that both the practitioners of Islamic and 'modern mathematics' share the view that there is no 'complete knowledge'. Although they do arrive at the same conclusion, the main reasoning underlying the conclusion is totally different. The chief reason for advocates of Islamic mathematics is the overiding belief in God as the only One who has complete knowledge whereas this aspect is not considered at all by the practitioners of modern mathematics in arriving at that conclusion. The same analysis applies to their concept of mathematical truth or mathematical certainty.[21]

[21]See for example, D. Kline, The Fall of Certainty., (New York; Oxford University Press, 1980).



Dia. 7.1 The Three Worlds Corresponding to Three Concentric Circles

The three worlds operating as the scheme of mathematical research imbedded in al-Bīrūnī's and Newton's philosophies of

mathematics point to an interesting aspect. Mathematics is claimed to function as a handmaiden of theology (mathematica ancilla theologiae). Our findings as reflected by the structure of the outline of their 'shared' philosophy also shows that it is theology that provides the foundation of their mathematics, eventhough it is less evident in Newton's compared to al-Bīrūnī's.

At least from the aspect of Divine Unity, theology is central in both al-Bīrūnī and Newton's overall conceptions of mathematics and thus it functions as the dominating factor in World 1 and consequently in the other two worlds. All there is has its roots in the Divine. Metaphysical principles residing in the metamathematical world are part of religious belief which is stongly entrenched in their hearts even before their lifelong engagement with mathematics. The doctrine of the existence of the levels of reality, the belief in hiearchy of truths, at least with the fundamental knowledge that Absolute Truth is the prerogative of God whose other names is The Truth, the uncertainty of mathematical knowledge at the level of sense experience and so forth is, in the first place, <u>not</u> a result of having mathematical knowledge alone. More important than that, it is a consequence of the deep-rooted belief <u>and</u> knowledge in the ever encompassing, ever knowing God; the Absolutely Infinite.

wa'l-ḥamdu li' Llāh wa bihi nasta ʿīn.