CHAPTER II

Al-BIRUNI'S MATHEMATIZATION OF NATURE

2.1 Introduction

Muslim mathematicians and natural scientists are linked to nature through the Holy Qur'ān and the traditions of the Prophet.[1] They are capable of studying nature because God has endowed them with the various faculties of knowing.

Nature, which is regarded in Islam as 'al-Qur'ān al-takwīnī', is objective and has several levels of reality. These are the spiritual (or angelic), animistic (or psychic) and material (or corporeal).[2] Where is the place of mathematics in the link between the mathematician and nature? In particular, how does al-Bīrūnī view the role of mathematics in solving problems discussed in the previous chapter? We believe that no proper philosophical study of al-Bīrūnī's philosophy of mathematics is possible without a prior investigation into some aspects underlying his concept of the

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[1]See Osman Bakar, Tawḥīd And Science, (Kuala Lumpur; 1991), p.17

[2]Ibid., pp.18-20

mathematization of nature such as the belief in the mathematical structure of the universe and that mathematical aspects of Nature are knowable. It is only from this investigation that we can unveil the true significance of mathematics to al-Bīrūnī.

2.2 Mathematical Structure of the Universe

As a scholar who preferred to think of himself as a mathematician, al-Bīrūnī embraces the view that mathematics is the key to unravel the mysteries of the universe. One can study the universe by way of mathematics is because all of the contents of the universe have mathematical structure. They are mathematically related. Let us consider for examples plants, leaves, beehives, and snowflakes. Al-Bīrūnī maintains that these things can be analysed mathematically. His examples of mathematical objects include even colours, motion, cycles and fever days. States al-Bīrūnī:

> Arguments can never be accepted unless there be a reason which properly connects that which is measured with that by which you measure, the proof with that which is to be proved. There exist, e.g. double formations or correlations in things opposite to each other (e.g. black and white), triple formations in many leaves of plants and in their kernels, quadruplications in the motion of the stars and in the fever days, quintuplications in the bells of the flowers and in the leaves of the most of their blossoms, and in their veins; sextuplications are a natural form of cycles, and occur also in bee-hives and snow flakes.[3]

Al-Bīrūnī claims that there is mathematical harmony in the structure of the universe. What he means by 'mathematical harmony' is that there is a uniformity, a mathematical regularity in every

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[3]See Chronology, pp.293-294

genus and species of nature. He does not mean by 'mathematical harmony' that there is a balance between regularity and irregularity. As far as the structure of the universe is concerned, it is the regularity and not irregularity which is more pervasive. Unlike regularity, irregularity happens only in isolated cases. That the contents of nature can always be classified to various genera and species points to this aspect. Thus:

> Possibly one may find among the species hitherto known such a number of leaves; but on the whole, one must say Nature preserves its genera and species such as they are. For if you would, just to cite an example, count the number of seeds of one of the many pomegranates of a tree, you would find that all the other pomegranates contain the same number of seeds of which you have counted first. So, too Nature proceeds in all other matter.[4]

Al-Bīrūnī's view with respect to the mathematical regularity of genera and species is shared by others, particularly the followers of Pythagoras. However we claim that al-Biruni certainly was not a Pythagorean. It is true that Pythagoras made statements similar to the one above such as the statement by Nicomachus who wrote that:

> All that has by nature and with systematic method been arranged in the Universe seems both in part and as a whole to have been determined and ordered in accordance with number, by the forethought and mind of Him that created all things; for the pattern was fixed, like a preliminary sketch, by the domination of number pre-existing in the mind of the world-creating God, number conceptual only and immaterial in every way, so that with reference to it, as to an artistic plan, should be created all these things, times, motions, the

[4]Ibid., p.295. All quotations underlined is by the author.

heavens, the stars, all sorts of revolutions.[5]

Nevertheless there are other aspects of Pythagorean mathematical philosophy concerning the structure of the universe such as the "virtues" and "personalities" of geometrical figures which are irrelevant to al-Bīrūnī. There is no evidence that has come to us to show that al-Bīrūnī accepts or rejects these qualitative notions.

The belief that the universe has a mathematical structure is also imbedded in al-Bīrūnī's concept of man. Man is created with the potential ability to study and understand nature. An important aspect concerning his potential capability is his capacity to count so much so that everything around him is countable. Man is endowed with this counting skill eversince he is born and those who develop the skill will know how to measure things, that is, to count in a more sophisticated way. Explains al-Bīrūnī:

> Counting is innate to man. The measure of a thing becomes known by its being compared with one another thing which belongs to the same species and is assumed as a unit by general consent. Thereby the difference between the object and this standard becomes known.[6]

There is another aspect underlying al-Bīrūnī's position that the universe has a mathematical structure. When he says that the universe has a mathematical structure, he is not only implying that

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[5]See Nicomachus, Introduction to Arithmetics, trans. M.L. D'Ooge (Chicago; Encyclopedia Brittanica, 1953) pp.813-814. See also S.H. Nasr, An Introductiion..., op. cit., p.49.

[6]See India, Vol. 1, pp.160-1.

the universe is such because everything can be associated with numbers. Rather he is also inferring that the structure of the universe can be analysed geometrically. In addition to numerical relations, things in the universe are likewise related geometrically. Al-Bīrūnī argues that such is the case because sensible objects take space and space has dimensions. Space has length, breadth and depth (or height). Says al-Bīrūnī:

> The dimensions of space (ab'ad al-makam) are three in number, length, breadth, and depth; these terms are not applied to the dimensions in themselves so as to be invariable, but relatively, so that one of them is called length, that which crosses it is breadth, and the third, which traverses both, depth, but it is customary to call the longer of the first two, length, the shorter, breadth ot width, and that which is extended downwards, depth [or thickness], while if its extension upwards is considered height.[7]

Therefore al-Bīrūnī's concept that the universe has a mathematical structure means that there are always mathematical relationship between things in the universe and that this relationship is both numerical as well as geometrical.

2.3 The Presence of Numbers and Geometrical Figures in Created Things

So far we have attempted to show that in al-Bīrūnī's philosophy of mathematics, the universe has a mathematical structure, that is the constituents of the universe are mathematically related. In this section, we want to make a stronger claim by arguing that in al-

[7]See Elements of Astrology, p.2

Bīrūnī's philosophy of mathematics, he perceives that things are not only mathematically related but they are also mathematical objects. In other words, one can always find the presence of numbers and geometrical figures in them.

The mathematical aspect of a plant is a case in point. Al-Biruni maintains that if one observes the plant carefully, one discovers that its shape follows the law of geometry; that the plant actually is an embodiment of geometry. Besides, the number of leaves too is in accordance with that law. Al-Bīrūnī argues further that such phenomena is true in most cases. Sensible objects in nature do have mathematical properties. States al-Bīrūnī:

> Among the peculiarities of the flowers, there is one really astonishing fact, that is, the number of their leaves, the tops of which form a circle when they begin to open, is in most cases conformable to the laws of geometry. In most cases they agree with the chords that have been found by the laws of geometry, not with conic sections. You scarcely ever find a flower of 7 or 9 leaves, for you cannot construct them according to the laws of geometry in a circle as isosceles. The number of their leaves is always 3 or 4 or 6 or 18. This is a matter of frequent occurence.[8]

Therefore a number (al-ʿadad), which according to al-Bīrūnī "is defined as a sum of units (aḥad)"[9] is always present in every sensible object. They invariably bear numerical aspects.

In addition to his belief that numbers are present in all physical objects, al-Bīrūnī claims that the numerical properties of all sensible objects follow a certain pattern. Each species has a

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[8]Chronology, pp.294-295.

[9]See Elements of Astrology, p.24.

distinct 'number-formation'. For example, a particular kind of flower always has six leaves whereas another kind has three. In other words, the presence of numbers is in an orderly fashion; numbers do not simply present. Thus:

> So all numbers are found in physical appearances of the works of the soul and life, and especially in flowers and blossoms. For the leaves of each blossom, their bells and veins, show in their formation certain numbers peculiar to each species of them.[10]

Since al-Bīrūnī believes that "Counting is innate to man" and numbers are present in every physical objects, it follows that numbers function as the foundation of mathematics in his philosophy. It is instructive here to refer to a statement of his on God and plurality of things. While discussing the Unity of God, al-Bīrūnī remarks:

> It is not impossible to think that the existing beings are not and that He is, but it is impossible to think that He is not and that they are.[11]

In similar vein, one can argue that "It is possible to think that nature is uncountable[12] and numbers exist, but it is impossible to think that nature is countable without numbers."

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[10]Chronology, p.294.

[11]See India, Vol. 1, pp.29-31. cf. H. Heras,"The Advaita Doctrine in al-Bīrūnī", in Al-Bīrūnī Commemoration Volume, op. cit., p.119.

[12]The word "uncountable" here means we cannot find a one-to-one relationship, for whatever reasons, between the contents of nature and natural numbers.

In addition to numbers, al-Bīrūnī embraces the view that geometrical figures are likewise present in sensible objects. Every physical objects possesses some kind of mathematical forms or figures. That is the reason that they have geometrical properties. The different shapes and sizes conform to different geometrical figures. There are circles in some flowers and triangles in some leaves. The leaves of the palm tree are triangular in shape whereas the interior of its fruit is spherical. Since these various forms and figures have geometrical properties, it is only proper that al-Biruni gives the following description about geometry:

> Geometry (al-Handasah) is the science of dimensions and their relations to each other and the knowledge of the properties of <u>the forms and figures found in solids</u>.[13]

Given that geometrical figures are present in every physical objects, they do not only occur in sensible things which are clearly observable but also in those which are not immediately observable. In order to illustrate this aspect of geometrical figures, al-Bīrūnī gives the example of the center of the earth. The earth is a physical object yet nobody has actually seen its center. Still al-Bīrūnī argues that it must have a center. When we think about the center of something, usually we think of it as a point which lies in the middle of the object.[14]It is the concept

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[13]See <u>Elements of Astrology</u>, p.1.

[14]We say "usually" because not all centers are located in the middle. The center of gravity of a falling object could be outside that object.

of center in this sense that al-Bīrūnī attempts to show. The center is nothing but purely geometrical. Explains al-Bīrūnī:

> The center [of the earth] is nothing but a point, and a part of the earth, no matter how small we conceive it to be, cannot fit at the center.[15]

The center of the earth is only one of the many geometrical figures present in al-Bīrūnī's concept of the earth. Al-Bīrūnī's belief that geometrical figures are present in sensible objects is also clear if we examine closely his concept of a geometrical figure. What is a figure to al-Bīrūnī? According to him, the concept of a figure is directly related to that of a line. Everything that we can observe has at least one line, from a simple instrument such as a pencil to the complex building, for example an observatory. All of them are figures. In more specific term, they are physical representations of some geometrical figures. "A figure (al-shakl)", writes al-Bīrūnī, "is that which is surrounded by one or more lines".[16]

Geometrical figures are related by some geometrical concepts. This is especially true when one compares geometrical figures. Al-Bīrūnī maintains that these geometrical concepts are present in every sensible things. In order to illustrate this aspect, let us consider an important concept in geometry which al-Bīrūnī has described. The concept is that of a ratio (al-nisbat). It is an

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[15]See the quotation from Dehkhodā in S.H. Nasr, An Introduction..., p.169

[16]See Elements of Astrology, p.4.

important concept to al-Bīrūnī especially because of his scholarly tendency to compare things, so much so that he is known as a comparitivist. In his discussion about geometry, al-Bīrūnī relates his concept of ratio:

> Ratio is the relation between two things of the same kind, by which we know the measure of the one as compared with the other. Thus we call a man 'father' when we contrast him with his son, and the latter 'son' when comparing him with his father. Similarly we call one thing half of another, which is double the former.[17]

Since numbers and geometrical figures are so fundamental in al-Bīrūnī's philosophy of mathematics, they play a significant role in his theory concerning the mathematical structure of the universe. Those who want to study the structure of the universe and like to delve in branches of science associated with mathematical knowledge for example astrology, should first familiarize themselves with numbers and geometry. It is his belief in their presence in the constituents of the universe that he writes in the preface of one of his book:

> I have begun with geometry and proceeded to Arithmetic and the Science of Numbers, then to the structure of the Universe and finally to Judicial Astrology, for no one is worthy of the style and the title of Astrologer who is not thoroughly conversant with these four sciences.[18]

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[17]See Elements of Astrology, p.11. We can see how easy it is for al-Bīrūnī to shift from one subject to another and to see the analogies between them from the above quotation. According to a well-known twentieth century mathematician, Banach, a great mathematician usually has this ability: "Good mathematician see analogies between theorems or theories, the very best ones see analogies between analogies". See M.C. Sharma, "Mathematics Learning Personality," Math Notebook, (7)(1&2)(1989), p.52

[18]See Elements of Astrology, p.1

And elsewhere:

> I say firstly, that the subject of this investigation can hardly be comprehended except after encompassing (knowledge of) the constitution of the universe according to what is shown by demonstration, excluding what the various groups of people apply to it of what they have heard from their ancestors, as well as recourse from the sects to their beliefs, and (also) after (attaining) the capability of dealing with its varying situations, in which one <u>cannot dispense with arithmetic and deep investigation of it by geometry</u>.[19]

In light of the above statements, we conclude that al-Bīrūnī views numbers and geometrical figures as mathematical entities that are deeply entrenched in sensible objects. From his point of view, one may also conclude that sensible objects are merely physical representations of these entities because it is by way of arithmetic and geometry that one can know more about them.

2.4 Nature of Mathematical Knowledge

Thus far we have examined al-Bīrūnī's belief in the mathematical structure of the universe and the presence of numbers and geometrical figures in created things. According to him these concepts serve as the foundation of mathematical knowledge, consequently mathematical knowledge is knowledge <u>derived</u> from our understanding of numbers, geometry and the view that the universe has a mathematical structure.

According to al-Bīrūnī, our understanding of numbers and

[19]See <u>The Exhaustive Treatise On Shadows</u>, p.6.

geometry in turn depends upon our ability to reason and memorize arguments. The interesting thing about this ability is that it cannot be acheived from manual labour. To use a phrase from education psychology, it is not a learned behaviour. Al-Bīrūnī believes that this ability is a gift from God. He writes:

> Memorizations of arguments and reasoning is an even more useful implement for the attainment of knowledge and all the proofs concerning the facts evolved will stay in memory very long and easy to draw upon at will but this is a gift from God. It cannot be attained by means of toil or sedulousness. God gives it to some and denies it to others[20]

Because one's expertise in mathematics depends heavily on this ability, his above statement points to the notion that mathematical knowledge is likewise a gift from God. Mathematical knowledge originates from Him.

That in al-Bīrūnī's philosophy of mathematics mathematical knowledge and its basic entities such as numbers and geometrical figures issues forth from God can also be established from a different angle. The soul of the mathematician does not have the actual mathematical entities. Other wise the soul would be conscious of them and consciousness in this case, implies 'automatic' mathematical intellection. If the mathematical entities are not in the soul, they must be located elsewhere. In order for

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[20]See al-Bīrūnī, Kitāb al-Saydanah fī al-ṭibb, transl. by H. Said, (Karachi, 1973), p.2 cf. to the Qur'ānic verse (2:269):

> "He (Allah) giveth wisdom (ḥikmah) unto whom He will, and he unto whom wisdom is given, he truly hath received abundant good"

them to exist elsewhere, they must exist by themselves or existing in an independent substance from which they originate, a substance which has the propensity to impose mathematical entities on sensible things and on the soul so that mathematization is possible. And to al-Bīrūnī, only God exists independently and the rest are nothing but His creations because "it is not possible to have an endless chain of succession..."[21] Consequently it follows that mathematical entities issues forth from God. It is no other than The Substance which al-Bīrūnī rightly and humbly admits, "All good comes from Him".[22]

In his philosophy of mathematics, al-Bīrūnī maintains that mathematical knowledge is governed by religion. He believes that both the theoretical and practical aspects of mathematics are guided by religion. Let us consider the theoretical aspects first, By 'theoretical aspects' we meant those values that are connected with formulation of mathematical theories such as 'simplicity' as opposed to 'complexity'. For example in his astronomical discussion concerning the formation of eclipse, al-Bīrūnī maintains that "the sentences of the Qur'ān on these and other subjects necessary for man to know are not such as to require unnecessary complexity (i.e. myths and the like)." He is referring to the Hindus who he believes had to create myth corresponding to the doctrine of the

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[21]See the translation of S.H. Nasr in his An Introduction..., pp.116-7 which is based on the text written by al-Bīrūnī in his Kitāb Taḥdīd Nihāyat al-Amākin. See also al-Bīrūnī's Chronology, p.116 and his Kitāb al-Jamāhir..., "Chapter on Pearls...," trans. and reproduced by F. Krenkow, Islamic Culture, (15;421)(1941).

[22]See India, Vol.II, p.246.

Purana in order to explain the astronomical phenomena.[23]

From the practical point of view, al-Bīrūnī believes that mathematics is governed by religion because mathematics is extremely useful in solving problems sanctioned by it. Mathematics enhance prayer, almsgiving, distribution of wealth and so forth. In this sense, mathematics is a subset of Islam. There is an organic relationship between mathematics and that of Islam. In response to those who disagree with this point of view, he writes:

> ...that prayer is the buttress of religion and that its perfection is restricted to (its observance) at its (proper) time and facing in the proper direction for it, and that both matters are connected with astronomy and a due amount of geometry; and almsgiving follows them, and [inherintances] there being no escape from them, just as there is no escape from buying and selling as a means of subsistance, in the Muslim Law and (since) all of them require arithmetic either in the lowest degree, in imitation of the method(s) of the computers, or else at its highest level, it being the deep investigation of geometry...for he is obliged to apply the two (arithmetic and geometry) in almsgiving for the manufacture of weights and measures, and in charity the making of standard units, and for the holy war numerous manufactures and various instruments of steel.[24]

As reflected in the above statement, al-Bīrūnī views mathematical knowledge as that knowledge which is extremely useful because it solves problems. Epistemologically speaking, problems of mankind have hardly changed. So do problems faced by the Muslims.

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[23]See al-Bīrūnī Commemorative Volume, p. 326

[24]See al-Bīrūnī, The Exhaustive Treatise On Shadows, p.8. It is interesting to note that wih the exception of music (ilm al-musiqa), al-Bīrūnī's enumeration of the mathematical enterprise closely resembles that of al-Fārābi. See O.Bakar, Classification Of Sciences..., op. cit., pp. 194-6.

In this space age, they need sophisticated time pieces and instruments to indicate the correct time and direction of qiblah in order to offer solat. Prayer is definitely not a trivial matter. As far as the economy of the ummah is concerned, certainly much more need to be done so that the distribution of zakāt (using computers) becomes more efficient. When we speak of preparation for holy war, military research in Muslim countries are definitely below satisfactory level. These are major problems faced by Muslims. It is not possible to solve them without sufficient knowledge in mathematics. In our opinion, al-Bīrūnī's emphasis on utility in the nature of mathematical knowledge definitely deserves more sympathy.

We have just described the 'power' of mathematical knowledge. According to al-Bīrūnī, there is another aspect in the nature of mathematical knowledge which some branches of knowledge may not have. As al-Bīrūnī has shown, mathematical knowledge has tremendous applications. This is true because it is very difficult to think of anything that cannot be mathematized. Yet mathematical knowledge is not easily grasped by everyone. Al-Bīrūnī maintains that most people have difficulty with it. Thus:

> The other two parties are of the common people, whose hearts are disgusted by the mentions of shadows, or altitude, or sines, and who get goose-pimples at the mere sight of computation or scientific instruments.[25]

Therefore it is the nature of mathematical knowledge that only few people are attracted to it. There is a reason for this. Al-

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[25]See al-Bīrūnī, On Transits, op. cit., p.75

Bīrūnī believes that mathematical knowledge involves a great deal of abstraction and not every people have this mental capability. Mathematical knowledge is not that kind of knowledge one acquires simply by 'seeing and believing'. In order to illustrate the processes of abstraction involved, let us consider the case of a point. The point is certainly a very basic mathematical entity. Yet to understand the concept of a point is not that easy. 'Seeing' a point is not like seeing an amoeba or a stag. A dot from the ink of a pen is not a point, however small it is. It is only one of its many physical representations. Points can only be grasped through our intellect. The same form of reasoning applies to al-Bīrūnī's concept of lines and surfaces. Says al-Biruni:

> If a line is finite, its extremities are points. Points have one dimension less than lines, viz., length; they have neither length, breadth, nor thickness, and are indivisible. The point of a sharp needle may be taken as an illustration from the sensible world, but surface, line, point, although they occur on solids which bear them, apart from them cannot be apprehended except by the intellect.[26]

Although al-Bīrūnī concedes to the importance of the intellect in acquiring mathematical knowledge, he does not submit to the philosophical position that mathematical knowledge is localized. Mathematical knowledge is not personal, so to speak. Mathematicians have their own academic comunities and al-Bīrūnī believes that this social aspect also contributes in the development of mathematical knowledge.

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[26]See Elements of Astrology, p.3

The social aspect is what we coin as the 'external factor'. It also determines the nature of mathematical knowledge. In more specific terms, al-Bīrūnī believes that there are basic agreements between mathematicians on some mathematical concepts and the progress of mathematical knowledge is based on these concepts. Let us take the case of numbers. Al-Bīrūnī defines numbers operationally and collectively. When we claim we know what numbers are, it is not the case that we can enumerate all of the natural numbers, just as we know what we mean when we say "al-Bīrūnī is a man" although we do not know all men individually. In defining numbers operationally, he writes:

> Since the unit of measure is not a natural unit, but a conventional one assumed by general consent, it admits both practical and imaginary division. Its subdivisions or fractions are different in different periods in one and the same country. Their names, too, are different according to places and times; changes which are reproduced, either by the organic development of languages or by accident.[27]

Although al-Bīrūnī's definition has a tinge of conventionalism,[28] he was far from being a conventionalist in his conception of the nature of mathematical knowledge in the sense this term is understood today. Admittedly mathematical conventionalism deserves substantial credit for the manner it has helped in clarifying the rapport between theory and experiment. It

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[27]See India, Vol. I, pp.160-161

[28]For other passages of his wherein convention is mentioned, see The Exhaustive Treatise On Shadows, pp. 40-1 and Elements of Astrology, p.2.

underscores the importance of the role played by the mathematician's activity which is executed in accordance with collective agreements, in conducting and interpreting scientific experiments. Conventionalists believe that laws of nature are our own free creations; our arbitrary decisions and conventions.[29] The laws of nature are simple but nature is not. For the conventionalist, mathematical description is not a picture of nature but simply a logical construction. It is not the properties of this world which define this construction; on the contrary, it is this construction that defines the properties of the world. A measuring rod is 'accurate' and a clock is 'precise' only if the measurements of these instruments satisfy the axioms of mechanics which physicists or mathematicians have agreed to embrace. In short, applications of mathematics come about by fiat and mathematical truths are merely agreements. We can see that in the the conventionalist's conception on the nature of mathematical knowledge, there is hardly any room for God. Yet God is fundamental in al-Bīrūnī's concept of mathematical knowledge since according to him it is religion that licenses mathematical knowledge. Therefore al-Bīrūnī's concept of the nature of mathematical knowledge is certainly not that of a conventionalist.

Morever al-Bīrūnī believes that there is an aspect in the nature of mathematical knowledge whereby conventionalism is out of

[29]The argument that natural laws are nothing more than shared definitions and therefore tautologies will ultimately support the belief that man, by himself, can know; thus solving the problem of induction.

place. It has to do wih the concept of mathematical truth. Al-Bīrūnī maintains that truths transcend the rule established by convention. For example in elaborating the astrologers view of the order of planetary spheres, he states:

> ...it was known that the people of this craft are (sic) agreed among themselves that the nearest sphere to us is the sphere of the moon and the farthest of the spheres of the planets from us is the sphere of Saturn. And if they said, it was regarding the transit of the moon, that it is above Saturn, it was denying their saying that one planet, the extreme distance from the earth of which is sixty four times its (earth's) radius, passes over another, the nearest distance of which from the earth is fourteen thousand eight hundred and eighty one times its radius. But it is an expression without leading to this meaning, which is well-known among them by agreeing on it by convention, although the order of the planets is not necessarily thus.[30]

However al-Bīrūnī believes that there are parts of mathematics wherein community decisions are needed. In more specific terms mathematicians agree on what assumptions are considered fundamental to a branch of mathematics. These assumptions are known as axioms or first principles. Al-Bīrūnī states:

> Arithmetic and geometry are impossible to understand unless one proceeds systematically from first principles, unlike other sciences in which he may acquainted with something of their middle (parts) or their ends without knowledge of their beginnings.[31]

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[30]See al-Bīrūnī, On Transits, (transl. by M. Saffouri & A. Ifram), (Beirut, 1959), p.14.

[31]See al-Bīrūnī, The Exhaustive Treatise On Shadows, p.6.

And elsewhere in his discussions on shadows:

> So that is not among the things which are incapable of being pictured among the first principles, like the impossibility of two bodies being in one and the same place together, or the presence of two opposites in one place together and at one time.[32]

As reflected in the above statements, al-Bīrūnī argues that there is a major difference between the nature of mathematical knowledge and the nature of other branches of knowledge such as history or philosophy. Admittedly all of these branches of knowledge have their underlying assumptions. But the relationship between these assumptions and the development of the corresponding branches of knowledge is different. This does not imply that mathematical knowledge accumulates or progresses through deduction and that the other branches by way of induction. Certainly this is not what he meant because there are induction and deduction in all branches of knowledge, mathematics or otherwise. For that matter, all branches of knowledge are built upon their underlying assumptions.

In our opinion, what he meant by the phrase "unless one proceeds systematically from first principles" is best understood from the perspective of acquiring mathematical knowledge. A student of history can begin his quest of knowledge by studying any aspect of history. This is due to the nature of history. Generally speaking, history is by and large about story and events. There is no definite order in reading storybooks. Such is not the case with

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[32] Ibid., p.36.

mathematics. From al-Bīrūnī's point of view, mathematical knowledge is knowledge about the structure of the universe. It is not so much about story and events. The underlying assumptions of mathematics are truths (such as two points determine a line or things equal to the same things are equal to one another) and mathematical knowledge is constructed upon these truths. Thus without first knowing what these truths or first principles are, how can the student "proceed" from them?

We have given examples of al-Bīrūnī's "first principles" in geometry. If the notion of "first principles" is integral in al-Bīrūnī's conception on the nature of mathematical knowledge, then there must be "first principles" in arithmetic too. Al-Bīrūnī gives the following account of arithmetic:

> Arithmetic (al-ḥisāb) is the treatment of numbers and their properties in solving problems by way of addition and subtraction, increases and decreases.[33]

As reflected in the above statement, according to al-Biruni arithmetic is about numbers. Since "first principles" are axioms or basic truths and al-Bīrūnī believes that "all numbers are found in the physical appearances of the works of soul and life," it follows that in al-Bīrūnī's philosophy of mathematics, one of the "first principles" in arithmetic is the belief in the encompassing presence of numbers.

Therefore both arithmetic and geometry have "first

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[33]See <u>Elements of Astrology</u>, p.31.

principles". Since al-Bīrūnī believes that mathematics is constructed on arithmetic and geometry, therefore in al-Bīrūnī's philosophy of mathematics it is the nature of mathematical knowledge that its branches have "first principles" too.

So far we have not established the claim that al-Bīrūnī's "first principles" are self evident truths. We have only mentioned that they are truths. We can examine the claim that they are self-evident truths by analysing one of his statements on geometry. Al-Bīrūnī believes that one can arrive at truth concerning the universe by way of mathematics. He states:

> By it [geometry] the science of numbers is transferred from the particular to the universal, and astronomy removed from conjecture and opinion to a basis of truth.[34]

In other words, if one "proceeds systematically from first principles," one can arrive at truth. But ontologically speaking truth cannot follow from falsehood.[35] In this sense, a true theory has to be a consequence of a true axiom. It is instructive to quote Aquinas who states:

> Plato said that unity must come before multitude; and Aristotle said that "whatever is greatest in being and greatest in truth, is the cause of every being and of every truth," just as "whatever is the greatest in heat is the cause of all heat".[36]

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[34] Ibid., p.1

[35] Surely in terms of pure logic, everything follows from falsehood.

[36] See under "Treatise On the Creation" in Aquinas, S.T. The Summa Theologica (transl. by Fathers of the English Dominican Province), (The University of Chicago, 1952), p. 238.

In similar vein, al-Bīrūnī's "first principles" are self-evident truths because it is "the greater truth" that is the cause of the "great truth".

Although it is the nature of mathematical knowledge to produce true theorems, al-Bīrūnī believes that it is also the nature of mathematical knowledge that whatever theorems it produces are not necessarily guaranteed truths. That they are not necessarily true laws is because they are ceterus paribus generalizations; generalizations that hold only under particular, usually ideal conditions. Ceterus paribus is translated as 'other things being equal' or 'other things being right'.[37] We sympathize with the latter because in my opinion, it describes al-Bīrūnī's position more correctly. Al-Bīrūnī believes that the truth or falsity of a theory is connected to the conditions attached to it. There are instances where theories could turn out to be false, especially when the conditions are not 'right'. Alluding to the significance of ceterus paribus in mathematical knowledge, he writes:

> Whichever of the two theories may be correct, whether the Anwa are to be traced back to the days of the year or to the rising and setting of the Lunar Stations, in any case there is no room for a third theory. To each of these theories, whichever you may hold to be correct, certain conditions attach, on which the correctness of the Anwā depends.[38]

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[37]See Cartwright, N. How the Laws of Physics Lie, (Oxford University Press, 1983) p.45.

[38]See Chronology, p.232.

Since mathematical laws have conditions attached to them, al-Bīrūnī is very much aware of the uncertainty aspect of mathematical knowledge. Let us take for instance, the law of the excluded middle. Al-Bīrūnī used it extensively in his mathematical reasoning. For example in his correspondence with Ibn Sīnā concerning infinite divisions, he gives the following argument:

> According to your view, it becomes necessary that the side of a square be equal to its diagonal; if you deny it you have opposed your own principles. Or you will say that between the parts there is a separation; in this case I ask if the separation is <u>greater or smaller</u> than the indivisible parts.[39]

Yet al-Bīrūnī did not elevate the law of the excluded middle, or for that matter any mathematical law, to the level of an absolute truth on a par with revelation. He is conscious of the realm of possibilities even in mathematics; that with the ever creating God, literally anything is possible. Between A and not A, there can exist things which are not only plausible, but also true. There can be two contradictory worlds existing simultaneously.

> A group of sages have been of the opinion that it is possible for another world to exist <u>which differs from this world in nature</u>. Aristotle has considered their views detestable, but his hatred is untimely and out of place. For we find information about natures and element of things when we observe them with our eyes like a man born blind who can find about sight only when he hears about nature from other people. And if there were no faculty of hearing he would not know that in the world there is such a sense as sight, the fifth

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[39]See S.H. Nasr. <u>An Introduction...</u>, p.171. See also F.A. Shamsi. "Ibn Sīnā's Argument Against Atomicity of Space/Time", <u>Islamic Studies</u>, 23(1984), pp.83-103.

sense, by means of which colours and heights and shapes become visible and observable. To sum up, what harm can there be if there is a world which as we say differs in the directions of motion and is separated from this world by an isthmus so that each is hidden from the other?[40]

To recapitulate, al-Bīrūnī believes that mathematical knowledge issues forth from God. Mathematical knowledge is invaluable because it is of great utility to religion. And not everyone can master mathematics. This is in part due to the variations in their intellectual capabilities. Al-Bīrūnī maintains that it is part of the nature of mathematical knowledge that it is built on axioms or "first principles". The axioms and some mathematical procedures in turn are agreed upon by way of convention. He also believes that mathematical knowledge has its own limitations.

2.5 How Mathematical Aspects of Nature are Knowable

In the foregoing discussions, we have focussed on al-Bīrūnī's belief in the mathematical structure of the universe, in the presence of numbers and geometrical figures in created things and his view on the nature of mathematical knowledge. In this section, we shall examine al-Bīrūnī's position with respect to the way mathematical aspects of nature are knowable. In other words, we want to analyse how mathematicians can arrive at mathematical methods and models.

In our opinion, al-Bīrūnī believes that there are levels of

[40]See the translation of S.H. Nasr in his <u>An Introduction....</u>, <u>op. cit.</u>, p.168.

mathematical representations based oh his statements about 'things' mathematized. These various levels correspond to the levels of reality in nature as we will show in due course. In more specific term, there are mathematical representations at both material and immaterial levels. Mathematical models are products of applying mathematical methods to these mathematical representations. Mathematical methods in turn are results from applying mathematical concepts to mathematical representations.

In order to illustrate his belief that there are levels of mathematical representations, let us consider his statement concerning lines. He writes:

> If a surface has boundaries, these are necessarily lines, and lines have length without breadth therefore one dimension less than the surface, as that one has one less than the solid; if it had breadth, it would be a surface, and we have assumed it to be the boundary of a surface. A line can be imagined by observing the oil and water at the side of the glass, or the line between sunshine and shadow, contiguous on the surface of the earth, or, also, it is possible to picture all that to oneself from a thin sheet of paper [although it has thickness], until the familiar sense-perception leads gradually to the intellectual concept.[41]

In the above statement, al-Bīrūnī's examples of the mathematical representations of lines at the material level are the boundary separating "oil and water at the side of the glass" and the boundary dividing "sunshine and shadow".

We will give another example to buttress my claim that al-Bīrūnī believes in levels of mathematical representations. In his explanation about the number 'one', he writes:

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[41]See Elements of Astrology, p.3

> Although 'One' (al-wāḥid) is in reality indivisible, nevertheless the unit one as a technical expression, employed in dealing with sense-objects (al-maḥsūsāt), ...is obviously capable of sub-division...[42]

So there are two mathematical representations of 'one'. The first is at the immaterial level whereby "one is in reality indivisible" and the other is at the level of material objects.

In al-Bīrūnī's philosophy of mathematics, a mathematician arrives at a mathematical method by applying mathematical concepts to these mathematical representations. An example of al-Bīrūnī's mathematical concept is 'ratio' (al-nisbat) which he describes as "the relation between two things of the same kind". The "two things of the same kind" include the mathematical representations of two numbers or two lines. It is the application of the mathematical concept of 'ratio' that a mathematician can arrive at the mathematical method of measuring. Thus his statement that "ratio is the relation between two things of the same kind, by which we know the measure of the one as compared with the other".

Al-Bīrūnī's concept of ratio is equivalent to his concept of comparing mathematical representations. Thus the concept of comparing in the the sense of al-Bīrūnī, is also a mathematical concept. When we measure something, essentially what we are doing is comparing it with another one of its kind. Says al-Bīrūnī:

> The measure of a thing becomes known by its being compared with another thing which belongs to the same species and is assumed as a unit by general consent. Thereby the difference

[42] Ibid., p.24

between the object and this standard becomes known.[43]

There is an explanation for al-Bīrūnī's insistence that a thing should be "compared with another thing of the same species". The reason is that a comparative concept gives the mathematician more information about a species. The comparative aspect such as "longer, taller, or shorter" relates the element of the same species. A comparative term gives us more information about 'a' line than just the predicate name 'line', just as an 'overweight man' gives us more information than the predicate name 'man'.

The more precise the comparison is, the more information the mathematician gets and according to al-Bīrūnī the best way to improve the precision is by assigning numbers, which he has defined as "sums of units". When numbers are assigned to the qualitative comparative concepts, the mathematician can compare the mathematical representations to the unit measure rather than any arbitrary element of that species and "thereby the difference between the object and this standard becomes known".

In addition to the mathematical method of measuring, there is another mathematical method which is a result of applying the mathematical concept of ratio. This is the method of weighing. According to al-Bīrūnī, weighing is similar to measuring. Al-Bīrūnī writes:

> By measuring (with dry measures) people determine the body and the bulk of a thing, if it fills up a certain measure which has been gauged as containing a certain quality of it, it

[43]India, Vol.1, p.161.

> being understood that the way in which their surface is determined, and the way in which, on the whole, they are arranged within the measure, are in every case identical. If two objects which are to be weighed belong to the same species, they then prove to be equal, not only in bulk, but also in weight; but if they do not belong to the same species, their bodily extent is equal, but not their weight[44]

In al-Bīrūnī's philosophy of mathematics, weighing is similar to measuring because there are instances wherein sensible things have the same measurements yet their weights are different. Such is the case where "the body and the bulk of a thing...fills up a certain measure" but "they do not belong to the same species". Consequently "their bodily extent [measurement] is equal, but not their weight".

Al-Bīrūnī was unequivocal with regard to the difference between the mathematical method of measuring and weighing. Mathematicians measure in order to "determine the body and the bulk of a thing" whereas they weigh so that they can "determine the amount of gravity of heavy bodies..." States al-Bīrūnī:

> By weighing, people determine the amount of gravity of heavy bodies, when the tongue of scales stands at right angles on horizontal plane...[45]

Al-Bīrūnī uses the mathematical method of weighing which is deeply rooted in the mathematical concept of ratio in order to explain another mathematical method, algebra (al-jabr waʾl-muqābalah). He describes algebra as follows:

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[44] India, Vol. I, pp.160-66.

[45] Ibid., p.162.

> If things of different nature in the scales of a balance are in equilibrium, the scales remain parallel, the tongue vertical and the beam level. It is obvious that if you take anything from one of the scales of one kind you must remove the like from the other both in kind and amount so as to preserve the equilibrium and the previous condition. Similarly if you add anything to one scale you must add a like amount to the other.[46]

As suggested by the name 'al-jabr wa'l-muqābalah', there are two operations involved. One is 'jabr' and the other is 'muqābalah'. "Should there be a minus quantitiy on one side [of the balance] it is necessary to remove it and to restore the equilibrium by adding a like amount to the other side. This is the operation of jabr",[47] says al-Bīrūnī. Concerning the operation of muqābalah, he writes:

> When the operation of jabr has been concluded, we turn to that of muqābalah which consists in comparing things of the same nature, (munājasat), which maybe on the opposite sides, and then deducting the smaller of these from both sides.[48]

In addition to mathematical methods such as measuring, weighing and algebra, al-Bīrūnī also used another method which is common to other branches of knowledge. Still, the method is connected to the concept of ratio or 'comparing things' of the same kind. The mathematician compares solutions to the same problem. We are referring to the usage of examplars[49] by al-Bīrūnī.[50] Examplars

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[46]See Elements of Astrology, p.37.

[47] Ibid., p.37.

[48]Ibid., p.38. The operation here is essentially the same to that given by Muḥammad ibn Mūsā al-Khwārizmī (d.863H), the first Muslim to work on algebra.

[49]The significance of examplars is stressed by Kuhn in his Structure of Scientific Revolution, op. cit.. Examplars, to Kuhn, are tools for scientists to solve "puzzles", that is, his term for

are analogical examples found in textbooks, functioning as heuristic guides. The examplar in the particular problem al-Bīrūnī seeks to solve is the work of Erathosthenes. Writes al-Bīrūnī:

> We have not so far been able to experiment with this dip, and its value in any high place. We were led to this method of Abū al-ʿAbbās al-Nayrizi who states that Erathosthenes has mentioned that the heights of the peaks of the mountains would be five and the half miles when the length of the radius of the earth is approximately 3,200 miles. For the solution of this problem, it is necessary mathematically that the dip of the horizon in the mountain wherein the perpendicular is so high should be about one third degree.[51]

Another mathematical method familiar to al-Bīrūnī which is likewise based on the concept of comparison is the graphic method. In his statement concerning those people who prefer rigorous calculation than the graphic method, he says:

> As some people have a predilection for calculations, and like to arrange them in tables, and prefer them to graphic method we shall also have to show them how we may find, by calculation, the diameters of the circles of longitude and latitude, and the distances of their centres from the centre of the (great circle).[52]

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scientific problems. Al-Bīrūnī's view on scientific problems as elaborated in the previous chapter, however, is by no means identical to that of Kuhn because of the 'sacredness' associated with them.

[50]A. Saidan has shown that al-Bīrūnī compares the trigonometrical works of his contemporaries. See A. Saidan, "The Trigonometry of al-Bīrūnī", in Commemorative, pp. 681-683.

[51]For computational details, see S.H. Barani, "Muslim Researches in geodesy", in Al-Bīrūnī Commemoration Volume, p.33.

[52]See Chronology, p.360.

When the mathematician applies mathematical methods such as weighing and measuring to mathematical representations in order to solve a problem, he will arrive at mathematical models. A mathematical model is a solution to a problem. We will illustrate al-Bīrūnī's approach with a well-known problem which he solved at Nandana on 19th September 1024 AD.[53] The problem was to find the earth's dimensions.

Although the problem is not literally sanctioned in the Holy Qur'ān, it is a legitimate consequence of God's injunction to travel and observe the signs of God.[54] A correct measurement of the circumference of the earth and its diameter, coupled with his skill in finding latitudes and longitudes and computation of distances, enables al-Bīrūnī to chart longitudes and latitudes of the globe and determine location of cities and countries. It also helps in determining the direction of the 'qiblah'. It is of tremendous help to everyone, particularly to preachers, soldiers, travellers and Muslim rulers. Explaining the way he solves the problem, he writes:

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[53]We pick out this example because in my opinion, it is a major work of al-Bīrūnī since he describes it in three of his major works; Tahdīd Nihāyat al-Amākin, Kitāb-ul-Tafhīm-Li-Sinā'at-Tanjīm, and al-Qānūn al-Mas'ūdī. For further details of his solution, see S.H. Barani, "Muslim Researches in Geodesy," Al-Bīrūnī Commemoration Volume, pp.11-22. See inter alia, S.Husain Rizvi, "A Newly Discovered Book of al-Bīrūnī 'Ghurrat-uz-Zījāt' and Al-Bīrūnī's Measurements of Earth's Dimensions", Al-Bīrūnī Commemorative Volume, pp.605-619; S.H. Nasr, An Introduction..., pp.128-130; and H. Said & A. Zahid, Al-Bīrūnī- His Life, Times and Works, pp.164-171.

[54]See al-Qur'ān; (10: 5-6), (12: 109), (22:46), (6:99) and (13:3).

You climb a mountain situated close to the sea or a level plain, and then observe the setting of the sun and measure the dip of the horizon we have already mentioned, and then measure the value of the perpendicular of the mountain. You multiply this height into the sine of the complementary angle of the dip, and divide the total by the verse sine of this dip itself. Then multiply (the double of) the quotient into 22 and divide the results of this multiplication by 7. You will get the length of the earth's circumference (in the same proportion) in which the height of the mountain has been found.[55]

In the above problem, the earth, mountain and the sun, (of which their mathematical representations consist of a circle, perpendicular line and a point respectively), are finally organised into a mathematical model as shown in the next page.[56] Similar analysis can be given to other mathematical problems.[57]

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[55]See Barani, Commemoration Volume, p.35. See also Validi Togan's Picture of the World-- Kitāb al-Taḥdīd, op. cit., p.245.

[56]Cf. S.H.Naṣr, An Introduction..., (Cambridge, 1964) under the chapter "al-Bīrūnī". Al-Bīrūnī's mathematical models should not be construed as efforts resulting from the Greek's approach of "saving the phenomena" chiefly because of his emphasis of the empirical import, especially the aspect of experimentation involved in testing the model. His mathematical models are more than geometric devices of describing the phenomena. With respect to "saving the phenomena", Simplicius says: "Eudoxus of Cnidos was the first Greek to concern himself with the hypotheses of this sort, Plato, having, as Sosignes says, set it as a problem to all serious students of this subject to find what are the uniform and ordered movements by the assumption of which the phenomena in relations to the movements of the planets can be saved (sozein ta' phainomena)". See Simplicius, In II De Caelo, 12, comm. 43 (Venice, 1548), fol. 74r-v.

[57]See for example, Chronology, pp. 360-365 and Elements of Astrology, p.61. Other examples abound throughout his mathematical writings, in particular the contents of al-Qānūn al-Masʿūdī.

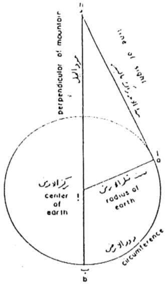

Fig. 1 Al-Bīrūnī's mathematical model to solve the problem of the Earth's dimensions.

Al-Bīrūnī discovered that the circumference of the earth is 80,780,039 cubits 1' 38" which is equivalent to 24, 778.5 miles, 33 miles less than the modern values.[58]

2.6 Mathematical Abstraction by the Internal Senses.

In our previous discussions concerning al-Bīrūnī's philosophy of mathematics, we have shown, inter alia, that mathematics is

[58]For computational details, see Barani, "Muslim Researches in Geodesy", Al-Bīrūnī Commemoration Volume, pp. 40-42, 50-52

intellectual in nature and that there are levels of mathematical representations which correspond to the levels of reality in nature. The mathematician gains knowledge by processing these mathematical representations through his senses. We have dealt with al-Bīrūnī's position regarding the function of the external senses in Chapter 1. In this section we wish to focus on the cognitive aspect of al-Bīrūnī's concept of mathematical abstraction, especially the function of the internal senses.[59]

That al-Bīrūnī concedes to the significance of the internal senses is reflected in some of his comments concerning the way mathematical representations are abstracted and processed. There is a connection between the intellect and the imaginative faculty. He refers to the intellect in the course of mathematical abstraction at several places. For example, he writes about the line, that "it is possible to picture [a line] to oneself from a thin sheet of paper until the familiar sense perception leads gradually to the intellectual concept".[60] He also states that the point "cannot be apprehended except by the intellect", even though "they occur on solids which bear them".[61] So there is a process which is not carried out by the external senses whereby mathematical

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[59]That the internal senses have been identified by earlier philosophers such as al-Fārābī, see O. Bakar, Classification of Knowledge in Islam: A Study in Islamic Philosophies of Science, (Kuala Lumpur, 1991) pp.49-64. Critics of al-Farabi's treatment can be found in ibid. p.54. On the internal senses as presented by Ibn Sina, see F. Rahman, Avicenna's Psychology, (London, 1952).

[60]See Elements of Astrology, p.3

[61] Ibid., p.24.

representations reached the mathematician in the manner that the intellect can arrive at the meaning of the mathematical representation.

From the above statements, we can derive an important principle in al-Bīrūnī's philosophy of mathematics; that the function of the intellect is crucial in the course of mathematical abstraction. Unlike al-Fārābī or al-Bīrūnī's contemporary Ibn Sīnā, al-Bīrūnī did not spell out in great detail the nature of the intellect as far as mathematical abstraction is concerned apart from the fact that it is the intellect which perceives the meaning of mathematical representations. He was vague concerning the manner by which the intellect operates although he knew that there is a connection between the intellect and the imaginative faculty.

Al-Bīrūnī believes that the imaginative faculty is just as important as the the sensitive faculty. Commenting on the way to solve the problems of finding the earth's dimension, he says that the solution "is quite conceivable in imagination and it rests on sound deduction".[62] And concerning the number 'one', he states: "Since the unit one is not a natural unit, but a conventional one assumed by general consent, it admits both practical and imaginary division".[63]

However al-Bīrūnī did not have a comprehensive account of the imaginative faculty in his philosophy of mathematics. Out of the

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[62]See A. Zeki Velidi Togan, Bīrūnī's Picture of the World, p.245. See also Barani, Commemoration Volume, p.35.

[63]India, Vol.1, p.162.

five well known internal senses,[64] he made a passing remark on the function of the faculty of memory (al-quwwat al-ḥāfiẓah); that "... all the proofs concerning the facts evolved will stay in memory very long and easy to draw upon at will...".[65] The main function of that faculty is to retain mathematical representations. According to al-Bīrūnī, the mathematical representations retained are used whenever the mathematician is involved in formulating mathematical arguments.

In short, the role of the internal senses in al-Bīrūnī's philosophy of mathematics is not clearly defined even though he alludes to the connections between the intellect and the imaginative faculty, especially the faculty of memory. We can only claim that in his concept of mathematical abstraction by the internal senses, the ascension of sensibles in the external world in the form of mathematical representations begins with sensual perception, through imagination then it attains to a higher level of abstraction by way of the internal senses which include the faculty of memory before the mathematical representations are finally abstracted by the intellect which is the seat of intelligence.

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[64]These are the faculty of representation (al-quwwat al-muṣawwirah), the faculty of estimation (al-quwwat al-wahm), the faculty of memory (al-quwwat al-ḥāfiẓah), the faculty of compositive human imagination and the faculty of compositive animal imagination (al-quwwat al-mutakhayyilah). See O. Bakar, Classification of Knowledge in Islam, op. cit., pp. 51-2.

[65]See Kitāb al-Ṣaydanah fi al-ṭibb, transl. by H. Said, op. cit., p.2.

2.7 Conclusion

The foregoing discussions on al-Bīrūnī's mathematization of nature shows that he firmly believes in the mathematical structure of the universe and the presence of number and geometrical figures in created things. The fundamental nature of the order pervading the cosmos such as harmony and uniformity is mathematical and this mathematical aspects of nature are knowable. It is due to all of these aspects of nature that problems can always be mathematized and solved mathematically, by the external as well as the internal senses.

Al-Bīrūnī's mathematization of nature enables us to see the organic relationship between mathematics and Islam and is very much relevant today. His perspective on the mathematization of nature surely enlightens those mathematicians who see mathematical activities as separated from religion, as well as those so-called 'ulamā' who consider mathematics to be of little value to the general well being of the 'ummah'.