CHAPTER III

ON THE STATUS OF MATHEMATICAL KNOWLEDGE

3.1 Introduction

In this chapter, we will examine in greater detail the nature of the existence of mathematical objects and representations in al-Bīrūnī's philosophy of mathematics. We are interested in the status of mathematical entities in his problem solving activity and the effects they have on his concept of mathematization.

According to al-Bīrūnī, mathematical activity and mathematical knowledge have a positive role to play in helping one to realize a consciousness of God. The status of mathematical knowledge in his philosophy of mathematics is also determined by the impact of that knowledge on the soul of the mathematician. Since there is an important relationship between the mathematical knowledge acquired and the soul of the mathematician, we shall focus on the necessity of safe-guarding the soul in so far as the soul is related to

acquiring mathematical knowledge.

Another aspect of al-Bīrūnī's conception concerning the status of mathematical knowledge that we will analyse in greater depth is his view of the notion of mathematical truth and the uncertainty of mathematical knowledge. The status of mathematical knowledge, for that matter any knowledge, surely depends on its ability to procure truths.

3.2 Numbers and Numbering, Zero, One and Many.

In this section, we shall examine al-Bīrūnī's thought on numbers and we will show that there are three levels of existence of numbers imbedded in al-Bīrūnī's philosophy of mathematics. The three levels correspond to the physical, mental and metaphysical level of existence.

In order to show the various levels of existence, first of all we need to distinguish two levels of perception in al-Bīrūnī's philosophy of mathematics. Al-Bīrūnī maintains that the two levels of perception, for example, brought forth dualism in belief. The scholars have different belief from the masses because they can percieve through the second level. The perceptions of the masses, by and large, do not go beyond the first level. They are more attracted to the sensible world than the world of abstract thought favoured by their scholars. They perceived God as idols[1] or the idols as intercessors with God.[2] The Greek masses, for example,

[1]See India, p.53.

[2]This is the belief of the pagan Arab. See India, p.59.

believed their idols to be mediators between them and the First Cause.[3] The educated among them who can percieve through the second level, however, disapproved idolatory. Instead they believed God to be eternal, omnipotent, and beyond gross representations.[4]

In other words, the sensible mathematical objects at the first level of perception become part of the veil between him and God. The mathematician does not realise that in the final analysis, there is no such separation as he has initially perceived at the rational and empirical level of ordinary experience.

At the second level of perception, al-Bīrūnī believes that the mathematician ascends to a higher hierarchy of mathematical truth. He understands that gross mathematical objects which include physical representations of numbers in the external world are not the ultimate level of reality. He is convinced that there is in fact another pervasive reality, the One which continuosly dominates the Many. He believes this higher reality to be God. Furthermore he is convinced that although it is He Who creates mathematical objects and everything else, He is not underdetermined by them. He is 'separated ' from them. At this level too, the whole processes of mathematical abstraction and the resulting mathematical experience integrates with the mathematician's spiritual insight by way of inner witnessing (Shuhūd), presence (ḥuḍūr) and trans-

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[3]See India, pp.59-60.

[4]See ibid., Vol. 1, p.18. Regarding al-Bīrūnī's interpretation on the origin of idolatory, please refer to ibid, p.111., cf., A. Jeffery, "Al-Bīrūnī's Contribution to Comparative Religion", Al-Bīrūnī Commemoration Volume, pp. 36-37.

empirical awareness (aḥwāl)[5]. It is at this level that numbers, for instance, have metaphysical existence. At this higher horizon, mathematical knowledge implies unification (tawḥīd) of the soul of the mathematician with the Absolute Truth which underscores all mathematical meanings. According to al-Bīrūnī, the mathematician will then understand the meaning of the Quranic verse:

> "We shall show them Our portents on the horizons and within themselves until it will be manifested unto them that it is the Truth" (xli: 53)[6]

It is the epitome of mathematical experience. As a matter of fact, the mathematician gains intuitive knowledge of the One, in which the number one is nothing but a very pale mathematical manifestation of one of His Divine Names and Qualities (al-wāḥid), never His Essence (dhāt). In this state, the mathematician experiences God's revelation of an Aspect of Himself through His Beautiful names (Asmā') or Attributes (Ṣifāt).

Mathematical objects in the external world in the second level of perception, according to al-Bīrūnī, are reflections of something real. Mathematical objects exist at each cosmic moment independent of the mind of the mathematician. Each moment is discontinuos, thus making counting and mathematization of problems possible. According to al-Bīrūnī;

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[5]See O. Bakar, Tawḥīd And Science, op. cit., p.50. Other terminologies used are ishrāq, mukāshafah, baṣīrah, naẓar, badīhah, ḥads and firāsah. See ibid, p.38.

[6]See Kitāb al-Jamāhir, p.5.

> Duration, or time in general, only applies to the Creator as being his age, and not determinable by a beginning and an end. In fact, it is his eternity... But as regards common time, which is determinable by motion, the single parts of it apply to beings beside the Creator...[7]

And elsewhere he elaborates in greater detail the 'discontinuity' of time with respect to creations. Says al-Bīrūnī;

> Also, verily time is the extension between two assumed instants, the two being two times of two known states, and because of the existence of these two situations, one after the existence of the other, the extent (of time) betweeen the two may include length or shortness, and (whatever) situation may exist in it <u>in succession</u> capable of having smallness and largeness.[8]

In accordance with al-Bīrūnī's belief that nature is countable, we posit that in al-Bīrūnī's philosophy of mathematics the creations are like the series of natural numbers. If we assigned the continuosly created mathematical objects according to the series of natural numbers, we will find that percieved as numbers, no matter how big they are, they can be derived from one without the one ever losing its original identity in the sense of 'losing' part of itself. In fact, every created mathematical object in the unending series of creation have its identity, which is a reflection of its archetype in the 'lawḥ maḥfūẓ' .[9]

Al-Bīrūnī maintains that at the second level of perception, the mathematician realizes that mathematical objects which include physical representations of numbers are nothing but creations of

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[7] <u>Chronology</u>, p.118.

[8] <u>The Exhaustive Treatise...</u>, p. 12. All quotations underlined is by autho

[9] See al-Quran, (6:59)

the One which have descended to the level of sense experience. He sees with his heart (qalb) the One characterizing the Many without ever imperfecting Itself, the Many appearing and disappearing perpetually. In his mathematical experience, he sees God through His Attributes. The mathematician experiences the revelation of God: "Wheresoever you turn, there is the Aspect of God"[10] and he is ever more convinced that "to Him shall they all be returned".[11]

In mathematizing and solving the problem he will admit that it is Him that made it all possible and "above every knowing man, there is God all-wise!"[12] Thus al-Bīrūnī, in the process of mathematizing the problem on Shadows, reminds himself that:

> God, be He exalted! is the Helper, and the Praised at the beginning of each treatise and at its end. By the praise of God and His help, finished is "The Exaustive Treatise on Shadows", ... and to God be the Praise,[13]

Elsewhere, having solved the problem posed to him by "a learned man",[14]concerning the eras observed by different nations, al-Bīrūnī concludes by saying:

> Let us finish our book with the praise of God, who afforded me help and guidance, and who taught me to distinguish the path of truth from the path of blindness.[15]

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[10]See al-Qurʾān, 2:115.

[11] Ibid., 3:83.

[12]See Chronology, p. 335.

[13]See The Exaustive Treatise On Shadows, p. 281.

[14]See Chronology, p.2

[15]See ibid., p.365.

Therefore the 'union' with God in al-Bīrūnī's mathematical experience does not mean union with God as He is in Himself, instead it is when God manifests Himself through His Beautiful Names and Sublime Attributes. The mathematician is ever aware that his mathematical experience, is incomplete and limited because God in His Essence is beyond any mathematization. Commenting on the Hindu whose mathematical experience has been led astray, al-Bīrūnī writes that they began to think of God as a point,[16] which to al-Bīrūnī is just a mathematical representation in geometry.

Moreover, al-Bīrūnī considers those people who claim union with God in His Essence as those who are misguided and who have deluded their followers. He classifies them under the Chapter entitled "On the Eras of the Pseudo-Prophets and Their Communities Who Were Deluded by Them, The Curse of the Lord be Upon Them."[17] An example that he gives is Ḥūsayn bin Manṣūr al-Ḥallāj who "preached that the Holy Spirit was dwelling in him, and he called himself God".[18] According to al-Bīrūnī, al-Ḥallāj writes letters to his followers describing himself as :

> "The He, the Eternal, the first He, the beaming and shining light, the original origin, the proof of all proofs, the Lord of the Lords, the Lord of the mountain (Sinai), who is

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[16]See India, Vol. 1, p.32. This is an interesting example because it shows that mathematics is indeed influenced by one's world view.

[17]See Chronology, p.186.

[18]See ibid., p.195.

represented in every shape".[19]

Al-Bīrūnī, however, admits that it is difficult to understand what the Sufi (al-Ḥallāj) really means for in his other work he states:

> The saying (sic) of the Sufis is hardly understood among them(selves), much less among others, and especially the word of Ḥusayn b. Manṣūr al-Ḥallāj.[20]

In al-Bīrūnī's philosophy of mathematics, mathematical experience as part of contemplative action of Divine Presence, must be under God's guidance. The mathematician must be aware while solving problems that he is in need of God. Otherwise he might end up believing that everything in reality is God, or that nature is God and God is nature. He might believes this in the monistic or pantheistic sense. He might even be led astray further by convincing himself that He is God, or a reincarnation (ḥulūl) of God. The mathematician forgets that whatever mathematical experience that he had is incomplete, imperfect knowledge of the world and its Creator, for Perfection and Completeness belong only to Him.

Now that we have elaborated two aspects of the external world believed by al-Bīrūnī, our task now is to analyse the manner whereby the mathematician can ascend through the levels of reality. We maintain that the answer lies in al-Bīrūnī's analysis on the status of numbers in his view of mathematics since numbers are to

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[19]See ibid., p.195.

[20]See The Exhaustive Treatise On Shadows, p.66.

him the very essence of counting. His statements concerning the nature of the main constituents of numbers; one, zero, many and infinity in his philosophy of mathematics are tremendously important.

Al-Bīrūnī believes that numbers do exist and they possess ontological status. They are not, so to speak, absolute non-existence (ʿadam muṭlaq). They are far from mere illusions. But it does not mean that they are in the state of existence, subsisting by themselves because absolute existence is the prerogative of the Creator of numbers alone. Number and its series are nothing but a series of accidents. They are manifestations (tajallī) of Aspects of God. It is for this very reason that al-Bīrūnī observes "all numbers are found in physical appearances of the works of the soul and life"[21] and the physical appearances "demonstrate the being of the Creator from His creation".[22]

That "counting", to al-Bīrūnī, "is innate to man" is because man in his primordial self, in the state before he came into this temporal world, understands the perfect concept of One. As a matter of fact, God reminds them of this pre-historical state of utmost importance when He says:

> "When the Lord drew forth from the Children of Adam, from their loins, their descendents and made them witness unto theirselves: Am I not your Lord? They said: Yes indeed! we do witness" (7:172)

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[21]See Chronology, p.294.

[22]See Kitāb al-Jamāhir, p.5.

The soul of the mathematician understands in that state, for he knows by means of direct vision (shuhūd), a kind of knowledge that is beyond any doubt, of God as his Lord, the Master, the Sustainer. He witnesses The One (al-wāḥid) and His Attributes. It is because of the primordial experience of the Oneness of God that in this world the mathematician can intuit the number one (aḥād). Also in the primordial state wherein the mathematician sealed the covenant with God, the mathematician recognized that he is different from God, that God is sui generis but he is not, he understands that he is part of the Many because he is neither The First (al-awwal) nor The Last (al-Ākhir).[23] Hence his capability to intuit the Many.

Although he is part of the Many, he is not equal to the others. Unlike the rest of the Many, man is created different. Says al-Biruni, "it is undeniable that God has the power to combine the whole world in one individual (i.e to create a microcosmos in him)"[24], that man is created in His image. What does this fact mean to the mathematician concerning the levels of reality? It means that the mathematician has the capability, when he arrives at this world, to transcend himself, that is, to ascend from the world of gross mathematical objects into the realm of the Divine Immanence. The ultimate objective of the mathematical quest, eversince the primordial state of mathematical experience, is none other than to be with Him, to return to Him who is The One and The Truth.

In al-Bīrūnī's philosophy of mathematics, this union is the

[23]See al-Qur'ān, (57: 2-3).

[24]See Chronology, p.2

mathematical experience of the highest order, the highest level of mathematical truth which the mathematician seeks to strive for, again and again, through his problem solving activities in this mundane world, a world of generation and corruption. In fact, this is the first mathematical experience, the primordial-proto-quantification state which the primordial man undergoes.

When the rational man comes to this world, he already has a limited insight of the One, remnants of his mathematical experience in the primordial state. Thus in defining 'One' (wāḥid), al-Bīrūnī writes:

> One is that to which the term Unity is applied. Complete (kamil) in itself, it does not admit of being added to or subtracted from, nor is it altered in substance from its original condition by multiplication or division. It has the powers of all numbers and all the properties pertaining to these, and has in addition a special technical function to discharge with regard to things which are numbered. In this sense it occupies an intermediate position between the higher numbers, which result from the continuous addition of units, and the lower fractions into which it may be divided, and differs from both in that it does not alter by being multiplied or divided by itself, whereas the former are respectively increased or diminished, and the latter diminished or increased by these processes while 'one' occupies its own position between the two.[25]

How close is al-Bīrūnī's description reminding the mathematician of some Aspects of God! Since the idea of 'one' is so crucial in his understanding of mathematics, it is best to turn to another passage of his own work about it. Al-Bīrūnī continues:

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[25]See al-Bīrūnī, The Book of Instruction in the Elements of the Arts of Astrology (EA), op. cit., p.23 Note that 'One' refers to God whereas 'one' denotes the numerical unit one.

> Although 'One' is in reality indivisible, nevertheless the unit one (aḥad) as a technical expression, employed in dealing with sense-objects (maḥsūsāt), whether by weighing, measuring by bulk, or length or number, or merely in thought, is obviously capable of sub-division (tajziyah) for as a technical expression one (al-wāḥid) only means unity (waḥdaniyyah).[26]

The notion of 'One' is so significant in his philosophy of mathematics that to al-Bīrūnī, "The One (al-wāḥid) is not called a number (ʿadad)", in fact, "The One is excluded from the category of numbers" because "a number is defined as a sum of units".[27] That the Many emanates from the 'One', a reflection of the fact that everything originates from Him, is evident from al-Bīrūnī's definition of natural numbers (al-ʿadad al-tabiy yah). States al-Bīrūnī: "The natural series of numbers results from the successive addition of a unit to one and is, therefore known as consecutive (mutawali'), for examples, 1,2,3,4,5." We can discern that al-Bīrūnī's contemplative knowledge of one of the Attributes of God (waḥdaniyyah), and His Name, The One (al-wāḥid), characterizes his penetrating insight into the foundation of mathematics.

It is interesting to note here that al-Bīrūnī's conception of 'one' is very similar to that subscribed to by the Ikhwān al-Ṣafā' (ca. 960). According to this group:

> The most general expression or name is thing, and a thing may be one or more than one. One is used in two ways:in its proper usage and in its metaphor. In its proper usage, it is a thing

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[26]See ibid., p.24.

[27] Ibid.

> which cannot be partitioned and divided and everything which cannot be divided is one when looked upon from the aspect by which it cannot be divided. One is that in which there is nothing but itself, by which it is one. As for metaphor, it is every aggregate which is considered a unity... One is the epitome of oneness as black is the epitome of blackness.[28]

Unlike al-Bīrūnī, the Ikhwān did not differentiate as lucidly as al-Bīrūnī on the distinction between 'wāḥid' and 'aḥad'. The Ikhwān only maintain that one is analogous to The One. The latter states:

> The relation of the Creator to the universe is analogous to the relation of the number one to the numbers; as one is the origin of the numbers and that which generates them, their beginning and their end, similarly God is the cause of all things and their Creator, their beginning and their end. And as one cannot be divided, nor can it be compared to any other number, so God cannot be compared or likened to anything in His creation; and as one encompasses and accounts for all the numbers, so God knows all things and their natures. Hence god is exalted over what the unjust say in grandeur and magnificience.[29]

It is interesting to note that the work of the Ikhwān as-Ṣafā' was known to al-Bīrūnī.[30]

In al-Bīrūnī's philosophy of mathematics, the essence and the existence of the natural numbers (al-ʿadad al-ṭabiyyah), are actually mental objects of the mathematician which tend to exist in his mind. In the extra-mental reality, essence <u>is</u> existence. Such is the case because in the final analysis, the natural numbers

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[28]See Goldstein, B.R. "A Treatise on Number Theory from a Tenth Century Arabic Source", <u>Centaurus</u>, 10(1964), p.136. Altogether, Ikhwān As-Safā' wrote fifty epistles. Goldstein's article contain the translation of the first epistle.

[29]See <u>ibid</u>, p.140

[30]See al-Bīrūnī's comment on the Ikhwān in his <u>The Exhaustive Treatise On Shadows</u>, p.79.

which are part of the Many are merely a series of the act of creation of Existence. The creation of numbers is like a person and his shadow.[31]The speed of light from the Sun is beyond the imagination of the mathematician so much so that he thinks his shadow exists simultaneously, and that the shadow is not created since to him creation necessarily involves time. In similar vein, the mathematician's ability to perceive the natural numbers might fool him to believe that all numbers, for that matter all mathematical entities, exist at once. He might even believe that they have independent existence. He overlooks the fact that just as there is a time lapse between him and his shadow, there is a time lapse between one, two, three and so forth. He also forgets that just as his shadow under the Sun needs the ever presence of the man and the presence of the man does not require the existence of his shadow, so is the existence of natural numbers and other mathematical objects. Analogous to shadows, the existence of the latter is never Absolute Existence because they are continuously existing and perishing. Thus that which is construed as essence or quiddities (māhīyah) of numbers, perceived at the level of gross mathematical objects in the world of sense experience are simply accidents (aʿrāḍ) and not Existence (wujūd) itself.

So far we have treated several aspects of the number one and certain aspects of the Many that we think are imbedded in al-

[31]It is interesting to note that in al-Bīrūnī's time, when the current research was on "investigation of the actual light and what is connected with it", he preferred rather to study "what (is connected with) its absence, that is, shadow". See ibid., p. 1.

Bīrūnī's philosophy of mathematics. There is more to be said concerning them but before we delve deeper into the subject, we want to examine al-Bīrūnī's conception of zero (ṣifr) and infinity. We maintain that his view of zero has a profound impact on issues pertaining to the Many.

What is zero to al-Bīrūnī? Zero to him is not nothing. Rather zero in al-Bīrūnī's terminology refers to a place where something is not created there yet. Man never experiences nothingness because since his creation, there is always something. God is always there. Zero symbolizes that emptiness which God creates in order to be filled. It alludes to a state of precreation. Zero, to al-Bīrūnī, is that which "has to be written in places lacking a number"[32] and he says elsewhere, "Should any group lack a number, a sign is used to indicate the vacancy. We employ for this purpose a small circle, o, and call it zero, but the Hindus use a point".[33] Hence, in a sense, zero is not a number like 1, 2 or 3. Zero points to the Divine Presence and nothingness besides Him. This observation reminds us of the hadith of the Holy Prophet: "God was, and there was nothing with Him".[34]

Concerning the notion of infinity, we believe that al-Bīrūnī does not subscribe to the idea of something infinite in so far as that something is countable in the world of sense experience. In other words, everything that can be mathematized from the external

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[32]See EA, p.42.

[33]See ibid., p. 36

[34]Ṣaḥīḥ al-Bukhārī.

world has a limit. Otherwise man is incapable of describing a point because one of al-Bīrūnī's descriptions of points is that "if a line is finite (countable), its extremeties are points".[35] But lines can never be infinite because they are objects of creation and thus not eternal since eternity, to al-Bīrūnī, belongs only to God. Thus points must exist (by his description). Since points exist, lines must also exist because, by his description, they exists together. Therefore lines must be finite. All mathematical representations, then, are finite.[36] The concept of infinity, in so far as mathematics is concerned, could only means 'as finite as you want'. There is no such thing, mathematically speaking, as 'actual infinite'[37] in the world of sense experience. In fact the largest natural number that we have come across in al-Bīrūnī's writing is "nine thousand thousand thousand thousand and eight thousand thousand thousand, and six hundred and seventy five thousand thousand, and thirty four thousand and one hundred and two"[38] which

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[35]See EA, p.3

[36]It is interesting to note that al-Bīrūnī's predecessor Yaʿqūb ibn Isḥāq al-Kindī (c. 805-873) has furnished mathematical arguments to support the view that there are only two possibilities of things; either finite or infinite. "It is not possible that there can be an infinite thing greater than some other infinite thing", writes al-Kindī. See al-Kindī's third thesis in his epistle Fi idah tanahi jirm al-ʿālim which was translated and reproduced by Nicholas Rescher and Haig Khatchadourian as "Documents and Translation: Al-Kindī's Epistle on the Finitude of the Universe", ISIS, Vol. 56 (4)(No. 186)(1965), pp.426-433. Apparently al-Bīrūnī shares the same view.

[37] I think this is the underlying belief of al-Bīrūnī in one of his responses to Ibn Sīnā about indefinite division. See al-Bīrūnī's reply in S.H. Nasr, An Introduction..., p.171.

[38]See EA, p.37

corresponds to nine billion eight thousand six hundred and seventy five million, thirty four thousand one hundred and two.

Absolute Infinitude belongs to God, whose other name is al-Ākhir.[39] Just as the number one never represents the Essence (dhāt) of God which is the One (al-wāḥid) from al-Bīrūnī's point of view elaborated earlier, so is the mathematical infinite. The mathematical concept of infinity can only be understood as a pale reflection of one of the Names of God, Who is The Absolute Infinite and the Ultimate Reality.

Moreover according to al-Bīrūnī's conception of mathematics, if a mathematician wanted to number the Many, all the mathematical objects in this world that is, it is sufficient for him to use the twenty eight Arabic letters. The one-to-one mapping could be done based only on those letters since "there are nine units, nine tens, nine hundreds, which, with a sign for a thousand, are provided for by the twenty-eight letters".[40] Note that all the natural numbers can be written in terms of the four divisions, just as all materials there is can be traced back to the four elements; earth, water, air and fire.[41] In order to illustrate the numbers, he constructs the following table under the sub-title "kaif ithbat al-ʿadad bi ḥurūf al-ʿarāb".[42]

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[39]See al-Qurʾān, (57:2-3).

[40]See EA, p.40.

[41]That al-Bīrūnī upholds the concept of the four elements, see ibid., pp. 45-46, 119-120.

[42] Ibid., p.41.

Units	ا	ب	ج	د	ه	و	ز	ح	ط
	a	b	j	d	h	w	z	ḥ	ṭ
	1	2	3	4	5	6	7	8	9
Tens	ي	ك	ل	م	ن	س	ع	ف	ص
	y	k	l	m	n	s	r	f	ṣ
	10	20	30	40	50	60	70	80	90
Hun-dreds	ق	ر	ش	ت	ث	خ	ذ	ض	ظ	غ
	q	r	sh	t	th	kh	dh	ḍ	ẓ	gh
	100	200	300	400	500	600	700	800	900	1000

It is interesting to note that the twenty eight letters of the Arabic alphabet likewise form the Divine Word in the Holy Qurʾān. Others consider it as an expression of the Divine Breath (nafas al-raḥmān).[43] The fact that the cosmos can be numbered based on the twenty letters lends support to the contention that mathematical objects are handiworks of God and that nature may be construed as a mathematical book in which Divine Mysteries are revealed.[44] Al-Bīrūnī, however, does not elaborate this mathematical argument which points to Divine Transcendence. He only states that "The object in using these letters is economy of space and ease of

[43]See for example, S.H. Nasr. An Introduction..., p.162.

[44]See O. Bakar, (transl.), "Mengembara ke Alam Matematik", Kurier, (7)(1990).

writing numbers especially in astronomical tables".[45] But there is every reason to believe that he is aware of the metaphysical connection because he fully upholds the idea that nature is a creation of God as we have shown in the previous chapters.

Al-Bīrūnī believes that when the mathematician contemplates the Many, he realizes that the existence of the Many is similar to that of the natural numbers; they are multiples of Unity, Diversities within Unity. The more he contemplates at the first level, the more he becomes unaware of his physical body since he increases his attention to the objects of contemplation. He will witness the continuing disappearance of the forms that he initally percieves as the essence of the mathematical objects since, at the level of gross mathematical objects, it is the forms that separate the mathematical objects from each other. The mathematical experience that he undergoes changes accordingly. The mathematician transcends the first level and enter the second level of reality. He will see, ever-increasingly with his heart (qalb), the disappearance of the varieties of the mathematical objects of the external world. Instead of the Many, now the mathematician sees the One. He will see that all mathematical abstractions involved in solving the problem are increasingly unified into a single, pervasive Unity. The mathematical experience will climax with a vision whereby the mathematician experiences the instant of God's manifestation of Himself (tajallī) through some of His Names or Attributes. He will see various Aspects of God in every stage of

[45]See EA, p.41

his mathematical abstraction. This, is the ultimate fruit of his mathematization.

Al-Bīrūnī's activity of problem solving, in light of the discussion above, is not so much to find the correct answer as to taste His Divine Presence, as an act of ʿibādah to Him, because the mathematican knows that only God has the correct answer.

In that instant where God reveals an aspect of Himself to the mathematician, it is not the case that the Essence (dhāt) of God is united with him. The mathematician, for that matter, will never know the Essence of God, at least not in this temporal world, because God is Perfect whereas his mathematical experience is not. It is impossible for the mathematician to know the Essence of God because God is above all particularizations (lā taʿayyun).

Moreover in terms of al-Bīrūnī's philosophy of mathematics, unity with God can only means that the mathematician witnesses the greatness of God as his Lord (rabb) and he is merely his humble servant (ʿabd). Unity with the One does not imply unity in Essence, rather the experience of unification strengthens the distinction, the disparity, between the Creator and the created. He understands even more than he ever had, the meaning of His names and Attributes. According to al-Bīrūnī, the experience will "show that the Creator ...is infinitely sublime, beyond everything which we poor sinners may conceive and predicate of Him".[46]

The illuminative experience of the mathematician, unfortunately, does not last but it is not lost. How could he lose

[46]See Chronology, p.295.

it when the experience of unity is with aspects of God, The Eternal? Now he has the knowledge derived from his illuminative experience; certainty which is acquired by way of direct experience (ḥaqq al-yaqīn) after he transcends himself. God will give him his consciousness of the material world and he will return to the first level. Al-Bīrūnī argues that it is not possible for the mathematician to concentrate on a particular problem continuosly. Says al-Bīrūnī:

> This is that the human, when he is charged with an affair, whether practical or theoretical, will not be devoid of (some) thoughts, and the rememberance of (certain) situations which endanger his heart for a time. It passes as a (sic) water of a river, through his consciousness and heart, it being a category, an example of which is dreams (sic). Discussion regarding it can be lengthy. (Indeed) it is not possible to free the heart from it and to compel the imagining force to forsake it, except for a moment, after which it comes back.[47]

The mathematician, falling from the state of illuminative experience, will again confront the Many, the world of mathematical objects. Although physically he is the same mathematician, spiritually he is different. Unlike the state he was in before the illuminative experience, now he knows that everything else besides God (mā siwā Allāh); mathematical objects, mathematical representations, mathematical models and indeed mathematical

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[47]See al-Bīrūnī, The Exhaustive Treatise on Shadows, p.228. The analogy of "it passes as water through a river" reminds us of the Quranic verse: "Thenceforth were your hearts hardened: they became like a rock and even worse in hardness. For among rocks there are some from which rivers gush forth; others there are which, when split asunder, send forth water; and others which sink for fear of God." (2:74).

abstraction, have only perpetual existences given by God. God is always creating and annihilating, organizing and disorganizing, constructing and destroying et cetera, without in anyway affecting Himself and His Unity. The mathematician witnesses the dynamicity of God's act.

Quoting approvingly from a Hindu text, al-Bīrūnī says; "All things are one, and whether allowed or forbidden, equal". And commenting on the statement, he continues; "However, such views come to the inteligent man only by knowledge".[48]

3.3 Necessity of Safe-guarding the Soul

After 'descending' from the second level in the course of mathematization, al-Bīrūnī believes that the mathematician has now both seen and felt the Unity in the Multiplicity and the Multiplicity in the Unity. His spiritual capacity, however, fluctuates. Concerning the instability of the soul; that the soul has the tendency to be both pure and impure, God says in Sūrah 12:53, "The soul is inclined to evil unless my Lord has had mercy". There are other Quranic verses that do show the soul indeed vacillates.[49] To al-Bīrūnī, "the soul is in matter like the rider in a carriage, being attended by the senses, who drive the carriage, according to the rider's intention [which varies]", [50]

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[48]See India, Vol. II, p.153.

[49]For example, please read al-Qur'ān, (91:7-10), (79:40-41), (16:112), (89:27-28) and (39:43).

[50]See India, Vol. I., p. 49. See also M.S. Khan, "Al-Bīrūnī and Indian Metaphysics", Islamic Culture, lv(3)(1981), p. 165.

[parenthesis mine].

Moreover, al-Bīrūnī also believes that the joy of the spirit in discovering the true nature of things is better than the pleasures of the flesh since the former is more lasting.[51] Therefore in order for the soul to "have proper insight into the nature of that which is false and idle"[52] in the course of mathematical abstraction and for the soul to experience the pleasure of being immersed in Divine Presence while mathematizing, al-Biruni emphasizes cleanliness of the heart and intention of the mathematician, that cleanliness is part of faith (al-naẓāfah min al-īmān).[53]

Since the mathematician does vacillate between the two levels, al-Bīrūnī gives some advice for the mathematician so that the latter can always be in the blessed state.[54]

Al-Bīrūnī maintains that as a seeker of a sacred knowledge, the mathematician should live according to a set of virtues

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[51]See S.K. Hamarneh's discussion on al-Bīrūnī's concept of futuwwah in his "Evaluation of Al-Bīrūnī's Book on Precious Stones and Minerals (Al-Jamāhir Fī Maʿrifat Al-Jawāhir", Hamdard Islamicus, (21)(2)(1988), p. 12.

[52]See India, Vol. II, p.246

[53]See al-Bīrūnī, Kitāb al-Jamāhir..., pp. 12-22. Also al-Qurān, 2:22, 27, 126. cf. S.H.H. Nadvi, "Al- Bīrūnī and His Kitāb al-Jamāhir fī maʿrifat al-Jawāhir: Ethical Reflections and Moral Philosophy", in Al-Bīrūnī Commemorative Volume, p. 534.

[54]A contemporary of al-Bīrūnī who once served in the same Court with him, Ibn Sīnā, likewise emphasizes the importance of preparing the soul in order for the heart to see the truth for "the Spirit bloweth where it listeth." See also Osman Bakar, "The Question of Methodology in Islamic Science", in Tawhīd And Science, op. cit., p.27.

revealed by God through His Prophets (peace be upon them). According to him, the mathematician should be actively involved in solving problems for the society because man cannot live by himself.[55] His research priorities should not be decided with the objective of hoarding wealth because those who are blinded by wealth will suffer in the hereafter and he quotes verses from the Holy Qurʾān to buttress his view.[56] We believe that al-Bīrūnī humbly rejects Sultan Masʿūd's gift of an elephant load of silver upon the completion of Al-Qānūn al-Masʿūdī [57]because he wants to safeguard the purity of his intentions in studying God's creation.

In addition to the above, al-Bīrūnī recommends that the mathematician strives earnestly "for the suppression of evil and for welfare of truth".[58] Quoting ʿAlī b. al-Jaham's statement, al-Bīrūnī maintains that the mathematician should not feel shame if he loses his prosperity but "the real disgrace is if he loses courtesy, generosity and etiquette".[59] The mathematician should not labour for his personal fame. Instead, the mathematican should put the pleasure of God above everything else in his quest of mathematical knowledge. He should let the pleasure of God alone be

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[55]See Kitāb al-Jamāhir..., pp.6-8.

[56]See ibid., pp.8-10

[57]See H. Said & A.Zahid, Al-Bīrūnī: His Life, Times and Works, op. cit. p.95.

[58]See Kitab al-Jamāhir.., pp.10-12

[59]See ibid..

the arbiter between his choice of actions.[60]

The code of conduct described by al-Bīrūnī rests on the mathematician's constant remembrance of God. Al-Bīrūnī realizes this most important axis, the continuous consciousness of God as the most important aspect that binds and characterizes the mathematician's quest of mathematical knowledge. For instance, commenting on Abū Bakr bin Zakāriā' Al-Rāzī's book entitled The Secrets of Secrets, he quotes the verse from the Holy Qur'ān: "Any one who does not seek God for light on his path has no light in him (stays in darkness)".[61] The mathematician should be mindful of God constantly. His private and public life should be in accordance to the famous hadith of the Holy Prophet which was related by Umar ibn al-Khattab and transmitted by Muslīm and Abū Hurayrah: "that you should worship God as though you saw Him..." (an ta'buda Allāha ka 'annaka tarāhu...). In other words, the mathematician should always be in a state of gratitude to his Lord and al-Bīrūnī cites at least four Quranic verses to support his view.[62]

In light of al-Bīrūnī's view on righteous conduct for the mathematician, we should never interpret that his problem solving activity equals to the utilitarian normative ethical doctrine. Utilitarians maintain that if a mathematician is faced with a

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[60]See ibid., pp. 24-26.

[61]See, for example, S.K. Hamarneh, "Notes on Al-Bīrūnī's views of Al-Rāzī's works", Al-Bīrūnī Commemorative Volume, p.475.

[62]See Kitāb al-Jamāhir, pp.4-6.

number of mathematical problems related to the society, he should prefer solving the problem that can promote the greatest happiness of the greatest number irrespective of guidance from the Scripture. Choices are judged by their consequences and the amount of pleasure derived from those consequences. Clearly al-Bīrūnī's code of ethic cannot be called utilitarian because choices are never analysed entirely through actions and consequences. Rather, underlying motives and intentions are crucial in his problem solving approach. As we have shown in Chapter I, al-Bīrūnī believes that problems are religiously defined. From the external aspect, problems are solved for the betterment of the society but to al-Bīrūnī, the welfare of the society is never the endpoint. The endpoint, the ultimate cause, the foremost reason problems are solved by the mathematician is so that both he and the society will enjoy continuous Divine Blessing from the One in this transitory world and the Hereafter.

3.4 Truth and Truthing.

In this section, we will examine in more detail al-Bīrūnī's concept of mathematical truths. We want to analyse how his view of certainty derived from mathematical knowledge corresponds to his belief in levels of reality. With this end in mind, we will study the various contexts in which he uses the word "truth".

Since al-Bīrūnī believes in levels of reality as we have shown in the foregoing sections, we maintain that al-Bīrūnī does refer to different levels of "truth" which corresponds to each level. More importantly, the orientation of "truth" at one level is not

necessarily the same with the other level. It is his deep understanding of the orientation of levels of mathematical truths that saves him from the abyss of relativism.

Al-Bīrūnī uses the word "truth" to characterize some products of mathematical endeavour in the realm of sense experience. He will say that a theory is true if it can be proven physically so. This is what he means when he argues that a mathematical theory should have empirical import. All that a mathematician needs to show that the theory is false is for him to come up with at least a single contradiction. The mathematician can obtain the contradiction by devising an experiment or simply by his observation of similar cases in which the theory is false. An example is the experiment conducted by al-Bīrūnī to show "that which is next to the surface of the earth is conditioned by them (heat and cold) more than the conditioning of the parts which are farther from it", thus contradicting al-Sarakhsi's claim that the freezing of hot or cold water is due to motion of the parts.[63] Other examples abound in his geographical researches, one of which is the measurement of the circumference of the earth which we have discussed at length earlier.

"Truth" at the level of sense experience in al-Bīrūnī's pattern of discovery can also mean that a mathematical theory <u>works</u> better. In view of the problem solving approach, the solution of a particular problem, for example a mathematical model, is the "true" model compared to its competitors because it is the most compatible

[63]See <u>The Exhaustive Treatise On Shadows</u>, p.33.

with the problem. Consequently the mathematician will claim that he has found the mathematical truth with respect to the particular problem since by applying the model, the problem is solved. A clearly relevant example pertaining to this conception of truth is his preference of the geocentric model instead of the heliocentric.

There is another meaning of truth in the realm of gross mathematical objects imbedded in al-Bīrūnī's philosophy of mathematics. It has to do with the concept of consistency. Throughout his research on nature, al-Bīrūnī acts as if Euclidean geometry can give an accurate representations of figures in this world in so far as mathematical geography is concerned. We say 'as if' because al-Bīrūnī is aware that there are certain features of this world which cannot be explained by using Euclidean geometry. To cite an example, concerning parallel lines he says: "Even that is better than those who believe in the flatness of the earth and in the parallelism of vertical lines, which belongs to confused information".[64] Nevertheless in view of his position on Euclidean geometry, we include his conception of geometrical truth as another meaning of truth in the world of sense experience. A mathematician will proclaim that he has found a true solution (that the others are false) because the solution is consistent with the mathematical axioms that he believed. An example would be trigonometrical proofs within Euclidean geometry that al-Bīrūnī did in his Al-Qānūn Al-

[64]See ibid., p.280. Contra al-Bīrūnī, according to Euclid's Parallel Postulate, vertical lines can be parallel and they do not intersect.

Masudi.[65] The mathematician will believe that he has discovered the truth because he cannot find a logical contradiction.

Thus far, the various meaning of truth in his philosophy of mathematics elaborated above are subjective in the sense that they are products from the basic subjectivist position which conceives knowledge as special kind of mental state, or as a special kind of belief. Yet there are statements by al-Biruni implying that a mathematical theory may be true although nobody believes in it and eventhough we have no strong reason to believe that it is true. Commenting on the belief that numbers "are found in physical appearances", in the external world, he says: "If anybody wants to support his theory by referring to one of these species, he can do so, but who will believe him?"[66] Likewise another theory may be false even though the mathematician has comparatively good reasons for accepting it. It is the awareness that his mathematical findings could turn out to be false, even though he believes that they are true, which leads al-Biruni at the end of his research which include Hindu mathematics to pray "We ask God to pardon us for every statement of ours which is not true".[67] Is there a contradiction here?

In order to answer the question, we maintain that the various meanings of truth at the level of sense experience, point to al-

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[65]See A. Saidan, "The Trigonometry of Al-Biruni", in Al-Biruni Commemorative Volume, pp. 682-685.

[66]See Chronology, p.294.

[67]See India, Vol. II, p.246.

Bīrūnī's belief that mathematical truths which are fruits of his intense mathematical labour are sophisticated conjectures.[68] To this effect, al-Bīrūnī writes:

> As to the laws of natural conditions, they do exist. So if true knowledge of them is attained, these laws are then called natural sciences. But is not human knowledge, together with what the parts make it, to be reckoned by its amount in absolute investigations? Rather it is like the mountains and the observers' conjecture, and we ask God for increase of goodness; verily He is the guardian of goodness.[69]

That by and large the so-called mathematical truths are not simply conjectures but sophisticated conjectures which are products of assiduous research is also clear from another passage of his where he states:

> The Chaldeans were the people of Babylon, whose share in science cannot be concealed, to the extend that they were called its sorcerers, even though nothing came down to us of their science except their opinion (sic) concerning the motion of the heaven which is based on a continuous solicitude in observing it for thousands of years (together with) what the observers, Ptolemy and the others, relate concerning them.[70]

Thus in view of the above two quotations, mathematical laws of nature to al-Bīrūnī do exist, in the mode of existence described

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[68]Although we are not saying that knowledge is a conglomerate of guesses, it is interesting to note that the idea of knowledge as guesses can be traced back into antiquity. Xenophanes, for example, states: "For all is but a woven web of guesses". See the quotation in K.R. Popper, "The Beginnings of Rationalism", in Popper Selections, (edited by D. Miller), (New Jersey; Princeton Univ. Press, 1985), p.31.

[69]See The Exhaustive Treatise on Shadows, p. 34.

[70]See ibid., p.281.

before that is. As sophisticated conjectures, they are not the same as doubt. Doubt is defined as "a wavering between two opposites without preponderating over either one of them" whereas conjectures is when "the heart (which is the intuitive faculty) inclines more toward the one and not toward the other while yet not rejecting the other",[71] (parenthesis mine).

In a more elementary form, al-Bīrūnī's sophisticated conjectures are merely doubts as defined above. Alluding to this aspect, al-Bīrūnī writes:

> It is equally conceivable that the shape of the Universe be spherical, or oval or elliptical or cylindrical or canonical or consisting of several sides, Ptolemy's arguments from the stars retaining the same magnitudes in all the parts of the Heavens and keeping the same direction is no sufficient reason by itself, but it precludes the other forms owing to the nature of the motion itself as well as the figures that the heavenly bodies describe in their movements.[72]

When they are mathematized, they developed into theorems and corollaries. Therefore in their extensive forms, they are not merely any conjectures. Rather they are reasonable[73] conjectures which means that their probabilities of turning out false are continuously minimized. At the level of gross mathematical objects,

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[71]See S.M. Naquib al-ʿAttas, Islam and The Philosophy of Science, (Kuala Lumpur; 1989), p.8. cf. Osman Bakar, "The Meaning and Significance of Doubt in Al-Ghazzālī's Philosophy", The Islamic Quarterly, 30(1) (1986), pp. 20-31.

[72]See the translation of a passage from al-Qānūn al-Masʿūdī in the general introduction of Qānūn-i-Masʿūdi, op. cit., p.xxii.

[73]I do not use the word 'rational' because two mathematicians can disagree about a theorem or an axiom without either of the two being irrational.

the mathematician knows that his developed theory is right with always the possiblity of error and he opines that the competing theory is wrong but it still has the possibility of being right. Commenting on followers of Aristotle, al-Bīrūnī says:

> The misfortune for these people is from their exaggeration in taking sides with the opinions of Aristotle entirely, and in their belief, excluding the possibility of error in it, in spite of their knowledge that he was one of the deep thinkers but not one of those who are infallible.[74]

Those sophisticated mathematical conjectures in al-Bīrūnī's philosophy of mathematics have several characteristics. The dominant characteristic is that they, by themselves, are not Ultimate Truth, if by truth we mean that which can never be revised. In all disciplines which are directly related to mathematics like astrology, geography, astronomy,[75] history of mathematics and to a lesser extent, mineralogy which al-Bīrūnī has mastered, he always makes mathematical improvements to the previous discoveries wherein the measurements are more accurate than ever, using more precise instruments[76] and better method of

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[74]See The Exhaustive Treatise On Shadows, p.32.

[75]That astronomy and astrology is not quite the same field of study in the time of al-Bīrūnī, see S. Pines, "The Semantic Distinction Between the Terms Astronomy and Astrology According to al-Bīrūnī", ISIS, 55(1964), pp.343-349.

[76]An example is an instrument that he constructed to determine time for prayers. See his The Exhaustive Treatise On Shadows, p. 76.

calculation.[77]

There is yet another feature of the so-called sophisticated mathematical conjectures in al-Bīrūnī's philosophy of mathematics and that is that some of them are eventually rejected.[78] As a matter of fact, ever since their discoveries they are not indubitable. Therefore mathematical models are never final. They may be be construed as the ultimate mathematical models only because better models have not been discovered. The reason that a better mathematical model for a particular problem could not be found is because a particular mathematical problem related to it is not yet solved. For example, there is no satisfactory mathematical model as a solution to any problem that needs the prior solution to the trisection of angle.[79]

There is also another important aspect of al-Bīrūnī's sophisticated conjectures and that is, they can never give the mathematician absolute certainty. This is so because once the mathematician comes up with a mathematical model describing the external world, he can only perform experiments to cover a finite

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[77]For example, see the calculations that he did in his Istikhrāj al-Awtār fi' al-Dā'irah which are reproduced in M. Saud, "A Part of Al-Bīrūnī's Istikhrāj al-Awtār fi'al-Dā'irah", in Al-Biruni Commemorative Volume, pp. 691-705.

[78]This is consistent with the thesis that Muslim scientists apply the approach of studying 'sharīah' to the method of studying nature because in the study of 'sharīah', it is well known that statements other than Quranic verses and sayings of the Holy Prophet can be rejected.

[79]The problem of the trisection of an angle was only solved mechanically by al-Kindī and the Greeks. Al-Bīrūnī gave an approximate geometric solution for it. See A. Saidan, "The Trigonometry of al-Bīrūnī", op. cit.. p.689.

number of cases to test the validity of the model. He can never be sure that the next experiment will produce the same result. For the sake of argument, let us suppose that the mathematician could cover all the relevant cases to test his mathematical model. Al-Bīrūnī still maintains that the mathematical model pertaining to a particular problem is never absolute truth because he believes that the condition of the world changes through time. Writes al-Bīrūnī:

> And what is told by the Hindus and other people regarding mountains after investigation and study seems on the surface to be in vain because they have recorded their observations thinking that whatever they have seen has always been and will always be the same. This is done in consideration of the fact that by examination it has been discovered that in the course of evidence the mountains have not remained in the same state, and that all of a sudden, or in the course of time changes and variations have occurred. Therefore, we cannot consider the present state and conditions which we have discovered by observation [the initial process of mathematical modelling] to have been always the same.[80]

It is important to note that although the world changes, the problems found in the Holy Qurān which are related to man as God's vicegerent on earth remain the same. Al-Bīrūnī was clear about this aspect of problems. The factors that vary are the problem situations[81] and not so much the problems themselves. Consequently the mathematical models also are subject to change. To cite an

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[80]See the quotation in S.H.Nasr, An Introduction..., p.120. See also A.A. Dekhoda, Sharh-i hāl-i nābighih-i shahīr-i Irān, op. cit., p.35.

[81]I consider determining the first of Ramadhan as a problem and the society, the instruments available, the school of 'fiqh' to which the mathematician belongs as the problem situation. The problem is always 'objective' whereas the corresponding problem situation is not. Problem situation consists of factors 'external' to the problem.

example, people have to pray facing the Kaaba since the time of the Holy Prophet (pbuh) and his noble companions right until the time of al-Bīrūnī and so on. There were several different mathematical models constructed to solve the same problem until the time of al-Bīrūnī[82] and he was aware of some of them.[83] Yet none of the mathematical models pertaining to the problem can be claimed to be the absolute model.

The claim that mathematical knowledge at the level of sense experience, according to al-Bīrūnī's philosophy of mathematics, can be construed only as sophisticated conjectures follows also from his usage of mathematical entities like Pi and irrational roots (asamm).[84] To al-Bīrūnī, "there does not exist any method of arriving at it except approximately".[85] Accordingly mathematical models based on these entities can only provide approximate answers to the problem, never an exact answer.

The functioning of mathematical instruments likewise reflects the uncertainty of mathematical models. A scientific instrument is a physical representation of a mathematical model. Mathematicians use them to measure and taking measurements, from al-Bīrūnī's point of view explicated earlier, is an important aspect of mathematization. However, mathematicians can only take measurements

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[82]See D.King, "The Sacred Direction in Islam", Interdisciplinary Science Reviews, Vol.10 (4), (1985), pp.315-328.

[83]See The Exhaustive Treatise on Shadows, pp.226-229 & p.232.

[84]See EA, p.32.

[85]See ibid.

of finite accuracy although mathematical statements make infinitely precise claims. Consider the mathematical statement that the value of Pi, according to al-Bīrūnī, is 3.1417482.[86] This value cannot be verified empirically because the mathematician's instrument cannot discriminate between 3.1417482 and 3.1417483.

In addition to the nature of existence of mathematical objects, the constraint of the ability of the mathematician's imaginative faculty, also set a limit to mathematical models that he can discover. Says al-Bīrūnī:

> So that is not among the things which are incapable of being pictured among the first principles, like the impossibilty of two bodies being in one and the same place together or the presence of two opposites in one place together and at one time. Verily these fail to exist only because of the contradiction in their existence itself, like the earth, (which) the imagination does not picture as touching the atmosphere, but outside it, nor whiteness in the feathers of the crow.[87]

That mathematical knowledge at the level of sense experience has the status of sophisticated conjecture in al-Bīrūnī's philosophy of mathematics is because the verification of mathematical models involves experiments and observations 'at a distance'. At stake in the experiment is not so much the mathematical objects themselves compared to the mathematical models connecting them. An example is the verification of mathematical models concerning the size of the earth. What is tested is the validity of the trigonometrical model

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[86]See H. Said & A. Zahid, Al-Bīrūnī, His Life, Times and Works, op. cit., p. 139.

[87]See Exhaustive Treatise On Shadows, p.36.

used to calculate the size of the earth and not the earth itself. The earth, unlike a fruit of a tree, is not within the physical grasp of the mathematician. The same argument applies to measurements of very small mathematical objects. Says al-Bīrūnī concerning measurement 'at a distance':

> It is inevitable that there should be controversies regarding the dimensions of the earth for it is one of those matters which must be based on experiment at a distance and on reports on observations".[88]

In addition to the various arguments presented so far, al-Bīrūnī also realizes that mathematical laws ususally hold only under idealized conditions. we have treated this aspect briefly in Chapter II. There is another aspect related to idealization and that is the use of instruments. A mathematician uses instruments in order to measure as accurately as he can. For example, in weighing[89] the law of the lever assumed there is no friction at the fulcrum and that in finding densities of minerals, there is no residue left in the beaker.[90] Sure, one can minimize these interferences in conducting experiments but he will never achieve the perfect conditons which are assumed preliminary to mathematical abstraction.

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[88]See al-Bīrūnī, EA, p.118. See also his remarks on the necessity to repeat observations ad infinitum, (particularly in astronomy) in Al-Qānūn al-Maṣūdī, p.776. A translation of the passage is given in the Introduction, ibid., p.liv.

[89]See India, Vol. I, pp. 160-166.

[90]An illustration of the instrument which al-Bīrūnī used in measuring densities, see H. Said & A. Zahid, op. cit., p.146.

In our point of view, it is a consequence of al-Bīrūnī's deep understanding of the uncertainty of mathematical knowledge at the level of sense experience that in his mathematical quest, he deliberately tries to cover[91] all different ways known during his time to prove and whenever possible, to test his mathematical models. He strives to the utmost to arrive at the best mathematical model for a particular problem, an examplary industrious attitude indeed for those who come into this world after him.

Thus far we have seen that mathematical truths at the level of sense experience, as far as al-Bīrūnī is concerned, are imbued with fragments of philosophical truth propounded by western conceptions of mathematical truth. There are traces of conventionalism, pragmatism, coherence and correspondence theories of truth propounded by modern theories of truth such as pragmatism, formalism and empiricism. Yet interestingly, al-Bīrūnī's conception of mathematical truth cannot be classified categorically under any of those category. This is so because unlike those philosophical doctrines which are void of transcendence, the whole mathematical activity of al-Biruni is an offshoot of his understanding of the encompassing concept of 'tawḥīd' and 'ʿibādah' in Islam. Indeed, it is in aspects of Islam too that he claims to find justification for all of his mathematical endeavours.

What is then the epistemological status of sophisticated conjectures in al-Bīrūnī's philosophy of mathematics? Can

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[91]For examples of his words to this effect, see The Exhaustive Treatise On Shadows, p.244, p.262 and p.271.

sophisticated conjectures achieve the status of truth? Yes they can, but we will never know that we have arrived at the truth if our position is only based on mathematical arguments, void of His Blessings, because Truth is sanctioned by Him. In light of al-Bīrūnī's view on the orientation of sophisticated conjectures, their degree of verification does not amount to belief. At the very best, they can only lend support to truth. In his discussion on the mathematization of nature by way of numbers and geometry, al-Bīrūnī states:

> Geometry is the science of dimensions and their relations to each other and the knowledge of the properties of the forms and figures found in solids. By it the science of numbers is transferred from the particular to the universal, and astronomy removed from conjecture and opinion to a basis of truth.[92]

In al-Bīrūnī's philosophy of mathematics, the function of sophisticated conjectures which operate at the level of gross mathematical objects is to be 'signs' between the mathematician and his creator. Their uncertainty at the level of sense experience points to the ever knowing God and imperfection of human mathematical knowledge. If we grant them the status of truth, truth is illusive[93] indeed. It is merely for the purpose of practical

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[92]See EA, p.1

[93]It is remarkable that some modern philosophers believe that truth is in fact illusive and consequently treat truth as an irrelevant aspect of scientific quest. For example, it is not necessary to include truth in their analysis of scientific progress. See K.R. Popper, "Truth and Approximation to Truth", in D. Miller, (ed.), Popper Selections, op. cit., p. 181.

mathematical activity that it is reasonable to base a research programme on the most verified mathematical models because they are the best that the mathematician has with respect to a particular problem situation.

The status of mathematical knowledge at the level of sense experience in al-Bīrūnī's view is best illustrated by referring to a passage from his own work. In this passage, al-Bīrūnī describes the mathematician quest for mathematical certainty:

> So, if the muezzin is interested in deep investigation, and he abstains from (blind) imitation, and (if) his temperament is akin to the science of Ptolemy, and Archimedes, and Apollonius, and he never puffs himself up above these names, and he seeks schooling and education until he reaches this position, then verily he must take up the whole of the Book of the Elements (of Euclid) and the middle works between it and the Almagest, and he must give (himself over) to eight treatises of it. Thus he came as empty as the devil, but he goes away as victorious as (the prophet) Idris (pbuh). If it happens that he becomes fed up from the very first with studying what we have mentioned, then let him take the shortest distance away from the work, let him shorten the length of hope by giving the bow over to one who can draw it and surrendering the matter to the experts who do not loathe steady striving for the reform of these elements and their improvement (sic) and the production of their results to those who seek them.[94]

In other words, we could only say in so far as al-Biruni's philosophy of mathematics is concerned, that mathematical models are approximations to truth. They are never absolute (like Euclidean geometry once thought to be).[95]

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[94]See The Exhaustive Treatise on Shadows, p.230.

[95]Kant, for example, believes that Euclidean geometry is 'inherent in the structure of our mind'. See M.J.Greenberg, Euclidean and Non-Euclidean Geometries (Freeman and Co., 1974), p.145.

At another level which we term as the metamathematical level, mathematical truth corresponds to Absolute Truth. This is when the mathematical experience of the mathematician coincides with the illuminative experience. It is the state whereby the mathematician transcends the veil of multiplicity. It is a situation wherein God reveals an aspect of Himself to him and the mathematician sees nature as nothing but the self-manifestation of God. This level of reality can also be called the metamathematical level. That I say it is a metamathematical level is because it is an off-shoot of a mathematical quest.

It is at this metamathematical level that the mathematician realizes that all mathematical objects, entities, forms, representations and models are signs[96] pointing to Divine Presence and to some other Aspects of Him. The mathematician's consciousness is empty save the awareness of Divine Transcendence and Immanence. The mathematician's mathematical abstraction transforms into a contemplation of God whereby he sees, feels and consequently believes with his heart, more than anything else, the true nature of Nature whereby God, whose other name is The Truth, individuates Truth from the state of perfection until it reaches the gross level of the material world which is the level of generation and corruption. Accordingly, the mathematician understands that mathematics is not only a ladder from the sensible to the intelligible, but also from the intelligible to the sensible, as can be deduced from al-Bīrūnī's definition of "One"

[96]See al-Qurʾān (xli;53.)

(wāḥid) and the numerical one (aḥad) in the foregoing passages. Elsewhere, we believe that al-Bīrūnī alludes to this illuminative experience whereby he states:

> The recitation of praise in the essence of a thing is its rising to its perfection, and its seeking peace by nature with the divine object in its continuous existence for what it was created to perform. What transcends this about it is its indication by the various (forms) which it undergoes and the different appearances which it takes (in which case) it is restricted, derived and to be explained. One who seeks guidance from Him about it becomes a praiser like it and he does not have to percieve (directly). For it is like his saying, be He exalted!, "The stars (or herbs) and the trees prostrate themselves!"[97]

Just as the mercy of a mathematician which results from God's individuation of His Name (al-raḥmān) from the level of absoluteness to the level of sense experience, is an incomplete and imperfect copy of a name of God, which is the Most Merciful, so is mathematical truth. Truth at the realm of sense-experience is no longer absolute. God says in the Holy Qurān: "Such is God, our Real Cherisher and Sustainer; apart from Truth, what remains is (but) error. How then are you turned away?"[98]

That mathematical truth operating at the level of gross mathematical object, according to al-Bīrūnī's philosophy of mathematics, is not absolute does not mean that his notion of mathematical truth is compatible with the doctrine of relativism.[99]

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[97] See The Exhaustive Treatise On Shadows, p.41.

[98] al-Quran, (10:32).

[99] Relativism in the West is a progeny of the Greeks. In Aristotle's Nicomachean Ethics, it is stated : "Fire burns both in Hellas and in Persia but men's idea of right and wrong vary from

In fact, we believe that his notion of truth is diametrically opposed to it. Relativism, stripped to the bare essentials, makes the claim that there is no universal standard of right and wrong, good and bad, and that there is no such thing as objective knowledge of realities. On the contrary, al-Bīrūnī believes there is a universal code of ethics and that there are in fact objective realities, that is to say realities independent of the individual knower.[100]

With that note above, we shall sum up the discussions in this section with al-Bīrūnī's own account in his Magnum Opus Al-Qānūn al-Maṣūdī which, by and large, summarizes his erudition as a mathematician:

> And I have truly done what every one is bound to do in respect of any particular science, that is, to accept gratefully the original contributions of his predecessors, to correct fearlessly the errors that come to his notice and to preserve what he himself discovers and to leave it as a record for the future generation that are to follow him in time.[101]

3.4 Conclusion

In al-Bīrūnī's view, mathematical knowledge enables the mathematician to understand the ontological status of things in the hiearchy of creation, thus recognising the proper relation between

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place to place." (v(vii)2). See also Herodotus, History, III, pp.37-38.

[100]In light of this statement, I think that the philosophy of mathematics adopted by Muslim mathematicians should never be classified strictly under the banner of any Western philosophy of mathematics in order to do justice to them.

[101]See the text reproduced in Al-Biruni Commemoration Volume, p. 1.

God, mathematics and himself. The numbers such as zero, One, and the concept of infinity reflects the various levels of reality, and correspondingly, levels of truth. This is the status of mathematical knowledge.

Al-Bīrūnī emphasizes the importance of safe-guarding the soul in seeking mathematical knowledge. Although al-Bīrūnī was not a Sufi in the traditional sense of the word, yet he could transcend the veil of multiplicity (the Many) which nature seems to display endlessly, enjoying the presence of The One, acknowledging His Wisdom and understanding His Divine Names and Sublime Attributes by placing mathematics squarely within the sacred fold of Islam. Consequently, al-Bīrūnī knows the true place of himself and the Ultimate Reality in the order of being and existence as a result of the mathematical experience that he undergoes. Could there be a nobler product of mathematical research than this?